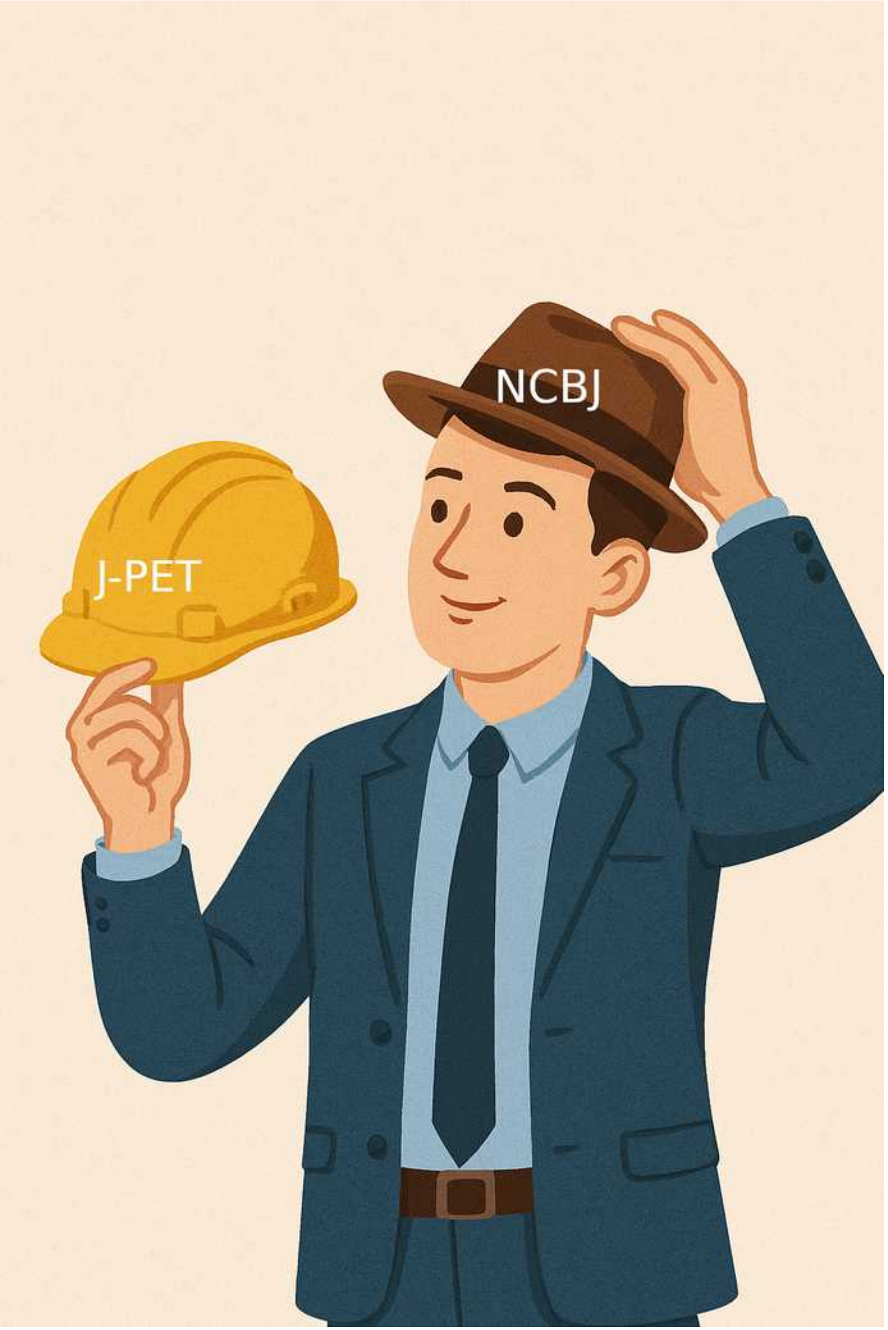
NATIONAL CENTRE
FOR NUCLEAR RESEARCH
ŚWIERK

GATE activities @ NCBJ

Wojciech Krzemień

Gate Scientific Meeting
01.04 2025


NCBJ
J-PET

National Center for Nuclear Research

Research sector in NCBJ

Nuclear Facilities Operations Department — Reaktor MARIA

Material Physics Department — LBM

Department of Fundamental Research

Department of Nuclear Techniques and Equipment

Complexity Center

One of the largest research institute in Poland

1195 employees, inc. 80 prof. & 170 PhD

PhD Study: ~45 students

EU projects: success rate: 30%

HR EXCELLENCE IN RESEARC

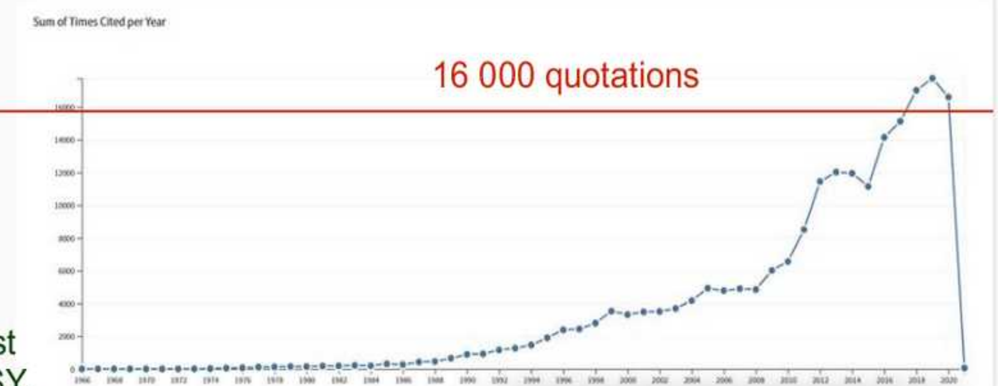
Scientific achievements:

~500 reviewed papers,

~16000 quotations each year,

5/4th position in Poland,

budget/year ~ 70 M€



124 different types projects

International collaborations with largest laboratories in the world (CERN, DESY, Grenoble, JParc, FAIR, Julich, ESS, JINR, T2K), cooperation in many universities around the world

International Research Agenda - NOMATEN

Radioisotope Centre POLATOM

Division of Nuclear Equipment HITEC

Science and Technology Park

Scientific-and-Industry Centers

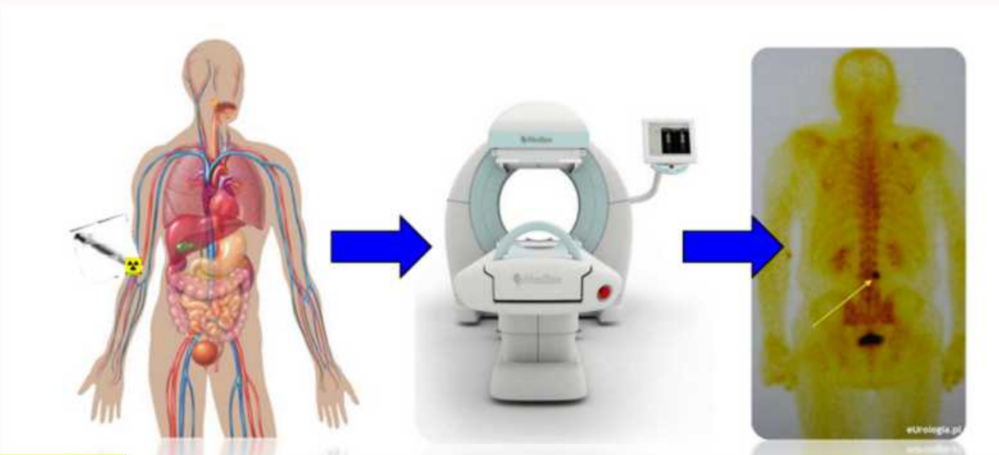
Radiopharmaceuticals with marketing authorisation

Quality Assurance System certified:

ISO: PN-EN ISO 9001:2015-10
cGMP and GLP

Radioisotope Centre POLATOM-NCBJ

POLATOM



Export to 80 countries
100% polish market
(except PET)

Medicines for 17 million
patients a year

I-131 Mo-99

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Department of Complex Systems

CIS Świerk Computing Centre

https://www.ncbj.gov.pl/en/department-complex-systems

- CIS computing cluster (1,4 PFLOPS, ~36 000 cores and 200 TB RAM)
- development of AI and Big Data processing methods
- data analysis in various applications
- development of novel imaging algorithms for medicine and industry
- more

IMPET – Industrial Multiphoton PET Tomography

Objectives:

- **AI-enhanced** solutions for **industrial multiphoton** imaging
- use of **quantum correlation** measurements as complementary information to classical spatial distributions and to PALS methods.
- Development of new **positron emission particle tracking** algorithms
- project of the **industrial scanner** and the library of the **reconstruction methods**

Timeline: **1.02 2025 – 31.01 2028**

contract number: **First Team FENG.02.02-IP.05-0152/23**

Principal Investigator: **Wojciech Krzemień (NCBJ)**

Scientific cooperation:

