

Collaborators



Philippe Brax

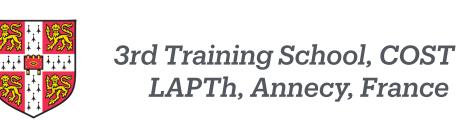


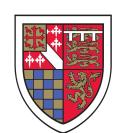
Cliff Burgess

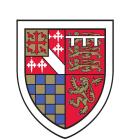


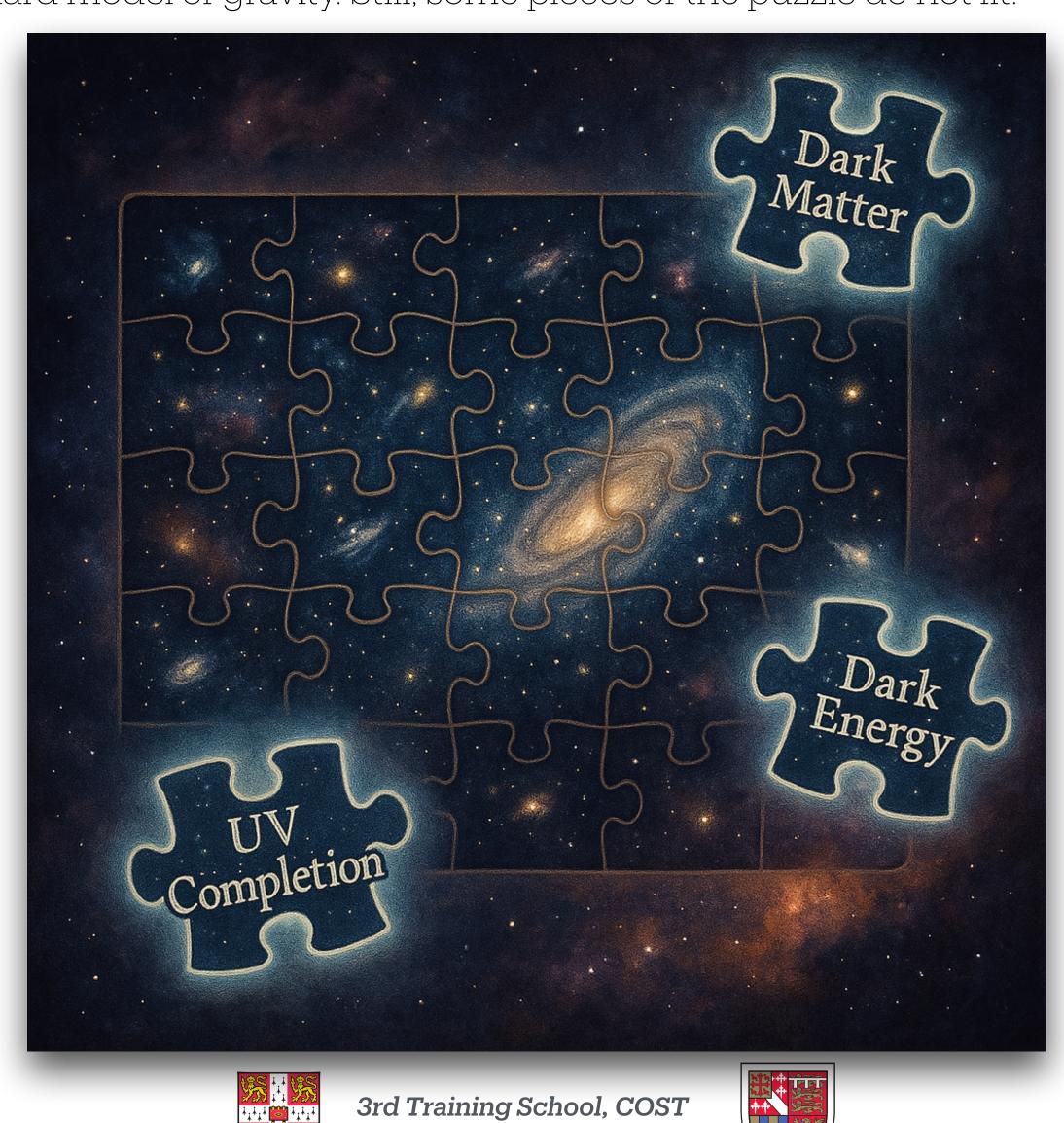
Fernando Quevedo

Sotivation

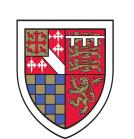


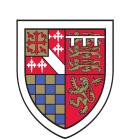






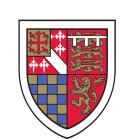






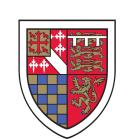
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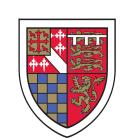
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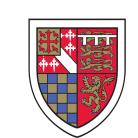


