Notes on Non-Axionic WISP theory

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Abstract

These are the lecture notes for the course "Non-Axionic WIMP Theoy" given at the 3rd Training School of the **Cosmic WISPers COST Action in September 2025 in Annecy, France.**

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1 Introduction

WISP ("Weakly-Interacting Slim Particle") is an umbrella term encompassing many types of hypothetical particle, united by having small (sub-eV) masses and feeble couplings. The QCD axion and its siblings, Axion-Like particles (ALPs), are canonical examples, where the weakness of their couplings arises from a high scale of new physics, and the smallness of their masses being typically due to a pseudo Nambu-Goldstone nature. Since they are (pseudo)scalar bosons they can mix with pions; and have Yukawa-type interactions – they can therefore mediate (very weak and short range) fifth forces. They also may couple to the electromagnetic field via a non-perturbative interaction; in this way, they can be detected in astrophysical and haloscope dark matter searches, when a magnetic field is present. These generally rely on the oscillation between axions and photons in the presence of a magnetic field.

It turns out that searches using this oscillation can also be applied to search for *hidden photons*. If they have a (slim) mass, a hidden photon can also oscillate into a photon without requiring a magnetic field to be present; hence experiments can be reused for hidden photon searches by ignoring or turning off the magnetic field. The properties of these hidden photons are then very similar to ALPs, and it is logical to group them together under the umbrella term "WISP." We can identify other fields which also belong in this taxonomy. These lectures will give an overview of these non-ALP WISPs, covering their theoretical aspects: what theories they come from, what sort of properties they might have, and how they might be produced.

2 A taxonomy

We can classify non-axionic WISPs by spin:

- Spin 0: dilatons, moduli,chamelions, ...
- Spin 1: hidden photons
- Spin 2: massive gravitons

We don't usually think of fermions as WISPs, even though weakly coupled and light fermions could exist. This is because:

- Slim fermions cannot explain dark matter, thanks to the Tremaine-Gunn bound [1] (of course there exist papers trying to evade this, e.g. by having multiple fermion species).
- Fermions can't oscillate into photons.

3 Spin 1 WISPs: Hidden $U(1)s$

I shall start with spin-1 WISPs because, out of the non-axionic WISPs, they have received the most attention. The basic construction is a lagrangian:

$$\mathcal{L} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{\chi}{2}\tilde{F}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{1}{2}m_{\gamma'}^2\tilde{X}_\mu\tilde{X}^\mu + ej^\mu\tilde{A}_\mu + g_X j_h^\mu\tilde{X}_\mu. \qquad (3.1)$$

This is just adding an extra (massive) $U(1)$ boson called X_μ to QED. If there are no fields charged under both $U(1)$s then the $U(1)$ is "hidden;" otherwise it is generally known as a Z'.

Hidden $U(1)$s are particularly interesting because the coupling χ is *renormalisable*. This means it could be generated at any scale and still be relevant for very light particles.

3.1 Kinetic mixing, mass mixing, millicharges

To be precise, the hidden photon should mix with the hypercharge rather than just with QED. Consider the lagrangian (using the same symbols for the field and field strength...)

$$\mathcal{L} = -\frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{\chi}{2}\tilde{B}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{1}{2}\tilde{m}^2\tilde{X}_\mu\tilde{X}^\mu + g_Y j^\mu \tilde{B}_\mu + g_X j_h^\mu \tilde{X}_\mu. \tag{3.2}$$

Here we have just written a mass for the hidden $U(1)$ without saying where it comes from. The first thing we do is to diagonalise the kinetic terms. This requires a *non-unitary* transformation:

$$\begin{aligned}\tilde{B}_\mu - \chi\tilde{X}_\mu \equiv& B_\mu \\ \tilde{X}_\mu \equiv& \frac{1}{\sqrt{1-\chi^2}}X_\mu \\ \tilde{B}_\mu =& B_\mu + \frac{\chi}{\sqrt{1-\chi^2}}X_\mu.\end{aligned} \tag{3.3}$$

This then gives

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{\tilde{m}^2}{1-\chi^2}\frac{1}{2}X_\mu X^\mu + g_Y j^\mu B_\mu + \frac{g_X}{\sqrt{1-\chi^2}}(j_h^\mu + \chi j^\mu)\tilde{X}_\mu. \tag{3.4}$$

In this way, hidden particles acquire no charge under the visible gauge field, but visible particles do directly couple to the hidden gauge field.

Note that this is just one convenient choice of transformation to diagonalise the kinetic terms. We could have alternatively rotated the hidden photon instead; this is actually desirable if the hidden photon is massless! In that case, the hypercharge picks up a coupling to the hidden coupling – and we get so-called millicharges.

We can also extend this to electroweak breaking, to include the couplings to the Z etc. Note that we can immediately see that the kinetic mixing will shift the mass of the Z:

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} - \frac{1}{4}\tilde{X}_{\mu\nu}\tilde{X}^{\mu\nu} + \frac{\chi}{2}\tilde{B}_{\mu\nu}\tilde{X}^{\mu\nu} - \frac{1}{4}\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu} + \frac{1}{2}\tilde{m}^2\tilde{X}_\mu\tilde{X}^\mu + \frac{1}{8}v^2(g_Y\tilde{B}_\mu - g_2\tilde{W}_\mu)^2 \\ &+ g_Y j_B^\mu \tilde{B}_\mu + g_2 j_W^\mu \tilde{W}_\mu + g_X j_h^\mu \tilde{X}_\mu \\ =& -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu} + g_2 j_W^\mu \tilde{W}_\mu + g_Y j_B^\mu B_\mu + \frac{1}{\sqrt{1-\chi^2}}(g_X j_h^\mu + \chi g_Y j_B^\mu)\tilde{X}_\mu \\ &+ \frac{\tilde{m}^2}{1-\chi^2}\frac{1}{2}X_\mu X^\mu + \frac{1}{8}v^2(g_Y B_\mu + \frac{g_Y\chi}{\sqrt{1-\chi^2}}X_\mu - g_2\tilde{W}_\mu)^2.\end{aligned} \tag{3.5}$$

You will see more about this in the tutorial by Mario.

3.2 Generation of the kinetic mixing

In a general QFT, kinetic mixing can arise either through integrating out heavy fields, or through the RGEs. If we integrate out heavy fields, we obtain through a one loop diagram:

$$\chi = -\frac{g_X g_Y}{8\pi^2}\left[\frac{2}{3}\mathrm{tr}_{\text{Weyl Fermions}}(Q_X Q_Y \log\frac{M}{\Lambda}) + \frac{1}{6}\mathrm{tr}_{\text{Real Scalars}}(Q_X Q_Y \log\frac{M}{\Lambda})\right] \tag{3.6}$$

In supersymmetric theories this becomes (see e.g. [2, 3]):

$$\chi = -\frac{g_Y g_X}{8\pi^2}\mathrm{tr}_{\mathrm{superfields}}\left[Q_X Q_Y \log\frac{M}{\Lambda}\right] \tag{3.7}$$

The scale Λ appears to make the logarithms dimensionless but actually plays no role since we should insist that $\mathrm{tr}(Q_X Q_Y) = 0$.

If there are fields charged under both $U(1)$s (so that the $U(1)$ is not hidden) then there will be RGEs that generate kinetic mixing terms. The canonical way to deal with this situation is to remove the kinetic mixing and instead treat the gauge couplings as a *matrix*; the result is not transparent but has been worked out in [4].

