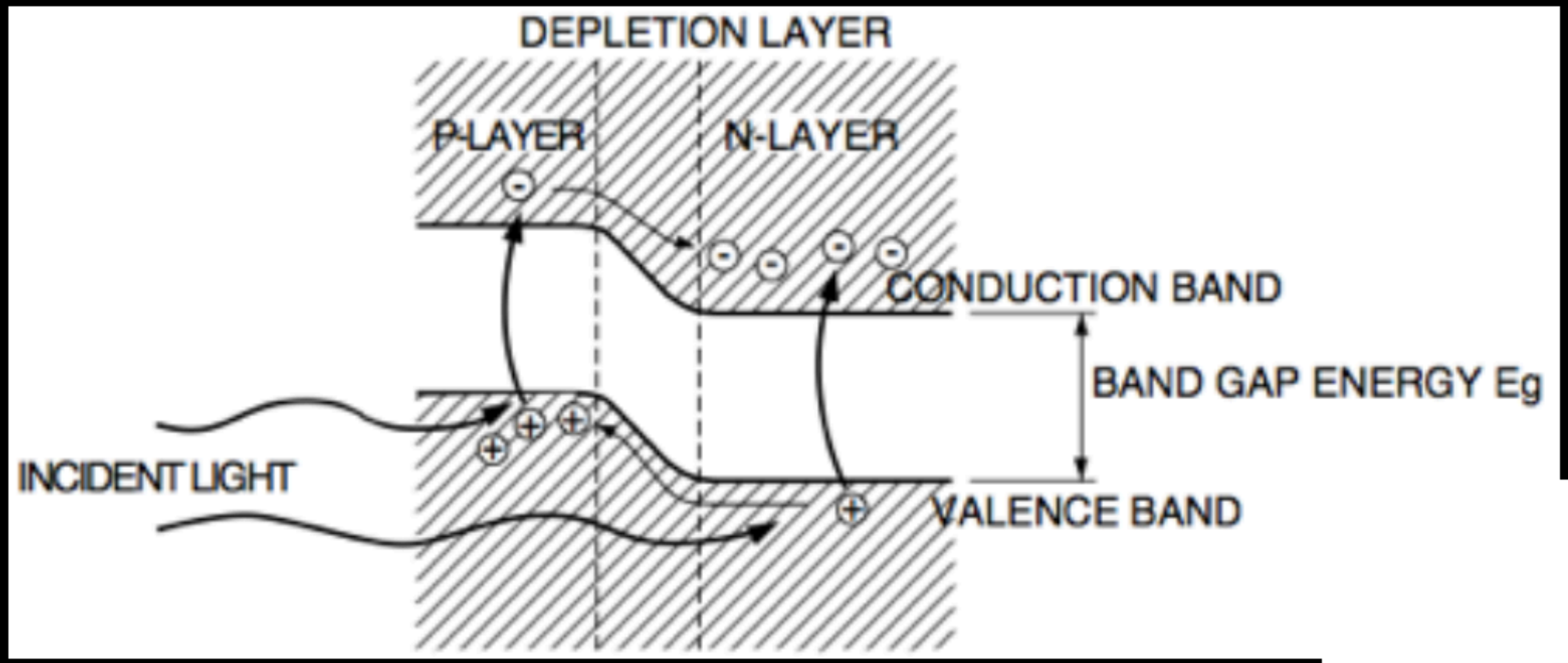
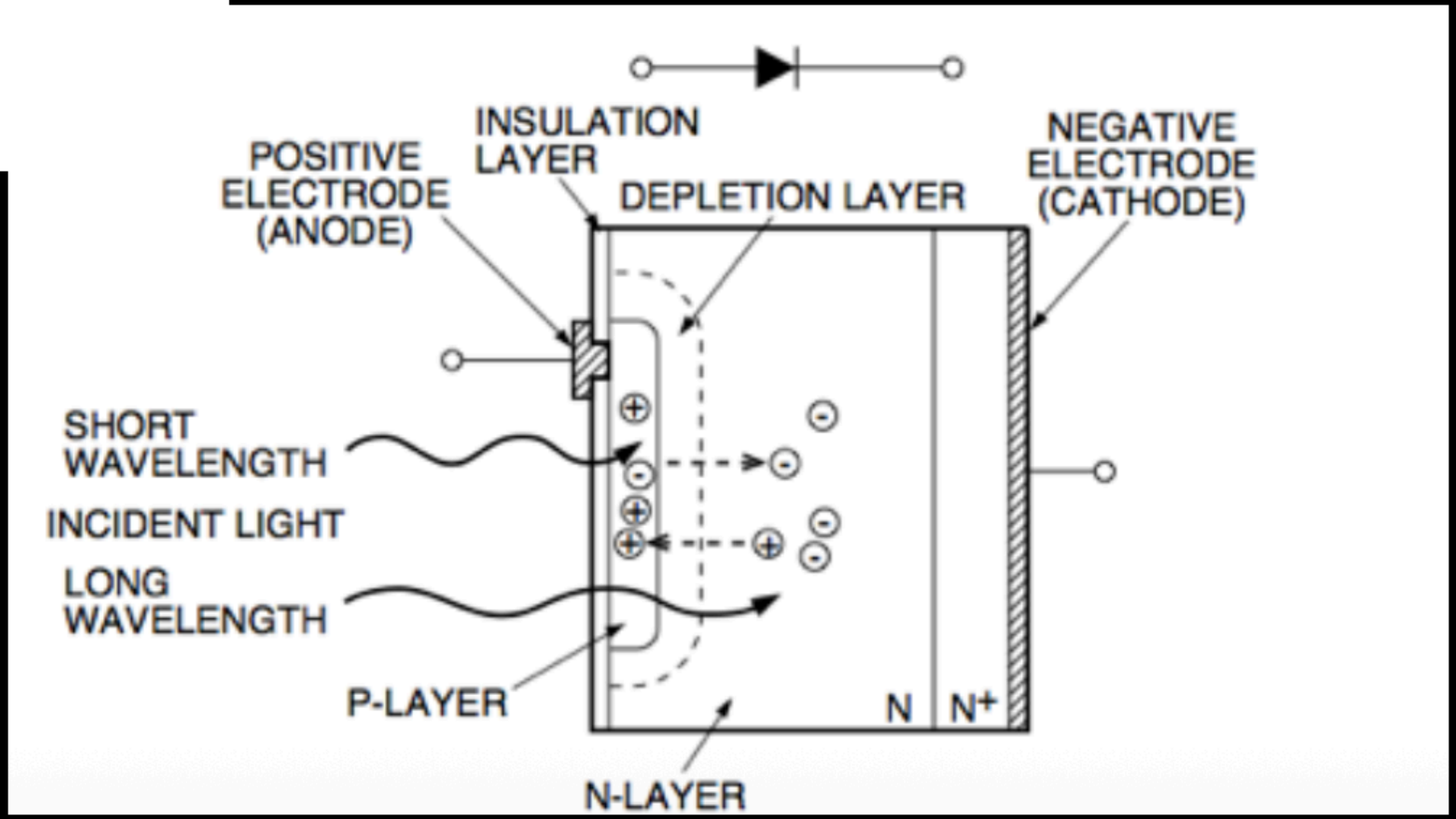
INTERFEROMETRY

HrZZ
Croatian Science Foundation

PHOTODETECTION

- WHEN LIGHT STRIKES A PHOTODIODE, THE ELECTRON WITHIN THE CRYSTAL STRUCTURE BECOMES STIMULATED. IF THE LIGHT ENERGY IS GREATER THAN THE BAND GAP ENERGY EG, THE ELECTRONS ARE PULLED UP INTO THE CONDUCTION BAND, LEAVING HOLES IN THEIR PLACE IN THE VALENCE BAND.





PHOTODETECTION

- RESPONSIVITY: $R_\lambda \ [A/W]$

  IS THE RATIO OF THE RESULTING PHOTOCURRENT EXPRESSED IN AMPERES (A), TO THE RADIANT ENERGY INCIDENT ON THE DEVICE EXPRESSED IN WATTS (W).