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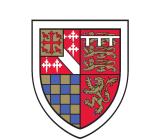
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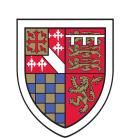
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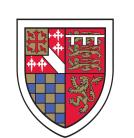
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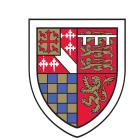
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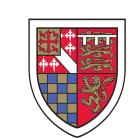
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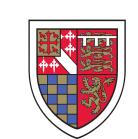
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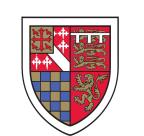
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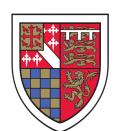
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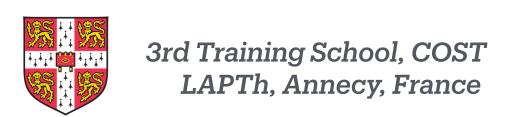
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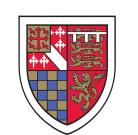
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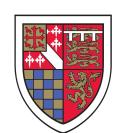
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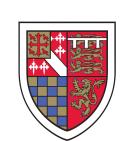


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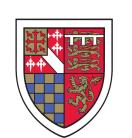
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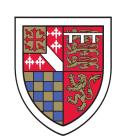


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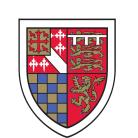
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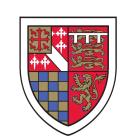
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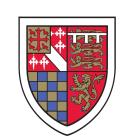
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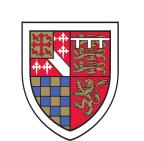


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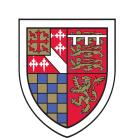
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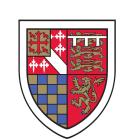
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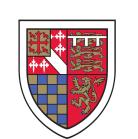
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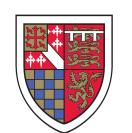
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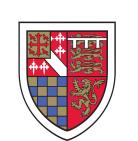
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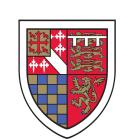
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New two-derivative interactions (not suppressed) appear when considering multi-scalars.



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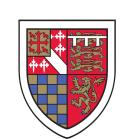
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$$\mathcal{G}_{ab}(\phi)\partial_{\mu}\phi^{a}\partial^{\mu}\phi^{b} = \partial_{\mu}\varphi\partial^{\mu}\varphi + W^{2}(\varphi)\partial_{\mu}\mathfrak{a}\partial^{\mu}\mathfrak{a},$$

where φ and α are pseudo-Goldstone bosons of and approximate spacetime scaling and internal shift symmetry, respectively. [C.P. Burgess, M. Cicoli, D. Ciupke, S. Krippendorf and F. Quevedo, 2020]

New two-derivative interactions (not suppressed) appear when considering multi-scalars.



Many proposals have been made to effectively screen the scalar coupling to matter:

• Cosmological evolution of the BD field

[T. Damour and A. M. Polyakov, 1994]

Chameleon mechanism

[J. Khoury and A. Weltman 2004]

Other mechanisms...

[C. Burrage and J. Sakstein, 2018]

These screening mechanisms usually assume fairly specific couplings and do not apply to scale-invariant scalar (dilaton).

[C.P. Burgess and F. Quevedo, 2022]

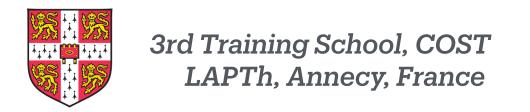
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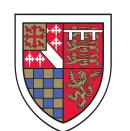
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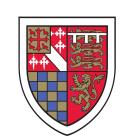
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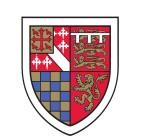
What is the effect of this coupling to matter in strong gravity environments?