3.3 Generation of masses

There are two main mechanisms for the generation of $U(1)$ masses: a (hidden) Higgs mechanism, and the Stückelberg mechanism. Z' models typically invoke a Higgs mechanism, where there is some scalar S charged only under X_μ and obtains a (large) vev. For hidden $U(1)$s, there is a problem associated with this: for the typical higgs mechanism the Higgs mass is

$$m_h \sim \sqrt{\lambda v_S^2}$$

whereas the hidden photon mass is

$$m_{\gamma'} \sim g_X v_S$$

So, the ratio of these masses is $\sqrt{\lambda}/\}_{\mathcal{X}}$. If g_X is very small, then this can be large; but in a typical supersymmetric theory we would have $\lambda \sim g_X^2$. This would mean that the hidden higgs would become very light too, and could potentially act as a millicharged particle!

The Stückelberg mechanism involves adding just a scalar to act as the longitudinal mode:

$$\mathcal{L} \supset \frac{1}{2}m_{\gamma'}^2(X_\mu - \frac{1}{m_{\gamma'}}\partial_\mu a)^2. \tag{3.8}$$

This includes the kinetic term for a, and is then the only part of the lagrangian where a appears. In the Higgs mechanism, this would be the would-be Goldstone boson; so sometimes people regard this as an effective theory after integrating out the (hidden) Higgs boson. However, this is not quite right. In the usual Higgs mechanism there would also be quartic interactions of the Goldstones:

$$\mathcal{L}_{\mathrm{Higgs}} \supset -\lambda|H|^4 \supset -\frac{\lambda}{4}a^4. \tag{3.9}$$

In order to remove the Higgs boson, we have to take $\lambda \to \infty$, and this would violate unitarity of the theory; alternatively we would impose a cutoff. In the Stückelberg lagrangian these couplings are absent and there is no problem. Interestingly, such axions which can couple to $U(1)$ gauge bosons are ubiquitous in string theory, with no trace of any corresponding " Higgs" bosons, and they can lead to interesting predictions for the hidden photon masses.

3.4 Kinetic Mixing in Supersymmetric Field Theories

3.4.1 Holomorphic vs Physical Couplings

In a supersymmetric theory, there is only one possible operator that can yield kinetic mixing. It appears in the gauge kinetic part of the supergravity Lagrangian, and is thus a holomorphic function of other fields:

$$\mathcal{L} \supset \int d^2\theta \left\{ \frac{1}{4(g_a^h)^2} W_a W_a + \frac{1}{4(g_b^h)^2} W_b W_b - \frac{1}{2}\chi_{ab}^h W_a W_b \right\}, \tag{3.10}$$

where W_a, W_b are the field strength superfields for the two U(1) gauge fields and $\chi_{ab}^h, g_a^h, g_b^h$ are the holomorphic kinetic mixing parameter and gauge couplings that must run only at one loop. The well known expression for the holomorphic gauge running is

$$\frac{1}{(g_a^h)^2(\mu)} = \frac{1}{(g_a^h)^2(\Lambda)} - \sum_r \frac{n_r Q_a^2(r)}{8\pi^2} \log \mu/\Lambda, \tag{3.11}$$

Here, $Q_a(r)$ denotes the charge under group a carried by n_r fields. The physical gauge couplings are given in terms of the holomorphic quantities by the Kaplunovsky-Louis formula [5, 6] (given here specialised to U(1) gauge groups):

$$g_a^{-2} = \Re\Big[(g_a^h)^{-2}\Big] - \sum_r \frac{Q_a^2(r)}{8\pi^2} \log\det Z^{(r)} + \sum_r \frac{n_r Q_a^2(r)}{16\pi^2}\kappa^2 K, \tag{3.12}$$

where $Z^{(r)}$ is the renormalised kinetic energy matrix of fields having charge $Q_a(r)$ (i.e. the renormalised Kähler metric $K_{\overline{\alpha}\beta}$), K is the full Kähler potential and $\kappa^2 = 1/M_P^2$.

We should expect that a similar formula should be obeyed for the kinetic mixing, too. Indeed, it can be shown that the relevant expression, exact to all orders in perturbation theory, is [3, 7]

$$\frac{\chi_{ab}}{g_a g_b} = \Re(\chi_{ab}^h) + \frac{1}{8\pi^2}\mathrm{tr}\Big(Q_a Q_b \log Z\Big) - \frac{1}{16\pi^2}\sum_r n_r Q_a Q_b(r)\kappa^2 K, \tag{3.13}$$

where χ_{ab} is now the parameter in the canonical Lagrangian density

$$\mathcal{L}_{\text{canonical}} \supset \int d^2\theta \left\{ \frac{1}{4} W_a W_a + \frac{1}{4} W_b W_b - \frac{1}{2}\chi_{ab} W_a W_b \right\}. \tag{3.14}$$

The first terms can be easily understood as being a consequence of rescaling the vector superfields from the holomorphic basis (where the gauge kinetic terms are as above) to the canonical basis, $V_a \to g_a V_a$; the shift in integration measure gives the second term. The third term is related to the Weyl (rescaling) symmetry of the classical supergravity theory. The above also has a simple interpretation in messenger-generated mixing; we can write

$$\chi_{ab}^h(\mu) = -\frac{1}{8\pi^2}\mathrm{tr}\Big(Q_a Q_b \log \mathcal{M}/\Lambda\Big) - \frac{1}{8\pi^2}\mathrm{tr}_{light}\Big(Q_a Q_b \log \mu/\Lambda\Big) \tag{3.15}$$

where μ is smaller than all the masses in the matrix $\mathcal{M} = W_{\alpha\beta} \equiv \partial_\alpha \partial_\beta W$, where W is now the superpotential, and tr_{light} denotes the trace over modes lighter than μ.

Then if we calculate the physical mixing directly at one loop we must consider the supergravity Lagrangian (given here for the scalar components):

$$\mathcal{L} \supset K_{\overline{\alpha}\beta} D_\mu \overline{\phi}^{\overline{\alpha}} D^\mu \phi^\beta - \exp(\kappa^2 K)(K^{\overline{\alpha}\beta}\overline{W}_{;\overline{\alpha}} W_{;\beta} - 3\kappa^2 |W|^2). \tag{3.16}$$

Extracting the mass terms, we find

$$\mathcal{L} \supset K_{\overline{\alpha}\beta} D_\mu \overline{\phi}^{\overline{\alpha}} D^\mu \phi^\beta - \exp(\kappa^2 K) K^{\overline{\alpha}\beta}\overline{W}_{\overline{\alpha}\overline{\gamma}} W_{\beta\delta} \overline{\phi}^{\overline{\gamma}} \phi^\delta \tag{3.17}$$

leading to physical kinetic mixing

$$\chi_{ab}(\mu) = g_a g_b \Big[-\frac{1}{16\pi^2}\mathrm{tr}\Big(Q_a Q_b \log \frac{K^{-1}\mathcal{M}^\dagger K^{-1}\mathcal{M}}{\Lambda^2}\Big) - \frac{1}{16\pi^2}\mathrm{tr}\Big(Q_a Q_b \kappa^2 K\Big)$$
$$- \frac{1}{8\pi^2}\mathrm{tr}_{light}\Big(Q_a Q_b \log \frac{K^{-1}\mu}{\Lambda}\Big)\Big] \tag{3.18}$$

which reproduces the above formula.

3.4.2 Implications

The bottom line is that the holomorphic kinetic mixing is

$$\begin{aligned}\chi^h_{ab} \sim & \frac{1}{16\pi^2} \times \mathcal{O}(1) \\ \sim & \mathcal{O}(0.01).\end{aligned} \tag{3.19}$$

because the logarithmic term is of order one. In principle, it is possible to engineer *almost* degenerate masses in order to suppress the logarithmic term. However, the natural way that that should appear, without the splitting being accidental, is via gauge symmetry breaking effects; e.g. the splitting could arise from the splitting of degenerate $SU(5)$ multiplets. But such effects are usually induced by supersymmetry breaking: and therefore this cannot appear in the holomorphic kinetic mixing formula.

