Lecture 1: ULDM

Definition and gravitational signatures

Elisa G. M. Ferreira

Kavli IPMU

3rd Training School COST Action *“Cosmic WISPers"*
September 18, 2025

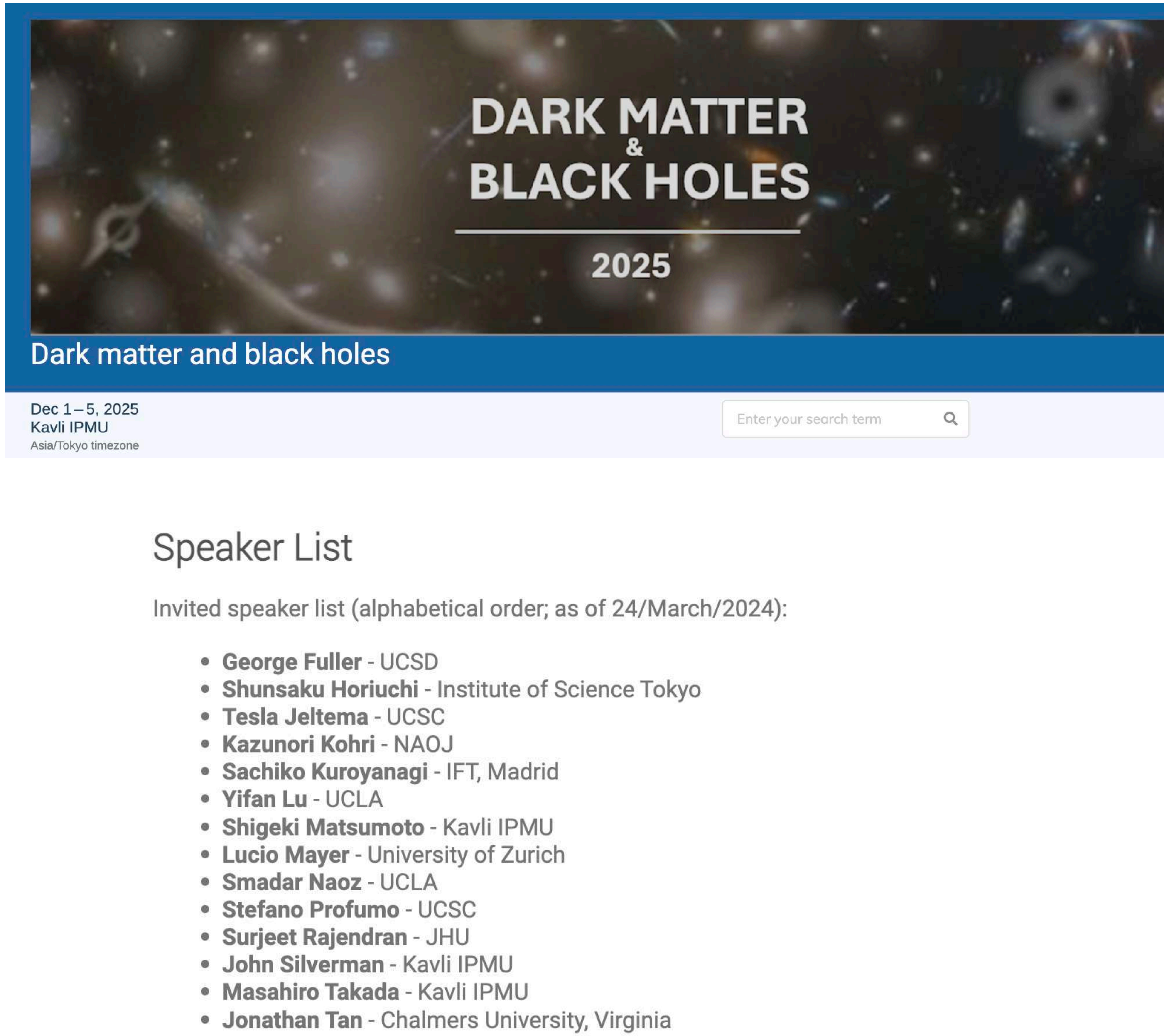
Image:NASA, ESA, CSA, and STScI

Axions in Japan

Nov 10 – 14, 2025
Kavli IPMU
Asia/Tokyo timezone

Invited speakers

- Andrey Kravtsov
- Atsushi Nishizawa
- Chanda Prescod-Weinstein
- Cora Uhlemann
- Francesca Chadha-Day
- Ippei Obata
- Jens Niemeyer
- Keir Rogers
- Masahiro Kawasaki
- Mustafa Amin
- Neal Dalal
- Philip Mocz
- Richard Easther
- Simona Vegetti
- Tomohiro Fujita
- Vera Gluscevic
- Yuko Urakawa
- Yuta Michimura



Dark matter and black holes

Dec 1 – 5, 2025
Kavli IPMU
Asia/Tokyo timezone

Speaker List

Invited speaker list (alphabetical order; as of 24/March/2024):

- **George Fuller** - UCSD
- **Shunsaku Horiuchi** - Institute of Science Tokyo
- **Tesla Jeltema** - UCSC
- **Kazunori Kohri** - NAOJ
- **Sachiko Kuroyanagi** - IFT, Madrid
- **Yifan Lu** - UCLA
- **Shigeki Matsumoto** - Kavli IPMU
- **Lucio Mayer** - University of Zurich
- **Smadar Naoz** - UCLA
- **Stefano Profumo** - UCSC
- **Surjeet Rajendran** - JHU
- **John Silverman** - Kavli IPMU
- **Masahiro Takada** - Kavli IPMU
- **Jonathan Tan** - Chalmers University, Virginia

Registration open!!!

A little bit about me…

I am originally from Brazil

Currently: Assistant Professor at Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo

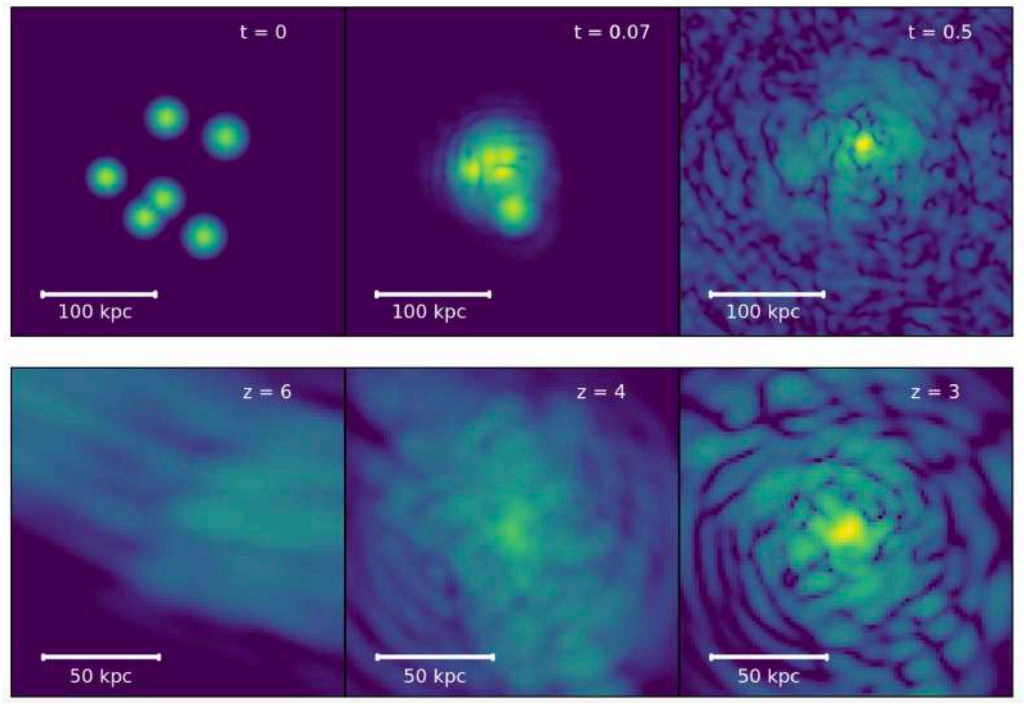
KAVLI IPMU
INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

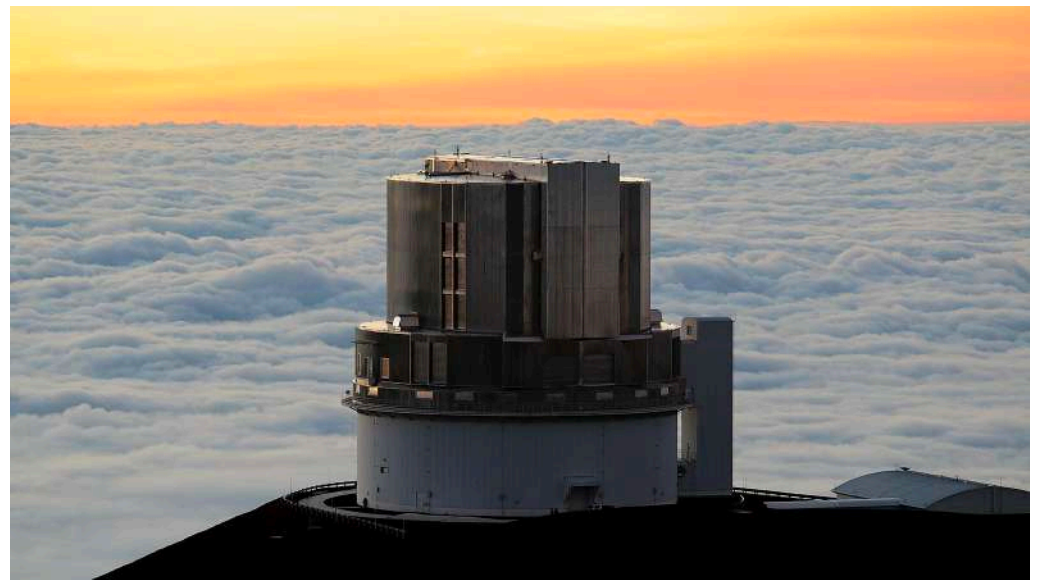
<ins>My research:</ins>

Theoretical cosmology

- Early universe
- Dark energy
- Dark matter
    - Ultra-light DM, axions

I also use observational data to test cosmological models and simulations.





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Currently: Assistant Professor at Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo

<u>Tutor: Andrew Eberhardt</u>

Kavli fellow at Kavli IPMU

Outline

Lecture 1: Cosm. Signatures

Part I:

- Evidences of DM
- DM model building
- DM models

Part II:

- ULDM definition
- ULDM models
- ULDM dynamics
- Observational signatures

Lecture 2: ULDM bounds

- ULDM gravitational bounds
- Interaction of ULDM with SM
    - Axion/ALPs interaction in astrophysical systems
    - Direct detection
- DM Superfluid

Disclaimer

- Impossible to cover all the possible searches there are now! But I will do my best to give a general view.
- Biased review of the ULDM field
- Field that is changing rapidly, so my apologies for not mentioning your model or reference
- Lectures are going to have a practical component. Hope it is useful in your research!

Units of mass, energy and momentum = eV
Length = eV^{-1}

←

Natural units$(c = \bar{h} = 1)$
$$1\,\mathrm{kg} \to 5 \times 10^{35}\,\mathrm{eV}$$
$$1\,M_\odot \to\sim 10^{66}\,\mathrm{eV}$$

BUT sometimes (astro/cosmology)
1parsec (pc) $\sim 3 \times 10^{16}\,\mathrm{m}$

Further reading

Reviews!!!

Main reference for gravitational searches:

- **Elisa Ferreira**, *Ultra-light dark matter*, The Astronomy and Astrophysics Review. 29 (2021) 1, 7, arXiv:2005.03254
- Andrew Eberhardt and Elisa Ferreira, *Ultralight fuzzy dark matter review*, arXiv: 2507.00705

Other very good reviews

- Lam Hui, *Wave dark matter,* Ann.Rev.Astron.Astrophys. 59 (2021) 247-289, arXiv: 2101.11735
- Jens C., *Niemeyer Small-scale structure of fuzzy and axion-like dark matter*, Prog.Part.Nucl.Phys. 103787, arXiv:1912.07064
- David Marsh, *Axion cosmology*, Phys.Rept. 643 (2016) 1-79, arXiv:1510.07633

Reference for non-gravitational searches:

- Francesca Chadha-Day et al., *Axion dark matter: What is it and why now?*, Sci.Adv. 8 (2022) 8, abj3618, arXiv: 2105.01406

+ many references *(in the slides)*

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Part I: dark matter

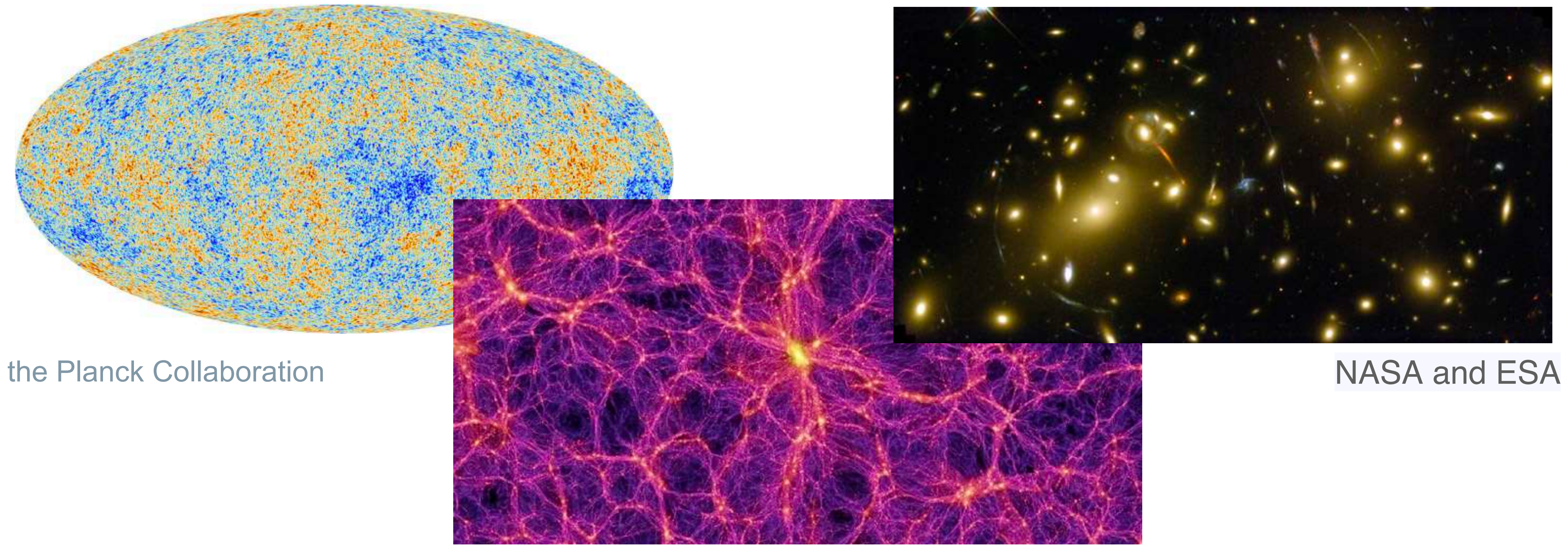
Evidences for dark matter

We can observe its effects in

Galaxies

NASA and ESA

CMB+LSS

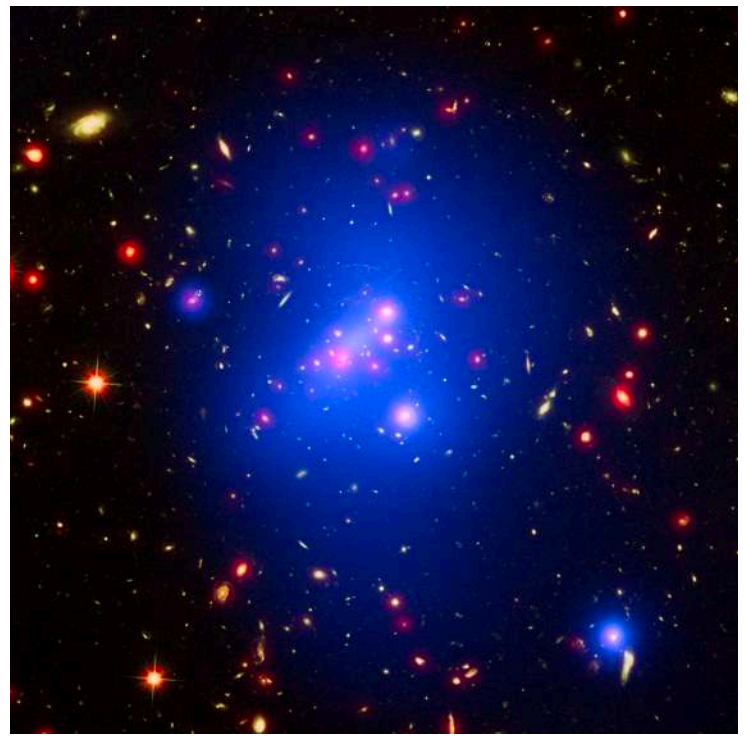


ESA and the Planck Collaboration

NASA and ESA

Springel & others / Virgo Consortium

Clusters

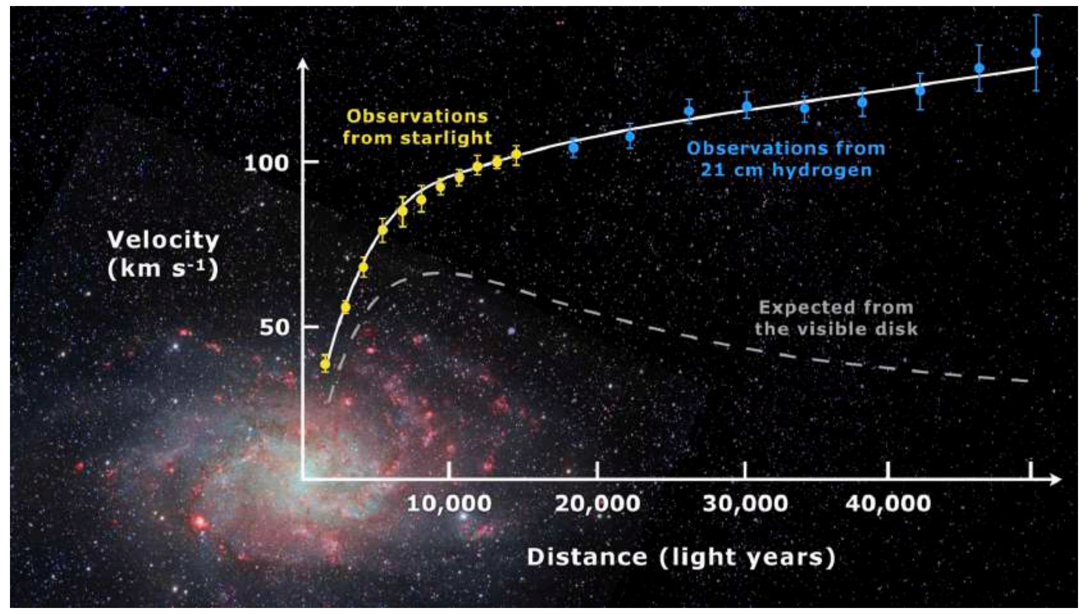


CC BY 4.0

Huge amount of evidence
From all scales

Evidences for dark matter - *properties*

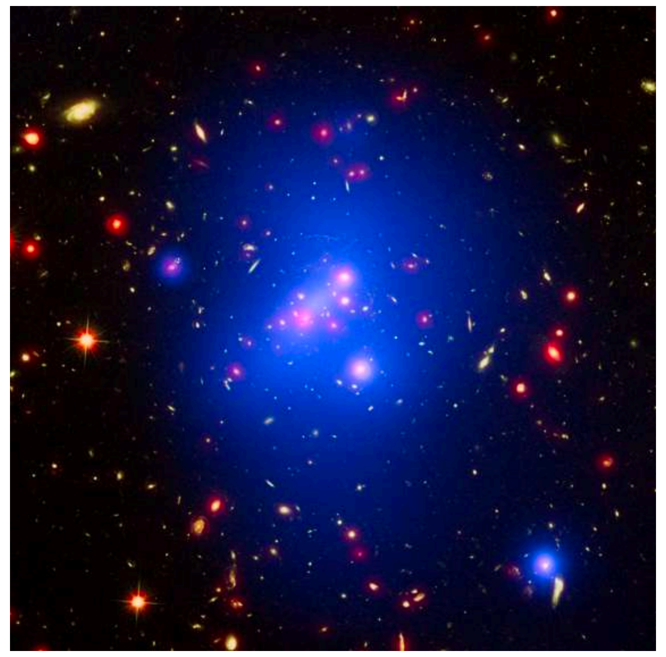
Galaxy rotation curves



Credit: Mario De Leo

- Mass fraction
- Distribution

Clusters



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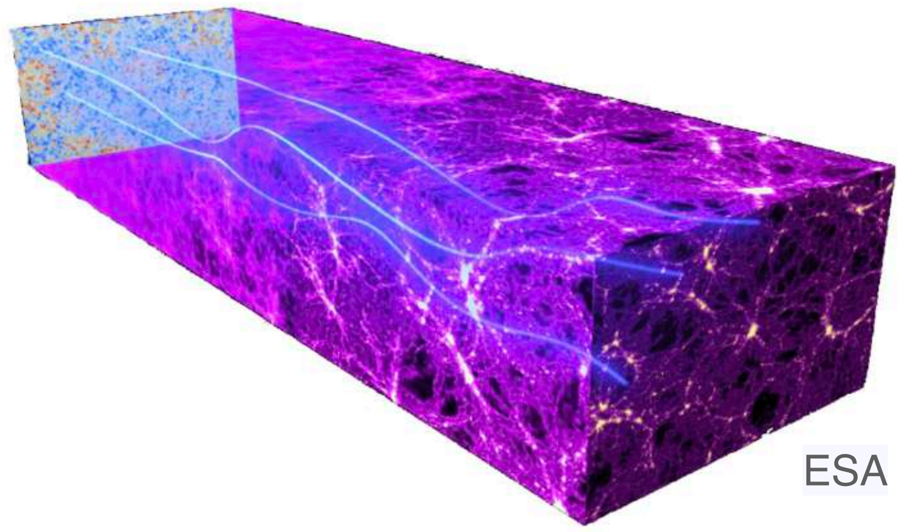
- Mass fraction
- Distribution

Lensing



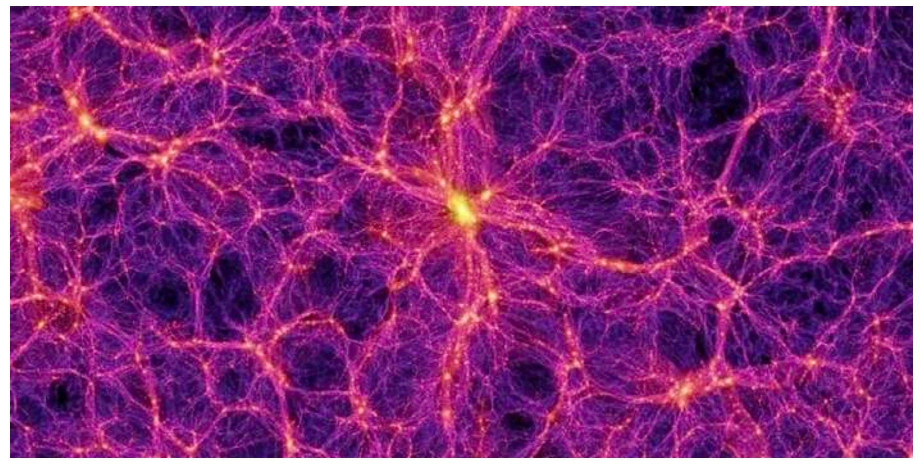
ESA

Strong lensing
- Mass fraction
- Distribution

Weak lensing
- Distribution
- Shape
- Structure

Micro lensing
- Mass fraction
- Smoothness

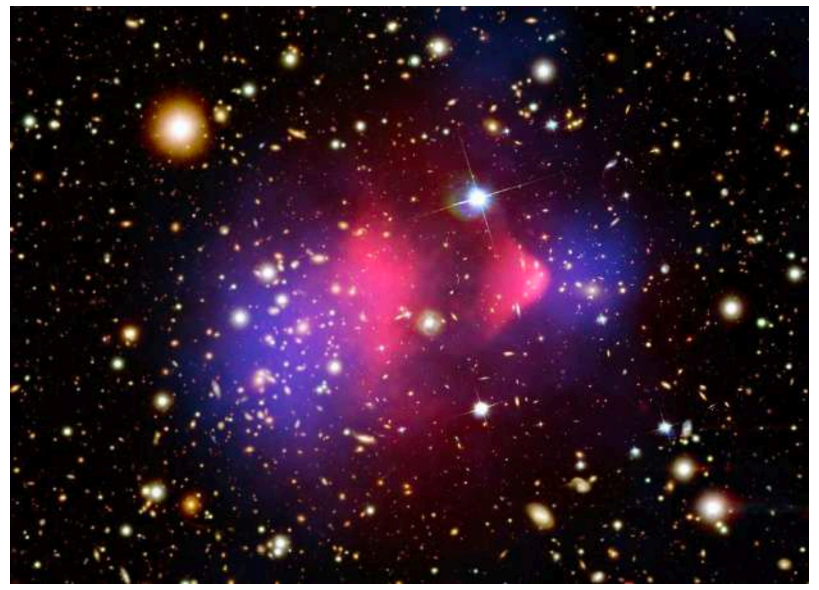
Large Scale Structure



Springel & others / Virgo Consortium

CMB/LSS
- Ratio of DM/collisional matter
- Thermal history

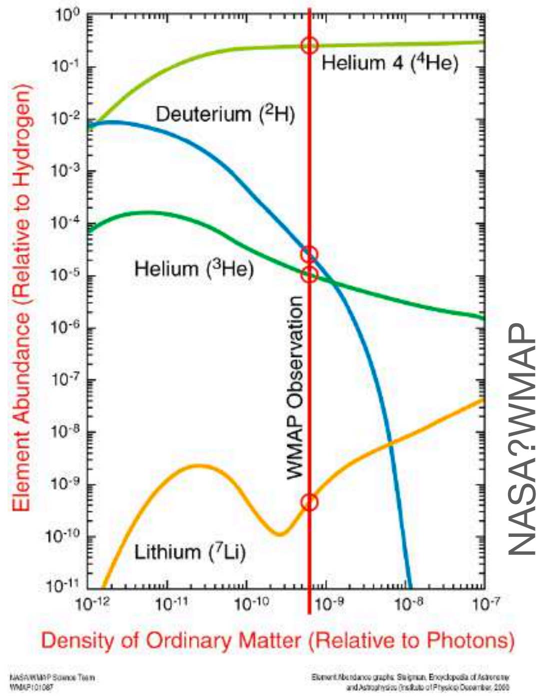
Cluster collision



NASA/CXC/CfA and NASA/STScI

- Distribution
- Separation from collisional matter
- Self-interaction

Big Bang Nucleosynthesis



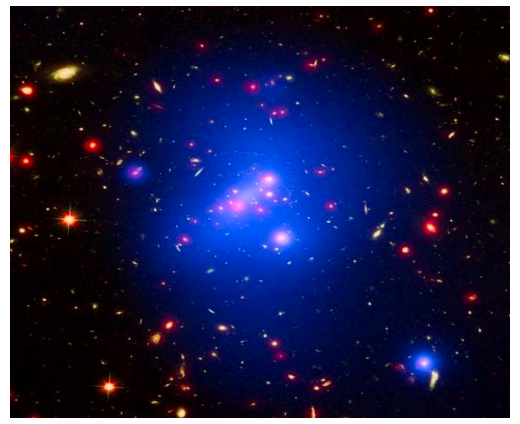
NASA?WMAP

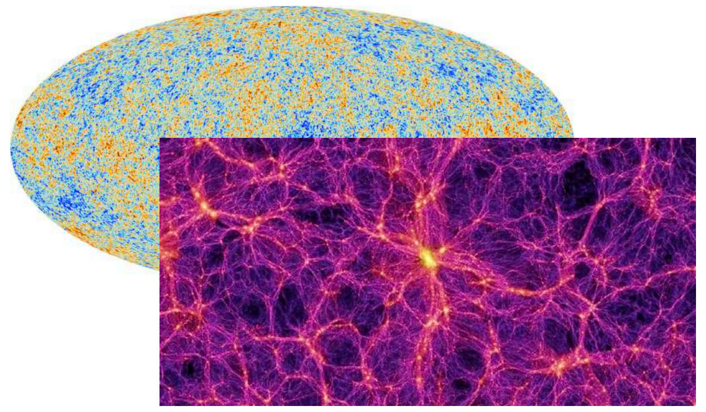
- Amount of baryons

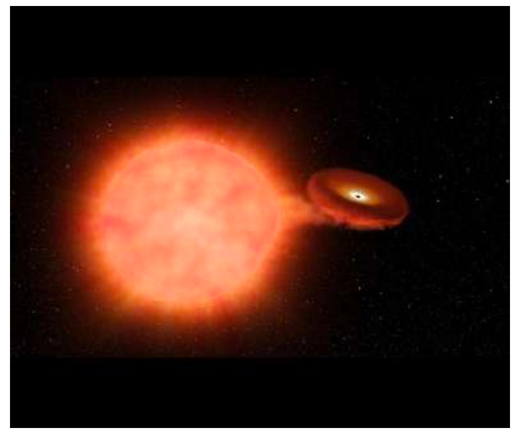
Based on K. Mack

What we know about dark matter


Galaxies

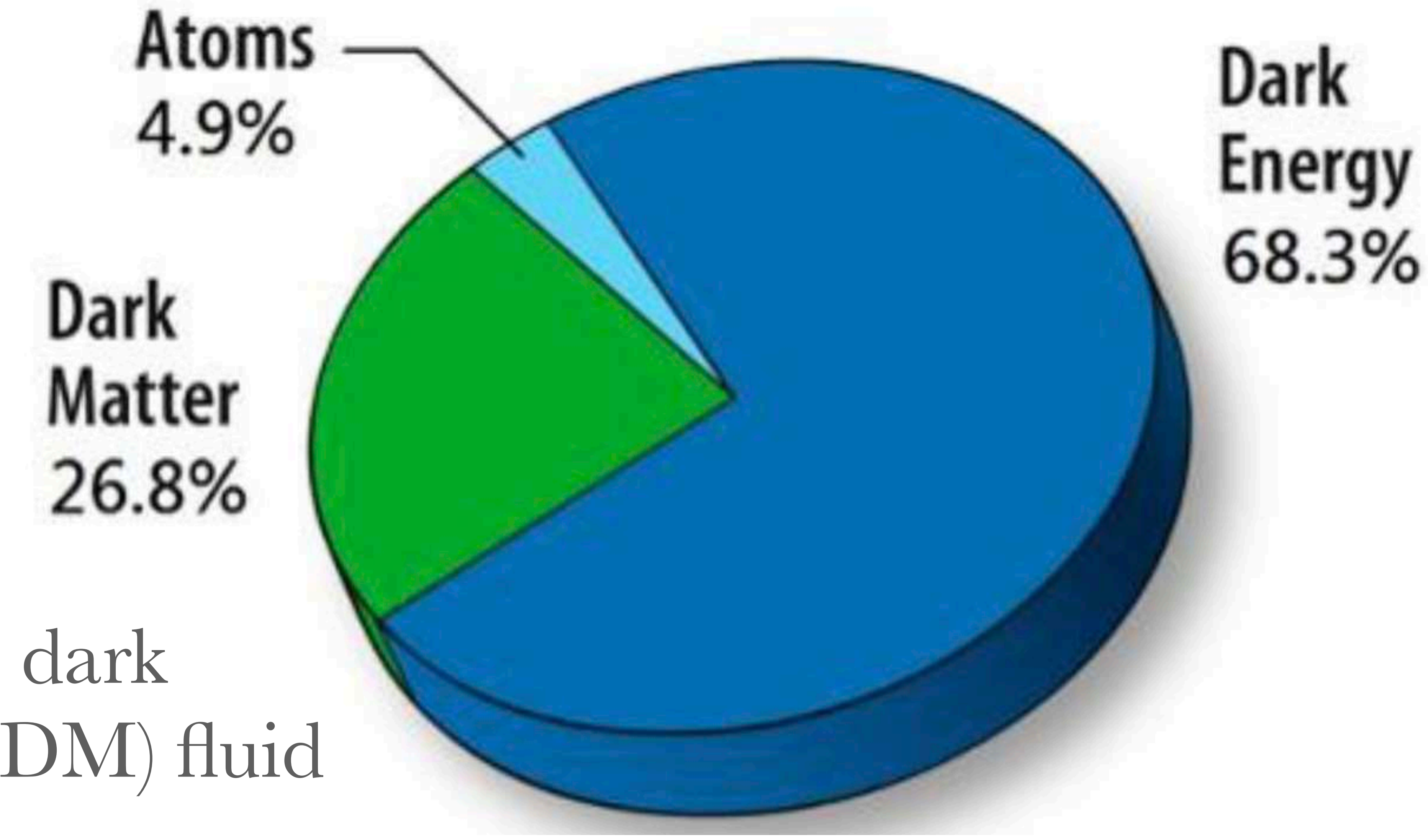

Clusters


Large scales


Sn Ia

ΛCDM – the standard cosmological model

Successful description of our universe with 6 free parameters, tested to sub-percent precision.



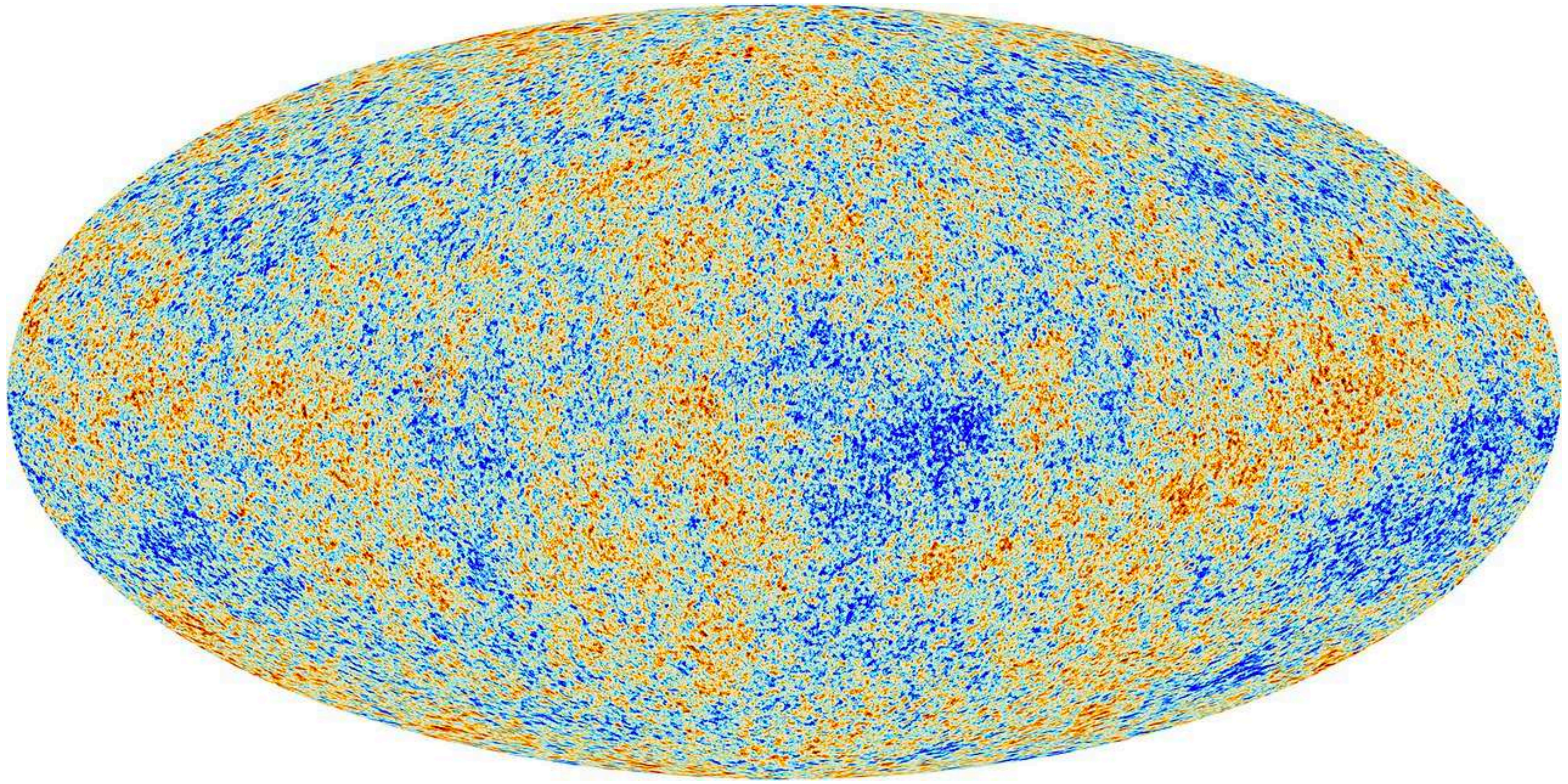
Λ

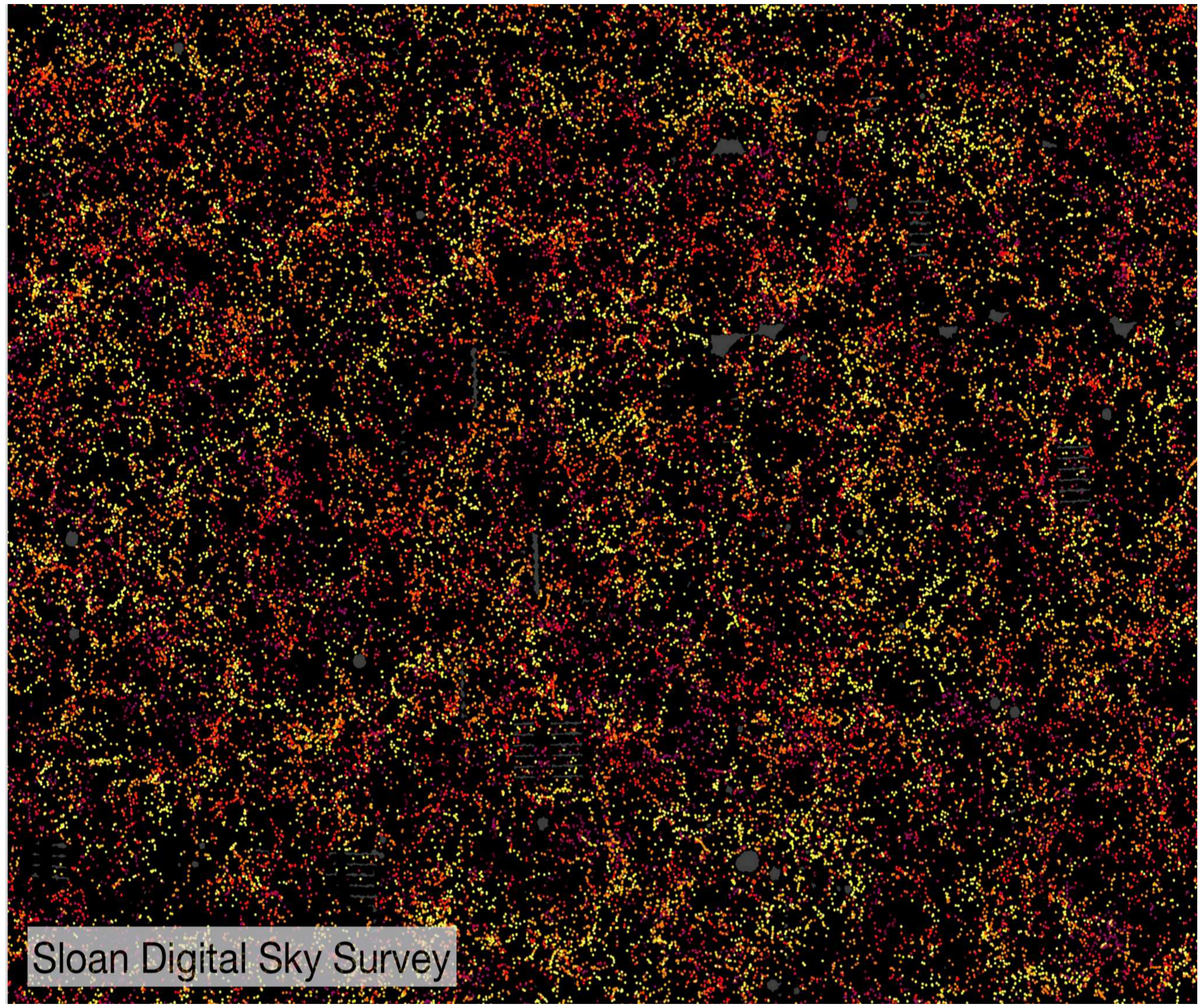
DM: cold dark
matter (CDM) fluid

Planck 2018

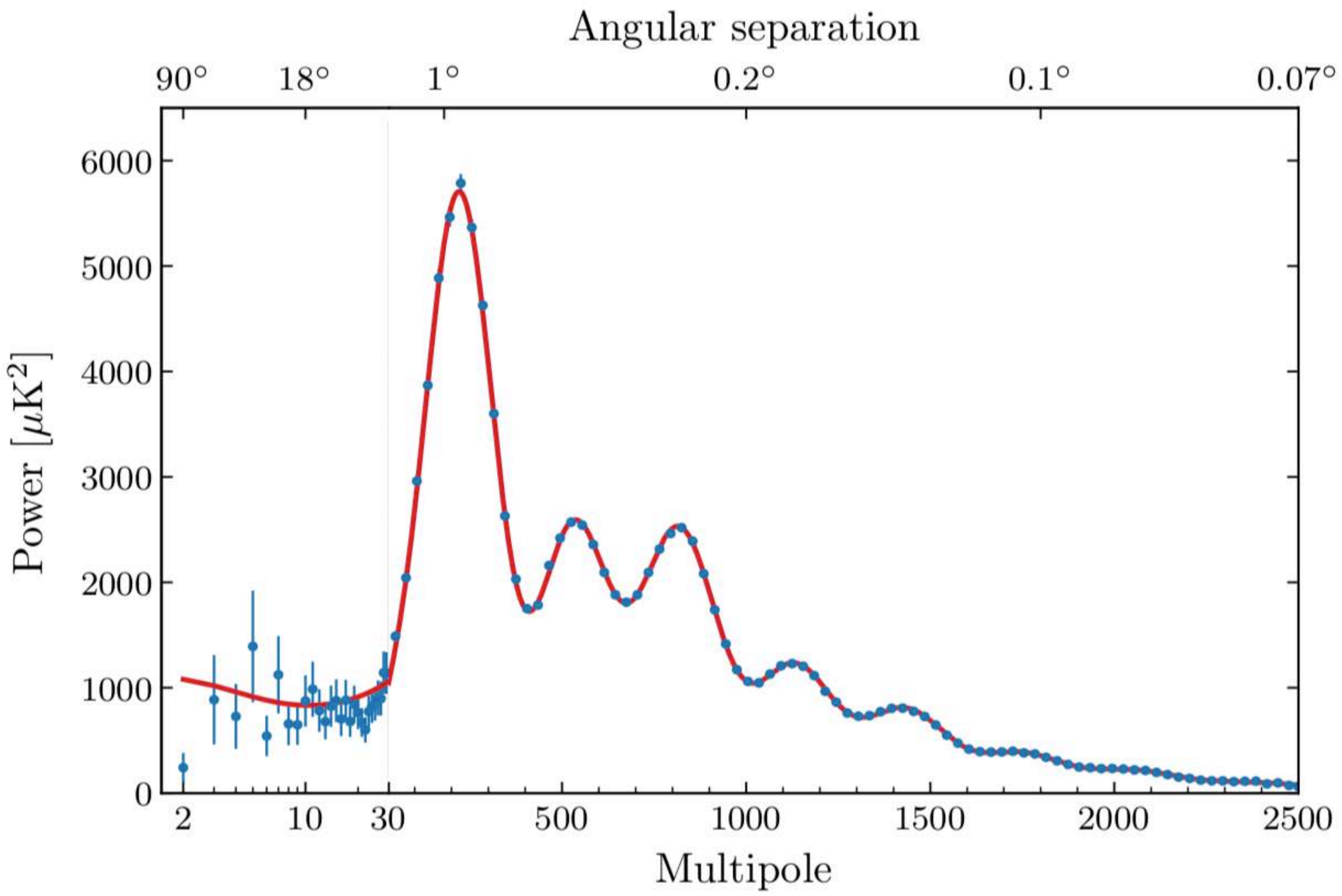
$$\Omega_b = 0.0484 \pm 0.0003$$
$$\Omega_m = 0.308 \pm 0.012$$

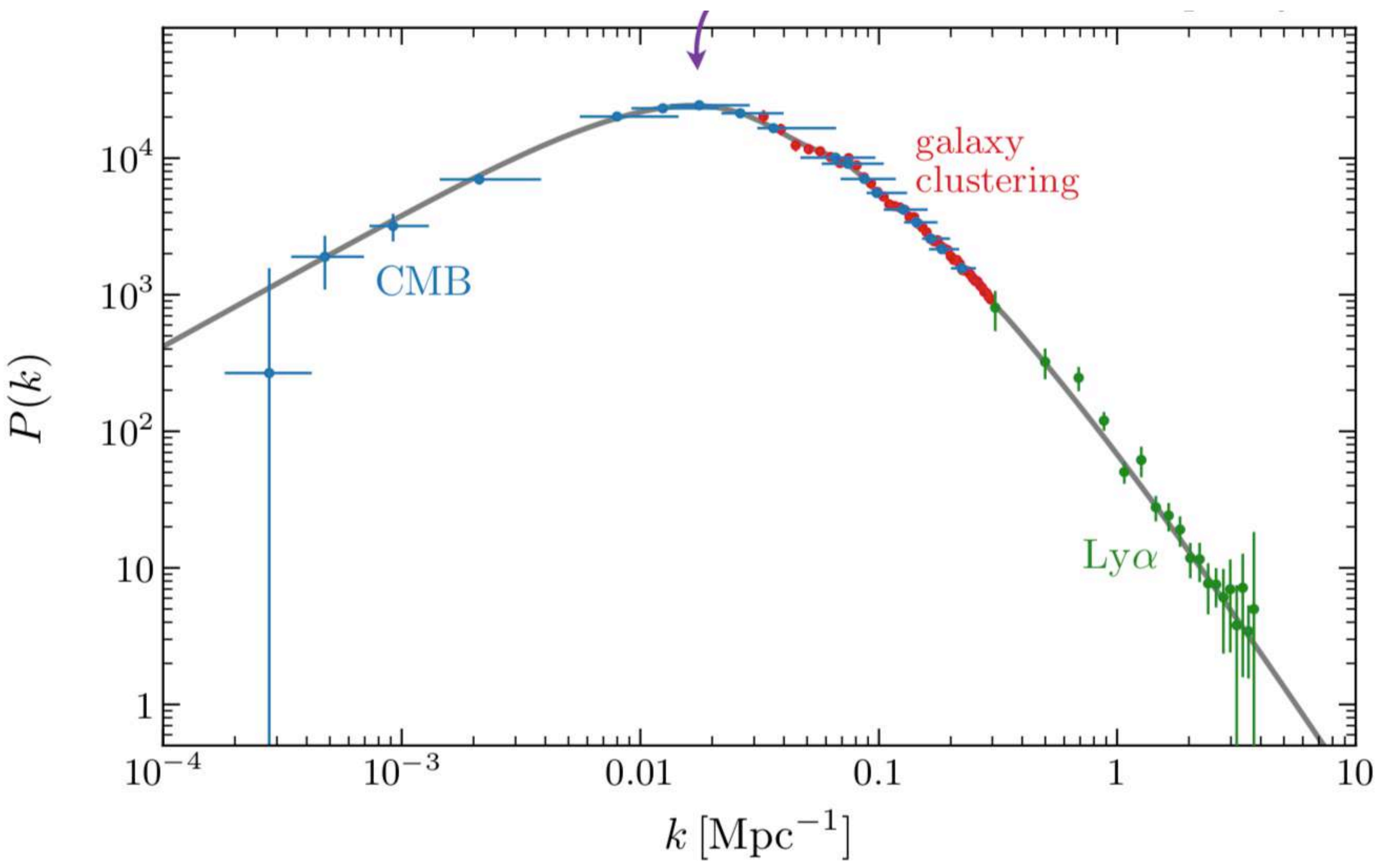
Large scale structure





Large scale structure





$$\Omega_m = 0.308 \pm 0.012$$

(Planck 2018)

Cold dark matter

- Cold: moves much slower than *c*
- Presureless: gravitational attractive, clusters
- Dark (transparent): no/weakly electromagnetic interaction
- Collisionless: no/weakly self-interaction or interaction with baryons

- Abundance: amount of dark matter today known

CDM on large scales described
by a ***perfect fluid***:

Backg.: ρ, P

$w = P/\rho$

with $P = 0 \Rightarrow w = 0$

$\Rightarrow \rho \propto a^{-3}$

Pert.: δ, θ

with $c_s \sim 0$

Cold dark matter

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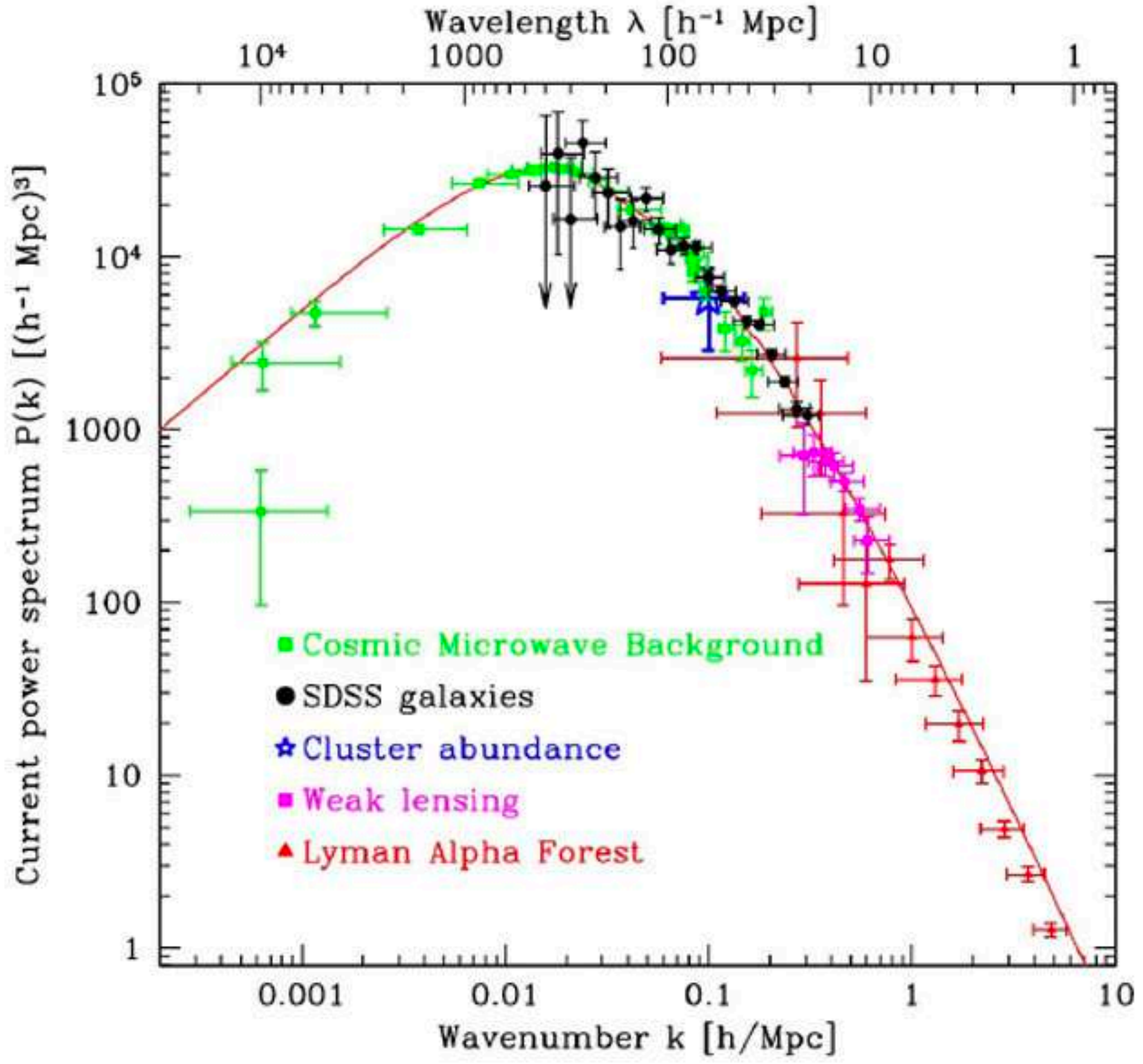


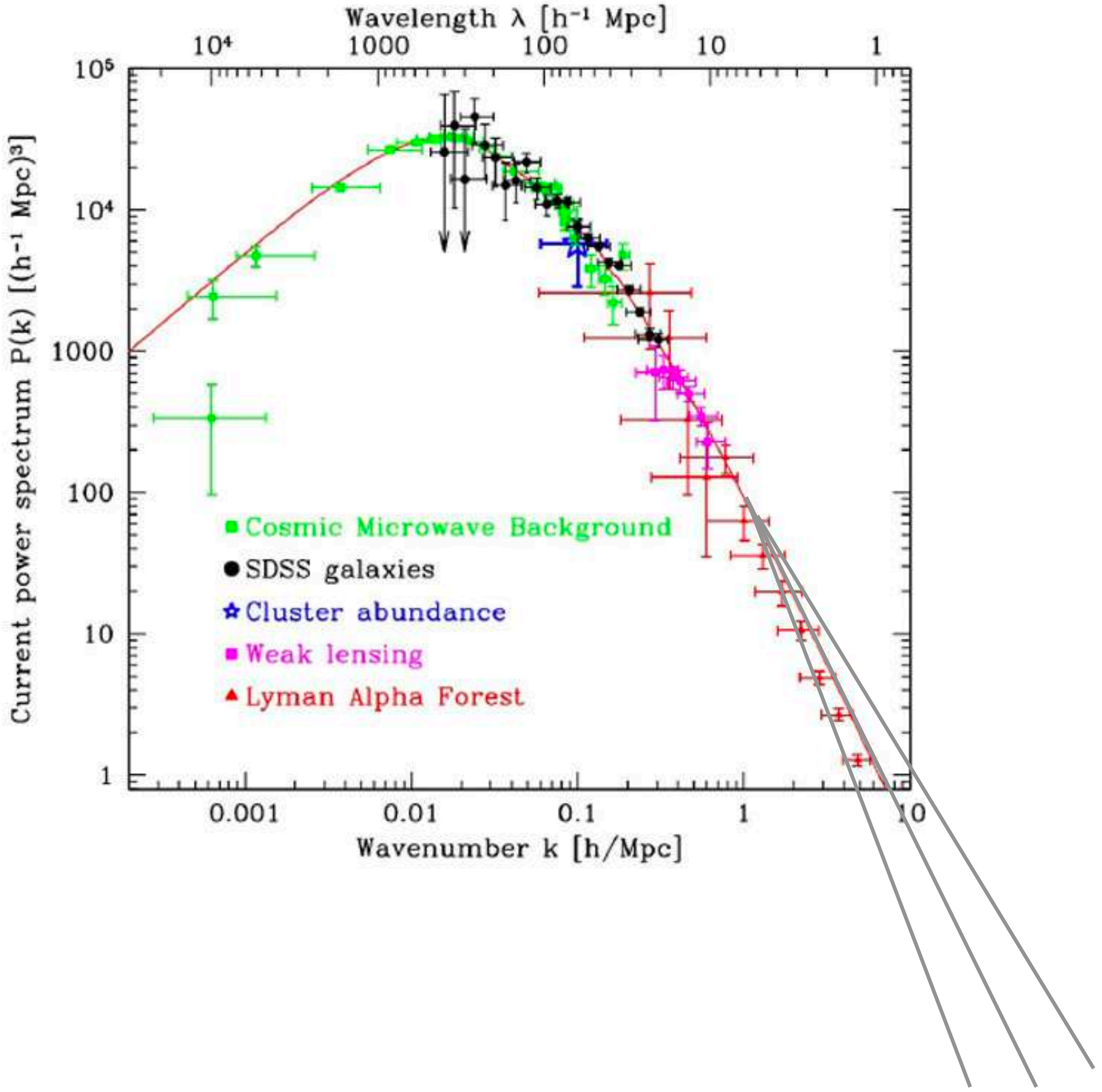
Many observational probes for $k \sim 10^{-3} - 10\,\mathrm{Mpc}^{-1}$

range of redshift $z < 3 - 4$

Incredible agreement to CDM!

What we don't know about dark matter

- ~~Cold~~ → How cold it is? WDM $m \sim \text{keV}$
- ~~Pressureless~~ → Cluster on all scales?
- ~~Dark~~ → Non-gravitational interaction? Milicharged DM
- ~~Collisionless~~ → How small sefl-interaction? SIDM



CDM on large scales

Small scale behavior: still weakly constrained and small scale challenges

Small scale curiosities: cusp-core, missing satellites, BTFR, …

What we know about dark matter

What we learned from observations

Properties:

What we know about dark matter

What we learned from observations

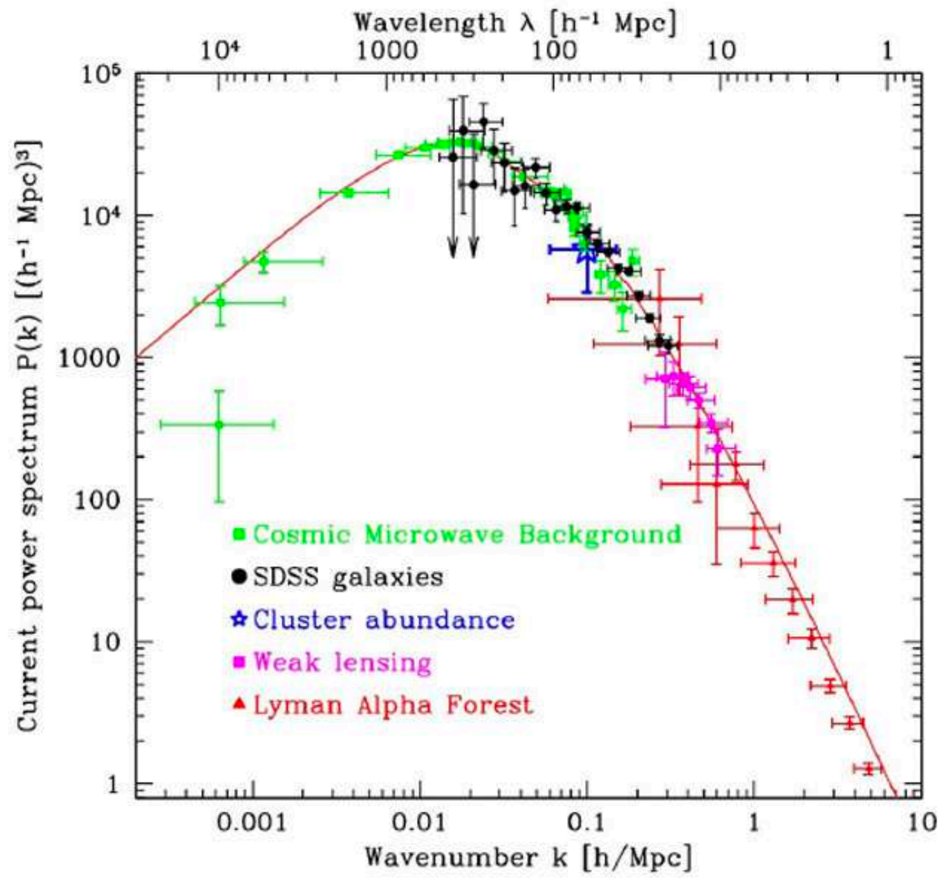
Properties:

- Cold
- Presureless

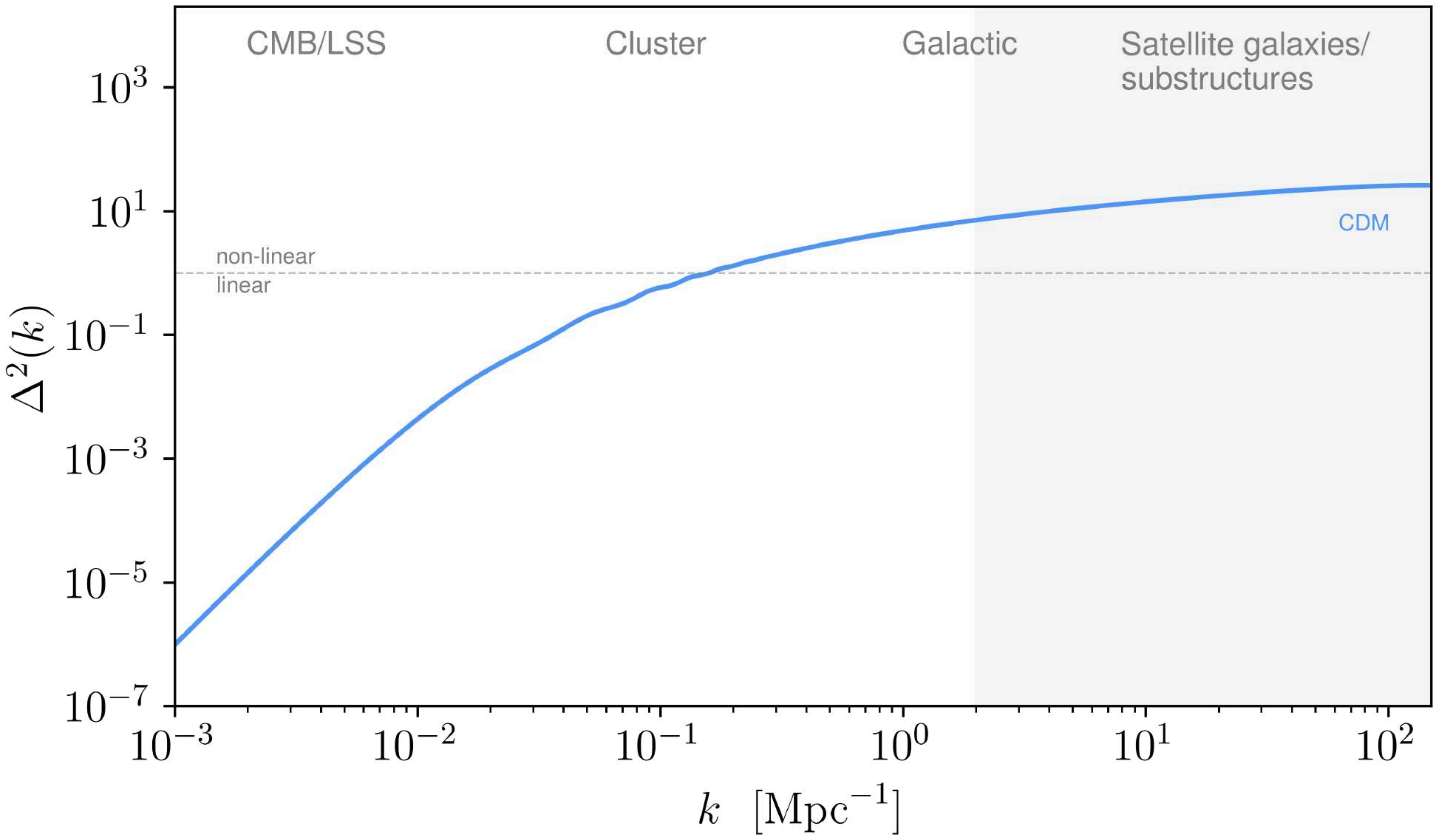
What we know about dark matter

Properties:

From LSS:



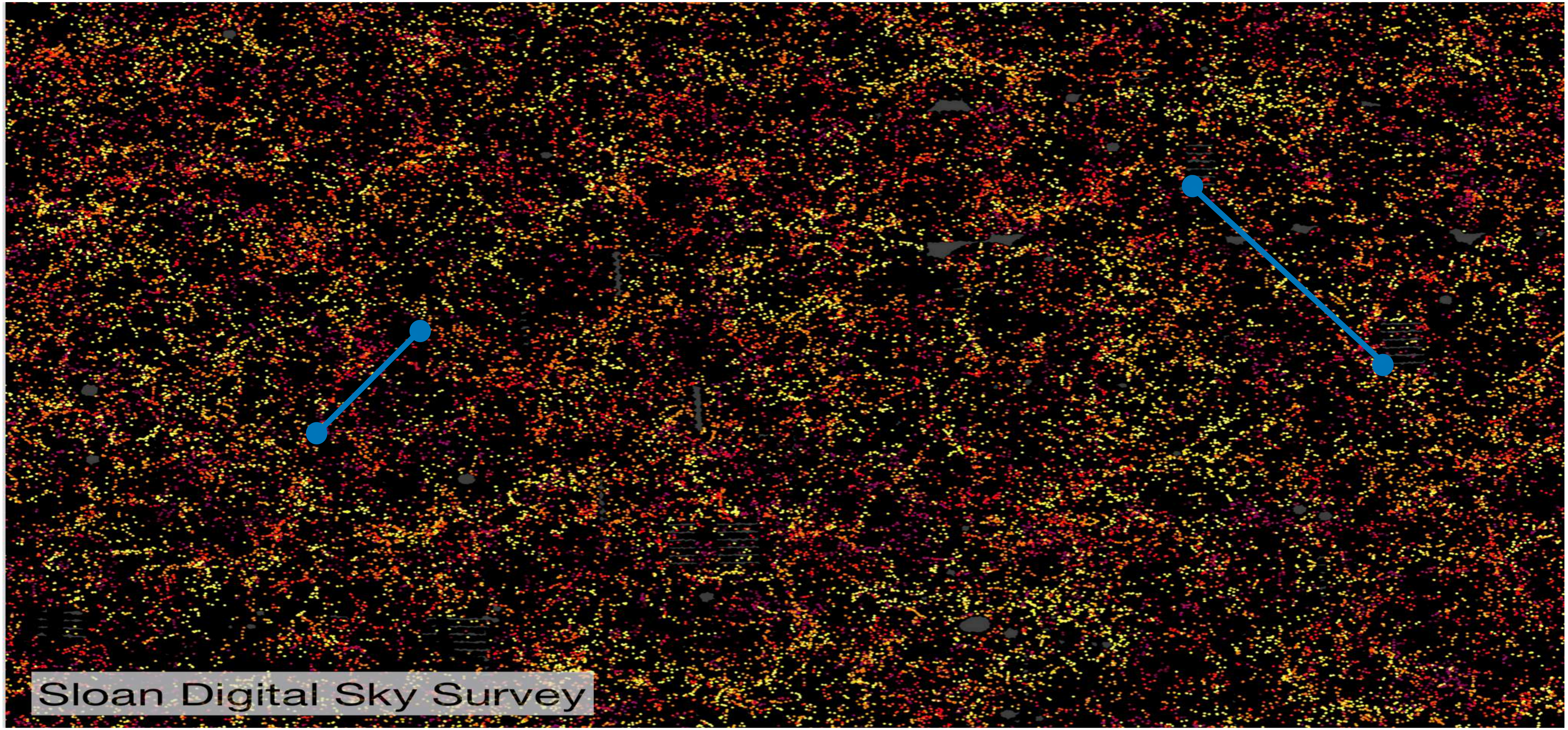
Measure PS well until scales $k \sim 10 - 20\,\mathrm{Mpc}^{-1}$



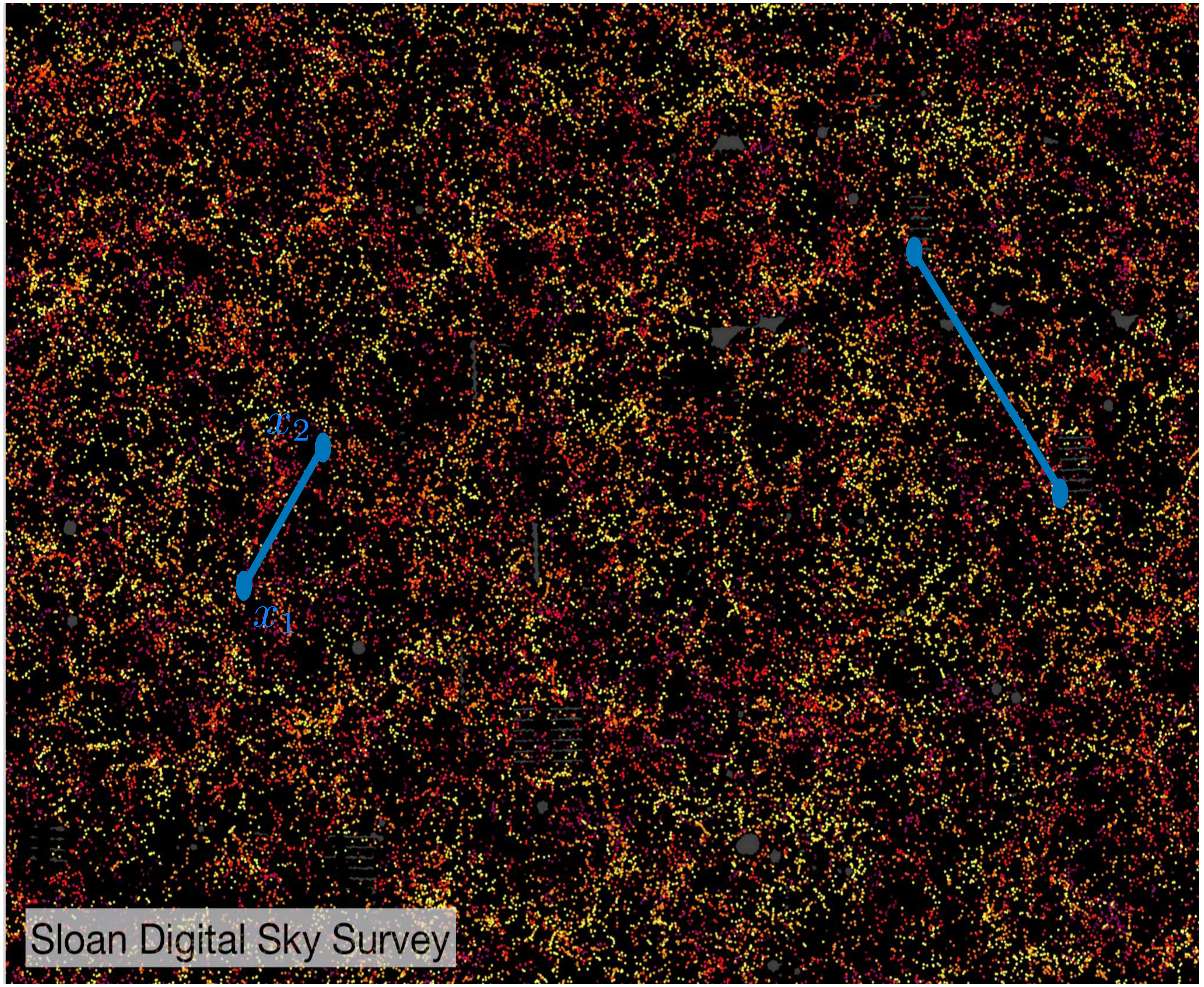
Dimensionless power spectrum

$$\Delta^2(k) = 4\pi(k/2\pi)^3 P(k)$$

How to measure *structures*



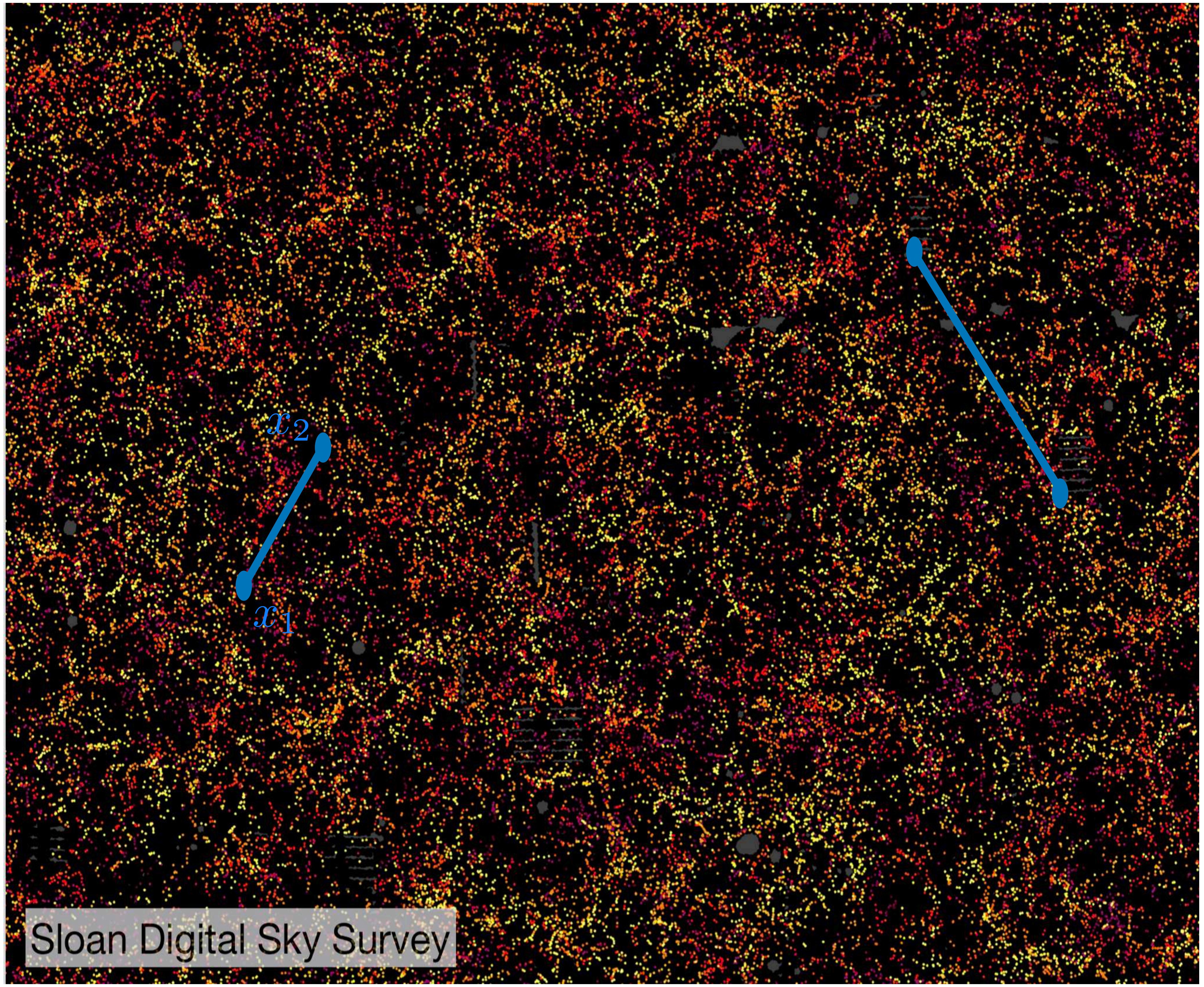
How to measure structures



2 point correlation function

$$\langle \delta(x_1)\delta(x_2) \rangle$$

How to measure structures



2 point correlation function

$$\langle \delta(x_1)\delta(x_2) \rangle$$

Decompose in Fourier modes:

$$\delta(x) = \sum_k \delta_k \, \sin(kx + \phi_k)$$

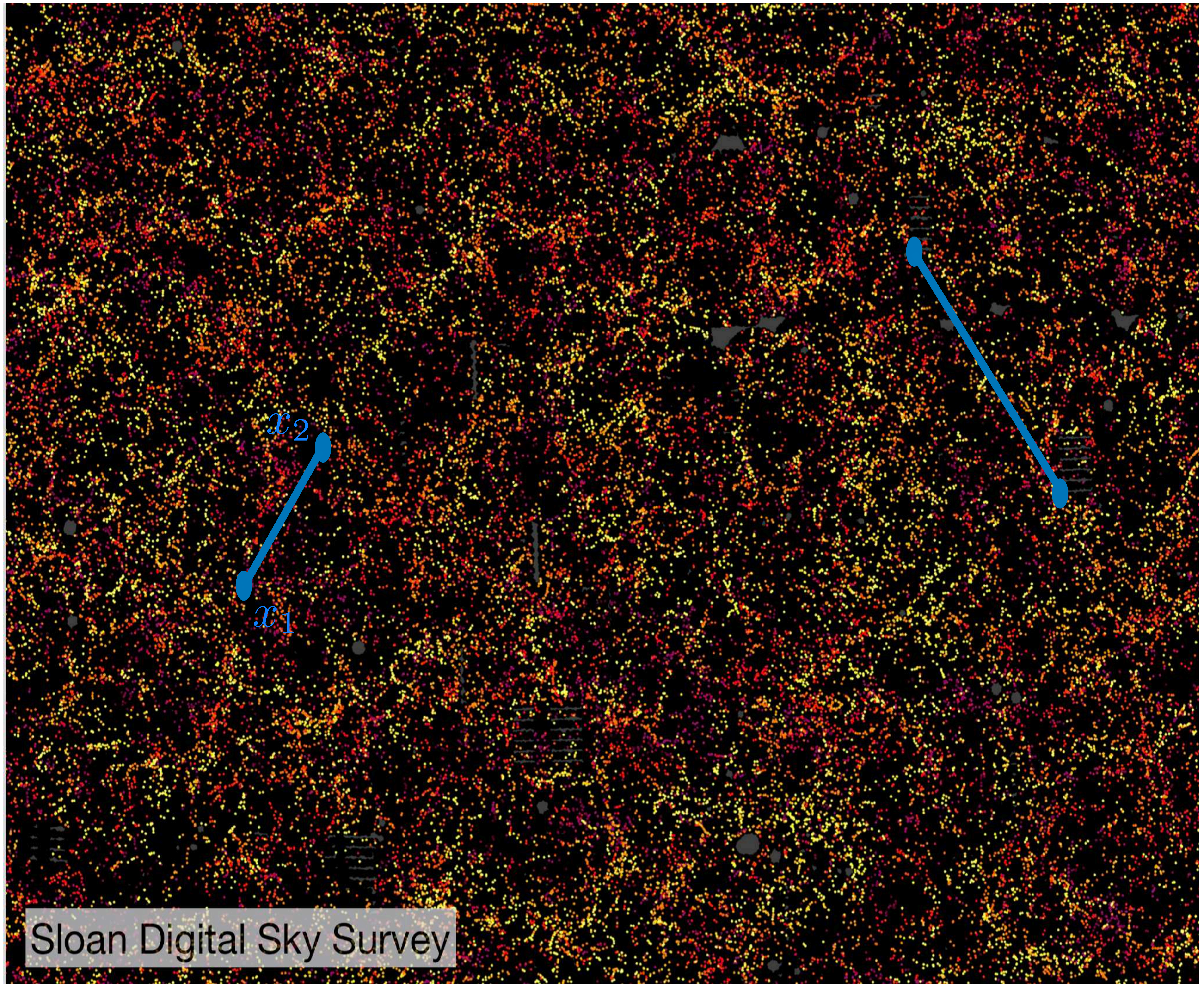
$$k = 2\pi/\lambda$$

$$\Longrightarrow \quad \boxed{P(k) = |\delta_k|^2}$$

Espectro de potências

Um dos principais objetos estatísticos da cosmologia!