- **Beatrix Hiesmayr (University of Vienna)**
- **David Sarrut (CREATIS - INSA Lyon)**

Commercial cooperation: **Creotech Instruments S.A.**

Total budget: **~ 940K euro**

- **Part of the simulations will be performed with GATE**
- **Also, contribution to GATE package planned**

European Funds for Smart Economy

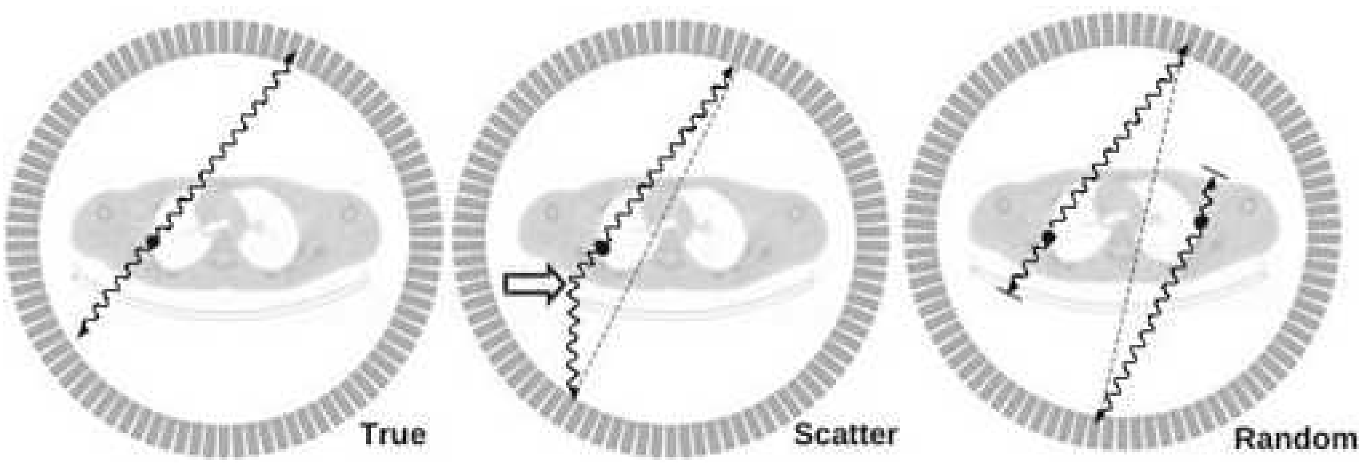
Republic of Poland

Co-funded by the European Union

FNP Foundation for Polish Science

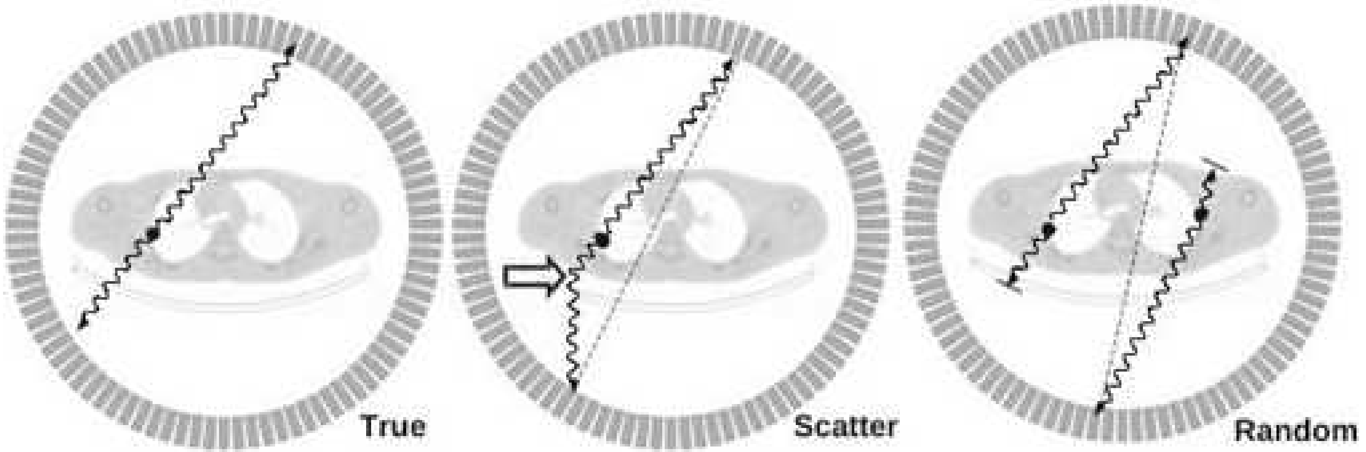
Coincidence classification (for total-body PET systems)

Coincidence selection as a classification problem



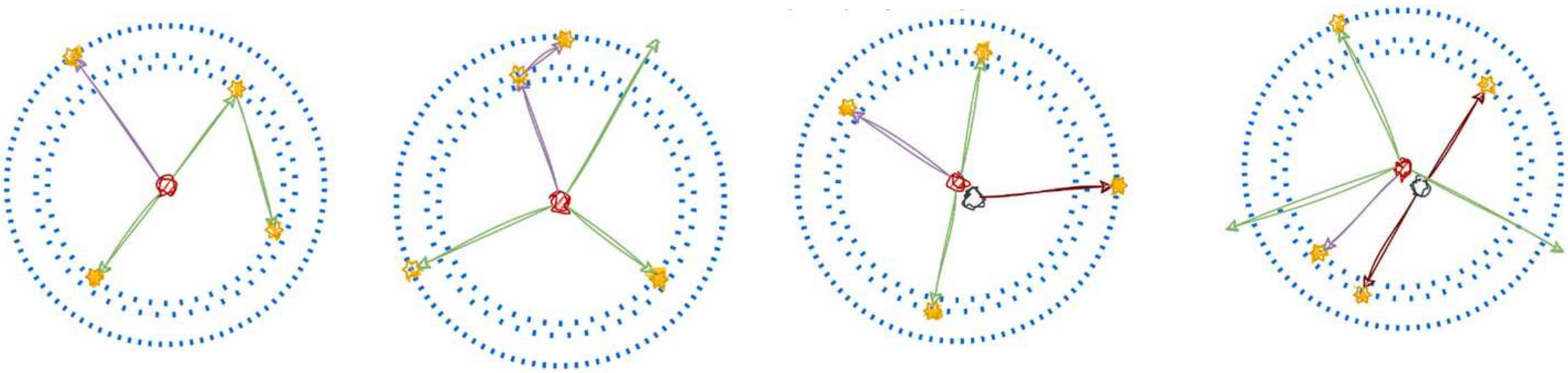
D. L. Bailey, Ed., Positron emission tomography: basic sciences. New York: Springer, 2005

Coincidence selection as a classification problem



D. L. Bailey, Ed., Positron emission tomography: basic sciences. New York: Springer, 2005

Multiphoton topologies becomes much more complicated:

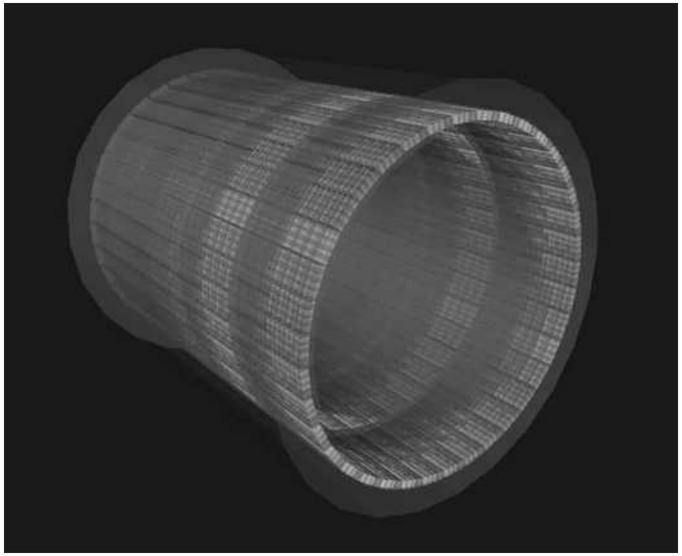


M. Obara, K. Klimaszewski et al.

Simulations with GATE 9.4

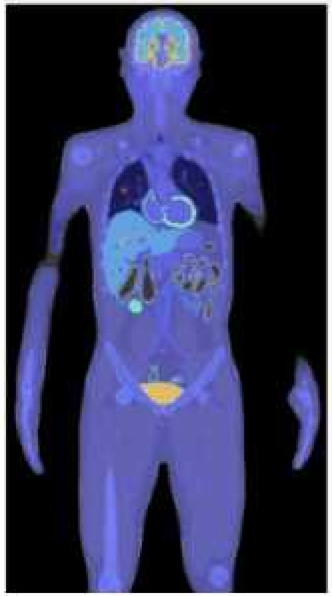
Biograph-Vision Quadra

https://www.siemens-healthineers.com/en-us/molecular-imaging/pet-ct/biograph-vision-quadra