- QUANTUM EFFICIENCY: QE

  IS THE NUMBER OF ELECTRONS OR HOLES THAT CAN BE DETECTED AS A PHOTOCURRENT DIVIDED BY THE NUMBER OF THE INCIDENT PHOTONS.

$$QE = \frac{R_\lambda \ [A/W] \times 1.24}{\lambda \ [\mu m]}$$

PHOTODETECTION

- NEP (NOISE EQUIVALENT POWER)

  NEP IS THE AMOUNT OF LIGHT EQUIVALENT TO THE NOISE LEVEL OF A DEVICE. SINCE THE NOISE LEVEL IS PROPORTIONAL TO THE SQUARE ROOT OF THE FREQUENCY BANDWIDTH, THE NEP IS MEASURED AT A BANDWIDTH OF 1 HZ.

$$\text{NEP } [W/\sqrt{Hz}] = \frac{\text{NOISE CURRENT } [A/\sqrt{Hz}]}{\text{RESPONSIVITY } [A/W]}$$

PHOTODETECTION

- NOISE CURRENT

  - THERMAL OR JOHNSON NOISE OF THE SHUNT RESISTANCE

$$i_{th} = \sqrt{4k_B T/R_{sh}} \quad [A/\sqrt{Hz}]$$

  - SHOT NOISE OF THE DARK CURRENT OF THE JUNCTION AND THE PHOTOCURRENT

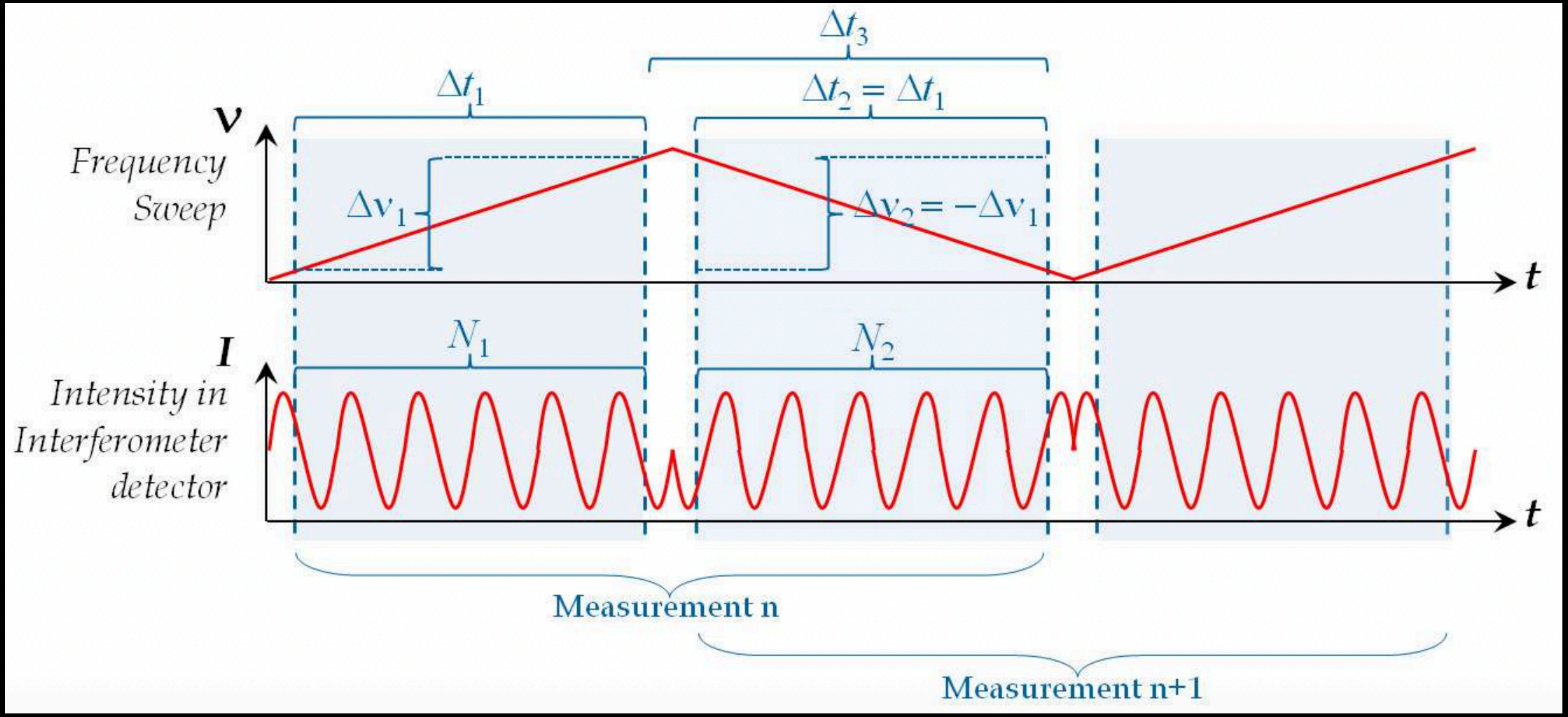
$$i_{sn} = \sqrt{2e(I_D + I_L)} \quad [A/\sqrt{Hz}]$$

  - THE TOTAL AMOUNT OF NOISE IS

$$i_n = \sqrt{i_{th}^2 + i_{sn}^2} \quad [A/\sqrt{Hz}]$$

INTERFEROMETER VISIBILITY

$$\mathcal{V} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$



INTERFEROMETER VISIBILITY

- IF THERE IS A PURELY ELECTRONIC OFFSET, MEASURE IT WHEN THERE IS NO LIGHT AND THEN

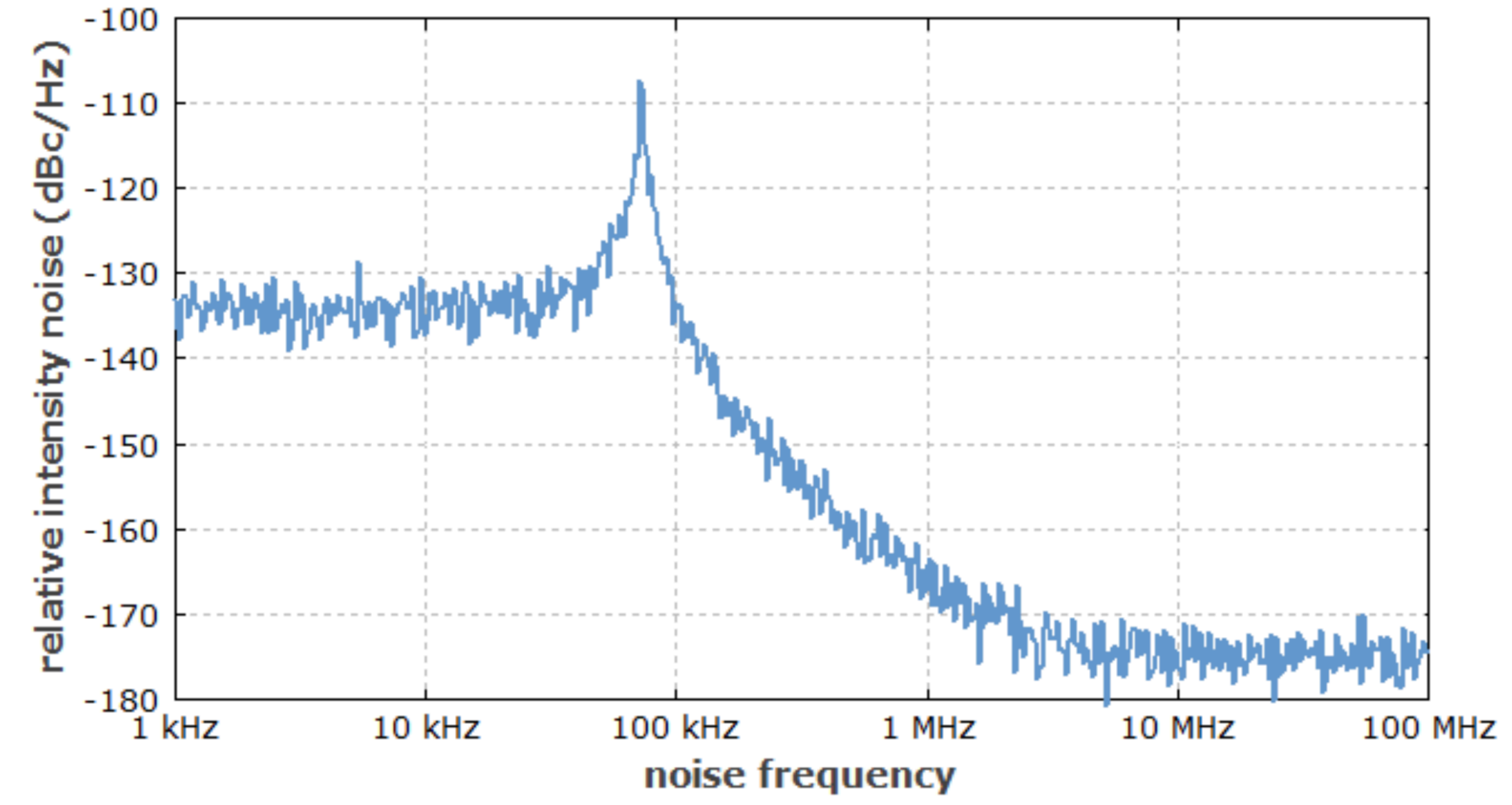
$$\mathcal{V} = \frac{(V_{\max} - V_{\text{off}}) - (V_{\min} - V_{\text{off}})}{(V_{\max} - V_{\text{off}}) + (V_{\min} - V_{\text{off}})}$$