How to measure structures



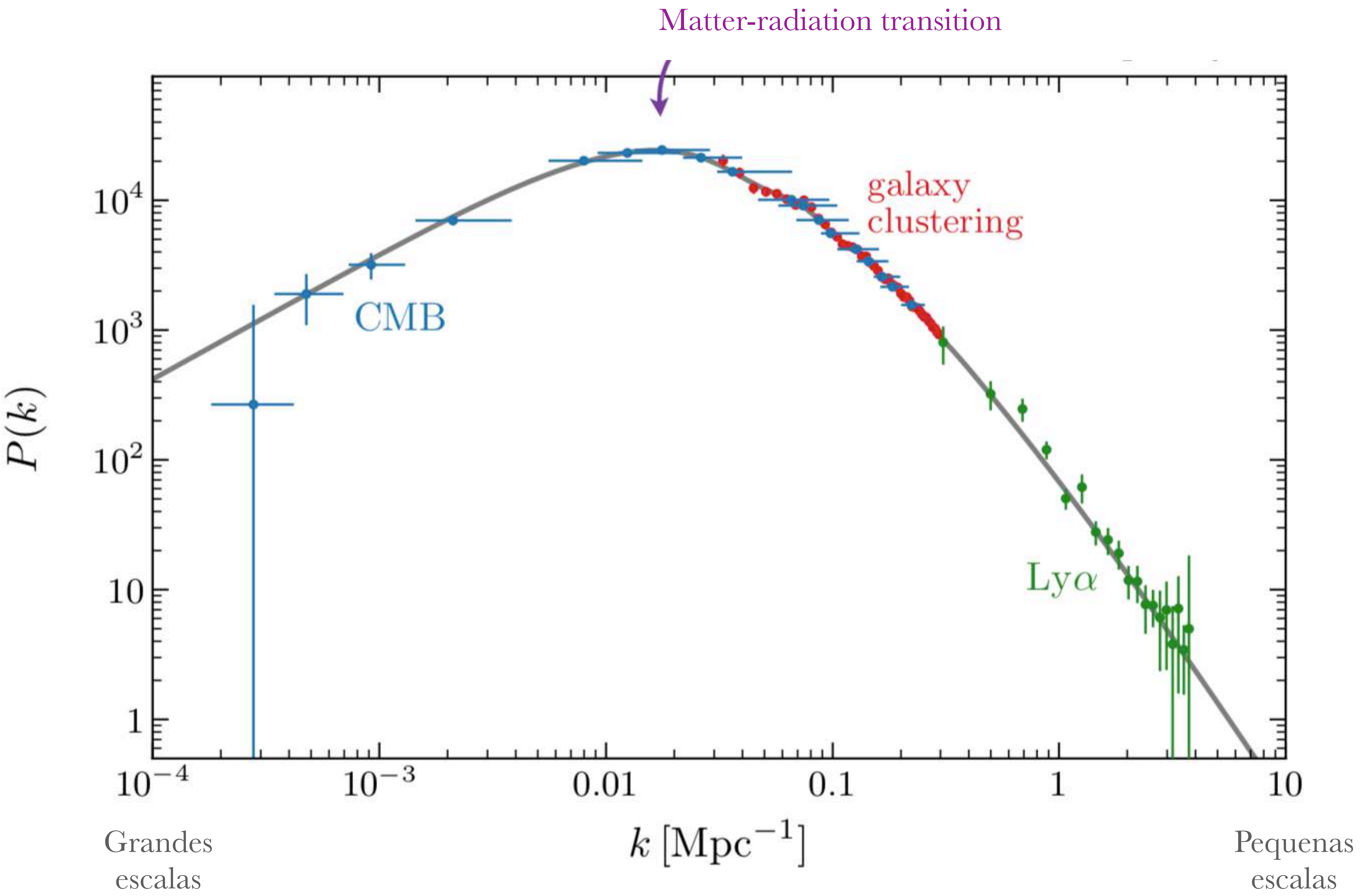
2 point correlation function

$$\langle \delta(x_1)\delta(x_2) \rangle$$

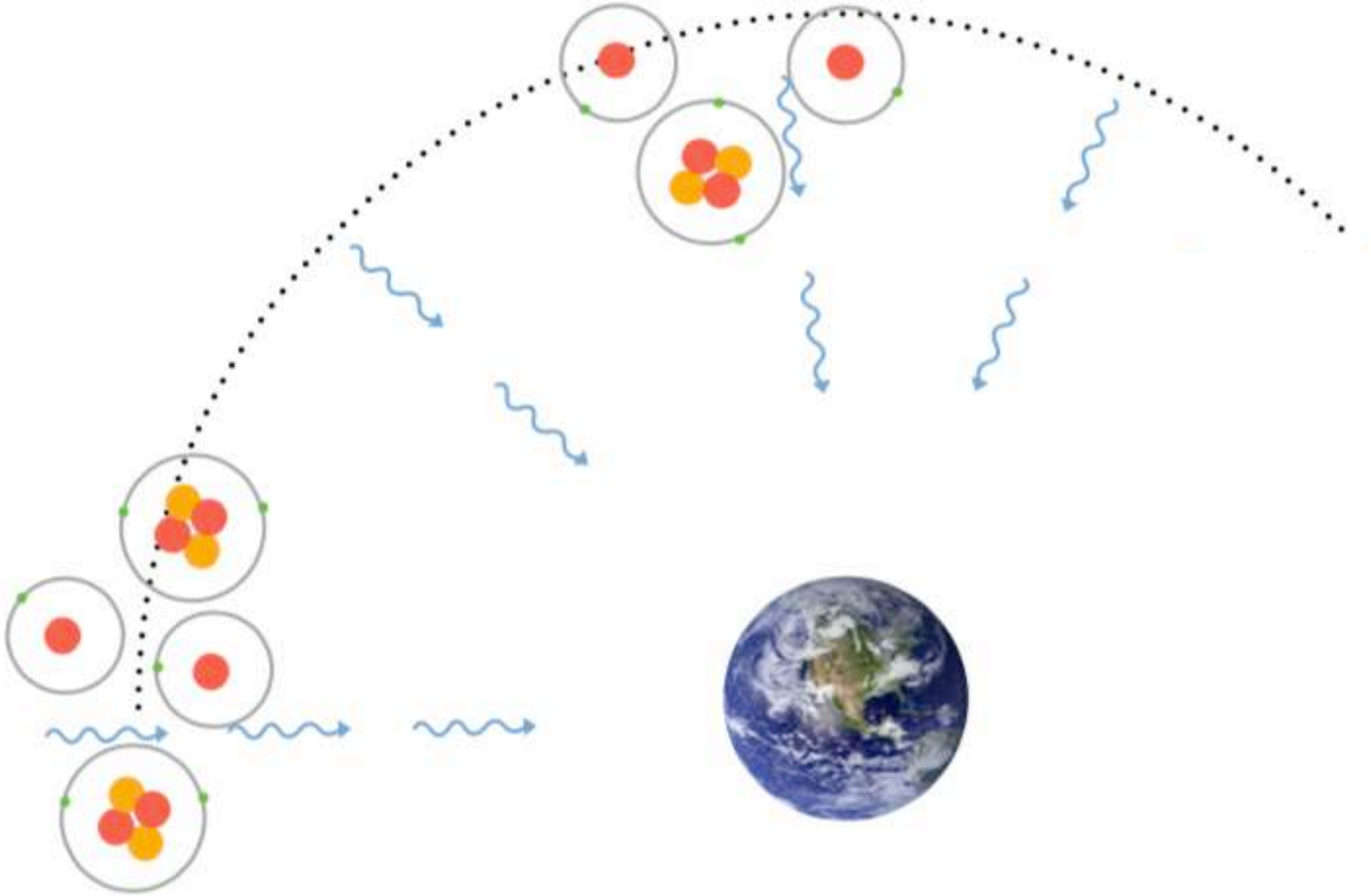
If Gaussian, all the information in the 2pt CF. If not, n-point correlation function:

$$\langle \delta\delta\delta \rangle \quad \langle \delta\delta\delta\delta \rangle \quad \langle \delta \dots \delta \rangle$$

Matter power spectrum



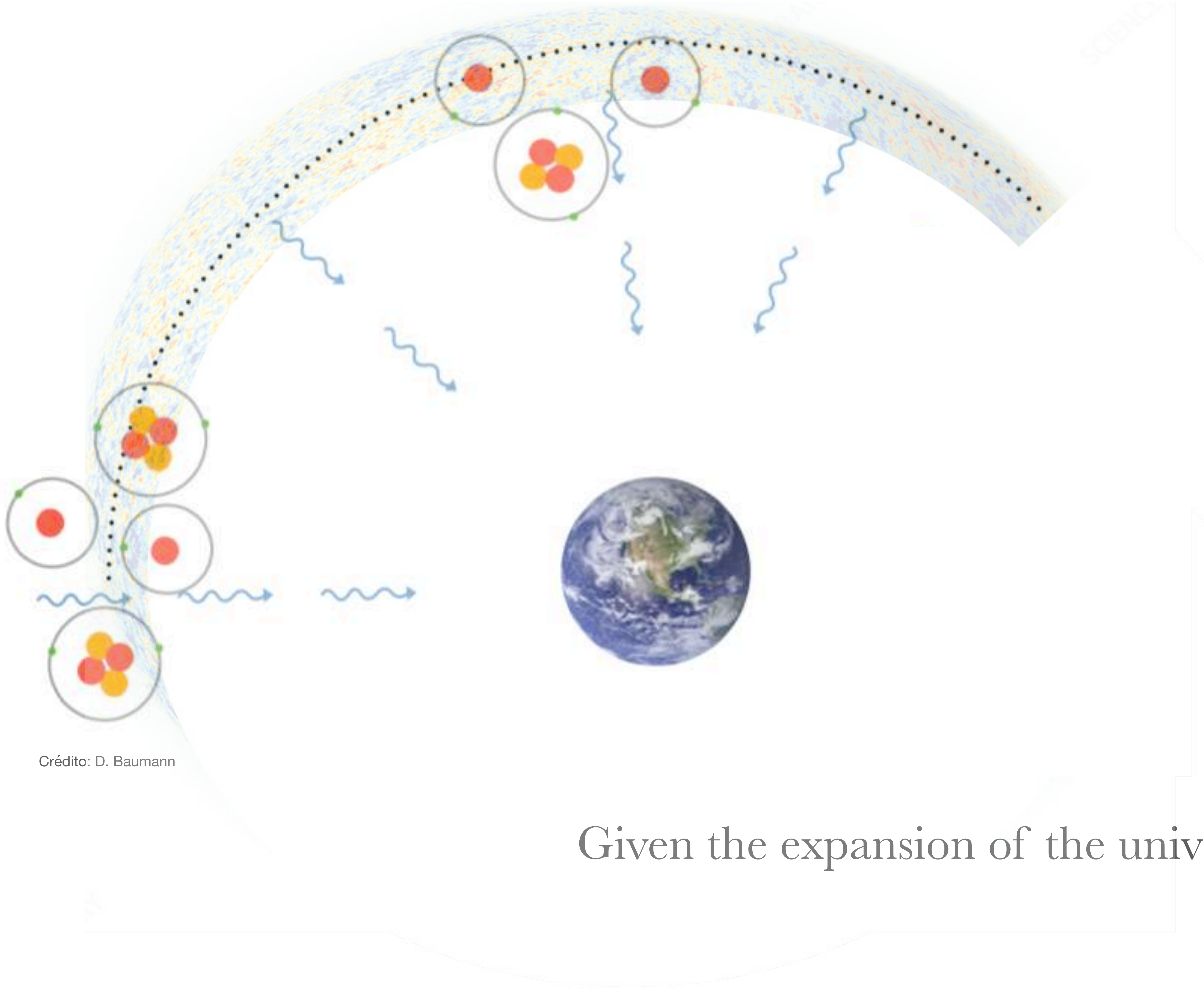
These photons are the first light of our universe…



Crédito: D. Baumann

… e tell us how the universe was at early times.

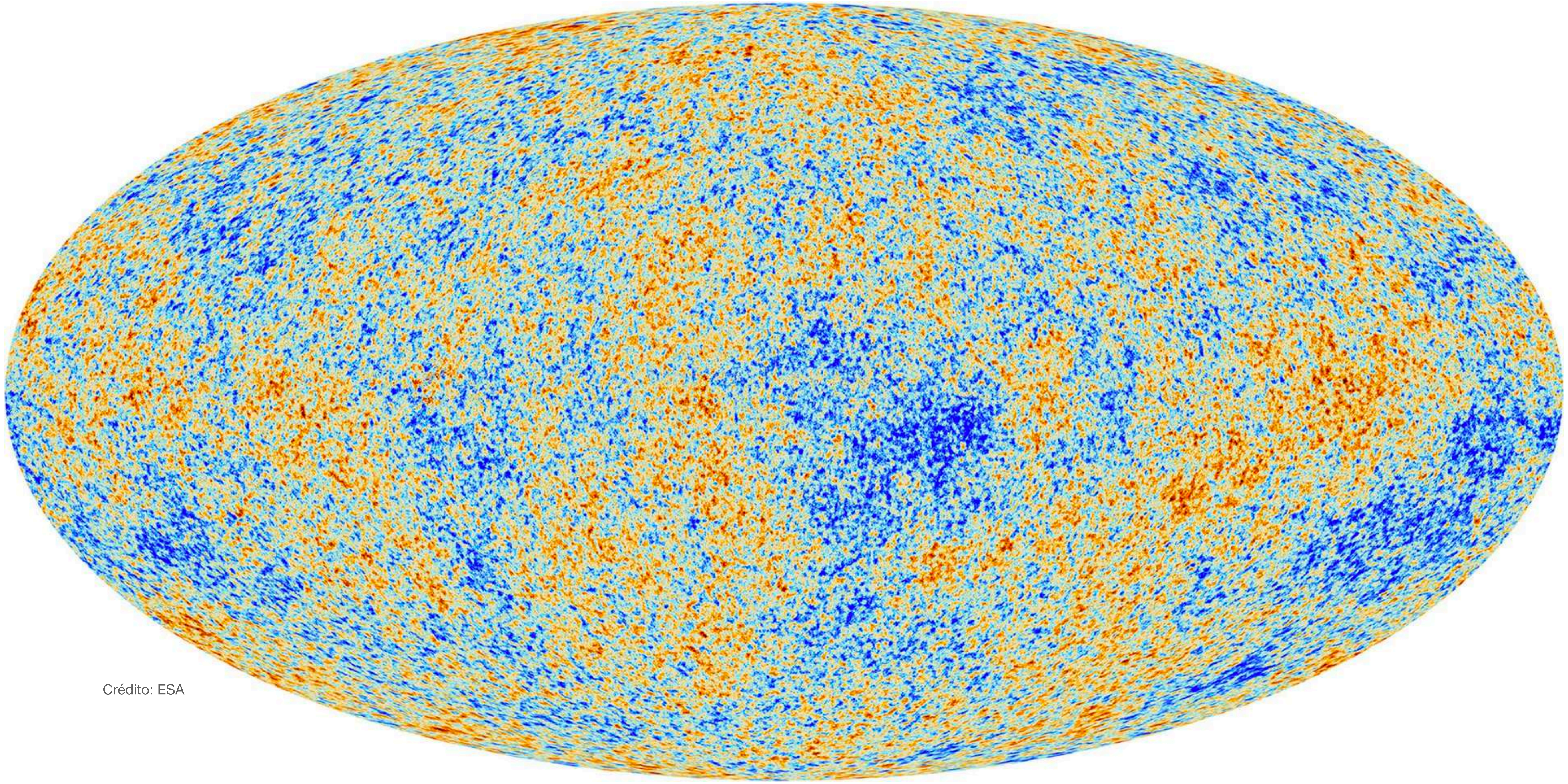
Cosmic **M**icrowave **B**ackground *(CMB)*



Crédito: D. Baumann

Given the expansion of the universe, we observe these photons in microwave.

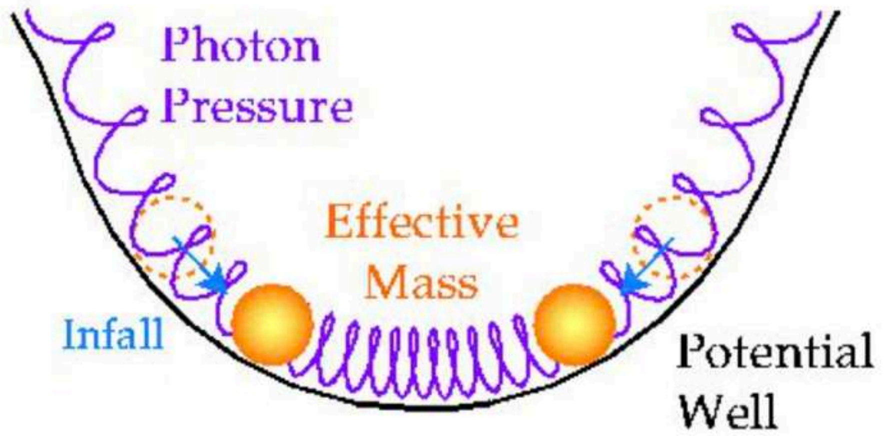
Cosmic **M**icrowave **B**ackground *(CMB)*


Crédito: ESA

Temperature 2.7 K. Small fluctuations - initial condition for the structures of our universe

Baryon Acoustic Oscillation (BAO)

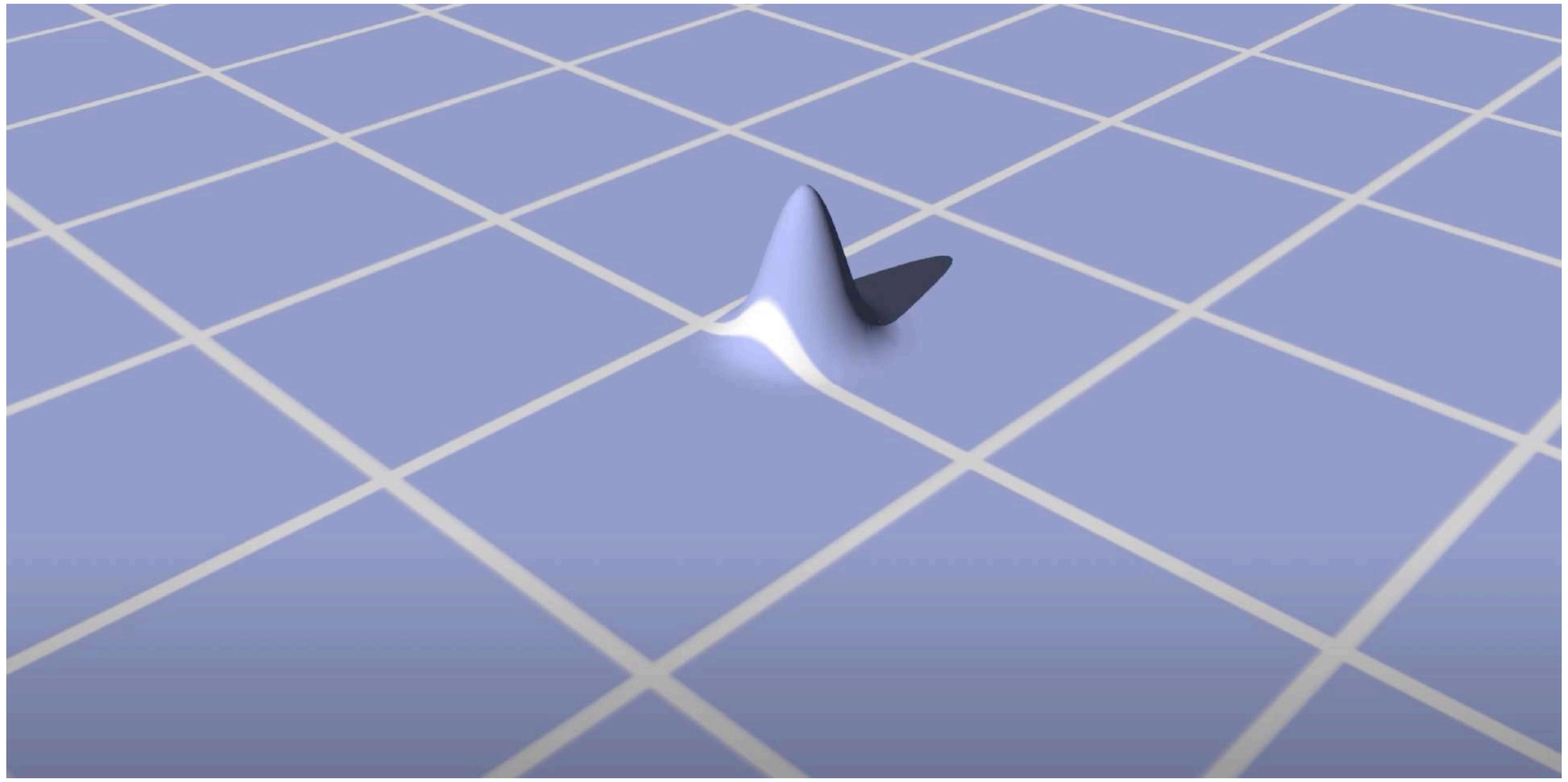
- Oscillation in the baryon-photon fluid: pressure vs gravity
- This wave propagates until matter/radiation decoupling



(Wayne Hu)

- Its signature is imprinted in the CMB and the distribution of galaxies

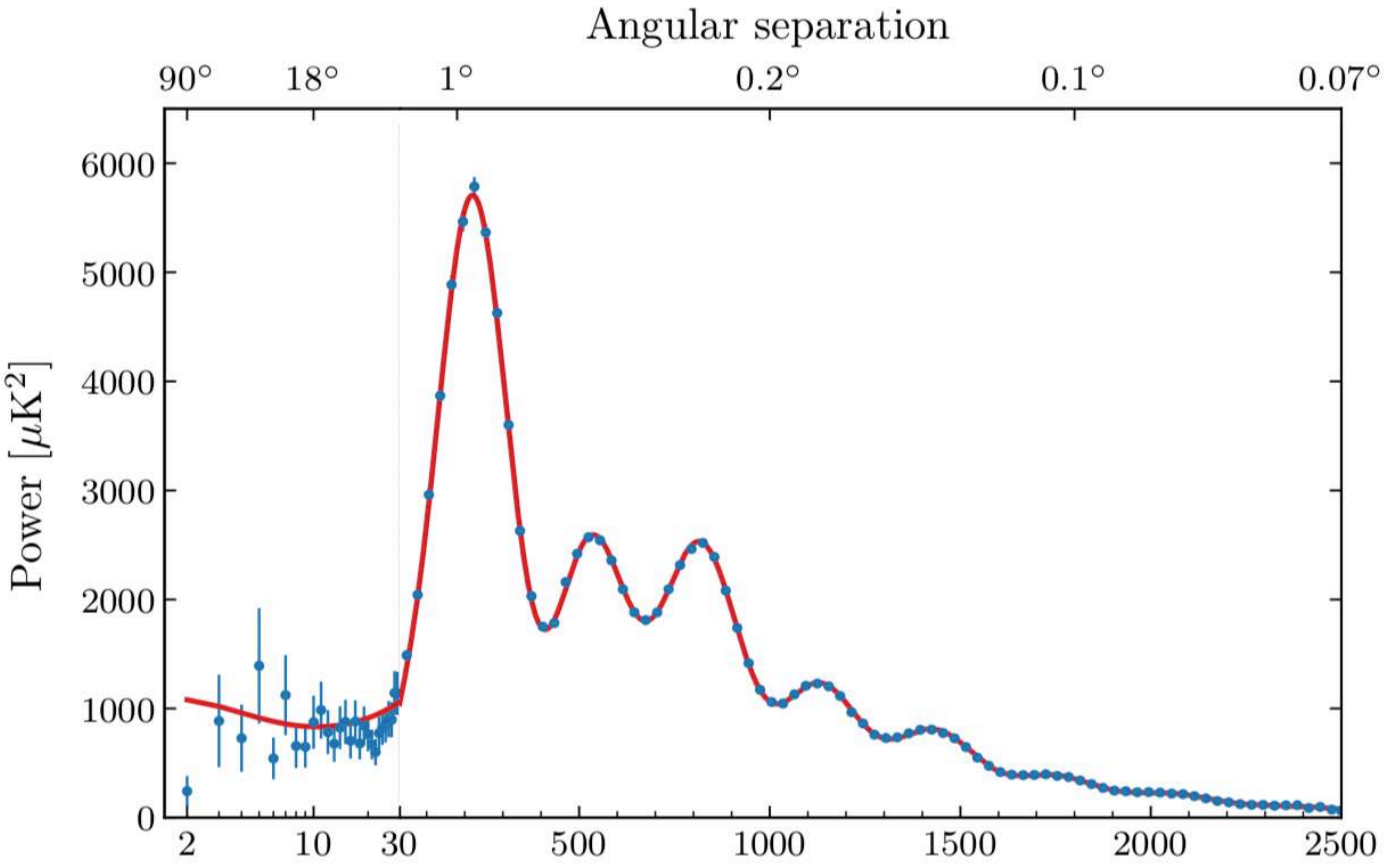
Scale known to 0.2% precision from
CMB power spectrum (147.4±0.3 Mpc)



Crédito: CASTRO

CMB

$$f(\theta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} C_\ell^m \, Y_\ell^m(\theta,\varphi)$$

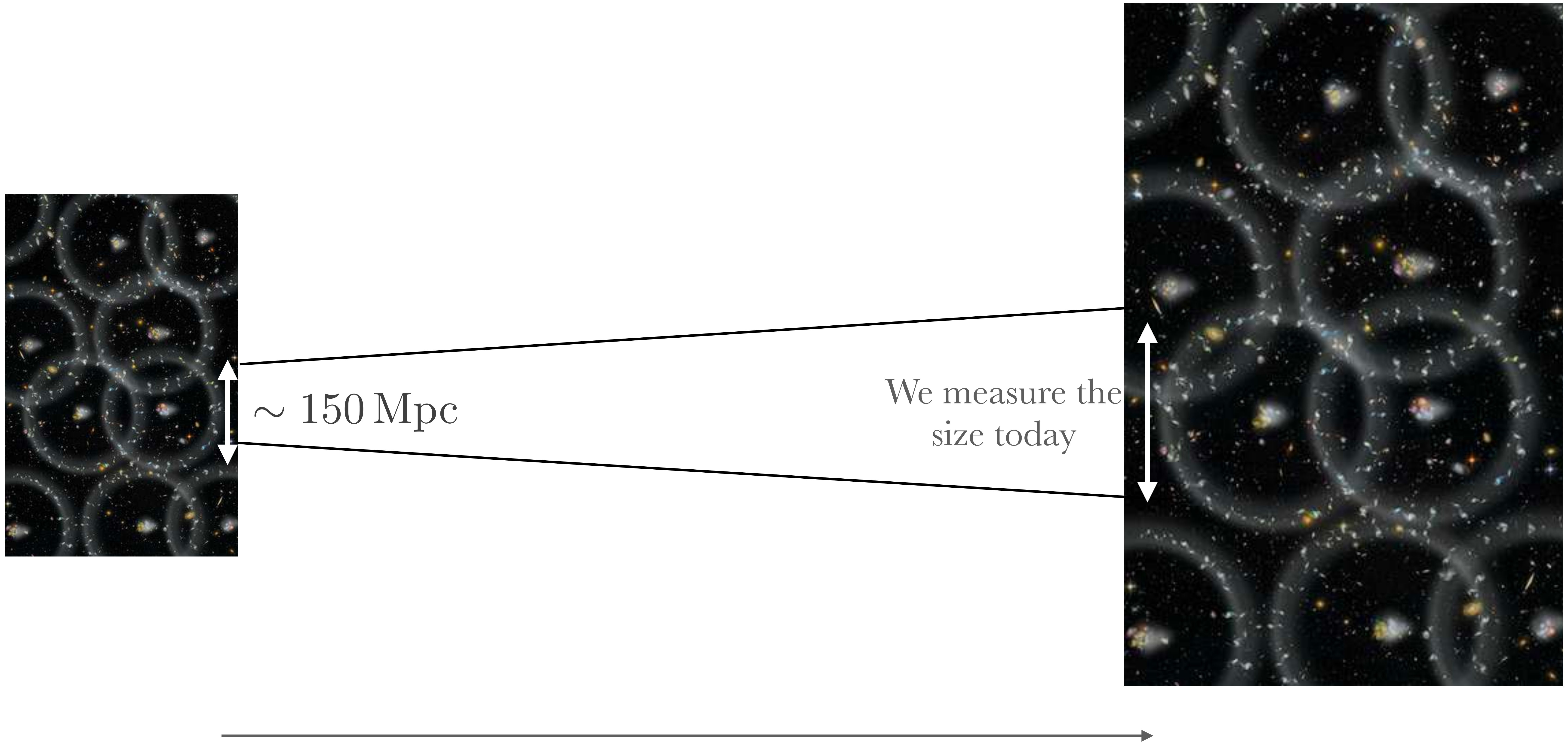


Grandes
escalas

Multipoles

Pequenas
escalas

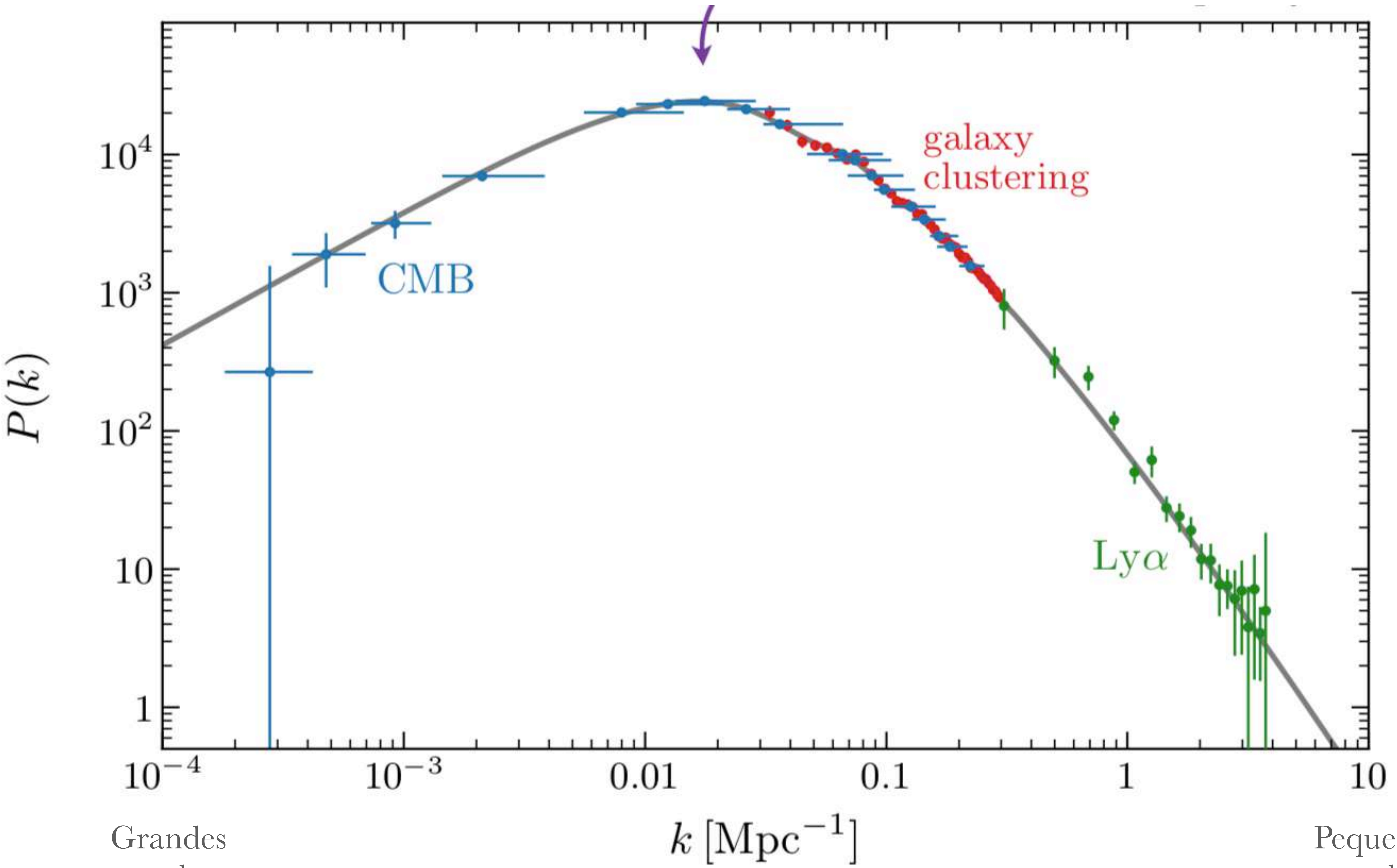
Baryon Acoustic Oscillation (BAO)



- BAOs are “standard rulers” to measure expansion

Matter power spectrum

Matter-radiation transition



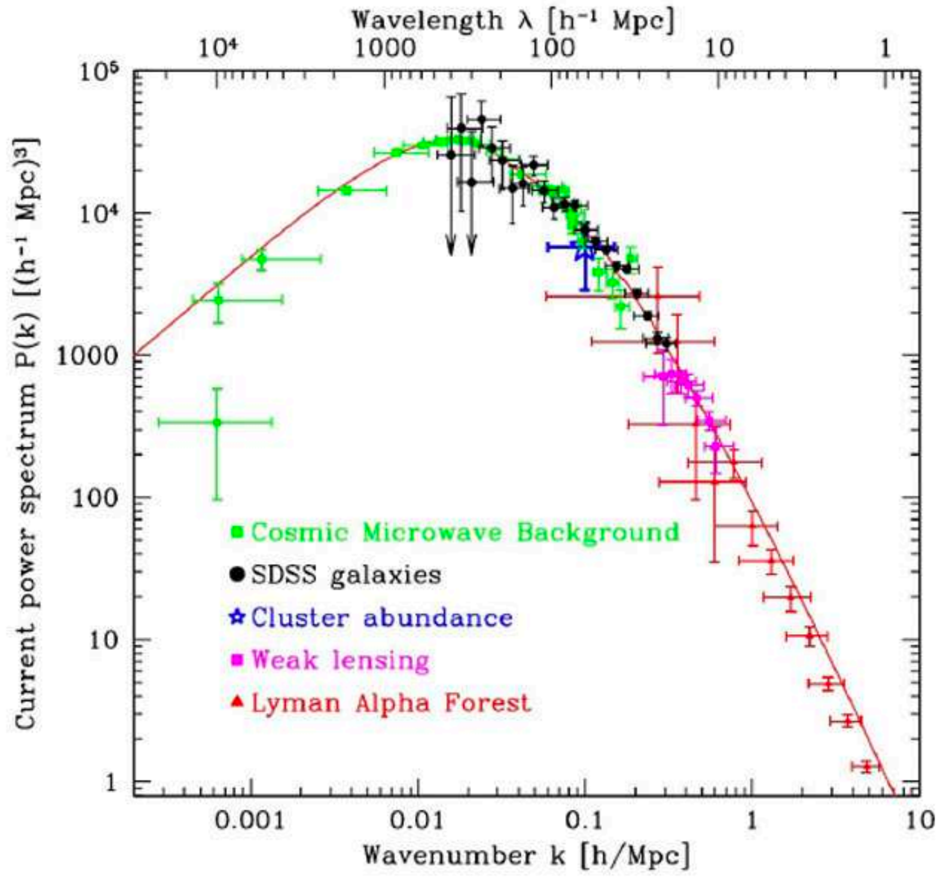
Grandes escalas

Pequenas escalas

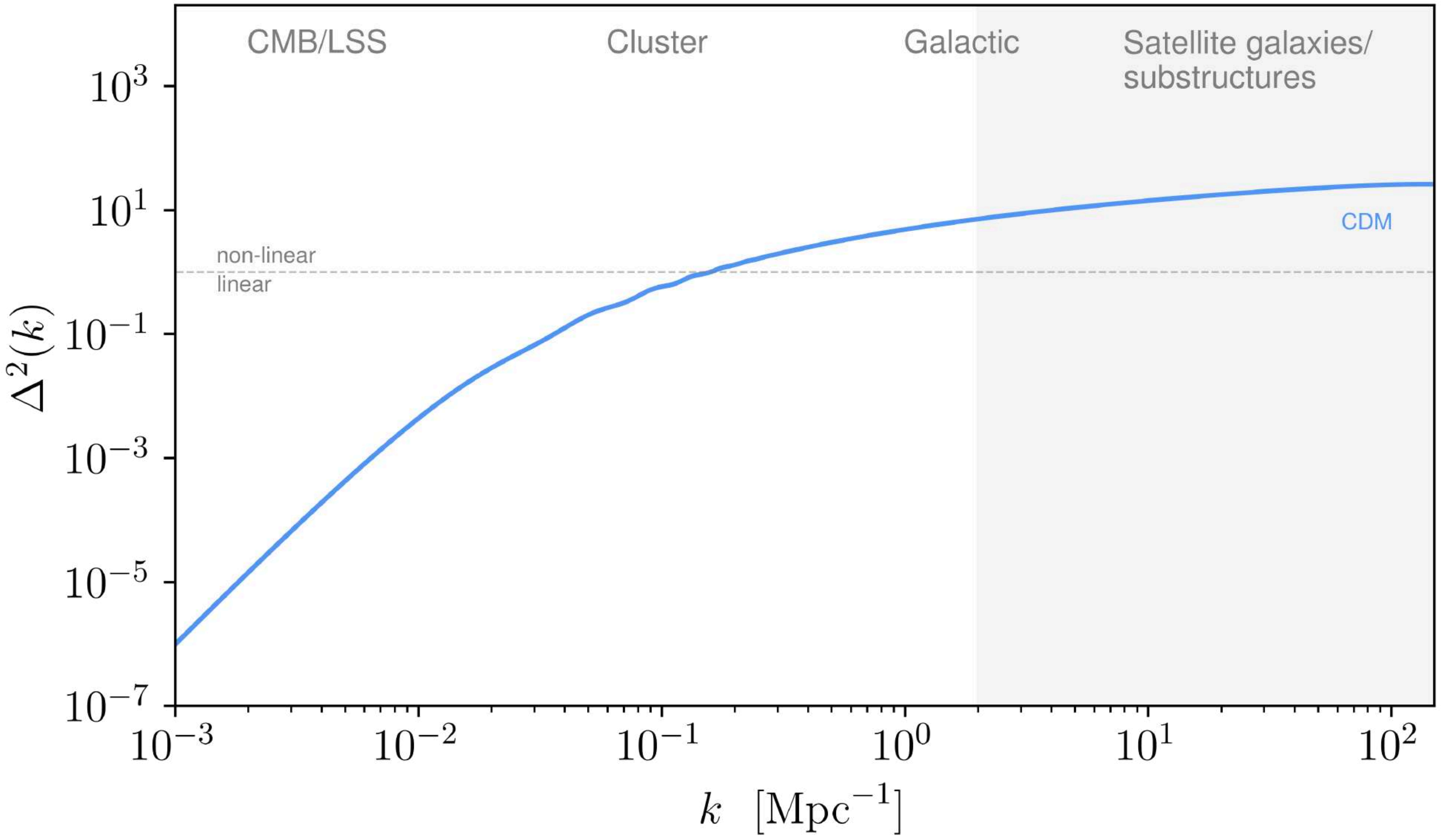
What we know about dark matter

Properties:

From LSS:



Measure PS well until scales $k \sim 10 - 20\,\mathrm{Mpc}^{-1}$



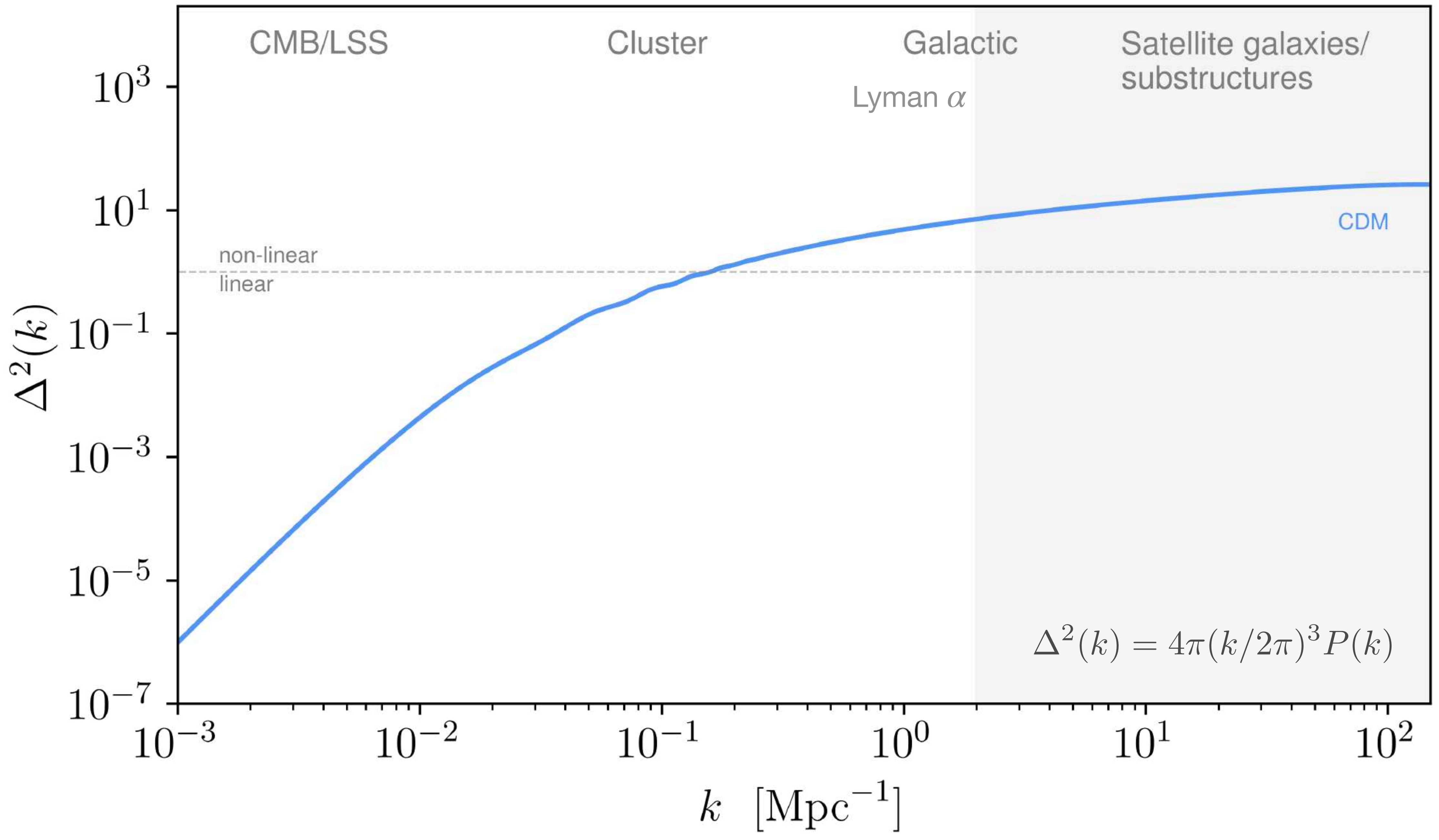
Dimensionless power spectrum

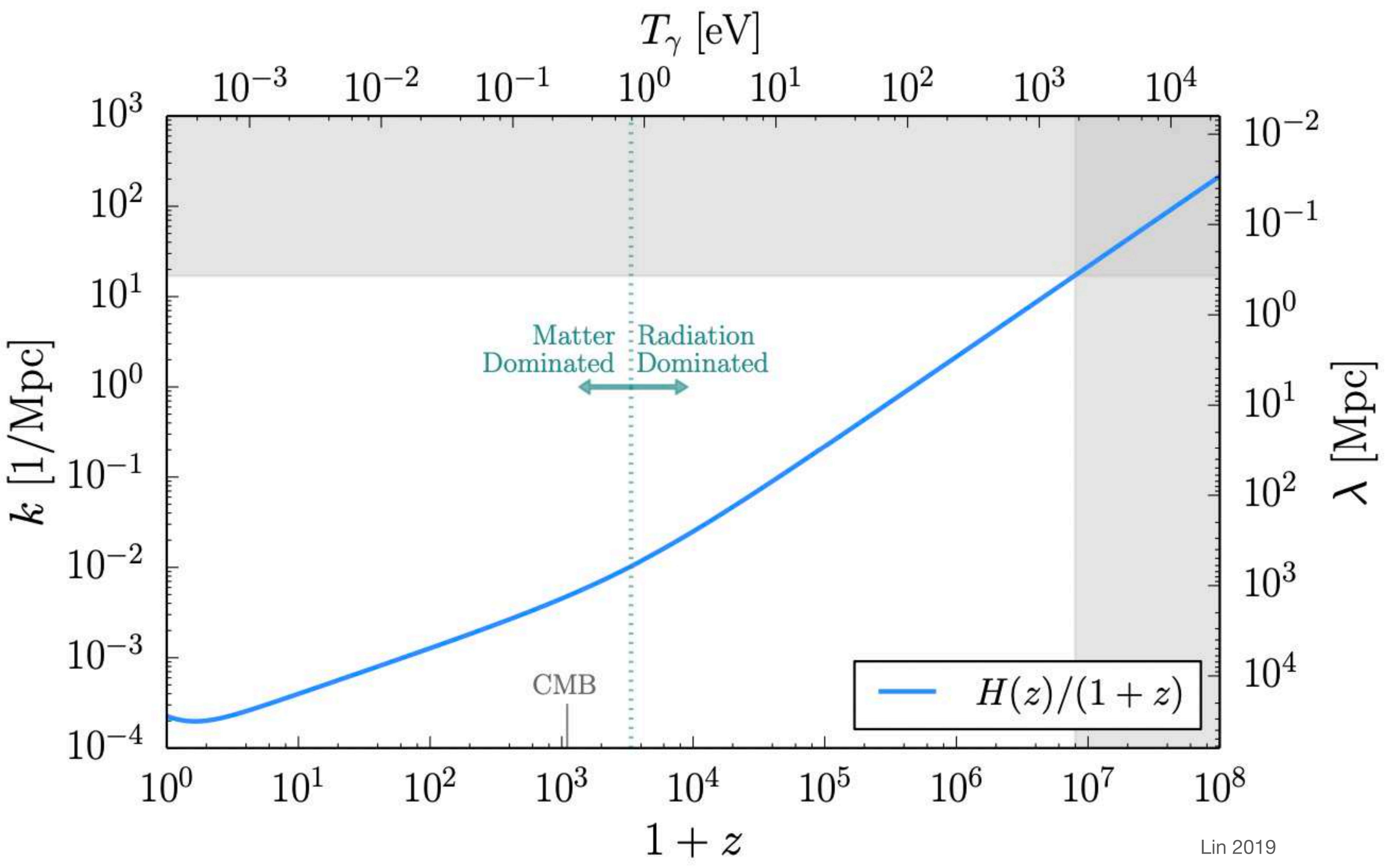
$$\Delta^2(k) = 4\pi(k/2\pi)^3 P(k)$$

What we know about dark matter

Properties:

$$T_\gamma = T_{\gamma,0}(1+z)$$





Lin 2019

Perturbation modes enter the Hubble radius $\lambda_{phys} = a/k = H^{-1}$

$k = aH = H/(1+z)$

After this, the density pert. of CDM start to evolve, **grow** - contribute to the PS

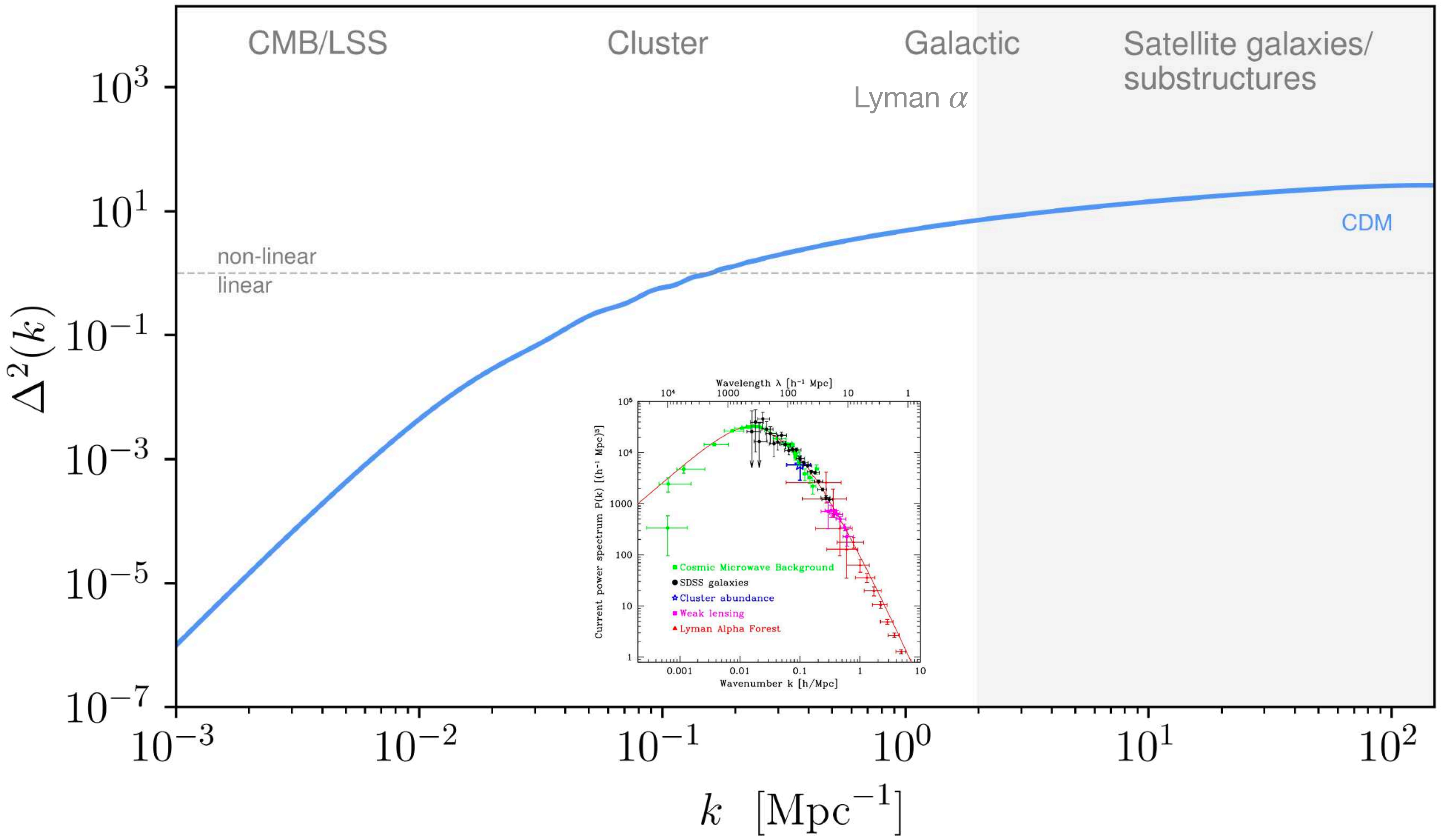
CDM pert. ($c_s = 0$) inside Hubble radius:

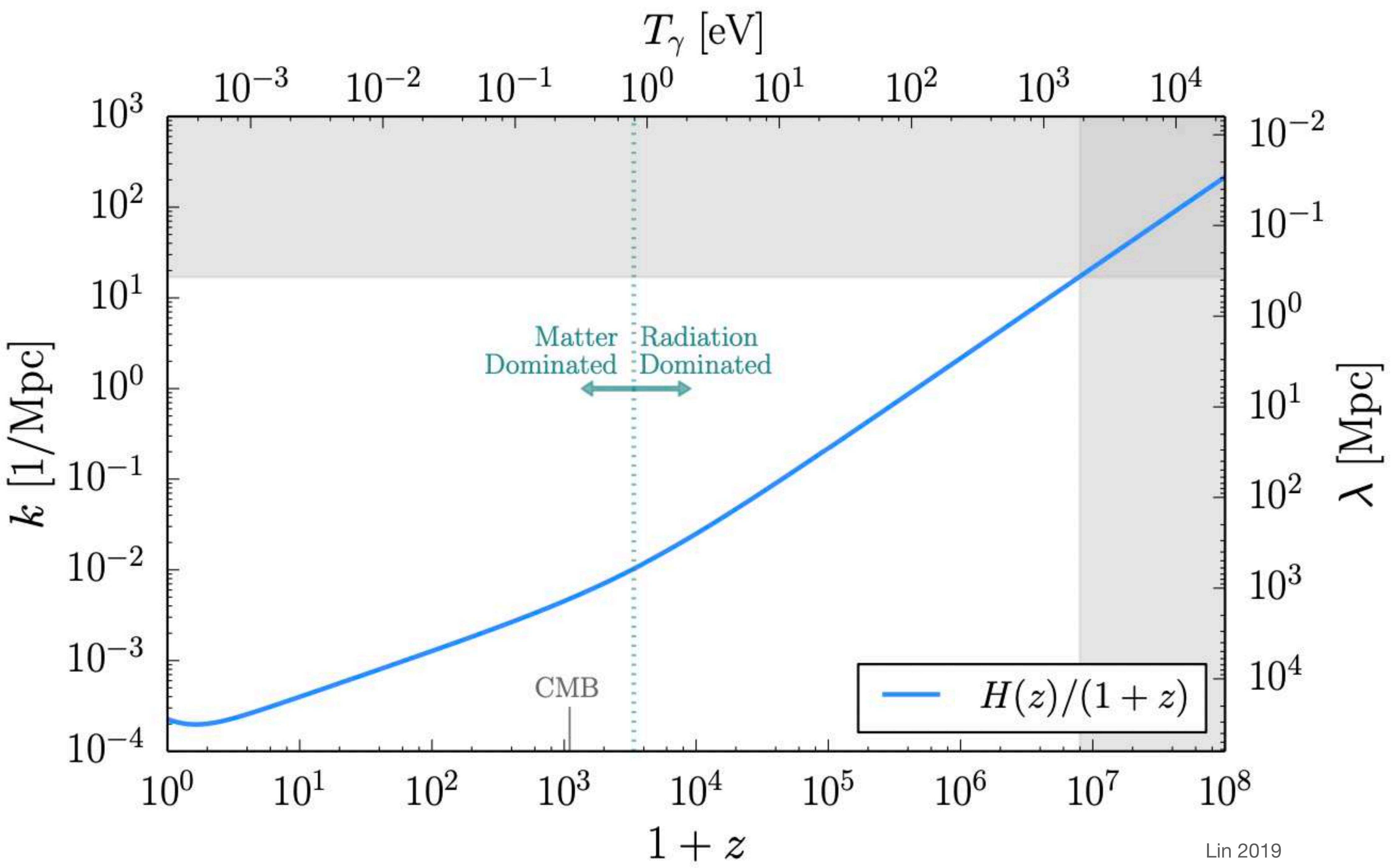
$$\delta \propto \begin{cases} \log a & \text{rad. domination} \\ a & \text{matter domination} \end{cases}$$

What we know about dark matter

Properties:

$$T_\gamma = T_{\gamma,0}(1+z)$$





Lin 2019

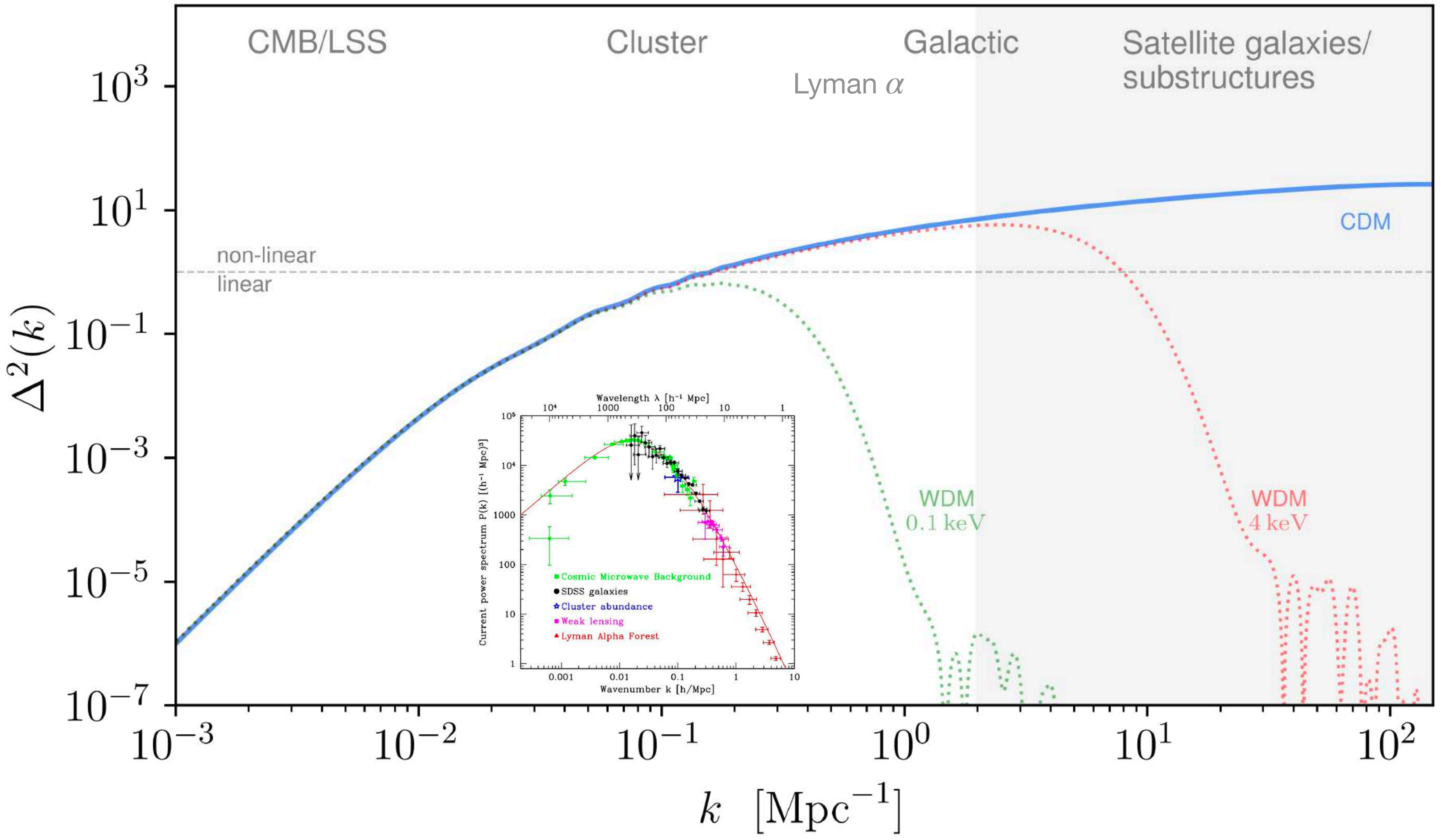
Perturbation modes enter the Hubble radius $\lambda_{phys} = a/k = H^{-1}$

$$k = aH = H/(1+z)$$

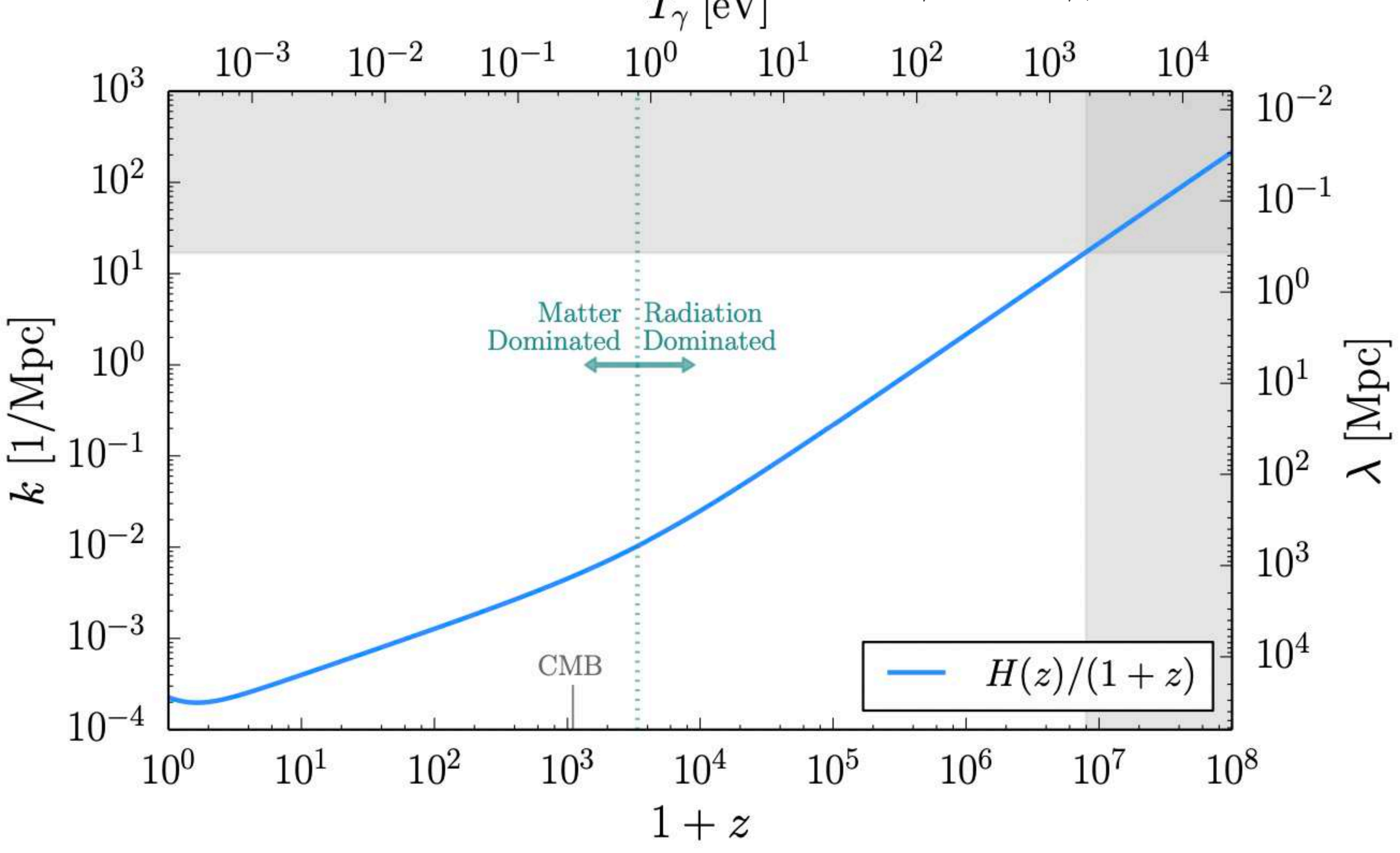
So we can describe the observations, all the modes in the white region ($< 10\,\mathrm{Mpc}^{-1}$) are inside the Hubble radius and contribute to the PS, and are very precisely described by CDM ⇒ cold and pressureless

What we know about dark matter

Properties:



$$T_\gamma = T_{\gamma,0}(1+z)$$



Lin 2019

If DM relativistic (or hot) when $z < 10^7$, this mode is inside R_H, so it will contribute to the PS - since relativ. pert. DO NOT cluster, we would have a **suppression in the power spectrum** for $k < 10 - 20\,\mathrm{Mpc}^{-1}$ - *not in agreement with observations!*

$\Rightarrow$ DM has to be non-relativistic before $z = 10^7$

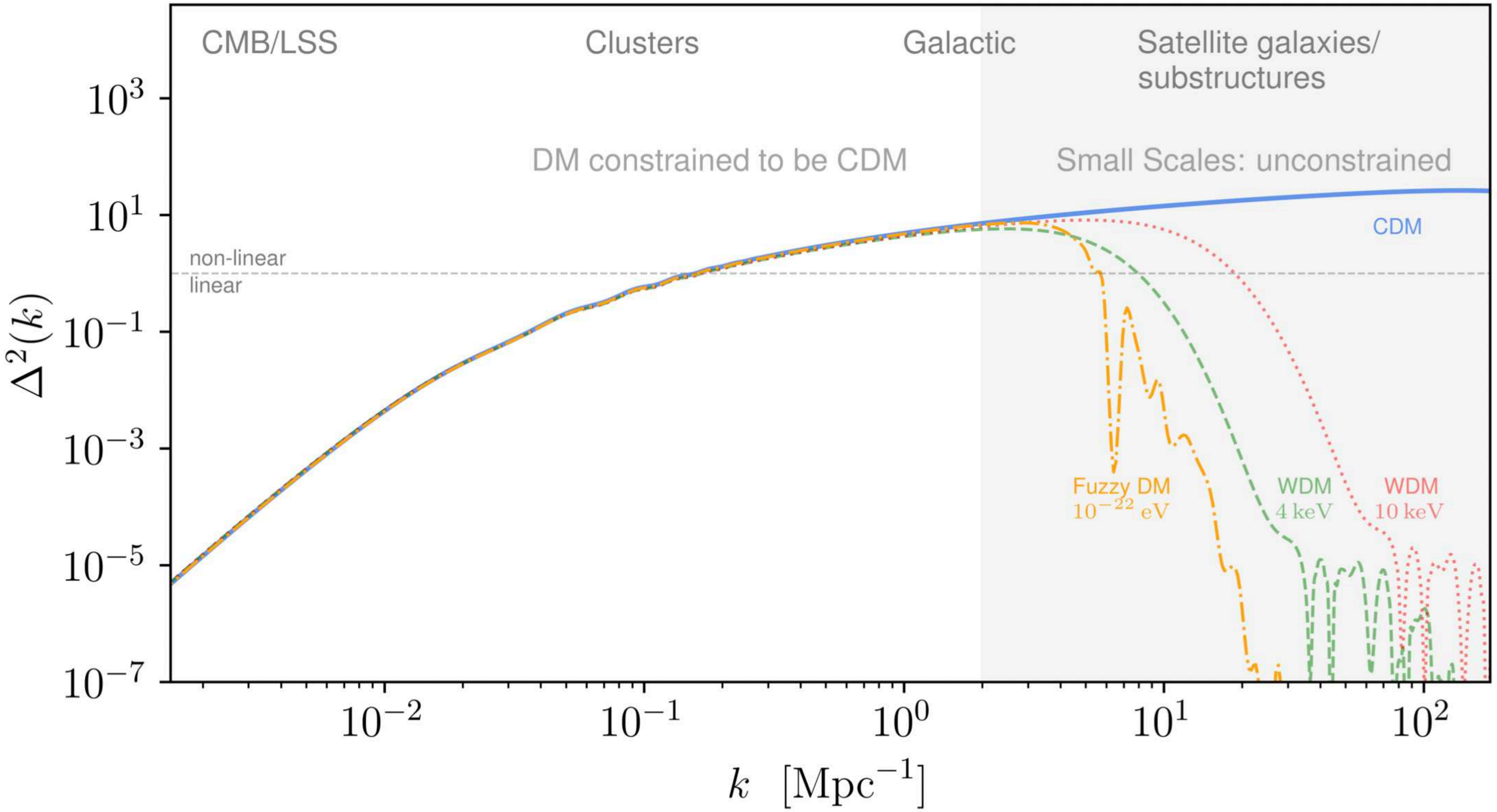
If **DM in thermal equilibrium** with the baryon-photon plasma ($T_{dm} = T_\gamma$)

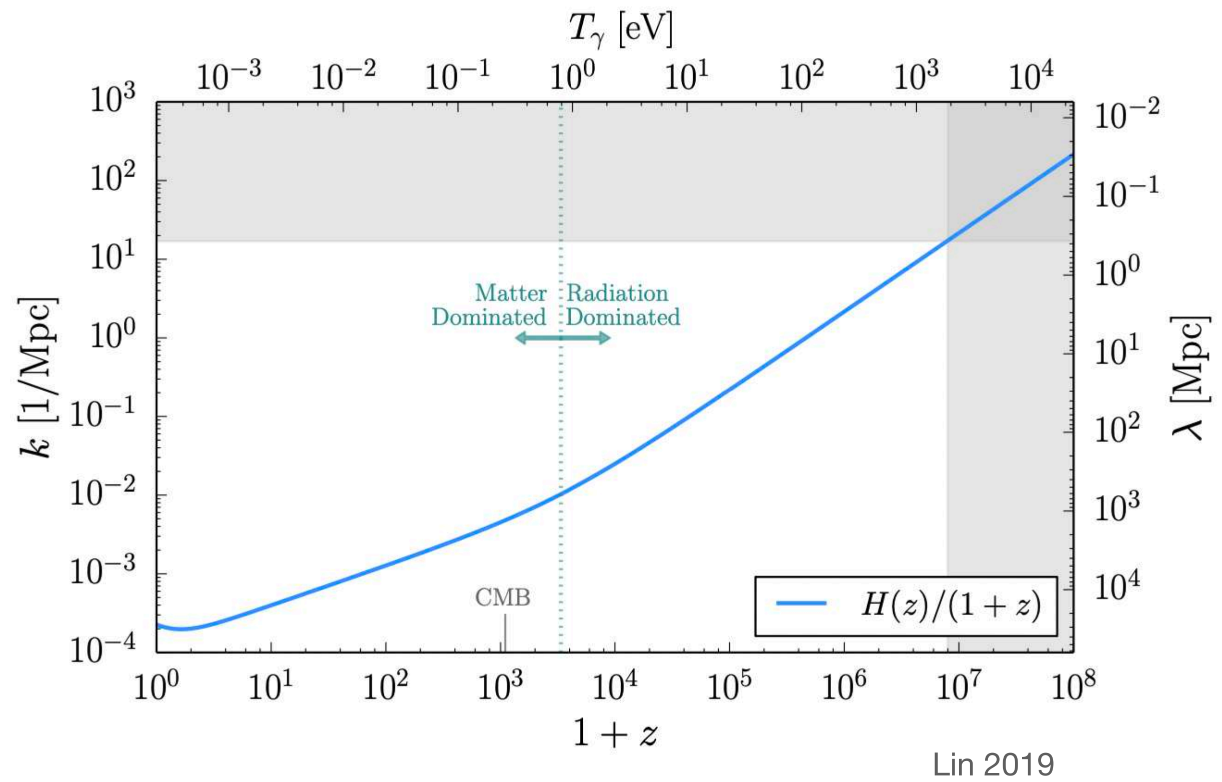
$\Rightarrow m_{dm} >$ keV

WDM bound

What we know about dark matter

Properties:





Lin 2019

Deviations from CDM in the highlighted region are allowed, since highly unconstrained!

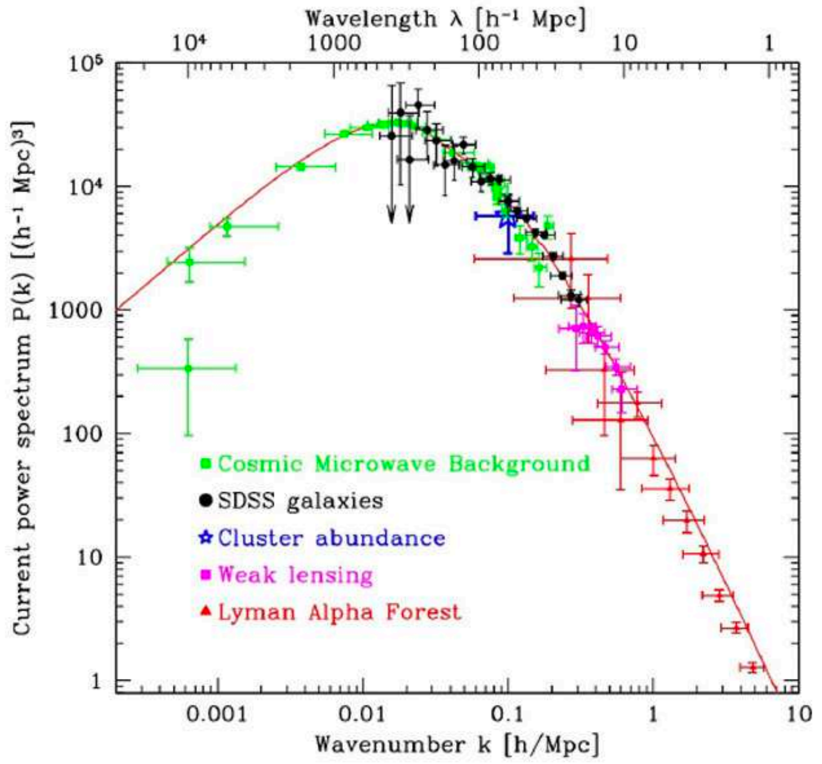
What we know about dark matter

What we learned from observations

Properties:

- Cold

- Presureless



What we know about dark matter

Properties:

- Cold
- Presureless
- Dark (transparent): DM does not interact electromagnetically

What we know about dark matter

- **Dark** (transparent/neutral): DM does not interact electromagnetically

Obviously: If DM interacted electromagnetically, interacted with photons, it would scatter light and thus not be dark

↘ *DM charge* $\epsilon\, e$

What we know about dark matter

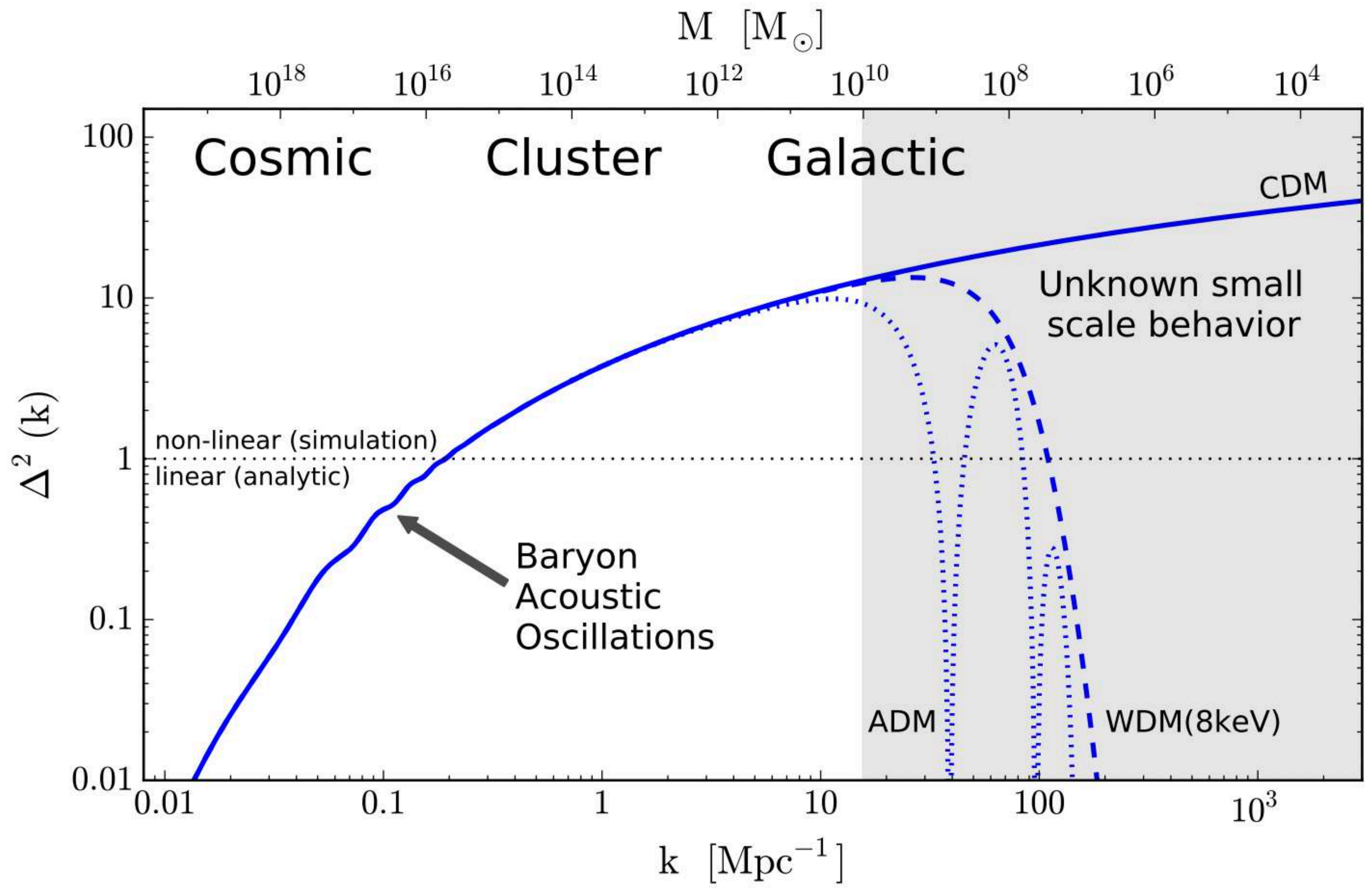
- Dark (transparent/neutral): DM does not interact electromagnetically

If DM had a *charge* $\epsilon\, e$:

- Suppression of the power spectrum

Charged DM particles interact with the Standard Model via a small coupling through the photon

If the DM is coupled with the baryon-photon plasma during *recombination*, the DM density fluctuations can be washed out due to the radiation pressure and the photon diffusion (Silk damping). The BAO structure will also be directly altered through the coupling.