Ring diameter	820 mm
Transaxial FOV	700 mm
Axial FOV	1060 mm
Crystal material	LSO
Crystal dimensions	3.2 × 3.2 × 20 mm
Total number of crystals	243,200
Crystals per ring	7600
Crystals per detector block	200
Number of detector rings	320
Energy window	435–585 keV
Coincidence window	4.7 ns
Coincidence resolving time	219 ps

XCAT Phantom

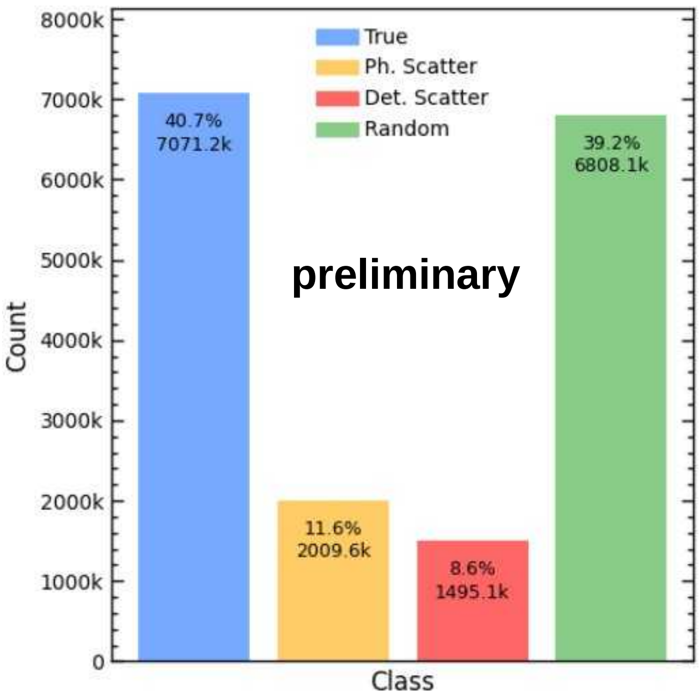


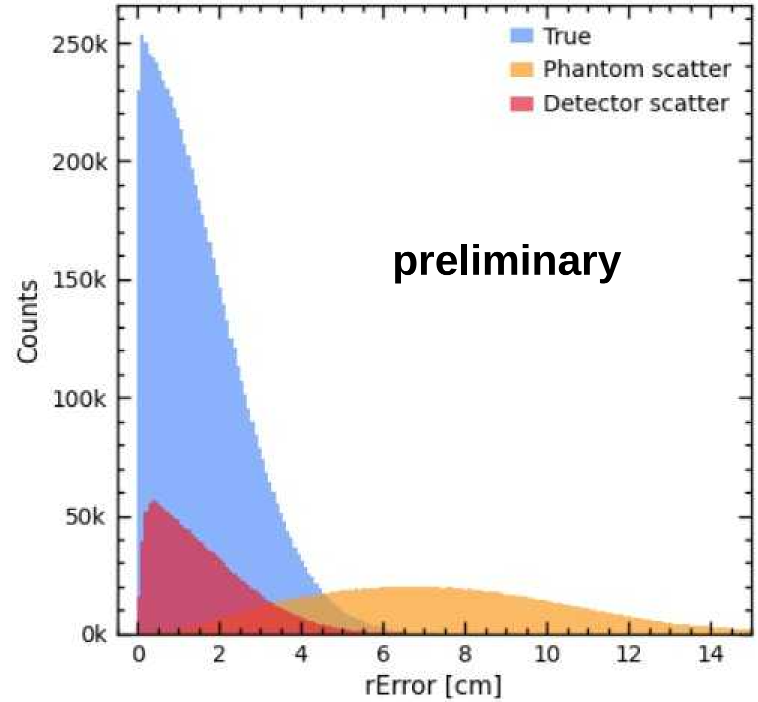
- Voxelized anatomical phantom
- Activity - 50 MBq
- FDG
- Acquisition time: 10 seconds

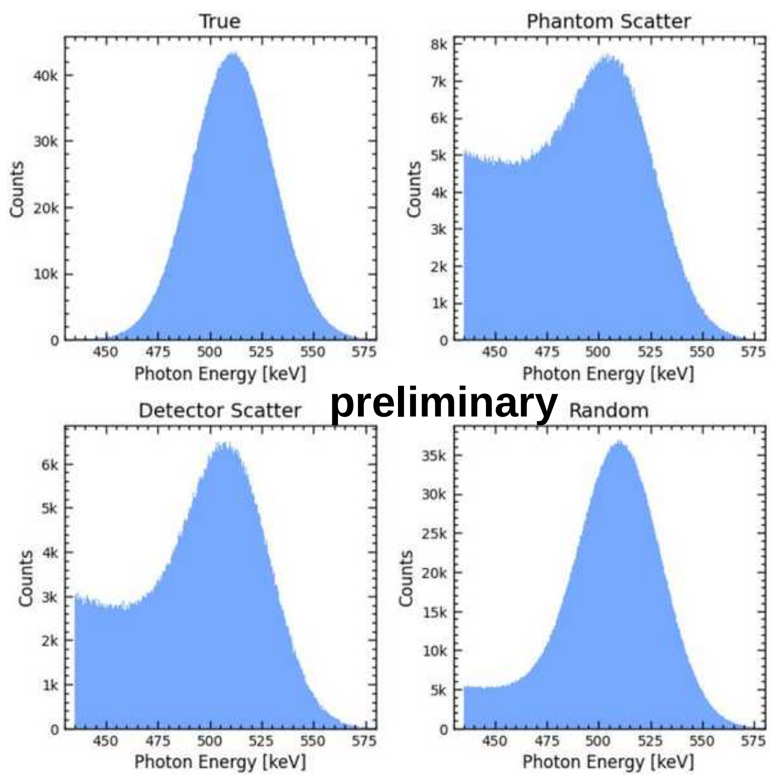
Post-processing

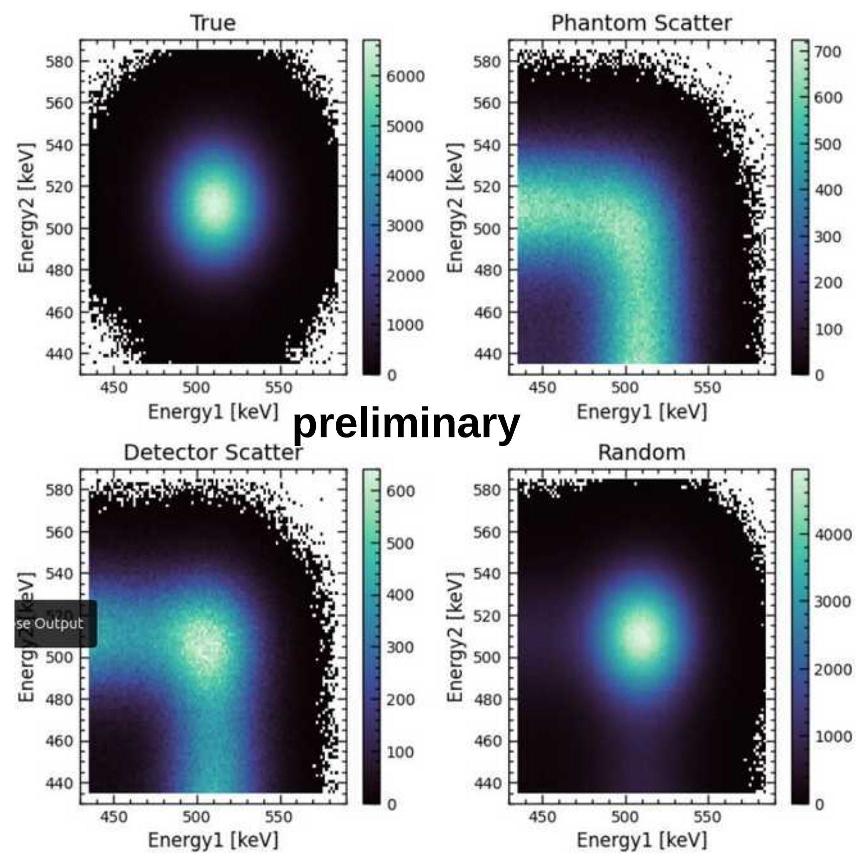
- Resolutions (energy, time etc) modeled in the post-processing
- Geometrical cuts to reduce the background contribution

Klimaszewski et al.