I.E.

$$\mathcal{V} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min} - 2V_{\text{off}}}$$

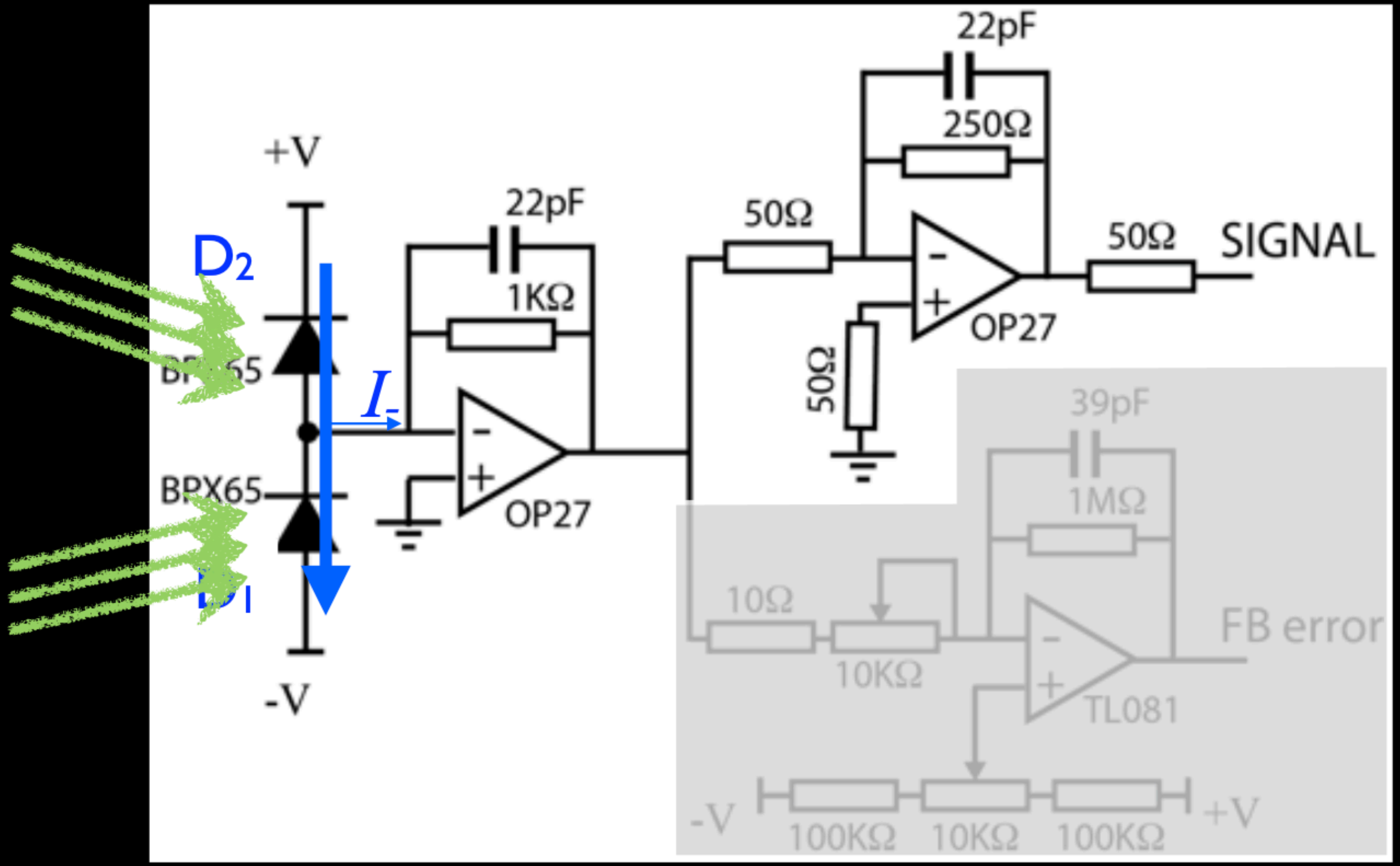
- NOTE THAT THIS QUANTITY ONLY REALLY MAKES SENSE WITH NO POWER BALANCING (THEREFORE, NOT FOR HOMODYNE, SEE FOLLOWING SLIDES)

CLASSICAL INTENSITY NOISE



Homodyne Detection

- Detection circuit



On photodetectors with resposivity R_λ $[A/W]$
the induced photocurrent is

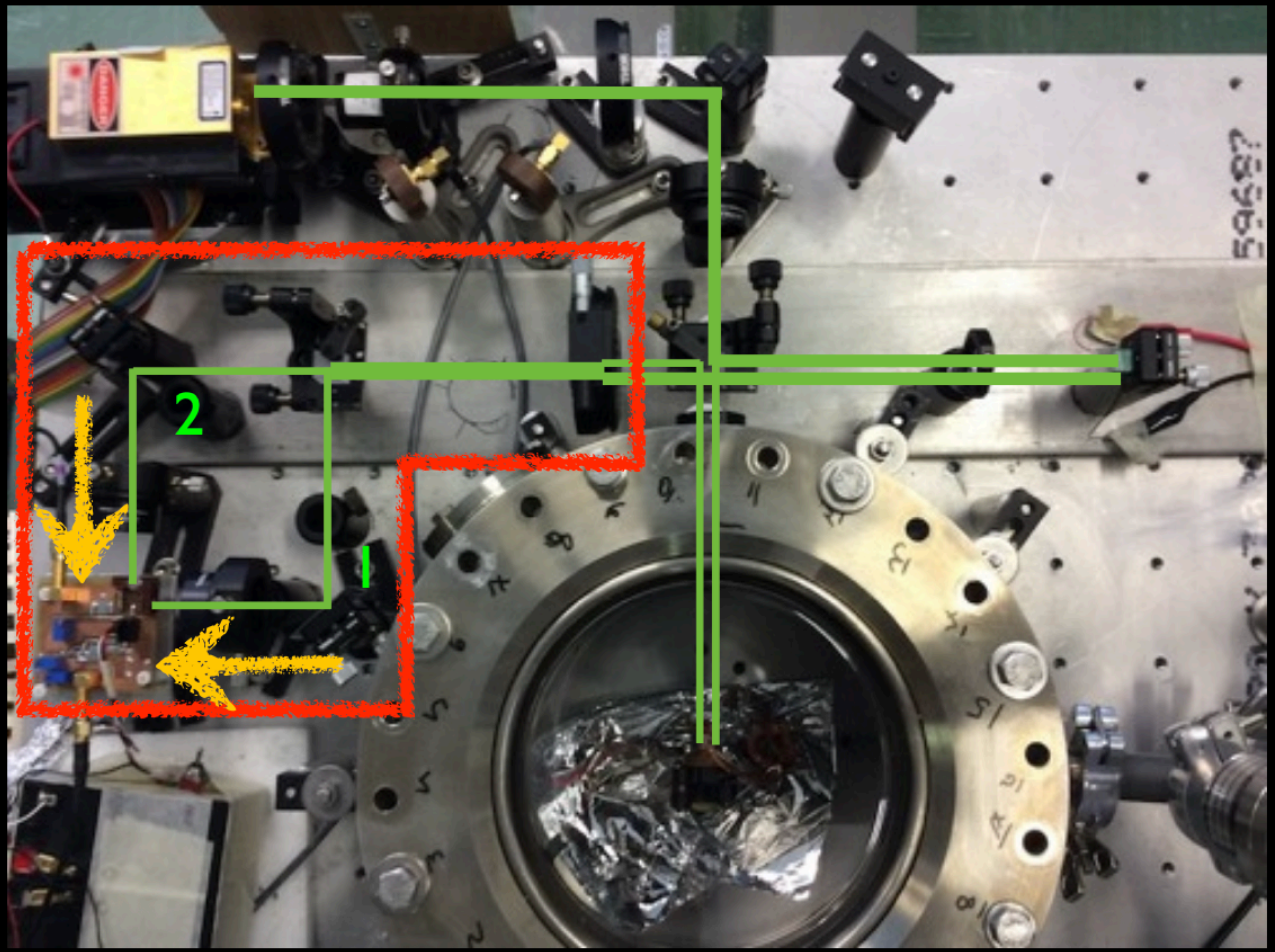
$$I_- = R_\lambda(\mathcal{I}_2 - \mathcal{I}_1) = R_\lambda \mathcal{I}_-$$