If we stick with the above estimate, then we see immediately a major implication: the *physical* kinetic mixing in supersymmetric theories is determined by the hidden gauge coupling:

$$\chi_{XY} \sim \frac{g_X g_Y}{16\pi^2} \sim 10^{-3} g_X. \tag{3.20}$$

3.5 Hidden photons in string theory

Hidden photons appear in string theory in many contexts:

- Type II theories: every stack of D-branes has a $U(1)$ associated with it.
- Heterotic theories: $E_8 \times E_8$ or $SO(32)$ broken to the SM gauge group leaves many additional generators, and it is common to find "unbroken" $U(1)$s. These can either be Z's or genuinely hidden $U(1)$s. See e.g. [8] for examples of these.
- R-R $U(1)$s (see below).

3.5.1 *RR* $U(1)$s

Section for the specialists

In type IIB string theory, the dimensional reduction of the Ramond-Ramond 4-form C_4 yields h^3 four-dimensional vectors (there is no similar contribution from the two-form as for a Calabi-Yau $h^1 = 0$). There is a similar story for C_3/C_5 in type IIA, but this is analagous and for simplicity we shall focus on type IIB. Writing

$$C_4 = D_2^\alpha(x) \wedge \omega_\alpha + V^\kappa(x) \wedge \alpha_\kappa + U_\kappa(x) \wedge \beta^\kappa + \rho_\alpha \tilde{\omega}^\alpha. \tag{3.21}$$

where $\omega_A, \tilde{\omega}^A$ are a basis of two and four-forms respectively. The vectors are V^κ, U_κ with $\alpha_\kappa, \beta^\kappa$ a symplectic basis of $H^{2,1}_+ \oplus H^{1,2}_+$ where the signs denote the parity under the orientifold projection. They are normalised so that $\int \alpha_\kappa \wedge \beta^\lambda = l_s^6 \delta^\lambda_\kappa$ with $\int \alpha_\kappa \wedge \alpha_\lambda = \int \beta^\kappa \wedge \beta^\lambda = 0$. In fact V^κ, U^κ are dual to each other due to the self-duality of the five-form field strength $F_5 = dC_4 - dB \wedge C_2 = \star_{10} F_5$ and so it is customary to eliminate U^κ from the action.

These $U(1)$s do not possess any charged matter, and in Calabi-Yau compactifications have no axionic couplings, and are thus massless. They can become massive in *non-Kähler* compactifications [9] by acquiring axionic couplings; in this case they may also exhibit *mass mixing* with the hypercharge. However, if we restrict our attention to the Calabi-Yau case, the only way that they can be detected is by their kinetic mixing with the hypercharge, supported on branes.

The kinetic mixing of RR $U(1)$s with $D7$-branes was calculated in [10], and with $D5$-branes in [11]. In the first case, it is necessary for there to be Wilson line moduli to obtain non-zero mixing, while in the second deformation moduli of the $D5$-branes are required. This means that the branes are not rigid prior to moduli stabilisation; the brane moduli appear as massles adjoint fields. Wilson line moduli are fluctuations of the gauge field on the brane; for x, y non-compact and compact coordinates respectively, we can write

$$A(x,y) = A_\mu(x) dx^\mu + a_I(x) A^I(y) + \overline{a}_{\overline{J}}(x) \overline{A}^{\overline{J}}(y) \tag{3.22}$$

where $A(x,y)$ is a one-form, $a_I, \overline{a}_{\overline{J}}$ are scalars and $A^I, \overline{A}^{\overline{J}}$ are one-forms on the compact portion of the brane; they thus take values in $H^1(S, \mathcal{O})$ where S is the manifold (divisor for a $D7$-brane, two-cycle for a $D5$ brane) wrapped by the brane and $\mathcal{O}$ is the sheaf supported on it. We thus find the action for the adjoint chiral mulitplets with scalar components $a_I, \overline{a}_{\overline{J}}$ from the standard dimensional reduction of the action on the brane.

The fluctuation moduli $\zeta^i, \overline{\zeta}^{\overline{j}}$ are counted instead by $H^0(S, NS)$ where NS is the normal bundle to brane. They are also adjoint fields, but their action is instead derived via the Dirac-Born-Infeld action from fluctuations of the metric pulled back to the brane:

$$\varphi^* g_{10} \supset 2 e^{\phi/2} g_{i\overline{j}} \partial_\mu \zeta^i \partial_\nu \overline{\zeta}^{\overline{j}} dx^\mu dx^\nu \tag{3.23}$$

where φ^* denotes the pullback, ϕ is the dilaton.

Concentrating now on the case of $D7$ branes (which is relevant for the large part of IIB model building, for example the LARGE volume scenario) the coupling of the RR $U(1)$ to the $U(1)$ on a $D7$-brane arises from the Chern-Simons coupling

$$S_{CS} \supset 2\pi l_s^{-8} \int_S \frac{1}{2} \frac{l_s^4}{(2\pi)^2} C_4 \wedge F \wedge F. \tag{3.24}$$

This leads to

$$S_{CS} \supset -\frac{l_s^{-4}}{(2\pi)^3} \int_4 (a_\kappa + \overline{a}_\kappa) dV^\kappa \wedge F + (a^\kappa + \overline{a}^\kappa) dU_\kappa \wedge F \tag{3.25}$$

where

$$\begin{aligned} a^\kappa &= a_I \int_S \beta^\kappa \wedge A^I \\ a_\kappa &= a_I \int_S \alpha_\kappa \wedge A^I. \end{aligned} \tag{3.26}$$

After eliminating the fields U^κ the kinetic mixing part of the action reads

$$S_{YM} \supset -\int_4 \frac{M_{Pl}^2}{32\pi}\text{Im}(\mathcal{M})_{\alpha\beta} dV^\alpha \wedge \star_4 dV^\beta - \frac{1}{\pi l_s^4} C_{\alpha\beta}\Big((a^\beta + \overline{a}^\beta) dV^\alpha \wedge \star_4 F\Big) \tag{3.27}$$

where here

$$\begin{aligned} A_\beta{}^\alpha =& -\frac{1}{l_s^6}\int_6 \beta^\alpha \wedge \star_6 \alpha_\beta \\ C_{\alpha\beta} =& \Big[-\frac{1}{l_s^6}\int_6 \beta^\alpha \wedge \star_6 \beta^\beta\Big]^{-1} \\ \mathcal{M} =& AC + iC. \end{aligned} \tag{3.28}$$

Note that V^κ are non-canonically normalised; to obtain a canonically normalised field strength for them we must define $dV^\kappa \equiv l_s F^\kappa$ (note a_κ, a^κ have dimension length^3).

3.5.2 Hidden photons from D-branes

The simplest way to engineer kinetic mixing in type II string theories is to consider brane-antibrane kinetic mixing [12–14]. However, using antibranes violently breaks supersymmetry. It turns out instead that supersymmetric mixing between $U(1)$s located on different D-branes is possible and generic, since every stack of branes has an associated $U(1)$. Many of these will be anomalous, and obtain large masses via the Green-Schwarz mechanism, which is nothing but the Stückelberg mechanism we saw earlier; but there can be additional $U(1)$s that are *not* anomalous (e.g. the hypercharge) ... and some of these can become massive too! And the interesting thing there, is that the masses for these $U(1)$s are typically *below* the string scale.