Ex: ADM - atomic dark matter

Ref.: Kaplan et al 2009, Cyr-Racine et al 2012

Interactions of DM with SM particles at early times would **suppress** the power spectrum, since the radiation pressure of the baryons and photons would prevent DM density perturbations from growing

What we know about dark matter

- Dark (transparent/neutral): DM does not interact electromagnetically

If DM had a *charge* ϵe :

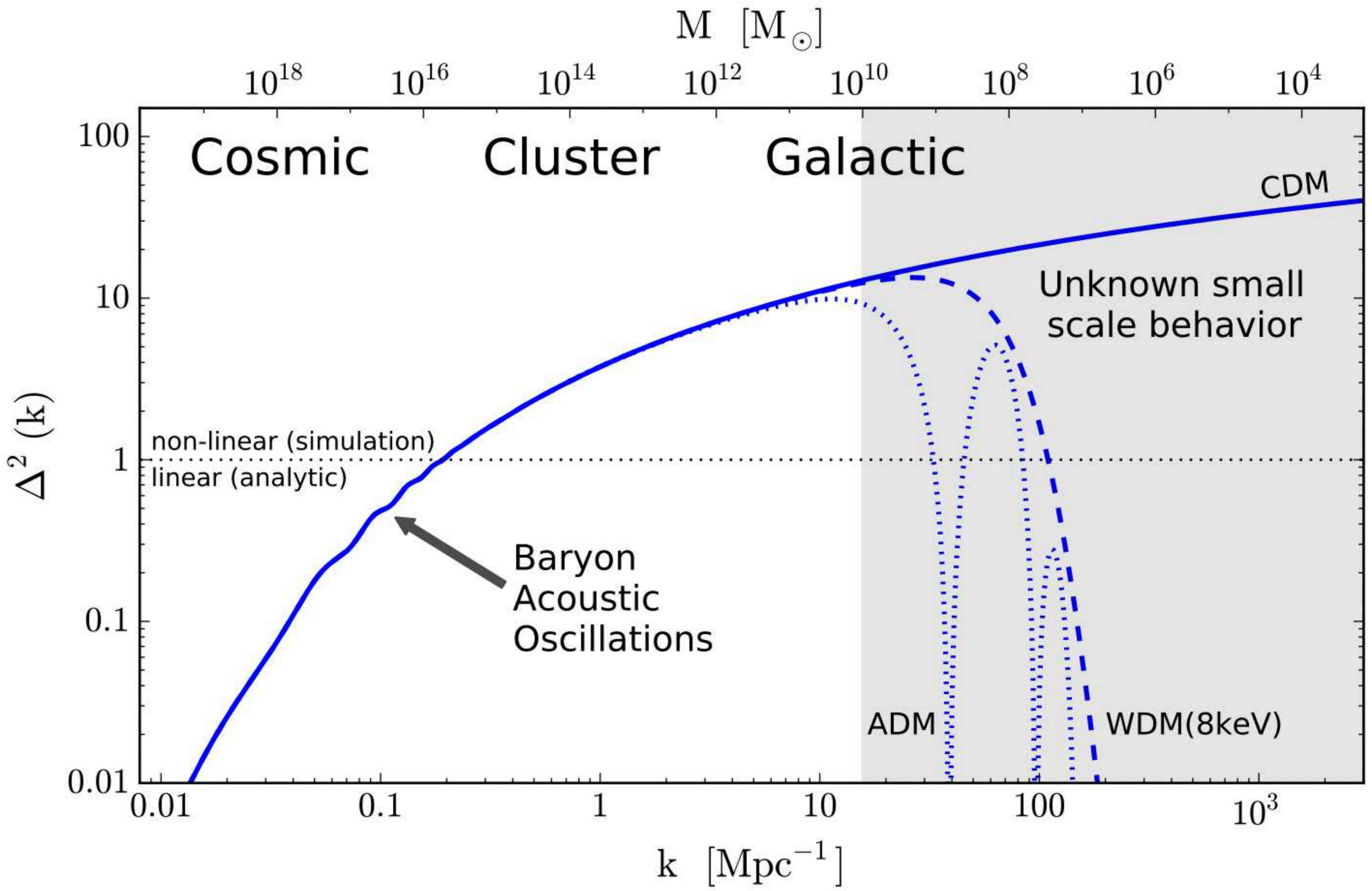
- Bound @ recombination

DM be completely decoupled from the baryon-photon plasma at recombination

$$\epsilon < 3.5 \times 10^{-7} (m_{dm}/1\,\mathrm{GeV})^{0.58} \text{ for } m_{dm} > 1 \text{ GeV}$$
$$\epsilon < 4.0 \times 10^{-7} (m_{dm}/1\,\mathrm{GeV})^{0.58} \text{ for } m_{dm} < 1 \,\mathrm{GeV}$$

* similar bounds from direct detection

DM has neutral or charge < mili-charge!



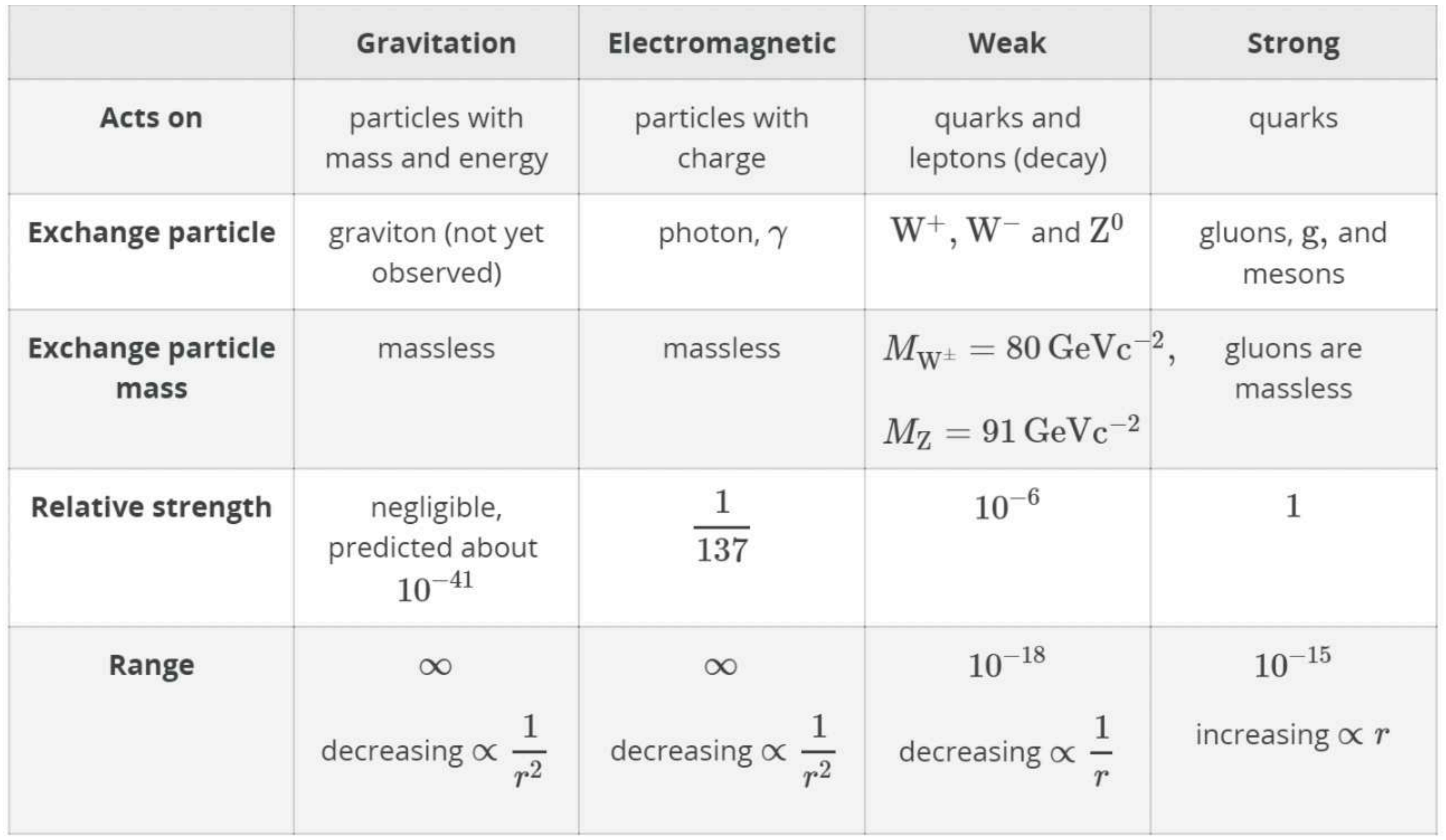
Ex: ADM - atomic dark matter

Interactions

1) DM interacts gravitationally - evidence for its existence

2) It **cannot** or have a small ***electromagnetic*** interaction

DM has neutral or charge < mili-charge!

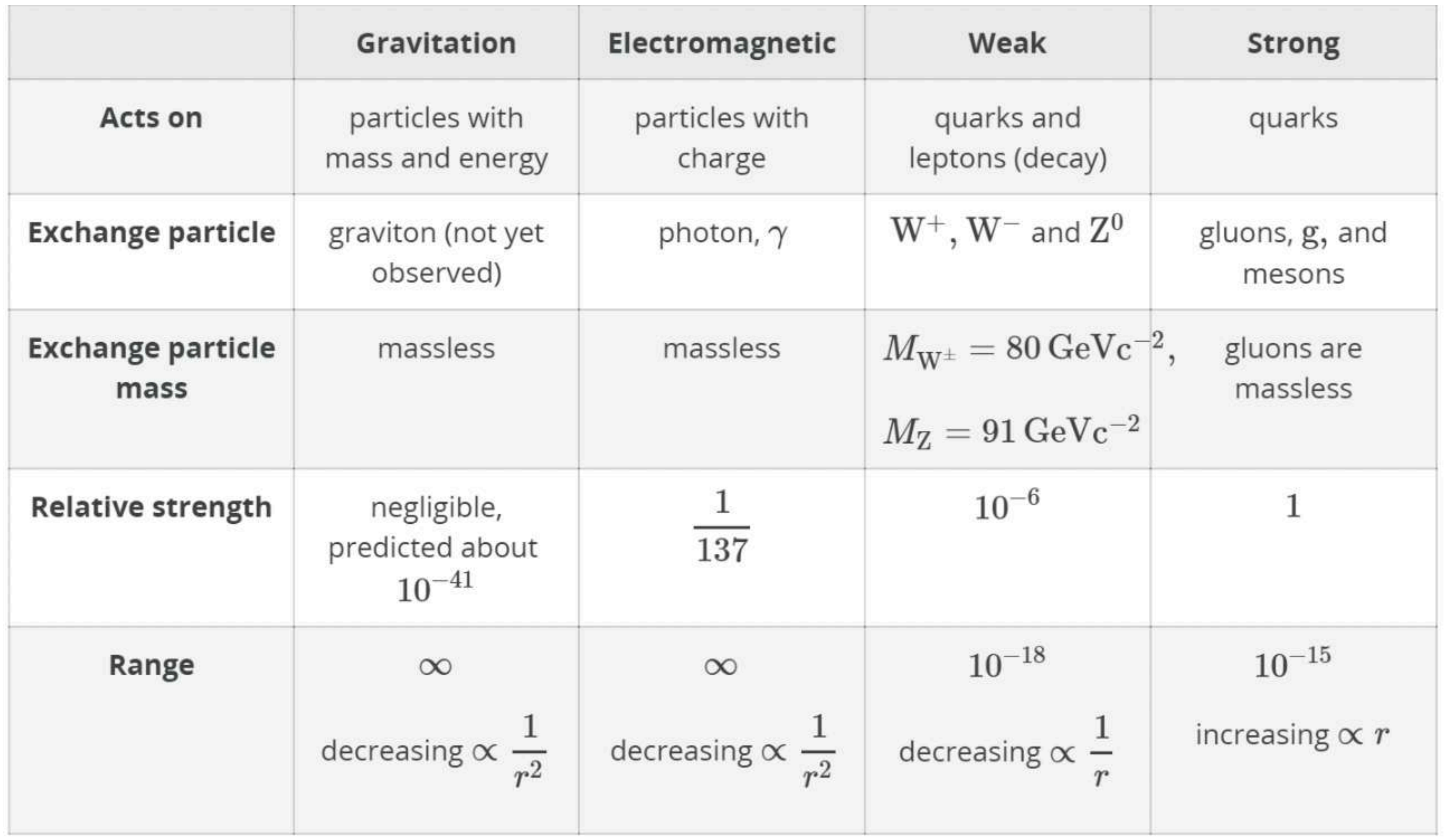


	Gravitation	Electromagnetic	Weak	Strong
Acts on	particles with mass and energy	particles with charge	quarks and leptons (decay)	quarks
Exchange particle	graviton (not yet observed)	photon, γ	W^+, W^- and Z^0	gluons, g, and mesons
Exchange particle mass	massless	massless	$M_{\mathrm{W}^\pm} = 80\,\mathrm{GeVc}^{-2}$, $M_{\mathrm{Z}} = 91\,\mathrm{GeVc}^{-2}$	gluons are massless
Relative strength	negligible, predicted about 10^{-41}	$\frac{1}{137}$	10^{-6}	1
Range	∞ decreasing $\propto \frac{1}{r^2}$	∞ decreasing $\propto \frac{1}{r^2}$	10^{-18} decreasing $\propto \frac{1}{r}$	10^{-15} increasing $\propto r$

Interactions

1) DM interacts gravitationally - evidence for its existence

2) It cannot or have a small electromagnetic interaction



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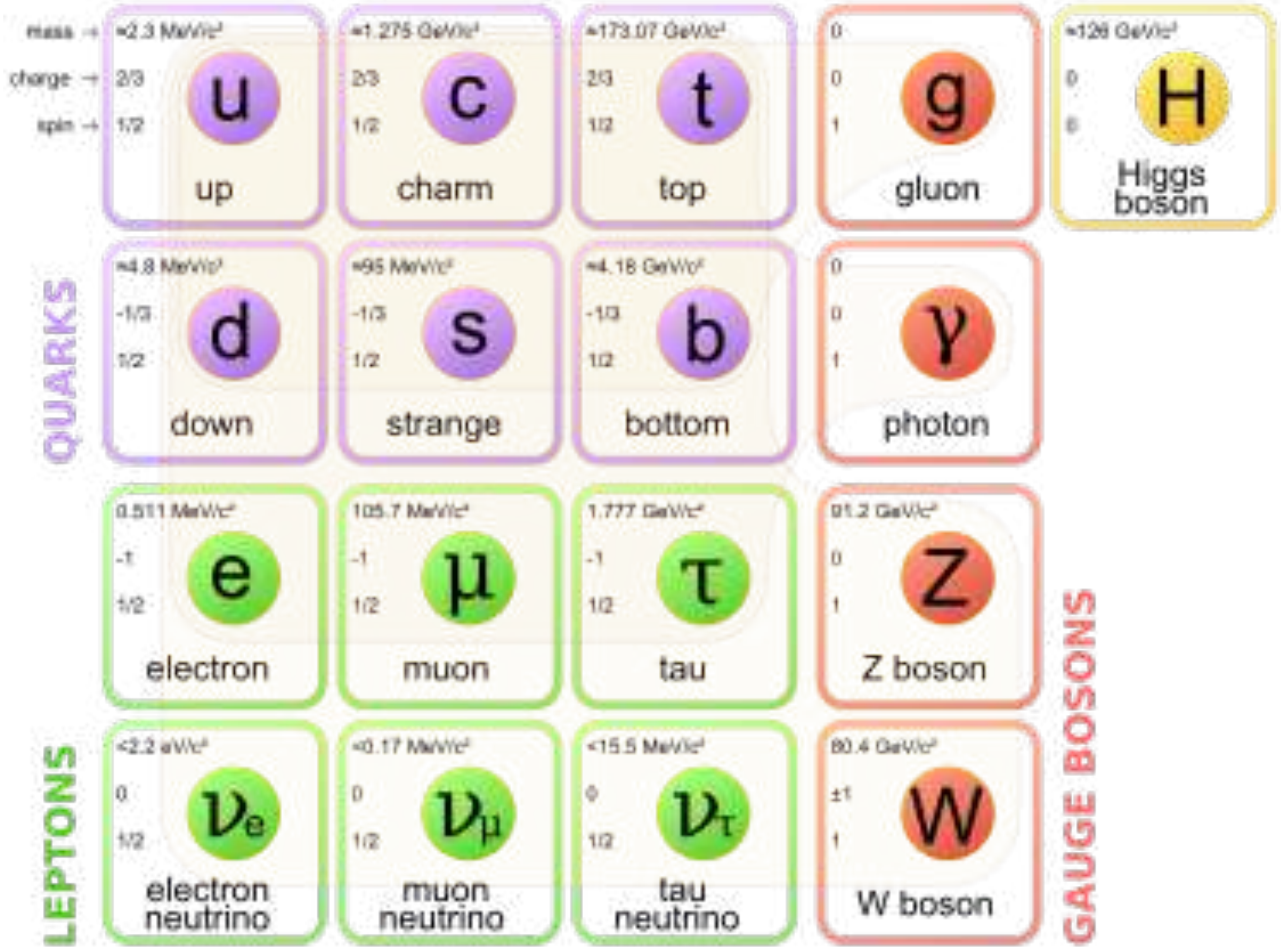
What about the **weak** and **strong forces**?

Strong force

The elementary particles of the DM that interact with the strong force are the quarks, interacting via gluons

And quarks also have electric charge!! This means that they also interact electromagnetically.

If DM interacted through the strong force: this would change the abundance of light elements.



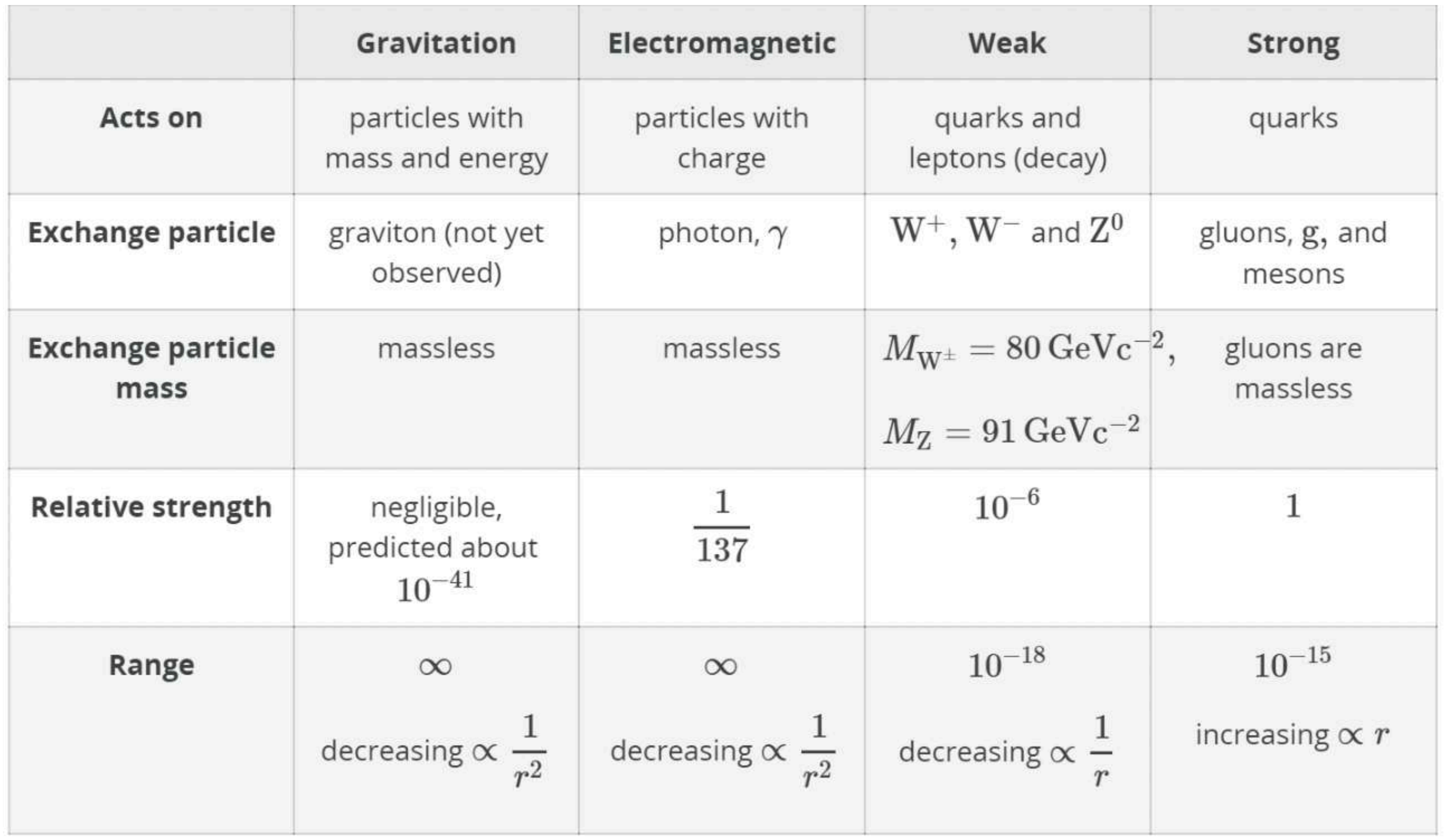
Interactions

1) DM interacts gravitationally - evidence for its existence

2) It cannot or have a small electromagnetic interaction

3) It cannot interact via the strong force

4) Weak force - DM *can* interact through the **weak force**



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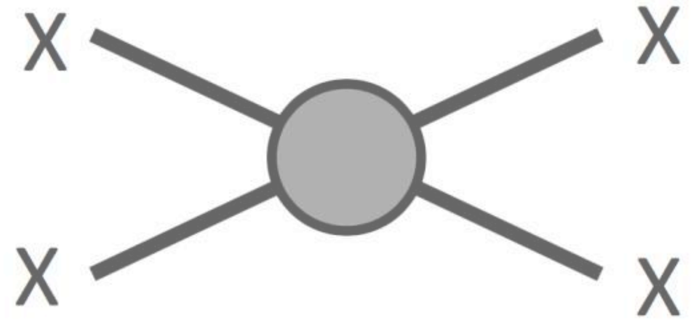
What we know about dark matter

Properties:

- Cold
- Presureless
- Dark (transparent): DM does not interact electromagnetically
- Collisionless: no/weakly self-interaction; non-interacting

What we know about dark matter



- Collisionless: no/weakly self-interaction; non-interacting

Self-interaction

Can DM interact with itself?

If dark-matter particles have a non-trivial probability of interacting there are implications for the distribution of DM: self-interaction allows ***energy and momentum to flow*** from one part of the dark matter halo to another beyond what is enabled by gravity.

Self-interacting can lead to changes in:

- Distribution of DM in the halo
- Halo shape
- Hierarchical assembly of structure on non-linear scales
- Matter power spectrum
- …

What we know about dark matter

- Collisionless: no/weakly self-interaction; non-interacting

Self-interaction

Self-interacting can lead to changes in:
- Distribution of DM in the halo
- Halo shape
- Hierarchical assembly of structure on non-linear scales
- Matter power spectrum
- …

Can be tested with:
- Mergers in groups and clusters
- Strong gravitational lensing in clusters
- Stellar streams in the Milky Way
- X-ray and weak lensing observations of clusters, groups and large ellipticals
- Dwarf galaxies
- Rotation curves of spiral galaxies
- LSS

What we know about dark matter

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Self-interaction

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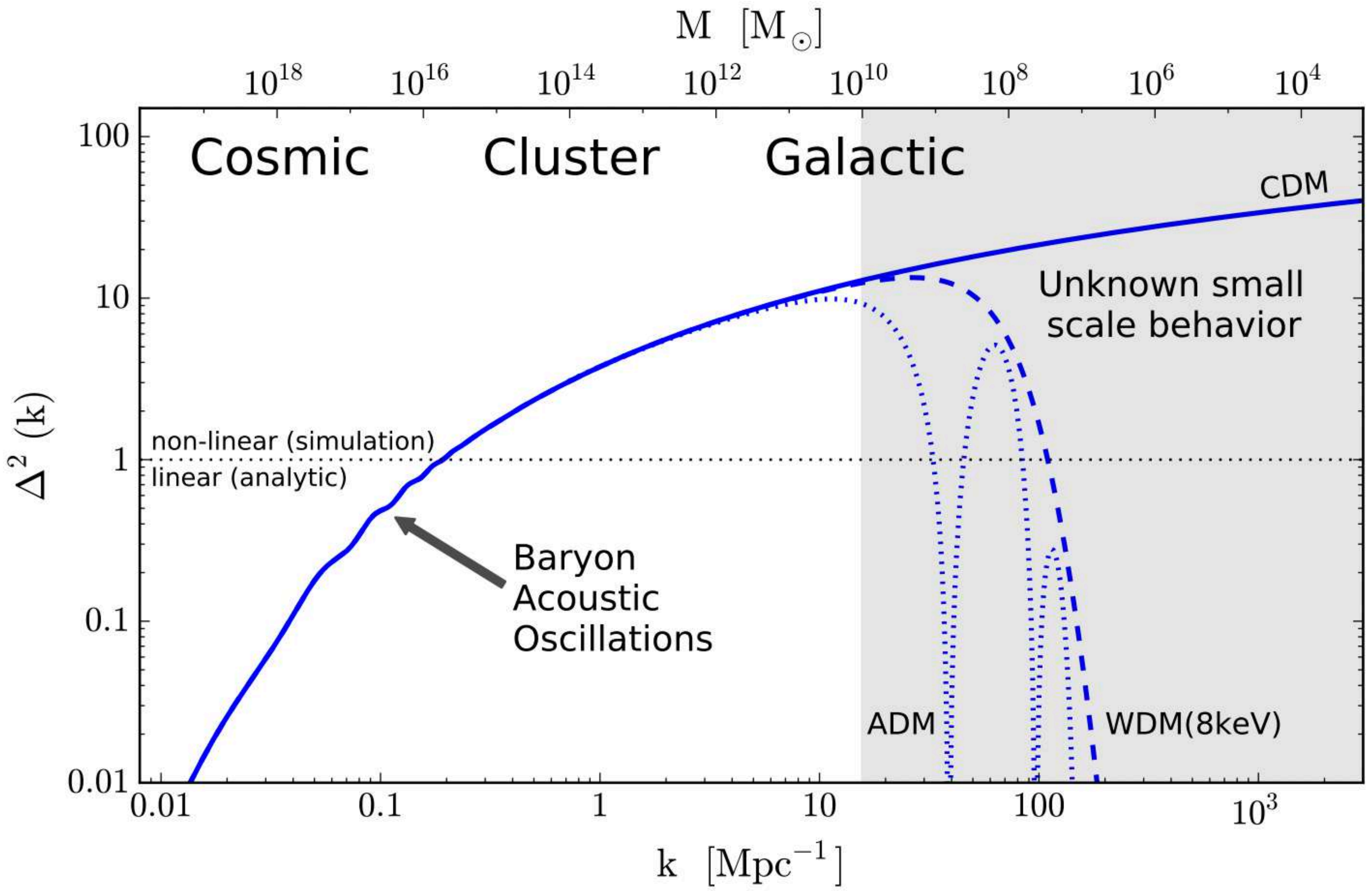

Ex: ADM - atomic dark matter

Presence of a "dark radiation" bath interacting withe the dark matter would delay growth of density perturbations and lead to the presence of "dark acoustic oscillations"

What we know about dark matter

- Collisionless: no/weakly self-interaction; non-interacting

Self-interaction

Self-interacting can lead to changes in:

- Distribution of DM in the halo
- Halo shape
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- Rotation curves of spiral galaxies
- LSS

Current bounds: $\sigma/m_{dm} < 0.13\,\mathrm{cm}^2/\mathrm{g}\,,\ \sigma/m_{dm} < 0.35\,\mathrm{cm}^2/\mathrm{g}$ Vel. independent

From: measured core densities from strong lensing

Ref.: Adhikari et al. 2022

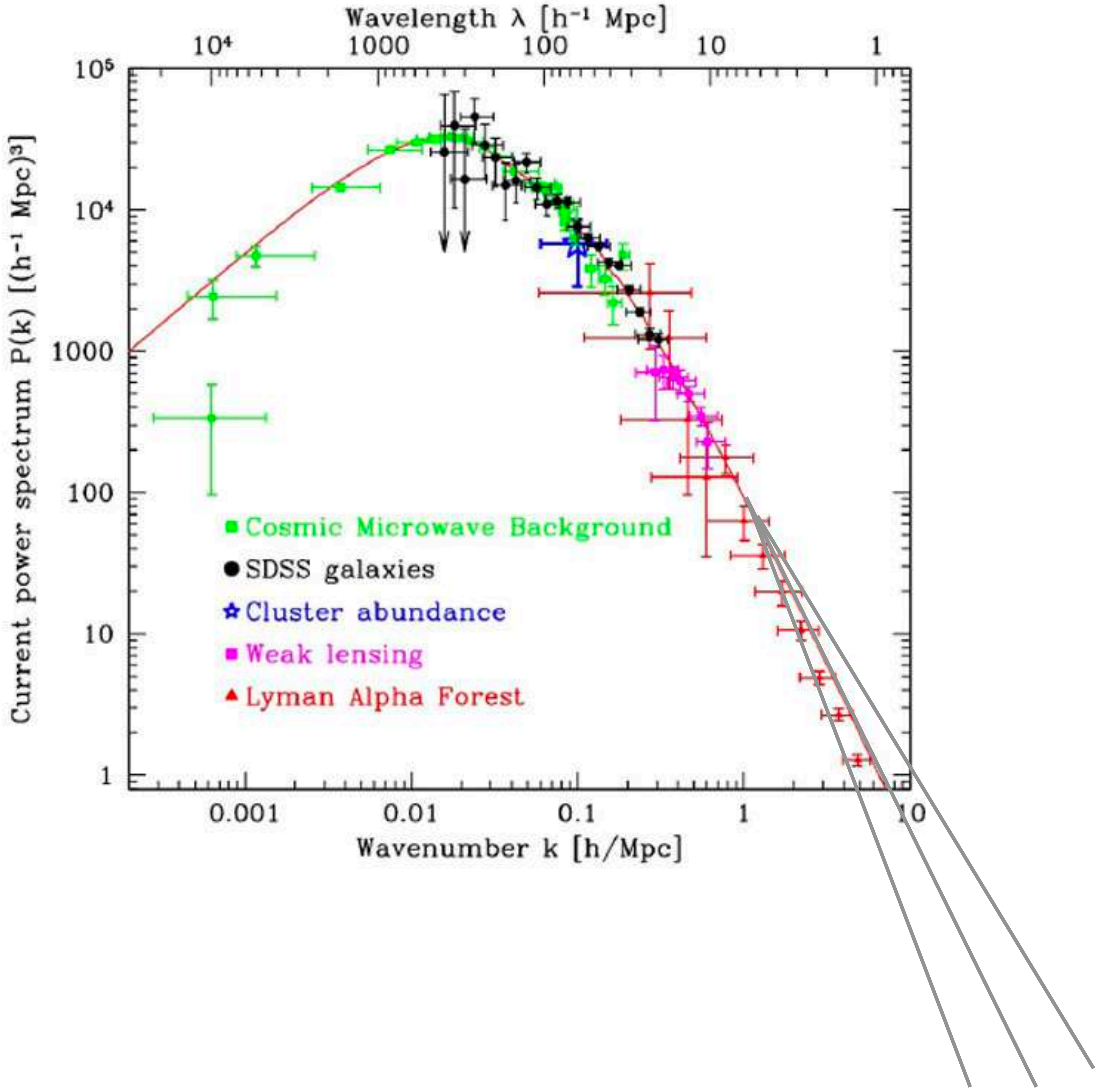
What we know about dark matter

Properties:

- Cold
- Presureless
- Dark (transparent): DM does not interact electromagnetically
- Collisionless: no/weakly self-interaction; non-interacting

What we don't know about dark matter

- ~~Cold~~ → How cold it is? WDM $m \sim \text{keV}$
- ~~Pressureless~~ → Cluster on all scales?
- ~~Dark~~ → Non-gravitational interaction? Milicharged DM
- ~~Collisionless~~ → How small sefl-interaction? SIDM



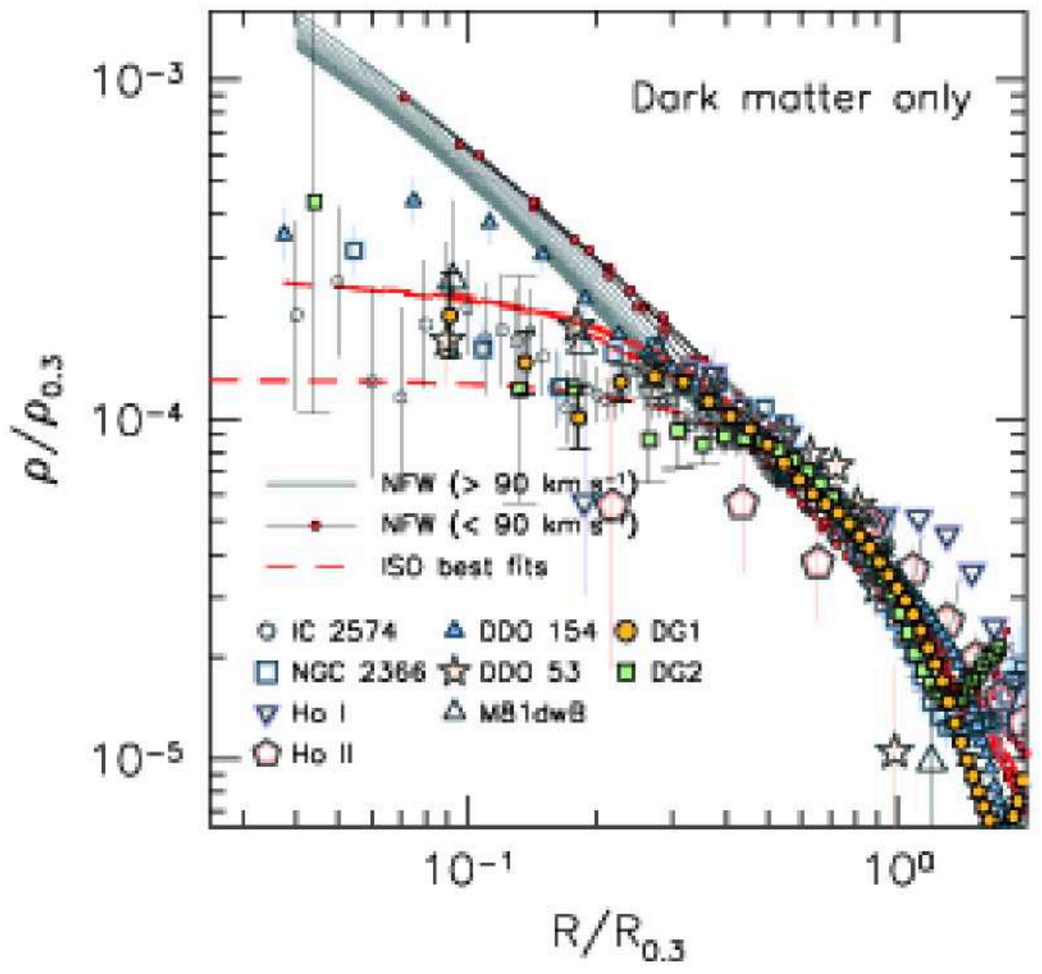
CDM on large scales

Small scale behavior: still weakly constrained and small scale challenges

Small scale curiosities: cusp-core, missing satellites, BTFR, …

Small scale challenges

Cusp-core

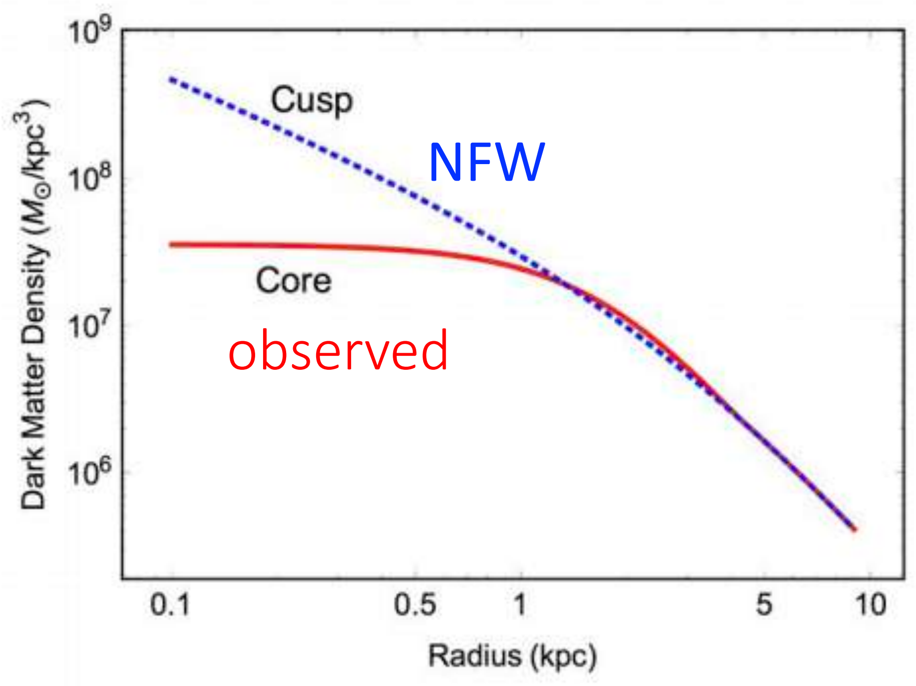


THINGS (HI Near Galaxy Survey)/LITTLE THINGS

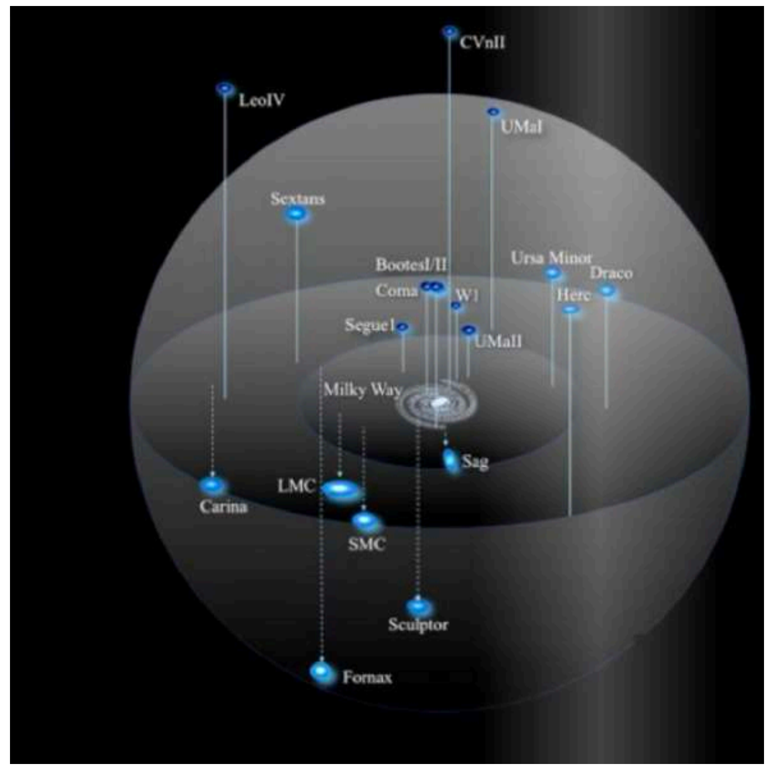
Density

Radius

CDM -
NFW profile



Missing satellites



Bullock, Baylon-Kolchin

Incompatibility between the # of satellites predicted by simulations using LCDM and the # of observed satellites

Regularity/diversity of rotation curves

But what is dark matter?
What is its nature/microphysics?

How can we build a model of DM?

Model building:

Pre-requisites for a dark matter candidate

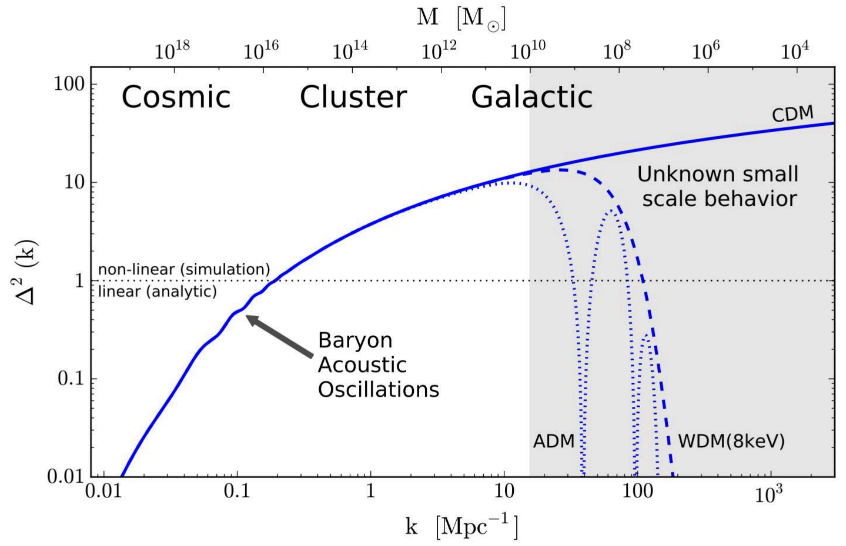
- Cold or warm Thermal candidate: $m_{dm} \geq \mathrm{keV}$ Or produced cold by a non-thermal mechanism

  Has to be non-relativistic at BBN

- Reproduce large and small scale distribution

  Clusters like pressure-less fluid on large scales $k \lesssim 10\,\mathrm{Mpc}^{-1}$

  Clustering on scales smaller than $k \gtrsim 10\,\mathrm{Mpc}^{-1}$ highly unconstrained



- Non-interacting or weakly interacting (~~Dark, colissionless~~)

  Can have a small electromagnetic interaction. Bound < **milicharge**

  Can have a **self interaction**. Bounds: $\sigma/m_{dm} < 0.13\,\mathrm{cm}^2/\mathrm{g}\,,\ \sigma/m_{dm} < 0.35\,\mathrm{cm}^2/\mathrm{g}$

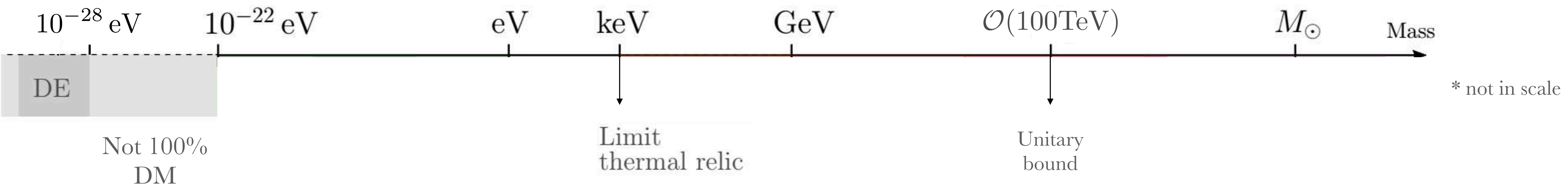
  Can interact via the ***weak force***

- Abundance $\Omega_m = 0.308 \pm 0.012$ (*Planck 2018*)

- Stable **If** it is a particle, it has to be stable with lifetime of DM should be much greater than the age of the universe

Mass scale of dark matter

Observations from both LSS and local, can put model-independent bounds on DM parameters, like mass and spin.



Observations:

- LSS
  - Recombination
- BBN
- Galaxy clusters
- Galaxy properties: namely galaxy densities must reach of order GeV cm−3, their velocity dispersions are of order 100 km s−1, and their sizes are of order kpc.
- Star clusters

LSS

Intermediary

Small scale structure

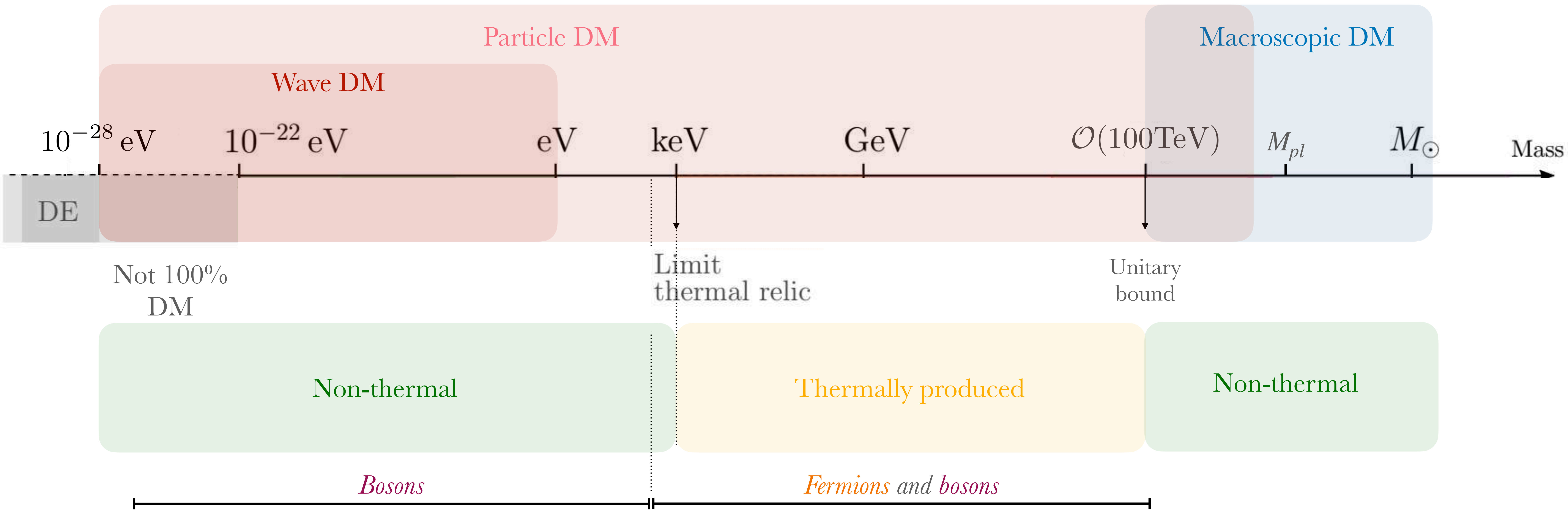
- electron mass 0.511 MeV

Natural units ($c = 1$)

$1\,\mathrm{kg} \to \sim 5 \times 10^{35}\,\mathrm{eV}$

$1\,M_\odot \to \sim 10^{66}\,\mathrm{eV}$

Mass scale of dark matter



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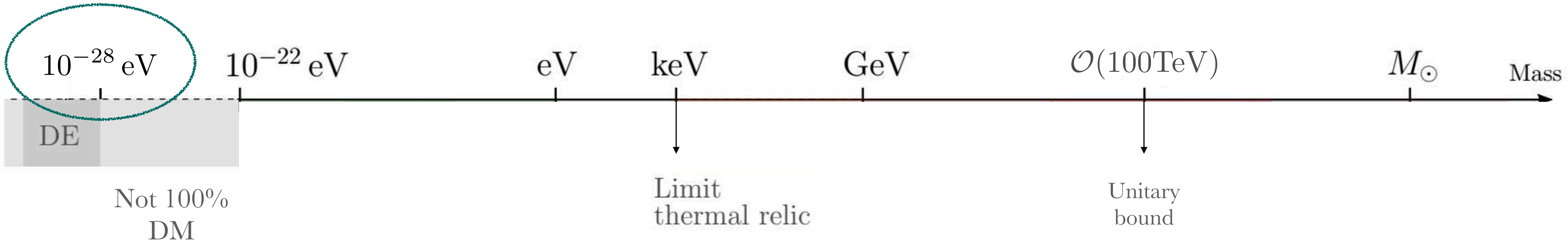


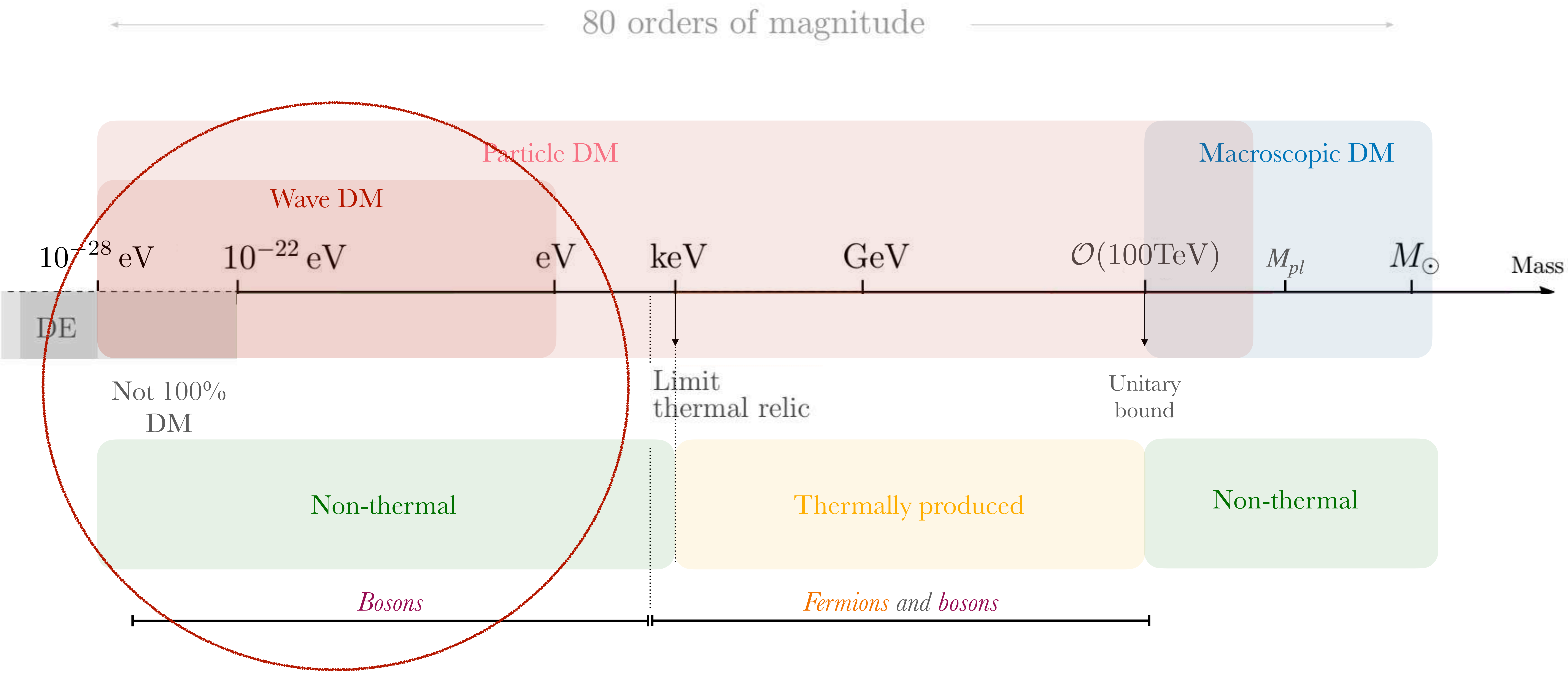
- Lower limit

This candidate is described by **bosons**. If for example we consider a ***spin 0*** particle, described by a **scalar field**.

Natural units ($c = 1$)
$1\,\text{kg} \to \sim 5 \times 10^{35}\,\text{eV}$
$1\,M_\odot \to \sim 10^{66}\,\text{eV}$

Mass scale of dark matter

⇒ We can use observations of LSS and galaxies to put bounds in the “particle" physics properties, like mass and spin, of the DM candidate



These lectures!

Natural units ($c = 1$)

$1\,\text{kg} \to \sim 5 \times 10^{35}\,\text{eV}$

$1\,M_\odot \to \sim 10^{66}\,\text{eV}$

Given these properties, what are the possibilities for a DM candidate?

Landscape of dark matter models

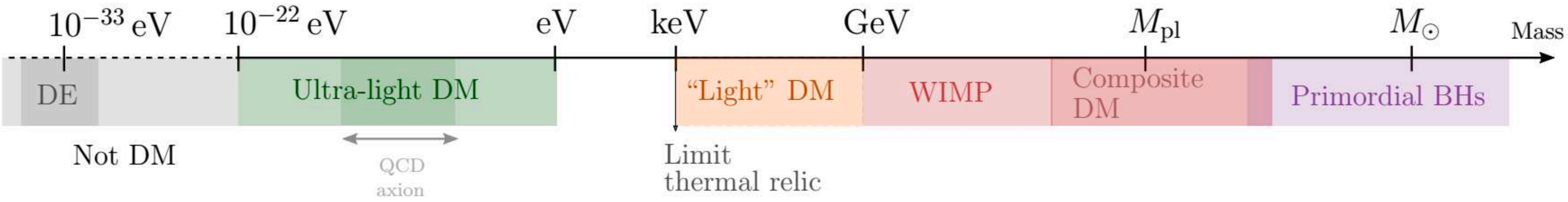
Landscape of dark matter models

- What is DM? What is the nature of DM?

State of the "art"

Mass scale of DM

80 orders of magnitude



Landscape of dark matter models

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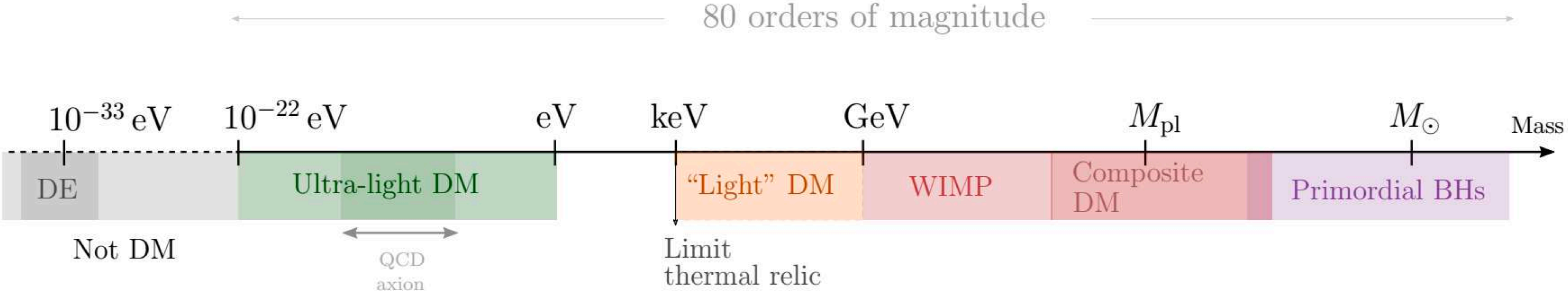
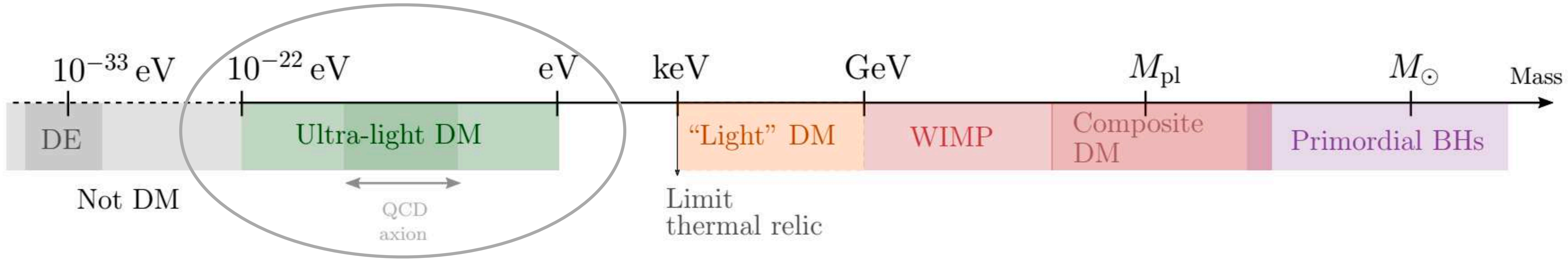


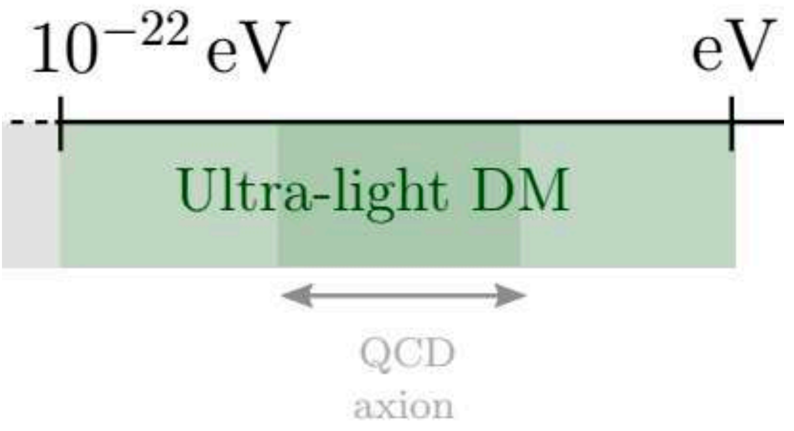
Ultra-light dark matter



Definition of ULDM

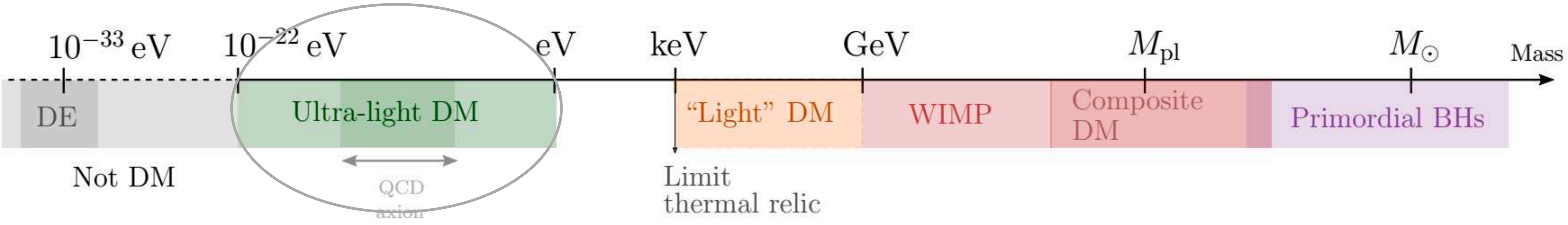
Mass range and wave behaviour

Ultra-light dark matter

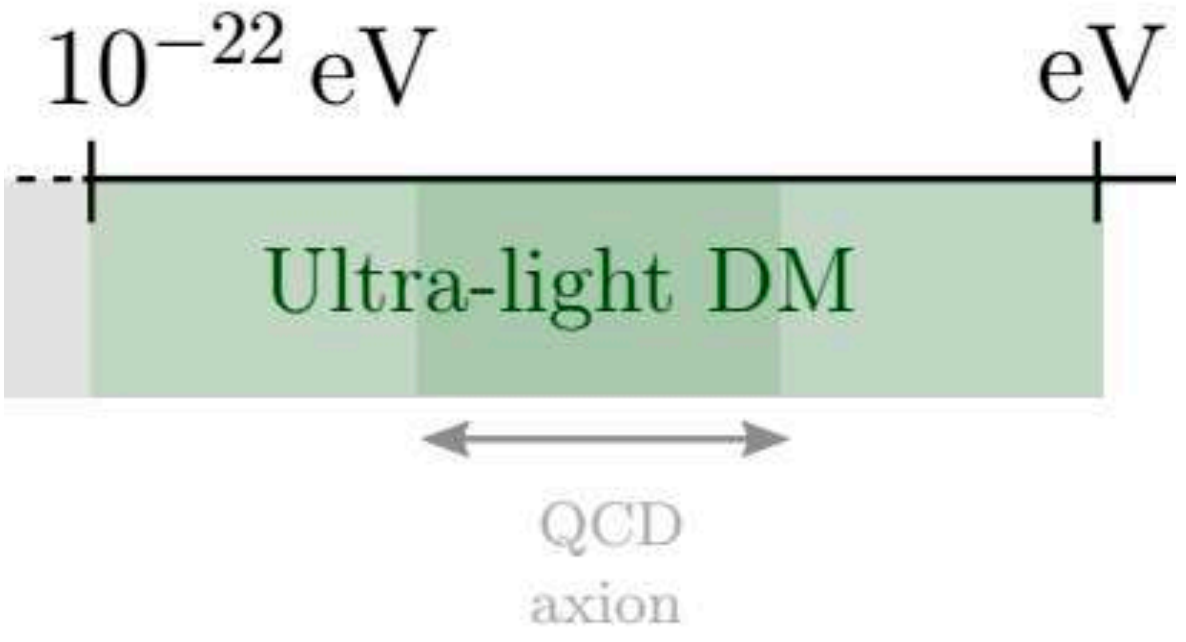


Ultra-light candidate, cold $\longrightarrow$ Large $\lambda_{\rm dB} \sim 1/mv$

Lightest possible candidate for DM



10^{-57} kg $\qquad$ 10^{-35} kg



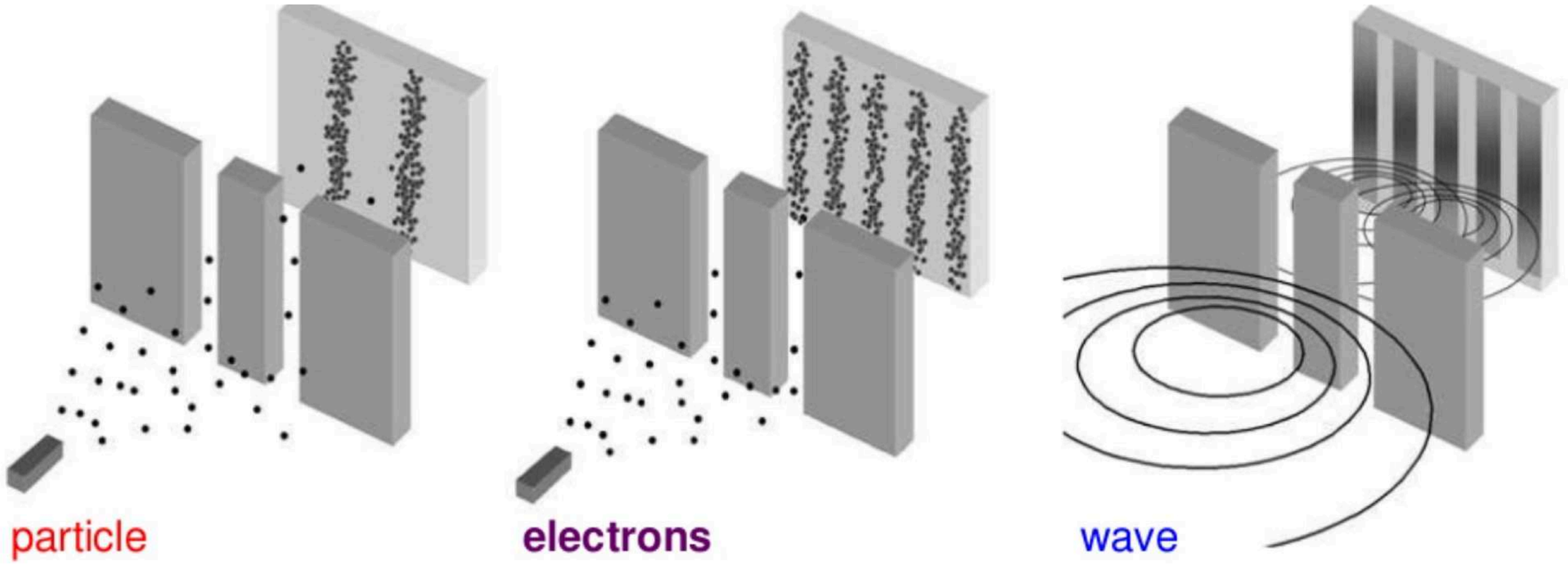
$\longrightarrow$ Bosons (scalar fields)

Non-thermally produced

Wave-Particle duality

All matter exhibits a wave behaviour

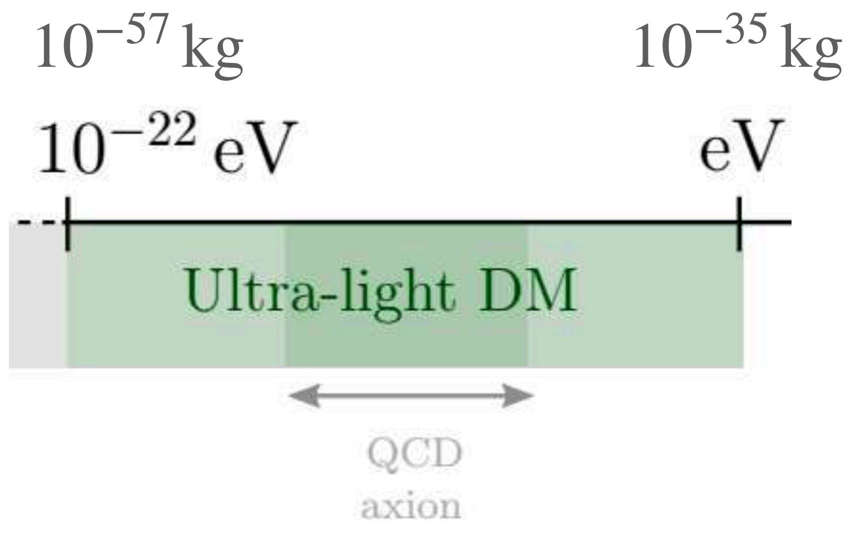
De Broglie 1924



$$\lambda_{dB} \sim \frac{1}{mv}$$

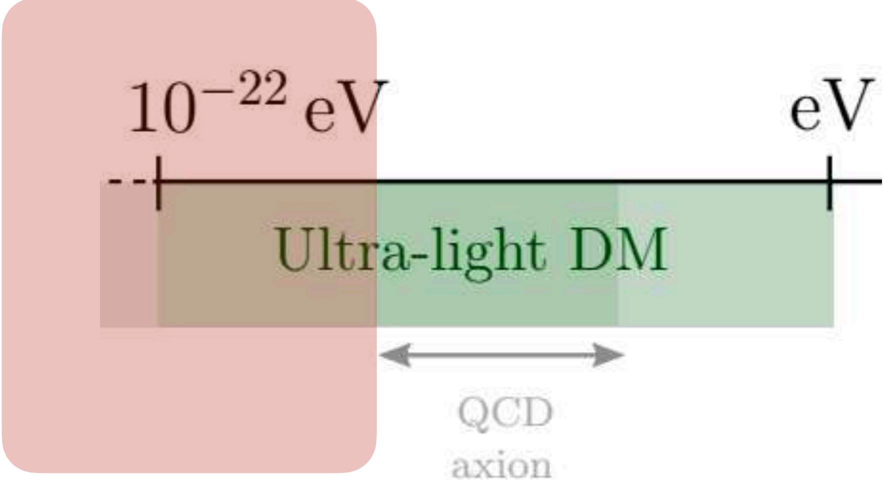
$$\lambda_{dB} \sim 1/\sqrt{2\pi m k_B T}$$

	Mass (kg)	Speed (m/s)	λ_{dB} (m)
Accelerated e-	9.1×10^{-31}	5.9×10^{6}	1.2×10^{-10}
Golf ball	0.045	220	4.8×10^{-30}



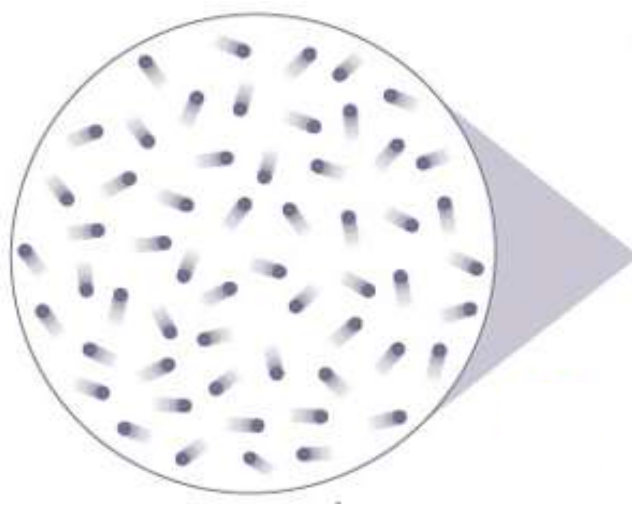
$$\lambda_{dB}^{ULDM} \sim \text{pc} - \text{kpc}$$

Ultra-light dark matter



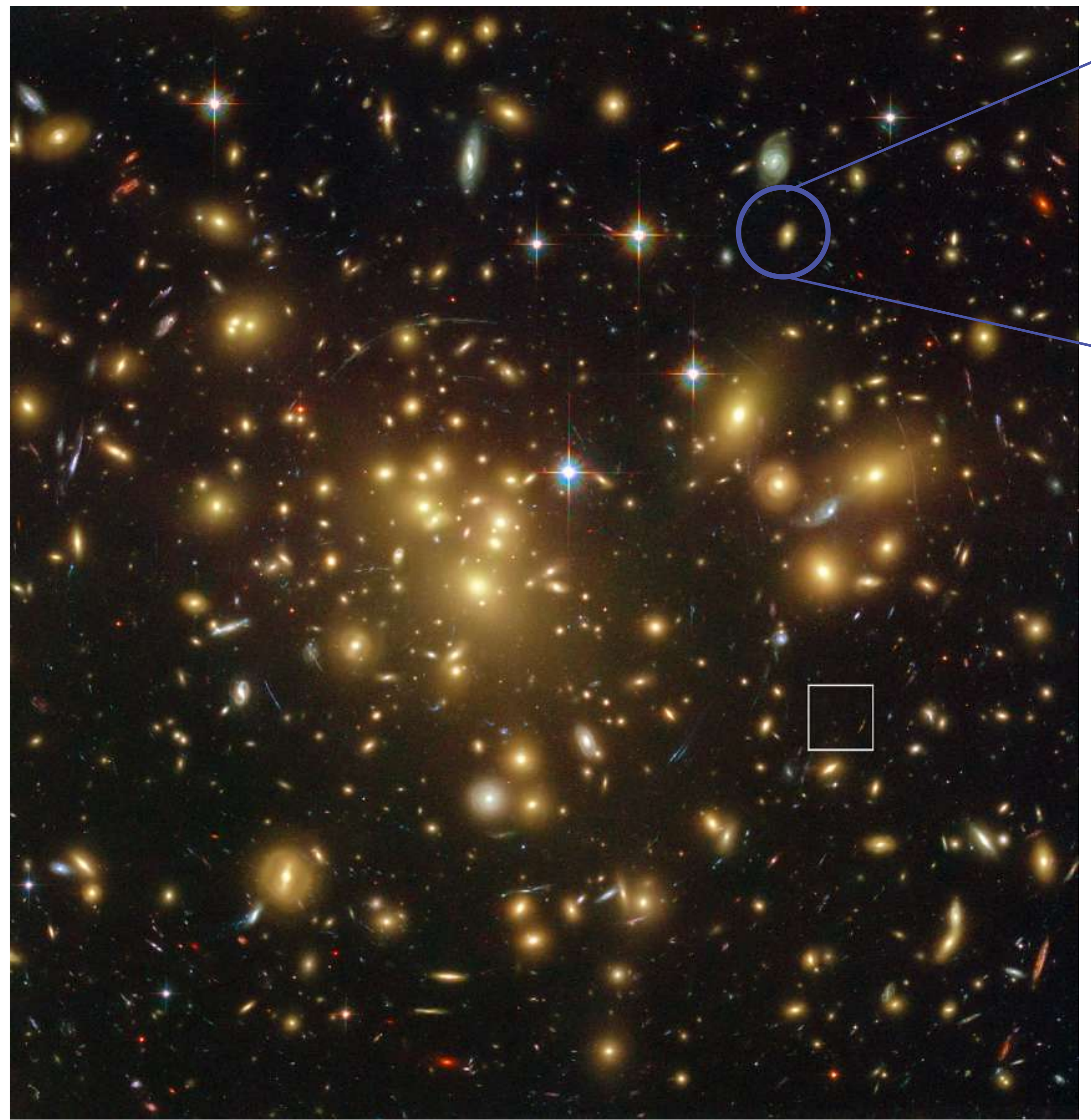
Ultra-light candidate $\longrightarrow$ Large $\lambda_{\mathrm{dB}} \sim 1/mv$

Large scales:
DM behaves like standard particle DM (CDM).

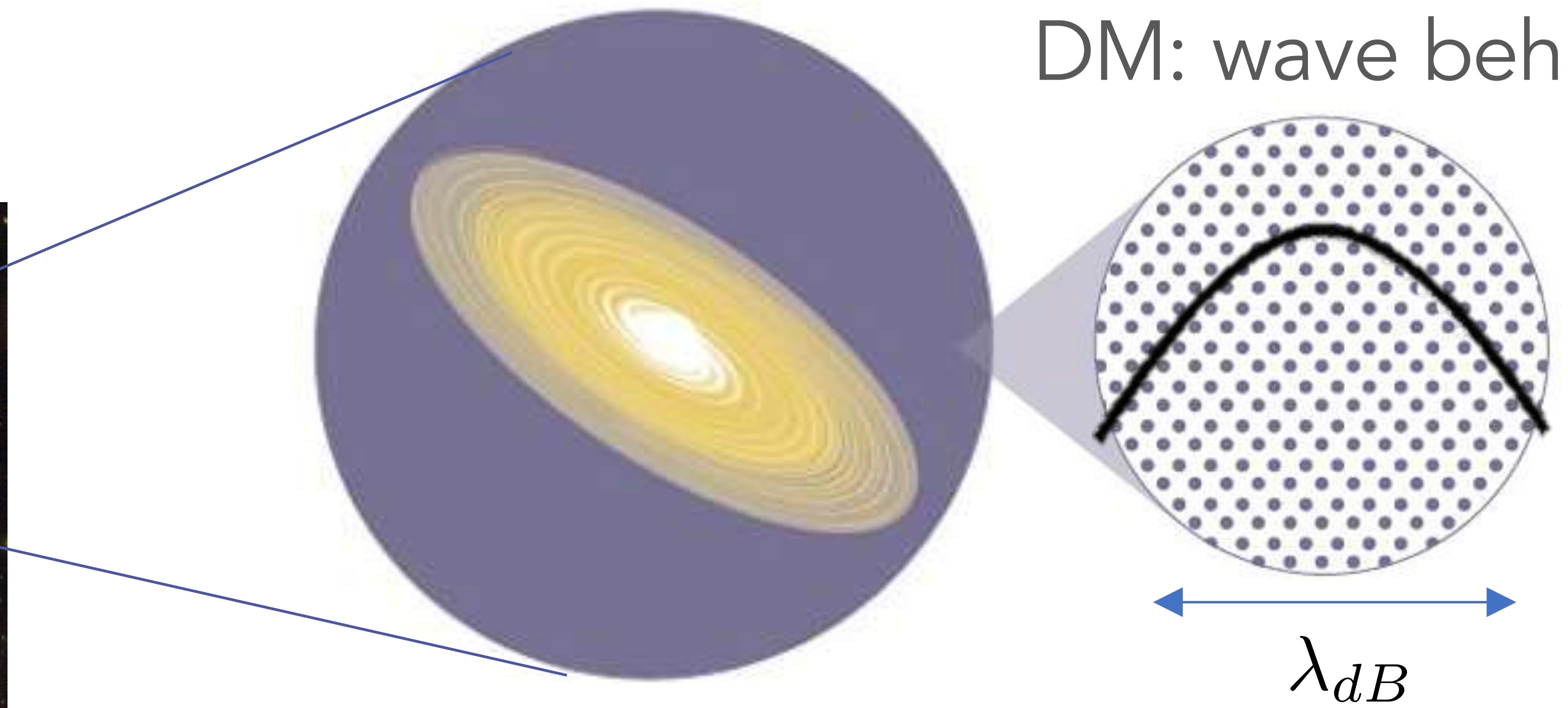


DM: particles

$d \gg \lambda_{dB}$



Adapted from Quanta



DM: wave behaviour

λ_{dB}

Galaxy halo

$d \ll \lambda_{dB}$

Small scales:
DM behaves like a wave

10^{-60} kg $\quad$ 10^{-35} kg

$$\boxed{10^{-25}\,\mathrm{eV} \lesssim m \lesssim \mathrm{eV}}$$

$$\lambda_{dB}^{ULDM} \sim \mathrm{pc} - \mathrm{kpc}$$

"Ultra-light dark matter", **EF**, 2020.

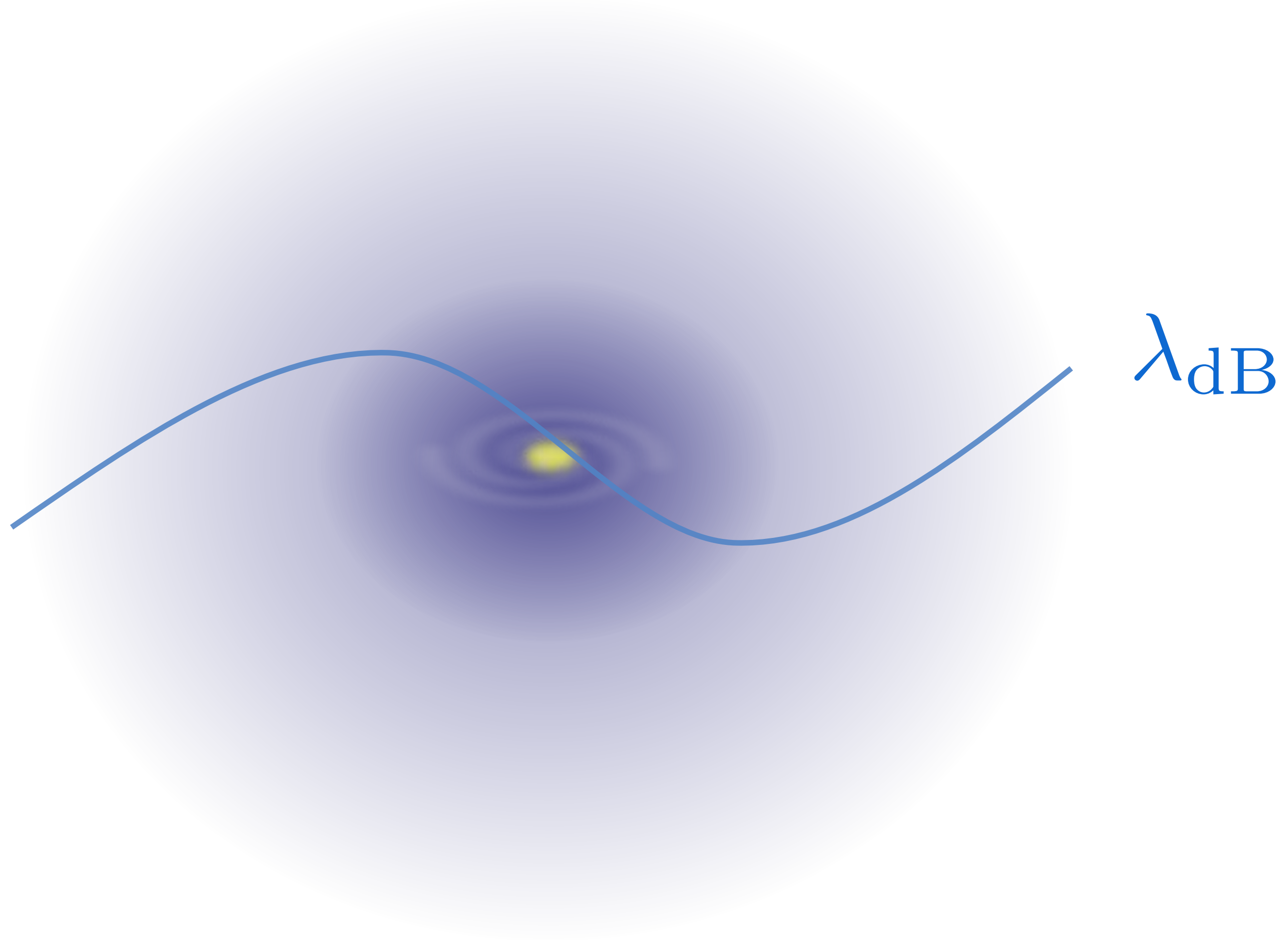
How light is ultra-light?