Homodyne Detection

- Intensities

$$\mathcal{I}_1 = |\mathcal{E}_1|^2 = \frac{|\mathcal{E}_0|^2}{2}\{\mathcal{R}\rho^2 + \tau^2 - 2\sqrt{\mathcal{R}}\rho\tau\cos[2k(l_2 - l_1)]\}$$

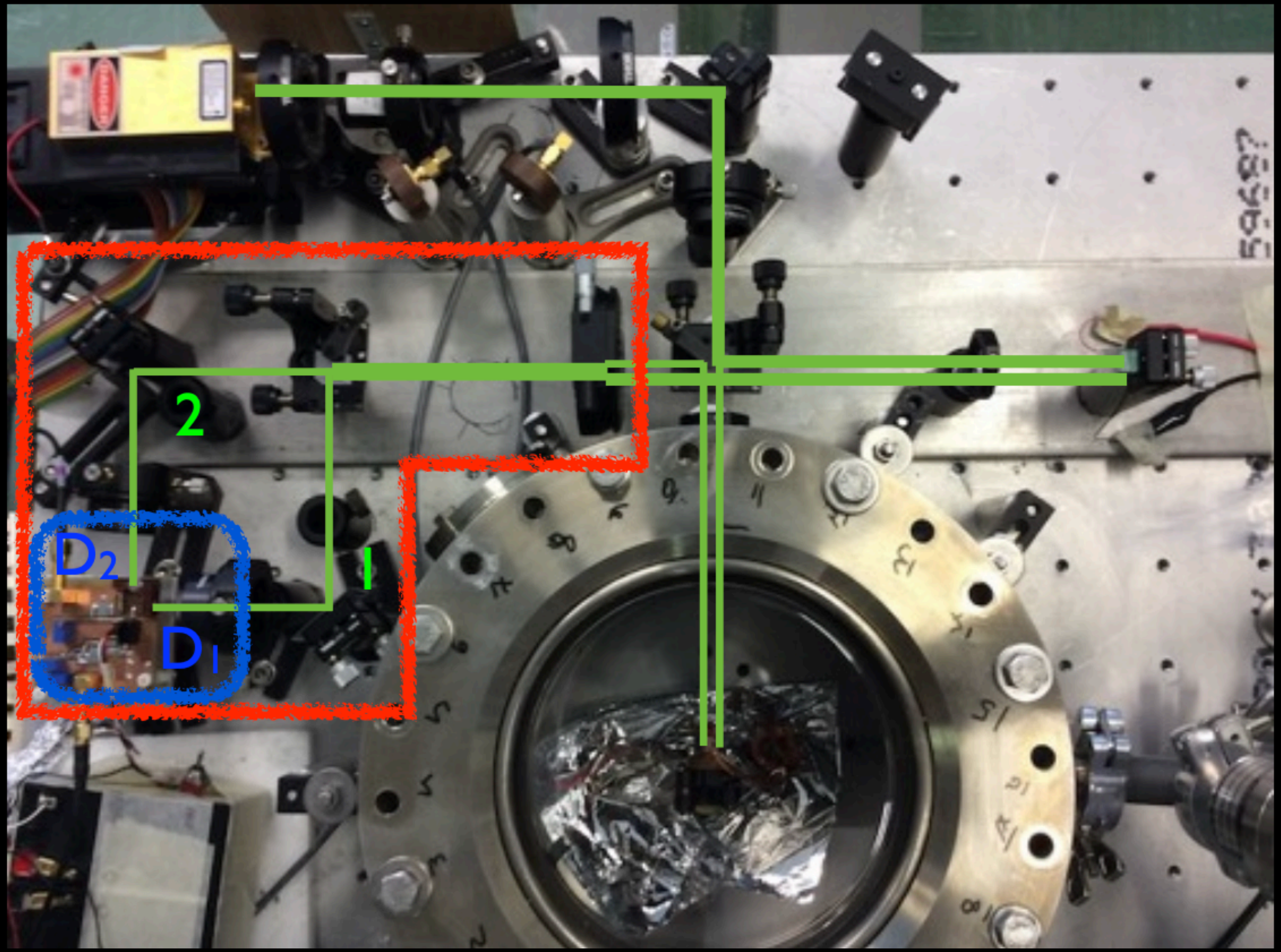
$$\mathcal{I}_2 = |\mathcal{E}_2|^2 = \frac{|\mathcal{E}_0|^2}{2}\{\mathcal{R}\rho^2 + \tau^2 + 2\sqrt{\mathcal{R}}\rho\tau\cos[2k(l_2 - l_1)]\}$$



HOMODYNE DETECTION

- DIFFERENCE OF INTENSITIES

$$\mathcal{I}_- \equiv \mathcal{I}_2 - \mathcal{I}_1 = 2|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\cos[2k(l_2 - l_1)]$$



HOMODYNE DETECTION

- LET'S CONSIDER NOW A SMALL TIME MODULATION OF THE PHASE $2k(l_2 - l_1)$ DUE TO A DISPLACEMENT OF THE MEMBRANE, $\delta x(t)$, (or mirror!) AROUND A STATIC POSITION, $\phi_0 = 2k(l_2 - l_1)$ SUCH THAT

$$2k(l_2 - l_1) = \phi_0 + 2k\delta x(t)$$

AND

$$\begin{aligned}\mathcal{I}_- &= 2|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\cos[\phi_0 + 2k\delta x(t)] \\ &= 2|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\{\cos(\phi_0)\cos[2k\delta x(t)] - \sin(\phi_0)\sin[2k\delta x(t)]\}\end{aligned}$$

HOMODYNE DETECTION

- IF WE CONSIDER A SIMPLE SINUSOIDAL MODULATION OF AMPLITUDE δx_m AT FREQUENCY ω_m, I.E. $\delta x(t) = \delta x_m \cos(\omega_m t + \chi_m)$ WE CAN EXPAND THE DIFFERENCE OF INTENSITIES IN TERMS OF BESSEL FUNCTIONS OF THE FIRST KIND

$$\begin{aligned}\mathcal{I}_- = 2|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\{&\cos(\phi_0)J_0(2k\delta x_m) - \\ &\sin(\phi_0)2J_1(2k\delta x_m)\sin(\omega_m t + \chi_m)\}\end{aligned}$$

$$\mathcal{I}_-^{DC} = 2|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\cos(\phi_0)J_0(2k\delta x_m) \quad \rightarrow I_-^{DC} = R_\lambda \mathcal{I}_-^{DC}$$

$$\mathcal{I}_-^{AC} = 4|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\sin(\phi_0)J_1(2k\delta x_m)\sin(\omega_m t + \chi_m) \quad \rightarrow I_-^{AC} = R_\lambda \mathcal{I}_-^{AC}$$