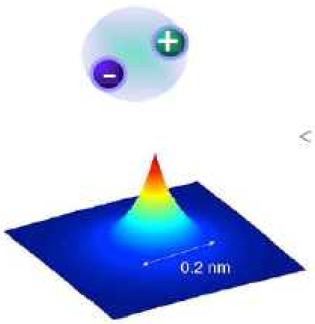


Trilateration „on steroids”

in collaboration with :
- Lech Raczyński (NCBJ),
- Aurelien Coussat (Creatis – INSA Lyon)

Motivation

Positronium tomography → positronium lifetime imaging

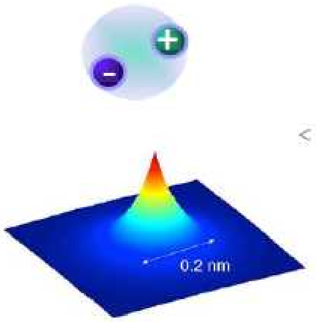


- P. Moskal et al. Science Advances 7 (2021) eabh4394
- P. Moskal et al. Science Advances 10 (2024) eadp2840
- S. Bass et al. Rev. Mod. Phys. 95 (2023) 021002

and more...

Motivation

Positronium tomography → positronium lifetime imaging



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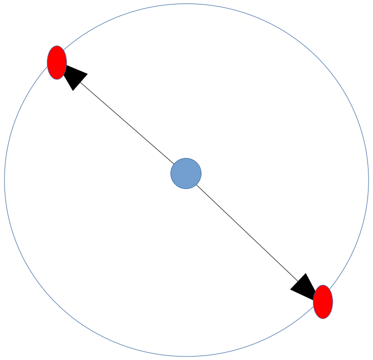
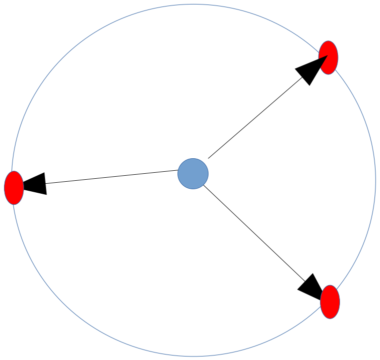


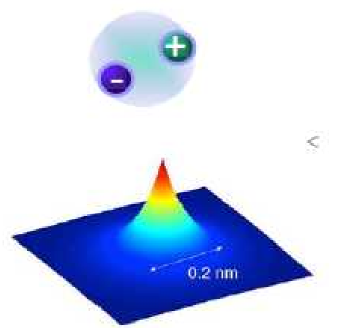
Ps→ 2g

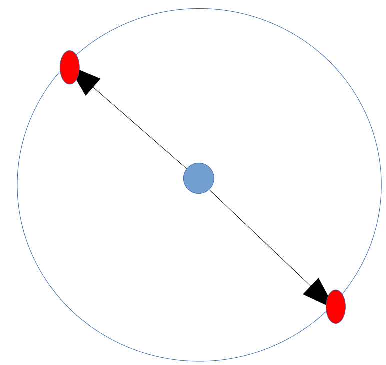


oPs→ 3g

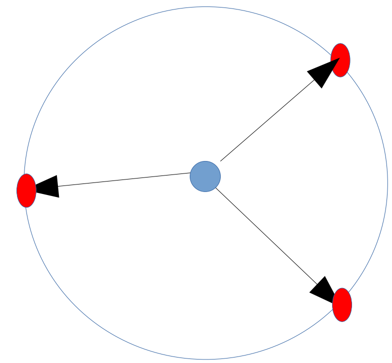
Motivation

Positronium tomography → positronium lifetime imaging





Ps→ 2g



oPs→ 3g

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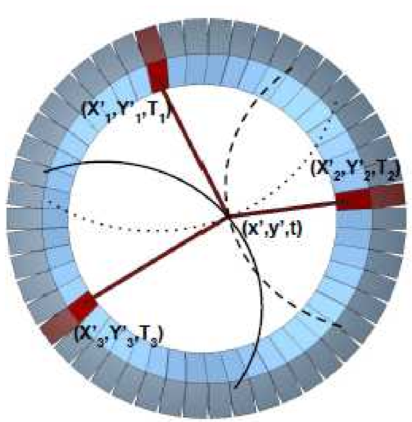
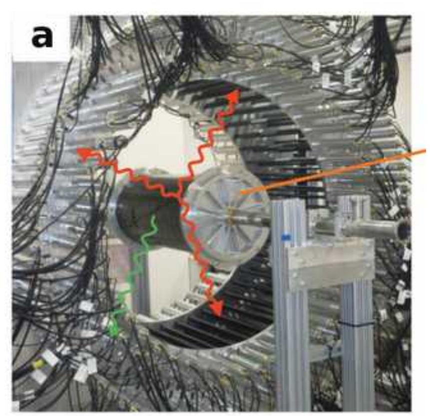
- S. Bass et al. Rev. Mod. Phys. 95 (2023) 021002

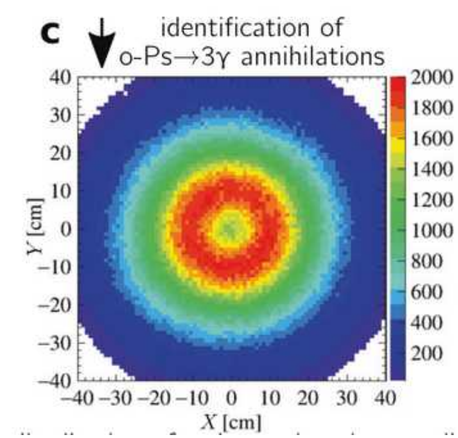
and more...

One can reconstruct the image formed of annihilation vertices in the oPs→ 3g



A. Gajos et al., NIM A 819 (2016) 54-59

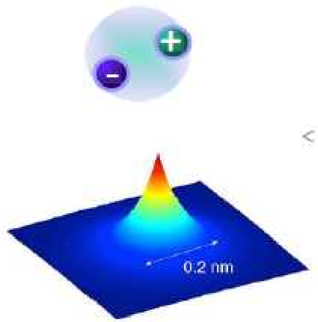


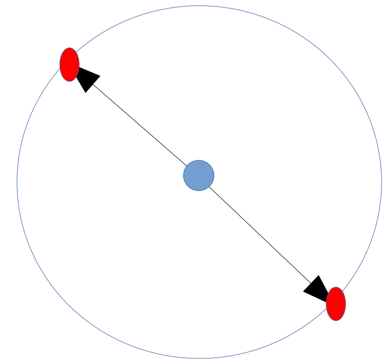


P. Moskal, A. Gajos et al. Nature Communications 12 (2021) 5658

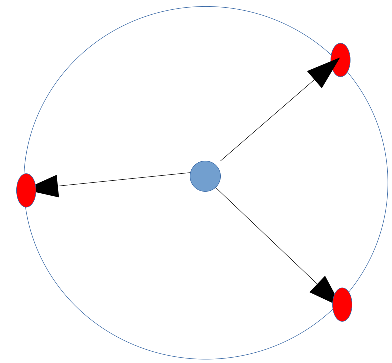
Motivation

Positronium tomography → positronium lifetime imaging





Ps→ 2g



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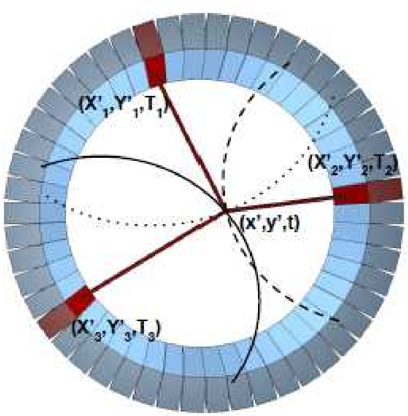

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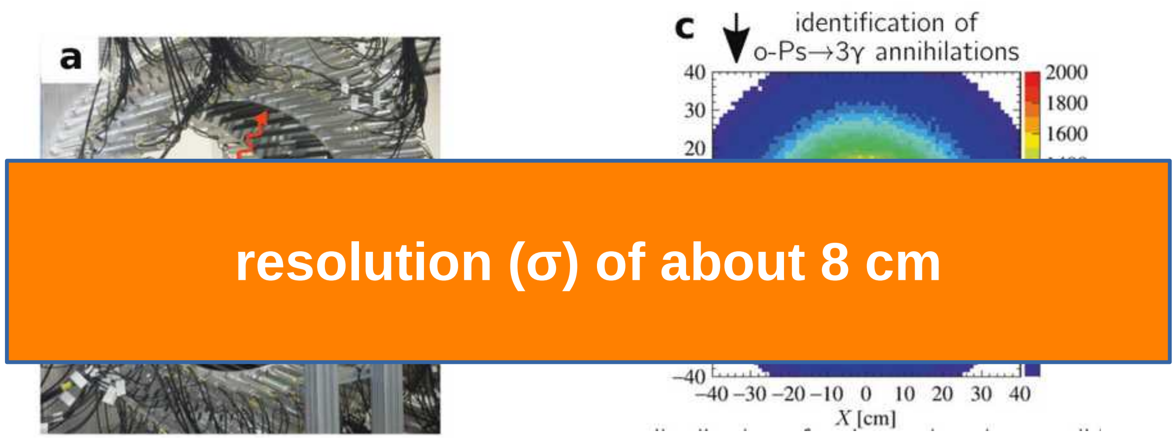
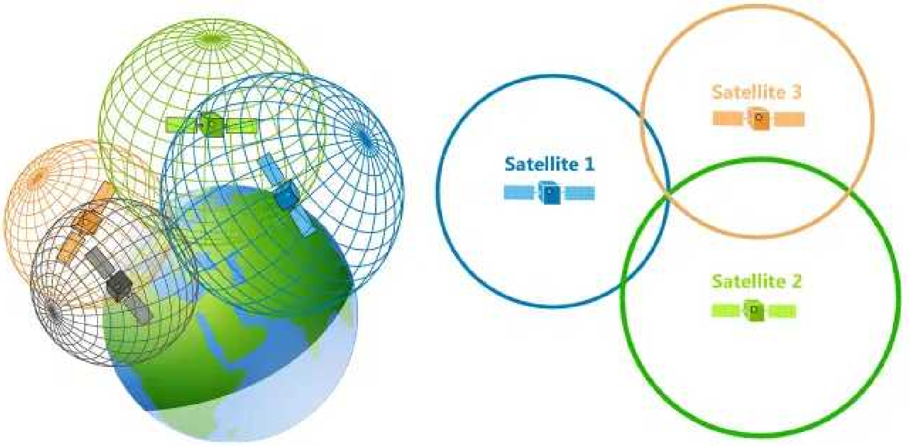