Behave as wave on galactic scales:

- λ_{dB} must be smaller than the halo

$$\lambda_{\mathrm{dB}} < R_{\mathrm{halo}}$$

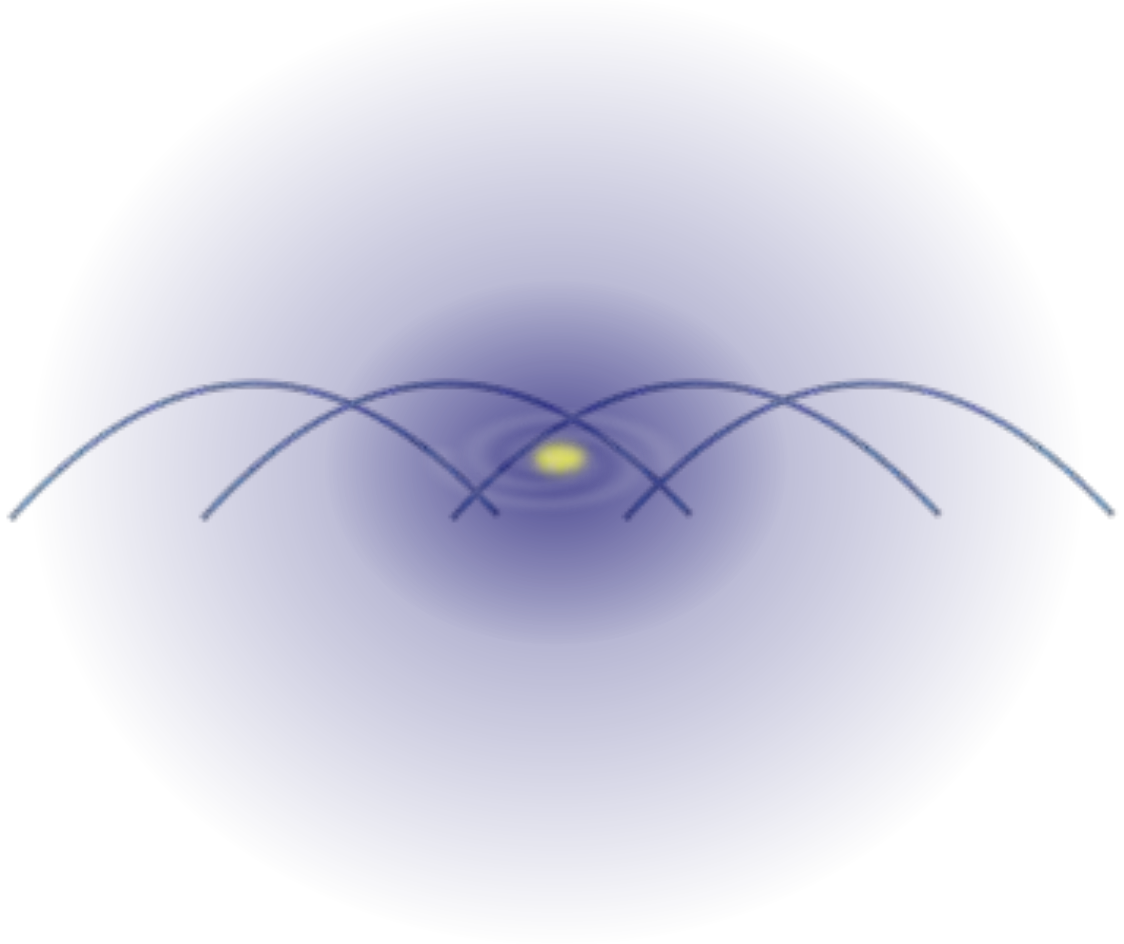
$$\implies \quad m \gtrsim 10^{-25}\,\mathrm{eV}$$



- λ_{dB} overlap to be of halo size

$$\lambda_b \sim \frac{1}{mv} \geq d \sim \left(\frac{m}{\rho_{vir}}\right)^{\frac{1}{3}}$$

$$\implies \quad m \leq 2\mathrm{eV}$$



$10^{-60}\,\mathrm{kg}$ $\quad$ $10^{-35}\,\mathrm{kg}$

$$\boxed{10^{-25}\,\mathrm{eV} \lesssim m \lesssim \mathrm{eV}}$$

$$\lambda_{dB}^{ULDM} \sim \mathrm{pc} - \mathrm{kpc}$$

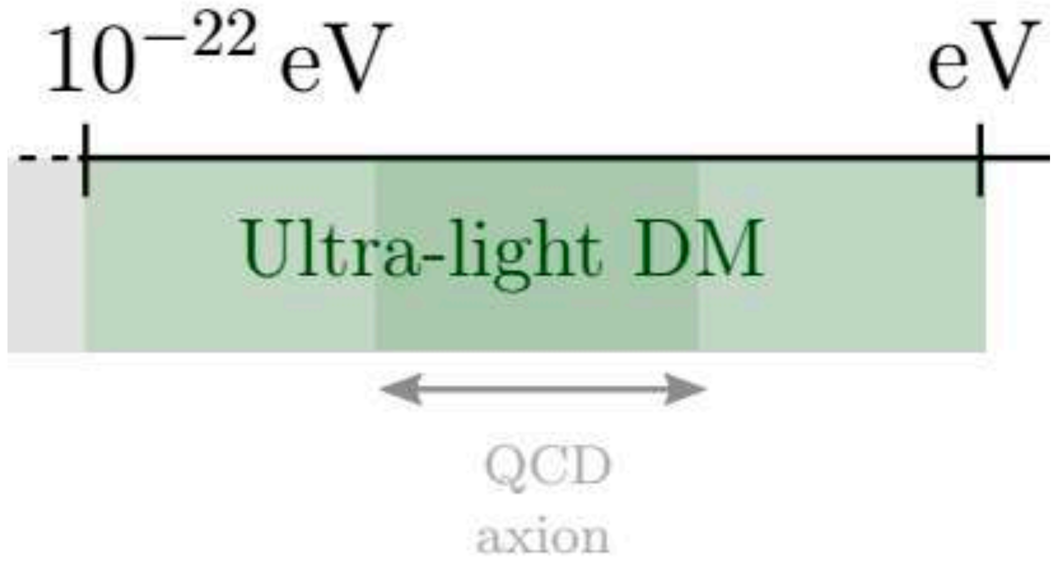
Definition of ULDM

Candidates

Motivation: particle physics

ULDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson (breaking of an approximate symmetry)



Known PNGB: QCD axion → Candidate for DM

(Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978)

Axion-like particles

Axions or Axion like particles (ALP)

Axions and ALPs are pseudo Nambu Goldstone bosons from the spontaneous symmetry breaking of a $U_{PQ}(1)$ (U(1)) symmetry, and are described by the complex field: $\Psi = v\, e^{i\phi/f_a}$

$$v_{0,ssb} = f_a/\sqrt{2} \qquad \longrightarrow \qquad \phi \to \phi + c$$

Non-perturbative effects (from string theory or instatons) induce a potential:

$$V(\phi) = \Lambda_a^4\,[1 - \cos(\phi/f_a)] \xrightarrow[\phi \ll f_a]{} \frac{1}{2}m^2\phi^2 + \frac{g}{4}\phi^4 + \cdots$$

Motivation: particle physics

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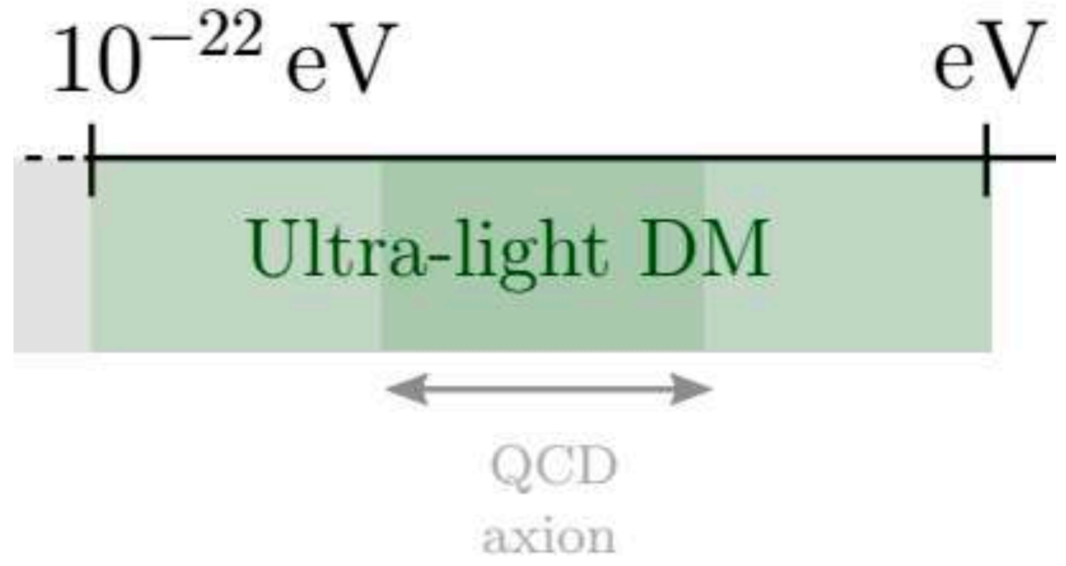
(Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978)



Axion-like particles or ultra-light axions:

- ALPs expected in string theory *(Arvanitaki et al., Svrcek, Witten)*
- Can generate PNGB that are ultra-light

- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance

Non-thermal mechanism (e.g. mis-alignement, decay of defects, …)

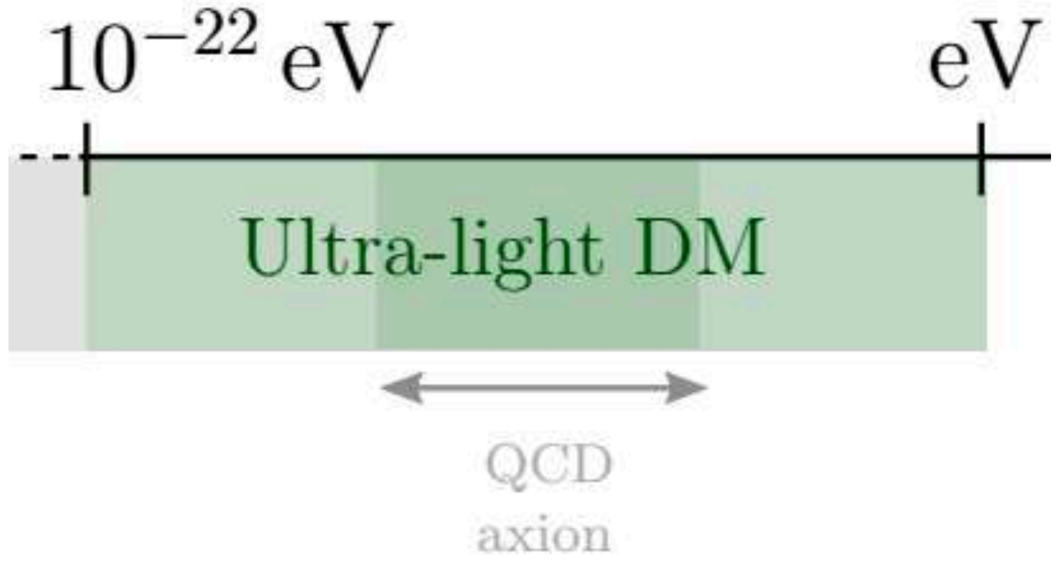
$$\Omega_{axion} \sim 0.15 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{7/6} \theta_1^2$$

$$\Omega_{ALP} \sim 0.1 \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^{2} \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)$$

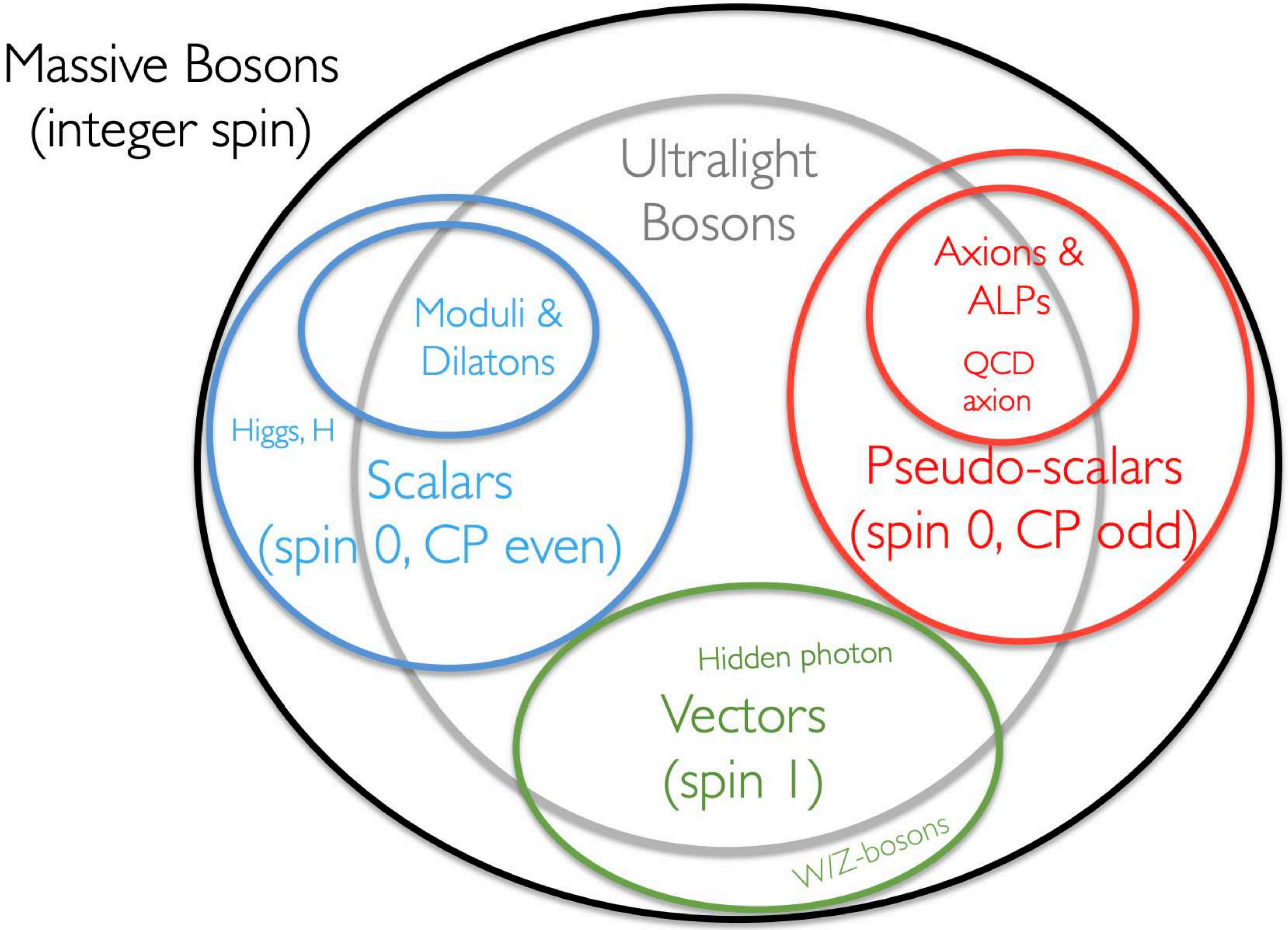
L. Hui

Motivation: particle physics

ULDM candidates



Many extensions of the Standard Model predict additional massive bosons



Ref.: Chadha-Day et al 2022

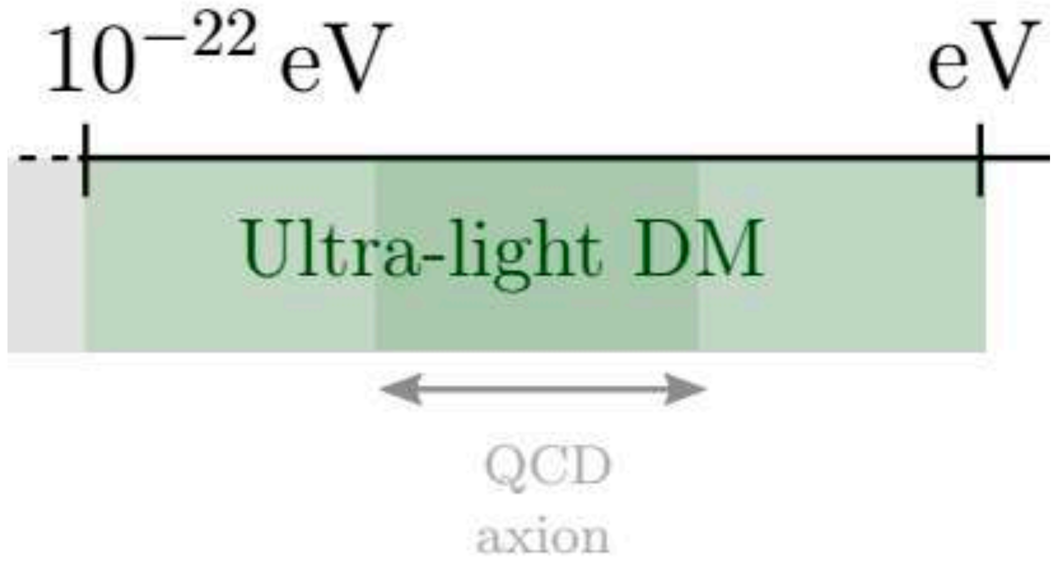
Motivation: particle physics

ULDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson

Known PNGB: QCD axion → Candidate for DM

(Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978)



Axion-like particles or ultra-light axions:

- ALPs expected in string theory *(Arvanitaki et al., Svrcek, Witten)*
- Can generate PNGB that are ultra-light

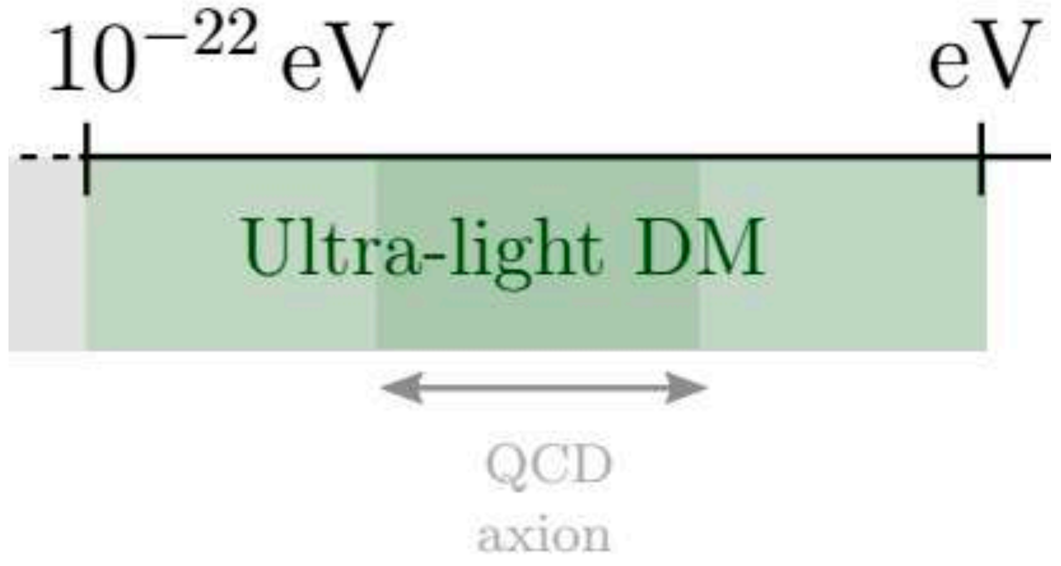
- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance

Spin-0: Non-thermal mechanism (e.g. misalignement)

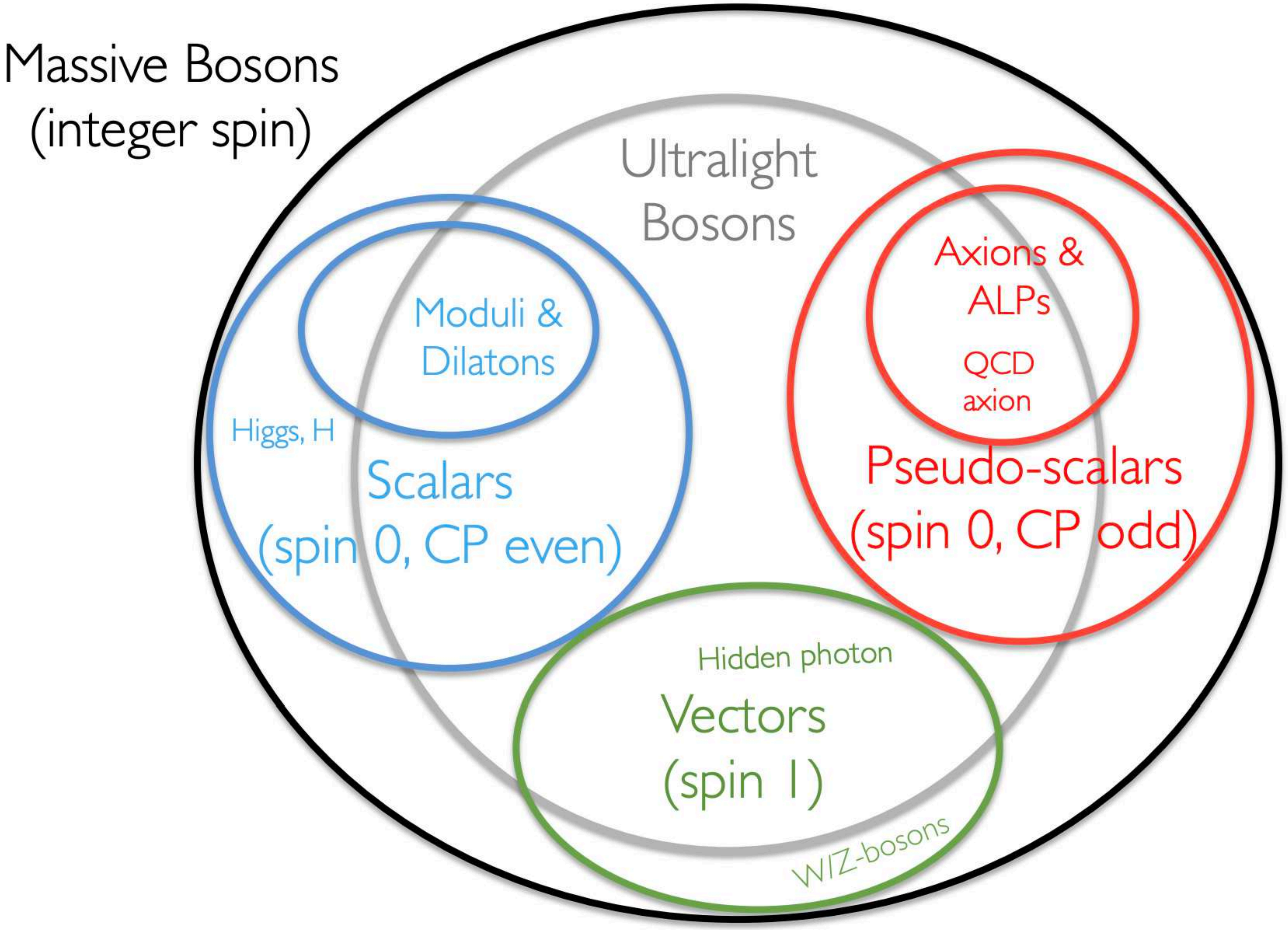
Vector FDM: challenging in the ultra-light regime
(e.g. from misalignment requires non-minimal couplings to Ricci scalar -> viol. of unitarity long. graviton-photon scattering; oscillating Higgs or oscillating misaligned axion - resonant production - choices for couplings for right abundance)
Spin 2 FDM: (e.g bigravity)

Motivation: particle physics

ULDM candidates



Many extensions of the Standard Model predict additional massive bosons

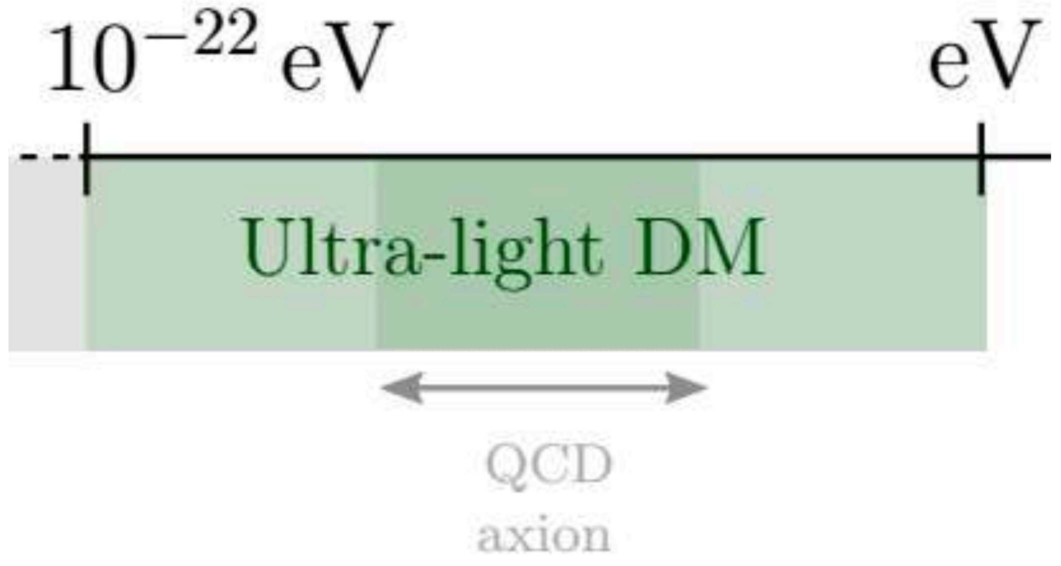


WISPs are a subset of ULDM

Ref.: Chadha-Day et al 2022

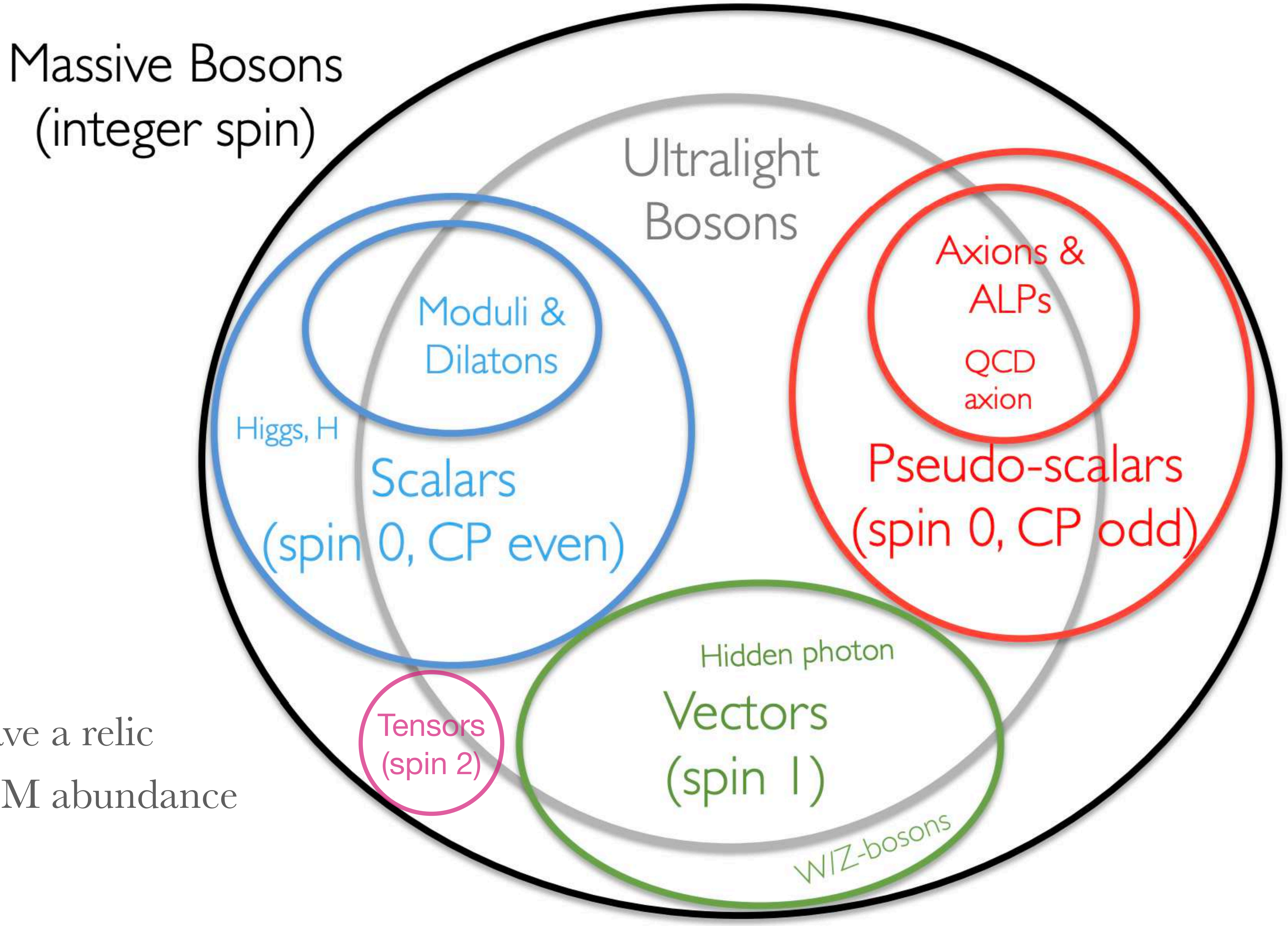
Motivation: particle physics

ULDM candidates



Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson

Many extensions of the Standard Model predict additional massive bosons

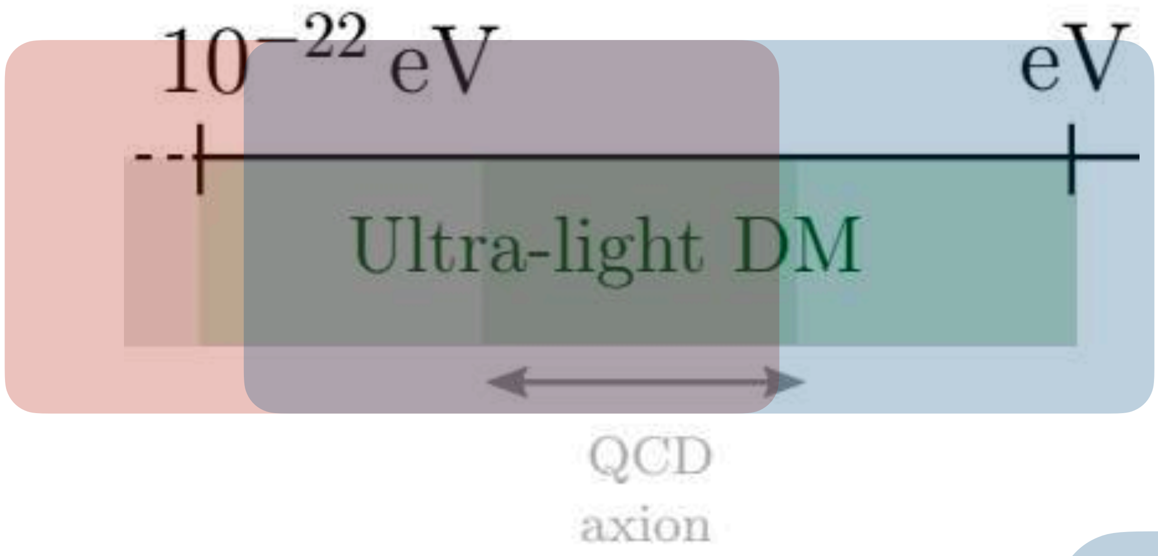


- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance

Ref.: Modified from Chadha-Day et al 2022

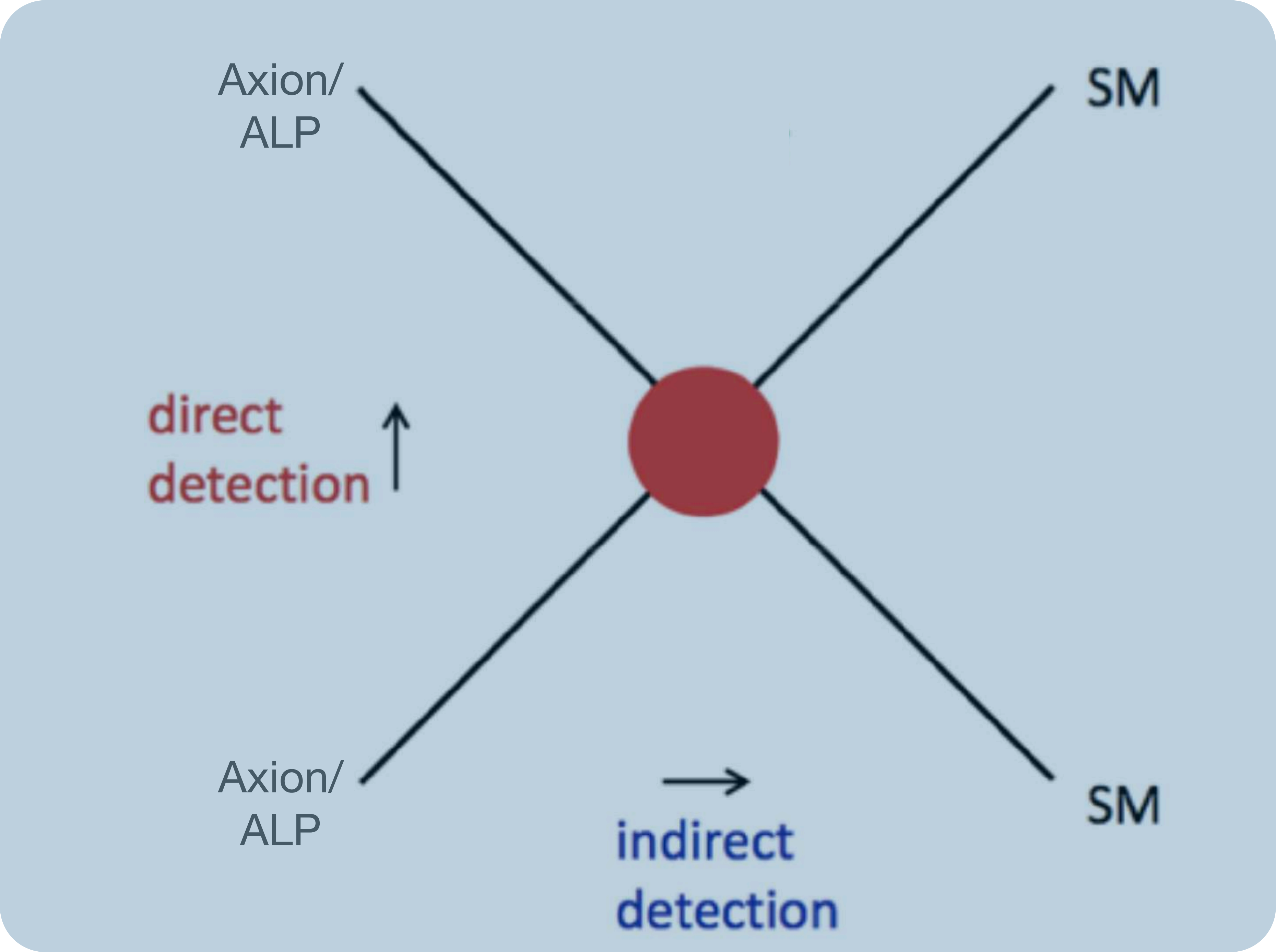
Searching for ULDM

How to search for ULDM?

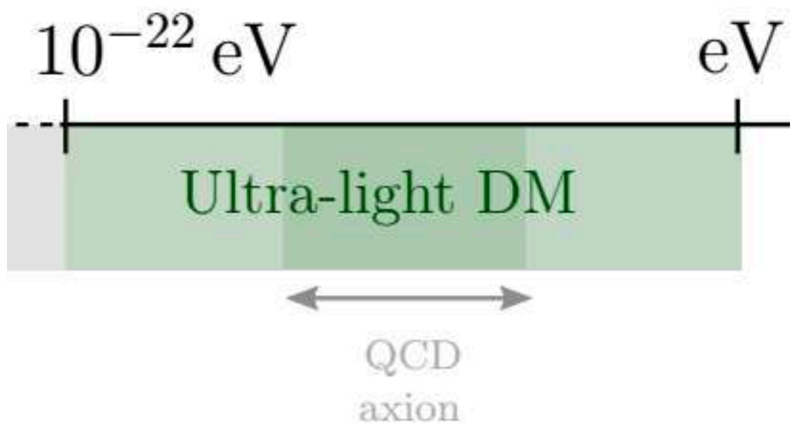


Gravitationally
Cosmological and astrophysical searches

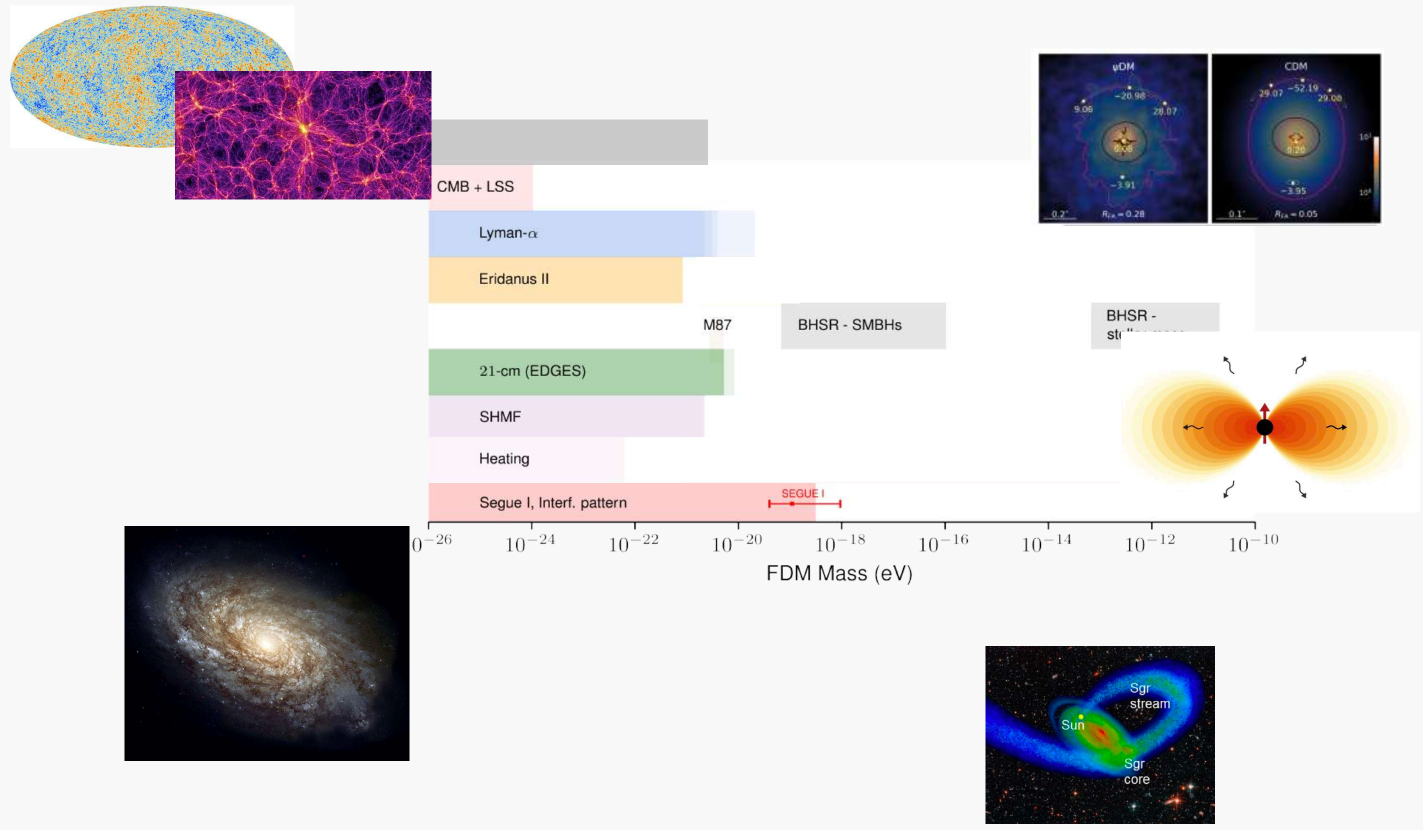
+



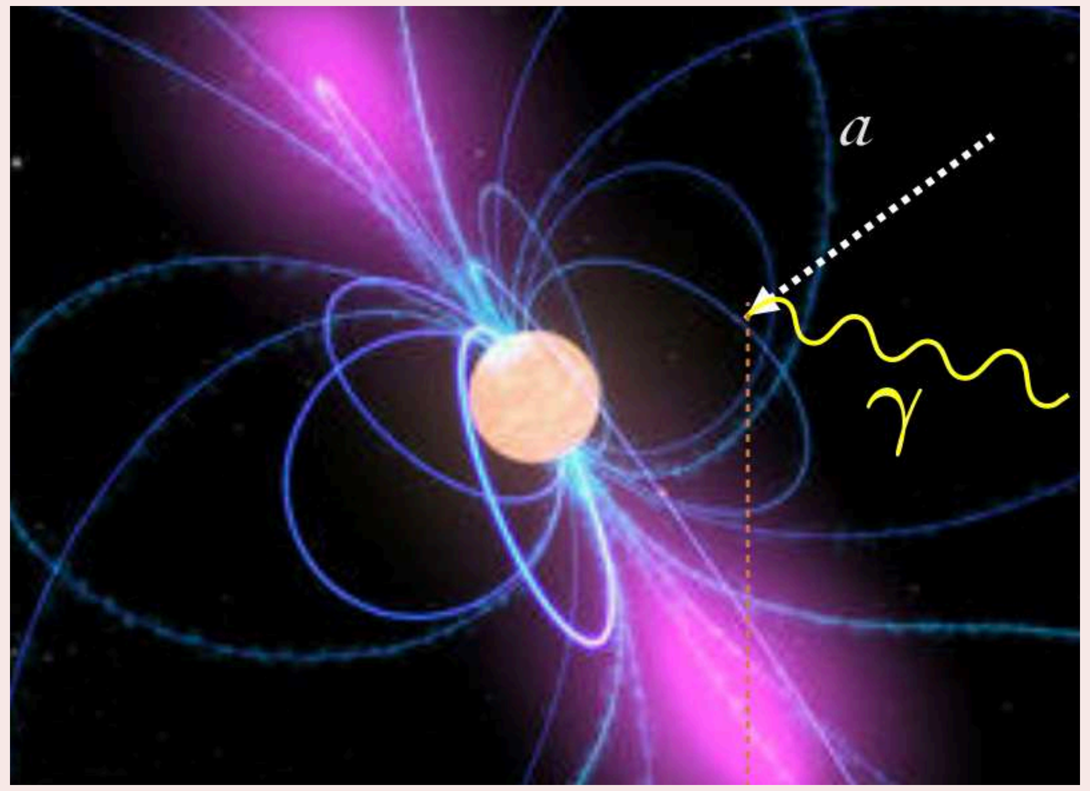
How to search for axions/ALPs?



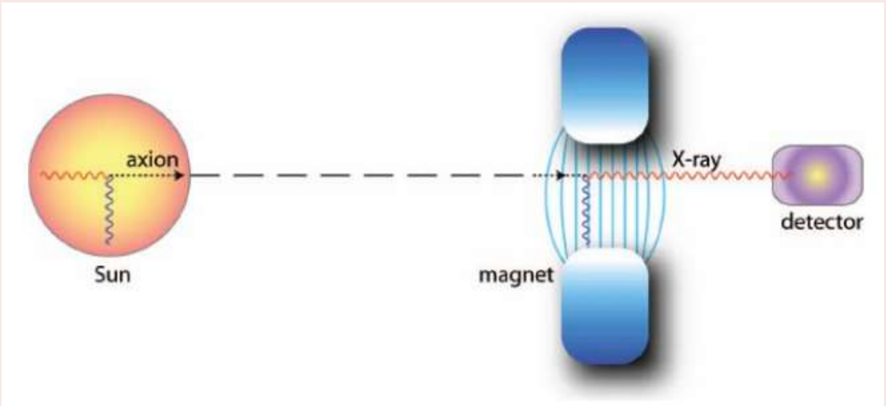
Cosmological and astrophysical searches

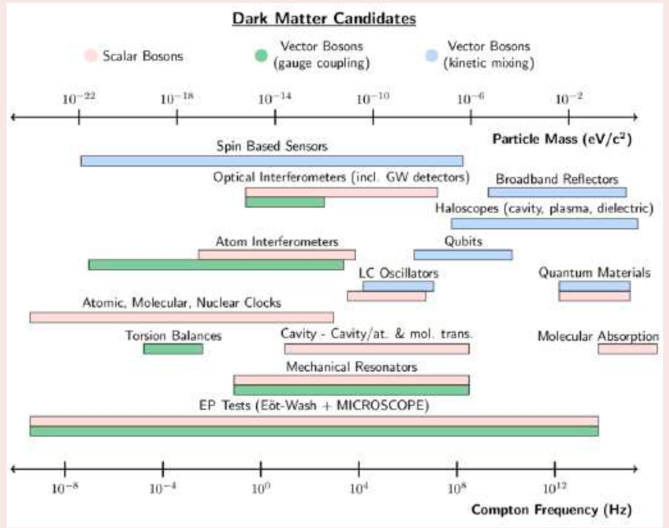


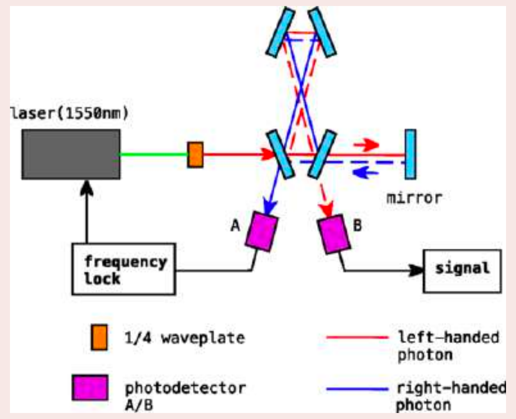
Indirect detection



"Direct detection"
Axion/ALPs experiments

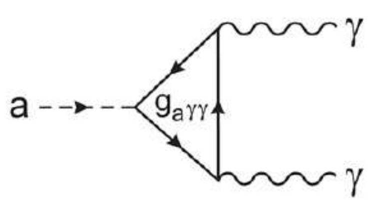


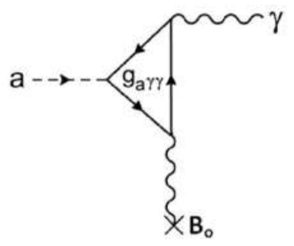




Gravitational

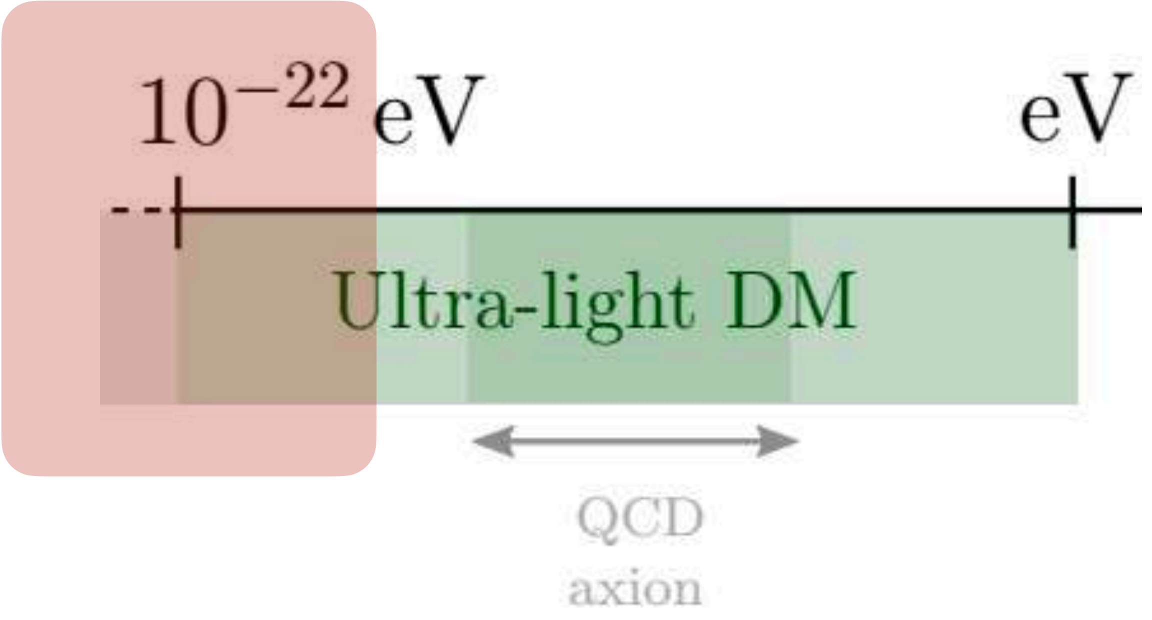
Interactions with the SM



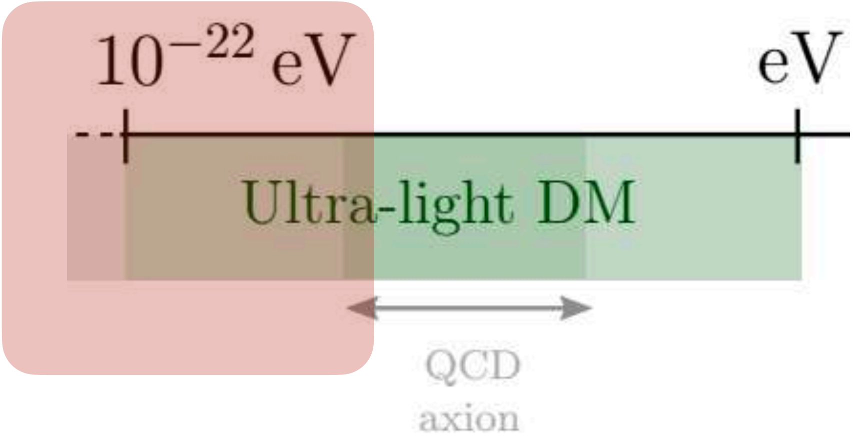


Gravitational signatures

Cosmological and astrophysical



Ultra-light Dark Matter -classes



3 classes:

DOFs

Fuzzy DM (FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

m

Self Interacting FDM (SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

m g

DM Superfluid

- Forms a superfluid in galaxies
- MOND behaviour interior of galaxies

Axion and ALP (axion like particles)

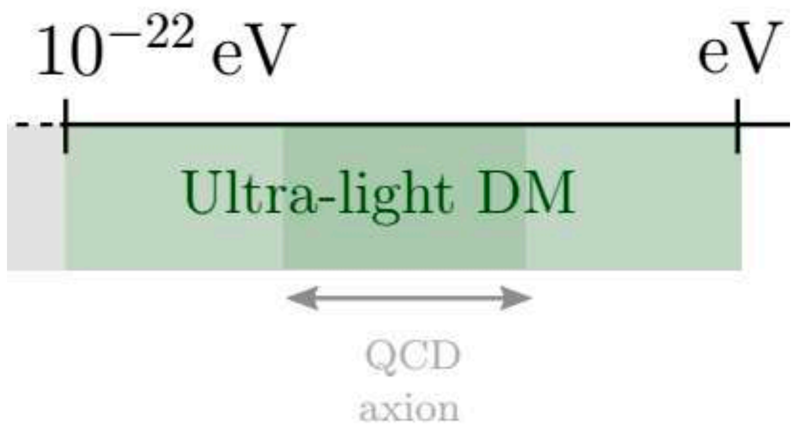
$$i\dot{\psi} = \left(-\frac{1}{2m}\nabla^2 + \frac{g}{8m^2}|\psi|^2 - m\Phi\right)\psi$$

$$\mathcal{L} = P(X)$$

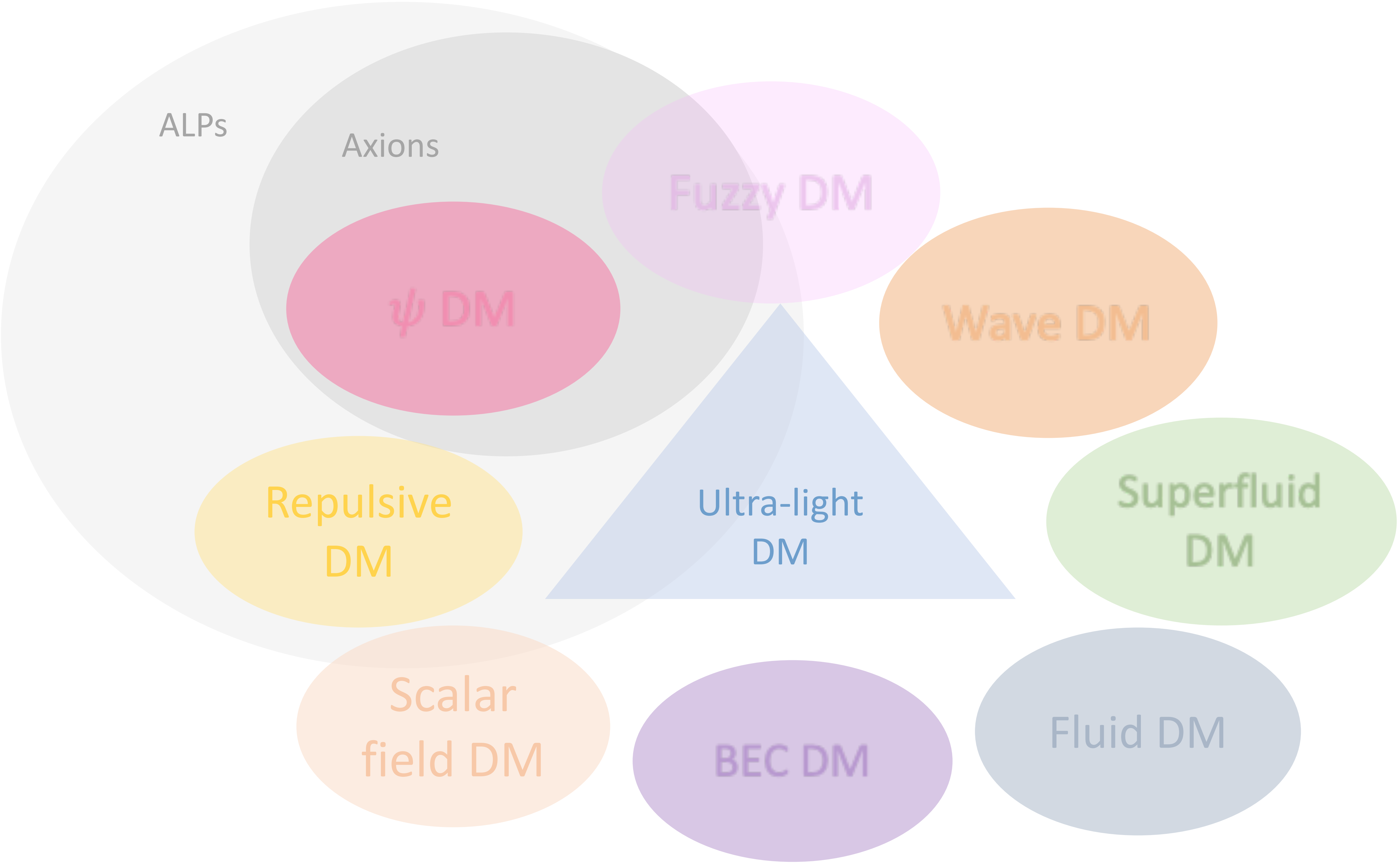
⟶ Connection with condensed matter and particle physics!

"Ultra-light dark matter", **E.Ferreira**, 2020. The Astronomy and Astrophysics Review.

Ultra-light Dark Matter - models

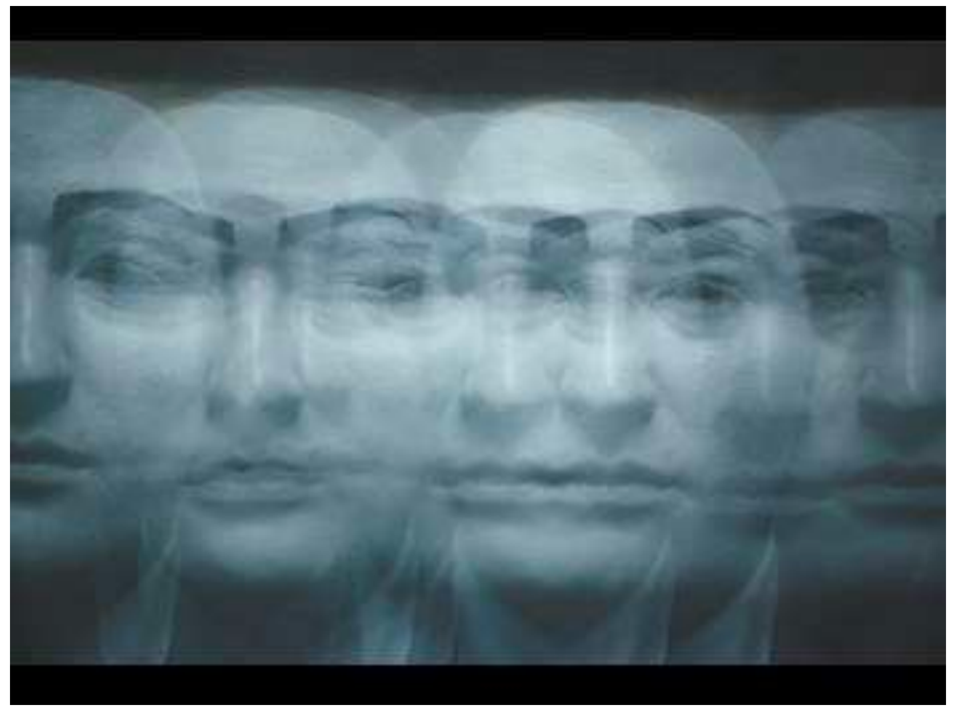


There are many ways to have a DM with this property → many ULDM models in the literature
However, each of these models presents a different dynamics on small scales - different phenomenology

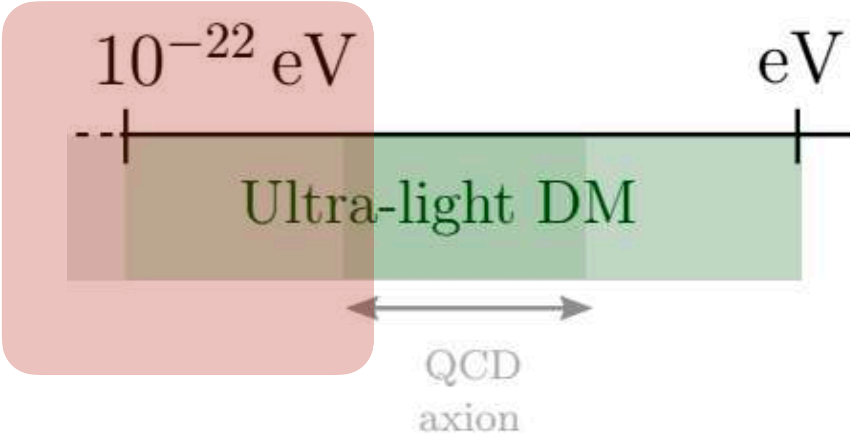


Fuzzy dark matter

Self interacting fuzzy dark matter



Fuzzy dark matter



Fuzzy DM
(FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

m

Wave DM
Ultra-light axions

Self Interacting FDM
(SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

m g

Hu W, Barkana R, Gruzinov A (2000 a,b)

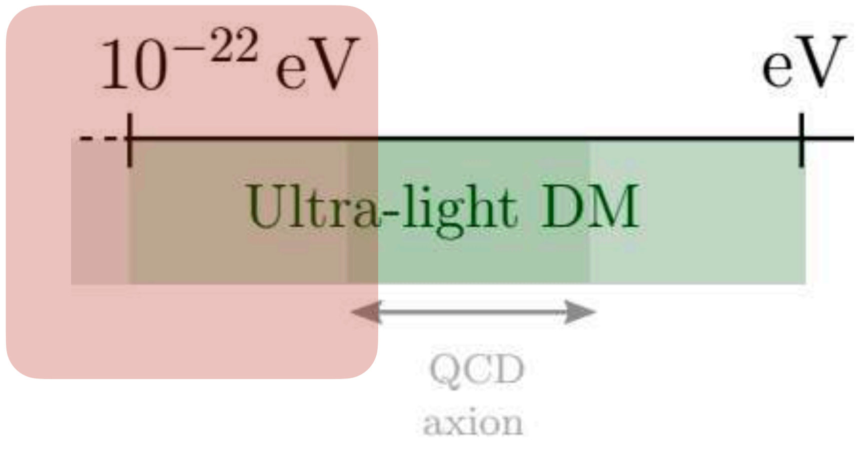
(Reviews: EF (2021), J. Niemeyer (2019), L. Hui (2021))

Idea:

$$m_{\rm fdm} \sim 10^{-22}\,\mathrm{eV}$$

address the small scale problems+ rich phenom.

Fuzzy dark matter



Fuzzy DM
(FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

m

Wave DM
Ultra-light axions

Focus more on spin 0 particles here!

(Some of the grav. phenom. is carried for vectors, for example)

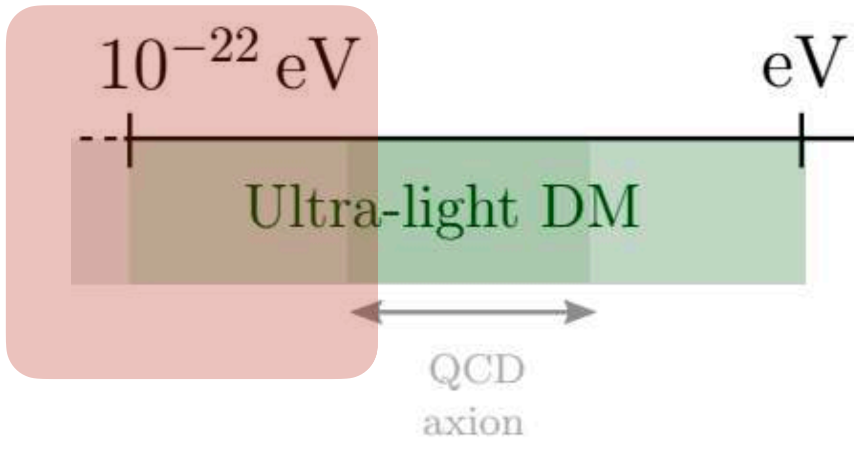
- Spin 0 - FDM
- Spin 1 - Vector FDM
- Higher spin FDM

Hu W, Barkana R, Gruzinov A (2000 a,b)

(Reviews: *EF (2021), J. Niemeyer (2019), L. Hui (2021)*)

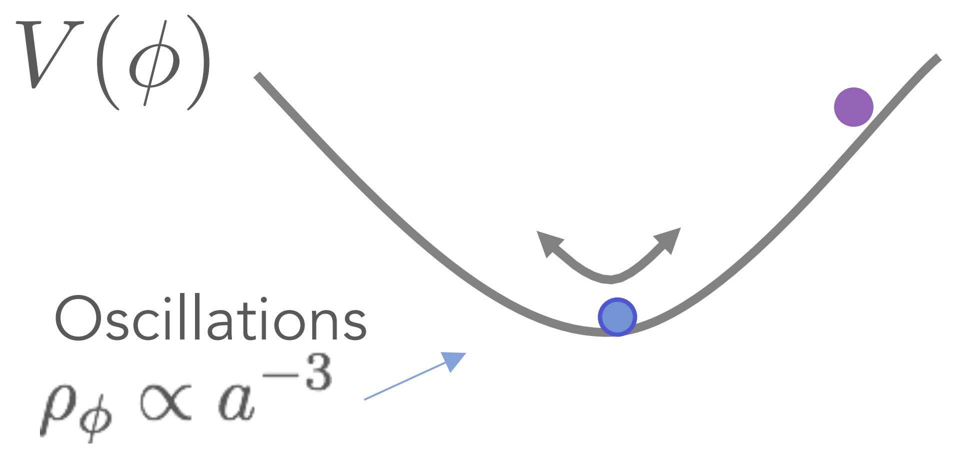
ULDM evolution

Cosmological *evolution*

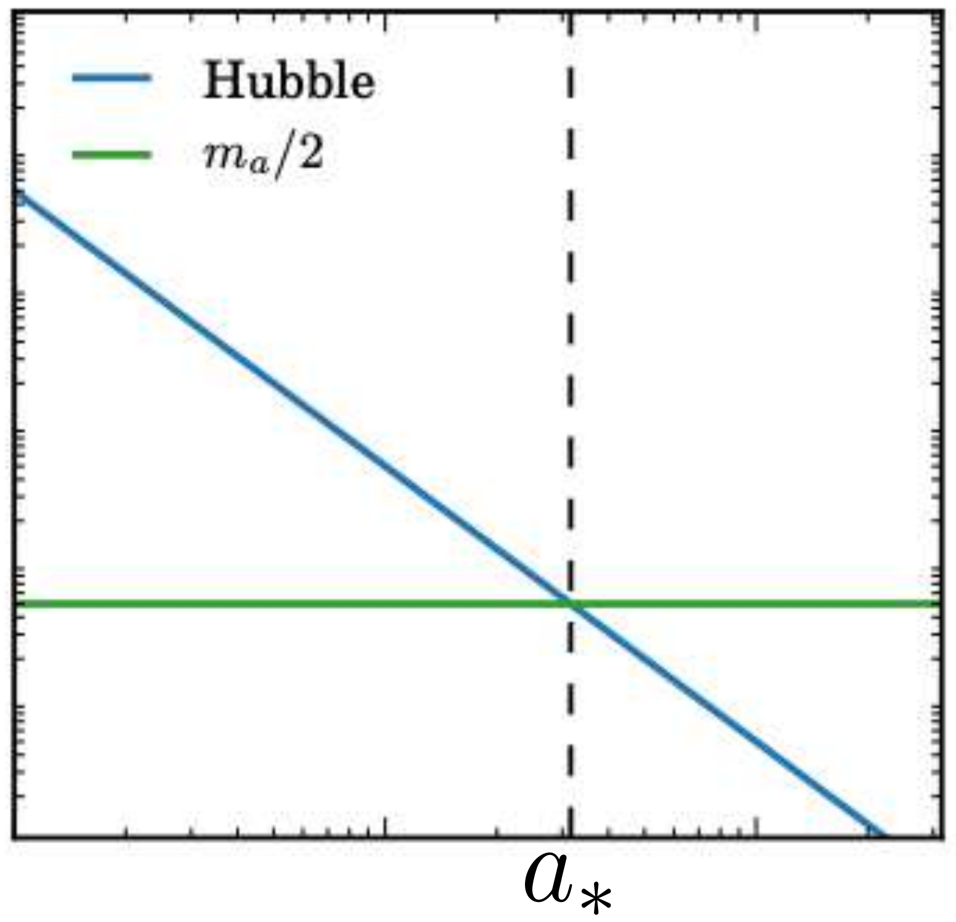


$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

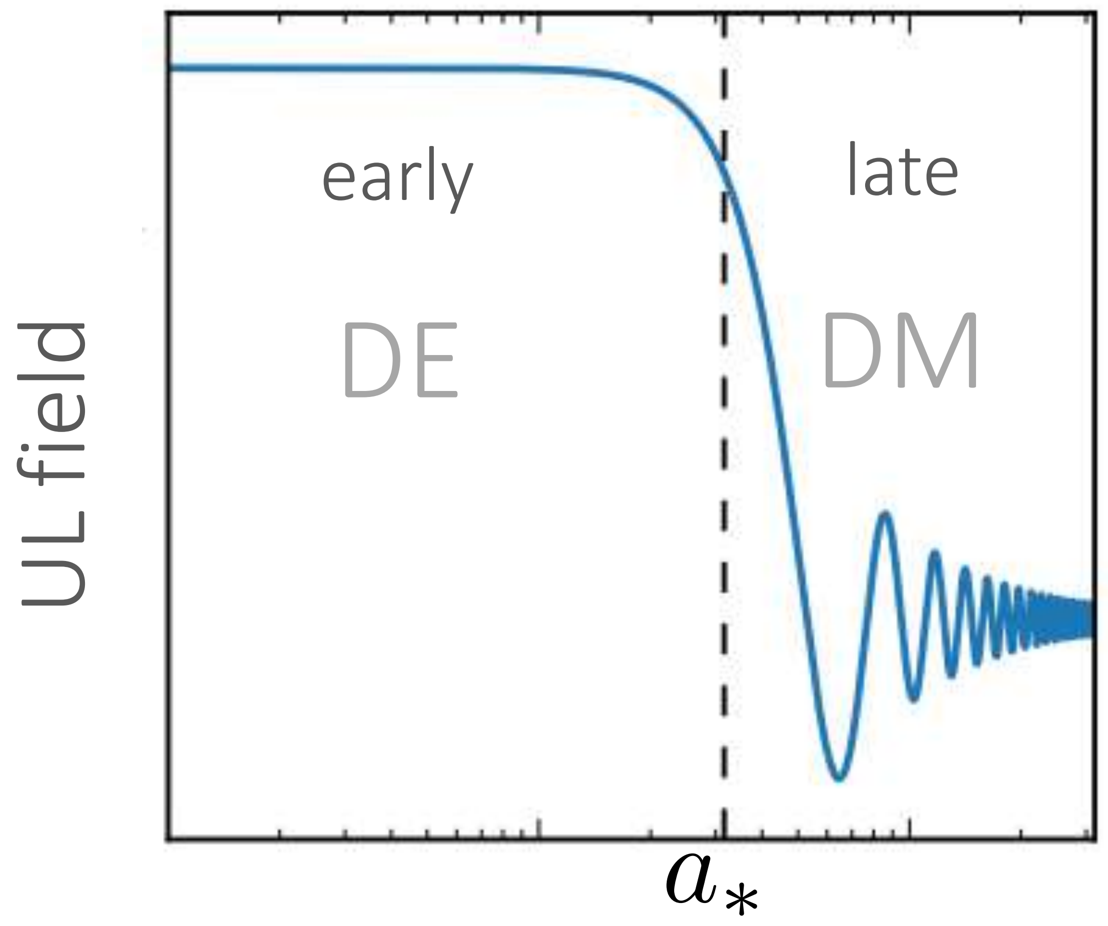
FDM



$$\begin{cases} H \gg m \implies \phi_{\text{early}} = \phi(t_i) \longrightarrow \omega = -1 & \text{DE} \\ H \ll m \implies \phi_{\text{late}} \propto e^{imt} \longrightarrow \langle \omega \rangle = 0 & \text{DM} \end{cases}$$

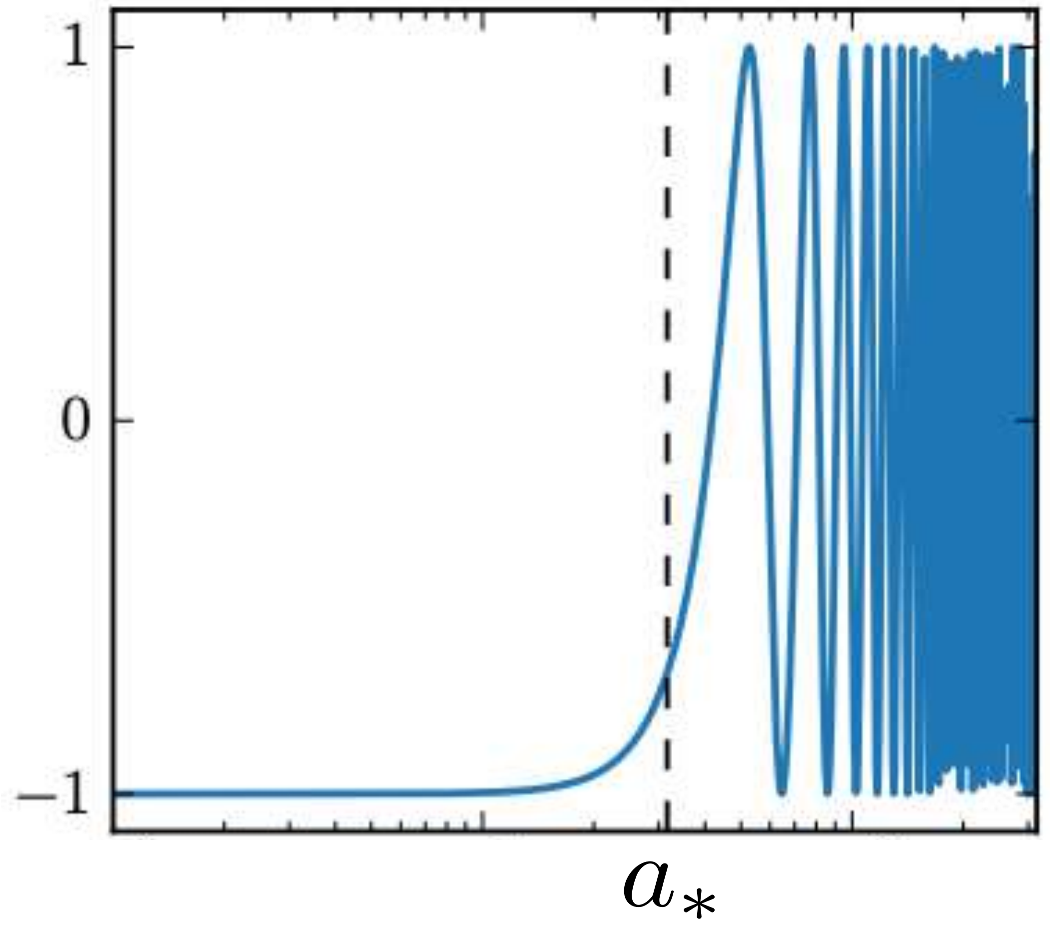


Scale factor a(t)



Scale factor a(t)

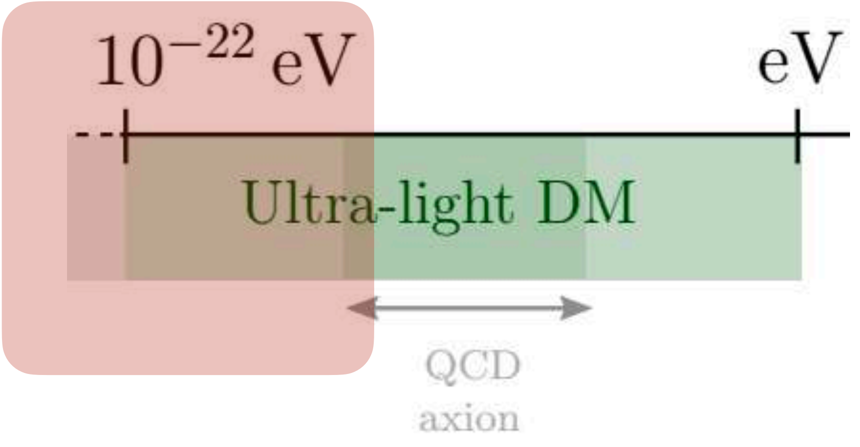
$H(t_*) = m$



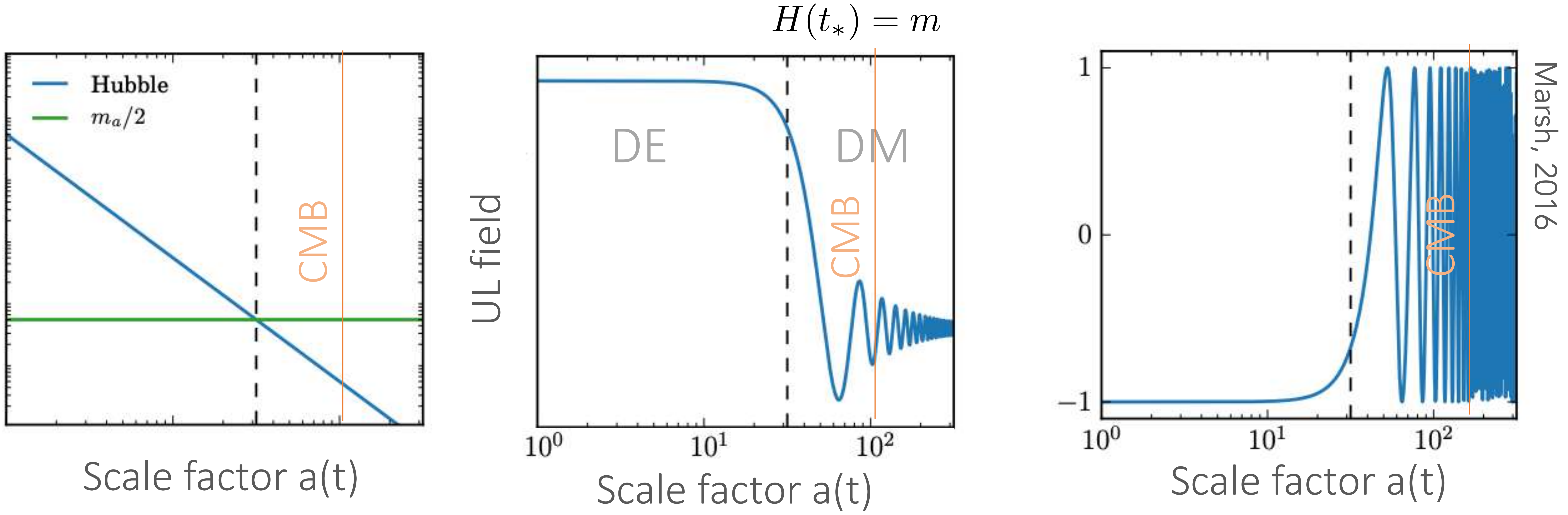
Scale factor a(t)

Marsh, 2016

Cosmological evolution



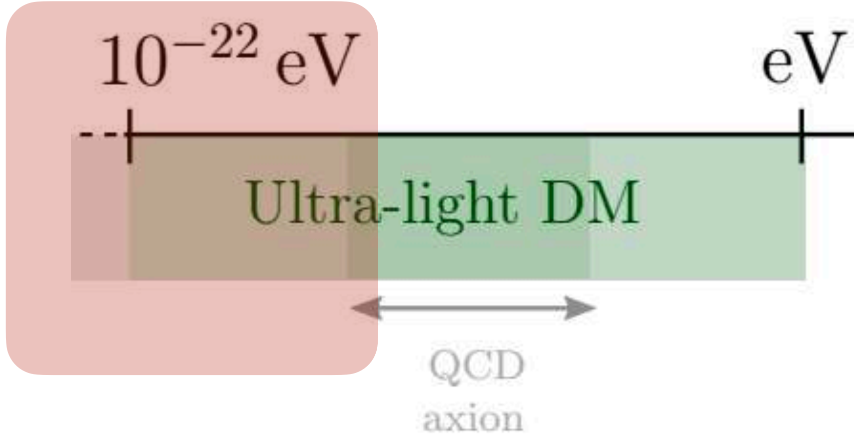
In order to behave like DM: start oscillating before matter-radiation equality



Marsh, 2016

$$m > 10^{-28}\,\mathrm{eV} \sim H(a_{\rm eq})$$

Structure formation - non-relativistic regime



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

Schrödinger-Poisson system : describe the FDM and the SIFDM

$$\begin{cases} i\dot{\psi} = \left(-\dfrac{1}{2m}\nabla^2 + \dfrac{g}{8m^2}|\psi|^2 - m\Phi\right)\psi \\ \nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho}) \end{cases}$$

Schrödinger equation
(Gross-Pitaevskii)

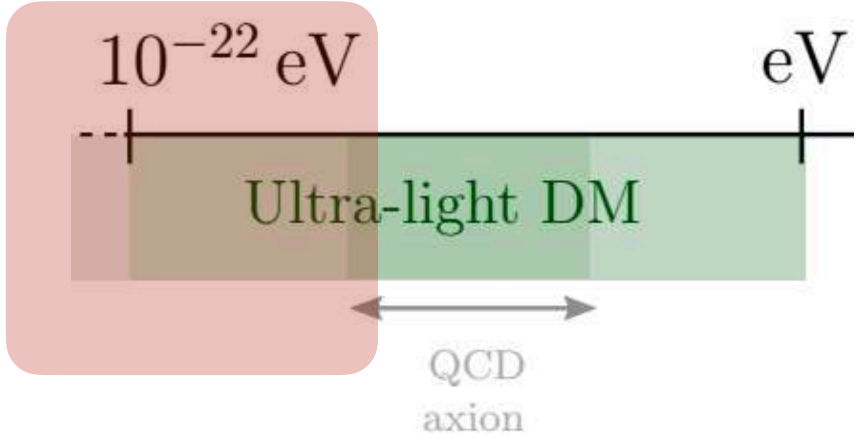
Poisson equation

$g = 0 \longrightarrow$ FDM

$g \neq 0 \longrightarrow$ SIFDM

Fundamentally different than
CDM/WDM/SIDM!