HOMODYNE DETECTION

- THE SIGNAL POWER DELIVERED TO A SPECTRUM ANALYSER WITH INPUT IMPEDANCE R_s IS

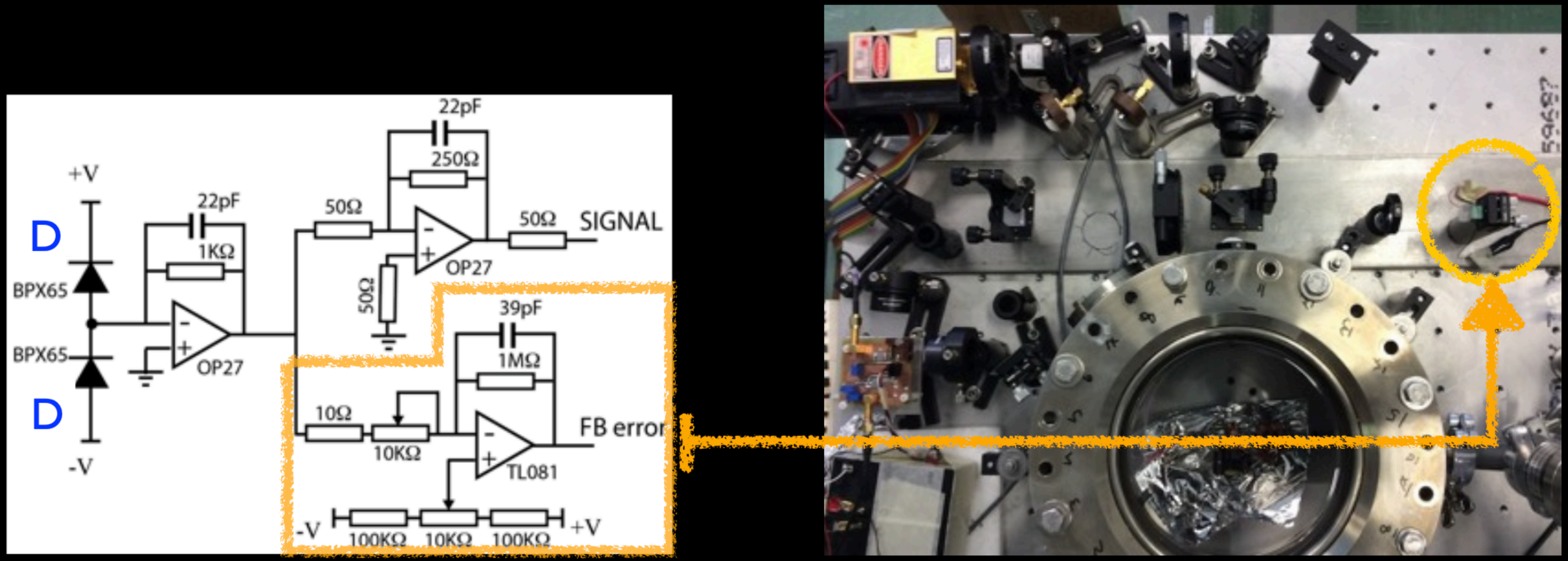
$$\mathcal{P}_{signal} = \overline{(I_-^{AC})^2}\, R_s$$

$$= [4R_\lambda|\mathcal{E}_0|^2\sqrt{\mathcal{R}}\rho\tau\sin(\phi_0)J_1(2k\delta x_m)]^2\frac{R_s}{2}$$

THE SIGNAL POWER IS STRONGEST IN GRAY-FRINGE WHEN THE DERIVATIVE OF THE TRANSMISSION WITH RESPECT TO PHASE IS LARGEST$(\phi_0 = \pi/2)$.

Homodyne Detection

- The gray-fringe working point is set by sending an FB-error to a piezo on the mirror



HOMODYNE DETECTION

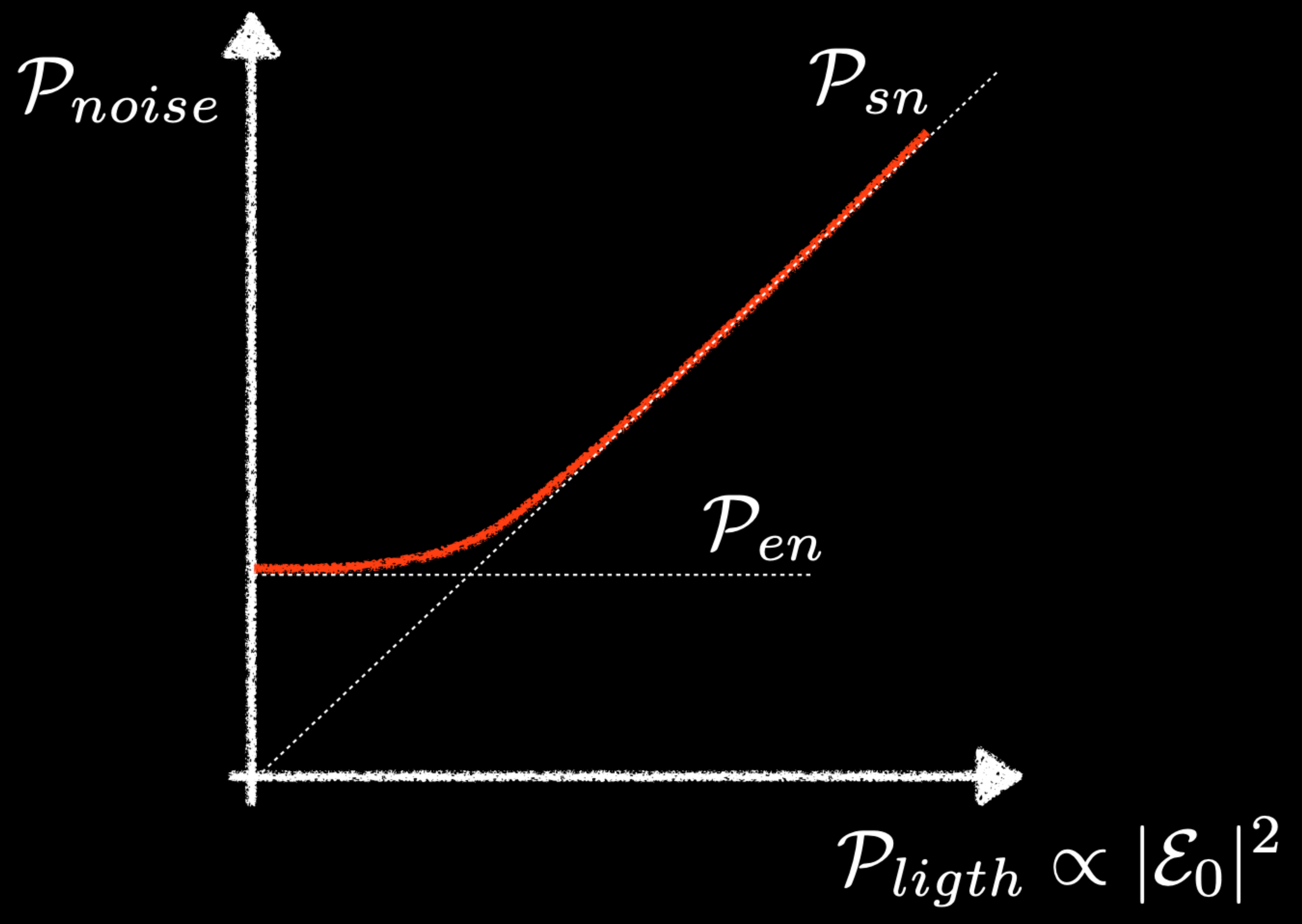
- ALTHOUGH THE DC TERM IS ZERO FOR $\phi_0 = \pi/2$, THE SHOT NOISE IS NEVER ZERO, AND IS LINEAR WITH THE INPUT LIGHT INTENSITY

$$\begin{aligned}\mathcal{P}_{sn} &= 2e(I_1^{DC} + I_2^{DC})\, R_s\, BW \\ &= 2eR_\lambda |\mathcal{E}_0|^2 (\mathcal{R}\rho^2 + \tau^2) R_s\, BW\end{aligned}$$