resolution (σ) of about 8 cm

P. Moskal, A. Gajos et al. Nature Communications 12 (2021) 5658

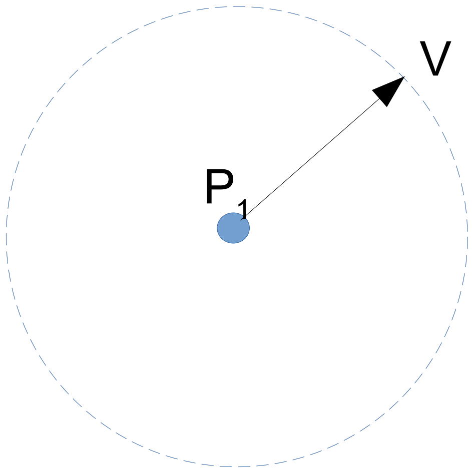
What is trilateration?

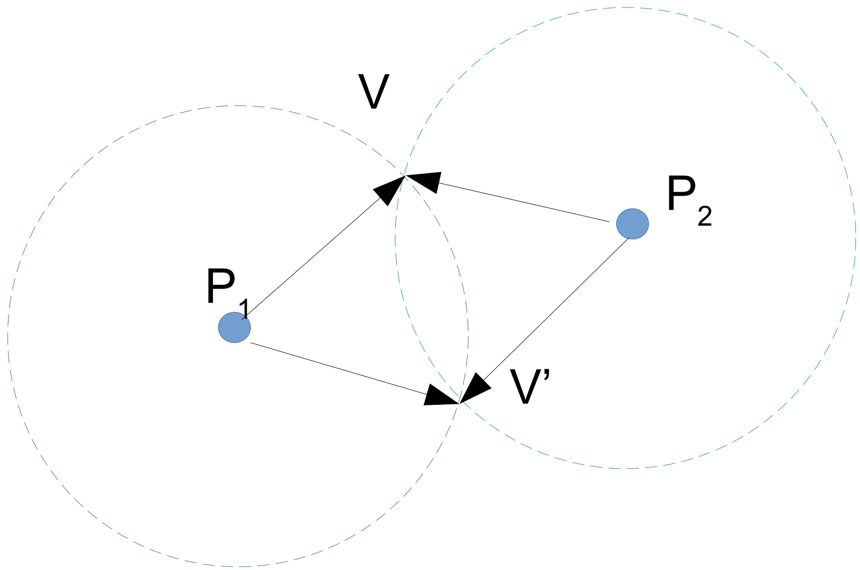
- **Pseudorange multilateration** - technique for determining an object position (e.g. vehicle) based on signal time of arrival from/to multiple system stations with known locations

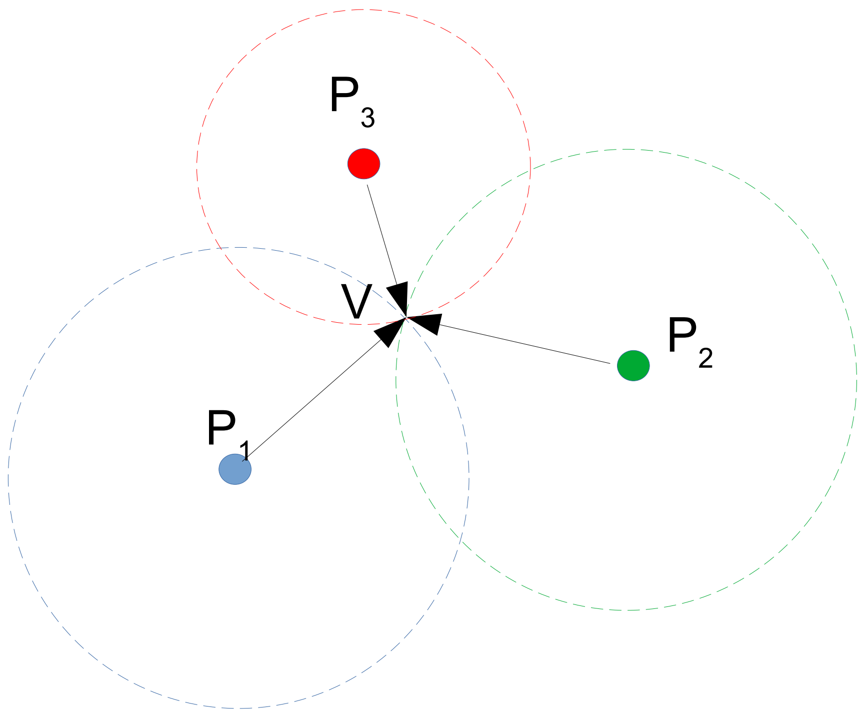


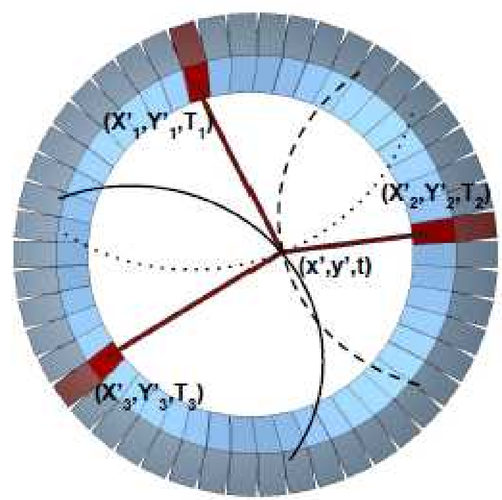
- problem known and studies for years in navigation (e.g. GPS)
- keywords: **multilateration** (MLAT), **pseudorange multilateration**, **hyperbolic positioning**
- Typically **over-constrained** systems (e.g. more stations than unknowns)
- Typically we assume **no errors** on station locations.

Basic idea





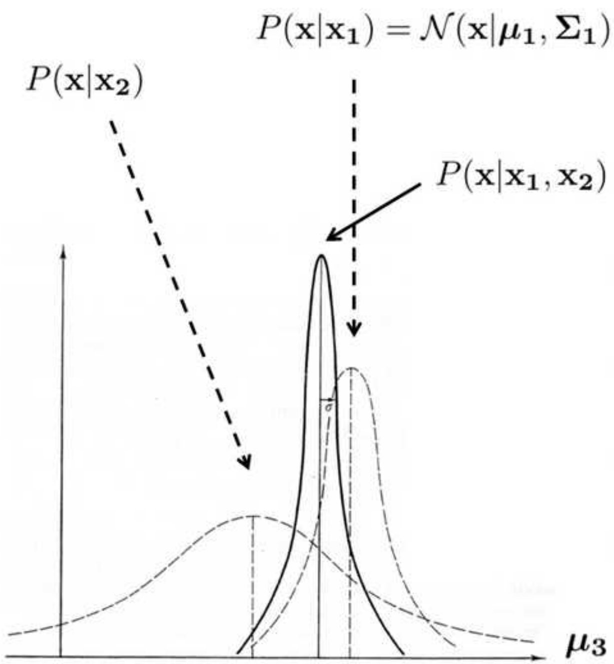




Trilateration “on steroids”
How to improve the trilateration algorithm?