Structure formation - non-relativistic regime



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

Schrödinger-Poisson system : describe the FDM and the SIFDM

$$\begin{cases} i\dot{\psi} = \left(-\dfrac{1}{2m}\nabla^2 + \boxed{\dfrac{g}{8m^2}|\psi|^2} - m\Phi\right)\psi \\ \nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho}) \end{cases}$$

Schrödinger equation
(Gross-Pitaevskii)

Poisson equation

$g = 0$ → FDM

$g \neq 0$ → SIFDM

Fundamentally different than
CDM/WDM/SIDM!

Madelung equations $\left(\psi \equiv \sqrt{\rho/m}\, e^{i\theta}\right.$ and $\left.\mathbf{v} \equiv \nabla\theta/m\right)$

$$P_{int} = K\rho^{(j+1)/j} = \frac{g}{2m^2}\rho^2$$

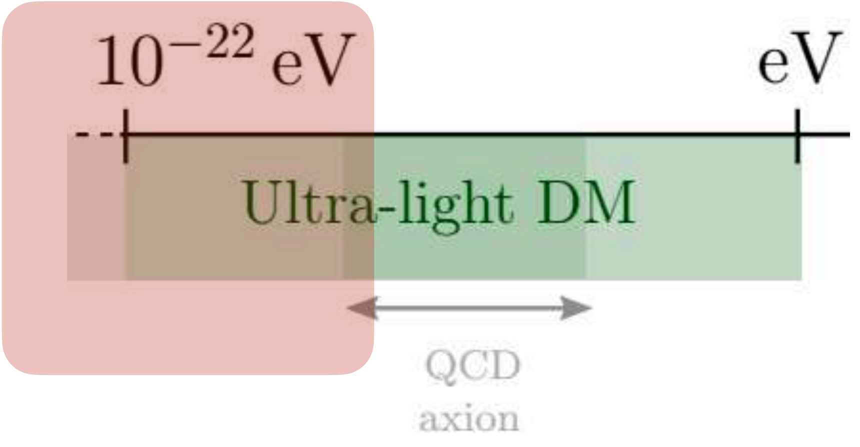
$$\dot{\rho} + \nabla\cdot(\rho\,\mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{m}\left(V_{grav} - \boxed{P_{int}} - \boxed{\frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}\right)$$

Quantum pressure

FLUID
DESCRIPTION

Structure formation - *non-relativistic regime*



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

<u>Schrödinger-Poisson system</u> : describe the FDM and the SIFDM

$$
\begin{cases}
i\dot{\psi} = \left(-\dfrac{1}{2m}\nabla^2 + \dfrac{g}{8m^2}|\psi|^2 - m\Phi\right)\psi \\[2ex]
\nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho})
\end{cases}
$$

Schrödinger equation (Gross-Pitaevskii)

$g = 0 \rightarrow$ FDM

$g \neq 0 \rightarrow$ SIFDM

Poisson equation

Fundamentally different than CDM/WDM/SIDM!

HOMEWORK

Refs.: EF, "ULDM" review
Niemeyer 2019 review

From the relativistic action of a scalar field in FRW

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

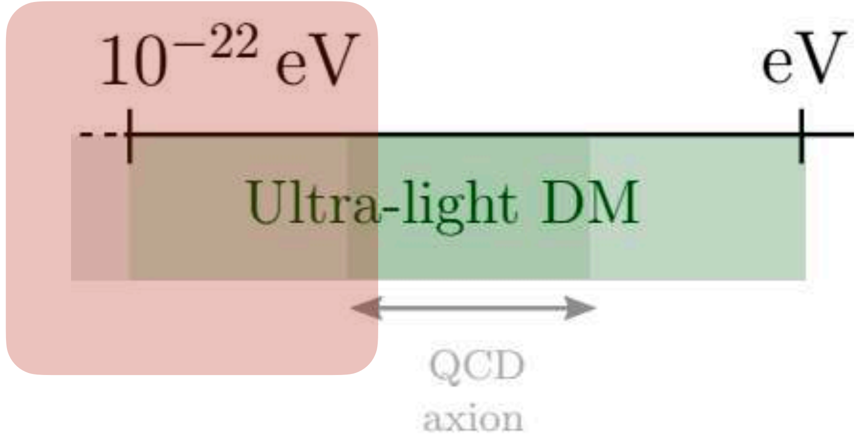
Write the perturbed equation in Newtonian gauge

$$S_\phi = \int d^4x\, a^3\left[\frac{1}{2}(1-4\Phi)\,\dot{\phi}^2 - \frac{1}{a^2}(\partial_i\phi)^2 - (1-2\Phi)V(\phi)\right]$$

Compute the non-relativistic action and EoM (Schrödinger-Poisson system)

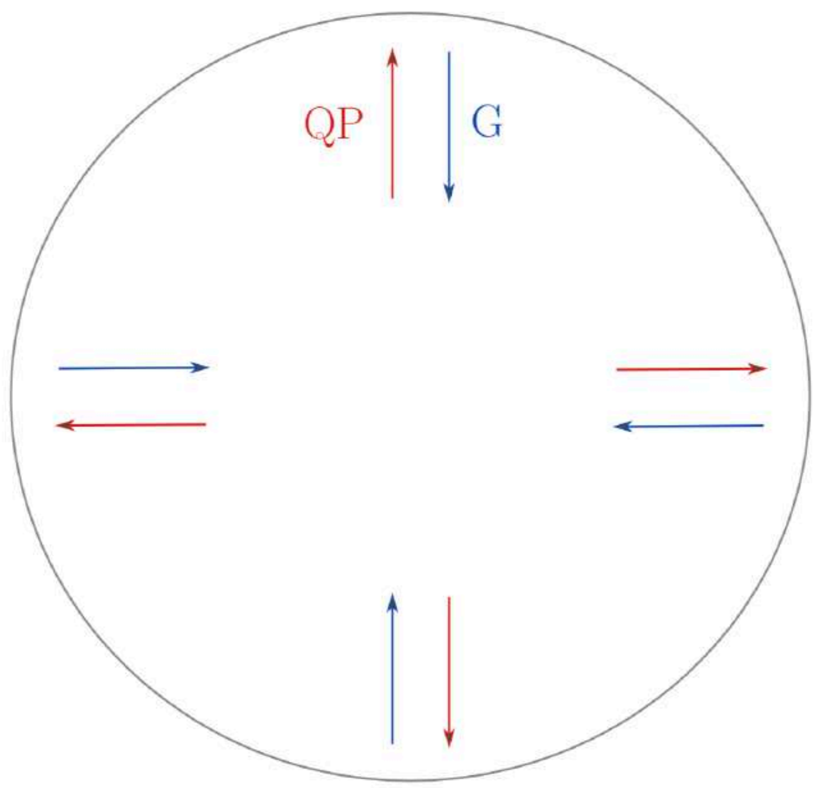
$$S = \int d^4x\left[\frac{i}{2}\left(\psi\partial_t\psi^* - \psi^*\partial_t\psi\right) - \frac{|\nabla\psi|^2}{2m} - \frac{g}{16m^2}|\psi|^4 - m(\psi\psi^* - \langle\psi\psi^*\rangle)\Phi - \frac{a}{8\pi G}(\partial_i\Phi)^2\right]$$

Structure formation - perturbation and stability



Competition between gravity and pressure (quantum pressure and interaction)

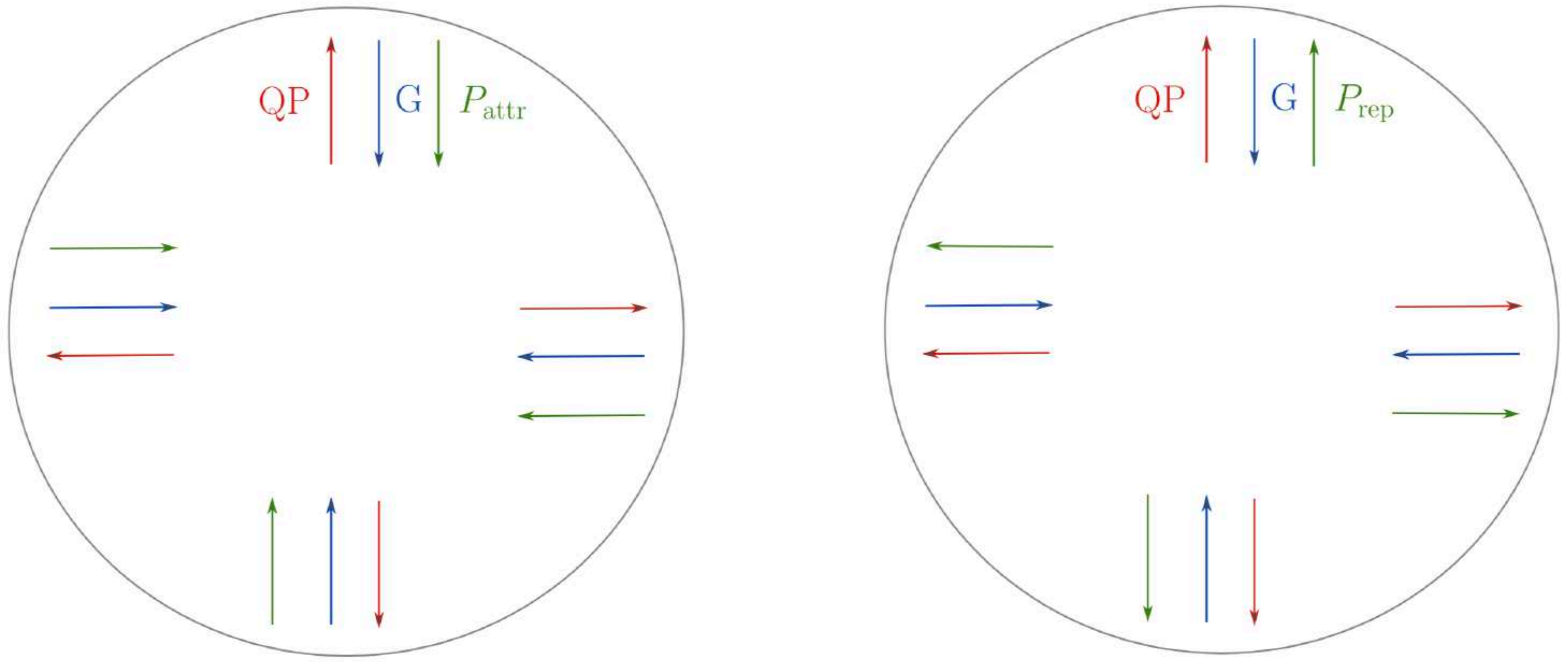
FDM



SIFDM

ATTRACTIVE

REPULSIVE



$g < 0$

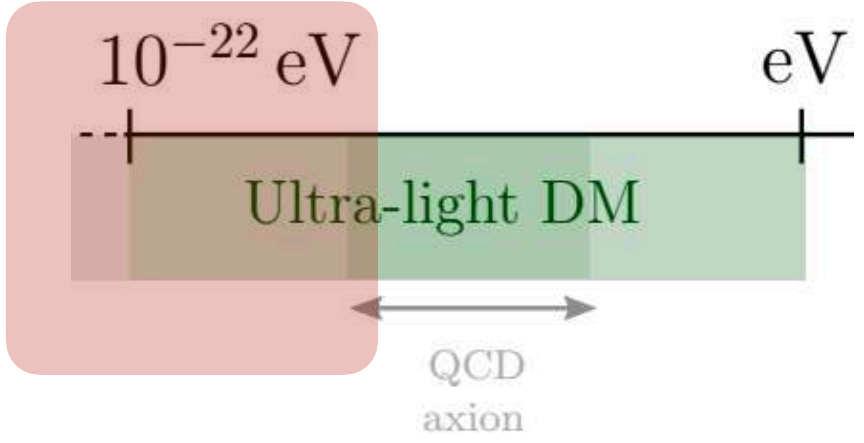
$g > 0$

$$\dot{\rho} + \nabla \cdot (\rho\, \mathbf{v}) = 0$$

$P_{\rm int} = \frac{g}{2m^2}\rho^2$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{m}\left(V_{grav} - P_{int} - \underbrace{\boxed{\frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}}_{\text{Quantum pressure}}\right)$$

Structure formation - *perturbation and stability*



Finite clustering scale (Jeans length)- no structure formation on small scales

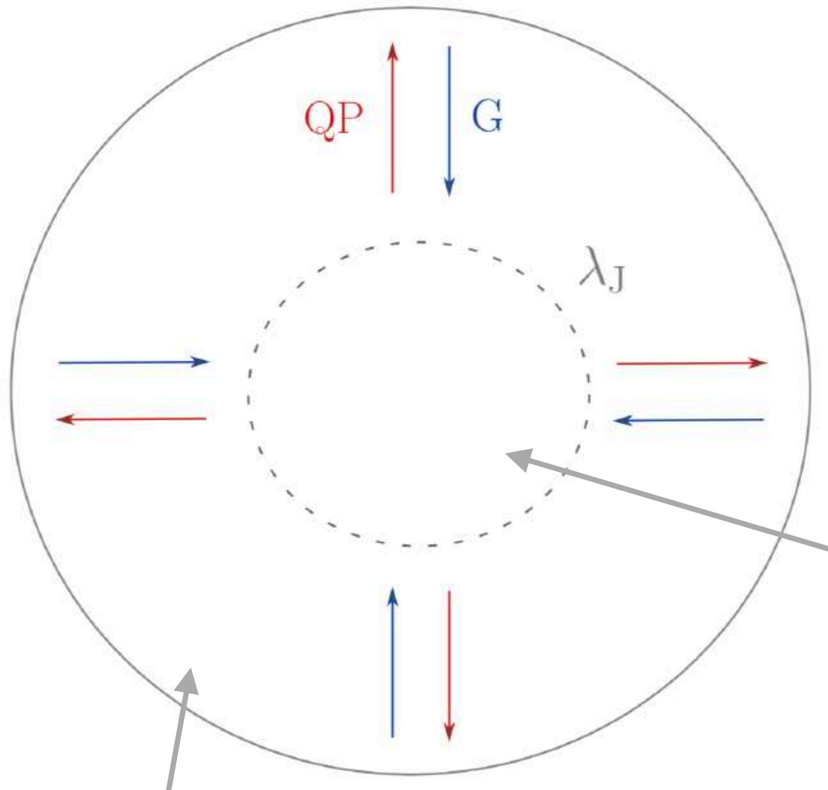
(CDM, λ_J effectively zero)

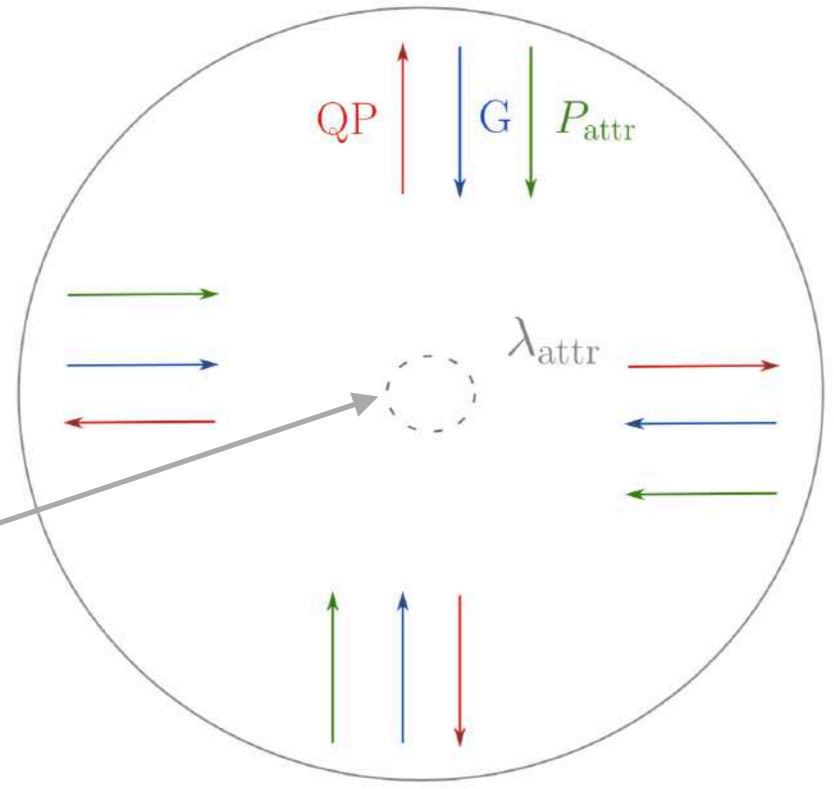
SIFDM

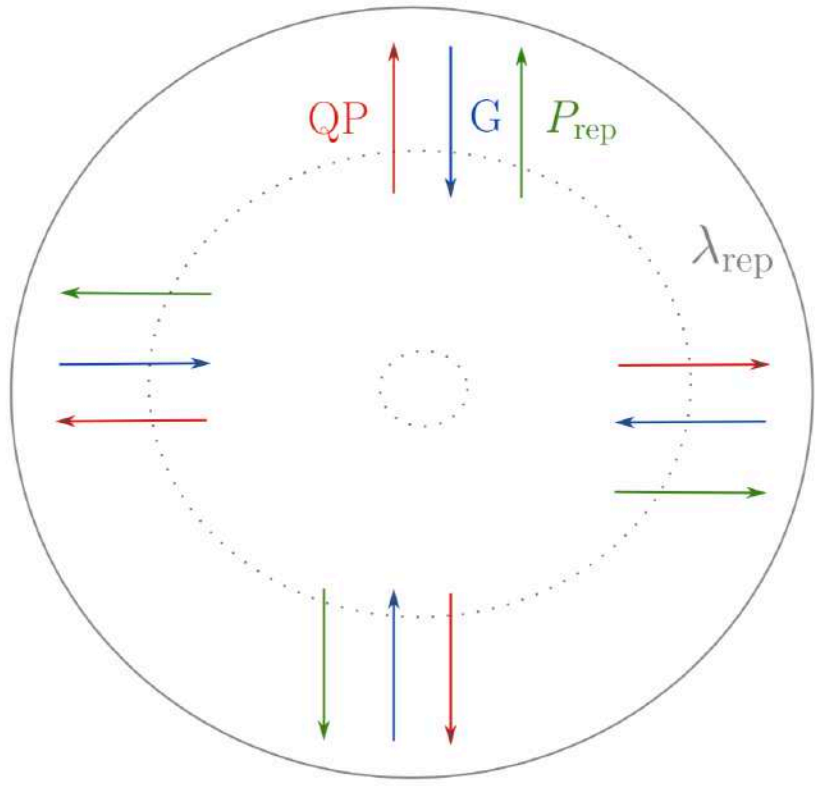
FDM

ATTRACTIVE

REPULSIVE







QP dominates -
Stable solution
NO structure formation

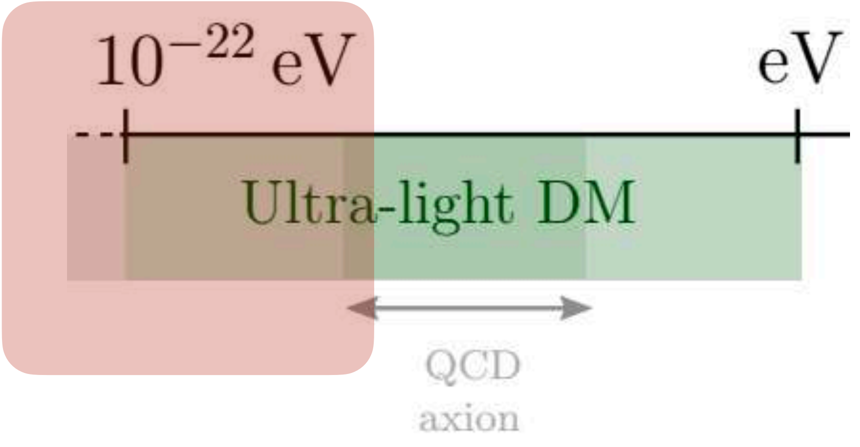
$$\lambda < \lambda_\mathrm{J},\ \lambda_\mathrm{attr},\ \lambda_\mathrm{rep}$$

$$g < 0$$

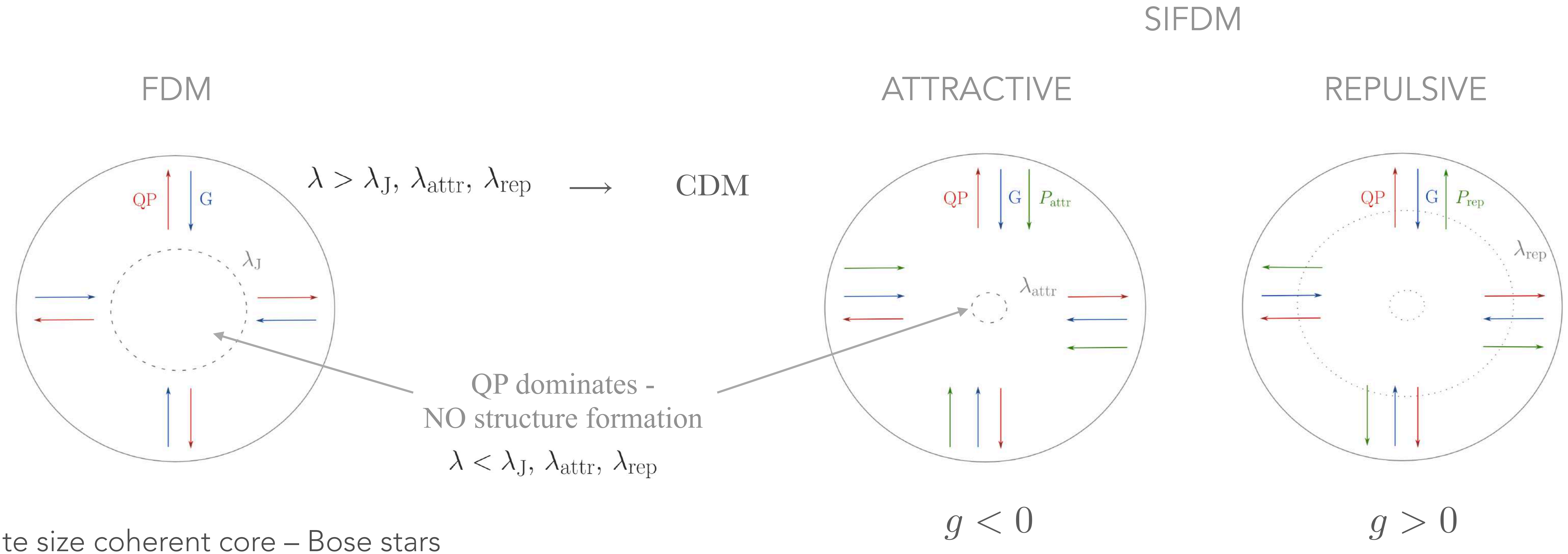
$$g > 0$$

For $\lambda > \lambda_\mathrm{J},\ \lambda_\mathrm{attr},\ \lambda_\mathrm{rep} \longrightarrow$ CDM

Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales



$\lambda > \lambda_{\rm J},\, \lambda_{\rm attr},\, \lambda_{\rm rep} \longrightarrow$ CDM

QP dominates -
NO structure formation

$\lambda < \lambda_{\rm J},\, \lambda_{\rm attr},\, \lambda_{\rm rep}$

$g < 0$

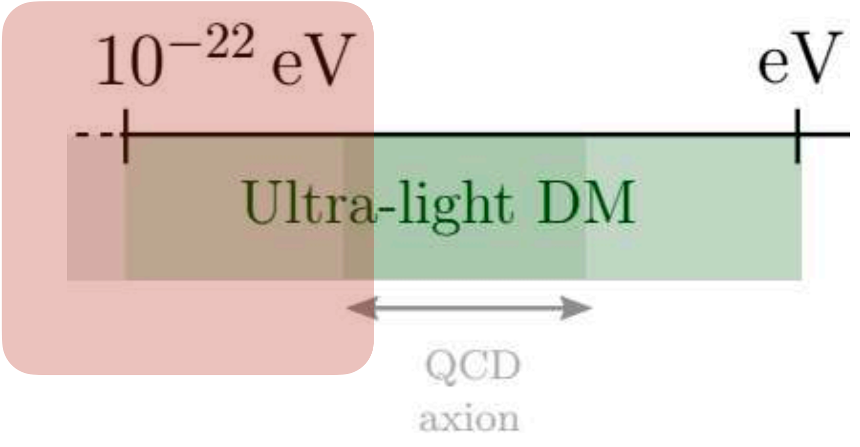
$g > 0$

Finite size coherent core – Bose stars

$$\lambda_J = 55 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-1/2} \left(\frac{\rho}{\bar{\rho}}\right)^{-1/4} (\Omega_m h)^{-1/4}\,\mathrm{kpc}$$

$m \leq 10^{-20}\mathrm{eV} \;\Rightarrow\; \lambda_{dB} > \mathcal{O}(\mathrm{kpc})$ Galactic scales

Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales

In the limit where only self-interaction is important:

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{g}{8m^2}|\psi|^2\psi\,.$$

We can decompose as: $\psi(\mathbf{x},t) = \psi_c(t) + \delta\bar{\bar{\Psi}}(\mathbf{x},t)$

<u>Homogeneous:</u> $$i\dot{\psi}_c = \frac{g}{8m^2}|\psi_0|^2\psi_c$$

Periodic solution $\psi_c(t) = \psi_0\, e^{-i\mu_c t}$

where $|\psi_0|^2 = n_0$ fixes the amplitude and $\mu_c = gn_0/8m^2$

<u>Perturbations:</u> $$i\delta\Psi = -\frac{1}{2m}\nabla^2\delta\Psi + \frac{gn_0}{8m^2}\left(\delta\Psi + \delta\Psi^*\right)$$

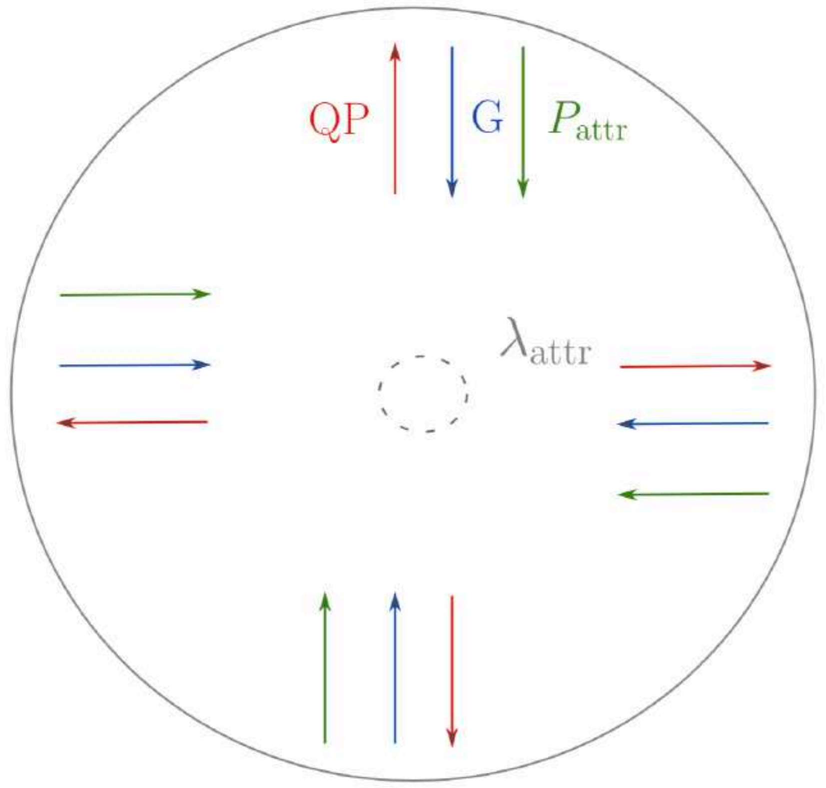
Rewriting $\delta\Psi$ as $\Psi = A + iB$

$$\Rightarrow \quad \frac{d}{dt}\begin{pmatrix} A_k \\ B_k \end{pmatrix} = \begin{pmatrix} 0 & \frac{k^2}{2m} \\ -\frac{k^2}{2m} - \frac{gn_0}{4m^2} & 0 \end{pmatrix}\begin{pmatrix} A_k \\ B_k \end{pmatrix}$$

↘ Dispersion relation $\omega_k^2 = \frac{gn_0}{4m^2}\frac{k^2}{2m} + \frac{k^4}{4m^2}\,.$

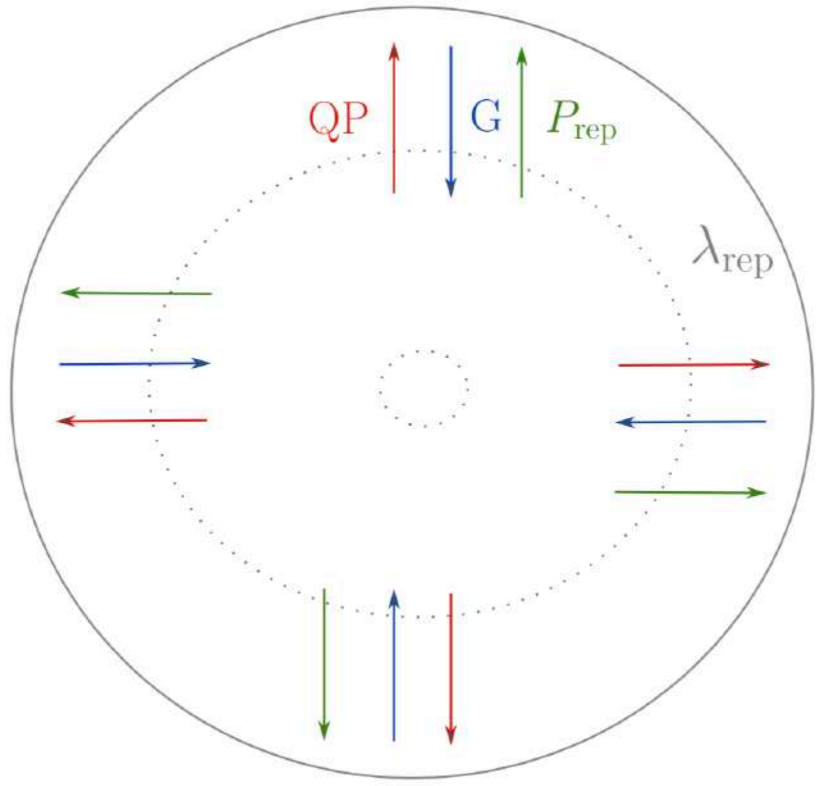
SIFDM

ATTRACTIVE



$g < 0$

REPULSIVE



$g > 0$

$$k_c^2 = -\frac{g\,n_0}{2m}$$

<u>Solution:</u>

$(\omega_k^2 > 0)$ $\quad \delta\Psi_k = Z\,(\omega_k + \zeta_k)\,e^{i\omega_k t} + Z^*\,(\omega_k - \zeta_k)\,e^{-i\omega_k t}$

$(\omega_k^2 < 0)$ $\quad \delta\Psi_k = c_1\,(\gamma_k - i\zeta_k)\,e^{\gamma_k t} + c_2\,(\gamma_k + i\zeta_k)\,e^{-\gamma_k t}$

Structure formation - perturbation and stability

HOMEWORK

EXERCISE:

In the limit where only self-interaction is important:

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{g}{8m^2}|\psi|^2\psi\,.$$

We can decompose as: $\psi(\mathbf{x},t) = \psi_c(t) + \delta\bar{\bar{\Psi}}(\mathbf{x},t)$

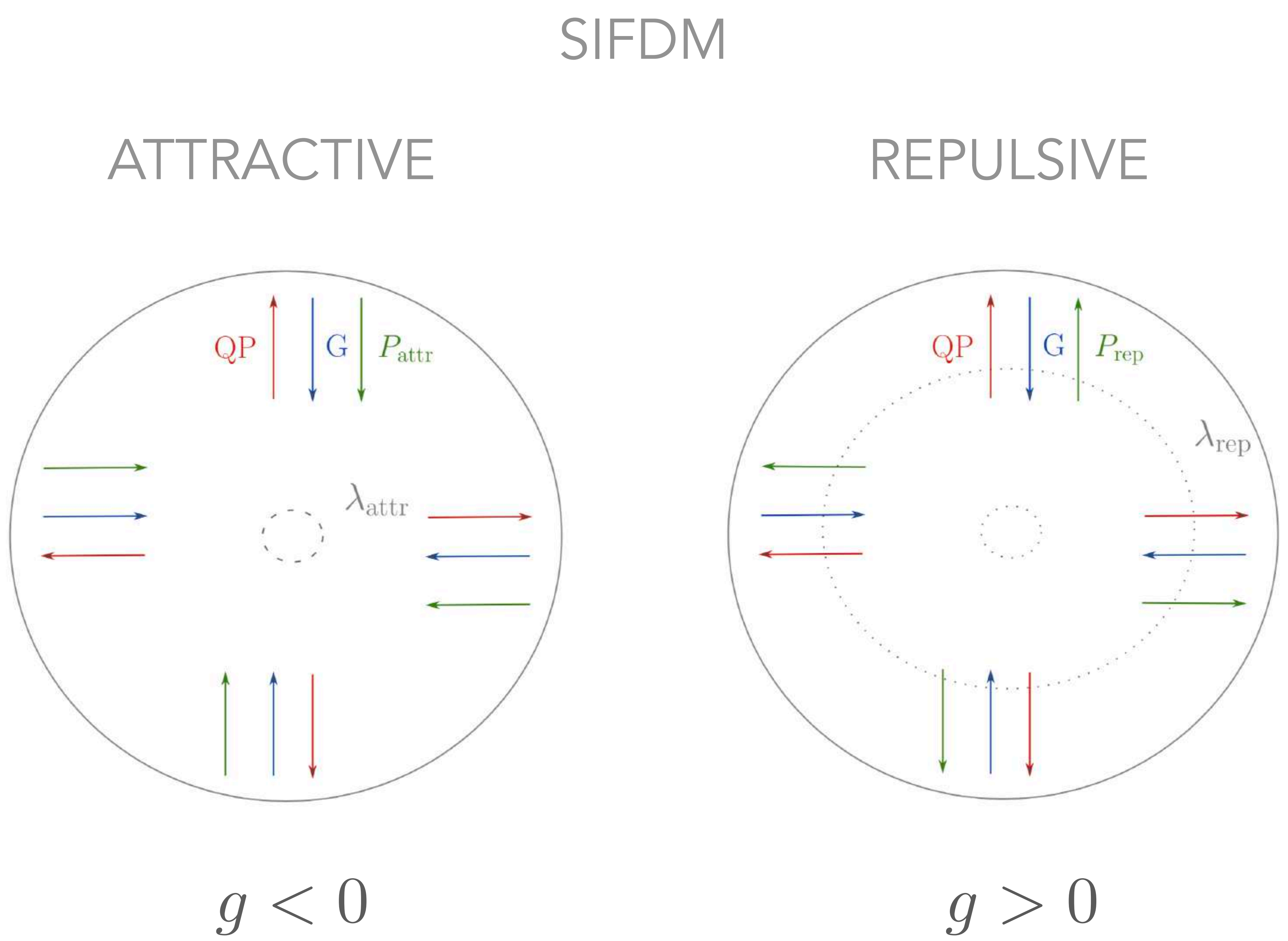
Homogeneous:

$$i\dot{\psi}_c = \frac{g}{8m^2}|\psi_0|^2\psi_c$$

$$\psi_c(t) = \psi_0\, e^{-i\mu_c t}$$

Perturbations:

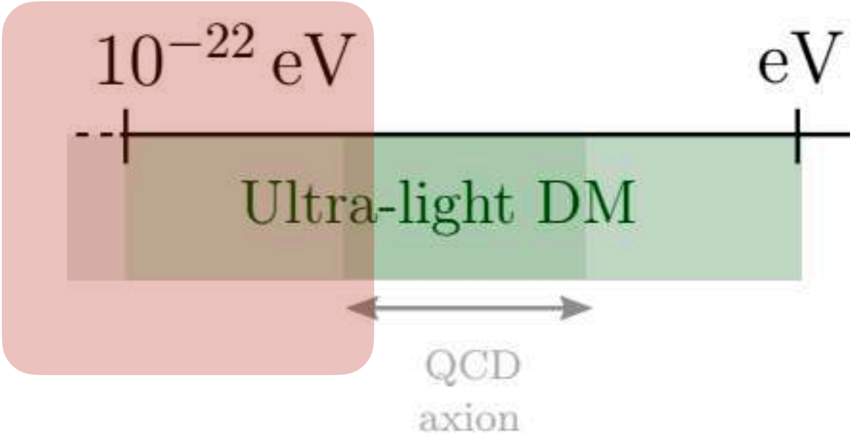
$$i\delta\Psi = -\frac{1}{2m}\nabla^2\delta\Psi + \frac{g n_0}{8m^2}\left(\delta\Psi + \delta\Psi^*\right)$$



$$k_c^2 = -\frac{g\,n_0}{2m}$$

Derive the solutions to the Schrodinger equation above for an attractive and repulsive potential.
Identify the different scales of the problem and where we have clustering or a stable, oscillatory solution.

Structure formation - *perturbation and stability*



Finite clustering scale - no structure formation on small scales

ATTRACTIVE

$$k_c^2 = -\frac{|g|\,n_0}{2m}$$

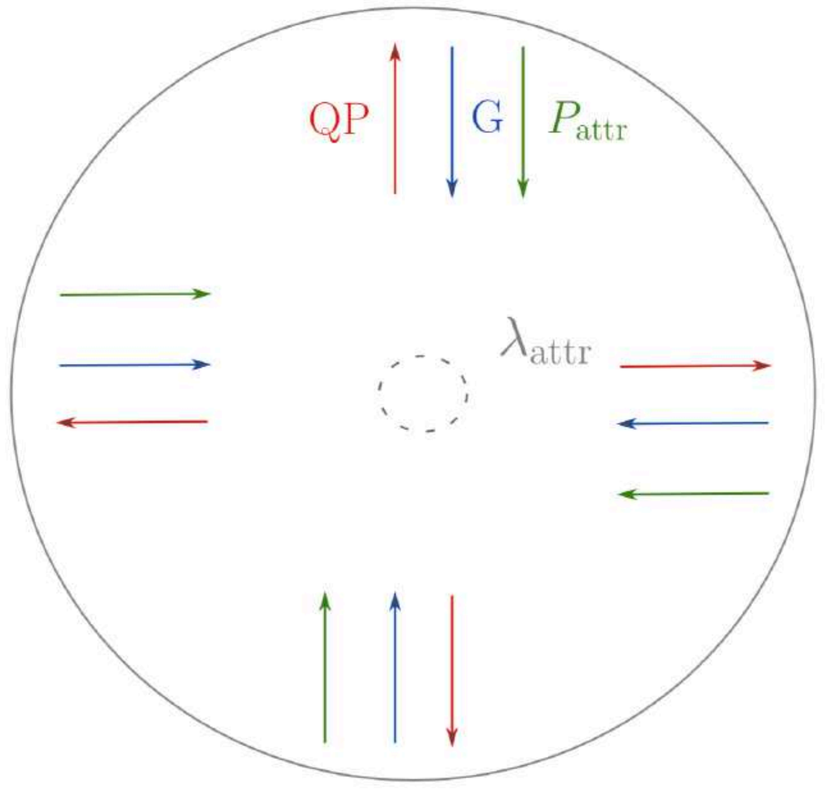
$k > k_c \, (\lambda < \lambda_c)$ $\Rightarrow$ Solution oscillates and is stable

This stable configuration, however, is different than in the case for repulsive interaction, forming a localized object, with maximum size given by λ_c

$k < k_c \, (\lambda > \lambda_c)$ $\Rightarrow$ Exponential growth (like CDM)

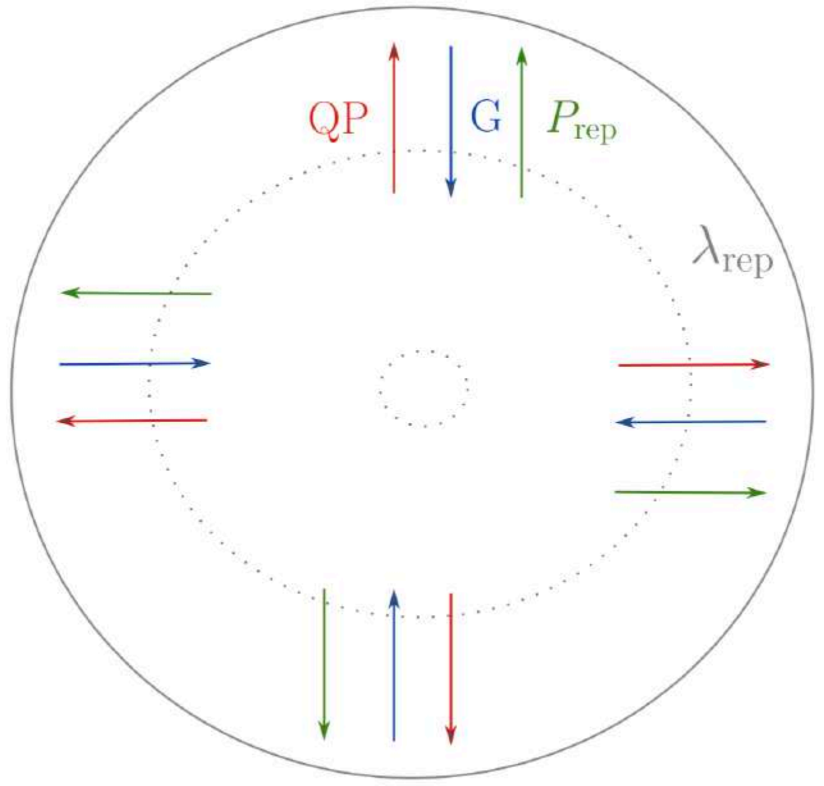
SIFDM

ATTRACTIVE



$g < 0$

REPULSIVE



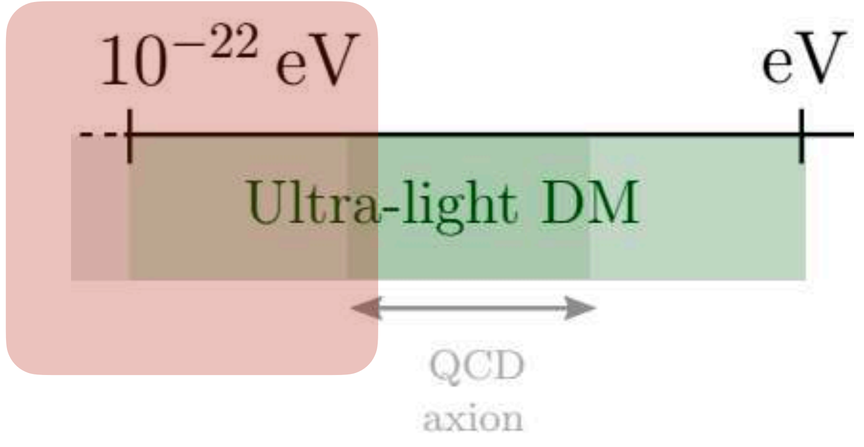
$g > 0$

$$k_c^2 = -\frac{g\,n_0}{2m}$$

For attractive interactions can only form localized clumps (solitons)

QCD axion: $m \sim 10^{-5}\,\mathrm{eV}$, $\lambda_a \sim -10^{-48}$ $\longrightarrow$ $l_{soliton} \sim 10^{-5}\,\mathrm{kpc}$

Structure formation - perturbation and stability



Finite clustering scale - no structure formation on small scales

REPULSIVE $g > 0$

$$k_c^2 = \frac{g\, n_0}{2m}$$

Homogeneous configuration is always stable, and it is always going to be described by an oscillatory solution, either if λ is bigger or smaller than λ_c

Dispersion relation

Long wavelength regime

$$\lambda \gg \lambda_c$$

$$\omega_k \simeq c_s k$$

Long range - superfluid

Short wavelength regime

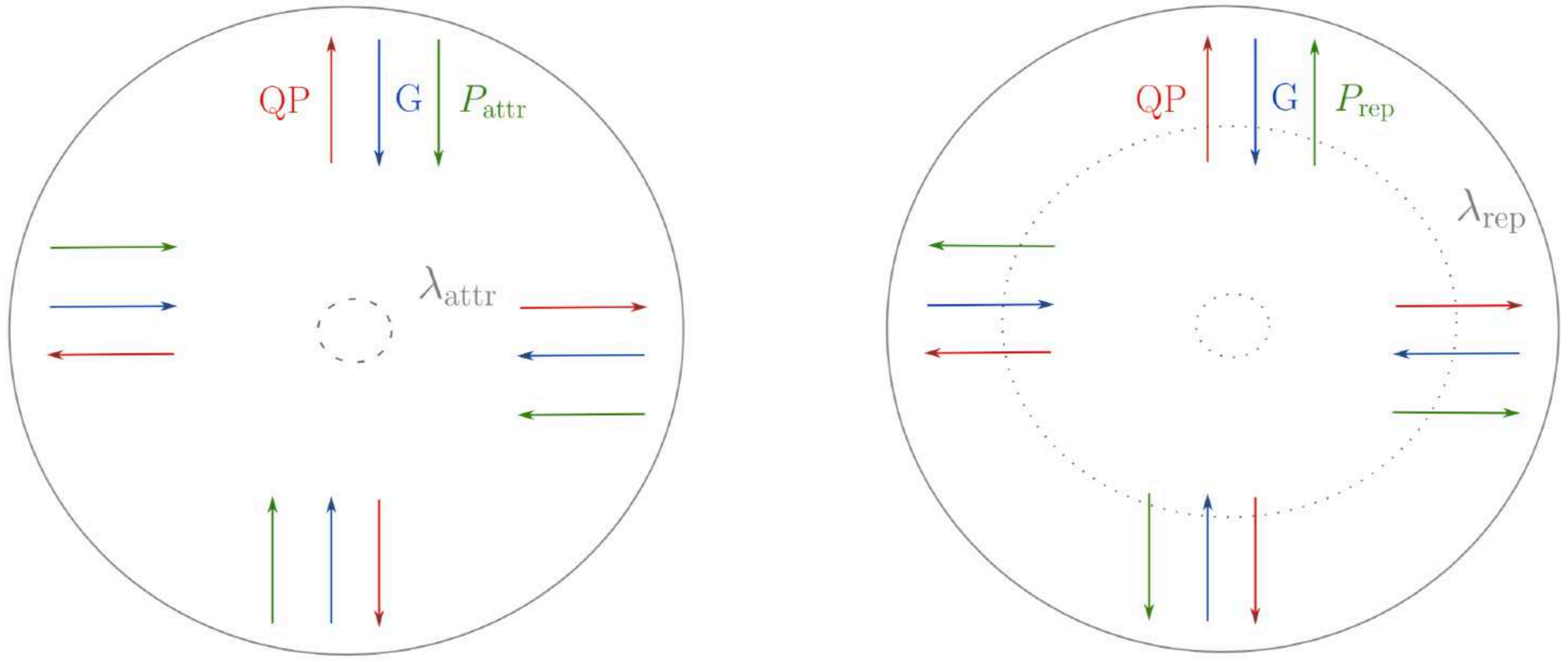
$$\omega_k \simeq k^2/2m$$

Free massive particle

SIFDM

ATTRACTIVE

REPULSIVE



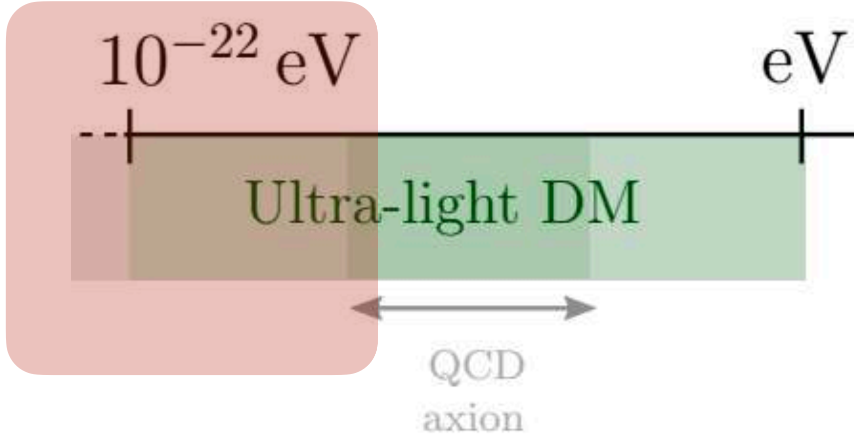
$g < 0$

$g > 0$

$$k_c^2 = -\frac{g\, n_0}{2m}$$

Superfluid!

Structure formation - *perturbation and stability*



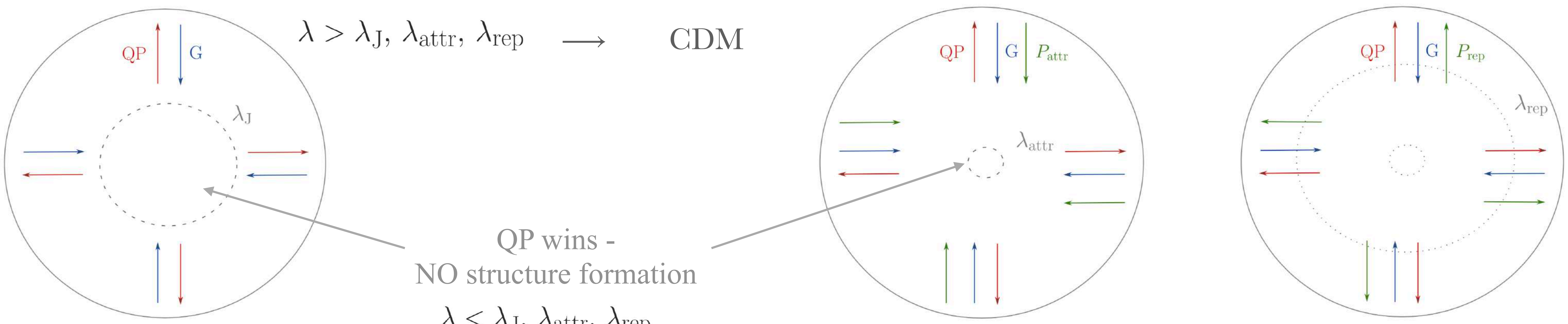
Finite clustering scale - no structure formation on small scales

SIFDM

FDM

ATTRACTIVE

REPULSIVE



$\lambda > \lambda_J, \lambda_{attr}, \lambda_{rep} \quad \longrightarrow \quad$ CDM

QP wins -
NO structure formation

$\lambda < \lambda_J, \lambda_{attr}, \lambda_{rep}$

$g < 0$

$g > 0$

$$k_c^2 = -\frac{g\, n_0}{2m}$$

Finite size coherent core – Bose stars

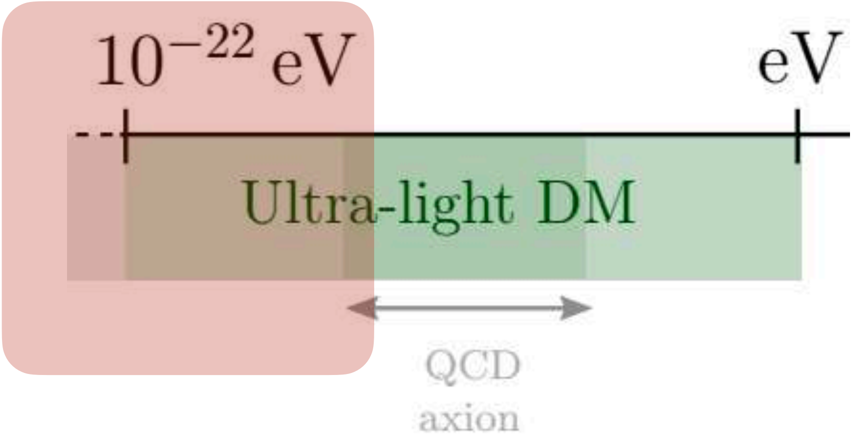
$$\lambda_J = 55 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-1/2} \left(\frac{\rho}{\bar{\rho}}\right)^{-1/4} (\Omega_m h)^{-1/4}\,\mathrm{kpc}$$

$$m \leq 10^{-20}\mathrm{eV} \;\Rightarrow\; \lambda_{dB} > \mathcal{O}(\mathrm{kpc})$$

Galactic scales

$g > 0$	$\longrightarrow$	$\forall\,\lambda$	Solution oscillates. Condensate (long range)
$g < 0$	$\longrightarrow$	$\lambda > \lambda_*$	Structures grow. No condensate.
		$\lambda < \lambda_*$	Solution oscillates. Condensate (finite size)

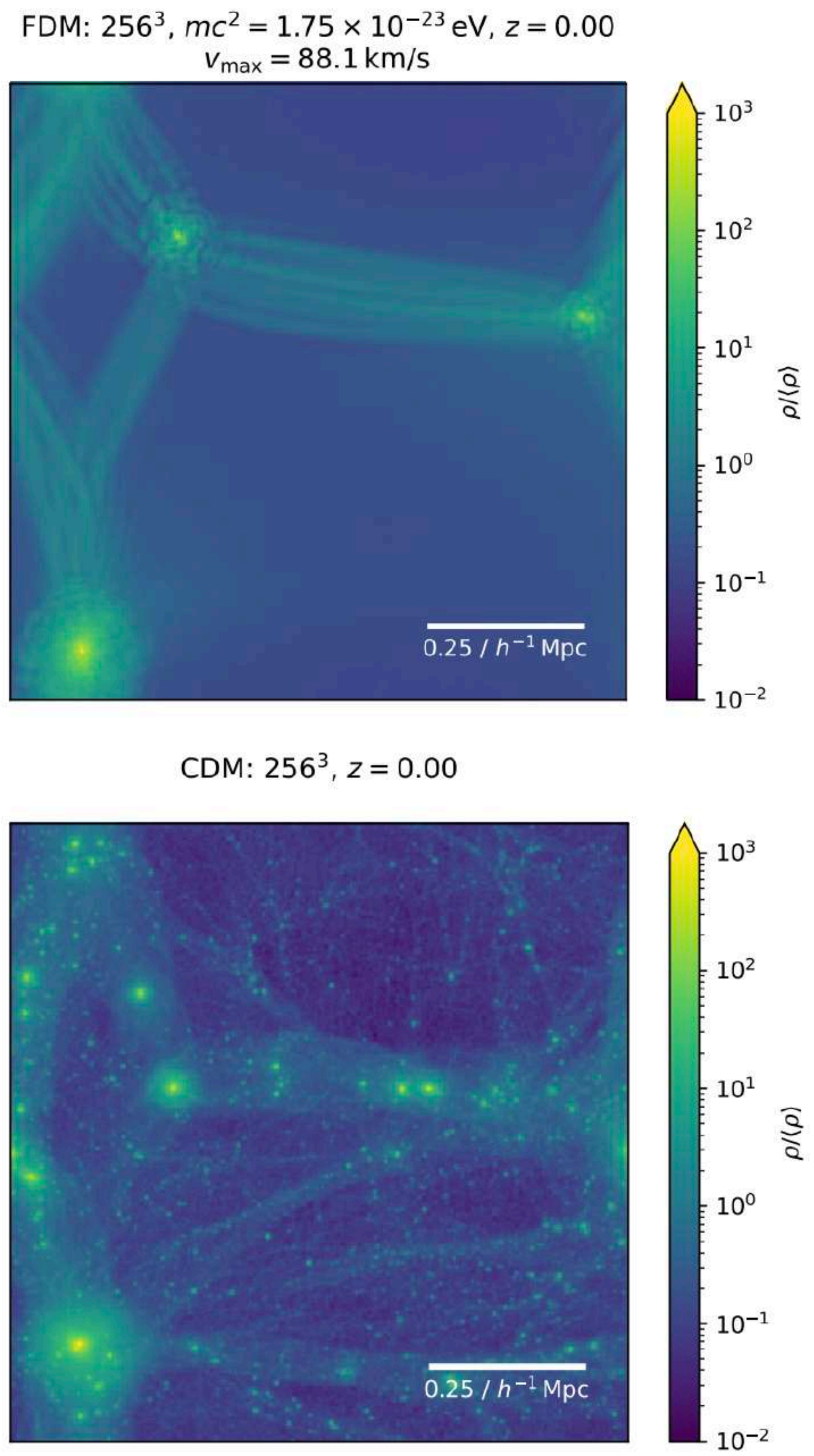
Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

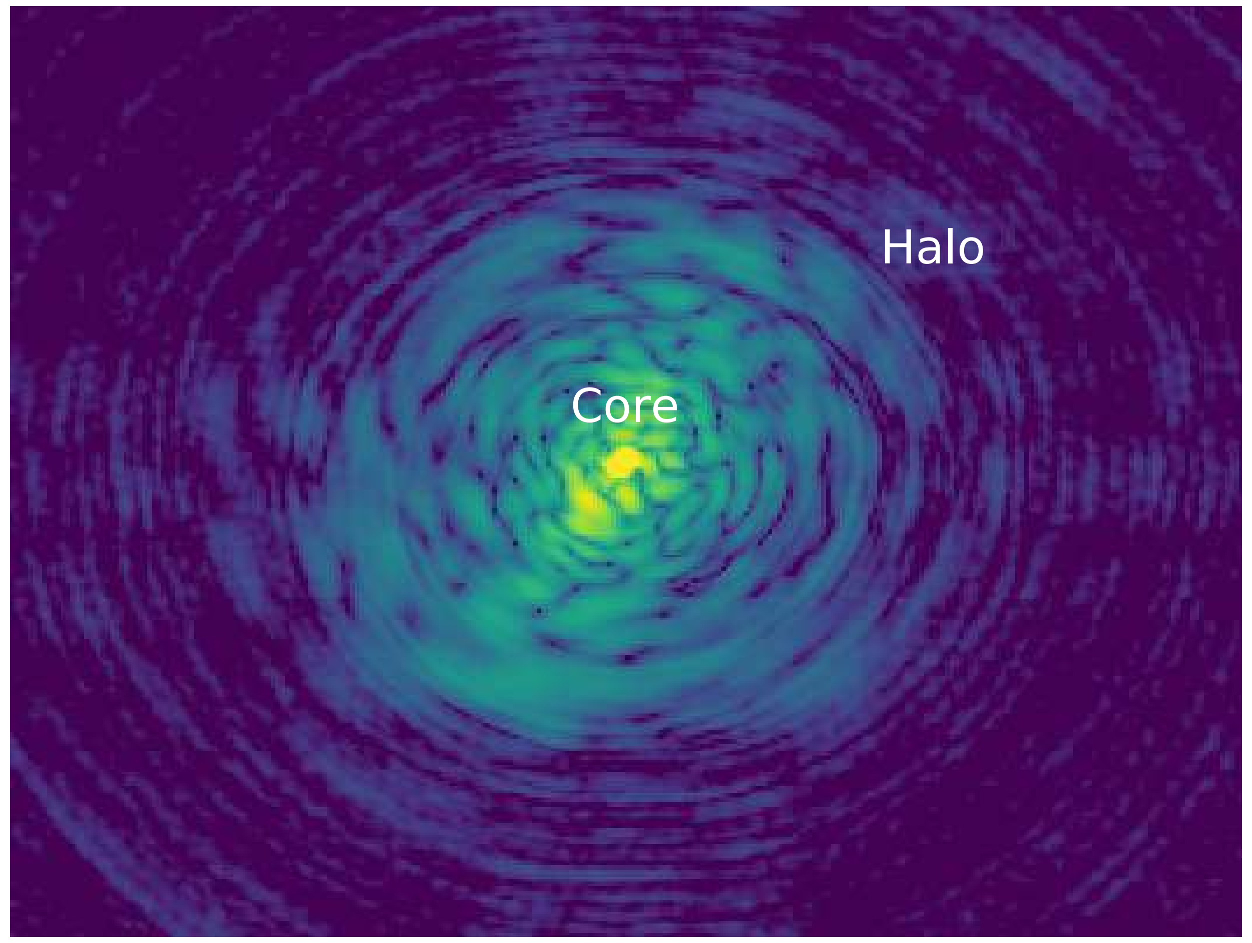
* Focus only in gravitational signatures

Suppression of small structures

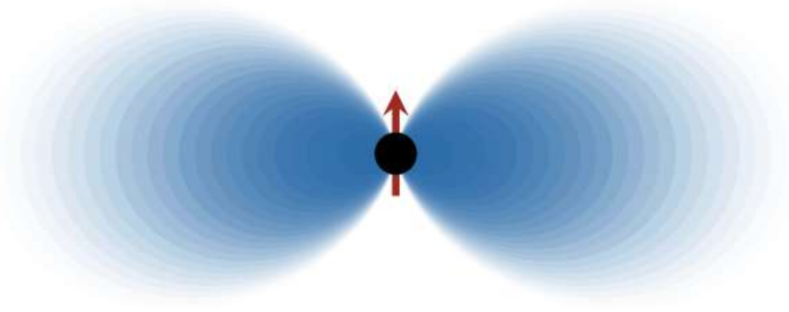


S. May et al. 2021

Formation of a solitonic core

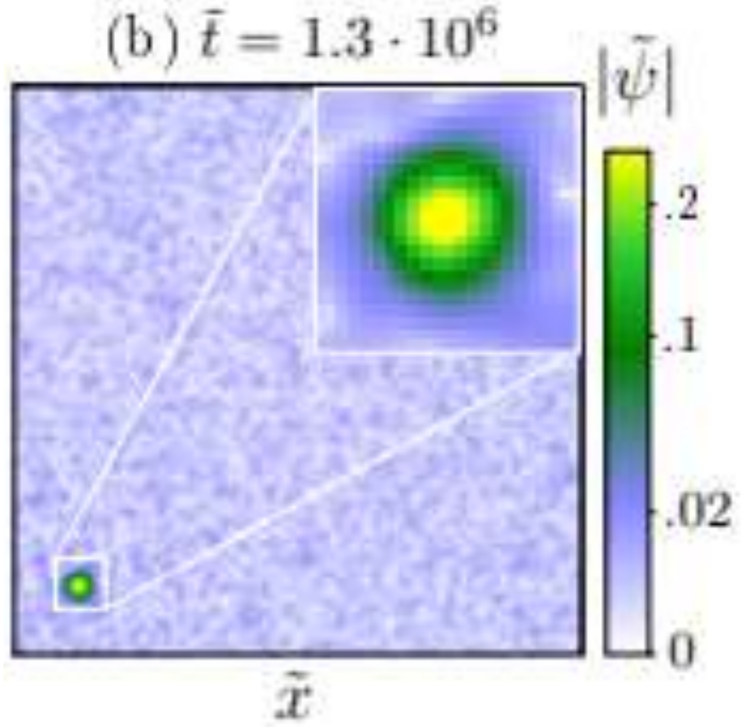


Axion clouds



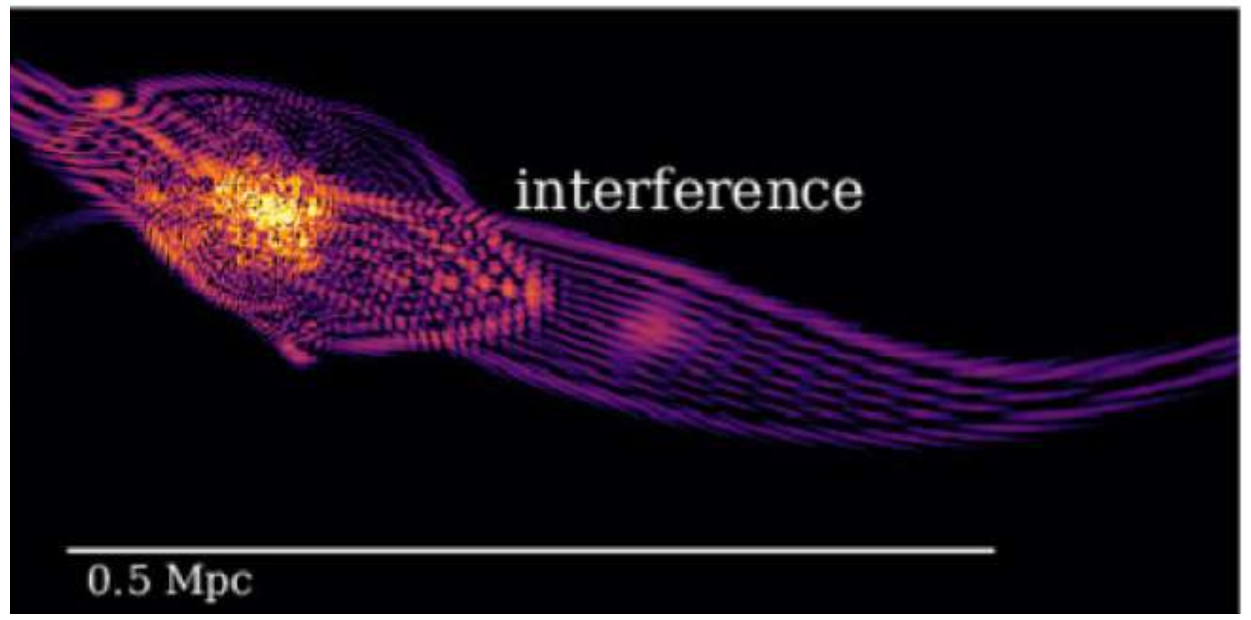
Baumann et al. 2019

Dynamical effects



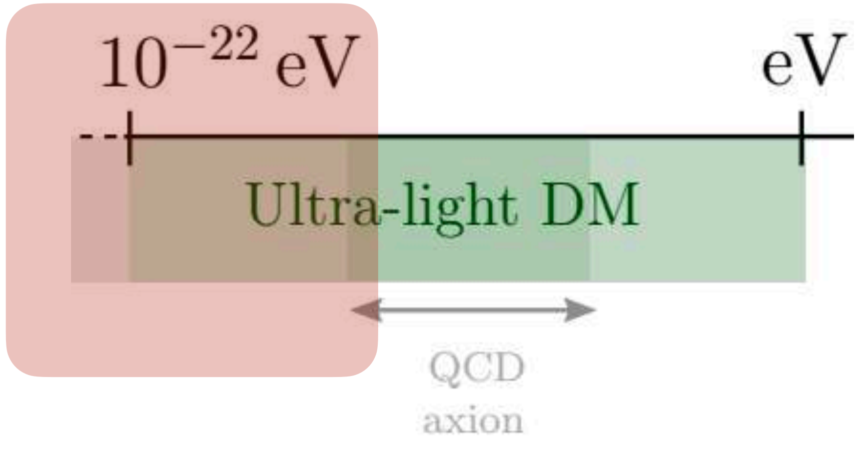
Levkov et al. 2018

Wave interference

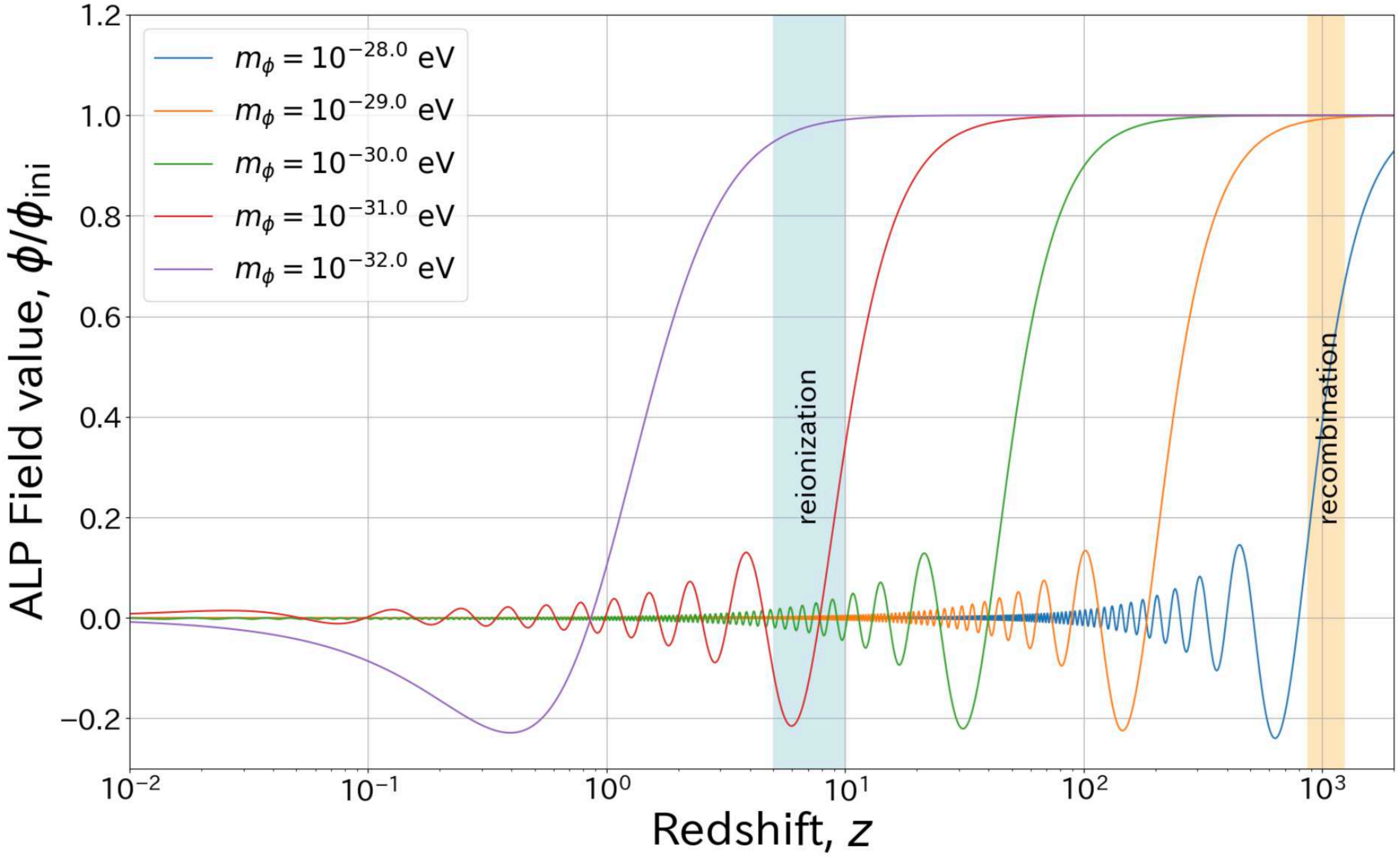


Mocz et al. 2017

Linear evolution



Boson/ Scalar field in a cosmological (FRW) background



Boltzmann codes: axionCAMB, axionECAMB, AxiCLASS

New emulator (to appear *soon*)

Condition DM today:

$$m > 10^{-28}\,\mathrm{eV} \sim H(a_{\rm eq})$$

Predictions

Non-linear evolution - simulations

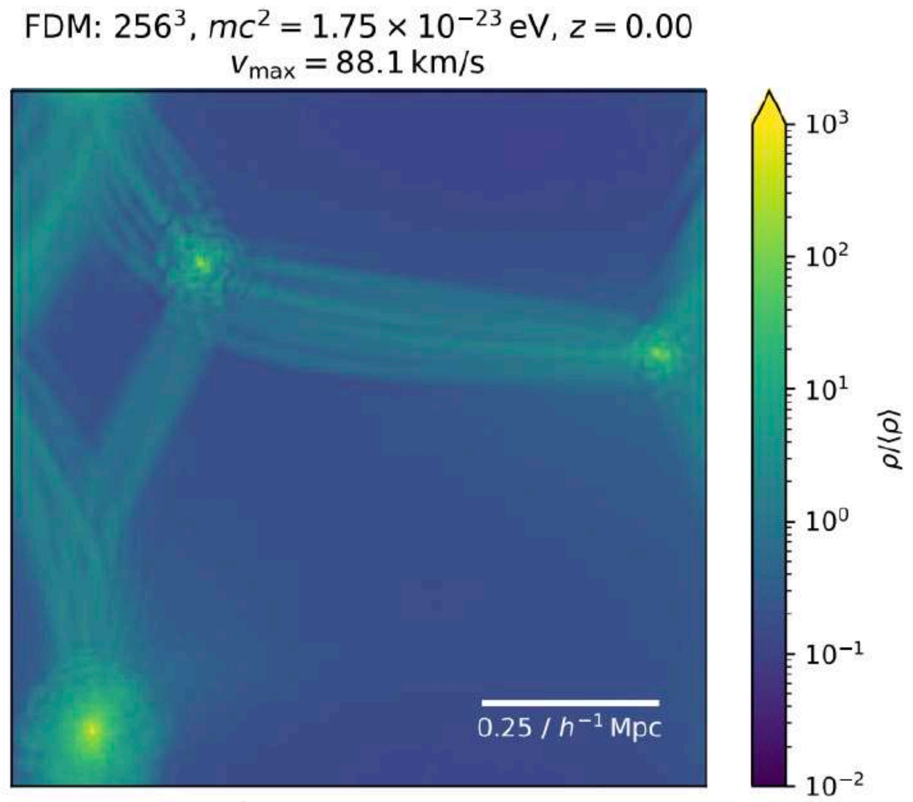
Pseudo-spectral methods

Solves the Schrödinger-Poisson equations.