- THE ELECTRONIC NOISE DUE TO THE PINS AND ELECTRONIC CIRCUIT IS INDEPENDENT ON THE INPUT LIGHT INTENSITY

Homodyne Detection

- Noise analysis



HOMODYNE DETECTION

- IF GRAY-FRINGE WORKING POINT $\phi_0 = \pi/2$, INTENSE LOCAL OSCILLATOR $\tau^2 \gg \mathcal{R}\rho^2$, AND SMALL DISPLACEMENT $2k\delta x_m \ll 1 \rightarrow$

$$J_1(2k\delta x_m) \simeq k\delta x_m$$

  - SIGNAL $\mathcal{P}_{signal} = 8R_\lambda^2 \mathcal{I}_{LO}\mathcal{I}_m [J_1(2k\delta x_m)]^2 R_s$

    WITH $\mathcal{I}_{LO} = |\mathcal{E}_0|^2\tau^2$ AND $\mathcal{I}_m = |\mathcal{E}_0|^2\mathcal{R}\rho^2$

  - SHOT NOISE $\mathcal{P}_{sn} \simeq 2eR_\lambda \mathcal{I}_{LO} R_s\, BW$

HOMODYNE DETECTION

- SIGNAL-NOISE-RATIO

$$S/N = \frac{\mathcal{P}_{signal}}{\mathcal{P}_{sn} + \mathcal{P}_{en}}$$

- IF GRAY-FRINGE WORKING POINT AND $\mathcal{P}_{sn} > \mathcal{P}_{en}$ THEN

$$S/N = \frac{4R_\lambda|\mathcal{E}_0|^2}{e\,BW}\,\frac{\mathcal{R}\rho^2\tau^2}{\mathcal{R}\rho^2 + \tau^2}[J_1(2k\delta x_m)]^2$$

Homodyne Detection

- Limits
  - Intense local oscillator $\tau^2 \gg \mathcal{R}\rho^2$

$$S/N = \frac{4R_\lambda \mathcal{I}_m}{e\,BW}[J_1(2k\delta x_m)]^2 \qquad \text{with } \mathcal{I}_m = |\mathcal{E}_0|^2\mathcal{R}\rho^2$$

  - Small displacement $2k\delta x_m \ll 1 \rightarrow$

$$J_1(2k\delta x_m) \simeq k\delta x_m$$

$$S/N = \frac{4R_\lambda \mathcal{I}_m}{e\,BW}\frac{4\pi^2}{\lambda^2}\delta x_m^2$$