- Incorporate experimental uncertanities (uncertanities of the measurement of hits positions and times)

- Incorporate the knowledge about the ortho-positronium decay probability distribution (type of regularization)



$$
\begin{aligned}
P(\mathbf{x}|\mathbf{x_1},\mathbf{x_2}) &= \frac{P(\mathbf{x},\mathbf{x_1},\mathbf{x_2})}{P(\mathbf{x_1},\mathbf{x_2})} \\
&= \frac{P(\mathbf{x}) \cdot P(\mathbf{x_1}|\mathbf{x}) \cdot P(\mathbf{x_2}|\mathbf{x},\mathbf{x_1})}{P(\mathbf{x_1}) \cdot P(\mathbf{x_2})} \\
&= \frac{P(\mathbf{x}) \cdot P(\mathbf{x_1}|\mathbf{x}) \cdot P(\mathbf{x_2}|\mathbf{x})}{P(\mathbf{x_1}) \cdot P(\mathbf{x_2})} \\
&= \frac{P(\mathbf{x}|\mathbf{x_1}) \cdot P(\mathbf{x}|\mathbf{x_2})}{P(\mathbf{x})} \\
&= \eta \cdot P(\mathbf{x}|\mathbf{x_1}) \cdot P(\mathbf{x}|\mathbf{x_2})
\end{aligned}
$$

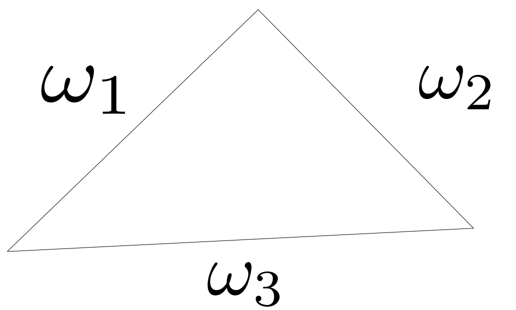
$$\boldsymbol{\mu_3} = \arg\max P(\mathbf{x}|\mathbf{x_1},\mathbf{x_2})$$

Icorporating ortho-positronium pdf

$$d\sigma_{3\gamma} = \frac{(2\pi)^4 |M_{fi}|^2}{4I} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\delta(\omega_1 + \omega_2 + \omega_3 - 2m) \frac{d^3k_1\, d^3k_2\, d^3k_3}{(2\pi)^9 2\omega_1 \cdot 2\omega_2 \cdot 2\omega_3}$$

amplitude – decides about the deviation from “isotropic” distribution

$$E_i = |k_i| = h\omega_i, \;\; for \; i \in 1, 2, 3$$

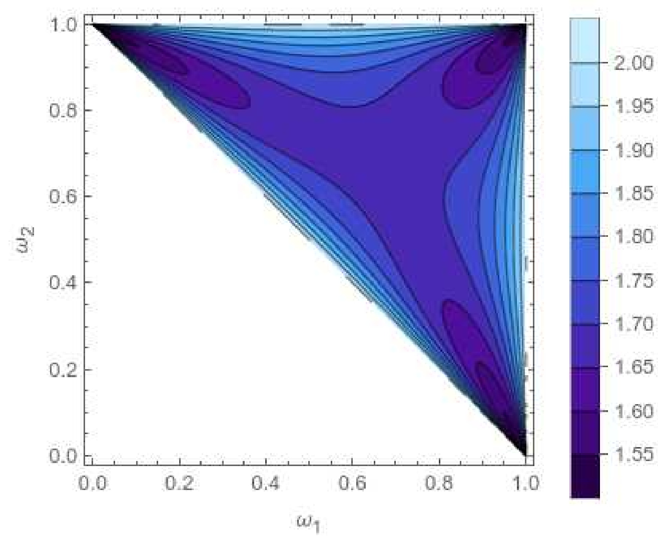


$$\omega_1 + \omega_2 + \omega_3 = 2m$$

only two independent variables

$$cos\theta_{12} = \frac{-\omega_3^2 + \omega_1^2 + \omega_2^2}{2\omega_1\omega_2}$$

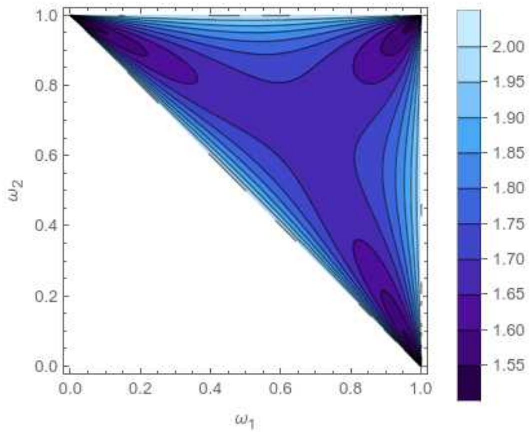
Berstetskii, Lifshitz and Pitaevskii, Quantum Electrodynamics

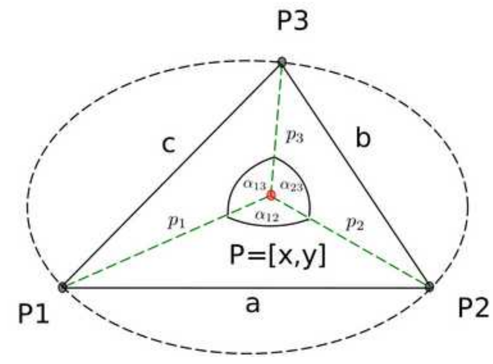


After calculating the the amplitude and getting rid of Dirac deltas:

$$d\bar{\sigma}_{3\gamma} = \frac{1}{6}\frac{8e^6}{vm^2}\left\{\left(\frac{m-\omega_3}{\omega_1\omega_2}\right)^2 + \left(\frac{m-\omega_2}{\omega_1\omega_3}\right)^2 + \left(\frac{m-\omega_1}{\omega_2\omega_3}\right)^2\right\} d\omega_1\, d\omega_2$$

From momentum to image space

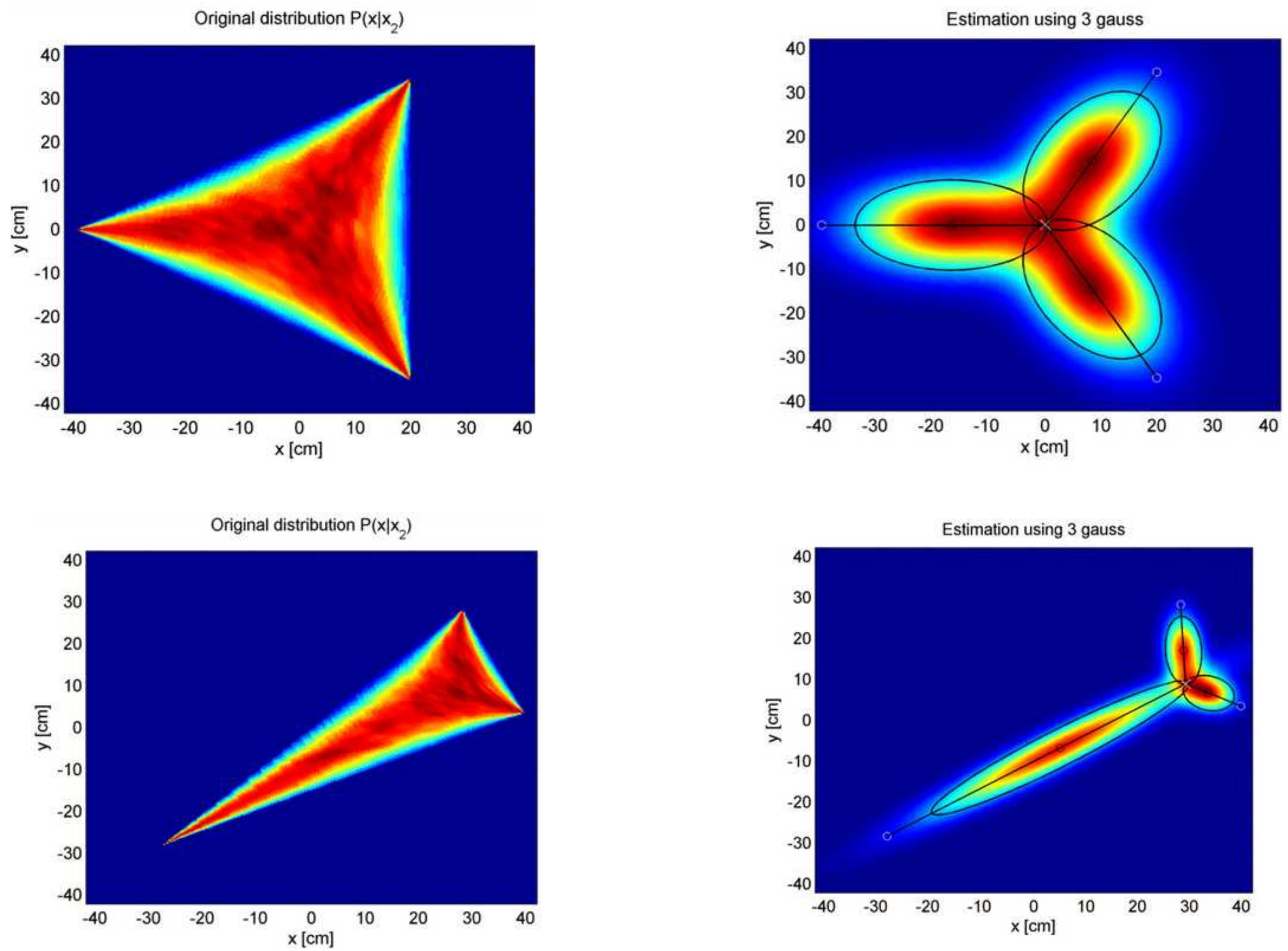




- „Interpolation” based on generated MC sample (slow, but accurate)
- Three Gaussian mixture parametrization (fast, but with some bias)
- Analytical closed-form (very ugly)