Focussing on type IIB string theory, we can explicitly derive these Stückelberg masses (see [3,15]) starting from the action involving $U(1)$s on D7 branes:

$$S \supset -\int \Big(dc_\alpha + \frac{M_P}{\pi}A_i q_{i\alpha}\Big)\frac{\mathcal{K}_{\alpha\beta}}{8}\wedge\star\Big(dc_\beta + \frac{M_P}{\pi}A_j q_{j\beta}\Big) + \frac{1}{4\pi M_P} r^{i\alpha} c_\alpha \text{tr}(F\wedge F) - \frac{r^{i\alpha}\tau_\alpha}{4\pi M_P}\text{tr}(F_i \wedge \star F_i).$$

Here

$$\mathcal{K}_0 \equiv \frac{\partial^2}{\partial\tau_i \partial\tau_j}(-2\log\mathcal{V}) \tag{3.29}$$

These are sensitive only to Kähler moduli τ_i, so the masses are diluted by volumes in compact space, and the gauge couplings:

$$m_{ab}^2 = g_a g_b \frac{M_P^2}{4\pi^2}\Big[r_{ac} r_{bd}\mathcal{V}^{-2}(\mathcal{K}_0^{-1})^{cd} + q_{a\alpha}(\mathcal{K}_0)_{\alpha\beta} q_{b\beta}\Big] \tag{3.30}$$

These appear because the gauge couplings on a D-brane are related to the volume of the brane via the DBI action

$$\frac{1}{g_a^2} \propto V_d. \tag{3.31}$$

For $D7$ branes, the volume of the branes can be large, and of course the larger the brane, the smaller the coupling, so the smaller the physical kinetic mixing.

This can be rather predictive; for "isotropic" scenarios where the six extra dimensions are all of similar size. E.g. for a swiss-cheese where $\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$, we heavier

$$\begin{aligned}
\partial_{\tau_b} K =& -3\frac{\tau_b^{1/2}}{\mathcal{V}} \\
\partial^2_{\tau_b} K =& -\frac{3}{2}\frac{\tau_b^{-1/2}}{\mathcal{V}} + \frac{9}{2}\frac{\tau_b}{\mathcal{V}^2} \\
=& \frac{3}{2}\frac{1}{\mathcal{V}^2 \tau_b^{1/2}}[3\tau_b^{3/2} - \tau_b^{3/2} + \tau_s^{1/2}] = 3\frac{1}{\mathcal{V}^2 \tau_b^{1/2}}[\tau_b^{3/2} + \frac{1}{2}\tau_s^{1/2}] \\
\partial_{\tau_s} K =& 3\frac{\tau_s^{1/2}}{\mathcal{V}} \\
\partial^2_{\tau_s} K =& 3\frac{1}{\mathcal{V}^2 \tau_s^{1/2}}[\frac{1}{2}\tau_b^{3/2} + \tau_s^{1/2}] \\
\partial_{\tau_s}\partial_{\tau_b} K =& -\frac{9}{2}\frac{\tau_s^{1/2}\tau_b^{1/2}}{\mathcal{V}^2}
\end{aligned} \tag{3.32}$$

and therefore

$$(\mathcal{K}_0)_{ij} \sim \frac{1}{\mathcal{V}}\begin{pmatrix} \frac{3}{\mathcal{V}^{1/3}} & -\frac{9}{2}\tau_s^{1/2}\mathcal{V}^{-2/3} \\ -\frac{9}{2}\tau_s^{1/2}\mathcal{V}^{-2/3} & \frac{3}{2}\tau_s^{-1/2} \end{pmatrix} \tag{3.33}$$

This has determinant $\sim \frac{9}{2}\tau_s^{-1/2}\mathcal{V}^{-5/3}$ so

$$(\mathcal{K}_0^{-1})^{ij} \sim \frac{2}{9}\tau_s^{1/2}\mathcal{V}^{5/3}\begin{pmatrix} \frac{3}{2}\tau_s^{-1/2} & \frac{9}{2}\tau_s^{1/2}\mathcal{V}^{-2/3} \\ \frac{9}{2}\tau_s^{1/2}\mathcal{V}^{-2/3} & \frac{3}{\mathcal{V}^{1/3}} \end{pmatrix} \tag{3.34}$$

Hence we see that the terms proportional to $(\mathcal{K}_0)_{ij}$ actually have the greater volume suppression of the masses.

We can therefore find cases for $U(1)$s on the 'large' brane, having $g^2 \sim \mathcal{V}^{2/3}$ that

$$m_{\gamma'} \sim \frac{M_P}{\mathcal{V}}. \tag{3.35}$$

Recalling that

$$M_{\rm string} \sim M_P/\mathcal{V}^{1/2},$$

if we have a large D-brane then in the isotropic case we have $V_d \sim V^{2/3}$ so $g_X \sim V^{-1/3}$. If we have a string scale of a TeV, this gives us $\mathcal{V} \sim 10^{30}$ and therefore

$$\chi \sim 10^{-13}, \qquad m_{\gamma'} \sim \text{meV}. \tag{3.36}$$

But we can do better with "anisotropic" scenarios where not all dimensions are the same.

3.5.3 Swampland conjectures

These arguments were extended to the swampland in [16] using the Weak Gravity Conjecture, that implies that the cutoff of a theory should be

$$\Lambda_{UV} \lesssim g M_P. \tag{3.37}$$

If the g is the coupling of a hidden $U(1)$, then insisting that the cutoff be around 1 TeV implies a minimum hidden gauge coupling of around 10^{-15}. In turn, this would give a minimal value for the kinetic mixing in the swampland of around 10^{-18}.

3.6 Hidden Photon Dark Matter

3.6.1 Production from misalignment

Famously axions and ALPs can be produced via the misalignment mechanism. In principle this works for any light scalar field, because any initial expectation value is trapped by "hubble friction" until the field starts oscillating, when $H \sim m$. In [17], Nelson and Scholtz proposed that this could also work for hidden photons. In fact, in [18] it was shown that it's necessary to add an extra gravitational interaction term (employed in *vector inflation* in [19]):

$$\mathcal{L}_{\text{grav}} = \frac{\kappa}{12} R X_\mu X^\mu. \tag{3.38}$$

Recall that during inflation with $ds^2 = dt^2 - a^2(t)dx_i^2$ we have

$$\begin{aligned} R =& -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \\ \dot{H} =& \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \\ \rightarrow R =& -6(\dot{H} + 2H^2). \end{aligned} \tag{3.39}$$

We only consider the spatial components of the vector field, and consider a homogeneous field, i.e. $\partial_i X_\mu = 0$. Then the relevant bits of the lagrangian become

$$\begin{aligned} \mathcal{L} \supset& \sqrt{-g}\left[\frac{\kappa}{12} R X_\mu X^\mu + \frac{1}{2} m_{\gamma'}^2 X_\mu X^\mu - \frac{1}{4} X_{\mu\nu} X^{\mu\nu}\right] \\ \supset& a^3\left[\frac{\kappa}{2}(\dot{H} + 2H^2) X_i^2/a^2 - \frac{1}{2} m_{\gamma'}^2 X_i^2/a^2 + \frac{1}{2a^2}(\partial_0 X_i)^2\right]. \end{aligned} \tag{3.40}$$

We already at this point see a problem with the $\kappa = 0$ case: the energy density per unit comoving volume would be $\frac{1}{2} m_{\gamma'}^2 X_i^2/a^2$ if we can neglect $\dot{X}$, and in that case it is rapidly diluted with the expansion.