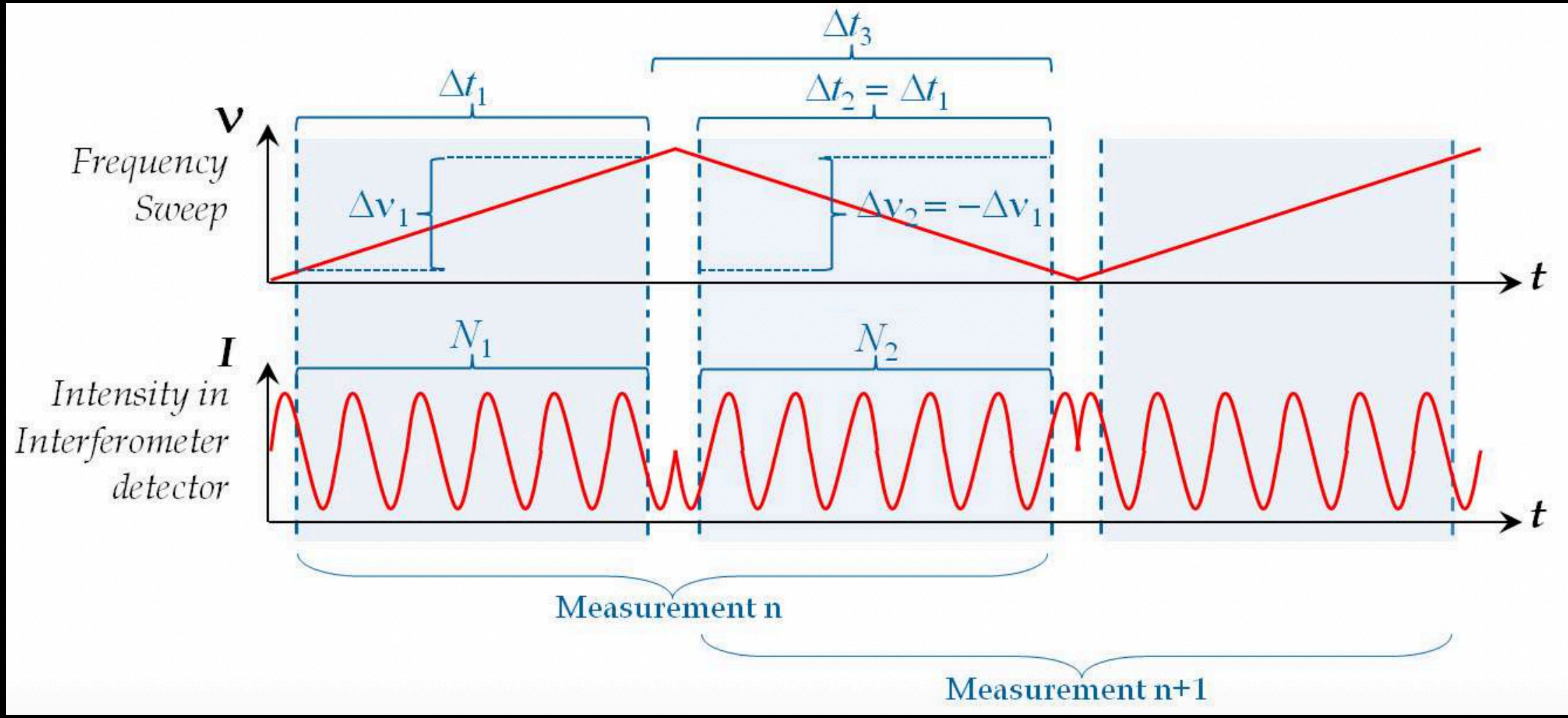
BETTER WAY TO CALIBRATE

- RAMP TO PIEZO, VOLTAGE GIVEN BY

$$V(x) = \frac{V_{\text{pp}}}{2}\sin(2\,k\,\delta x)$$

- SO, AT THE GRAY FRINGE

$$V(x) \approx V_{\text{pp}}\,k\,\delta x$$



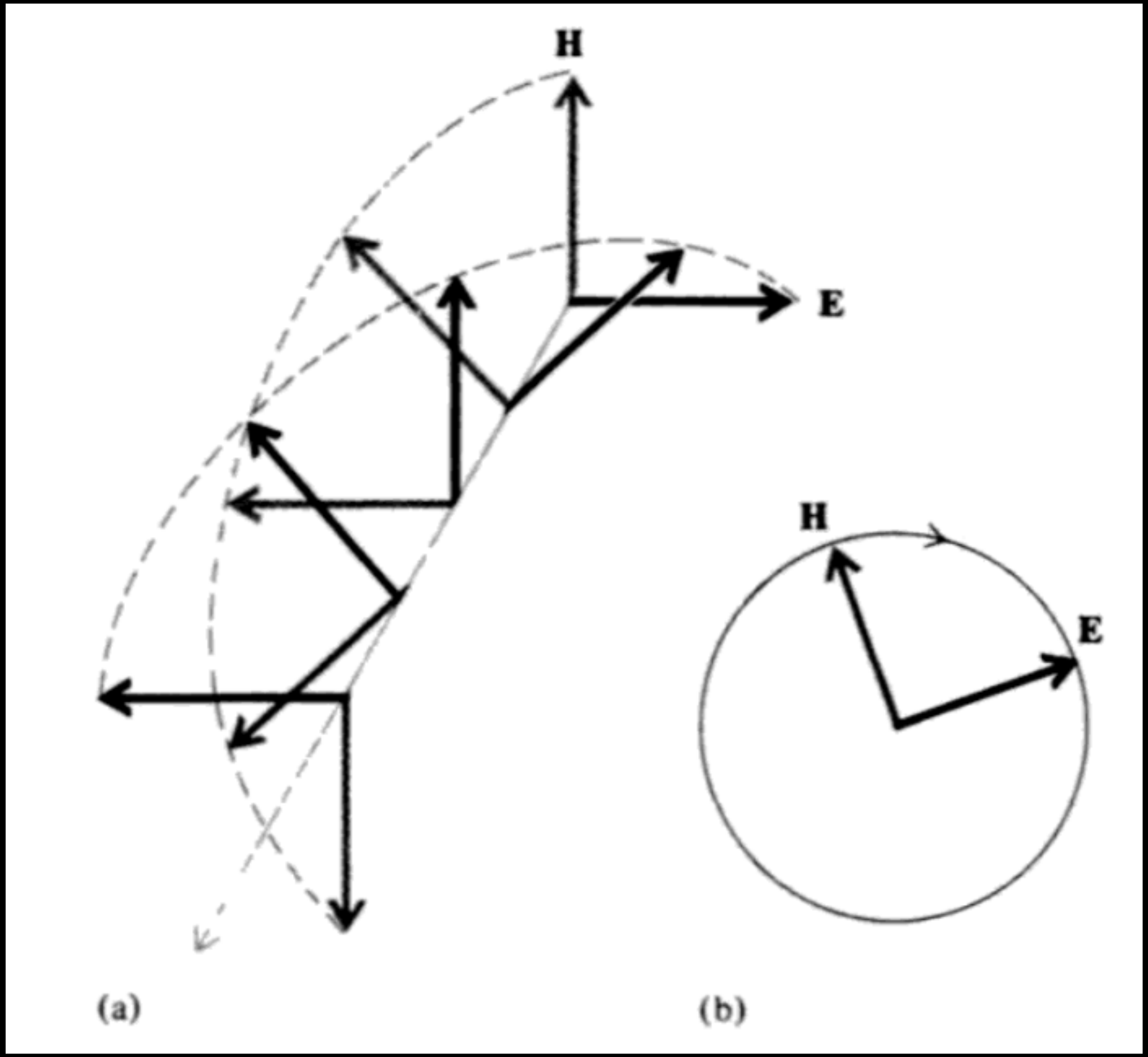
Note I: QWP, HWP, PBS

- circular polarization:

$\hat{\mathbf{x}} E_0 \cos(kz - \omega t)$

$\hat{\mathbf{y}} E_0 \sin(kz - \omega t)$

$$\mathbf{E} = E_0[\hat{\mathbf{x}} \cos(kz - \omega t) + \hat{\mathbf{y}} \sin(kz - \omega t)]$$



Note I: QWP, HWP, PBS

- waveplates: index of refraction depends on polarization, so they induce a relative phase between different polarization components

$$\Delta\phi = (k_1 - k_2)d = \frac{2\pi}{\lambda_0}(n_1 - n_2)d$$

- QWP: transforms linear polarization to circular (elliptical) and vice versa

$$\Delta\phi = \frac{\pi}{2} \quad \Rightarrow \quad d = \frac{\lambda_0}{4(n_1 - n_2)}$$

Note I: QWP, HWP, PBS

- polarizer output (not normalized):

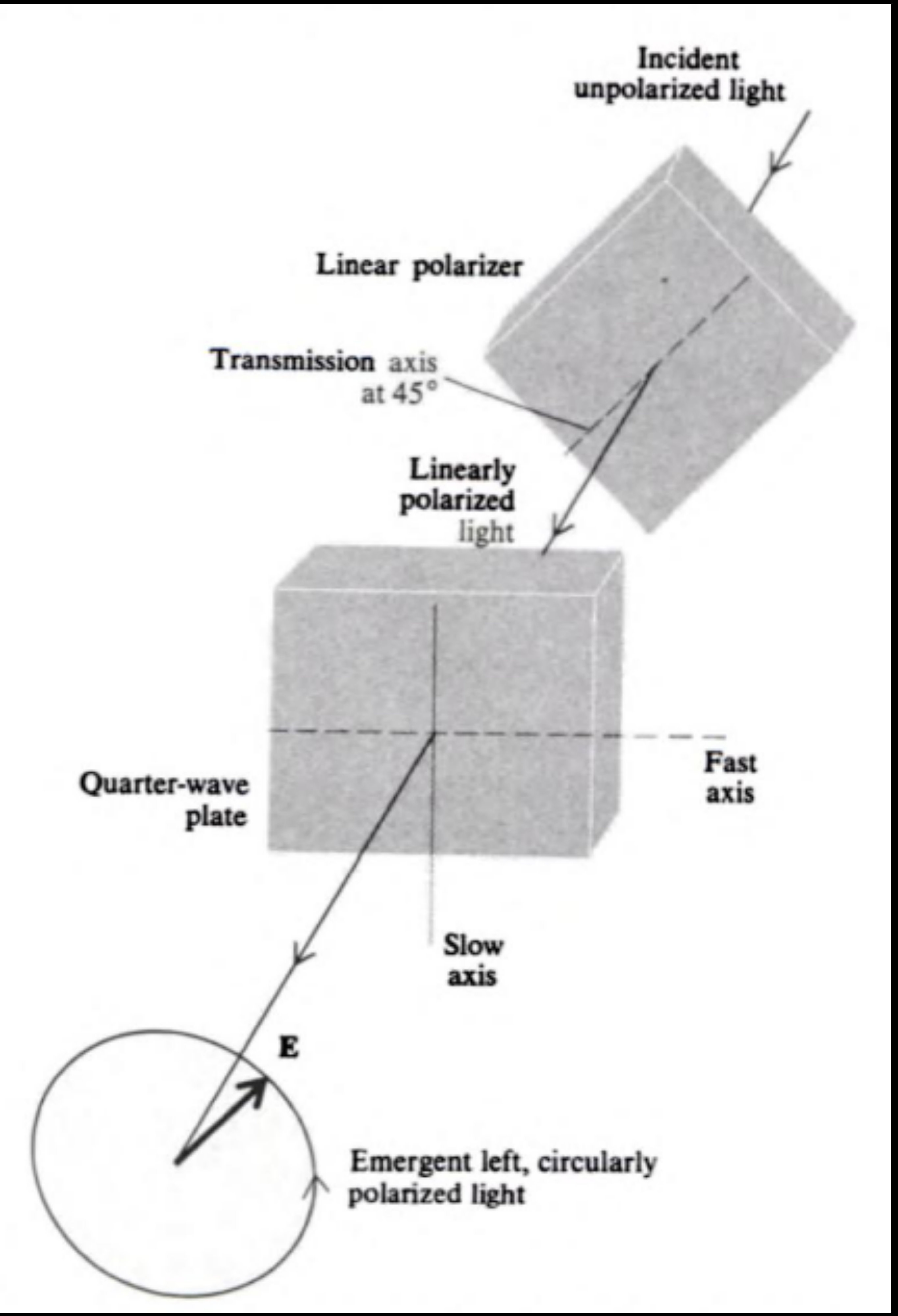
$$\mathbf{E} = E_0[\hat{\mathbf{x}}\cos(kz-\omega t) + \hat{\mathbf{y}}\cos(kz-\omega t)]$$

- QWP output:

$$\mathbf{E} = E_0[\hat{\mathbf{x}}\cos(kz-\omega t) - \hat{\mathbf{y}}\sin(kz-\omega t)]$$

- output after another QWP:

$$\mathbf{E} = E_0[\hat{\mathbf{x}}\cos(kz-\omega t) - \hat{\mathbf{y}}\cos(kz-\omega t)]$$