→ Used widely in the field to simulate: isolated halos, the formation of cores, and cosmological simulations

Expensive! $\Delta t \sim \Delta x^2$

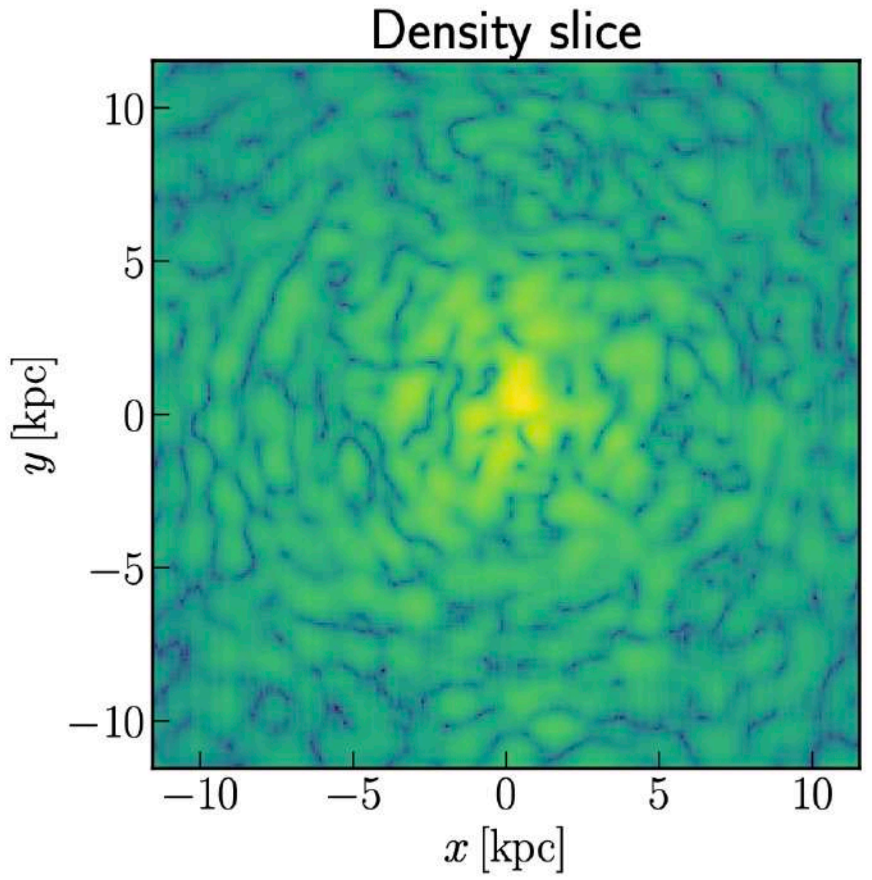
Largest to date: 10/Mpc/h



S. May et al. 2021

Mock halo generation

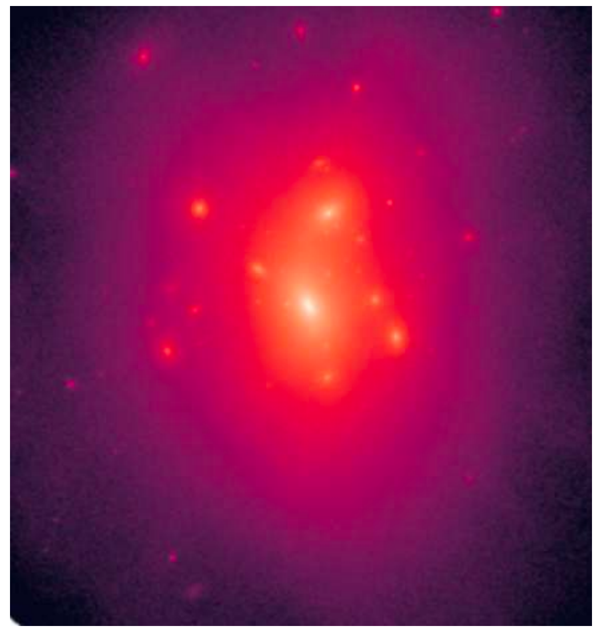
- Soliton collisions
- Eigenvalue decomposition: semi-analytical model to describe the halo



Eberhardt et al. 2024

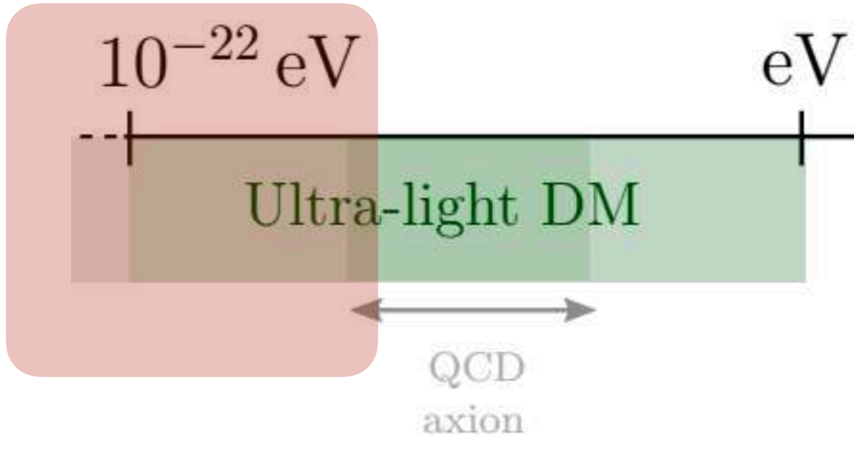
Approximation schemes

- Eigenvalue solvers
- Madelung simulations
- **N-body schemes** (or initial condition sims)



Nadler et al. 2024

Phenomenology



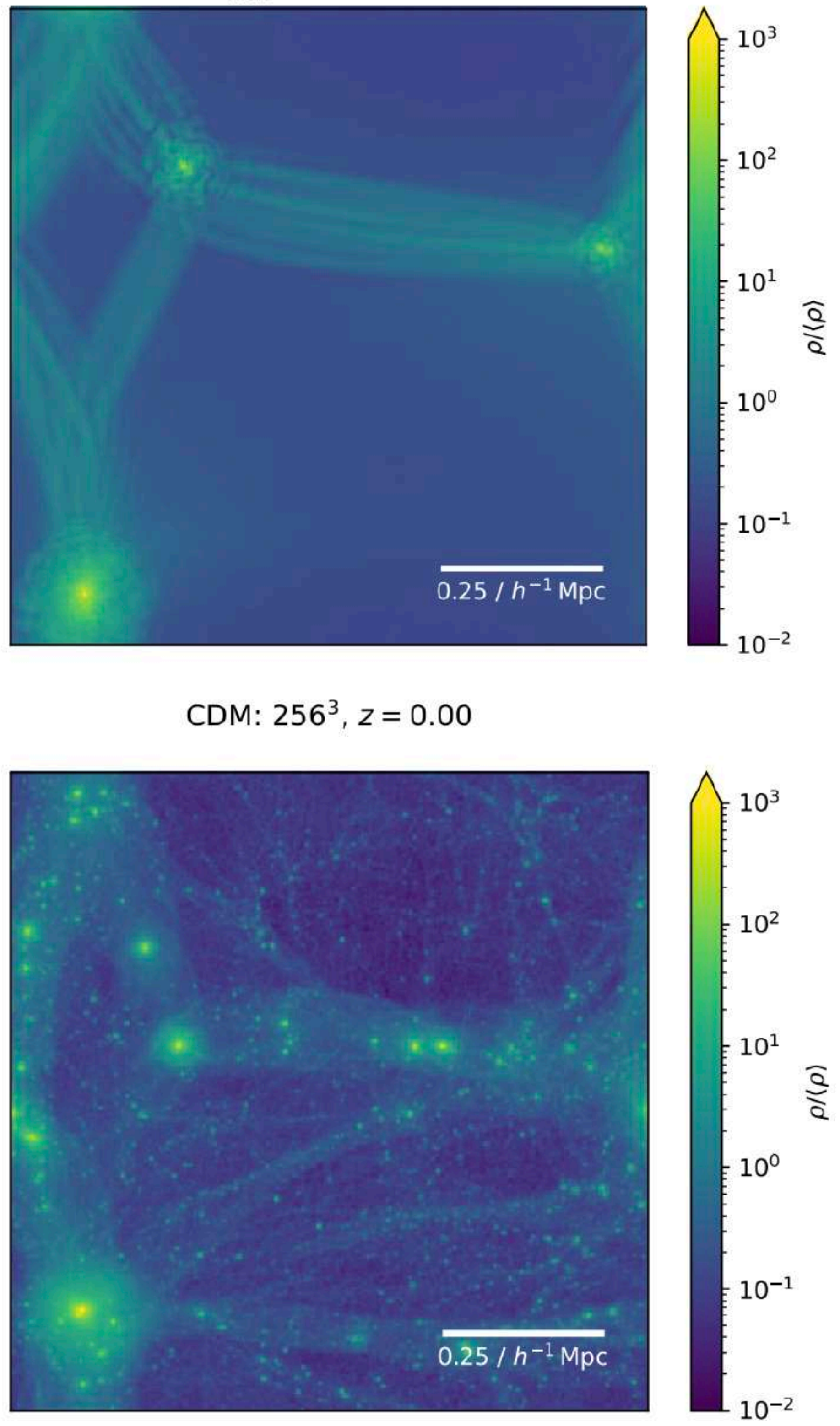
RICH PHENOMENOLOGY ON SMALL SCALES

* Focus only in gravitational signatures

Suppression of small structures

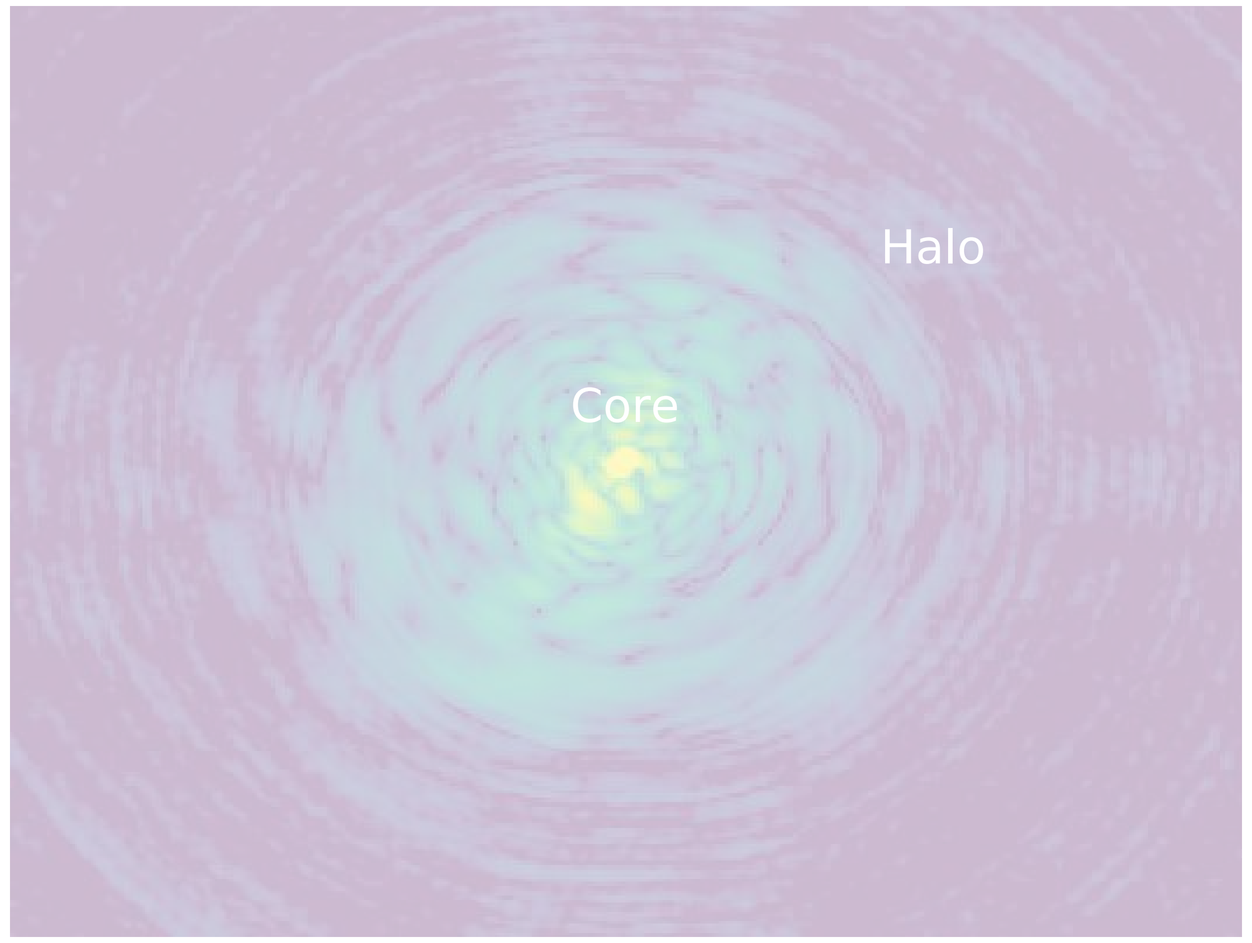
FDM: 256^3, $mc^2 = 1.75 \times 10^{-23}$ eV, $z = 0.00$
$v_{\max} = 88.1$ km/s

CDM: 256^3, $z = 0.00$

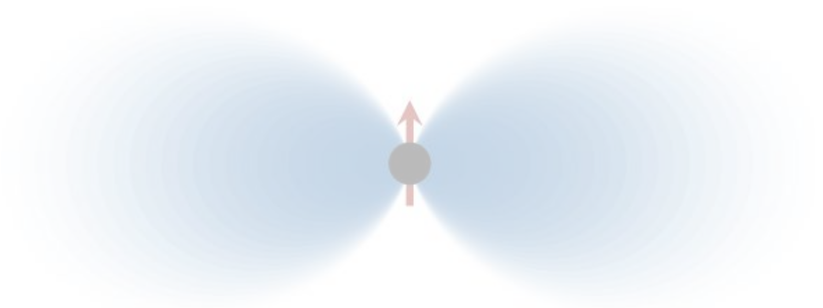


S. May et al. 2021

Formation of a solitonic core

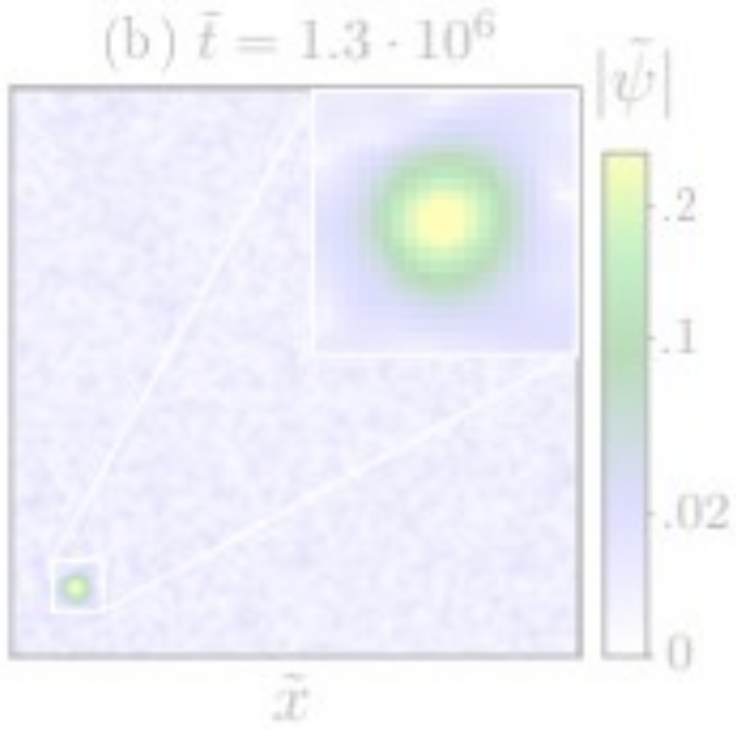


Axion clouds



Baumann et al. 2019

Dynamical effects



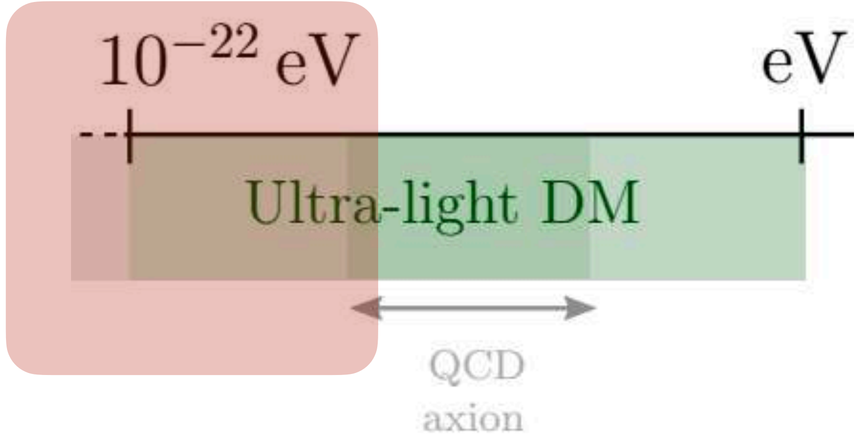
Levkov et al. 2018

Wave interference

Mocz et al. 2017

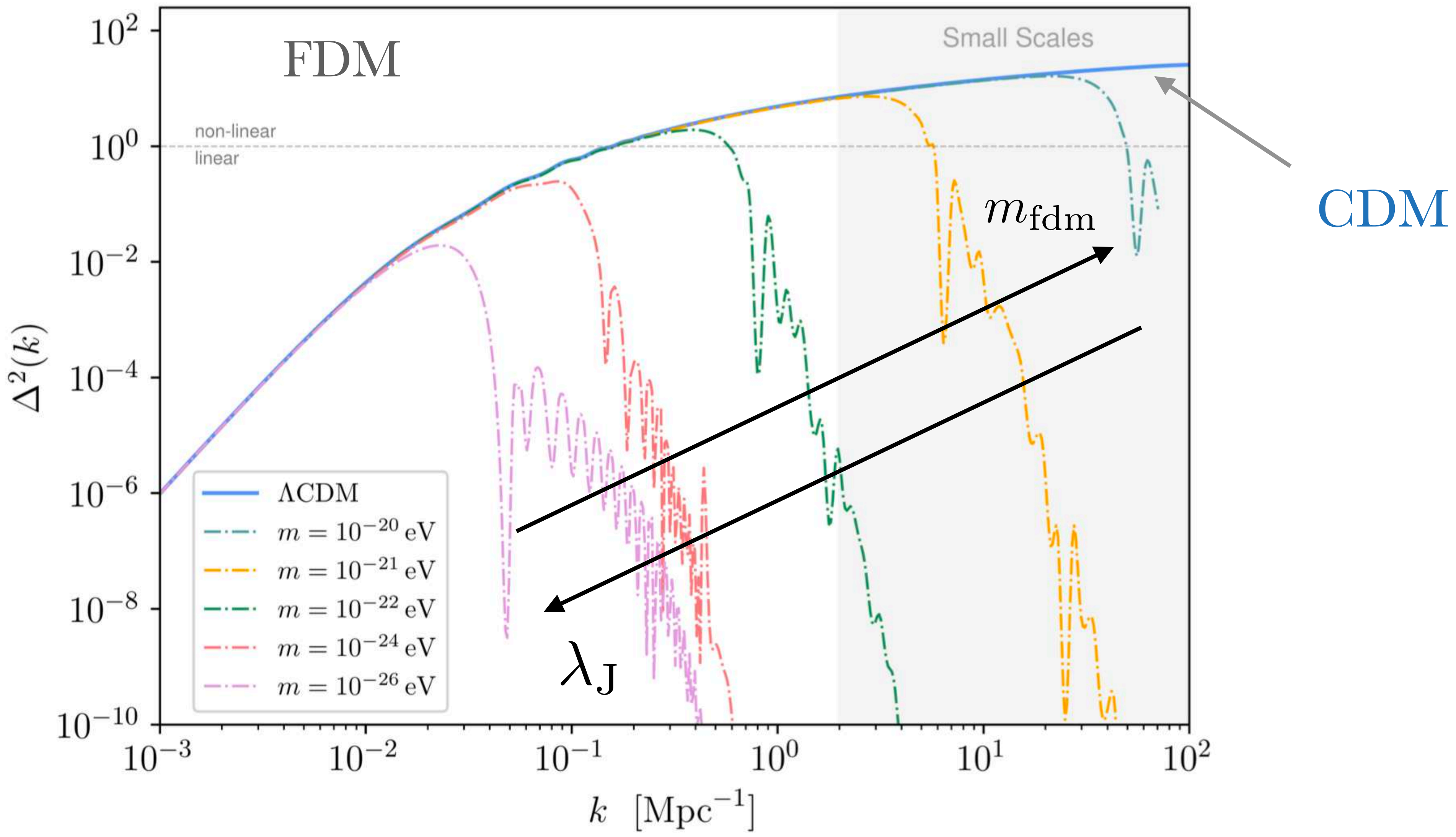
Phenomenology

Suppression of small structures

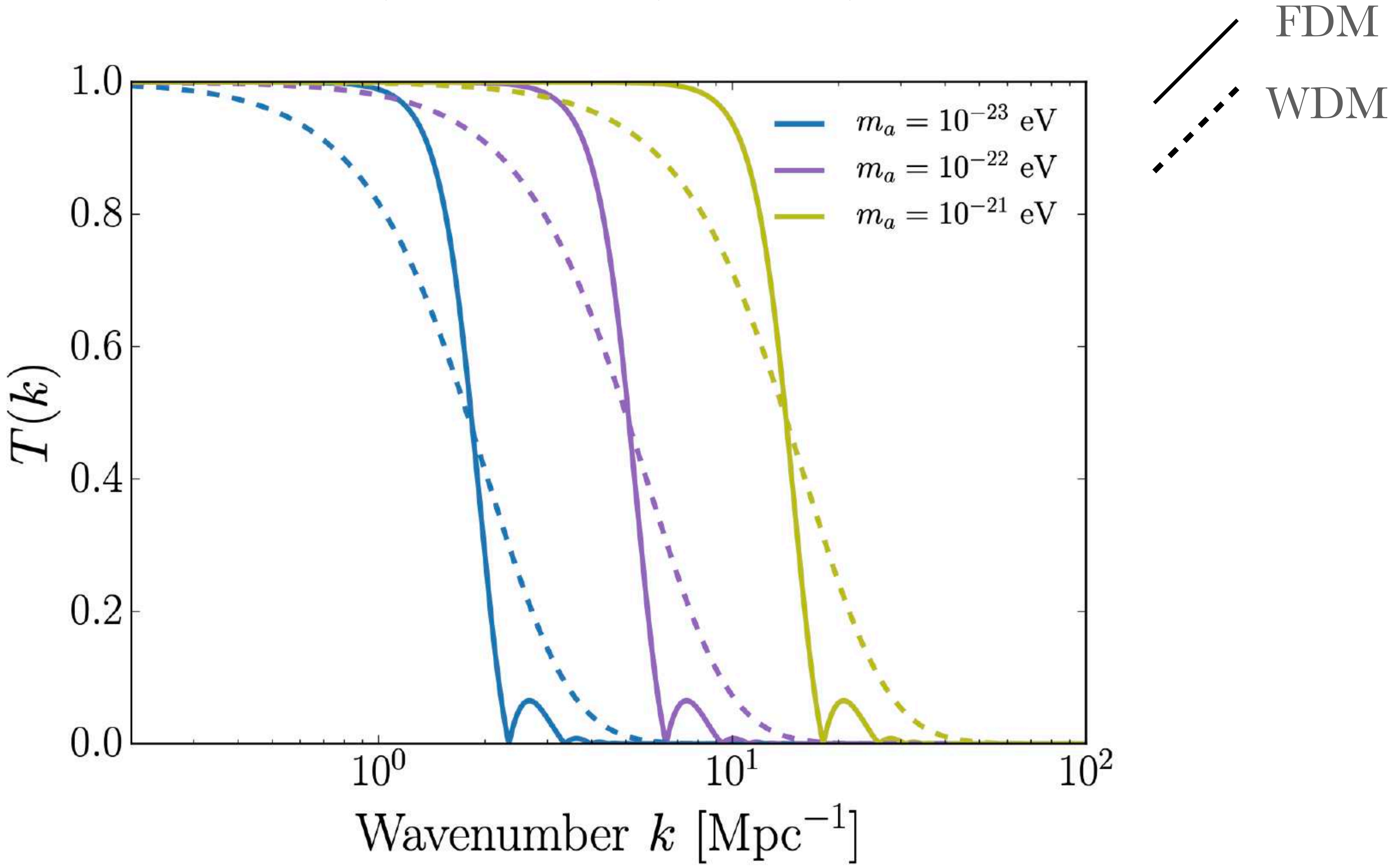


Finite Jeans length λ_J or $\lambda_{attr}, \lambda_{rep}$ → Suppresses small scale structure

POWER SPECTRUM



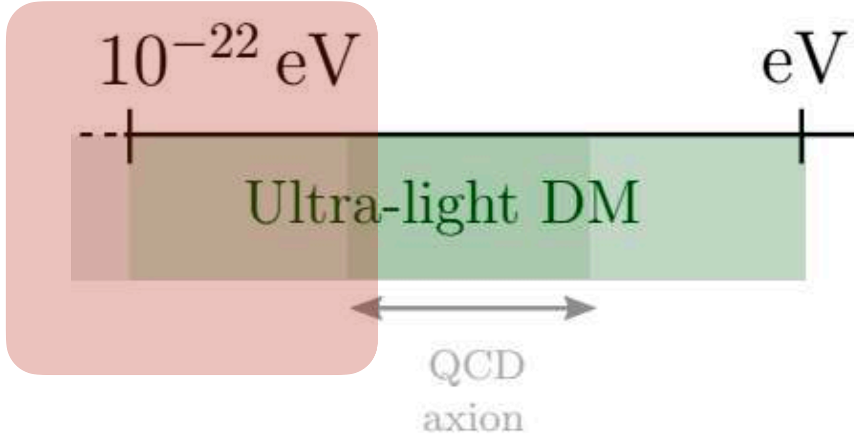
TRANSFER FUNCTION



- Degenerate with WDM

Phenomenology

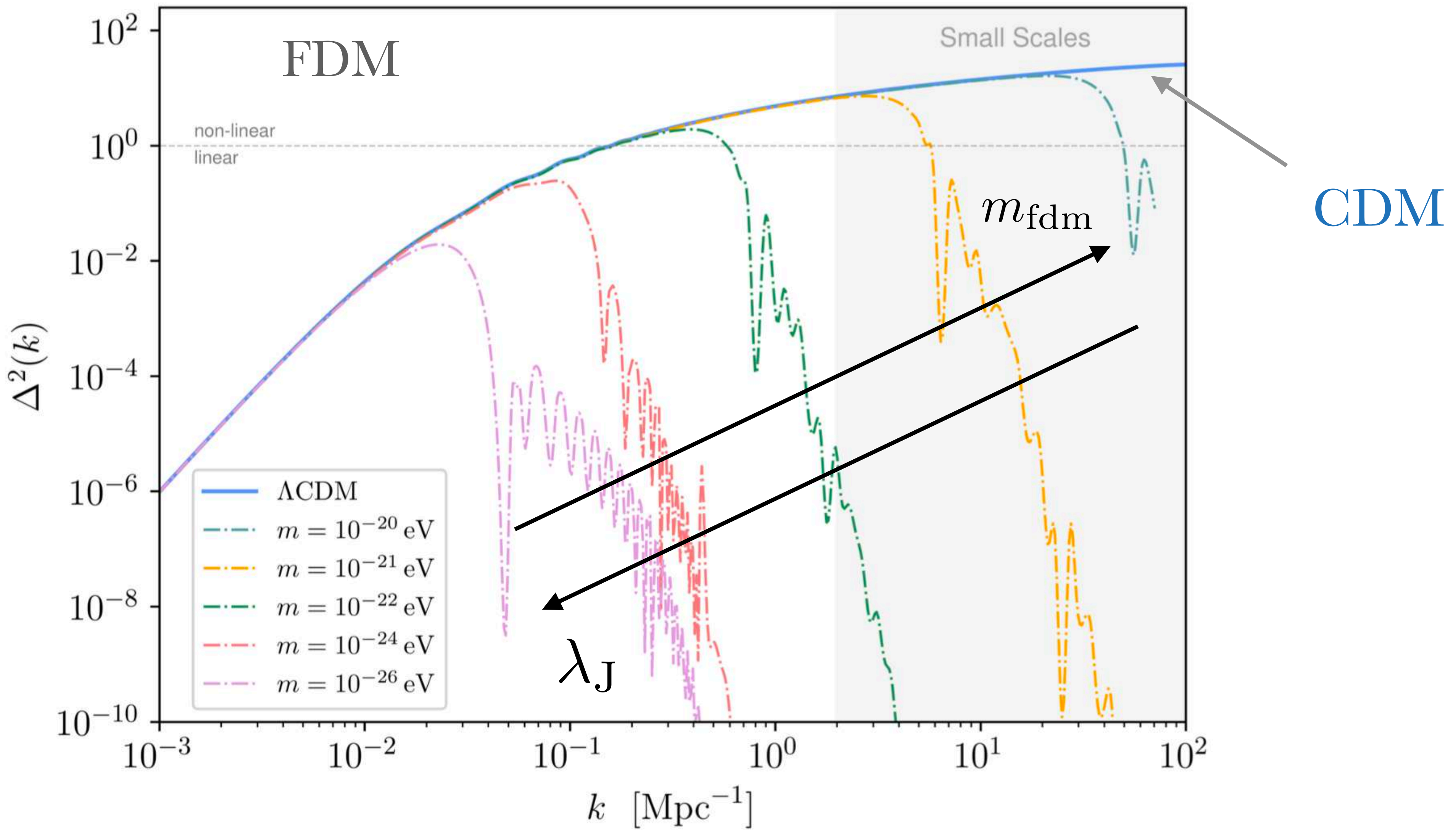
Suppression of small structures



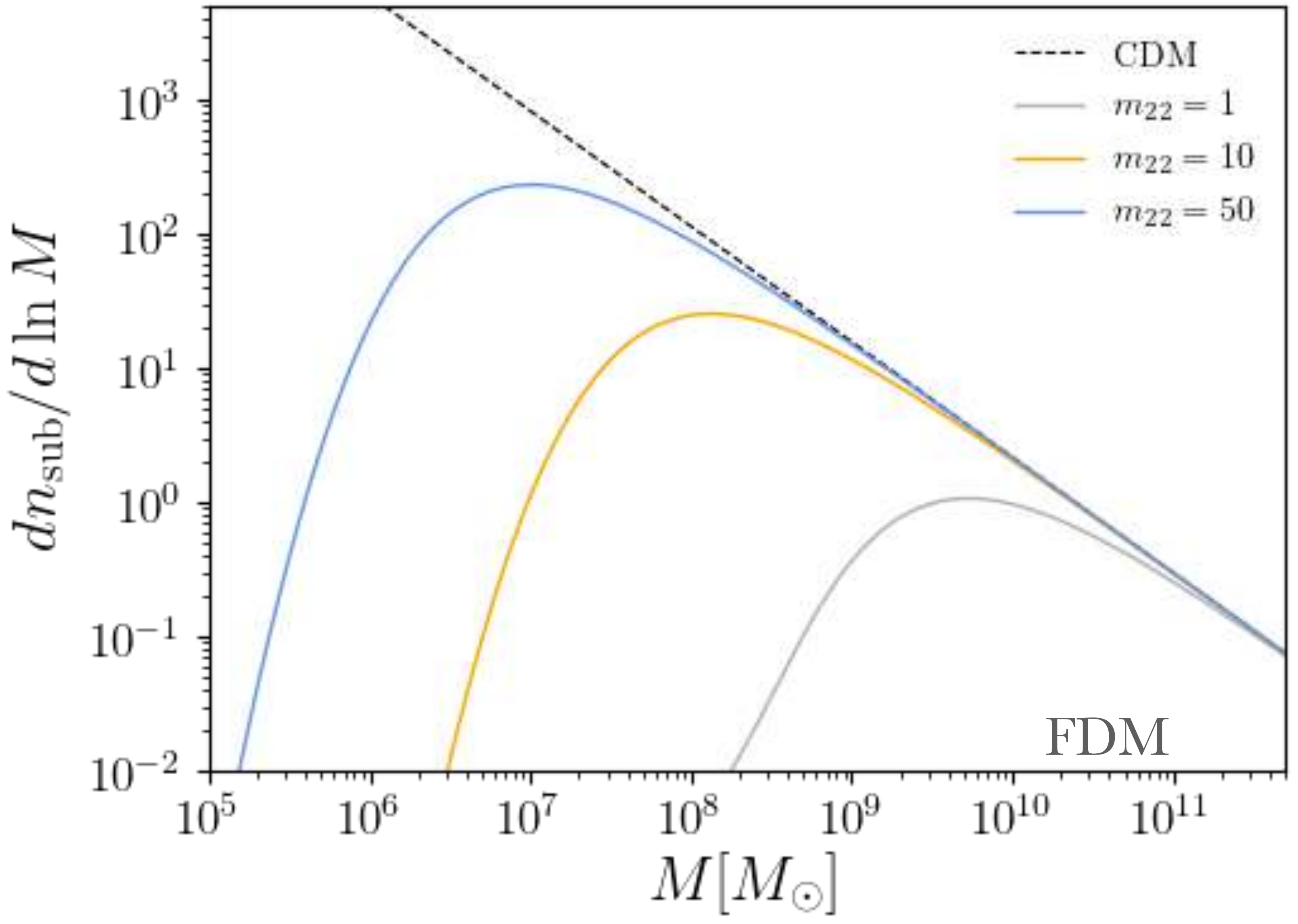
Finite Jeans length λ_J or $\lambda_{\rm attr}, \lambda_{\rm rep}$ → Suppresses small scale structure

$$\lambda_J = 55 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-1/2} \left(\frac{\rho}{\bar{\rho}}\right)^{-1/4} (\Omega_m h)^{-1/4}\,\mathrm{kpc}$$

POWER SPECTRUM



(sub) HALO MASS FUNCTION

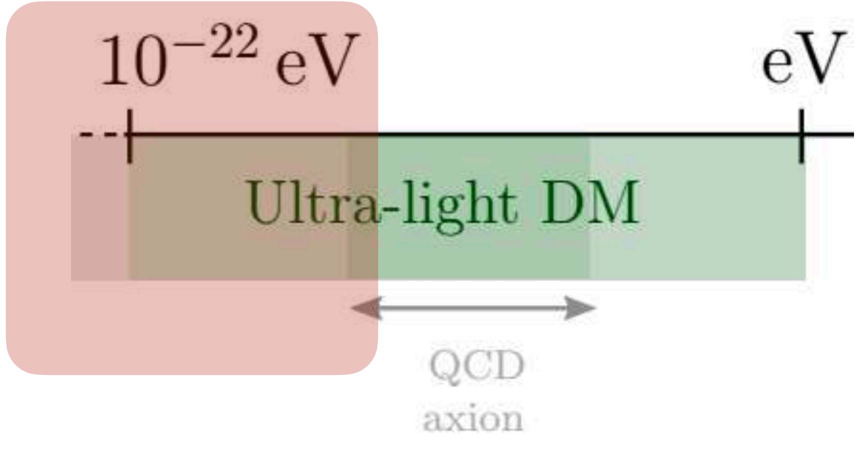


Power spectrum: highly constrained for $k > 10\,\mathrm{Mpc}^{-1}$
unconstrained for $k < 10\,\mathrm{Mpc}^{-1}$

* hard to get a proper prediction!

Practice:

Linear power spectrum

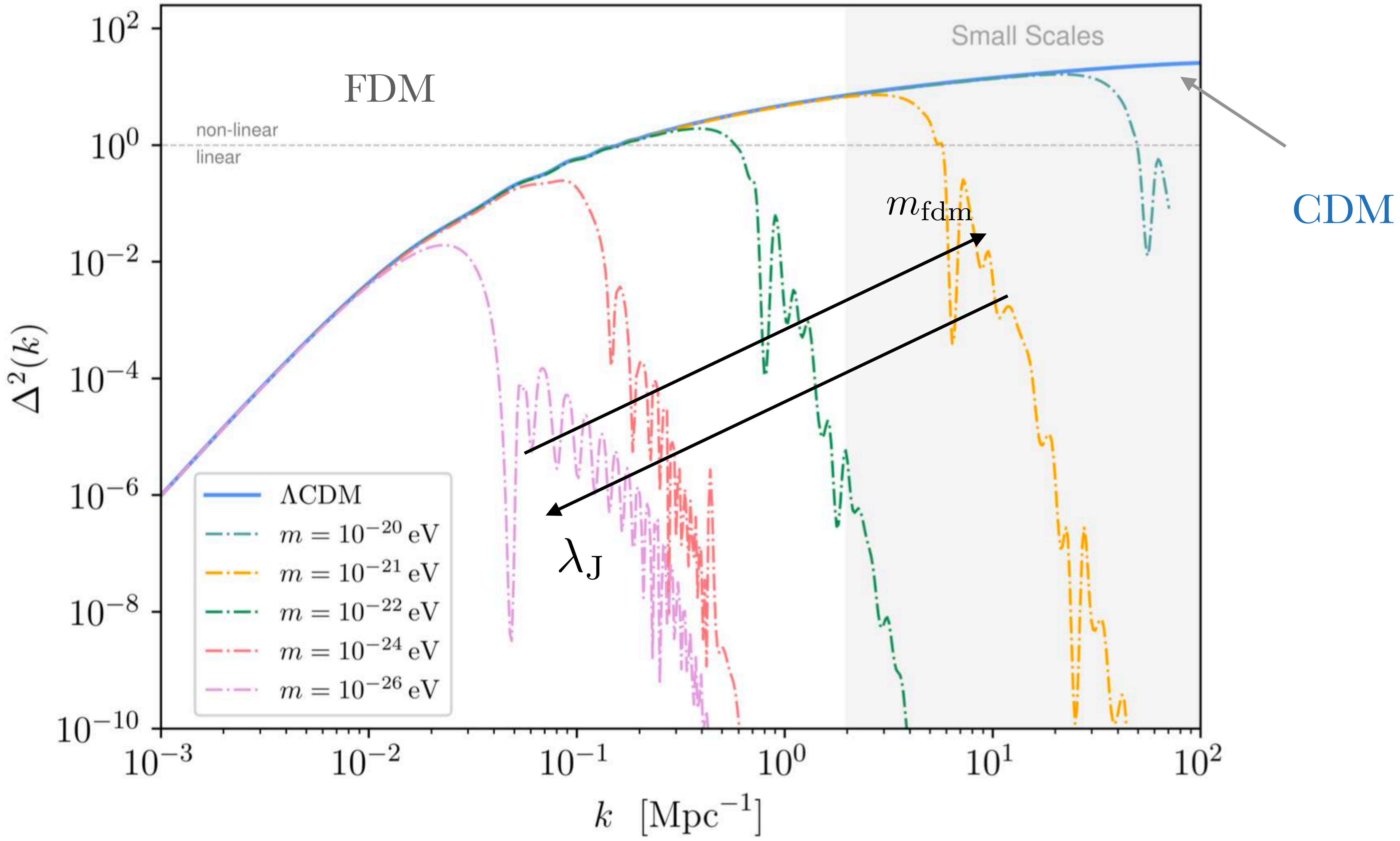


Learn how to generate the linear power spectrum for ULDM using Boltzmann codes.

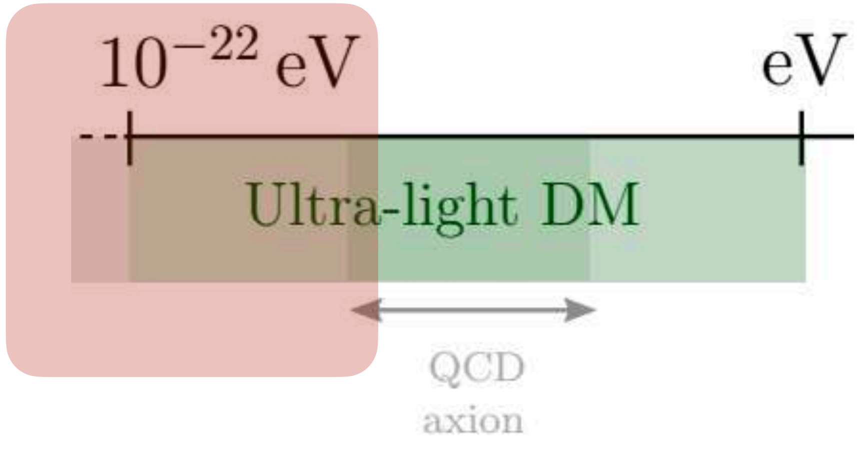
Follow the instructions in this notebook:

https://colab.research.google.com/drive/1364W_M8vl8a01lG7LYMnSUtDEQkuk1MY?usp=sharing

POWER SPECTRUM



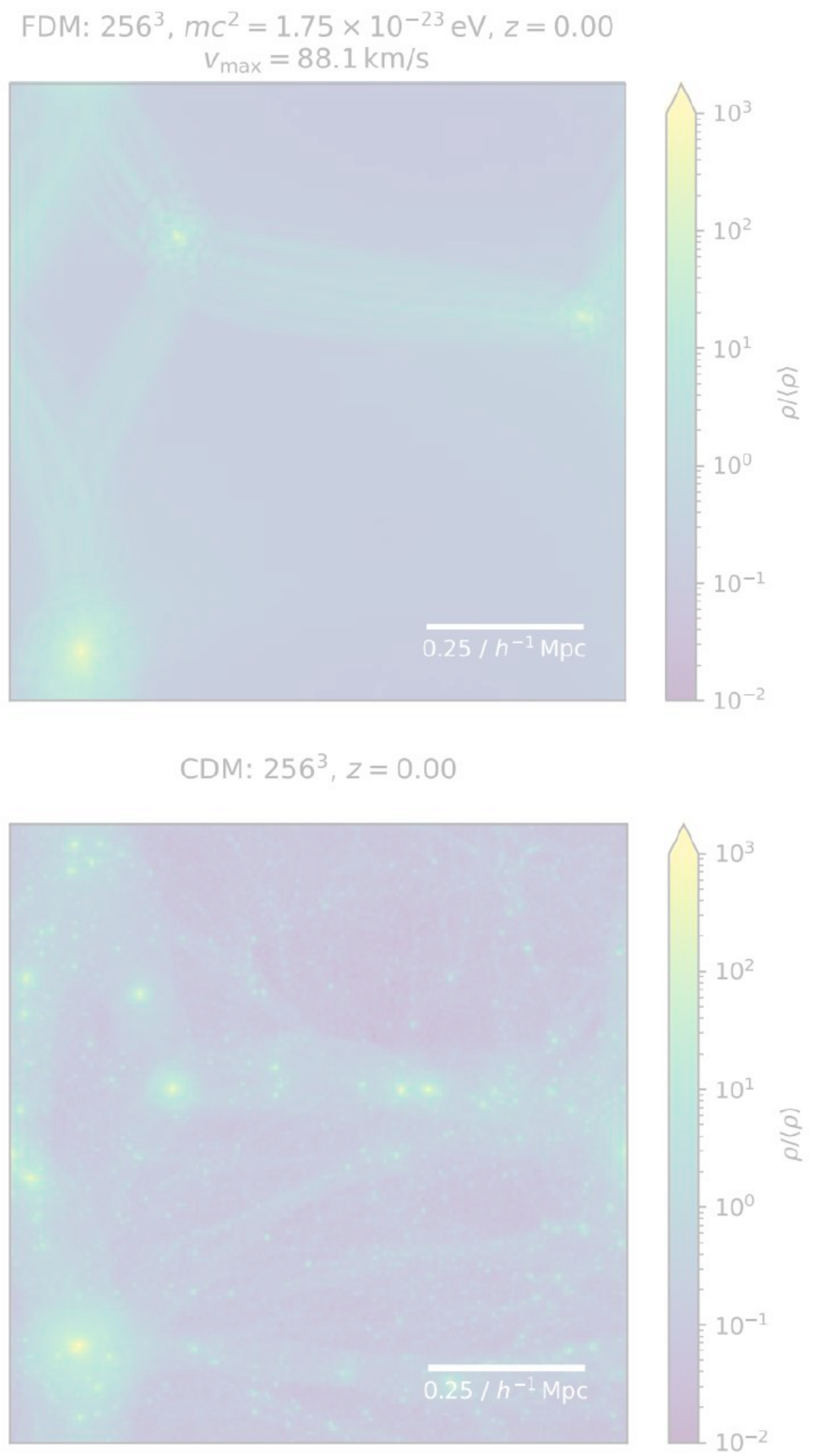
Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

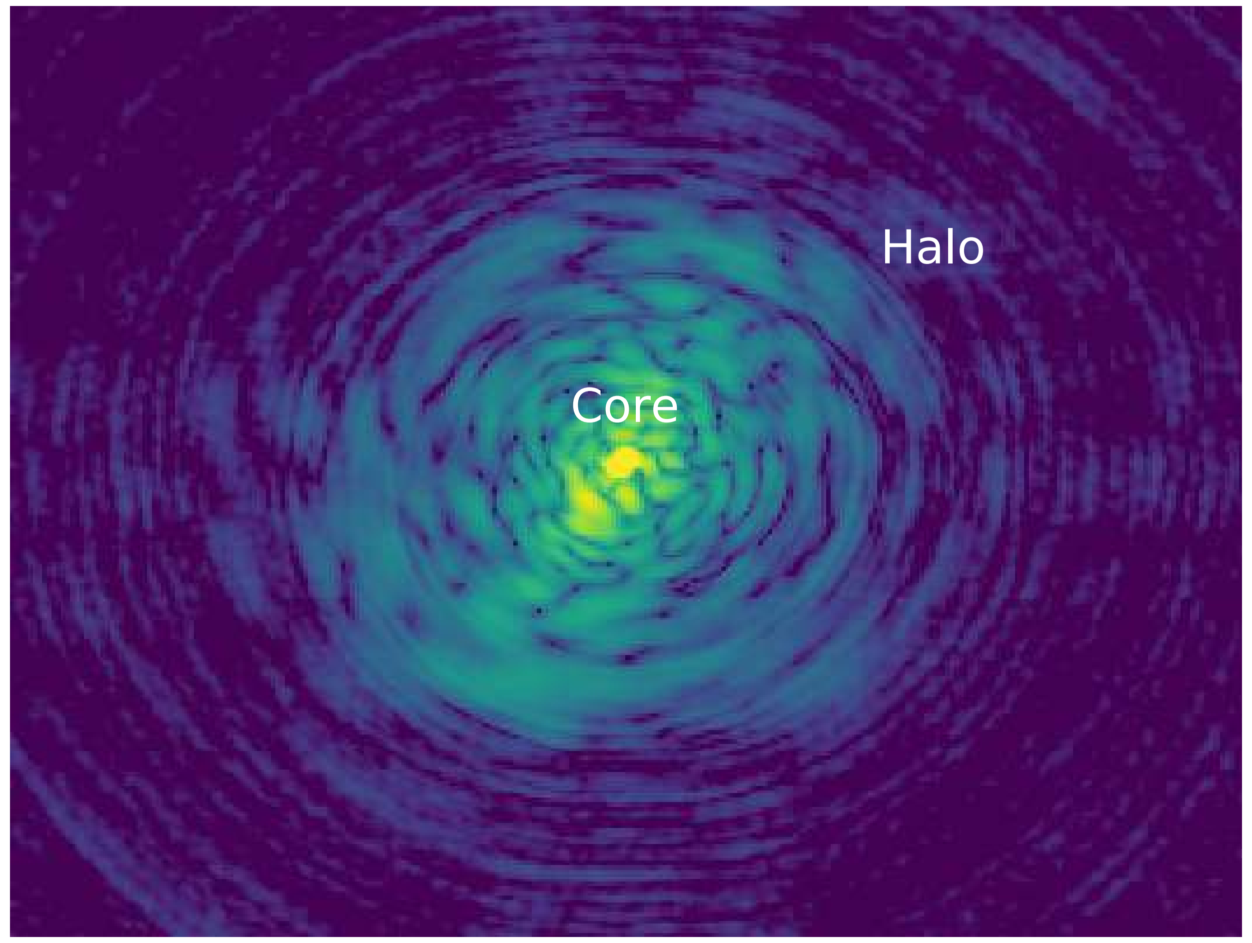
* Focus only in gravitational signatures

Suppression of small structures

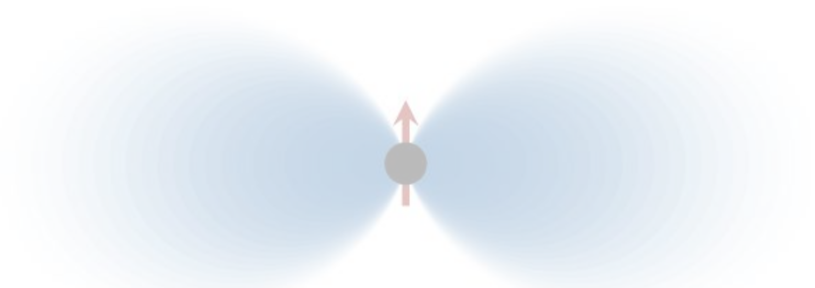


S. May et al. 2021

Formation of a solitonic core

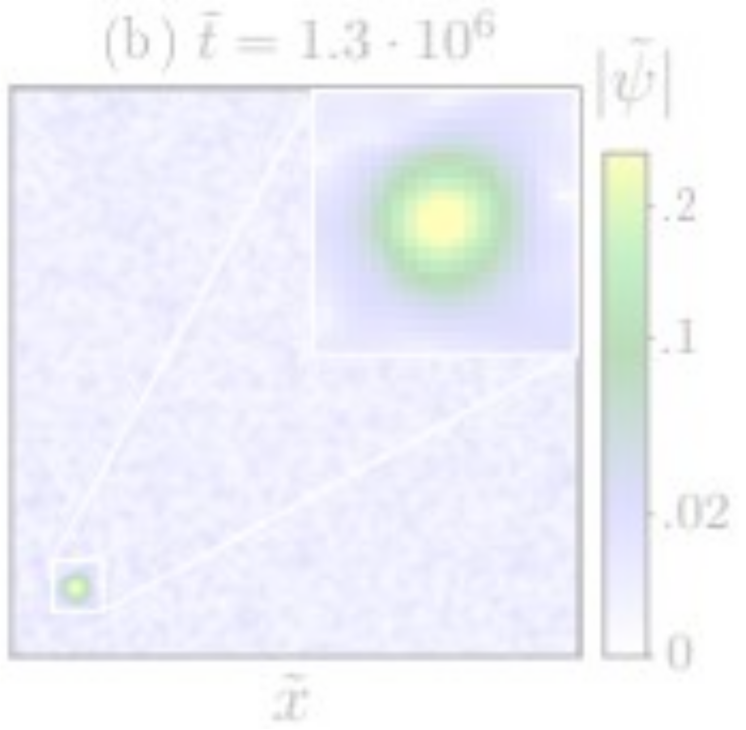


Axion clouds



Baumann et al. 2019

Dynamical effects



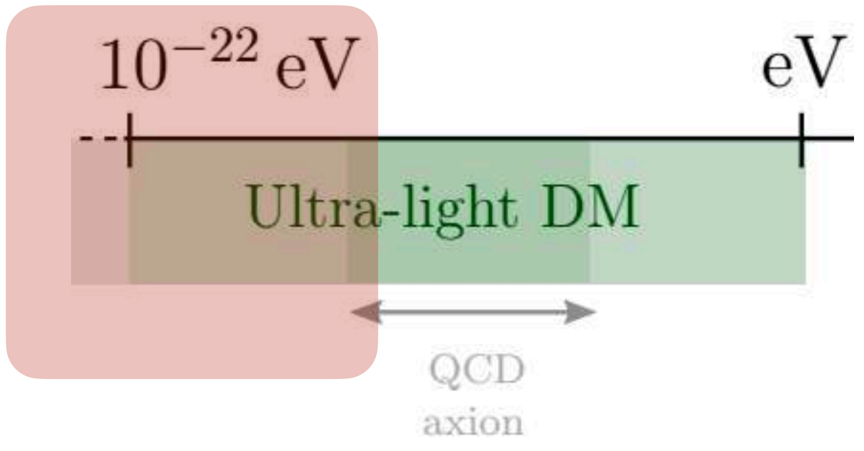
Levkov et al. 2018

Wave interference

Mocz et al. 2017

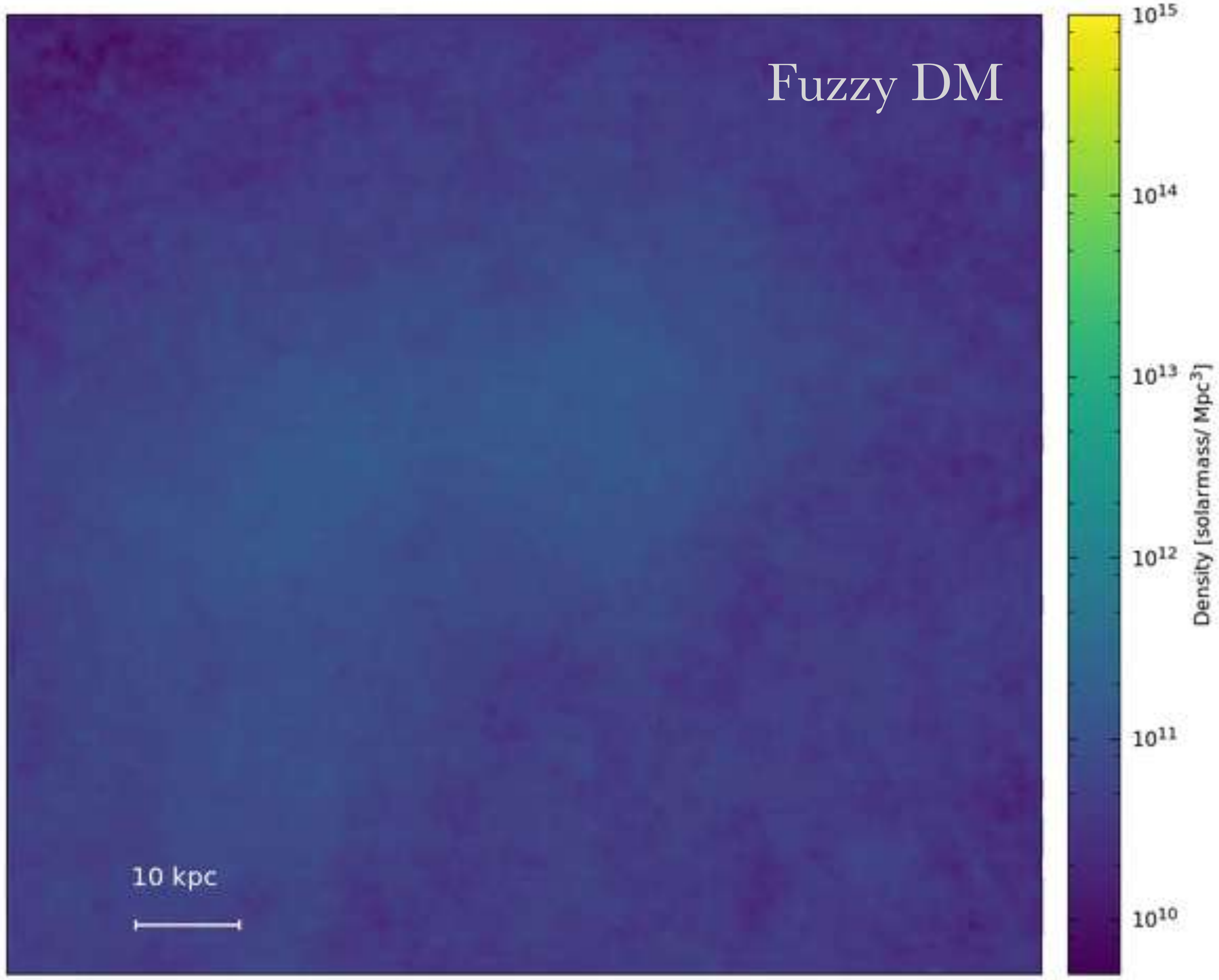
Phenomenology

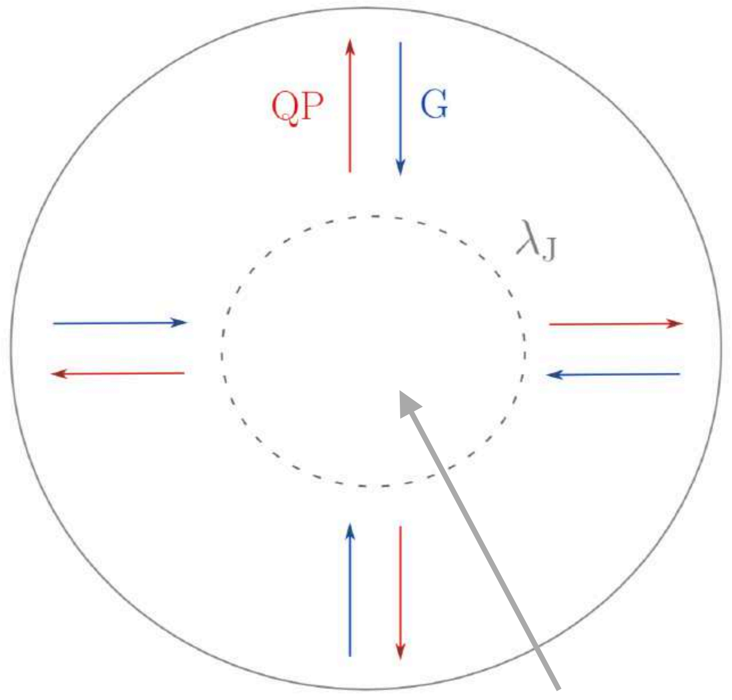
Formation of cores



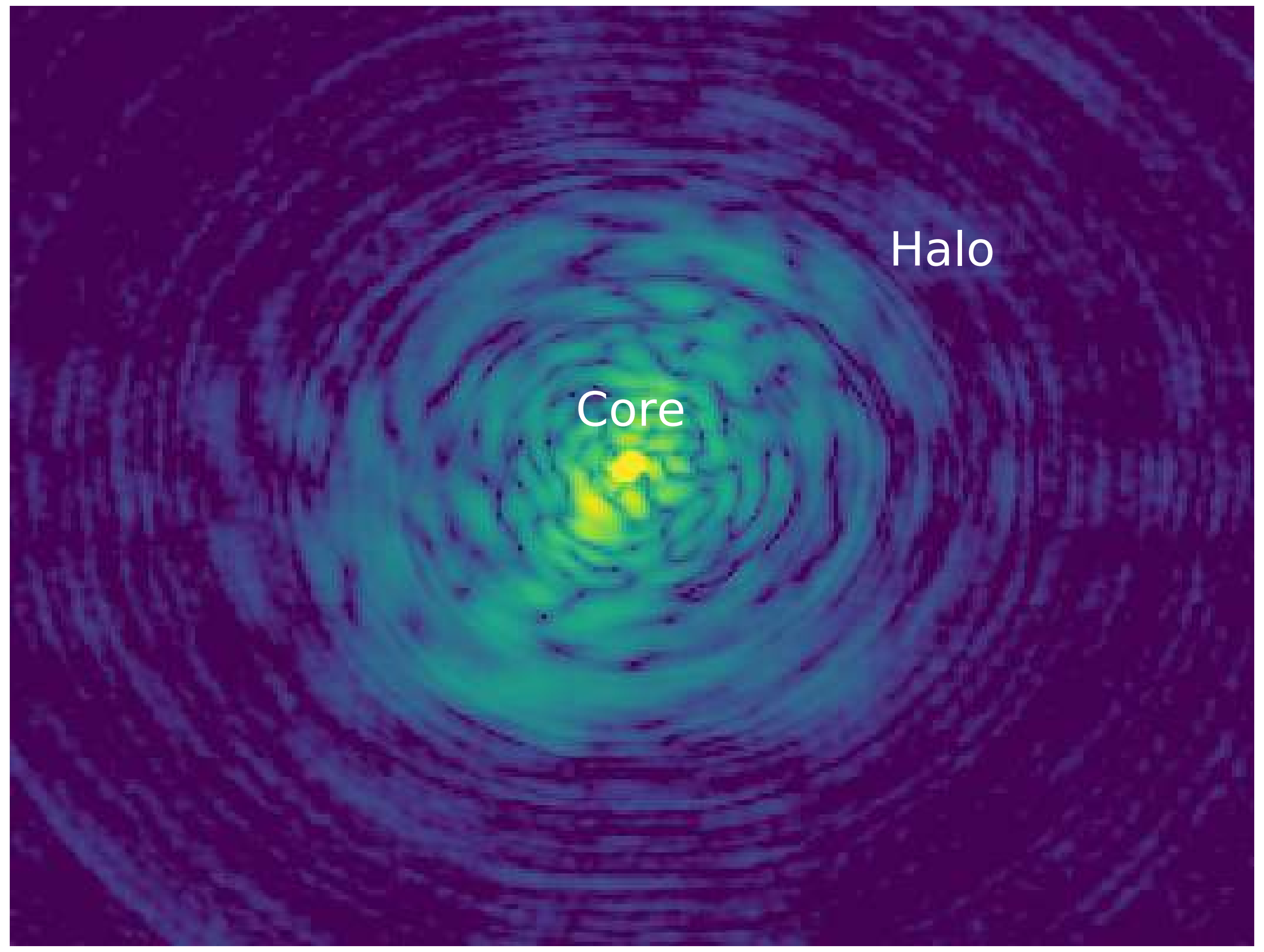
NON-LINEAR
evolution: need
simulations

$m = 10^{-22}\,\mathrm{eV}$ $N = 512^3$ $L = 300\,\mathrm{kpc}$



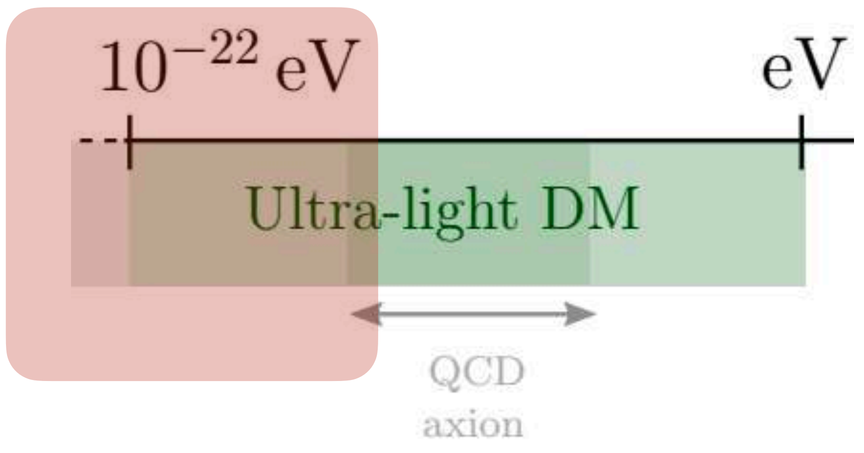


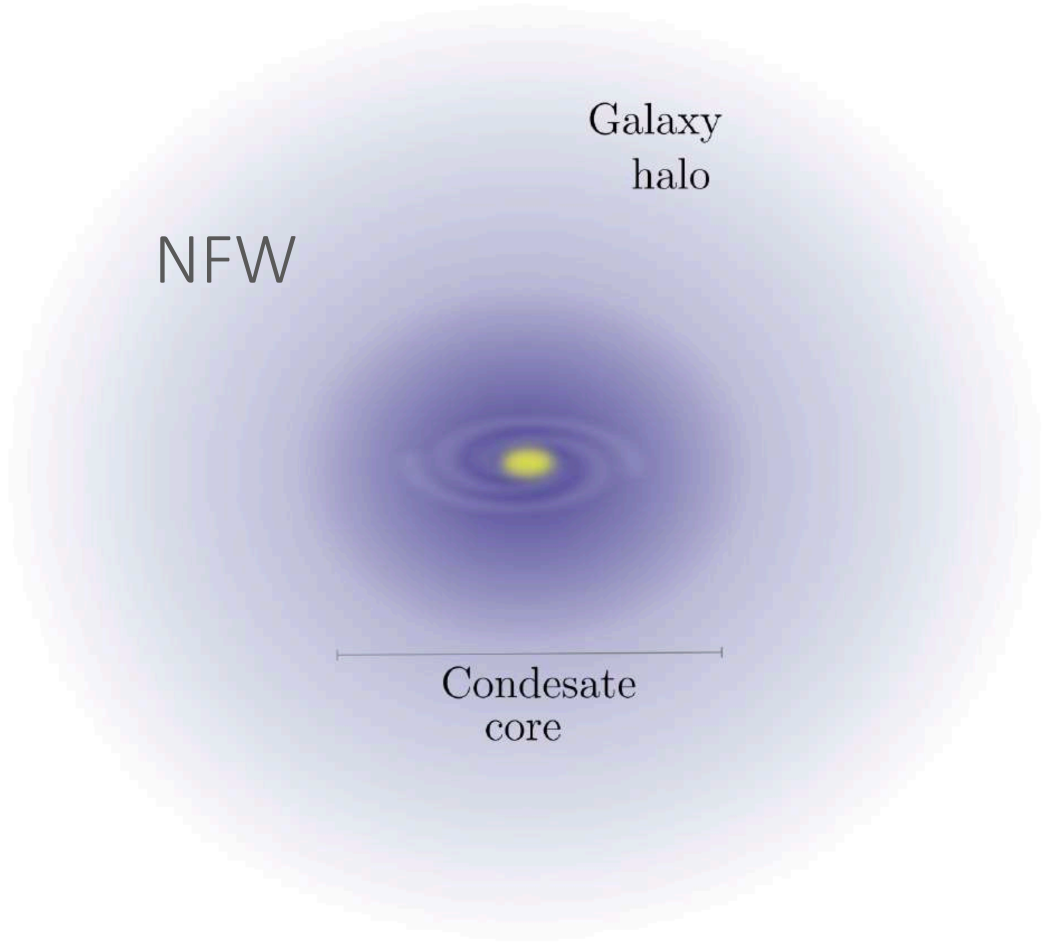
NO structure formation
Stable, oscillating solution



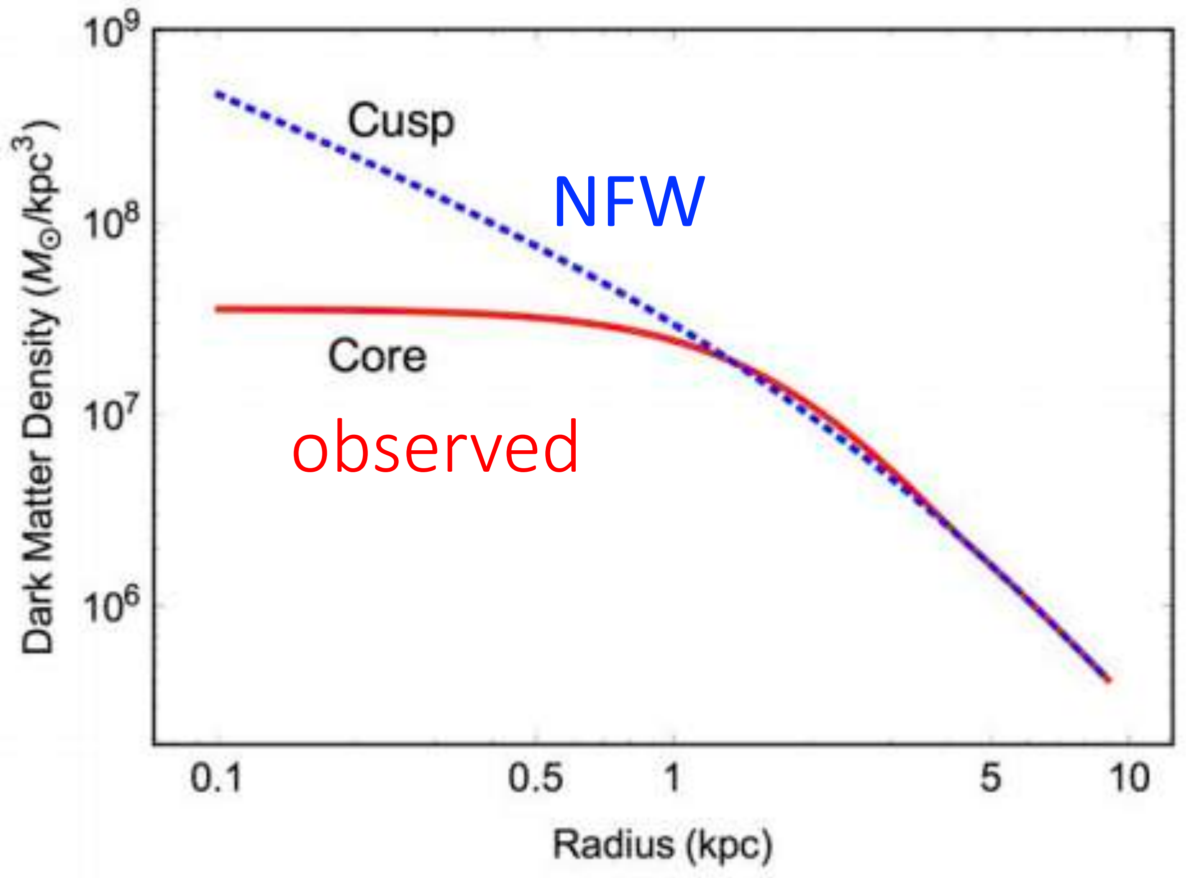
Phenomenology

Formation of cores





$$\rho(r) \simeq \begin{cases} \rho_c & \text{for } r \leq r_c \\ \rho_{\rm NFW} & \text{for } r \geq r_c \end{cases}$$



FDM From simulations Schive et al. 2014, fitting function:

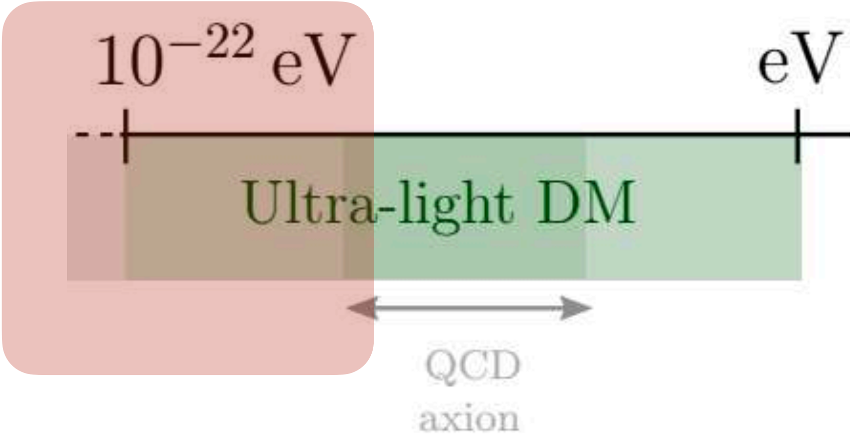
Updated in *Chan, EF et al 2021*

$$\rho_c \simeq \frac{1.9\times10^{-2}}{[1+0.091\,(r/R_{1/2,c})^2]^8}\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{-2}\left(\frac{r_c}{\rm kpc}\right)^{-4} M_\odot\,{\rm pc}^{-3}\,,$$

$$r_c \simeq 0.16\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{-1}\left(\frac{M}{10^{12}\,M_\odot}\right)^{-1/3}{\rm kpc}\,.$$

Relations used to compare with observations

Phenomenology



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S. May et al. 2021

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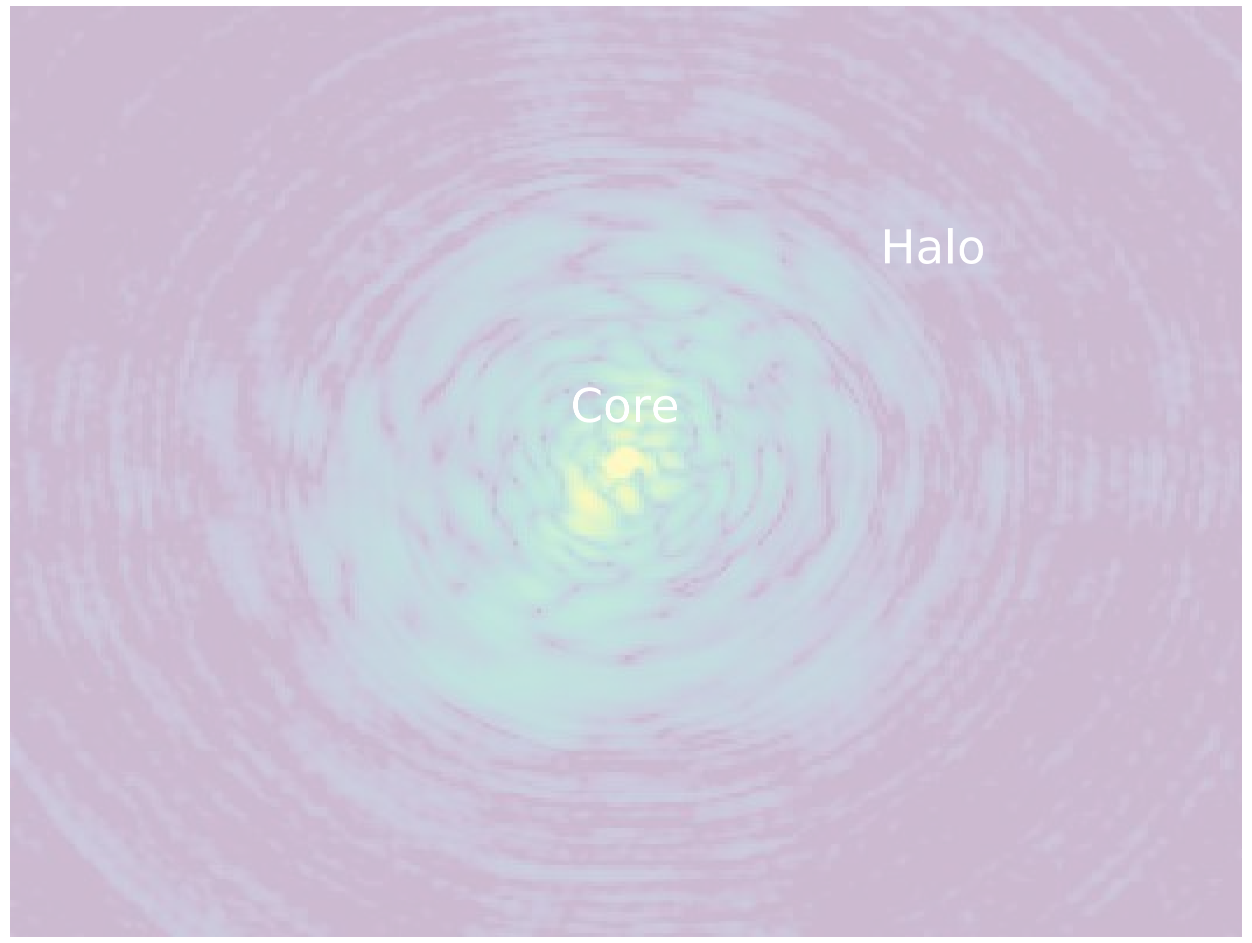
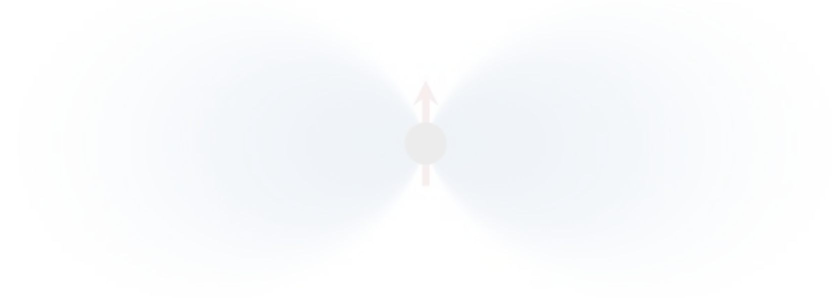
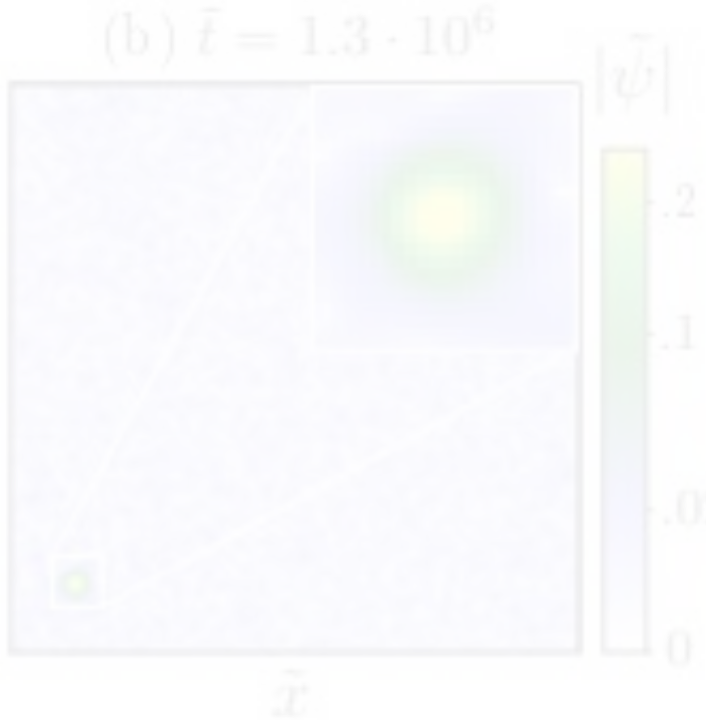


Axion clouds



Baumann et al. 2019

Dynamical effects



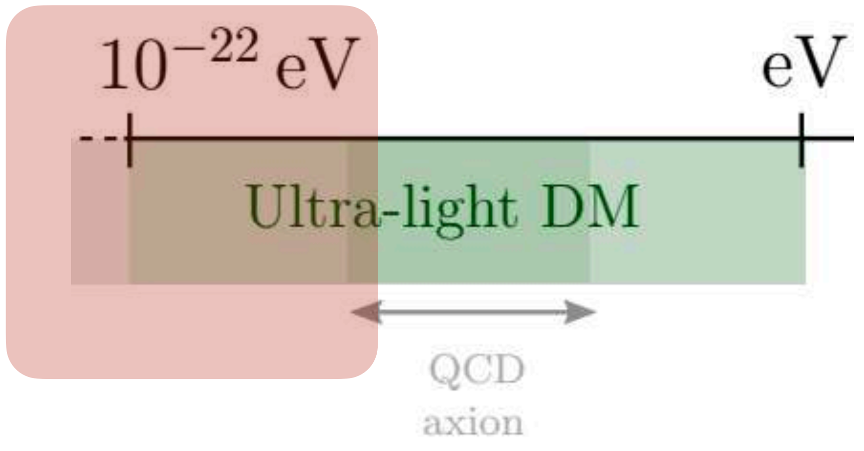
Levkov et al. 2018

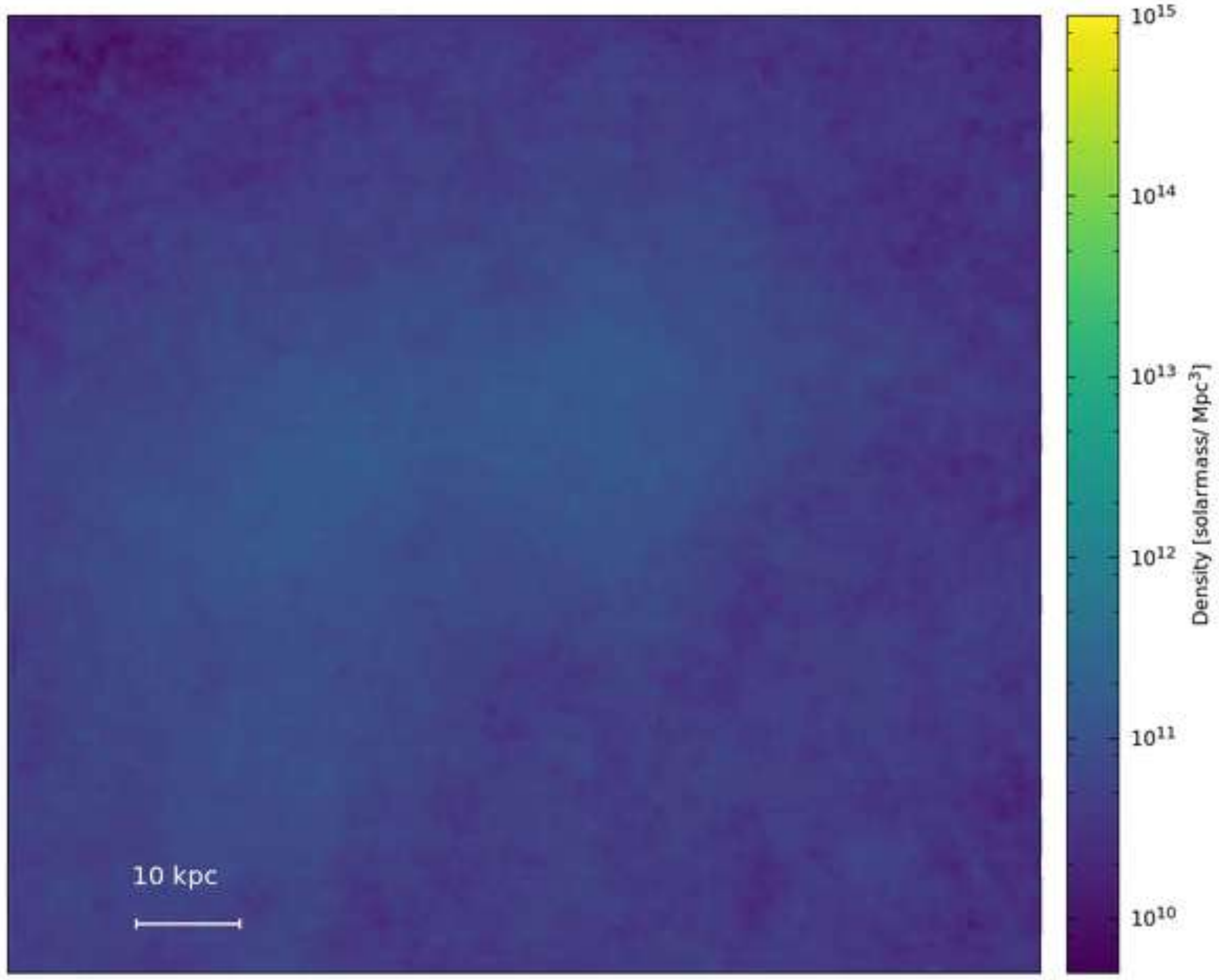
Wave interference

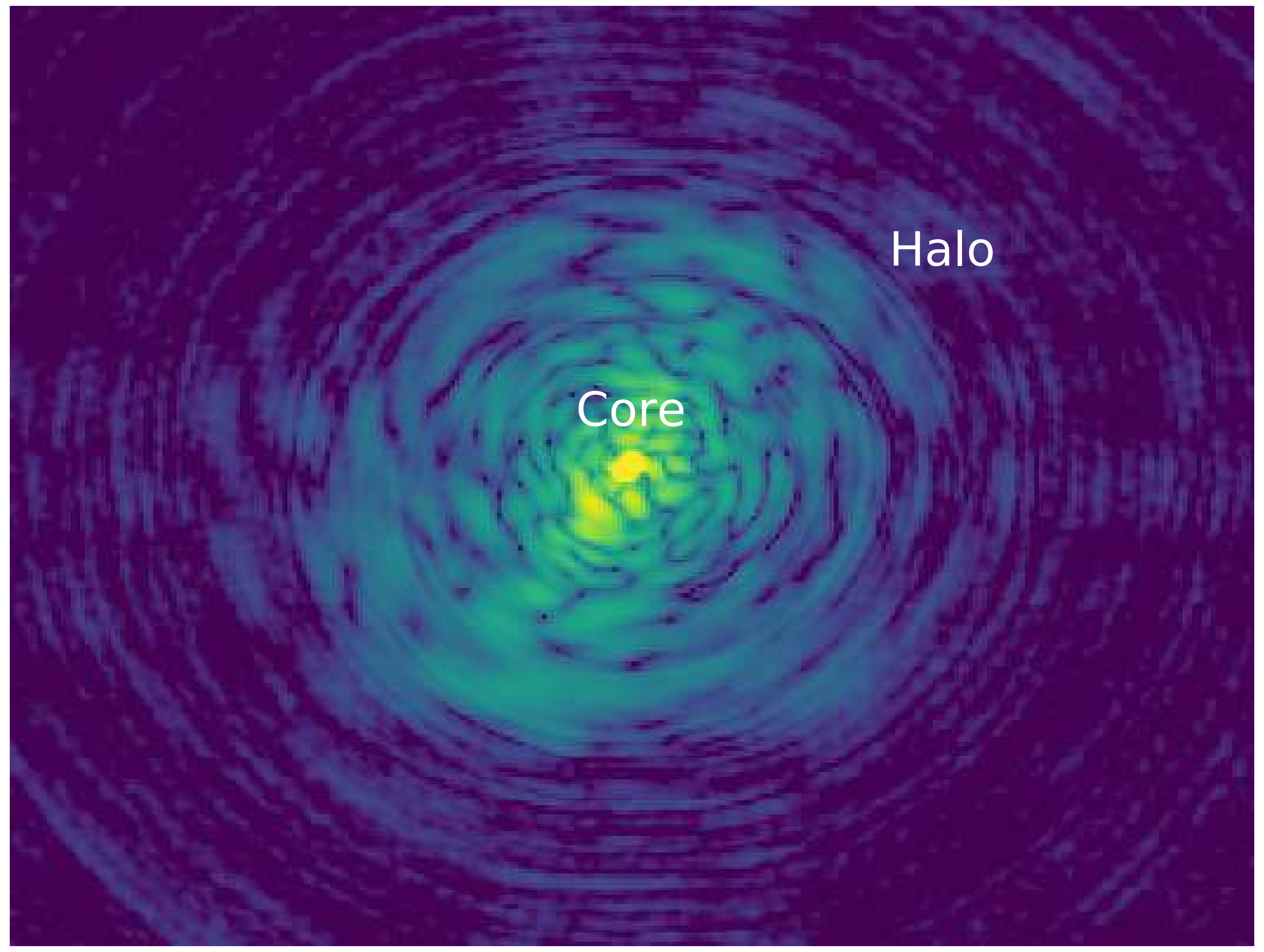
Mocz et al. 2017

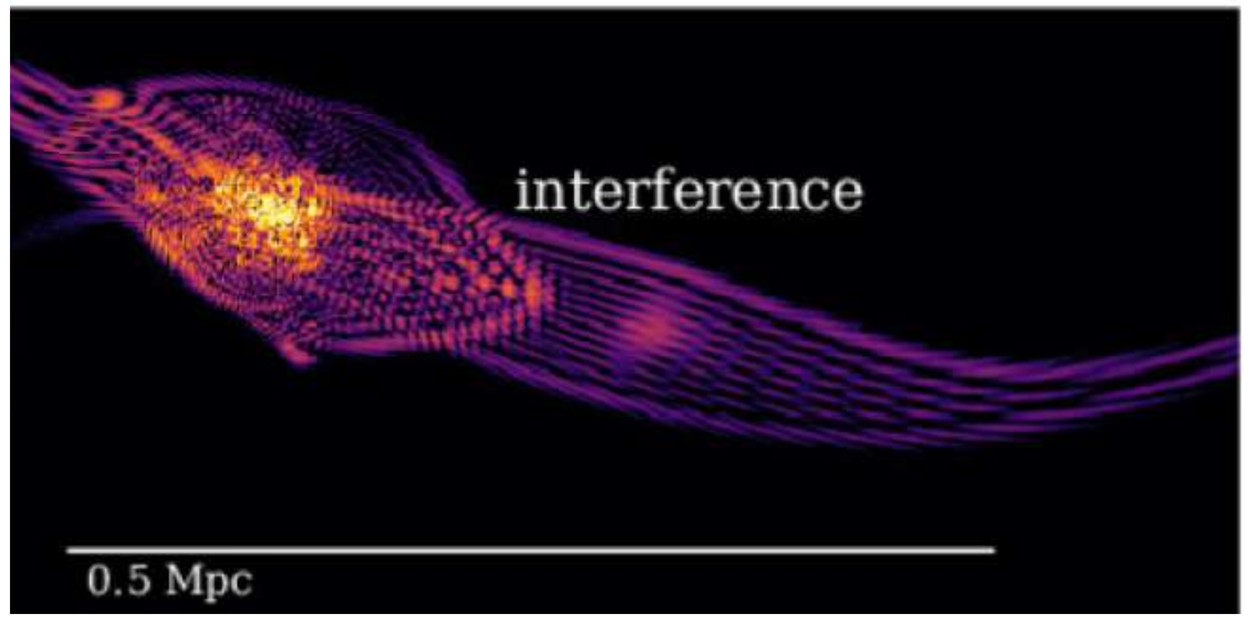
Phenomenology

Wave interference: granules and vortices









Mocz et al. 2017

Order one fluctuations in density ⟶ Constructive interference: granules

Destructive interference

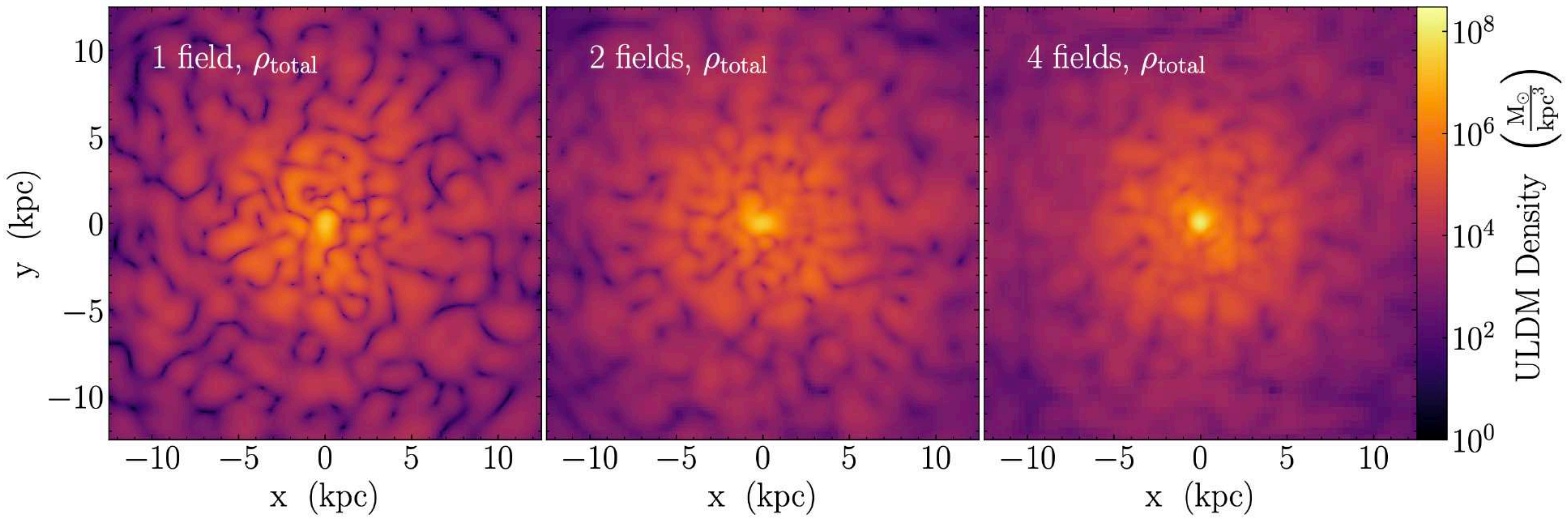
$\hookrightarrow \;\sim \lambda_{\mathrm{dB}}$

Vector, higher spin or multicomponent FDM

ULDM or ULA are a coherent wave - same frequency and constant phase difference

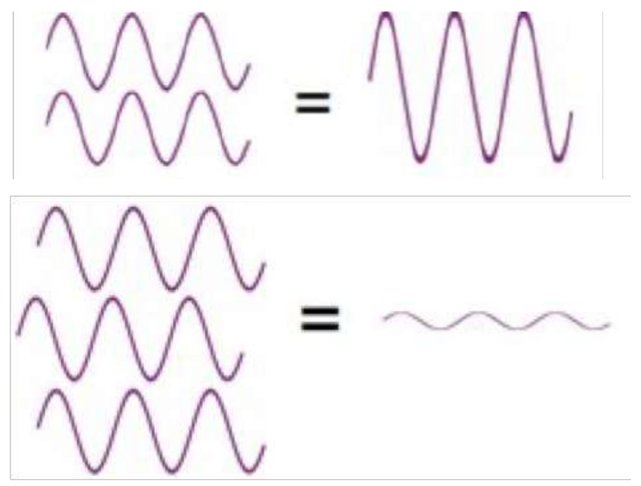
Multiple coherent waves

For ULDM:



Gonseca et al 2023

Interference patterns



Multiple FDM or VFDM (or higher spin s FDM) *attenuates* the granule amplitude by

$$\frac{[\delta\rho/\rho]_{\mathrm{nfdm,s}}}{[\delta\rho/\rho]_{\mathrm{fdm}}} \propto \frac{1}{\sqrt{(2s+1)}} = \frac{1}{\sqrt{N}}$$

(Amin et al 2022)

Vector (and higher-spin) FDM Amin et al 2022

(Vector FDM = 3 x same mass FDM (spin 0))

Multicomponent FDM Gonseca et al 2023

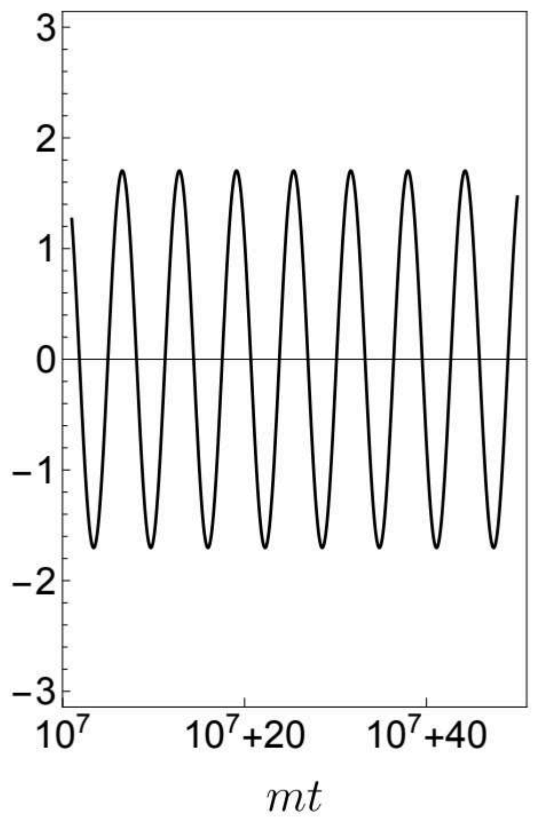
Modeling a granular halo

Coherent wave oscillation of ULDM

$$\phi(t,\vec{x}) = m^{-1}\sqrt{2\rho(t,\vec{x})}\,\cos[mt+\theta]$$

Fixed freq.