Since $X^\mu X_\mu = -1/a^2(t) X_i X_i$ is a coordinate independent measure for the size of the vector it is convenient to introduce $\bar{X}_i = X_i/a(t)$ and then find the equations of motion for that field. Noting that

$$\partial_0 \overline{X}_i = \dot{X}_i/a - H X_i/a, \tag{3.41}$$

we have

$$\begin{aligned} \mathcal{L} \supset& a^3\left[\frac{\kappa}{2}(\dot{H} + 2H^2)\overline{X}_i^2 - \frac{1}{2} m_{\gamma'}^2 \overline{X}_i^2 + \frac{1}{2}(\dot{\overline{X}}_i + H\overline{X}_i)^2\right] \\ =& a^3\left[\frac{\kappa}{2}(\dot{H} + 2H^2)\overline{X}_i^2 - \frac{1}{2} m_{\gamma'}^2 \overline{X}_i^2 + \frac{1}{2}(\dot{\overline{X}}_i)^2 + H\dot{\overline{X}}_i\overline{X}_i + \frac{1}{2} H^2 \overline{X}_i^2\right] \end{aligned} \tag{3.42}$$

Making a variation:

$$\delta\mathcal{L} = a^3\left[\left(\kappa(\dot{H} + 2H^2) - m_{\gamma'}^2 + H^2\right)\overline{X}_i \delta\overline{X}_i + (\dot{\overline{X}}_i)\partial_t \delta\overline{X} + H\dot{\overline{X}}_i \delta\overline{X}_i + H\overline{X}_i \partial_t \delta\overline{X}_i\right]$$

$$\rightarrow a^3\left[\left(\kappa(\dot{H}+2H^2)-m_{\gamma'}^2+H^2\right)\overline{X}_i\delta\overline{X}_i-(\ddot{\overline{X}}_i)\delta\overline{X}-3H\dot{\overline{X}}\delta\overline{X}+H\dot{\overline{X}}_i\delta\overline{X}_i-H\dot{\overline{X}}_i\delta\overline{X}_i-\dot{H}\overline{X}_i\delta\overline{X}_i-3H^2\overline{X}_i\delta\overline{X}_i\right]$$

$$=a^3\left[\left(\kappa(\dot{H}+2H^2)-m_{\gamma'}^2-\dot{H}-2H^2\right)\overline{X}_i\delta\overline{X}_i-(\ddot{\overline{X}}_i)\delta\overline{X}-3H\dot{\overline{X}}\delta\overline{X}\right] \tag{3.43}$$

This gives us the equations of motion:

$$\ddot{\bar{X}}_i+3H\dot{\bar{X}}_i+\left(m_{\gamma'}^2+(1-\kappa)(\dot{H}+2H^2)\right)\bar{X}_i=0. \tag{3.44}$$

We see now that the term $\kappa=1$ is special and allows the field to evolve in the same way as a standard scalar. Otherwise, at early times we can neglect the friction term and the energy will dissipate rapidly. With $\kappa=1$ however we can obtain the correct relic density of dark matter for appropriate values of the initial vev and hidden photon mass.

The interesting parameter space is shown in figure 1.

3.6.2 Production during inflation

It is perhaps difficult to justify the addition of the $RX_\mu X^\mu$ coupling from a UV theory. In [20] it was shown that, instead, inflationary fluctuations will lead to the production of an acceptable relic density of hidden photons, without modifying the lagrangian. They showed that the longitudinal modes are produced automatically through quantum fluctuations during inflation, and as a result the dark matter relic density is:

$$\Omega_{\gamma'}h^2=0.112\times\sqrt{\frac{m_{\gamma'}}{6\times10^{-6}\ \mathrm{eV}}}\left(\frac{H_I}{10^{14}\mathrm{GeV}}\right)^2 \tag{3.45}$$

Hence it is straighforward to obtain the correct relic abundance for a sufficiently high inflationary scale and hidden photon mass.

Unlike ALP misalignment production, the fluctuations are sharply peaked around

$$1/k_*\sim3.2\times10^{-10}\mathrm{Mpc}\sqrt{\frac{10^{-5}\mathrm{eV}}{m_{\gamma'}}} \tag{3.46}$$

... hence isocurvature fluctuations from the dark matter produced this way are never observable in the CMB.

4 Spin 0 WISPs

There are a large variety of spin-0 particles that could be slim and weakly coupled. A broad overview is given in [21].

4.1 Dilaton

Dilatons appear in many contexts in HEP. In field theory, they arise when we try to make a theory scale invariant. They appear automatically in string theory. They can be “WISPy” when they are light, in which case they can be strongly constrained by many observations such as fifth-force searches. For some phenomenological work see [22–26]

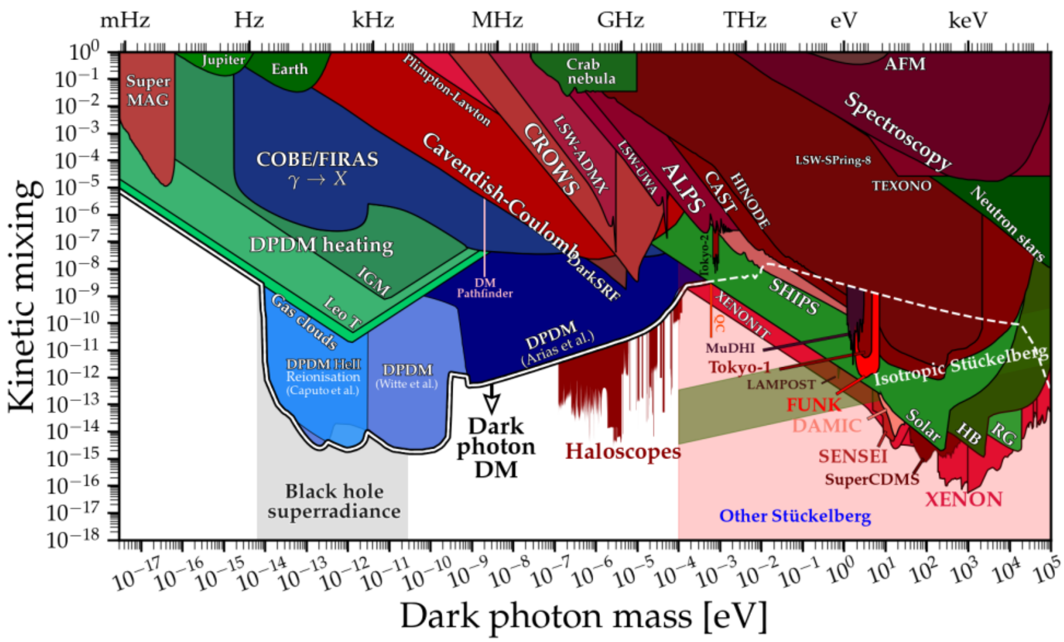

Figure 1: Hidden photon DM parameter space.

4.1.1 Dilatations

We first review dilatations. There is some review of this in Chapter 3 of Coleman's book.

Imagine that we make a shift of the coordinates by

$$x^\mu \to e^{-\alpha} x^\mu. \tag{4.1}$$

This is the opposite sign found in Coleman's book, and I explain why in appendix A. We induce a transformation on the fields associated with this; we can therefore implement it *instead* (i.e. without transforming x) for a field of dimension 1[1]:

$$\phi(x) \to e^\alpha \phi(e^\alpha x^\mu) \tag{4.2}$$

The relation between the two is explained in appendix A. Infinitesimally this is

$$\delta\phi = \alpha(1 + x^\mu \partial_\mu)\phi \tag{4.3}$$

A free field is invariant under such transformations; we have

$$\begin{aligned}\delta \int d^4x \frac{1}{2}(\partial_\mu \phi)^2 &= \int d^4x \alpha \Big[\partial^\mu \phi \partial_\mu (1 + x_\nu \partial^\nu)\phi\Big] \\ &= \int d^4x \alpha \Big[2\partial^\mu \phi \partial_\mu \phi + x_\nu \partial^\nu \partial^\mu \phi \partial_\mu \phi\Big] \\ &= \int d^4x \alpha \Big[4\mathcal{L} + x_\nu \partial^\nu \mathcal{L}\Big]\end{aligned}$$

[1]In general, for an operator of dimension d we will have $\delta\mathcal{O} = \alpha(d + x^\mu \partial_\mu)\mathcal{O}$.

$$
\begin{aligned}
&= \int d^4x \alpha \partial_\mu \left[x_\nu \partial^\nu \mathcal{L} \right] \\
&\to 0
\end{aligned} \tag{4.4}
$$

by integration by parts.