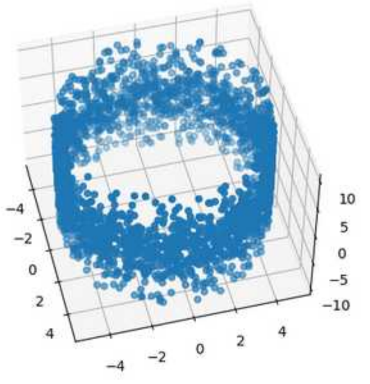
$$\boxed{P(\mathbf{x}|\mathbf{x_2}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_\mathbf{2}, \boldsymbol{\Sigma}_\mathbf{2})}$$

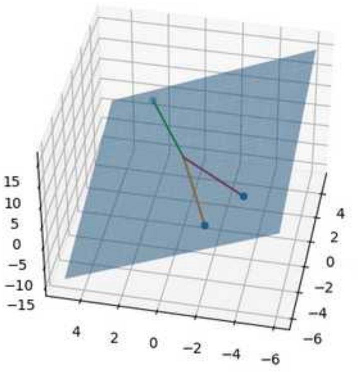
Approximation with Gaussian mixture



$$\boxed{P(\mathbf{x}|\mathbf{x_2}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu_2}, \boldsymbol{\Sigma_2})} \quad \theta_{12} = \theta_{23} = \theta_{31} = 120^o$$

First tests with point-sources on simplified MC



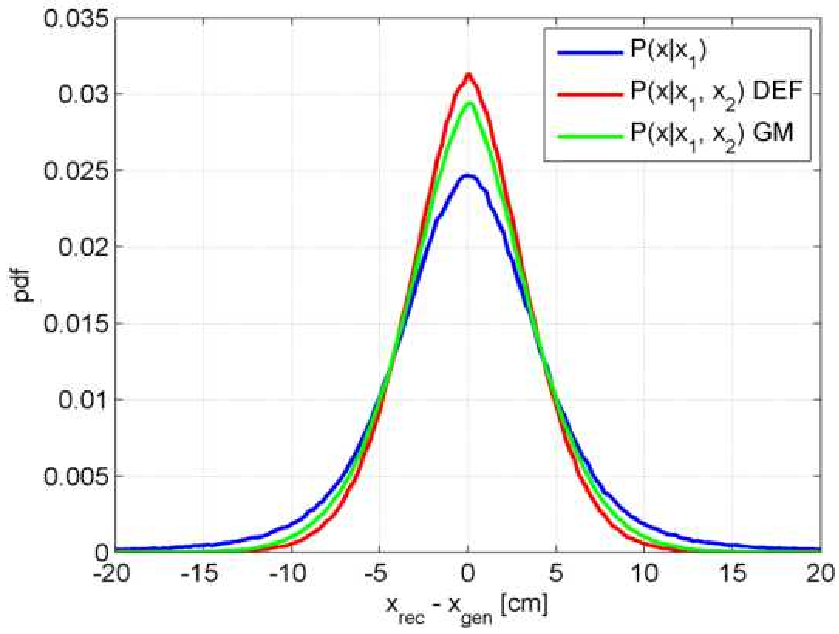


❑ Scanner size:	length 50 cm, radius 43.75 cm
❑ Scanner resolution:	TOF = 500 ps, CRTz = 4.5 cm
❑ Statistics:	~ 4 mln events

Additional preselection cuts:

❑ radius cut:	r < 40 cm	
❑ axial cut:	|z| < 23 cm	(after cuts ~**97.7%** data)

First tests with point-sources on simplified MC

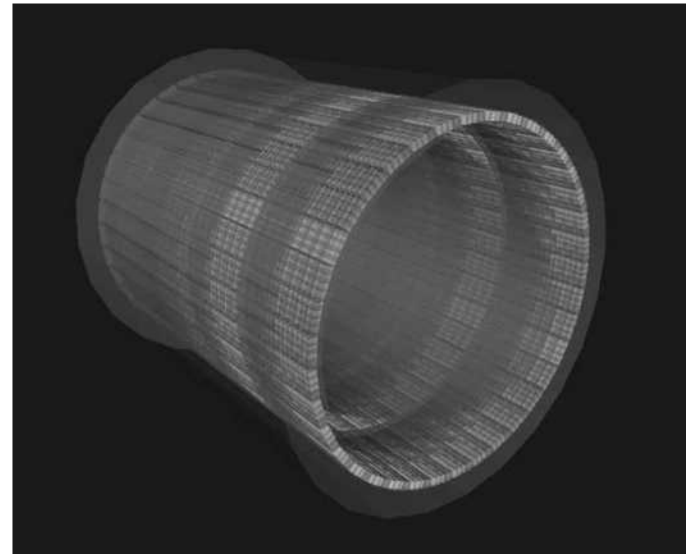


Preliminary results:
resolution improvement al least 4x

Test data → simulations

Biograph-Vision Quadra

https://www.siemens-healthineers.com/en-us/molecular-imaging/pet-ct/biograph-vision-quadra



- Simplified MC package with Biograph-Vision Quadra like geometry in use
- Spatial, time and energy resolution included

- Simulations with GATE 9.4 in preparation (o-ps → 3 g decay with ExtendedVSource model)

Other activities

Siemens Healthineers Academy

24 January 2025, Warsaw

One-day tutorial on MC techniques
in nuclear medicine

Jakub Baran (Siemens), Wojciech Krzemień (NCBJ), Szymon Parzych (UJ)

Hands-on with GATE 9.3

Geant 4 Workshop at
Warsaw University of Technology

24-28 February 2025, Warsaw

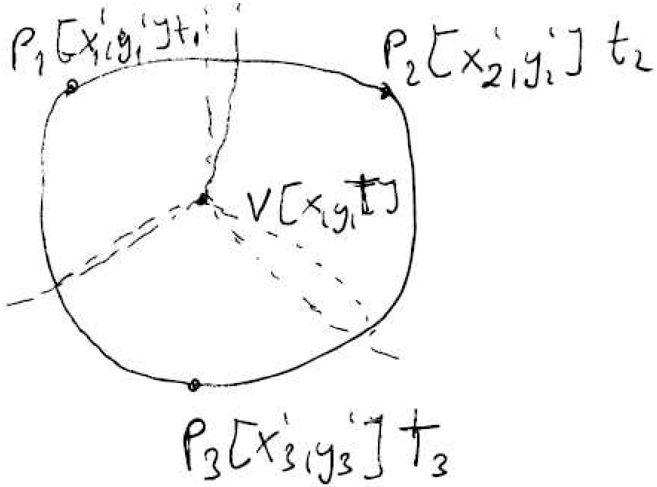
Half-day introduction to GATE for Geant4 users

Wojciech Krzemień (NCBJ), Szymon Parzych (UJ)

Hands-on with GATE 9.4

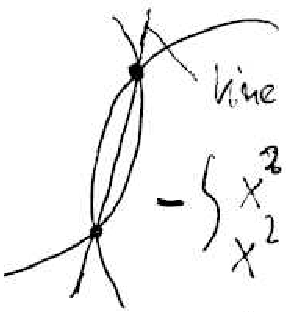
Thank you for attention

One look closer



$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 = d_1^2 \\ (x-x_2)^2 + (y-y_2)^2 = d_2^2 \\ (x-x_3)^2 + (y-y_3)^2 = d_3^2 \end{cases} \qquad d_i = c(t_i - T)$$

First step

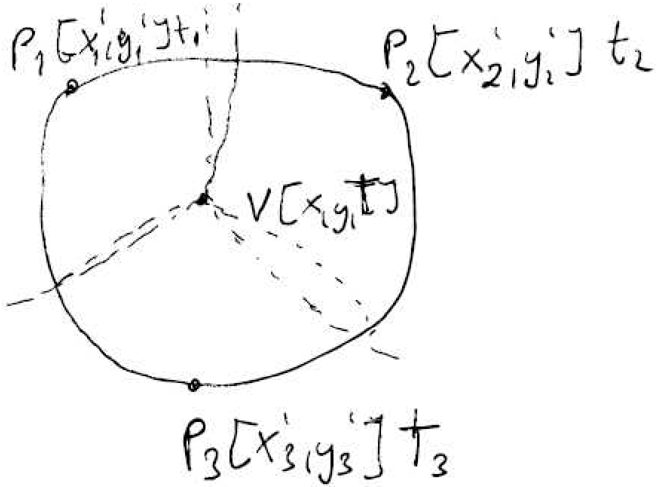


line equation

$$-\begin{cases} x^2 - 2x x_1 + x_1^2 + y^2 - 2y y_1 + y_1^2 = d_1^2 \\ x^2 - 2x x_2 + x_2^2 + y^2 - 2y y_2 + y_2^2 = d_2^2 \end{cases}$$