Constant phase



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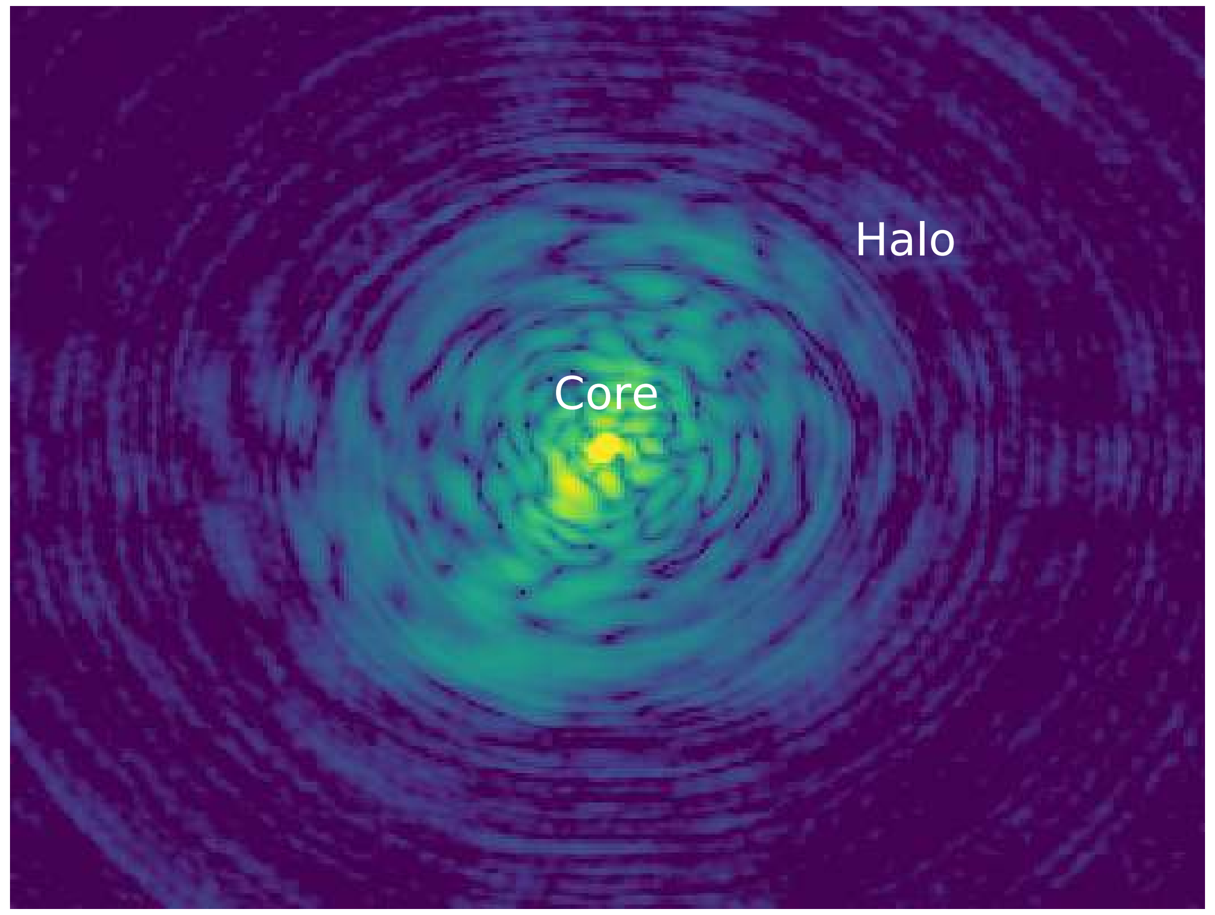
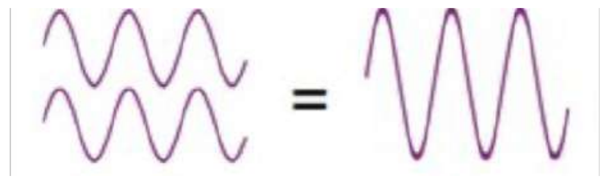
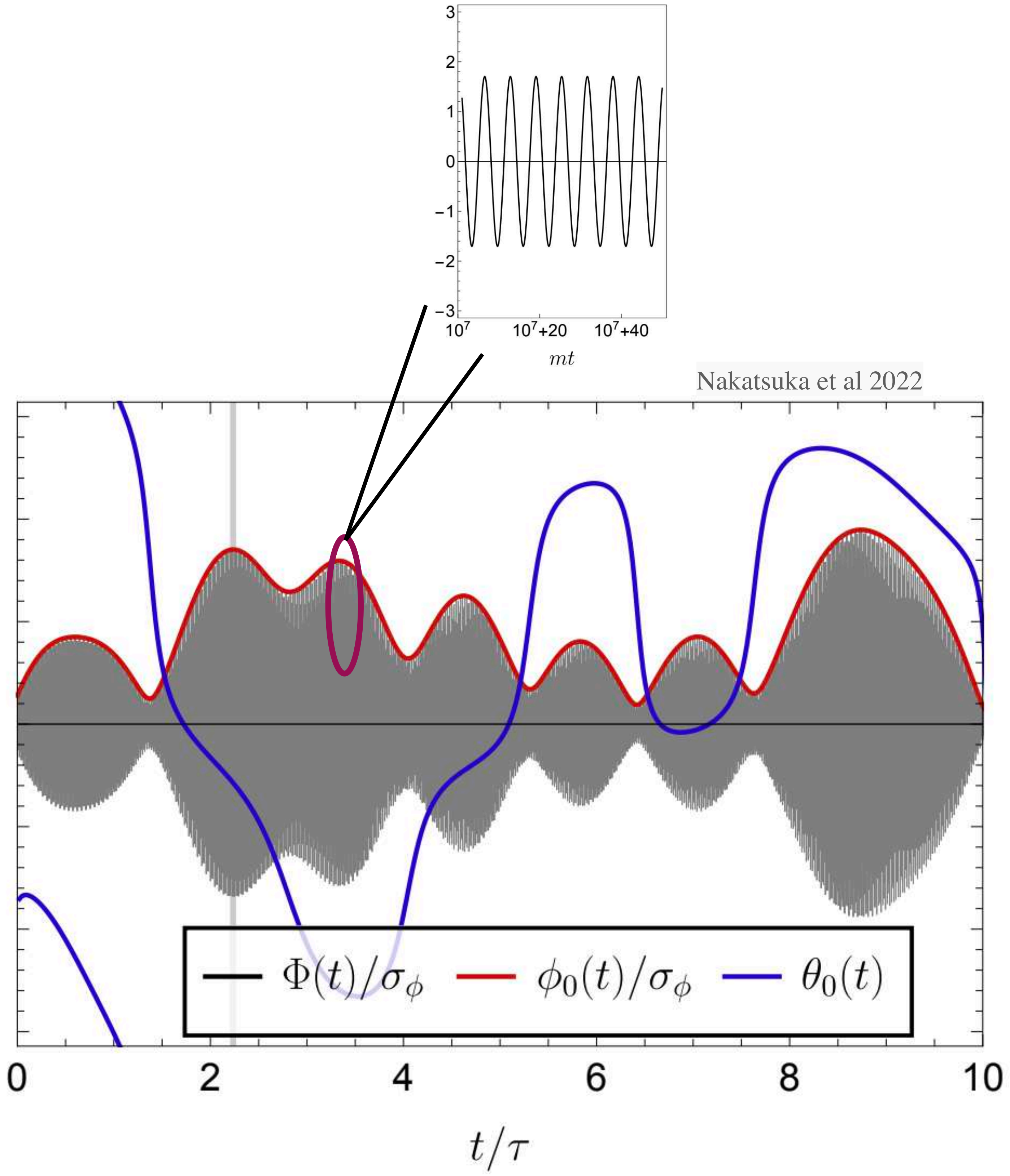


Superposition of plane waves





Nakatsuka et al 2022

Modeling a granular halo

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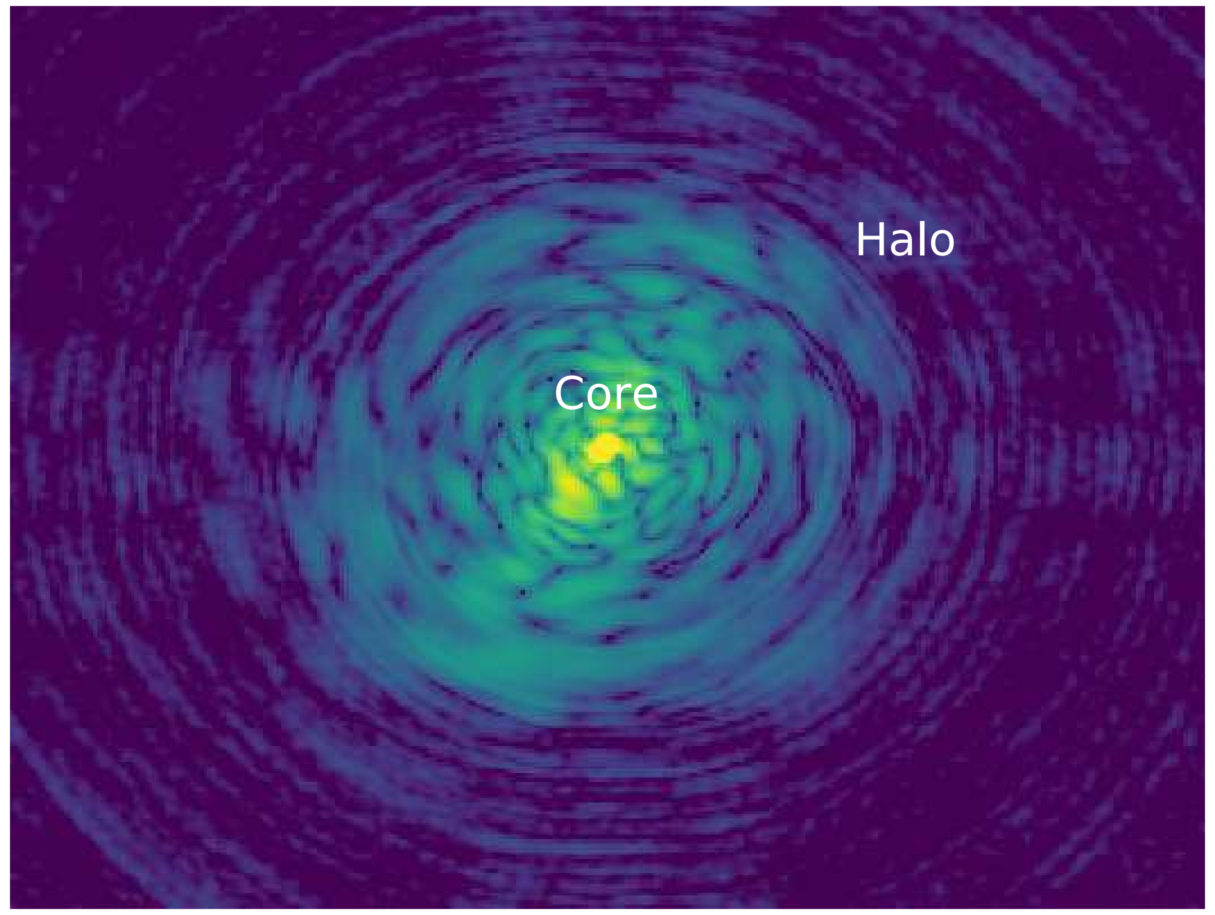

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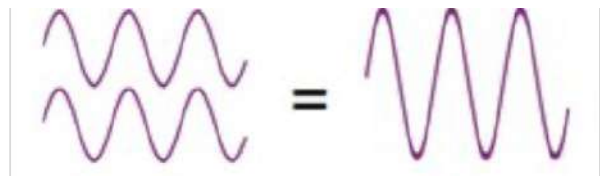
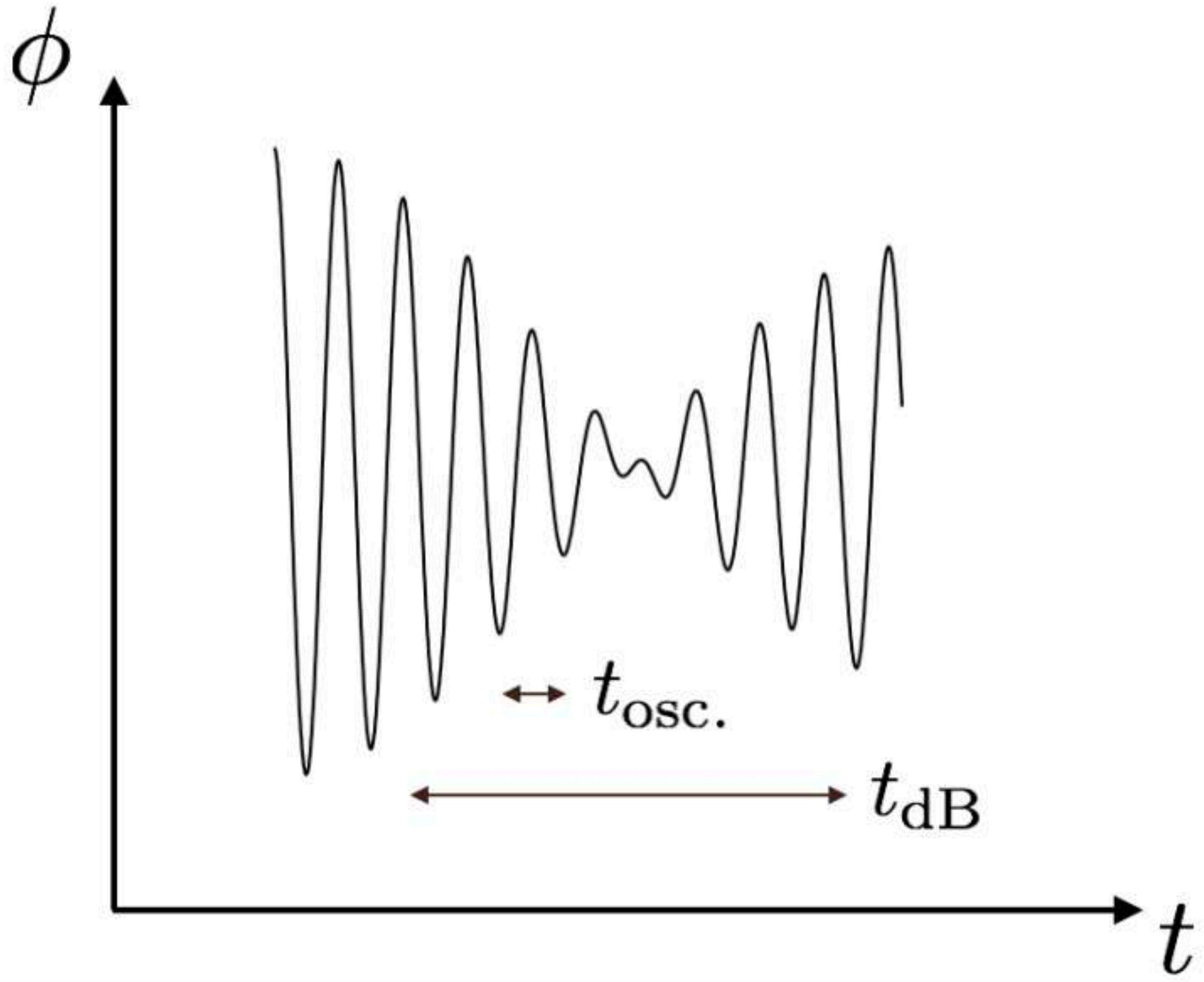




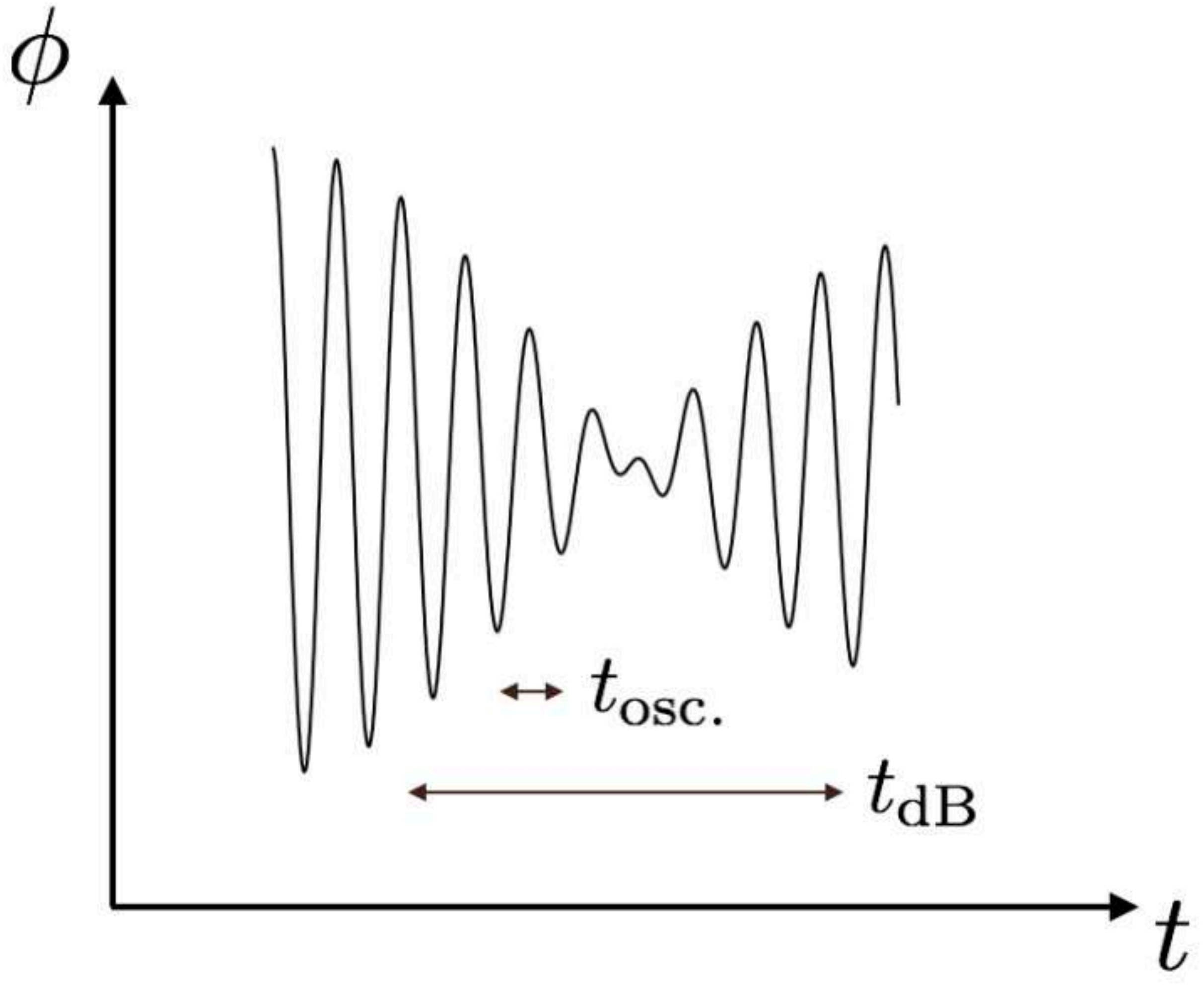
$$t_{\rm osc.} = 2\pi/m$$

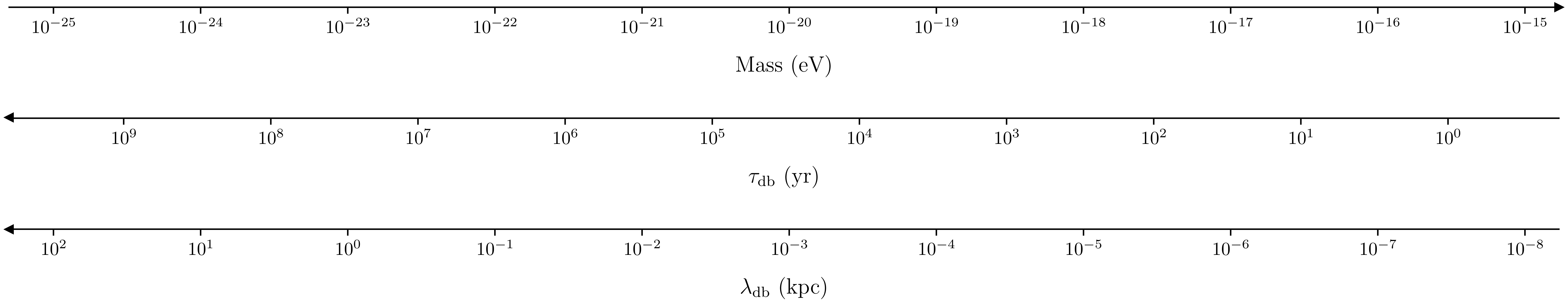
$$t_{\rm dB} = 2\pi/(mv^2)$$
$$= 1.9\times10^{6}\,{\rm yr}\left(\frac{10^{-22}\,{\rm eV}}{m}\right)\left(\frac{250\,{\rm km/s}}{v}\right)^2$$

Modeling a granular halo

$$t_{\mathrm{osc.}} = 2\pi/m$$

$$\begin{aligned} t_{\mathrm{dB}} &= 2\pi/(mv^2) \\ &= 1.9 \times 10^6\,\mathrm{yr}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\frac{250\,\mathrm{km/s}}{v}\right)^2 \end{aligned}$$



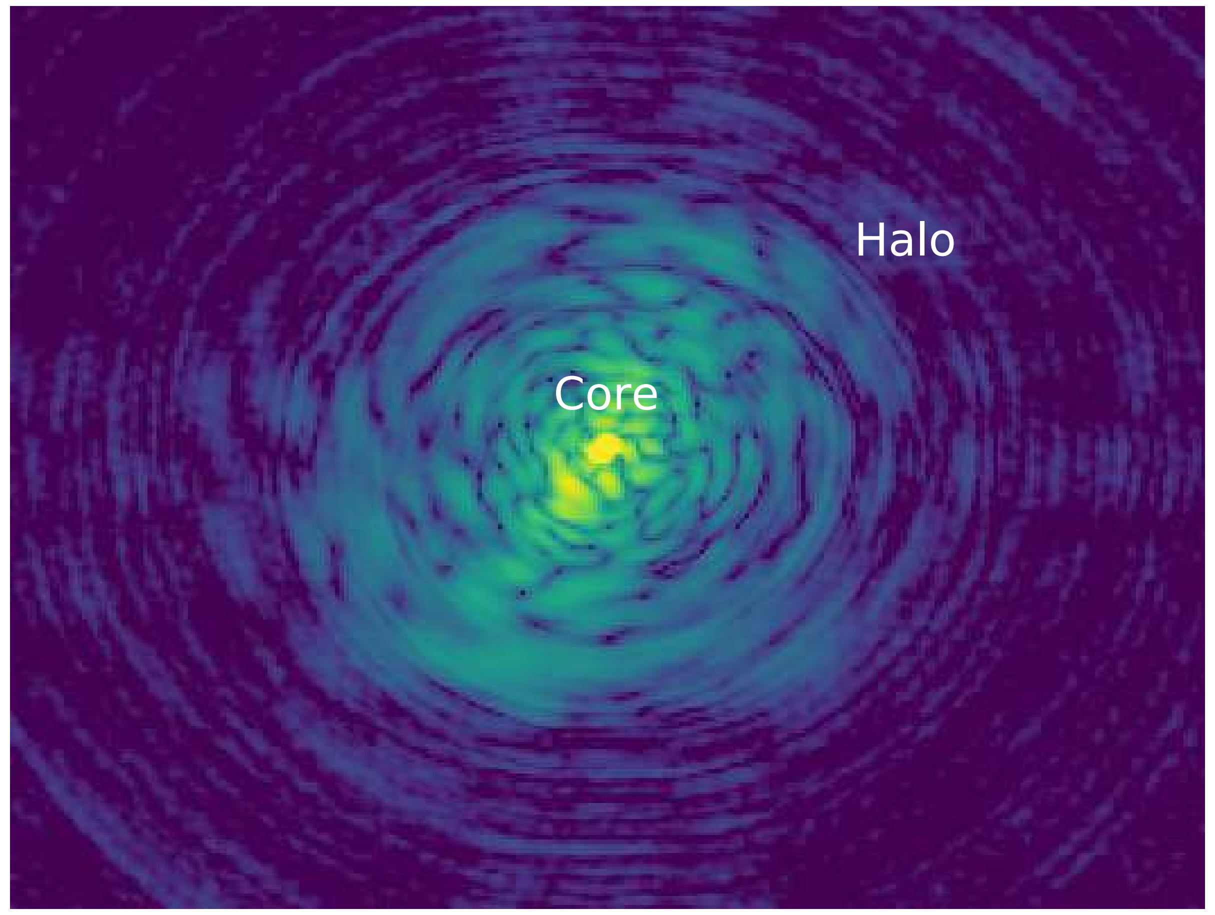


Modelling a granular halo

Full SP simulations can describe perfectly this interference pattern (while fluid ones *cannot* describe it)

OR

We can adopt simpler descriptions of the galactic halo to describe this effect.



1) A simple model of a galactic halo, consider a superposition of plane waves:

Randomly distributed

$$\psi(t,\vec{x}) = \sum_{\vec{k}} A_{\vec{k}} e^{iB_{\vec{k}}} e^{i\vec{k}\cdot\vec{x}-i\omega_k t}$$

Also known as *random phase halo model*

The amplitudes should reflect the velocity (or momentum) dispersion within the halo

$$\rho = m|\psi|^2 = m\sum_{\vec{k}} A_{\vec{k}}^2 + m\sum_{\vec{k}\neq\vec{k}'} A_{\vec{k}} A_{\vec{k}'} e^{i(B_{\vec{k}}-B_{\vec{k}'})} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}-i(\omega_k-\omega_{k'})t}$$

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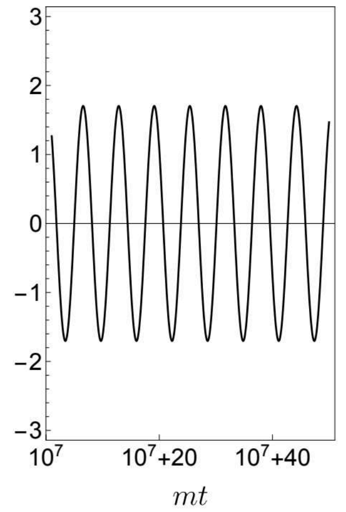
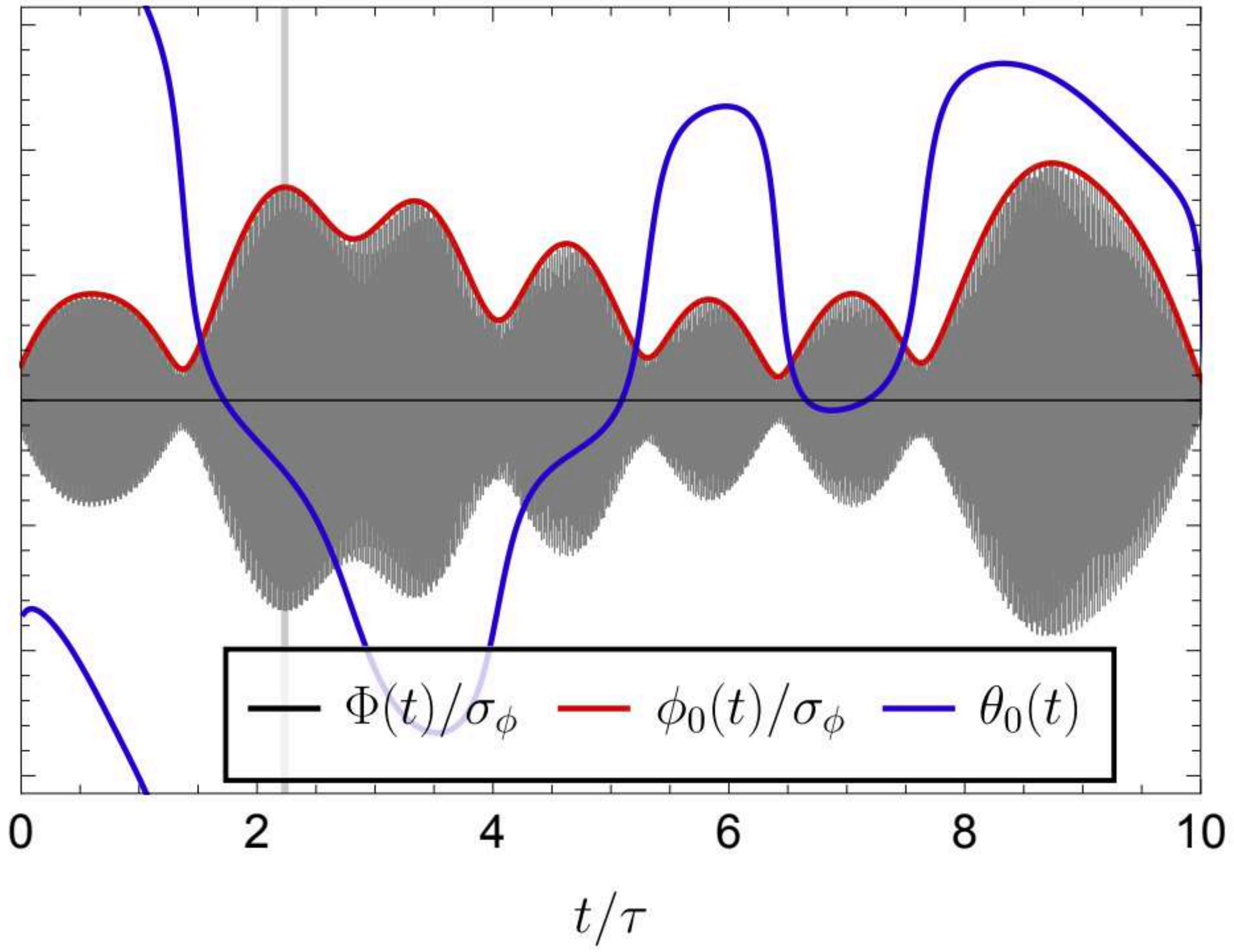


Nakatsuka et al 2022



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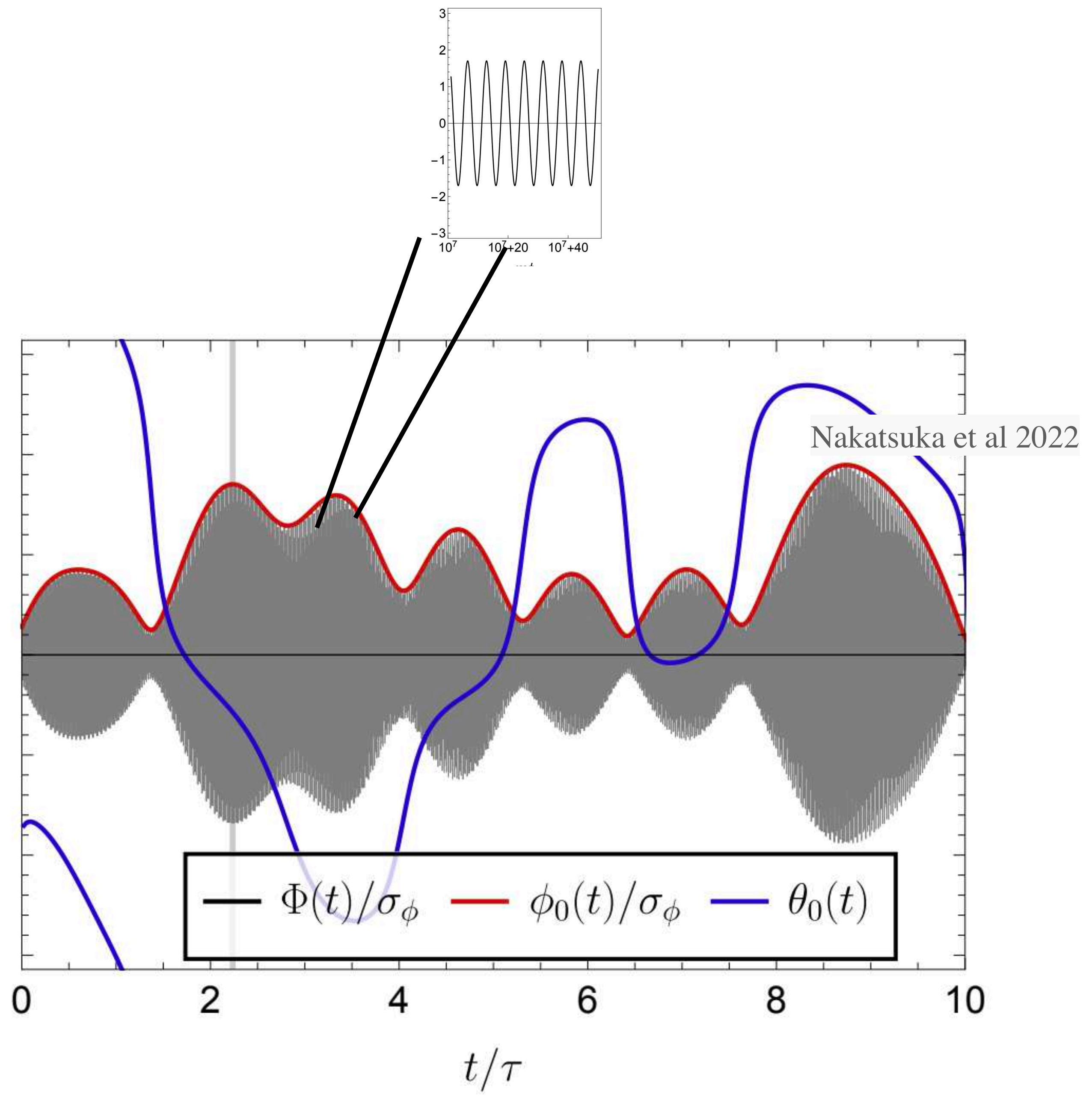
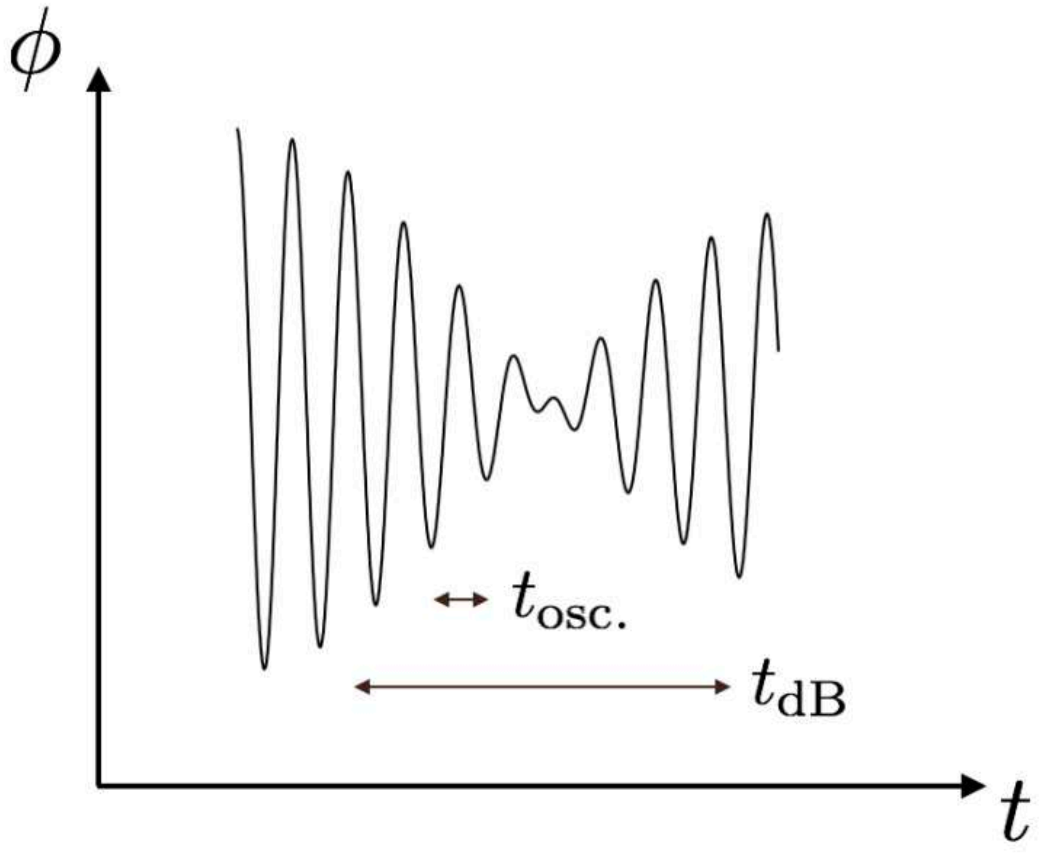


Nakatsuka et al 2022

1) A simple model of a galactic halo, consider a superposition of plane waves:

$$\phi(t,\vec{x}) = \sum_i^N \frac{\phi(0)}{\sqrt{N}} \cos\left(mt + \frac{m}{2}v_i^2 t - m\vec{v}_i\cdot\vec{x} + \theta_i\right)$$



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Wave interference produces de-Broglie-scale, order unity density fluctuations which vary on time scale of t_{dB}

This collection of plane waves can also be represented like this:

$$\phi(t,\vec{x}) = A(\vec{x})\cos(m\,t + \alpha(\vec{x}))$$

describes the interference patterns

2) Another model would superimpose eigenstates of a desired gravitational potential (Lin et al. 2018, Li et al. 2021)

Perform an eigenmode decomposition of the halo wavefunction, where the eigenmodes are for a fixed gravitational potential
→ ω_k is the energy of each eigenmode (labeled abstractly by k), with $e^{i\vec{k}\cdot\vec{x}}$ replaced by the corresponding eigenfunction.

Energy eigenvalue

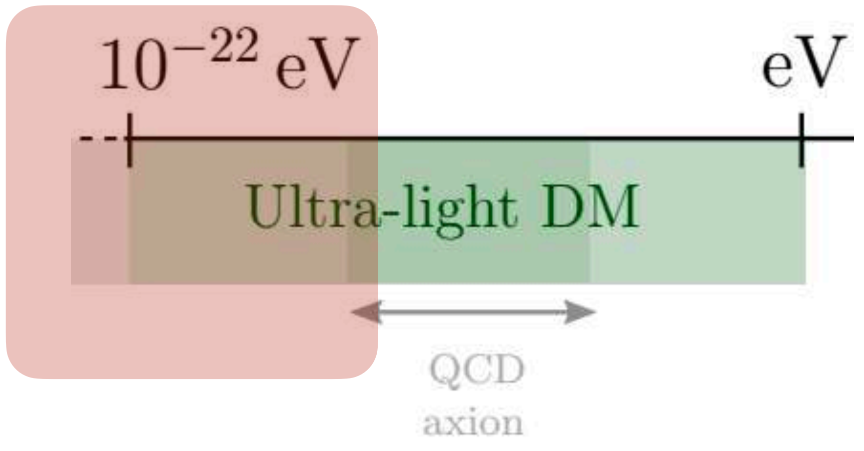
$$\psi(r,\theta,\phi,t) = \sum_{n,l,m} A_{nlm} F_{nlm}(r,\theta,\phi) e^{-iE_{nl}t/\hbar}$$

$$F_{nlm}(r,\theta,\phi) = R_{nl}(r) Y_l^m(\theta,\phi)$$

Radial eigenfunction

energy eigenmodes of the gravitational potential of the virialized halo

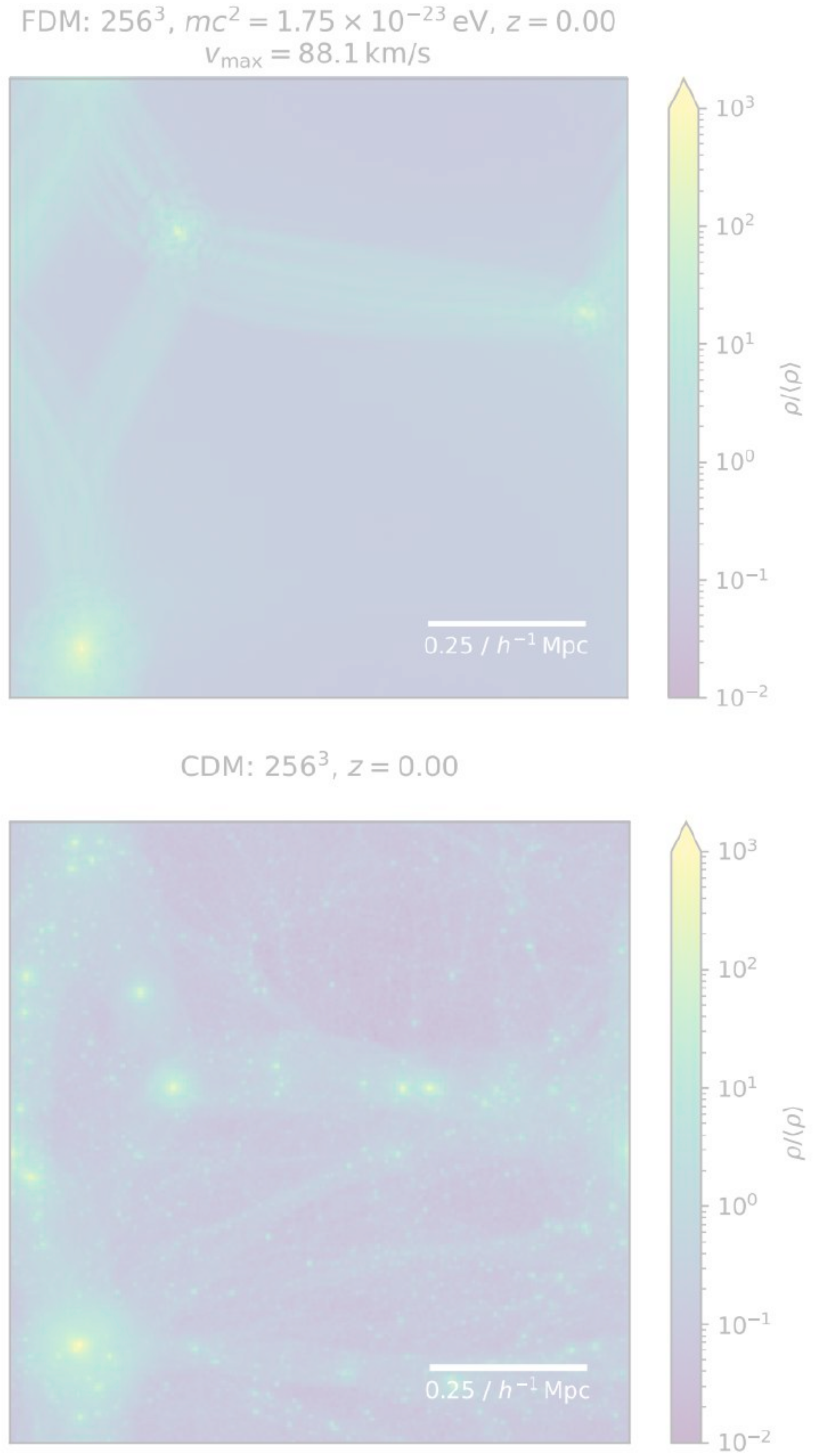
Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

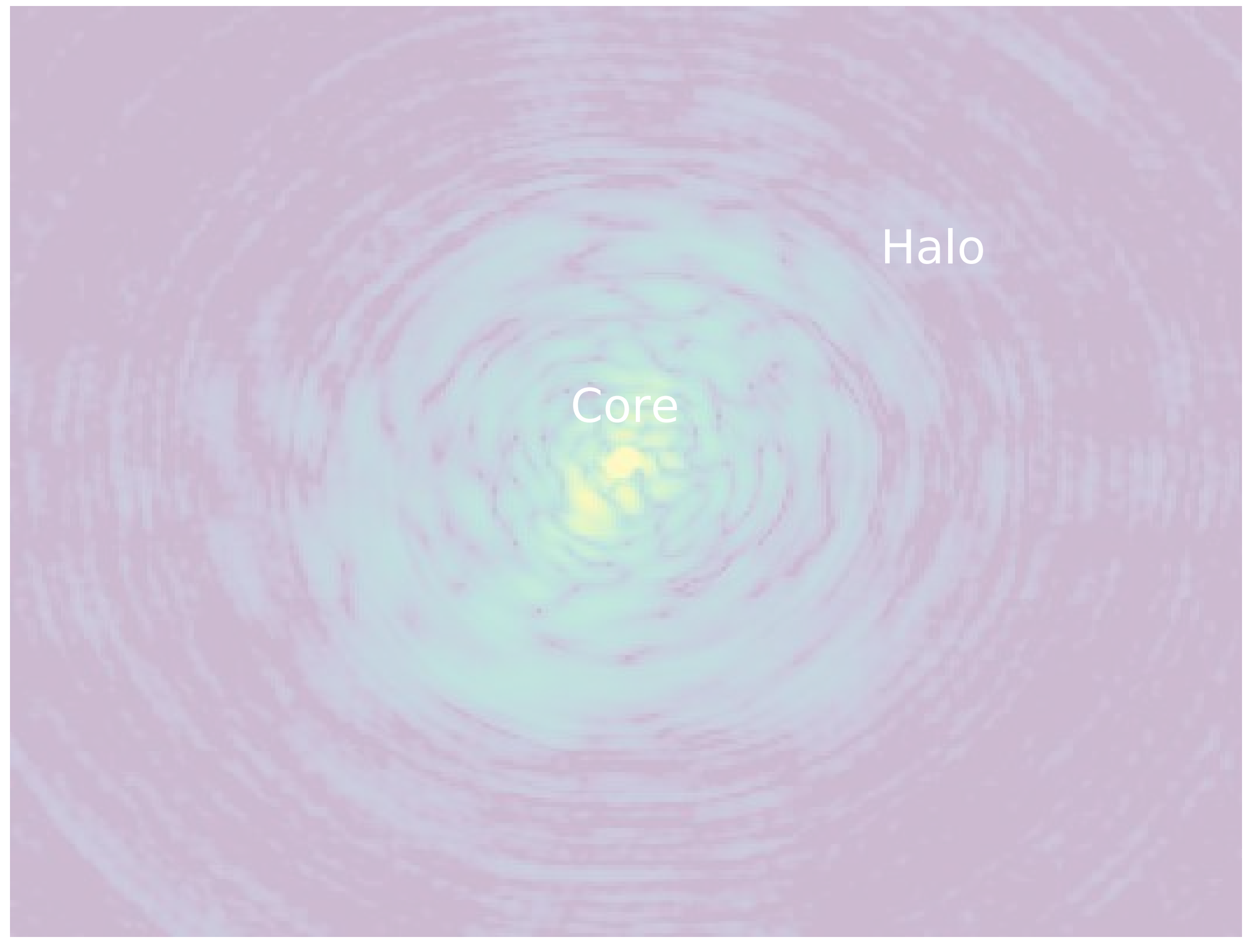
* Focus only in gravitational signatures

Suppression of small structures

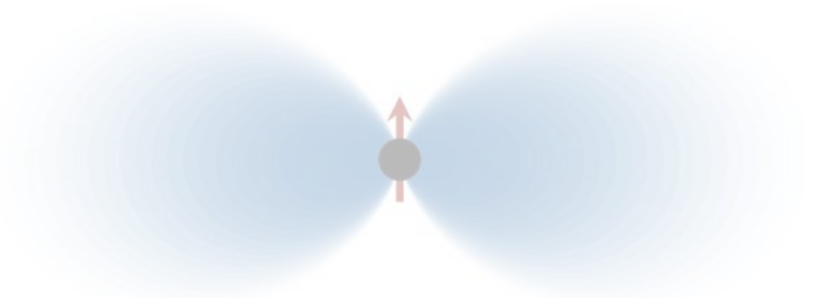


S. May et al. 2021

Formation of a solitonic core

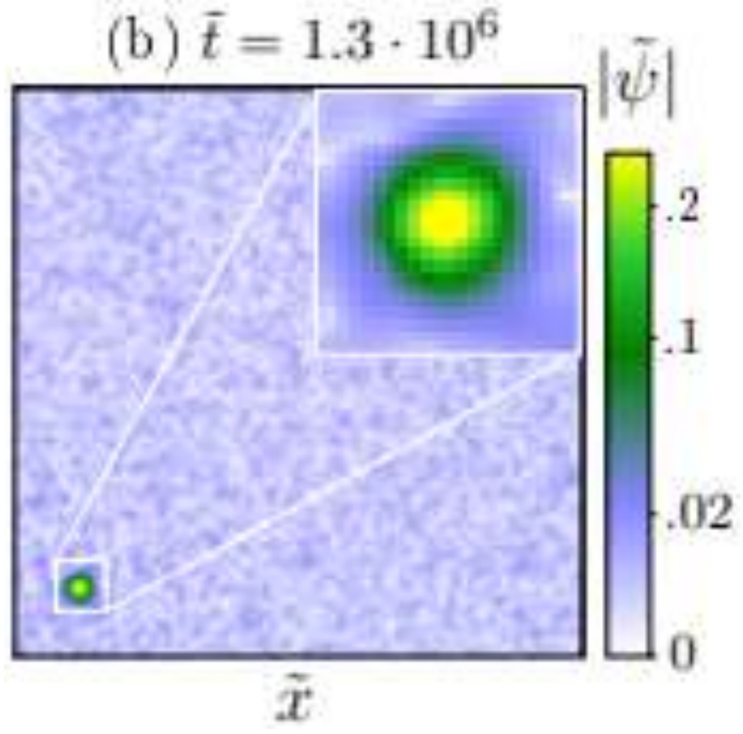


Axion clouds



Baumann et al. 2019

Dynamical effects



Levkov et al. 2018

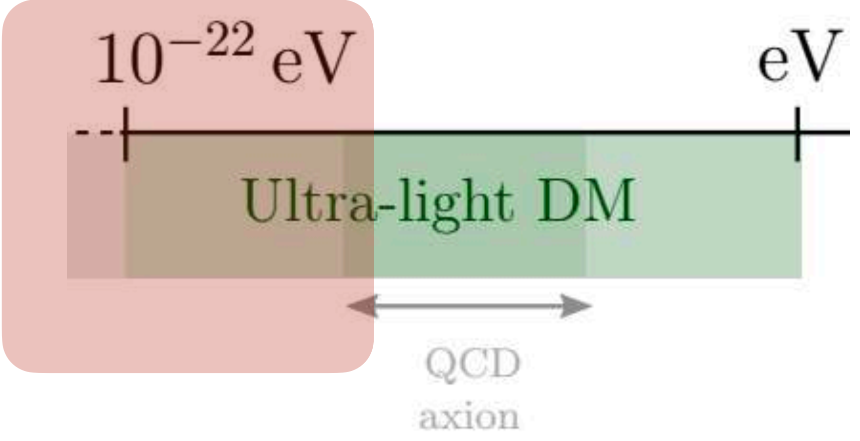
Wave interference

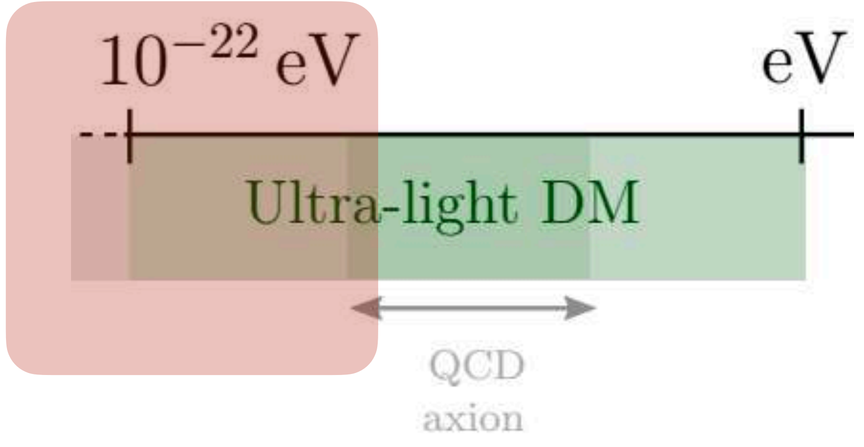
Mocz et al. 2017

Phenomenology

Dynamical effects

Relaxation, oscillation, friction, and heating

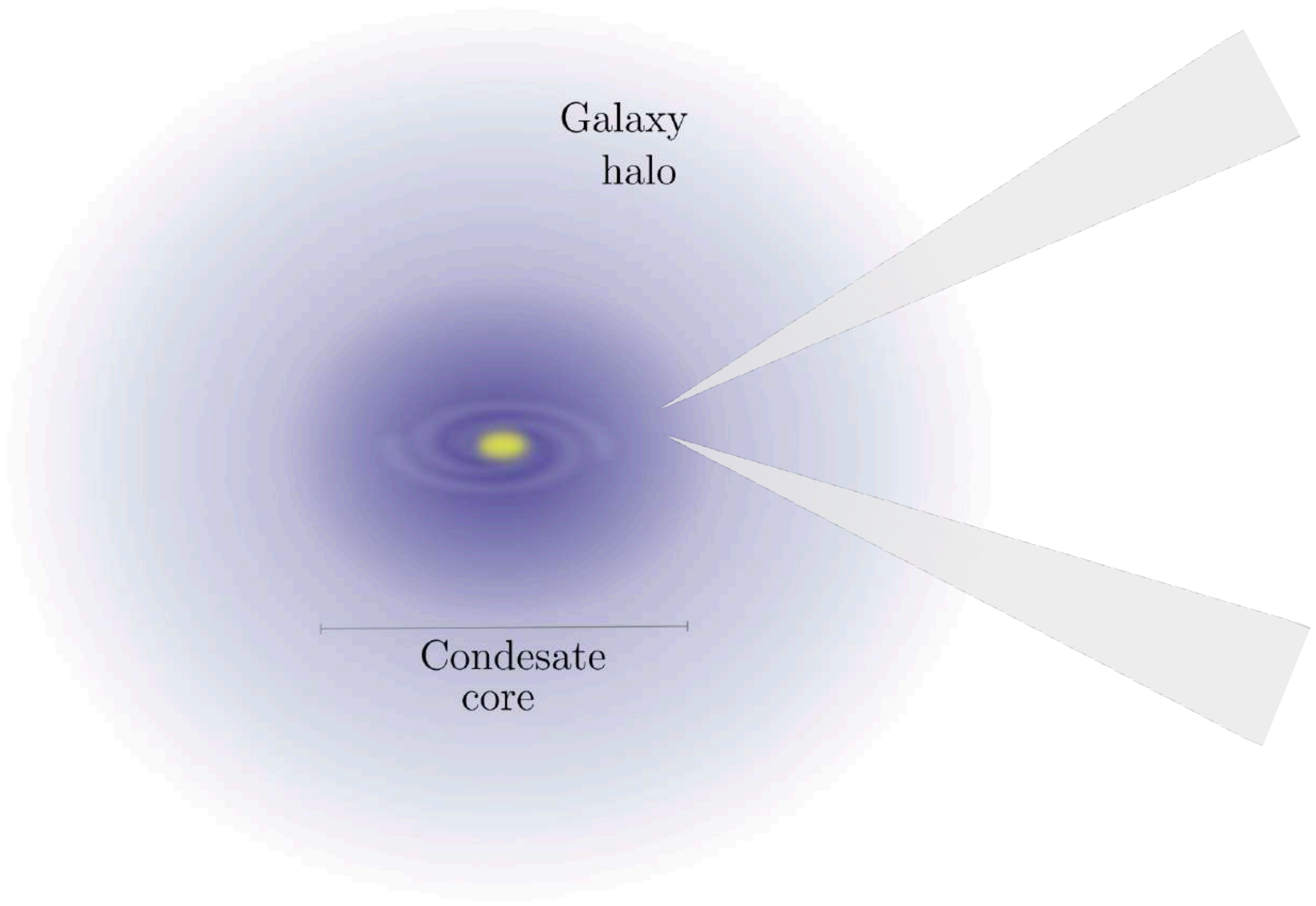




Phenomenology

Dynamical effects

Relaxation, oscillation, friction, and heating

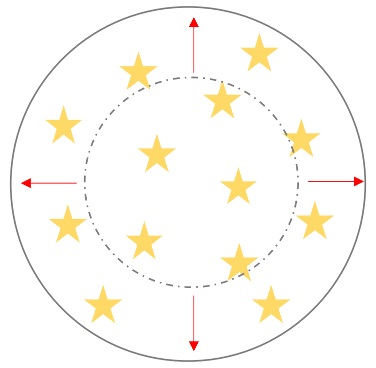


Heating

FDM granule

m_{eff}

System (star) gains energy



Friction

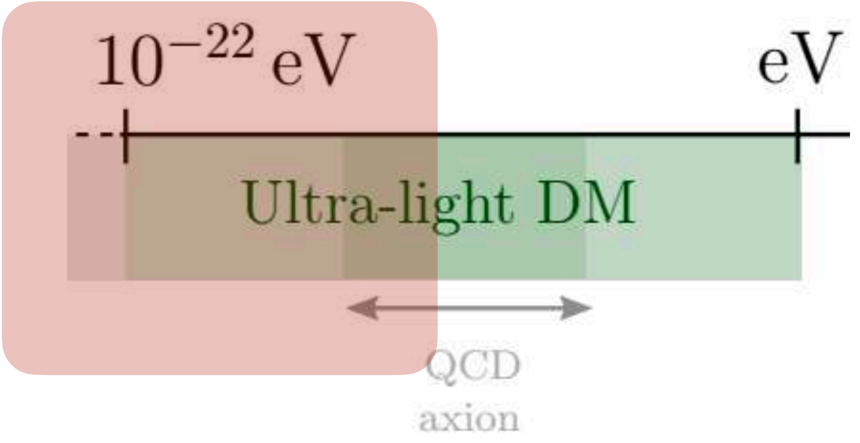
FDM granule

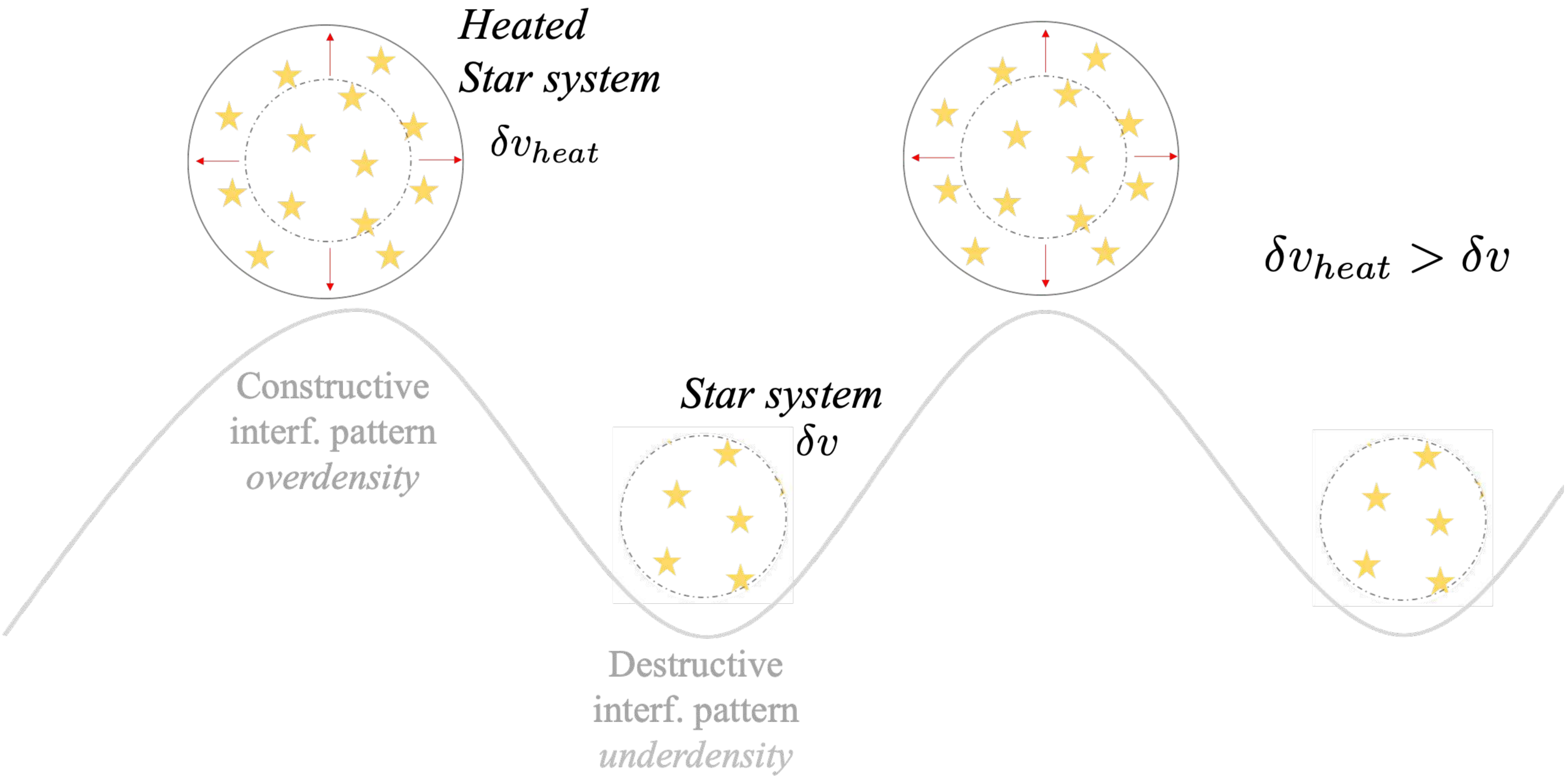
m_{eff}

Globular cluster

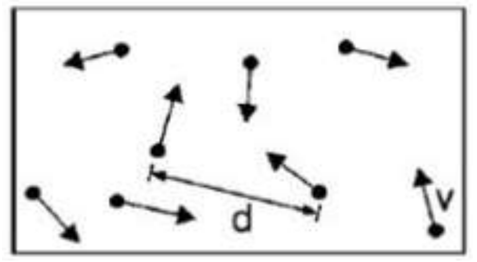
System (GC or BH) loses energy

Heating

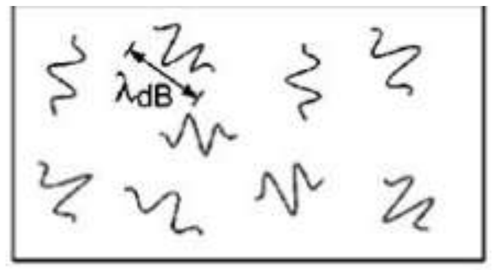

10^{-22} eV
eV
Ultra-light DM
QCD
axion


Heated
Star system
δv_{heat}
Star system
δv
$\delta v_{heat} > \delta v$
Constructive
interf. pattern
overdensity
Destructive
interf. pattern
underdensity

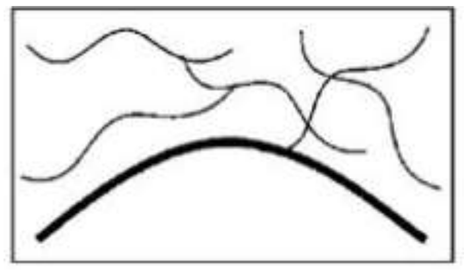
Bose Einstein Condensate



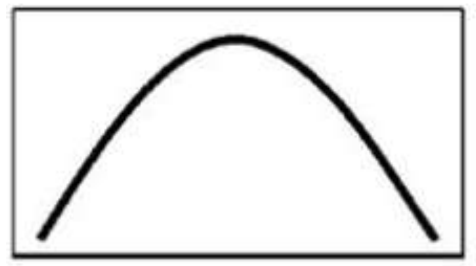
High temperature
Thermal velocities



Low temperature
$\lambda_B \sim T^{-1/2}$
"wave packets"



$T = T_c$
BEC
"matter wave overlap"
$d \sim \lambda_{dB}$



$T = 0$
Pure BEC
"giant matter wave"

- Bose Einstein condensate (BEC): macroscopic occupation of the ground state
- At low temperatures, each particle wave function overlap - single wave function describes the entire fluid.

Superfluid

- Appears at low T after the superfluid condenses into a BEC.
- Effective dynamics: fluid flows without friction

Description

Mean field approximation:
Large N, dilute

$$\hat{\Psi}(\mathbf{r},t) = \psi(\mathbf{r},t) + \delta\hat{\Psi}(\mathbf{r},t)$$

classical field
"wavefunction of the condensate"

small perturbation: describes depletion of the condensate

with $\psi(\mathbf{r},t) = \langle\hat{\Psi}(\mathbf{r},t)\rangle$

Fixed $n_0 = |\psi(\mathbf{r},t)|^2$

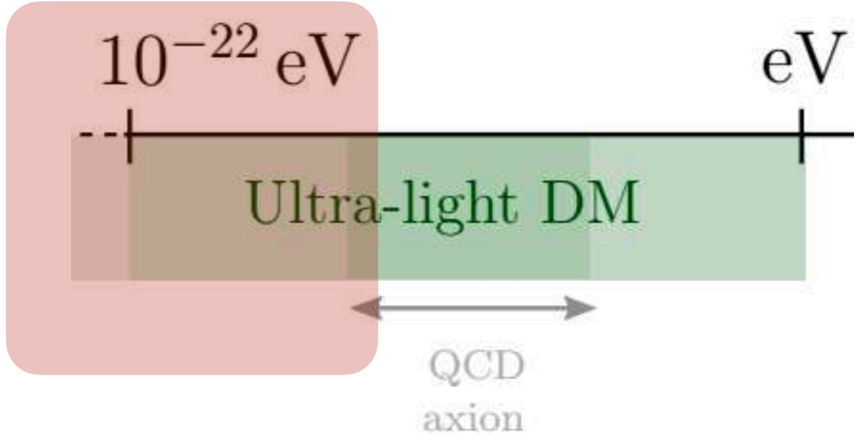
Credit: Peking University

$$\boxed{i\partial_t\psi(\mathbf{r},t) = \left(-\frac{\nabla^2}{2m} + V_{trap}(\mathbf{r}) + U_0|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t)}$$

Non-linear Schrödinger equation - Gross-Pitaevskii equation

Phenomenology

Dynamical effects



Relaxation, oscillation, friction, and heating

Formation of a BEC / superfluid

- Thermalization (and condensation) *seem* to happen inside the galaxy!
Formation of a soliton (ground state) or Bose star in the interior of galaxies

A. Guth M. Hertzberg, C. Prescod-Weinstein (2014)

- Formation of a condensate and a core occur from gravitational interaction.