We see however that dimensionful terms will violate scale independence:

$$
\begin{aligned}
\delta(-\frac{1}{2}m^2\phi^2) =& - \alpha m^2 \phi (1 + x^\mu \partial_\mu)\phi \\
=& - \alpha m^2 \left[\phi^2 + \frac{1}{2}\partial_\mu (x^\mu \phi^2) - 2\phi^2 \right] \\
\to& \alpha m^2 \phi^2 .
\end{aligned} \tag{4.5}
$$

For a scale-invariant theory there should be a conserved dilatation current. Recall from Noether's theorem:

$$
\begin{aligned}
\delta\mathcal{L} =& \frac{\delta \mathcal{L}}{\delta\phi}\delta\phi + \frac{\delta\mathcal{L}}{\delta \partial_\mu \phi}\delta(\partial_\mu\phi) \\
=& \frac{\delta \mathcal{L}}{\delta\phi}\delta\phi + \partial_\mu \left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi \right) - \delta\phi \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} \\
=& \delta\phi \left[\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} \right] + \partial_\mu \left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi \right).
\end{aligned} \tag{4.6}
$$

The first part in square brackets vanishes *on shell* (it is the equations of motion). If we suppose that we have a symmetry of the theory, then in general we can have

$$
\delta\mathcal{L} = \partial_\mu \Lambda^\mu , \tag{4.7}
$$

i.e. the change in the lagrangian leads only to a surface term. Then the current is

$$
s_\mu \propto \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\delta\phi - \Lambda^\mu
$$

and if the symmetry is exact then $\partial^\mu s_\mu = 0$ *on shell*, i.e. when we respect the equations of motion. Violation of scale invariance should be related to the non-vanishing of this divergence. Substituting the variations for a scalar field we have

$$
s_\mu = \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}[\phi + x^\nu\partial_\nu\phi] - \Lambda_\mu \tag{4.8}
$$

Recall that get the correct Einstein field equations we have

$$
\frac{\delta S_M}{\delta g^{\mu\nu}} = \frac{1}{2}\sqrt{-g}T_{\mu\nu} \tag{4.9}
$$

and for a Klein-Gordon field we have

$$
\begin{aligned}
T_{\mu\nu} =& - g_{\mu\nu}\mathcal{L} + \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\partial_\nu\phi \\
T^\mu_\mu =& \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu\phi - g^{\mu\nu}g_{\mu\nu}\mathcal{L} \\
=& \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu\phi - d\mathcal{L}.
\end{aligned} \tag{4.10}
$$

So we can recognise the second part:

$$s_\mu = \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\phi + x^\nu\left(T_{\mu\nu} + g_{\mu\nu}\mathcal{L}\right) - \Lambda_\mu \tag{4.11}$$

But for our dilatation we already showed that

$$\delta\mathcal{L} = \partial_\mu(x^\mu\mathcal{L}) \rightarrow \Lambda_\mu = x_\mu\mathcal{L} \tag{4.12}$$

Hence

$$s_\mu = \underbrace{\frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\phi}_{\text{virial current}} + x^\nu T_{\mu\nu} \tag{4.13}$$

The standard Dilatation current is the part

$$D_\mu = x^\nu T_{\mu\nu} \tag{4.14}$$

and it has divergence

$$\partial^\mu D_\mu = T^\mu_\mu + x^\nu \underbrace{\partial^\mu T_{\mu\nu}}_{=0}. \tag{4.15}$$

The divergence of the virial part is usually neglected; it can often either be written as a total derivative itself, in which case it can be absorbed into the stress-energy tensor, see [27].

An instructive alternative derivation of the effect of dilatations appears in chapter 19.5 of Peskin & Schroeder; we consider dilatations to be a shift in the metric $g_{\mu\nu} \rightarrow e^{2\alpha}g_{\mu\nu}$. Then for an infinitesimal shift $\delta g_{\mu\nu} = 2\alpha g_{\mu\nu}$ we have

$$\begin{aligned}
\delta S &= \int d^4x(2\alpha)\eta_{\mu\nu}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g_{\mu\nu}} \\
&= \int d^4x\sqrt{-g}(2\alpha)\eta_{\mu\nu}\frac{1}{2}T^{\mu\nu} \\
&= \alpha\int d^4x\sqrt{-g}T^\mu_\mu
\end{aligned} \tag{4.16}$$

So we see that

$$\partial_\mu s^\mu = T^\mu_\mu \tag{4.17}$$

as we obtained before.

4.2 Quantum shift

It should not come as a surprise that quantum corrections will modify the trace of the stress-energy tensor: if we regard it as the trace of the variation of the lagrangian under dilatations, which involve changes of scale, we can anticipate that anomalous dimensions and beta-functions arise. One naive derivation [28] (which does not appear in the published version ...) invokes assigns a shift in the RG scale μ to the dilatations $\mu \rightarrow e^\alpha\mu = (1 + \alpha + ...)\mu$, so that for a coupling g_i the variation will be

$$\delta g_i = \frac{dg_i}{d_\mu}\delta\mu$$

$$
\begin{aligned}
&= \mu\alpha \frac{dg_i}{d_\mu} \\
&= \alpha\beta_{g_i}
\end{aligned}
\tag{4.18}
$$

so for an operator $\mathcal{O}_i$ of dimension d_i with coupling g_i we have

$$
\begin{aligned}
\mathcal{L} &\supset g_i\mathcal{O}_i \\
\delta\mathcal{L} &= \alpha\Big[g_i(d_i + x^\mu\partial_\mu)\mathcal{O}_i + \mathcal{O}_i\beta_{g_i}\Big] \\
\to \partial_\mu s^\mu = \theta^\mu_\mu &= \alpha\Big[g_i(d_i - 4)\mathcal{O}_i + \mathcal{O}_i\beta_{g_i}\Big]
\end{aligned}
\tag{4.19}
$$

Peskin and Schroeder advocate a similar shift for renormalised couplings. A rigorous derivation can be found in [29]. This will be relevant fir us because classically conformal-invariant terms like $F_{\mu\nu}F^{\mu\nu}$ will appear in the trace thanks to their beta-functions!

4.3 Enter the dilaton

The aim will be to restore scale invariance to a theory by adding a field that compensates! We can then define a special field $\chi(x)$ where $\chi(x) \to e^\alpha\chi(e^\alpha x)$ under scale tranformations. We then just need to multiply every term by the appropriate power of χ. For example, for a massive scalar ϕ we have the lagrangian

$$
\mathcal{L} = \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2f^2}\chi^2\phi^2. \tag{4.20}
$$

Where f is some constant of dimension length. But now we have a theory with no mass scale: we only have m/f! So we must *spontaneously break scale invariance* by giving the dilaton a vev. In this way, the vacuum violates scale invariance (as the SM does) but the theory still respects it. Then there must be a Goldstone boson of scale invariance! To find it, we define

$$
fe^{\sigma/f} \equiv \chi \tag{4.21}
$$

so that now

$$
\begin{aligned}
e^\alpha\chi(e^\alpha x) &= e^\alpha f e^{\sigma(e^\alpha x)/f} \\
\sigma(x) &\to \sigma(e^\alpha x) + \alpha f \\
\delta\sigma &= \alpha(x^\mu\partial_\mu\sigma + f).
\end{aligned}
\tag{4.22}
$$

The field σ will be our Goldstone boson – the dilaton! This has been attractive as a potential solution to the hierarchy problem: by removing mass scales from the theory, we protect them from quantum corrections until the dilaton aquires its vev. And the dilaton can only obtain a mass from an explicit violation of the scale invariance in the theory, so its mass is protected. Typically we take a mass term of the form

$$
\begin{aligned}
\mathcal{L} &\supset -\frac{m_\sigma^2 f^2}{16}[e^{4\sigma/f} - 4(\sigma/f) - 1] \\
&= -\frac{1}{2}m_\sigma^2\sigma^2 + ...
\end{aligned}
\tag{4.23}
$$

The reason for the baroque form is that the variation of this gives

$$\begin{aligned}\alpha\theta^\mu_\mu = \delta\mathcal{L} =& -\alpha\frac{m_\sigma^2 f^3}{16}[4e^{4\sigma/f} - 4](x^\mu\partial_\mu\sigma + f)\\ \theta^\mu_\mu =& \frac{m_\sigma^2 f^2}{4}[e^{4\sigma/f} - 1] - \frac{m_\sigma^2 f^2}{16}x^\mu\partial_\mu[e^{4\sigma/f} - 4\sigma/f - 1]\\ =& \frac{m_\sigma^2 f^2}{4}[e^{4\sigma/f} - 1] + \frac{m_\sigma^2 f^2}{4}[e^{4\sigma/f} - 4\sigma/f - 1]\\ =& -(m_\sigma^2 f)\sigma,\end{aligned} \tag{4.24}$$

i.e. the divergence is linearly proportional to σ.