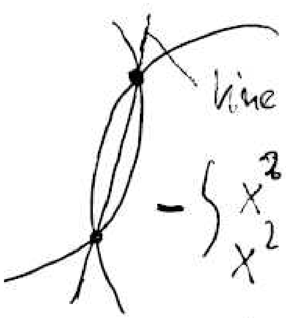
$$2x(x_2 - x_1) + 2y(y_2 - y_1) + y_1^2 - y_2^2$$
$$+ x_2^2 - x_1^2 = d_2^2 - d_1^2$$

One look closer



$$\begin{cases} (x-x_1)^2+(y-y_1)^2 = d_1^2 \\ (x-x_2)^2+(y-y_2)^2 = d_2^2 \\ (x-x_3)^2+(y-y_3)^2 = d_3^2 \end{cases} \qquad d_i = c(t_1 - T)$$

First step



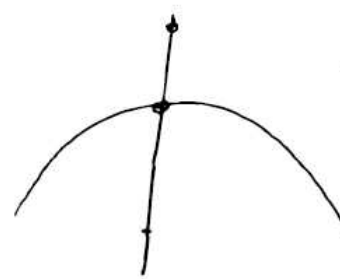
line equation

$$-\begin{cases} x^2 - 2x x_1 + x_1^2 + y^2 - 2y y_1 + y_1^2 = d_1^2 \\ x^2 - 2x x_2 + x_2^2 + y^2 - 2y y_2 + y_2^2 = d_2^2 \end{cases}$$

$$2x(x_2-x_1) + 2y(y_2-y_1) + y_1^2 - y_2^2 + x_2^2 - x_1^2 = d_2^2 - d_1^2$$

Second step

In the next step, we search for the intersection of the line wit the circle



Two solutions

$$(x-x_3)^2+(y-y_3)^2 = d_3^2$$

In collaboration with Lech Raczyński

Incorporation of experimental uncertainties

$$\begin{bmatrix} (x_1-x)^2+(y_1-y)^2 \\ (x_2-x)^2+(y_2-y)^2 \\ (x_3-x)^2+(y_3-y)^2 \end{bmatrix} = \begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \end{bmatrix}$$

$$\begin{bmatrix} 2(x_3-x_1) & 2(y_3-y_1) \\ 2(x_3-x_2) & 2(y_3-y_2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} d_1^2-d_3^2-x_1^2-y_1^2+x_3^2+y_3^2 \\ d_2^2-d_3^2-x_2^2-y_2^2+x_3^2+y_3^2 \end{bmatrix}$$

$$A\vec{x}=b \Rightarrow \vec{x}=(A^T A^{-1})A^T b$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$y = \frac{-a_{11} \cdot x + b_1}{a_{12}}$$

$$y = \frac{-a_{21} \cdot x + b_2}{a_{22}}$$

$$x = \frac{b_1 \cdot a_{22} - b_2 \cdot a_{12}}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$$

$$y = \frac{b_2 \cdot a_{11} - b_1 \cdot a_{21}}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$$

Incorporation of experimental uncertainties

Line equations

Point

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$y = \frac{-a_{11} \cdot x + b_1}{a_{12}}$$

$$y = \frac{-a_{21} \cdot x + b_2}{a_{22}}$$

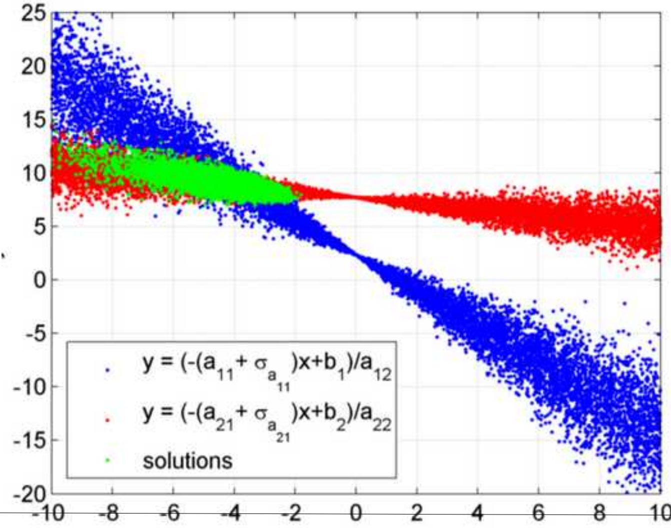
$$x = \frac{b_1 \cdot a_{22} - b_2 \cdot a_{12}}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$$

$$y = \frac{b_2 \cdot a_{11} - b_1 \cdot a_{21}}{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}$$

Uncertainties

$$a_{ij} \sim \mathcal{N}(\overline{a_{ij}}, \sigma_{aij})$$

$$b_i \sim \mathcal{N}(\overline{b_i}, \sigma_{bi})$$

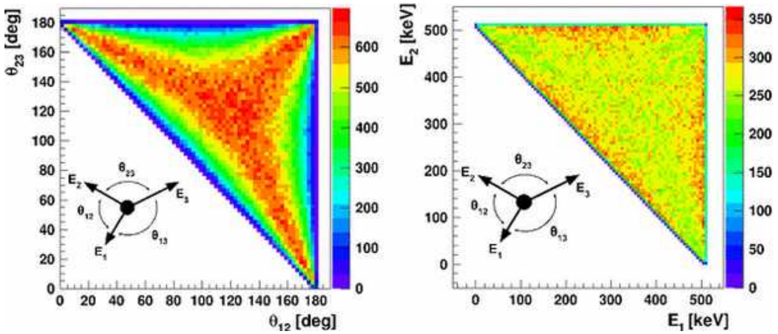


Estimation of covariance matrix

$$\tilde{f}(u,v) = f(u_0, v_0) + (u - u_0) \cdot \frac{\partial f(u_0, v_0)}{\partial u} + (v - v_0) \cdot \frac{\partial f(u_0, v_0)}{\partial v}$$

$$\boxed{P(\mathbf{x}|\mathbf{x_1}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)}$$

$$\boldsymbol{\mu}_1 = \begin{bmatrix} x \\ y \end{bmatrix} \quad \boldsymbol{\Sigma}_1 = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$



Implementation of QED-complient description of orto-positronium decay

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P. Moskal et al., Phys. Med. Biol. 64 (2019) 055017
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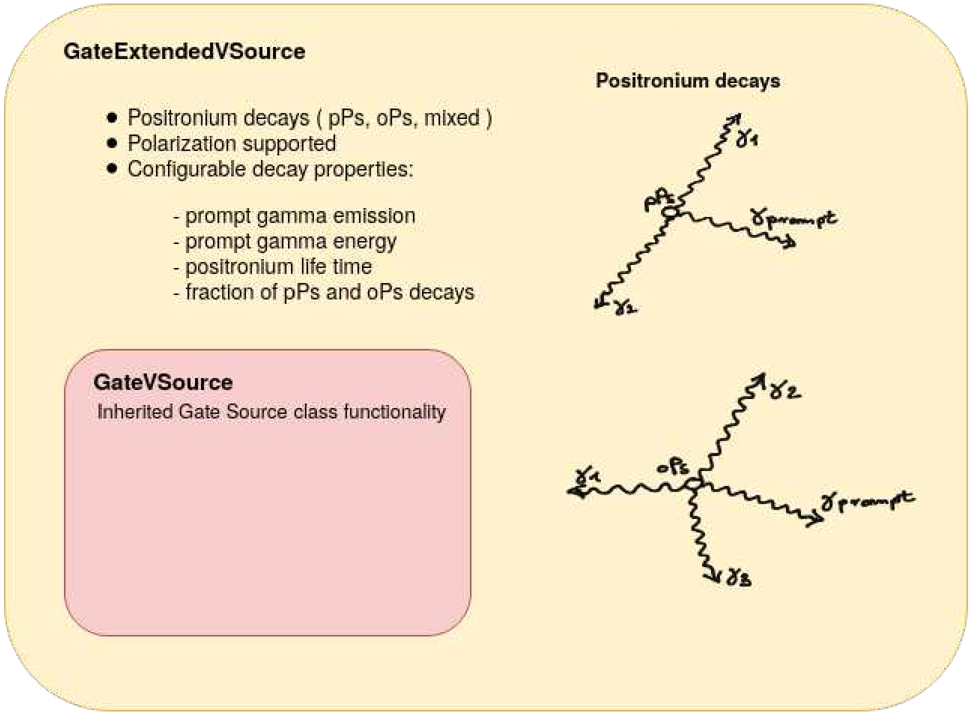

Mateusz Bała

available in GATE >= v9.0