- can direct detection (of a single field) yield phase information?

$$\hat{I} \propto \hat{n} = \hat{a}^{+}\hat{a}$$

$\hat{a} = \bar{a} + \delta\hat{a}$, $\bar{a} \in \mathbb{R}$ free to choose

$$\langle \hat{I} \rangle \propto \langle (\bar{a} + \delta a^{+})(\bar{a} + \delta\hat{a}) \rangle = \bar{a}^{2} + \bar{a}(\delta\hat{a} + \delta\hat{a}^{+}) + O(\delta a^{2}) \simeq$$

$$\simeq \bar{a}^{2} + \sqrt{2}\,\bar{a}\,\langle \delta\hat{X}(t) \rangle$$

amplitude quadrature!

$$\hat{X} \equiv \frac{\hat{a} + \hat{a}^{+}}{\sqrt{2}} \quad , \quad \hat{Y} \equiv \frac{\hat{a} - \hat{a}^{+}}{i\sqrt{2}} \quad \text{(phase quadrature)}$$

- "standard" interferometry:
  - copropagating two fields
  - we take one to be real, while the other has a relative phase θ

- for now, neglect the fluctuations, just add the mean fields, $\bar{a} \equiv \alpha$

$$I = (\alpha_1^* + \alpha_2^*)(\alpha_1 + \alpha_2) = |\alpha_1|^2 + |\alpha_2|^2 + \alpha_1^*\alpha_2 + \alpha_2^*\alpha_1$$

$$I = I_1 + I_2 + |\alpha_1||\alpha_2|\left(e^{i\theta} + e^{-i\theta}\right) \qquad I_i \equiv |\alpha_i|^2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$$

$\Rightarrow$ intensity dependent on the relative phase θ!

(1) for a given θ and fixed total power $I_1 + I_2 \equiv I_0$, when is the sensitivity the largest? (sensitivity to a change $\delta\theta$)

$$I_1 = x I_0 \quad , \quad I_2 = (1-x) I_0$$

$$\sqrt{x(1-x)} \xrightarrow[\text{monotonous}]{\text{sq. root}} \frac{d}{dx}\left[x(1-x)\right] = (1-x) - x = 1 - 2x \stackrel{!}{=} 0$$

$$\Rightarrow \boxed{x = 0.5}$$

(2) what about the angle for maximizing sensitivity?

$$\cos(\theta_0 + d\theta) \approx \cos\theta_0 - \sin\theta_0 \, d\theta$$

max. for $\theta_0 = \pi/2$! HOW IS THIS ENSURED?

- if intensities $I_1 = I_2 = 0.5 I_0$:

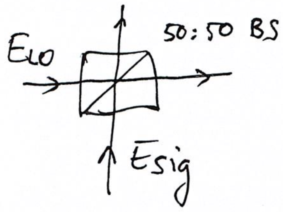
$$I = I_0(1 + \cos\theta) \begin{cases} \nearrow 0 \\ \searrow 2I_0 \end{cases}$$

sensitivity largest halfway ($\theta_0 = \pi/2$)

(3) big issue of these 'standard' schemes is that they do not discriminate between δI and $\delta\theta$!

Balanced homodyne detection (BHD)

→ now we'll write also the time-oscillating factor



better: PBS to combine and HWP + PBS to split (why?)

$$\hat{E}_\pm = \frac{1}{\sqrt{2}}\left[(\hat{a}_{LO} \pm \hat{a}_{sig})e^{-i\omega_L t} + (\hat{a}_{LO}^\dagger \pm \hat{a}_{sig}^\dagger)e^{i\omega_L t}\right]$$

notice: LO and sig of the same frequency!

$$\hat{E}_\pm^\dagger = \frac{1}{\sqrt{2}}\left[(\hat{a}_{LO}^\dagger \pm \hat{a}_{sig}^\dagger)e^{i\omega_L t} + (\hat{a}_{LO} \pm \hat{a}_{sig})e^{-i\omega_L t}\right]$$

photocurrent:

$$\hat{i}_\pm = \hat{E}_\pm^\dagger \hat{E}_\pm = \frac{1}{2}\Big[\left(\hat{a}_{LO}^\dagger \hat{a}_{LO} + \hat{a}_{sig}^\dagger \hat{a}_{sig} \pm \hat{a}_{LO}\hat{a}_{sig}^\dagger \pm \hat{a}_{sig}\hat{a}_{LO}^\dagger\right) +$$

$$+ \left(\hat{a}_{LO}\hat{a}_{LO}^\dagger + \hat{a}_{sig}\hat{a}_{sig}^\dagger \pm \hat{a}_{LO}\hat{a}_{sig}^\dagger \pm \hat{a}_{sig}\hat{a}_{LO}^\dagger\right)\Big] + O(2\omega_L)$$

→ strong LO, treat classically: $|\langle \hat{a}_{LO}\rangle| = |\alpha_{LO}| \gg |\langle \hat{a}_{sig}\rangle|$

$$\hat{i}_\pm \simeq \frac{1}{2}\left[2|\alpha_{LO}|^2 \pm 2|\alpha_{LO}|\left(e^{i\theta}\hat{a}_{sig}^\dagger + e^{-i\theta}\hat{a}_{sig}\right)\right] + O(2\omega_L)$$

$$\hat{i}_{diff} \equiv \hat{i}_+ - \hat{i}_- = 2|\alpha_{LO}|\left(e^{i\theta}\hat{a}_{sig}^\dagger + e^{-i\theta}\hat{a}_{sig}\right)$$

$$\hat{i}_{diff} = 2\sqrt{2}\,|\alpha_{LO}|\,\hat{X}_{sig}^\theta \qquad \hat{X}_{sig}^\theta \equiv \frac{e^{i\theta}\hat{a}_{sig}^\dagger + e^{-i\theta}\hat{a}_{sig}}{\sqrt{2}}$$

generalized quadrature

for $\theta = \pi/2$: Y_{sig} phase quadrature

notice: 1. no more classical intensity noise (i.e. highly suppressed)

2. signal scales with $|\alpha_{LO}|$

3. there is still shot noise

- to be clearer, let's treat the signal classically too:

$$\hat{i}_{diff} \propto e^{i(\theta-\varphi)} + e^{-i(\theta-\varphi)} \propto \cos(\theta-\varphi)$$

$\psi \equiv \theta - \varphi$ and consider a fluctuation $d\psi$:

$$\hat{i}_{diff} \propto \cos(\psi + d\psi) \simeq \cos\psi - \sin\psi \, d\psi$$

max. sensitivity $\psi_0 = \pi/2$ "gray fringe"