Now we can go further and work out the couplings of σ to matter fields; since the theory (other than the dilaton mass term) respects scale invariance then we expect that $\delta\mathcal{L}_{\mathrm{SM}+\sigma} = 0$; but we already discussed that $\delta\mathcal{L}_{\mathrm{SM}} = \theta^\mu_\mu$. So we should expect that the coupling of the dilaton should be

$$\mathcal{L} \supset -\frac{\sigma}{f}\theta^\mu_\mu. \tag{4.25}$$

Alternatively we can just insert the χ fields into the lagrangian and expand them!

For the coupling to gauge bosons, we have

$$\begin{aligned}\frac{\partial}{\partial\log\mu}\left(\frac{1}{g^2}\right) =& -\frac{b_0}{8\pi^2}\\ b_0 =& -\frac{11}{3}C_2(G) + \frac{4}{3}\kappa S_2(F) + \frac{1}{6}S_2(S) - \frac{11}{3}S_2(V).\end{aligned} \tag{4.26}$$

So the coupling to the dilaton to a general gauge boson becomes just

$$\begin{aligned}\mathcal{L} \supset& \frac{\sigma}{4f}\beta_{1/g^2}F_{\mu\nu}F^{\mu\nu}\\ =& -\frac{\sigma}{4f}\frac{b_0}{8\pi^2}F_{\mu\nu}F^{\mu\nu}\\ \to& -\frac{\sigma}{4f}\frac{b_0 g^2}{8\pi^2}F_{\mu\nu}F^{\mu\nu} = -\frac{\sigma}{f}\frac{b_0\alpha}{8\pi}F_{\mu\nu}F^{\mu\nu}\end{aligned} \tag{4.27}$$

where on the last line we go to the *physical basis* where the gauge bosons are canonically normalised.

In the SM we have (written in the canonical basis)

$$\begin{aligned}\mathcal{L}^0_{SM} \supset& -\frac{1}{4g_{EM}^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4g_s^2}G^a_{\mu\nu}G^{a,\mu\nu} - \sum_i \frac{m_f}{v}h\overline{\psi}_f\psi_f - \left(\frac{3m_h^2}{v^2}\right)\frac{h^4}{24}\\ \mathcal{L}^{\neq 0}_{SM} =& -\sum_i m_f\overline{\psi}_f\psi_f + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu\overline{W}^\mu - V_0 - \frac{1}{2}m_h^2h^2 - \left(\frac{3m_h^2}{v}\right)\frac{h^3}{6}\end{aligned} \tag{4.28}$$

For a dilaton much heavier than the SM we can use $b_0^{\mathrm{EM}} = 11/3, b_0^3 = -7$ so we have

$$\begin{aligned}\mathcal{L} \supset& \frac{\sigma}{f}\Bigg[-\sum_i m_f\overline{\psi}_f\psi_f + M_Z^2 Z_\mu Z^\mu + 2M_W^2 W_\mu\overline{W}^\mu - m_h^2h^2 - \left(\frac{3m_h^2}{v}\right)\frac{h^3}{6}\\ &- \frac{11\alpha_{EM}}{24\pi}F_{\mu\nu}F^{\mu\nu} + \frac{7\alpha_s}{8\pi}G^a_{\mu\nu}G^{a,\mu\nu}\Bigg].\end{aligned} \tag{4.29}$$

For a very light dilaton, we note a few things:

- For couplings to photons, we need only the QED beta function. At energies below the electron mass, we can integrate out the electrons and there will be no coupling!
- The dilaton couples to all matter, including nucleons! It therefore mediates fifth forces. See [23].
- It could be dark matter through the misalignment mechanism, similar to axions. See [26] for a very recent example. There they find dark matter consistent with all constraints with possible masses down to 10^{-10} eV.

4.4 String theory: dilaton, moduli, etc

Let us consider scale variations for gravity:

$$g_{\mu\nu} \to e^{2\alpha}, \qquad R \to e^{-2\alpha} R \tag{4.30}$$

The Einstein-Hilbert action transforms as:

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \to \frac{1}{16\pi G}\int d^4x\sqrt{-g}Re^{2\alpha}. \tag{4.31}$$

We can apply the dilaton trick to this theory too! We just multiply by χ^{-2}:

$$\begin{aligned} S_{\text{dilaton}} =& \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R(f/\chi)^2 + 8\pi G(\partial_\mu \chi)^2] \\ \propto& \int d^4x\sqrt{-g}Re^{-2\sigma/f} + ... \end{aligned} \tag{4.32}$$

Now we have introduced a scale f, and on the last line we note that the normalisation has no meaning until the dilaton aquires an expectation value.

This form is immediately familiar to string theorists, as essentially all string actions contain a dilaton. They were seen to arise because the graviton is traceless, but in string theory there is an on-shell physical degree of freedom corresponding to the trace part of the relevant vertex operators ($\partial X^\mu \overline{\partial} X_\mu e^{ik\cdot X}$ in the bosonic string). In the low-energy effective action we are obliged to include this as a scalar field.

There are also "moduli" associated with the graviton modes in higher dimensions, or equivalently deformations of the space that we compactify on. The simplest example is the "radion" if we compactify a 5-dimensional space; the metric decomposes as

$$g_{\mu\nu} = \begin{pmatrix} g^{4\text{d}}_{\mu\nu} & A_\mu \\ A_\nu & \phi \end{pmatrix} \tag{4.33}$$

If the space is a circle of radius R, then varying the vev of ϕ is equivalent to changing that radius, and if we do nothing to "stabilise" it then this field will be massless.

Typically in string theory we expect moduli to be much more massive than Standard Model fields. In supersymmetric string theories, the relevant mass scale is the supersymmetry-breaking scale, although their masses can be suppressed by this by powers of the volume in the Large Volume Scenario (see e.g. [30, 31]), it is hard to make the moduli light enough to be WISPs themselves. Such candidates have been discussed in the WISP-cost recently (see [32]).

5 Spin 2 WISPs

Recently at a Cosmic-WISPers event, the possibility of spin-2 WISPs was invoked: `https://indico.cern.ch/event/1485348/contributions/6360496/`. This is largely sparked by two works: [33] advocated searching for massive spin-2 particles via gravitational waves, and [34] pointed out that they can be searched for via their matter couplings in long-baseline atom interferometers.