- conclusion: a HWP (analogous to two QWP) rotates the polarization of linearly polarized light by twice the angle between that of the polarization and the plate axis

$$\Delta\phi = \pi \quad \Rightarrow \quad d = \frac{\lambda_0}{2(n_1 - n_2)}$$

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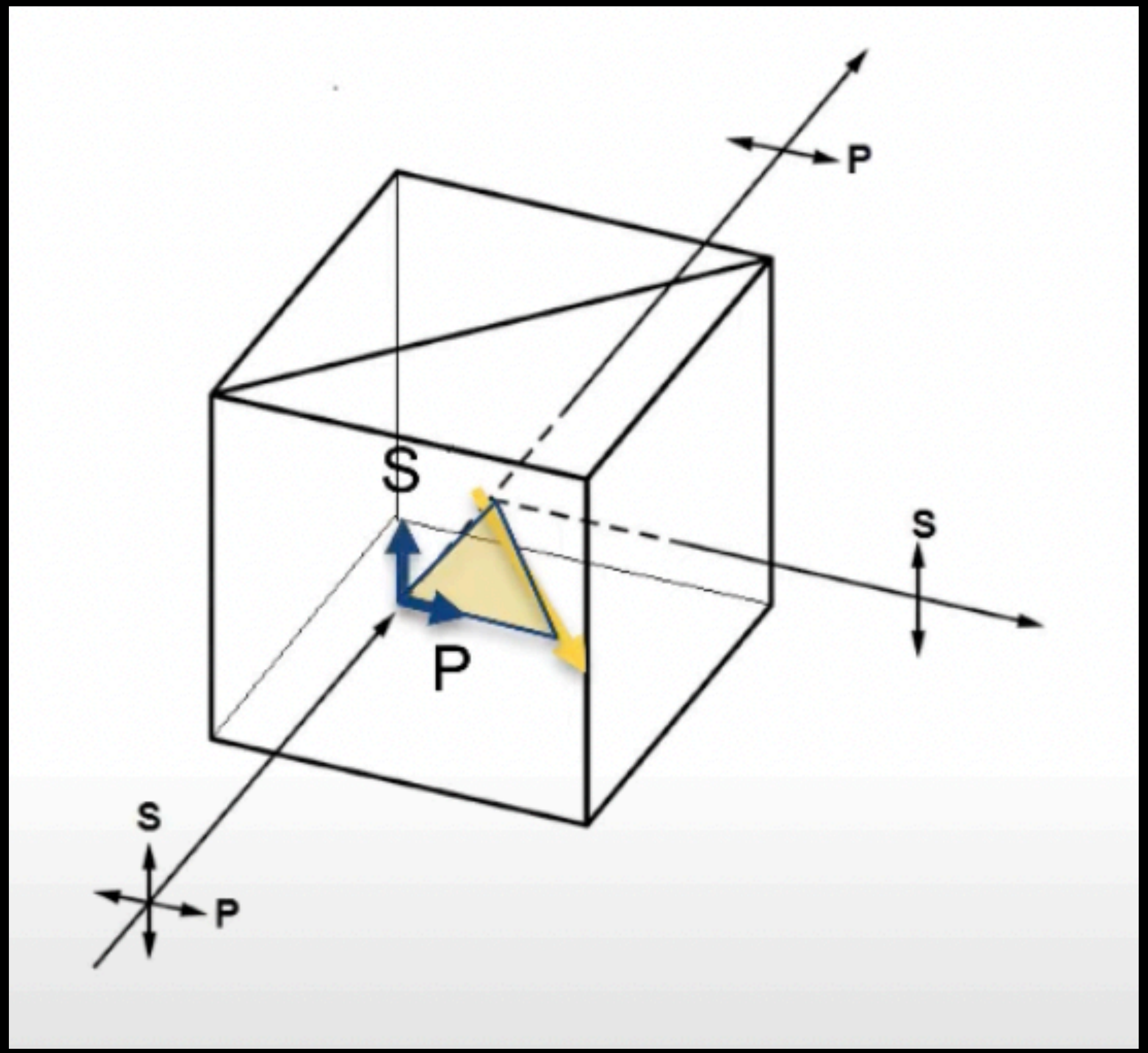
Note I: QWP, HWP, PBS

- PBS: polarizing beamsplitter



p - parallel
s - senkrecht

Note II: dBm

- typically, a spectrum analyzer will show data in dBm:

$$P[\mathrm{dBm}] = 10\log_{10}\frac{P[\mathrm{W}]}{1\,\mathrm{mW}}$$

- the power depends on how you bin the frequency, so the unambiguous unit is dBm/Hz:

$$P[\mathrm{dBm/Hz}] = 10\log_{10}\frac{P[\mathrm{W}]}{1\,\mathrm{mW}} - 10\log_{10}\mathrm{RBW}$$

- ok, but what is the measured power? $P[\mathrm{W}] = \dfrac{U^2}{R}$
- what about the conversion to meters?

By the way...

- There is a similar technique, heterodyne detection
  - LO and signal of different frequencies
  - Pro: low technical noise, no balancing needed
  - Con: noise penalty (quadrature rotates in time because of frequency difference)

By the byway...

- SAY YOU WANT TO DETECT PARTICLES LIKE CHAMELEONS, WHY DO ANY OF THESE TECHNIQUES WORK (AT LEAST IN PRINCIPLE)?

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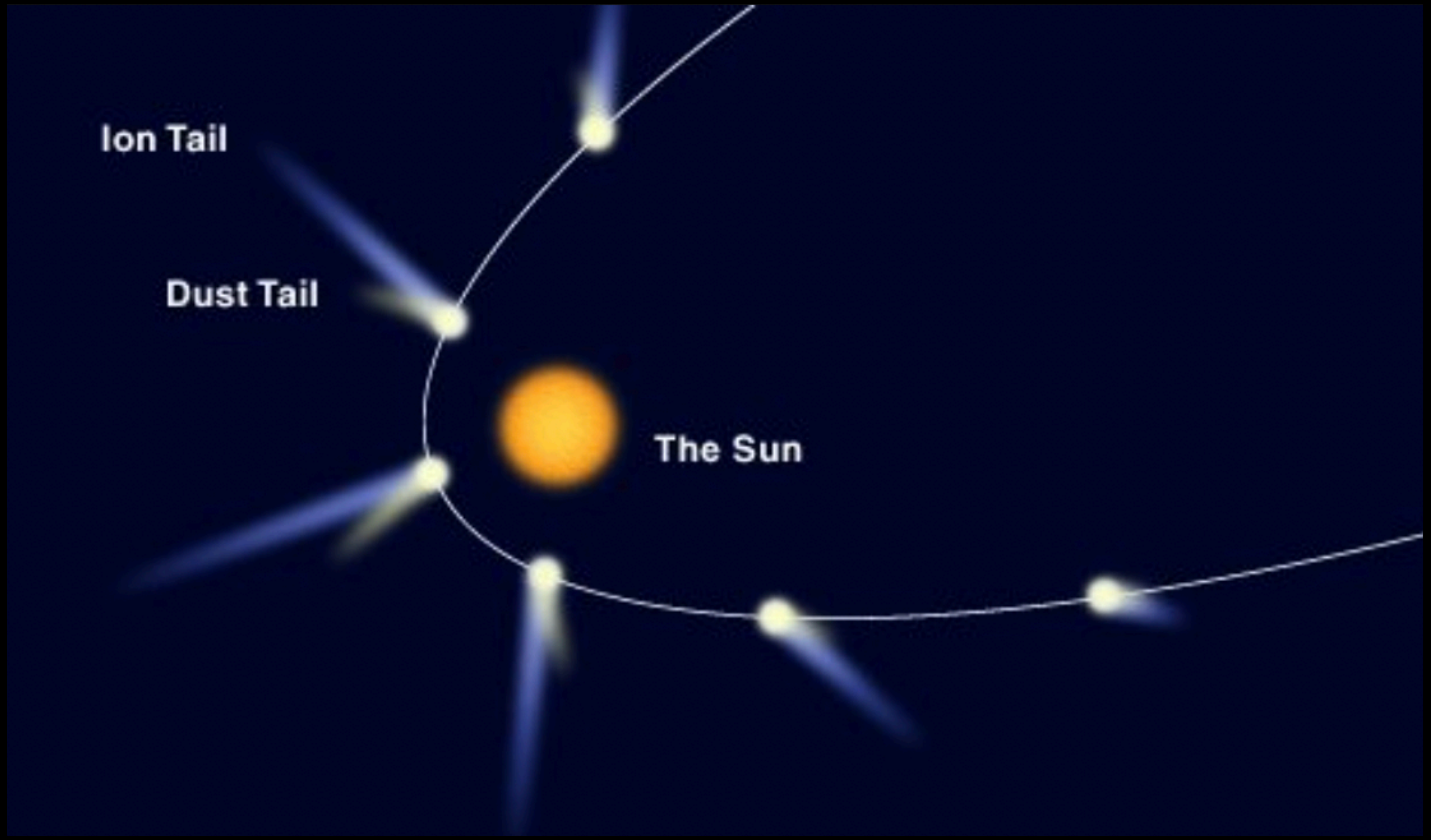

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- RADIATION PRESSURE
- OPTOMECHANICS

By the byway...

- HOW CAN YOU IMPROVE SENSITIVITY?

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