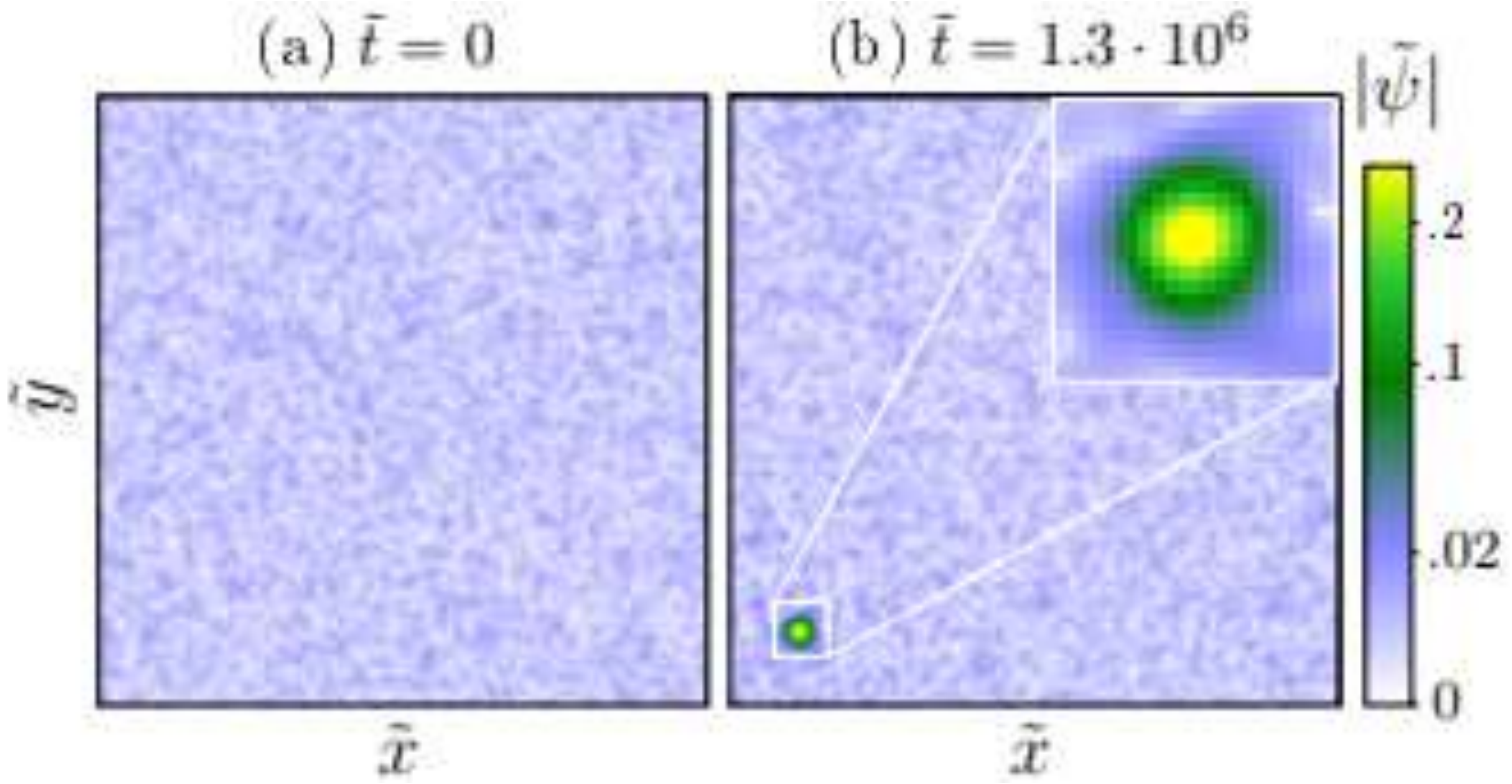
Condensation/relaxation time: $\tau_{\rm gr} \gg \tau_{\rm int}$

$$\tau_{\rm gr} \sim 10^6\,{\rm yr}\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^3\left(\frac{v}{30\,{\rm km/s}}\right)^6\left(\frac{\rho}{0.1\,M_\odot/{\rm pc}^3}\right)^{-2}$$

$$\tau_{\rm int} = \frac{1}{\sqrt{8}|g|n}$$

Smaller than the age of the universe!

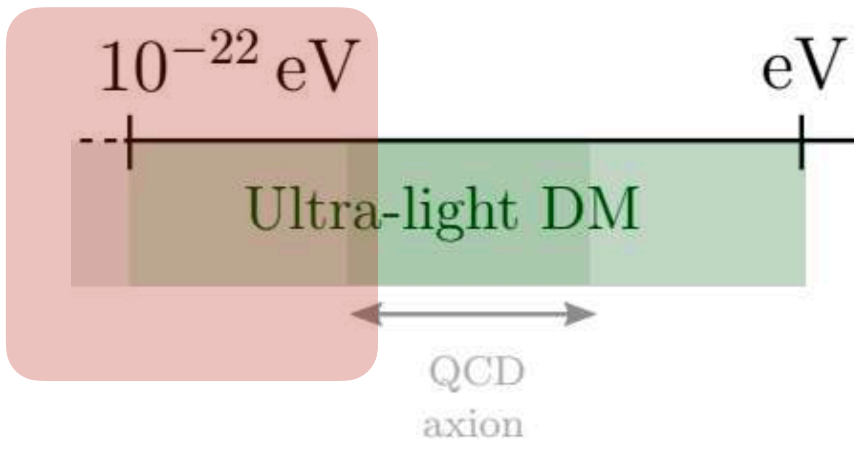
Wave turbulence theory



Levkov et al. 2018, Kirpatrick et al. 2020

Phenomenology

Dynamical effects



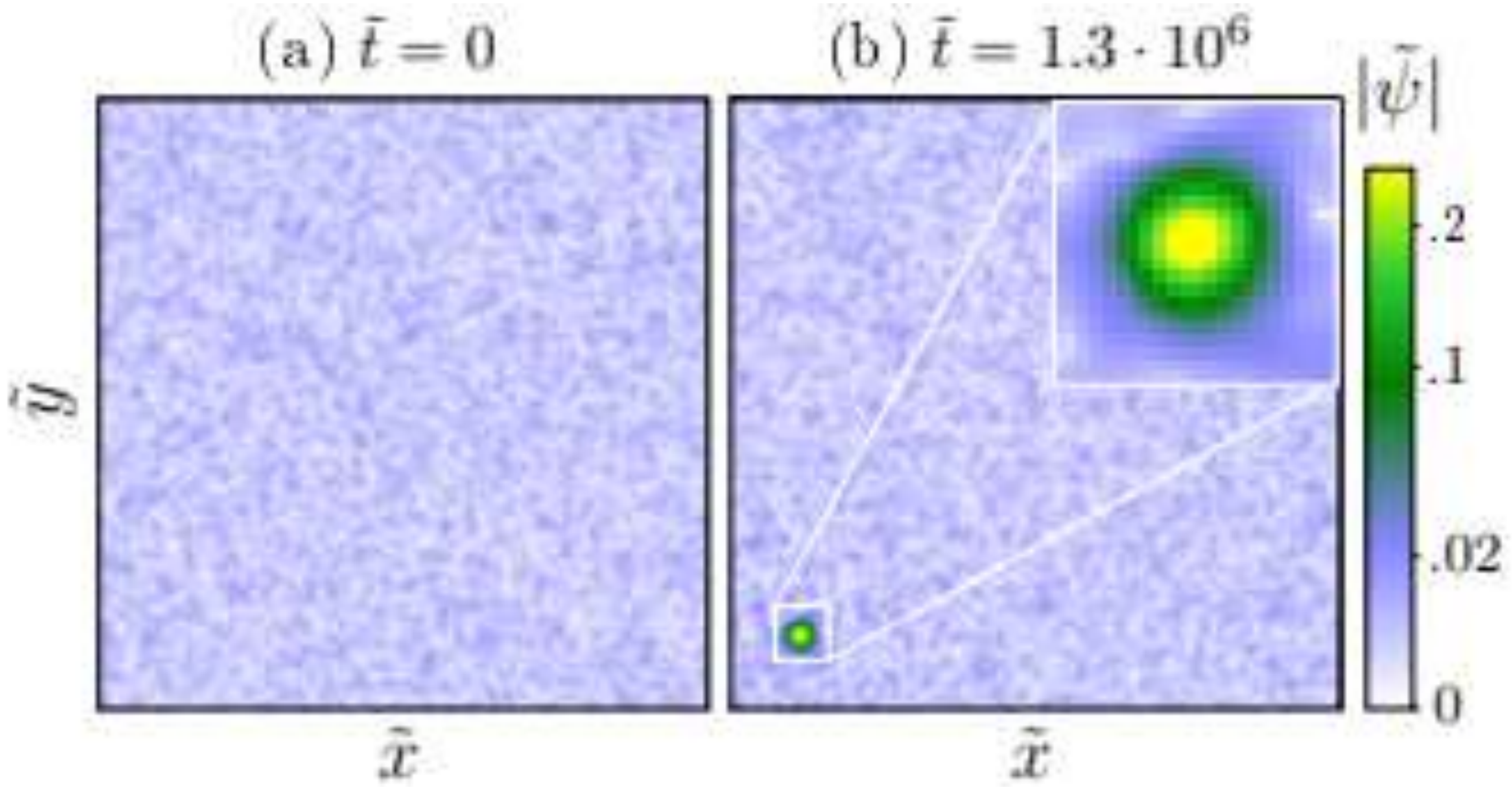
Relaxation, oscillation, friction, and heating

Formation of a BEC / superfluid

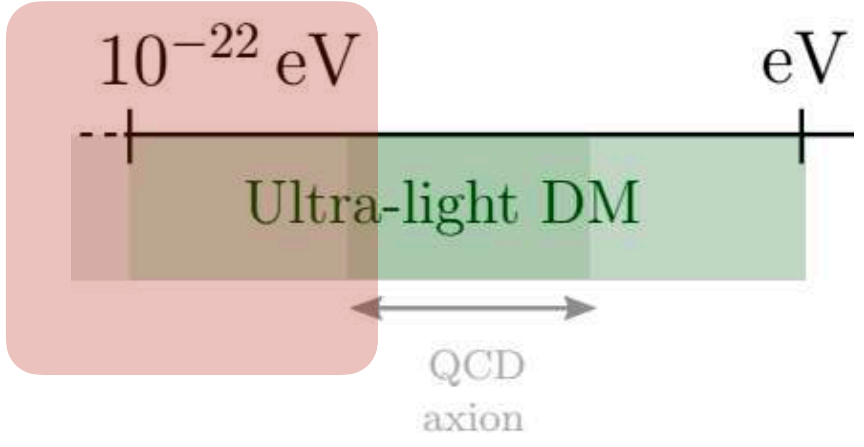
Open question!

- Need theoretical work to describe *analytically* the formation of these solitons

- Cosmologically, classical or quantum field?



Phenomenology



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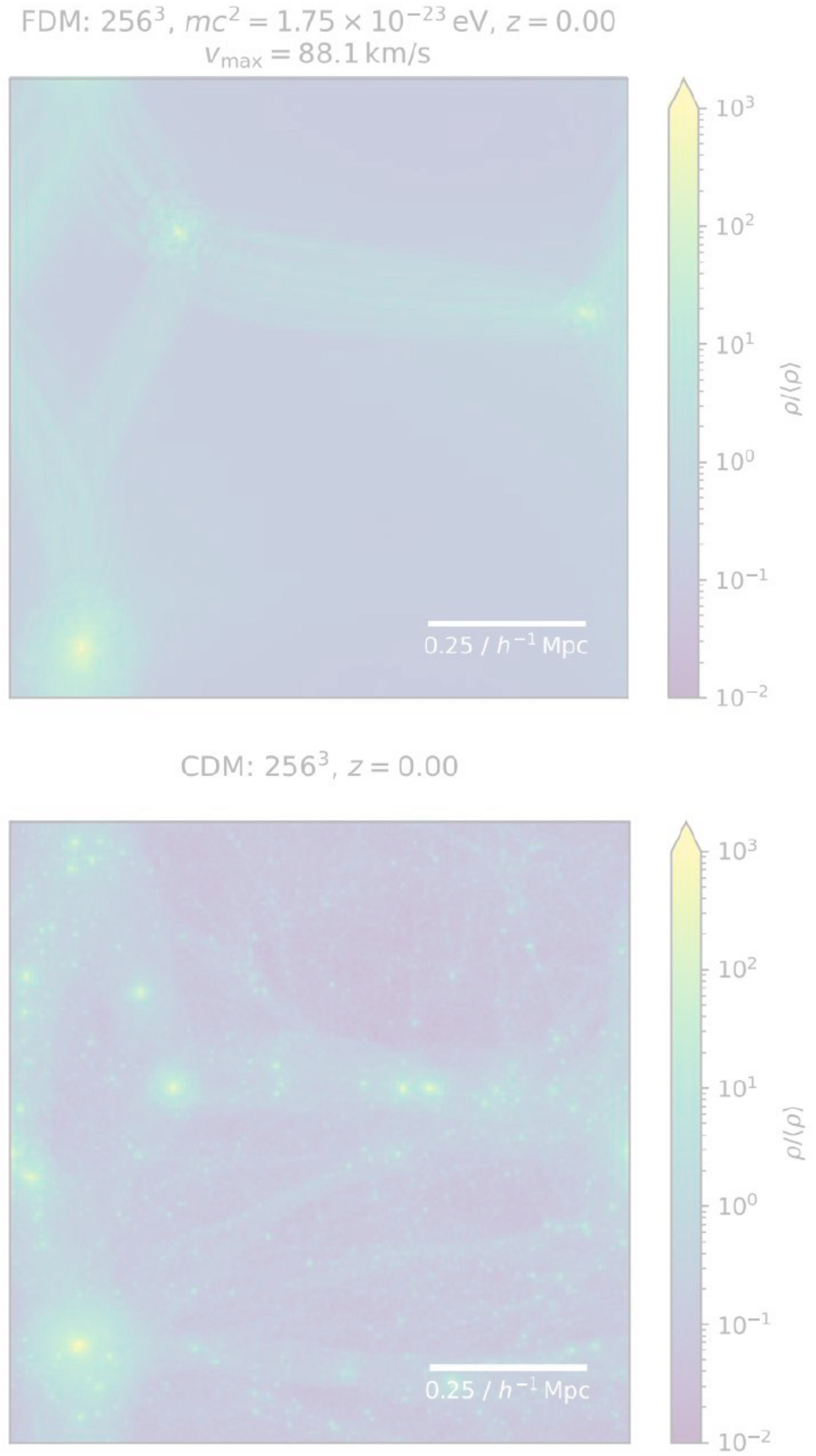
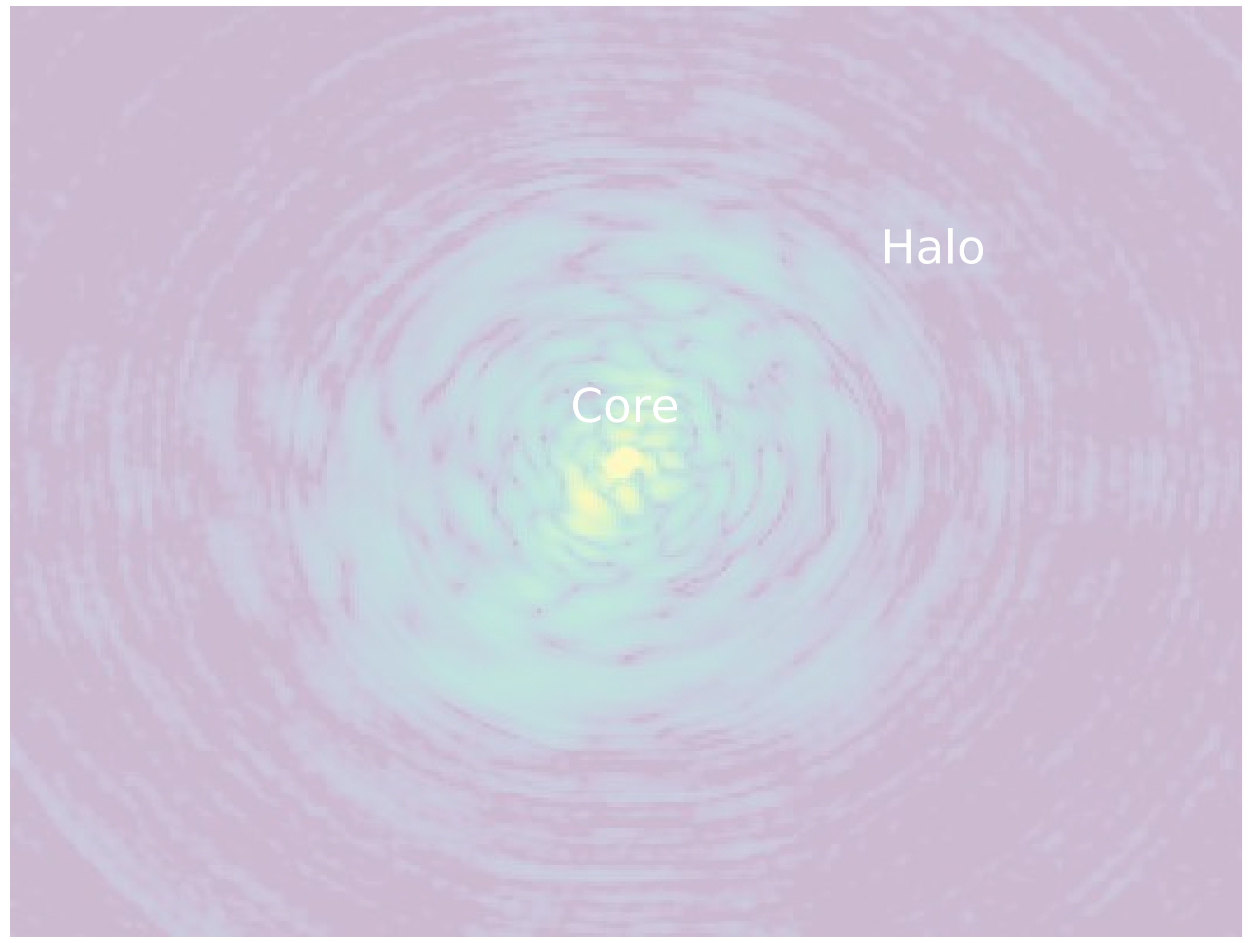


S. May et al. 2021

Formation of a solitonic core



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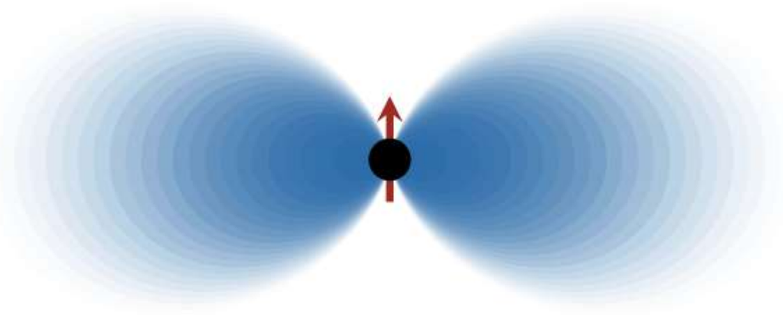
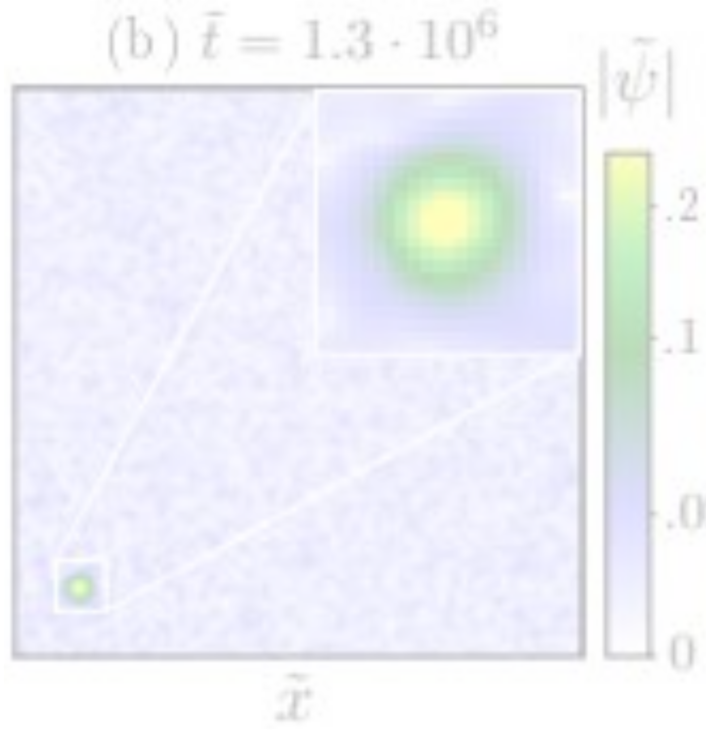


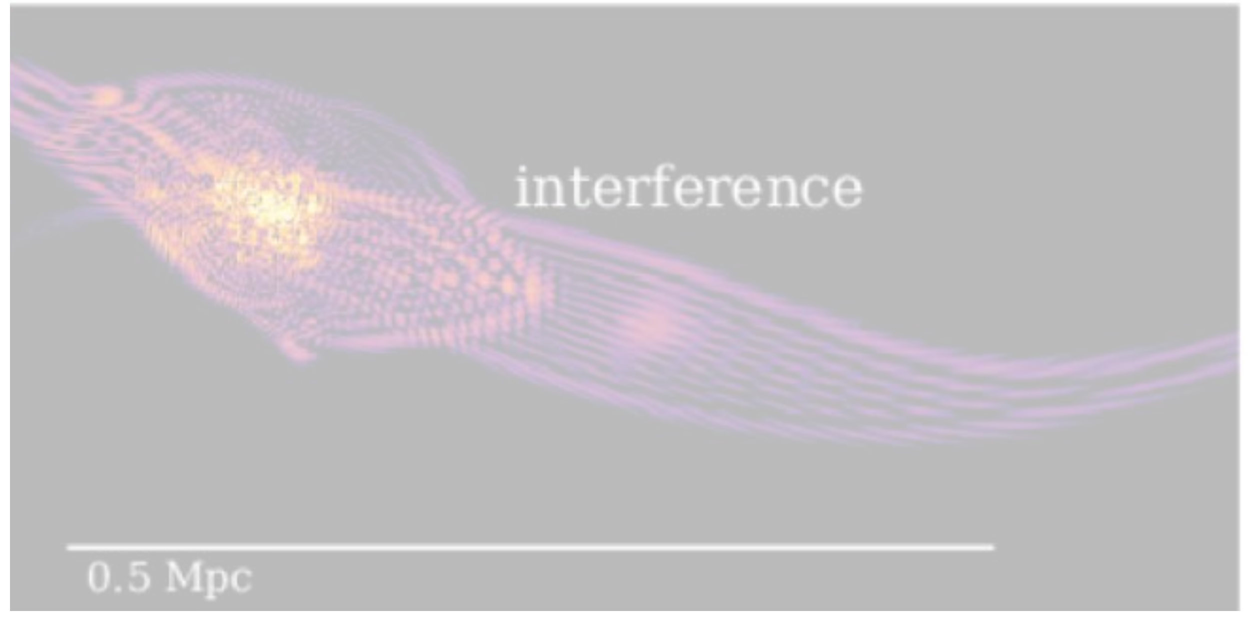
Baumann et al. 2019

Dynamical effects



Levkov et al. 2018

Wave interference



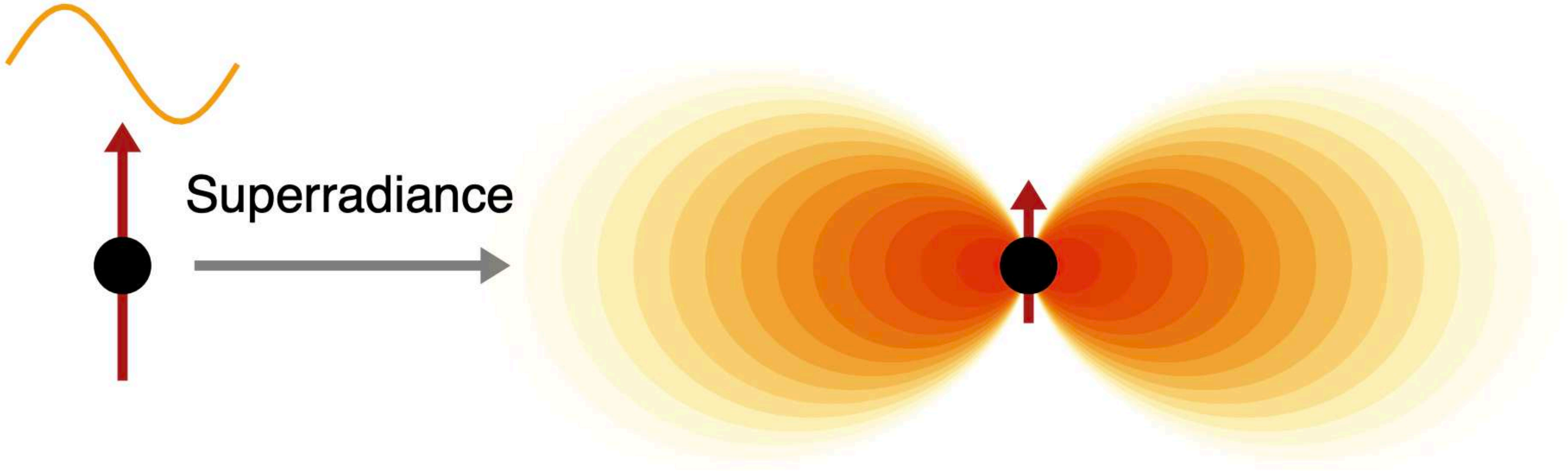
Mocz et al. 2017

Black Hole Superradiance

Zeldovich (1972) Starobinsky (1973) Arvanitaki et al. [0905.4720]

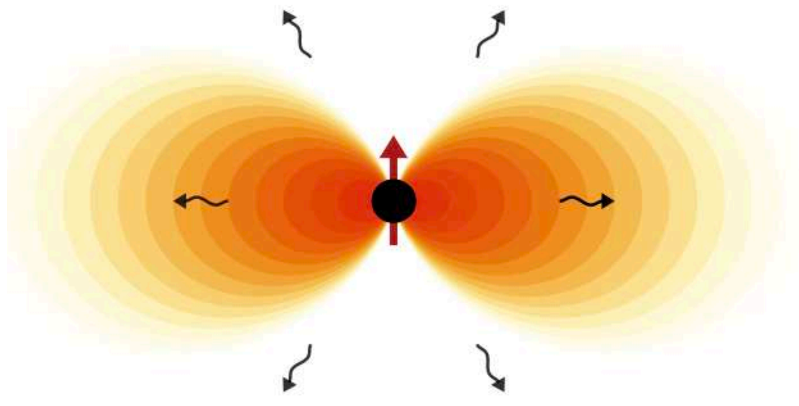
A cloud of ultra-light bosons (and vector fields) can be created around rotating black holes - if the particle Compton wavelength is of the order of the size of the BH

Structure like a "gravitational atom"



H. Chia et al, 2018

Emits gravitational waves



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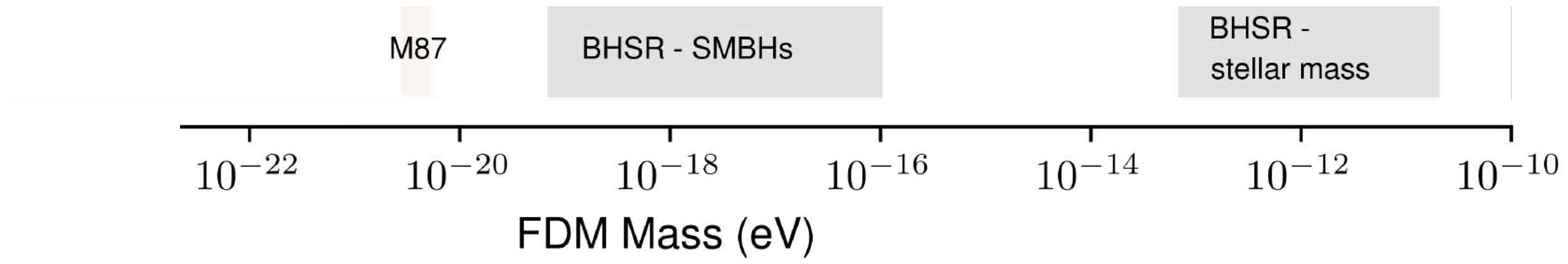
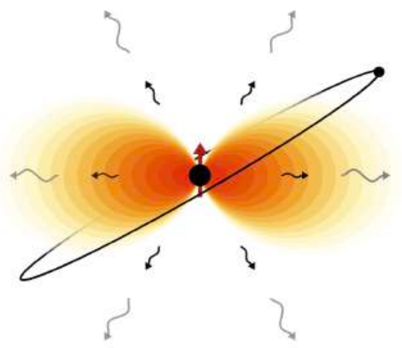
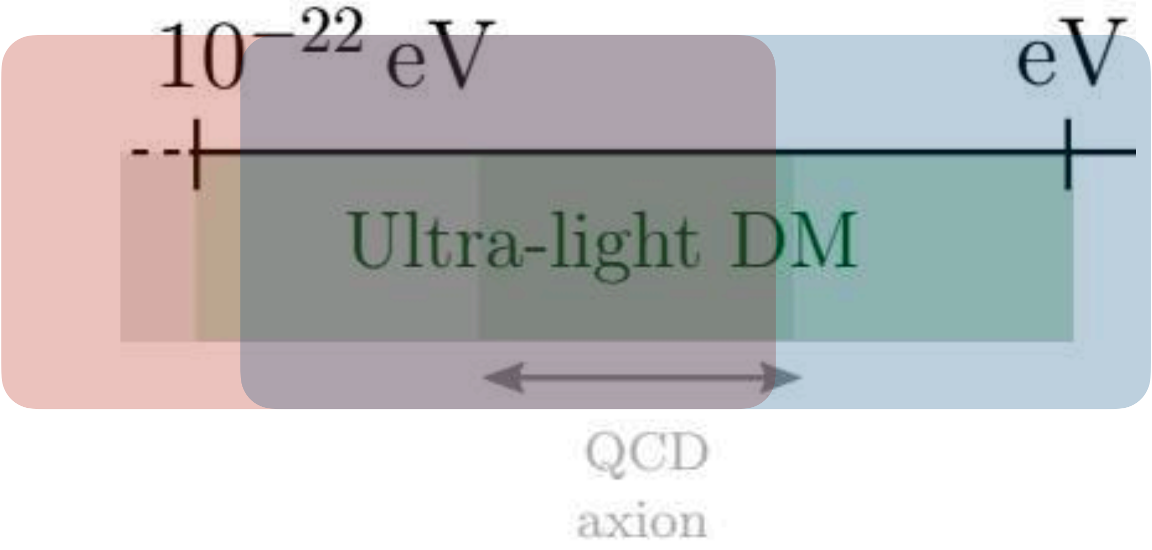


Dynamics can be altered by the presence of a companion - binary



H. Chia et al, 2018

How to search for ULDM?



Poisson Gravity
$\nabla^2 V_g = 4\pi G m^2 \left|\psi\right|^2$
(Attractive)

Coherent state → Oscillates
Leading time dependence
$\dot{\psi} \sim (m-\omega)\psi \ll m\psi$

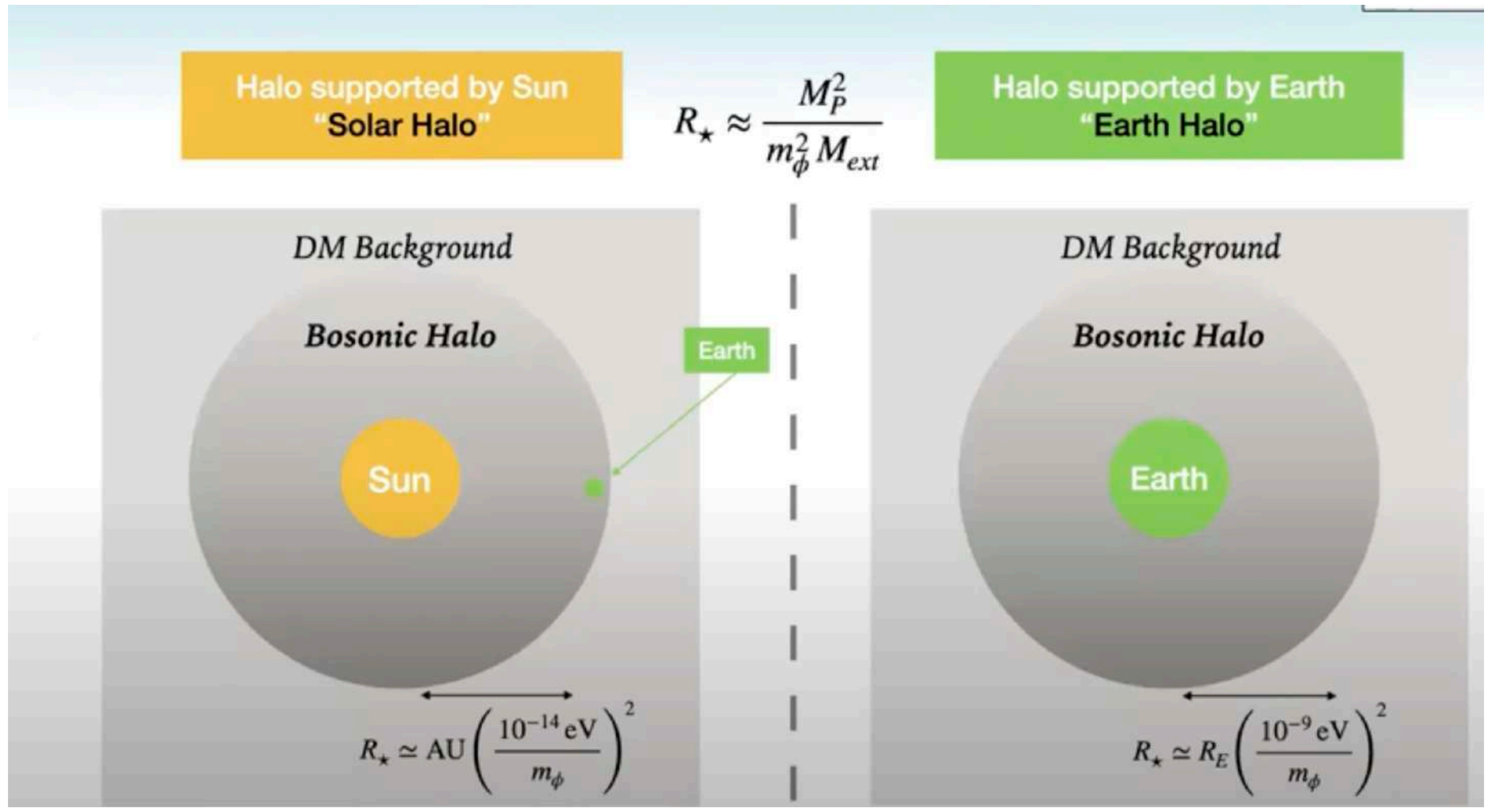
$$\boxed{i\frac{\partial\psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V_g\left(|\psi|^2\right) + V_{int}\left(|\psi|^2\right)\right]\psi}$$

Kinetic energy
(Repulsive)

Self-interactions
For axion potential,

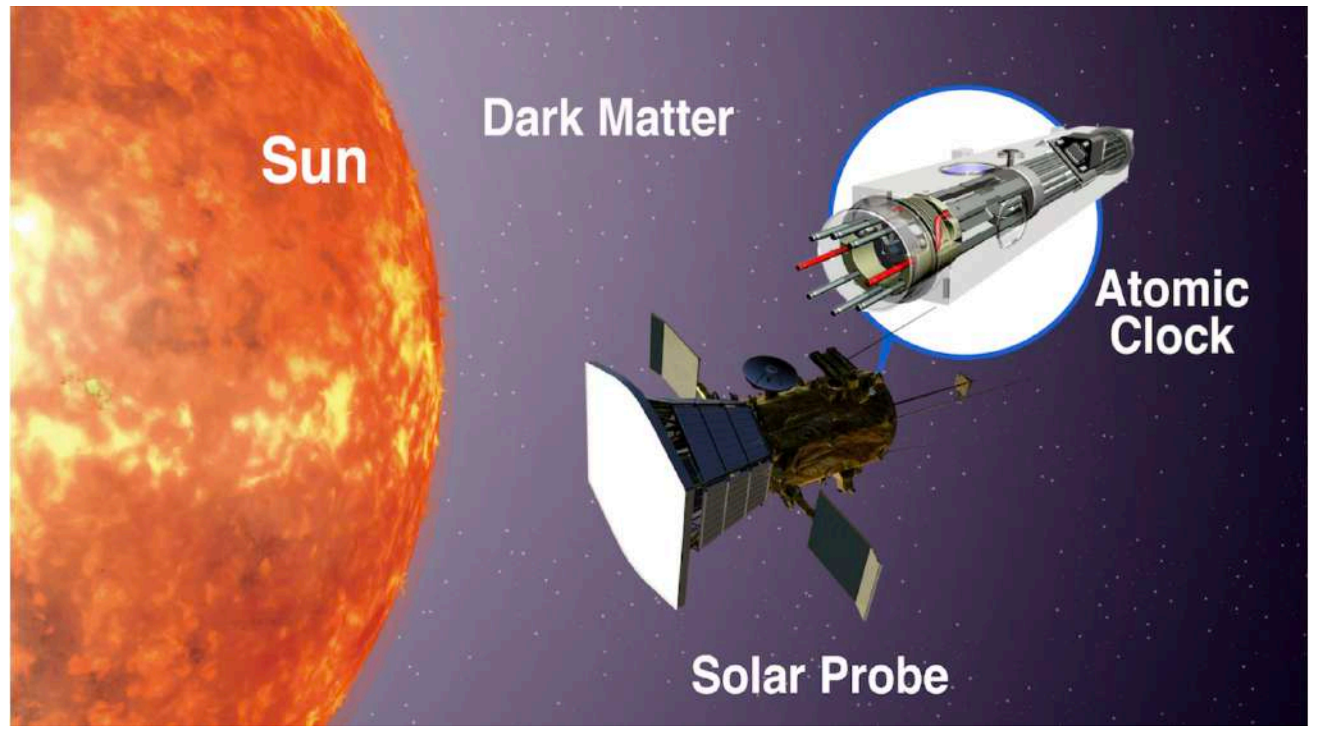
$$V(\phi) = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] = \frac{m^2}{2}\phi^2 - \underbrace{\frac{1}{4!}\left(\frac{m}{f}\right)^2 \phi^4}_{\text{(Attractive)}} + \underbrace{\frac{1}{6!f^2}\left(\frac{m}{f}\right)^2 \phi^6}_{\text{(Repulsive)}} - \dots$$

Normalization
$m\int d^3r\, |\psi|^2 = M_\star$



10^{-12} eV$\lesssim m_\phi \lesssim 10^{-7}$

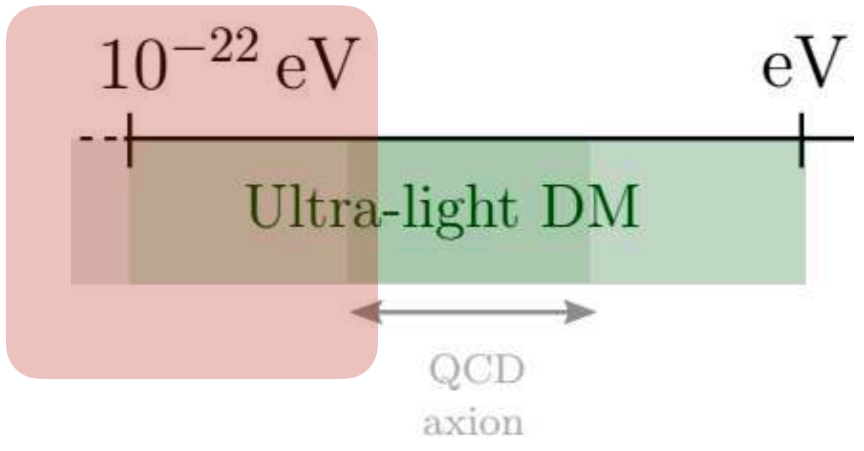
Direct detection of ultralight dark matter bound to the Sun with space quantum sensors
Yu-Dai Tsai et al, *Nature Astron.* (2022)



** Cannot be misaligment mechanism

Figures by Josh Eby

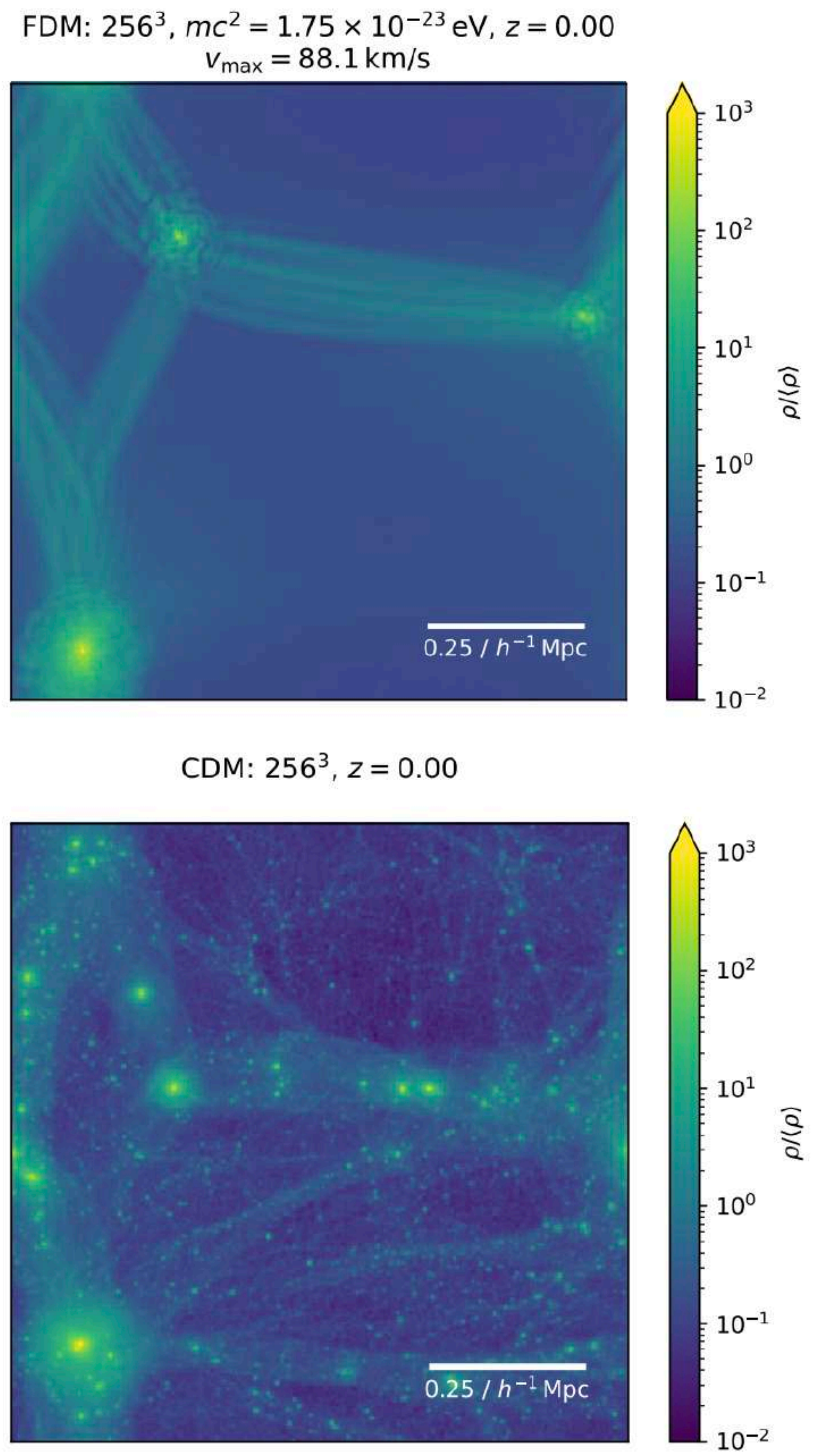
Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

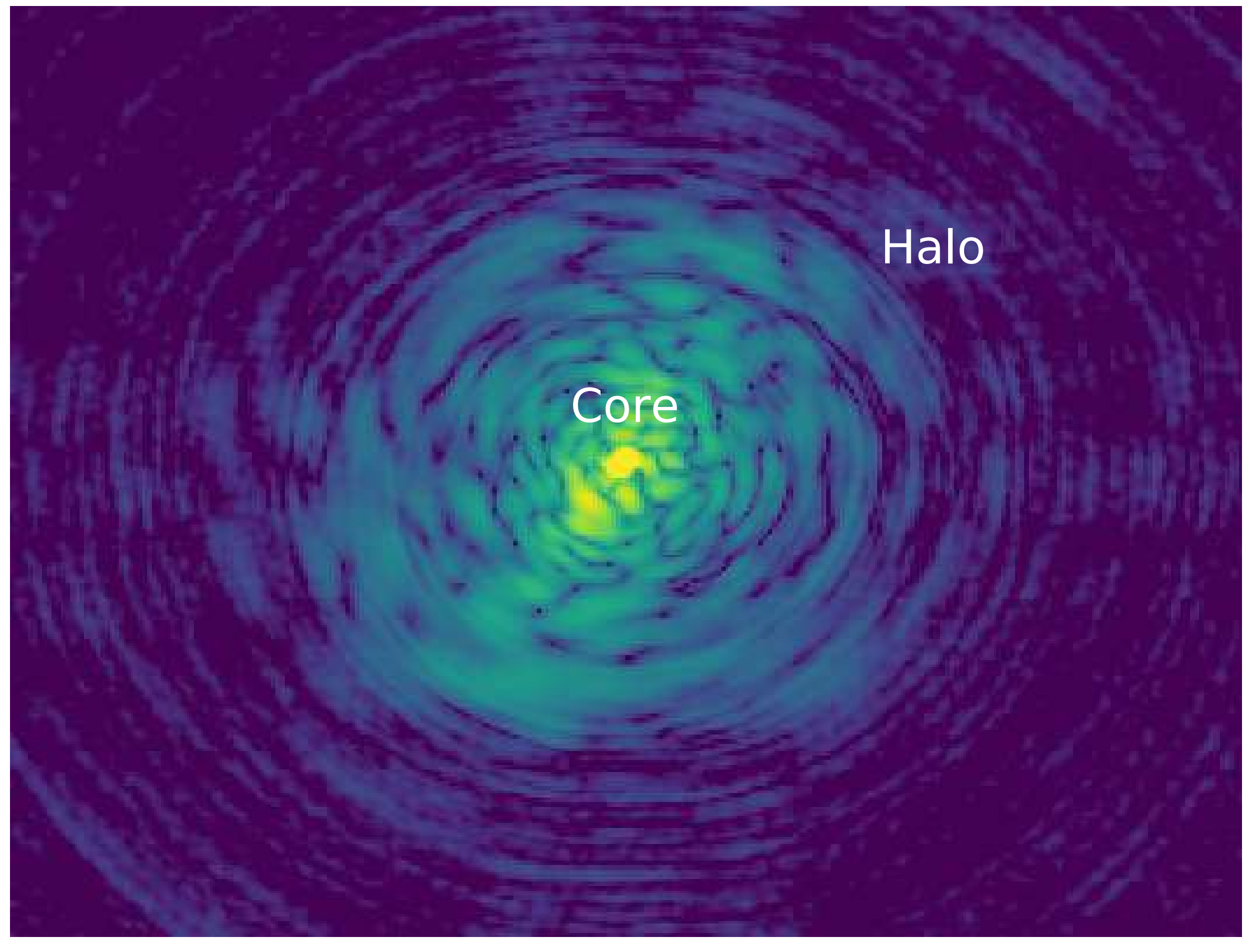
* Focus only in gravitational signatures

Suppression of small structures

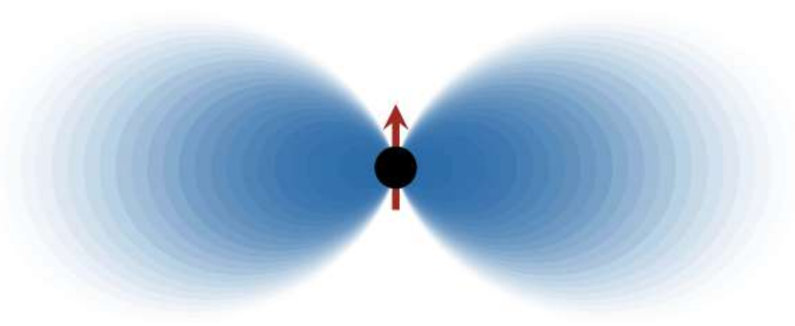


S. May et al. 2021

Formation of a solitonic core

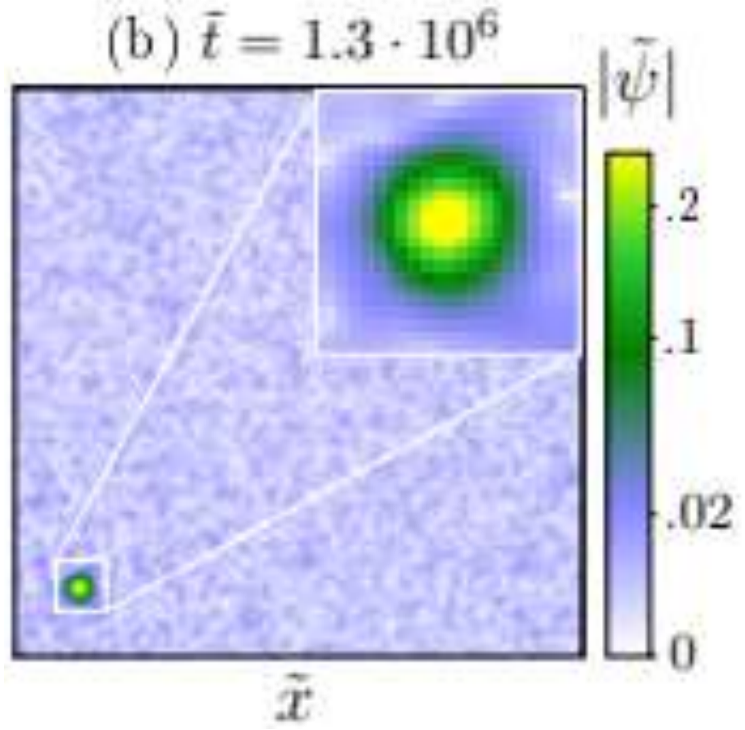


Axion clouds



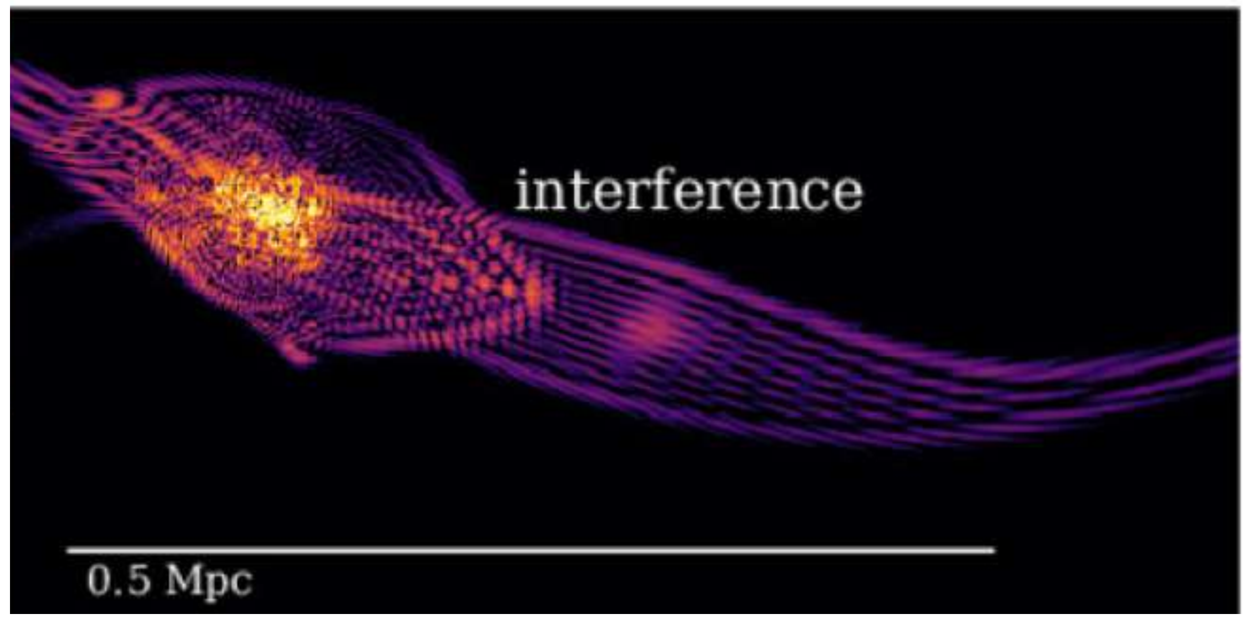
Baumann et al. 2019

Dynamical effects



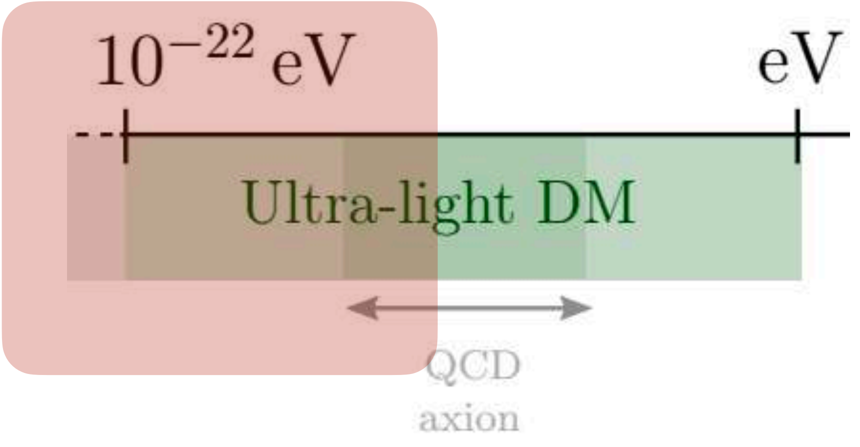
Levkov et al. 2018

Wave interference



Mocz et al. 2017

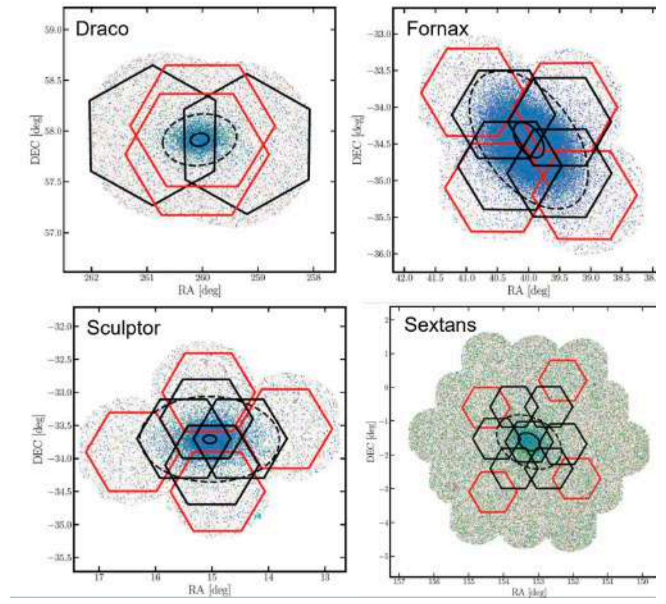
Observational implications and constraints



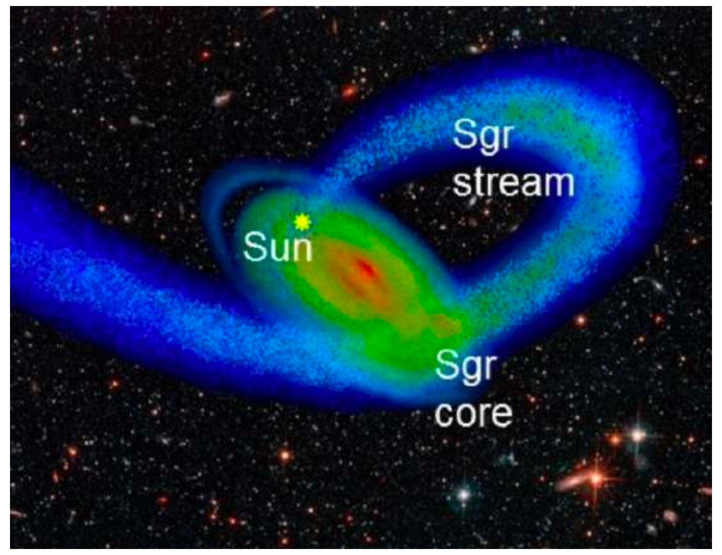
Galaxies

NASA and ESA

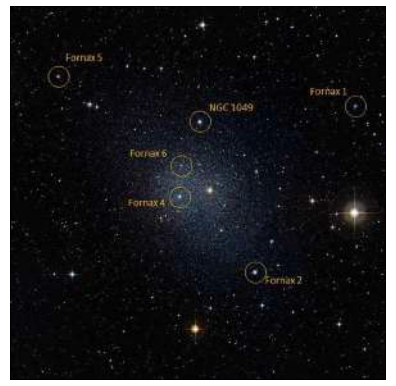
Dwarfs



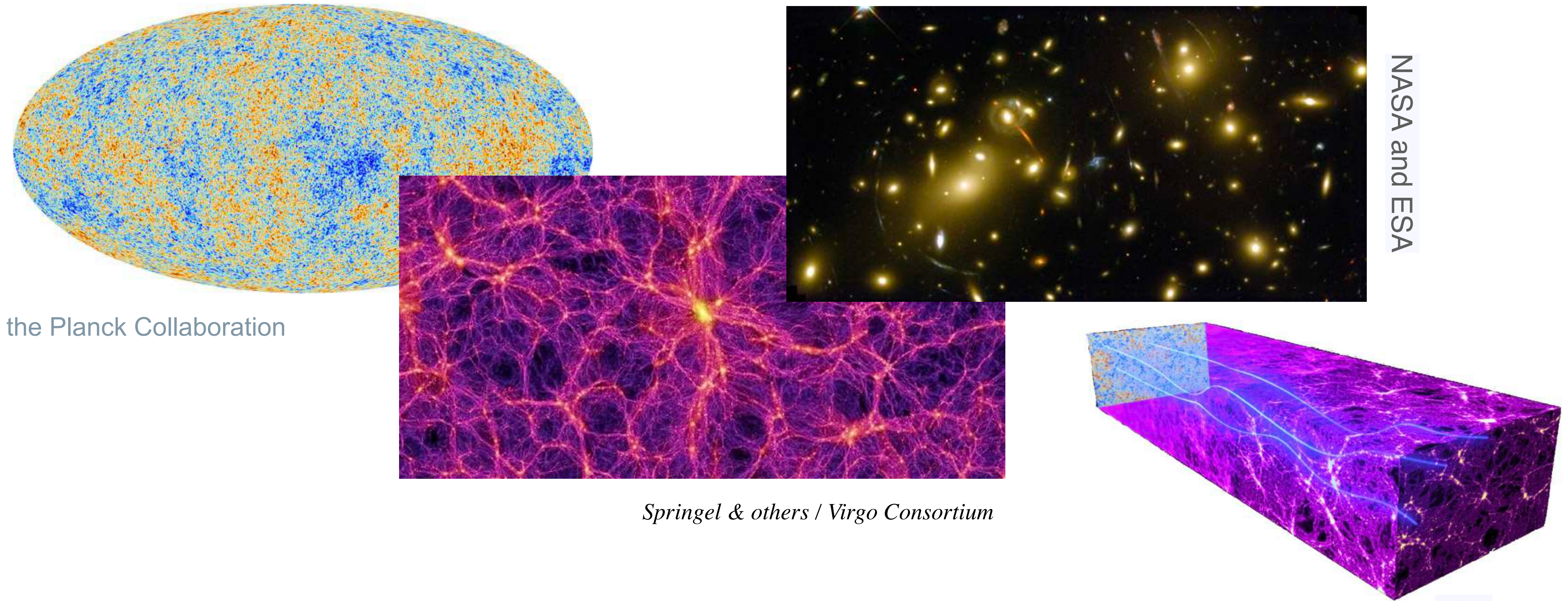
Stellar stream



Globular clusters



CMB+LSS



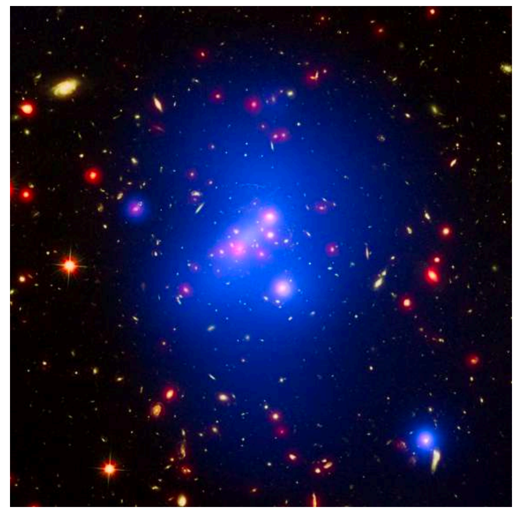
ESA and the Planck Collaboration

NASA and ESA

Springel & others / Virgo Consortium

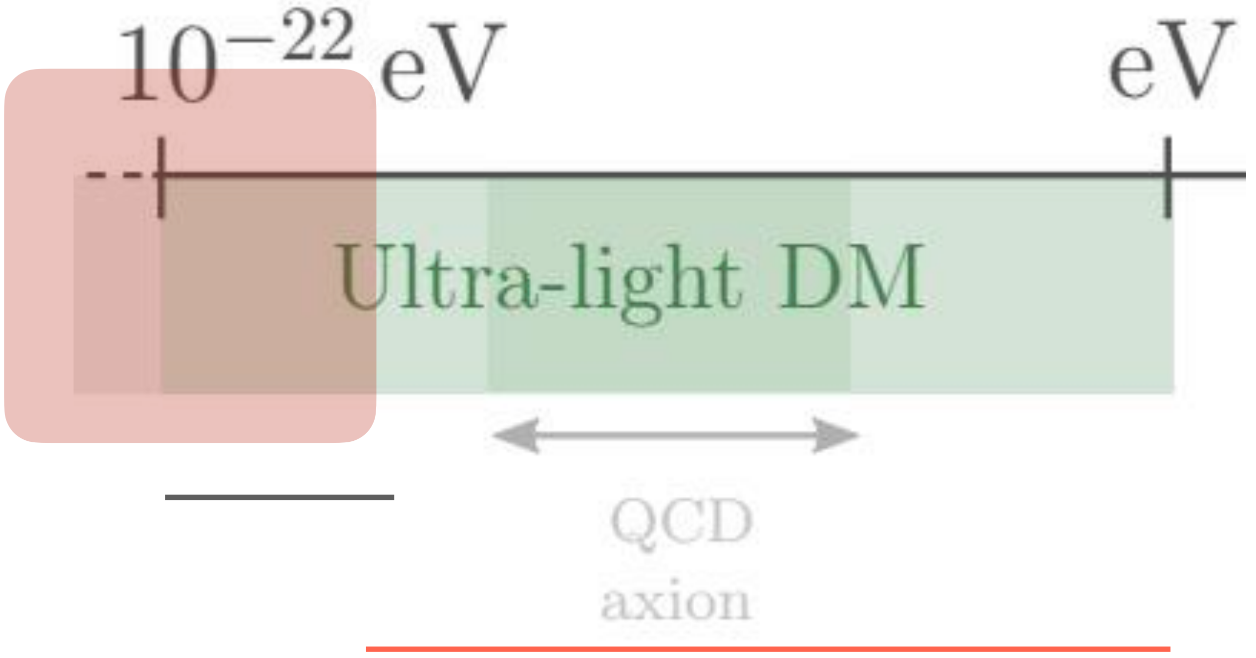
ESA

Clusters

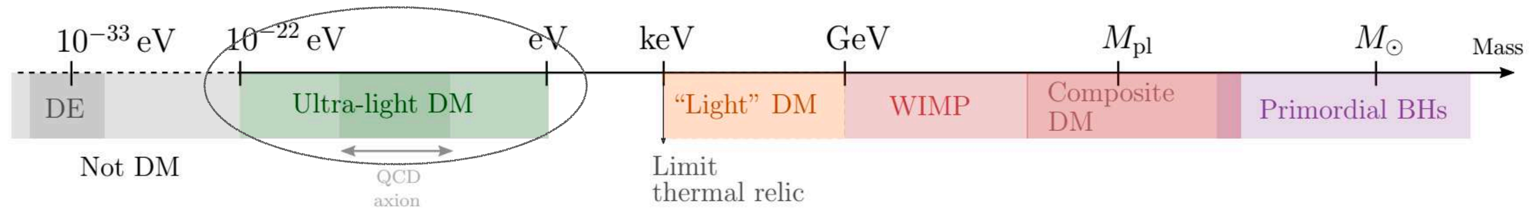


CC BY 4.0

Next lecture

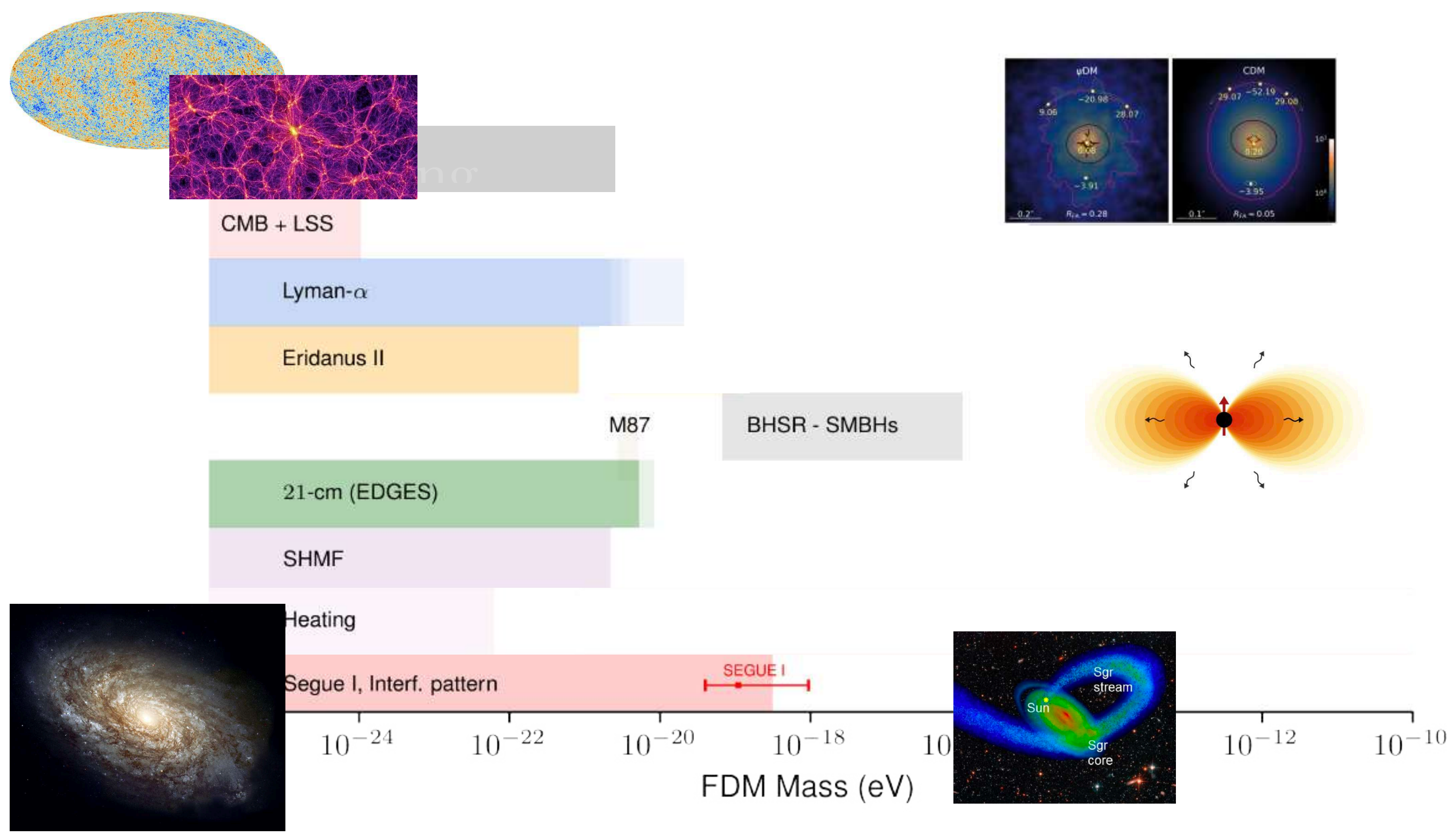


Mass scale of DM

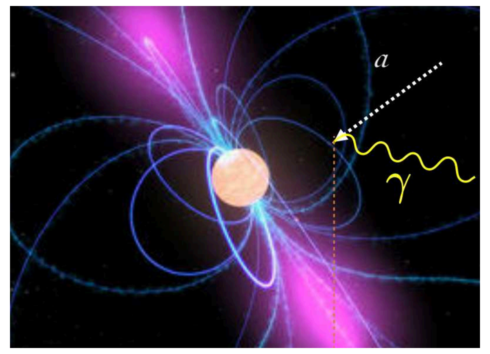


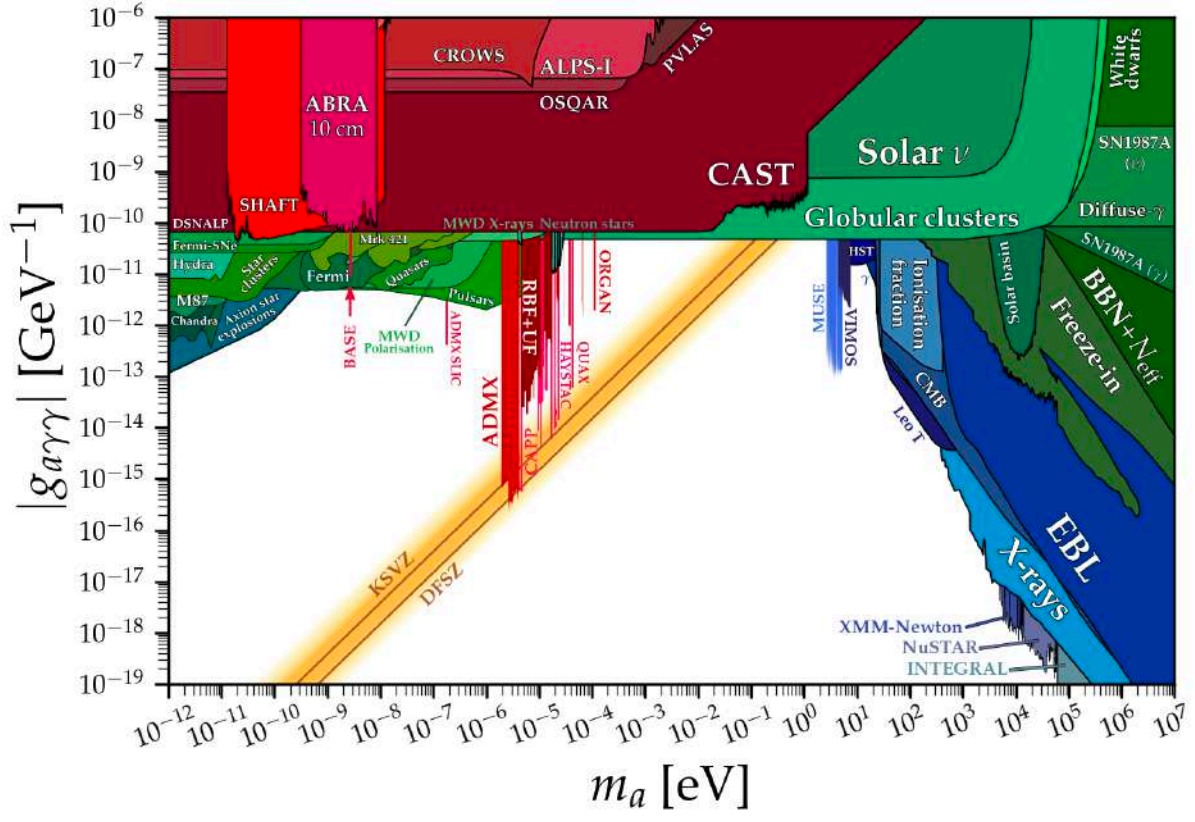
Interactions with the SM

Cosmological and astrophysical searches

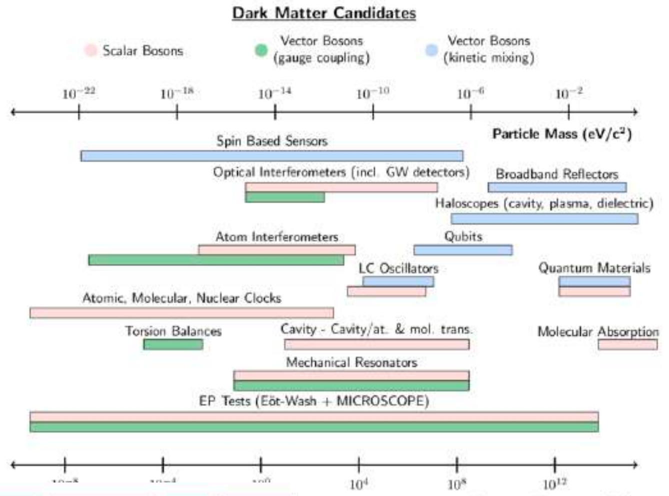


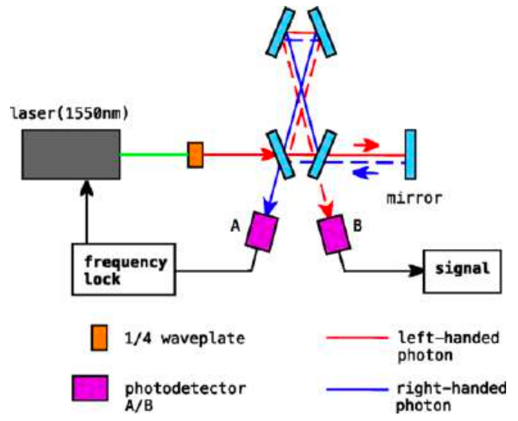
Indirect detection

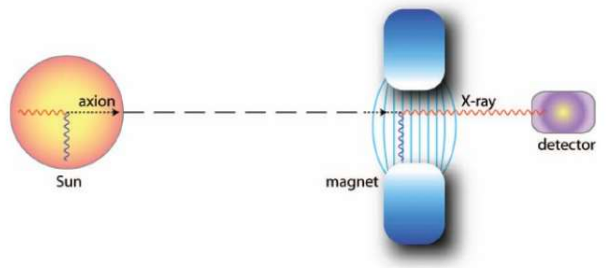




"Direct detection"
Axion/ALPs experiments





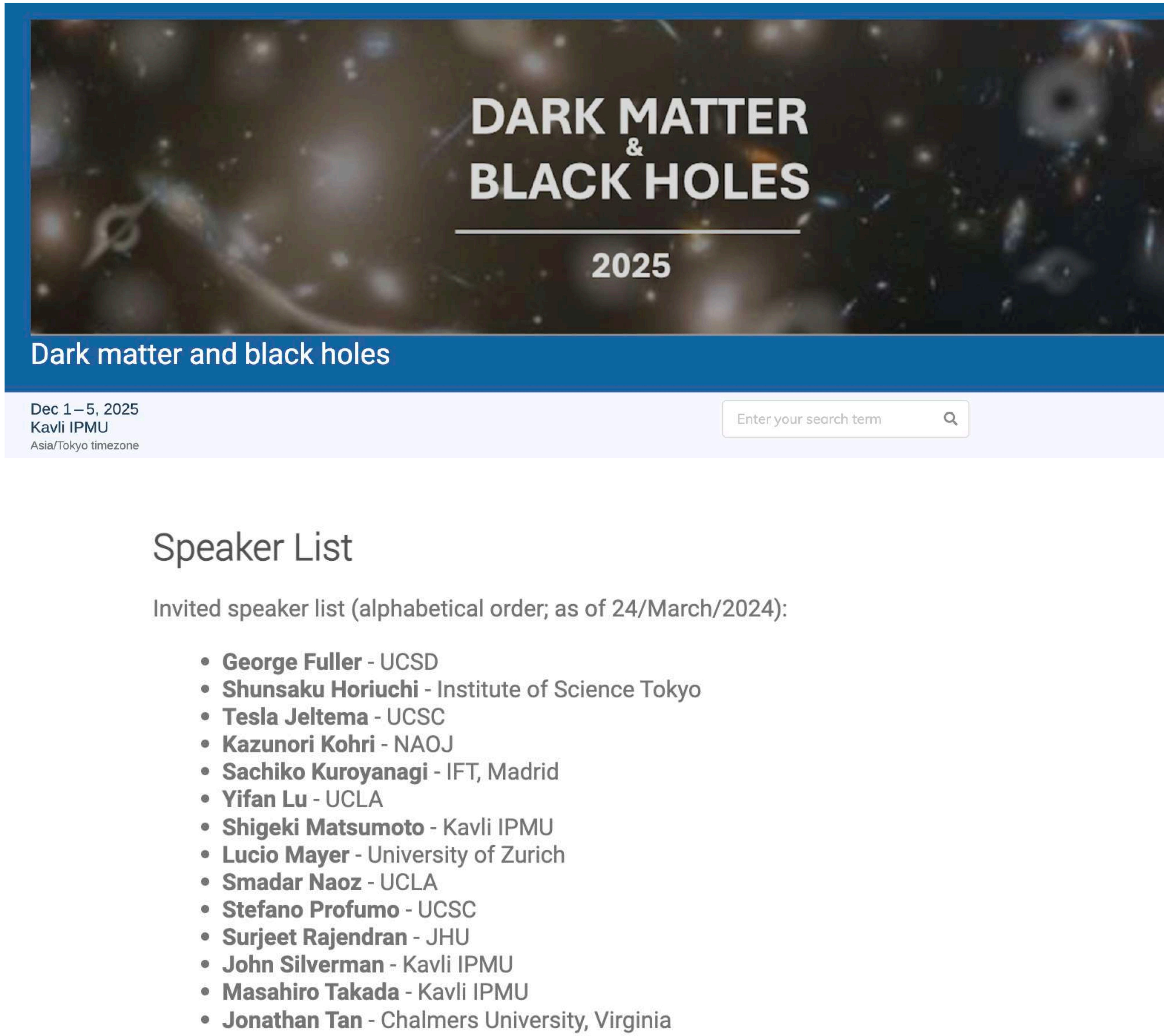


Axions in Japan

Nov 10 – 14, 2025
Kavli IPMU
Asia/Tokyo timezone

Invited speakers

- Andrey Kravtsov
- Atsushi Nishizawa
- Chanda Prescod-Weinstein
- Cora Uhlemann
- Francesca Chadha-Day
- Ippei Obata
- Jens Niemeyer
- Keir Rogers
- Masahiro Kawasaki
- Mustafa Amin
- Neal Dalal
- Philip Mocz
- Richard Easther
- Simona Vegetti
- Tomohiro Fujita
- Vera Gluscevic
- Yuko Urakawa
- Yuta Michimura



DARK MATTER
&
BLACK HOLES

2025

Dark matter and black holes

Dec 1 – 5, 2025
Kavli IPMU
Asia/Tokyo timezone

Speaker List

Invited speaker list (alphabetical order; as of 24/March/2024):

- **George Fuller** - UCSD
- **Shunsaku Horiuchi** - Institute of Science Tokyo
- **Tesla Jeltema** - UCSC
- **Kazunori Kohri** - NAOJ
- **Sachiko Kuroyanagi** - IFT, Madrid
- **Yifan Lu** - UCLA
- **Shigeki Matsumoto** - Kavli IPMU
- **Lucio Mayer** - University of Zurich
- **Smadar Naoz** - UCLA
- **Stefano Profumo** - UCSC
- **Surjeet Rajendran** - JHU
- **John Silverman** - Kavli IPMU
- **Masahiro Takada** - Kavli IPMU
- **Jonathan Tan** - Chalmers University, Virginia

Registration open!!!