The standard coupling of such a particle to matter

$$\mathcal{L} \supset \kappa_\phi \phi_{\mu\nu} \mathcal{O}^{\mu\nu}. \tag{5.1}$$

Here, $\phi_{\mu\nu}$ is the hypothetical particle, and $\mathcal{O}^{\mu\nu}$ could be the matter stress-energy tensor, or some other tensor involving matter fields. As stressed in [34], once we introduce a mass term for the new particle, there is no need for this coupling tensor to be conserved, but typically they expect it to have terms like:

$$\mathcal{O}^{\mu\nu} \supset \eta_{\mu\nu} M, \quad p_\mu p_\nu, \quad \eta_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \quad F_{\mu\alpha} F^\alpha_\nu, ...$$

So it could couple to matter, light, etc

The theoretical basis for such constructions rests on massive gravity theories, see e.g. [35, 36]. They suffer, however, theoretical objections, in that they cannot be UV complete theories and must fail at some cutoff scale, see e.g. [37,38]. The basic original argument is that the mass terms in such theories have to come along with *interactions,* which have an associated scale, and this induces a breakdown at that scale. E.g. for the Fierz-Pauli mass term

$$\mathcal{L} \supset f^4 [\phi_{\mu\nu}\phi^{\mu\nu} - (\eta^{\mu\nu}\phi_{\mu\nu})^2] \tag{5.2}$$

The problem is that the fluctuations of these massive fields contain the Goldstone bosons:

$$\phi_{\mu\nu} = \frac{h_{\mu\nu}}{M_P^2} + \partial_\mu \pi_\nu + \partial_\nu \pi_\mu - \eta_{\alpha\beta}\partial_\mu \pi^\alpha \partial_\nu \pi^\beta \tag{5.3}$$

This gives $m_g = f^2/M_P$. Furthermore, the Goldstones π_μ can be decomposed into vector and scalar modes:

$$\pi^\mu = \eta^{\mu\nu}(A_\nu + \partial_\nu \phi). \tag{5.4}$$

so considering only the scalar parts:

$$\phi_{\mu\nu} \supset 2\partial_\mu \partial_\nu \phi - \partial_\mu \partial^\alpha \phi \partial_\nu \partial_\alpha \phi. \tag{5.5}$$

and the Fierz-Pauli action contains terms such as

$$\sim (\partial^2 \phi)^3, \qquad (\partial^2\phi)^4, ... \tag{5.6}$$

which then lead to a breakdown of the theory at high enough energies. To do this properly we need to canonically normalise the field ϕ – it only gets a kinetic term proportional to f^4 from mixing with the graviton – and then we find the cutoff is $(m_g^4 M_P)^{1/5}$.

This is the basic original argument that [35, 36] managed to relax. However, recent works (e.g. [38]) argue that positivity constraints yield a cutoff proportional to the mass of the graviton.

The bottom line is that, most likely, we would still need some kind of UV completion for such theories.

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A Field theory formulation of Dilatations

Wess [39] derived expressions for conformal transformations. For dilatations he defined:

$$\begin{aligned} x'_\mu =& \lambda x_\mu \\ \phi'(x') =& \lambda^{-1}\phi(x) \end{aligned} \tag{A.1}$$

I use λ as in the original reference here so that we don't get confused with e^α at this stage.

He defines the shift in the action:

$$\delta S = \int d^4x' \frac{1}{2}(\partial'_\mu \phi')^2 - \int d^4x \frac{1}{2}(\partial_\mu \phi)^2 \tag{A.2}$$

To evaluate this, we can substitute in the transformations:

$$\begin{aligned} \int d^4x' \frac{1}{2}(\partial'_\mu \phi')^2 =& \int d^4x' \lambda^{-2} \frac{1}{2}(\partial'_\mu \phi(x))^2 \\ =& \int d^4x' \lambda^{-2} \frac{1}{2}(\partial'_\mu \phi(\lambda^{-1}x'))^2 \\ =& \int d^4x \lambda^{-2} \frac{1}{2}(\partial_\mu \phi(\lambda^{-1}x))^2 \end{aligned} \tag{A.3}$$

where on the last step we just relabel the integration variable. In this way, we just see that we have made the transformation

$$\phi \to \lambda^{-1}\phi(\lambda^{-1}x) \tag{A.4}$$

exactly as Coleman does, once we identify λ with $e^{-\alpha}$, except the transformations for x are the other way round compared to his book; I defined eq. (4.1) to agree with the expressions here. In this way, we need only make the transformation on the *fields* without transforming the measure. Of course, instead we could persist in working with x' and then we'd have to worry about shifts in the measure.

B Stress-energy tensors

The Einstein field equations are

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G/c^4 T_{\mu\nu}. \tag{B.1}$$

Recall that

$$\begin{aligned} M_P =& \sqrt{\hbar c/G} \to M_P^2 \leftrightarrow G^{-2} \\ 8\pi G \leftrightarrow& \overline{M}_P^{-2} \end{aligned} \tag{B.2}$$

where $\overline{M}_P$ is the reduced Planck mass ($\simeq 2.4 \times 10^{18}$ GeV).

The Einstein-Hilbert action is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R = \frac{\overline{M}_P^2}{2}\int d^4x\sqrt{-g}R. \tag{B.3}$$

The field equations are derived by making a variation with respect to the metric:

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \tag{B.4}$$

Now, using

$$\frac{\partial}{\partial A_{ij}}\det(A) = \mathrm{adj}(A)_{ji} = \det(A)(A^{-1})_{ji} \tag{B.5}$$

we have

$$\begin{aligned}\delta\sqrt{-g} =& -\frac{1}{2}\frac{1}{\sqrt{-g}}\delta g\\ =& -\frac{1}{2}\frac{1}{\sqrt{-g}}\frac{\delta g}{\delta g_{\mu\nu}}\delta g_{\mu\nu}\\ =& -\frac{1}{2}\frac{1}{\sqrt{-g}}(g)g^{\nu\mu}\delta g_{\mu\nu}\\ =& \frac{1}{2}\sqrt{-g}g^{\nu\mu}\delta g_{\mu\nu}.\end{aligned} \tag{B.6}$$

But

$$\begin{aligned}0 =& \delta(g_{\mu\nu}g^{\mu\nu})\\ =& g^{\mu\nu}\delta g_{\mu\nu} + g_{\mu\nu}\delta g^{\mu\nu}\end{aligned} \tag{B.7}$$

so we have (using the symmetry of $g_{\mu\nu}$)):

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu} \tag{B.8}$$

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$$\frac{\delta S_G}{\delta g^{\mu\nu}} = \frac{\overline{M}_P^2}{2}\sqrt{-g}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) \tag{B.9}$$

To get the correct Einstein field equations we therefore need

$$\frac{\delta S_M}{\delta g^{\mu\nu}} = \frac{1}{2}\sqrt{-g}T_{\mu\nu}. \tag{B.10}$$

$$\begin{aligned}S_M =& \int\sqrt{-g}[\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)]\\ T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} =& \frac{2}{\sqrt{-g}}\left[-\frac{1}{2}\sqrt{-g}g_{\mu\nu}\left[\frac{1}{2}(\partial_\alpha\phi)^2 - V(\phi)\right] + \frac{1}{2}\sqrt{-g}\partial_\mu\phi\partial_\nu\phi\right]\\ =& -g_{\mu\nu}\mathcal{L} + \partial_\mu\phi\partial_\nu\phi\end{aligned} \tag{B.11}$$

We can use

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} =& g^{\mu\nu}\partial_\nu\phi \\ \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi} =& g_{\mu\nu}\partial^\nu\phi = \partial_\nu\phi\end{aligned} \tag{B.12}$$

if we define the action as

$$S_M \equiv \int d^4x\sqrt{-g}\mathcal{L}. \tag{B.13}$$

so we have

$$\begin{aligned}T_{\mu\nu} =& -g_{\mu\nu}\mathcal{L} + \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\partial_\nu\phi \\ =& -g_{\mu\nu}\mathcal{L} + \frac{1}{2}\frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\partial_\nu\phi + \frac{1}{2}\frac{\delta\mathcal{L}}{\delta\partial^\nu\phi}\partial_\mu\phi\end{aligned} \tag{B.14}$$

If we take the trace of this we get

$$\begin{aligned}T^\mu_\mu =& \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu\phi - g^{\mu\nu}g_{\mu\nu}\mathcal{L} \\ =& \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu\phi - d\mathcal{L}\end{aligned} \tag{B.15}$$

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