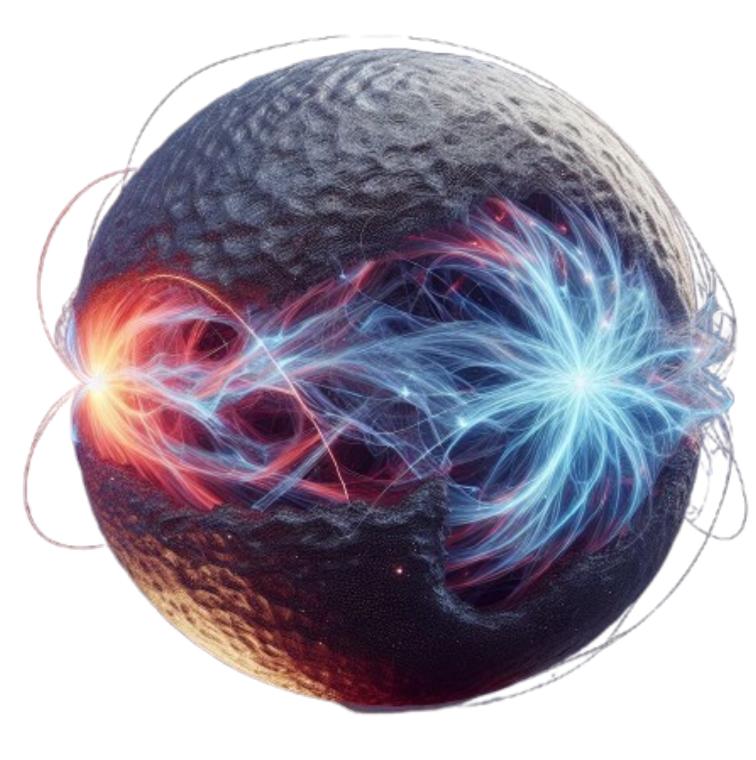




$$\mathcal{L} = -\sqrt{-g} \left[\frac{M_p^2}{2} \mathcal{R} + \frac{M_p^2}{2} (\partial \varphi)^2 + \frac{M_p^2}{2} W^2(\varphi)(\partial \mathfrak{a})^2 + \mathcal{V}(\mathfrak{a}) \right] + \mathcal{L}_m(\psi, \varphi, \mathfrak{a}, \tilde{g}_{\mu\nu}).$$



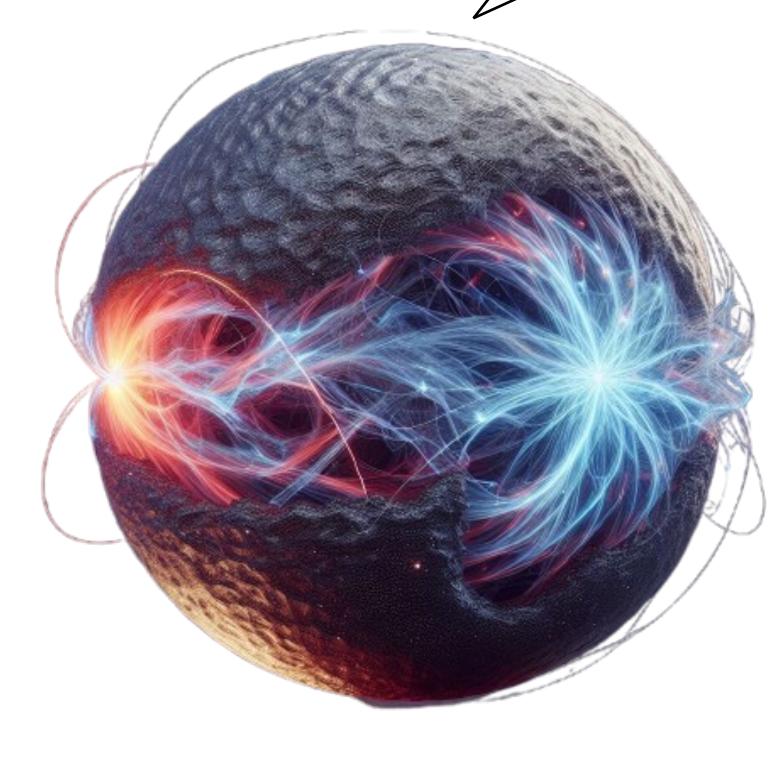
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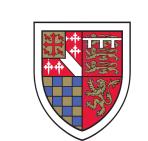




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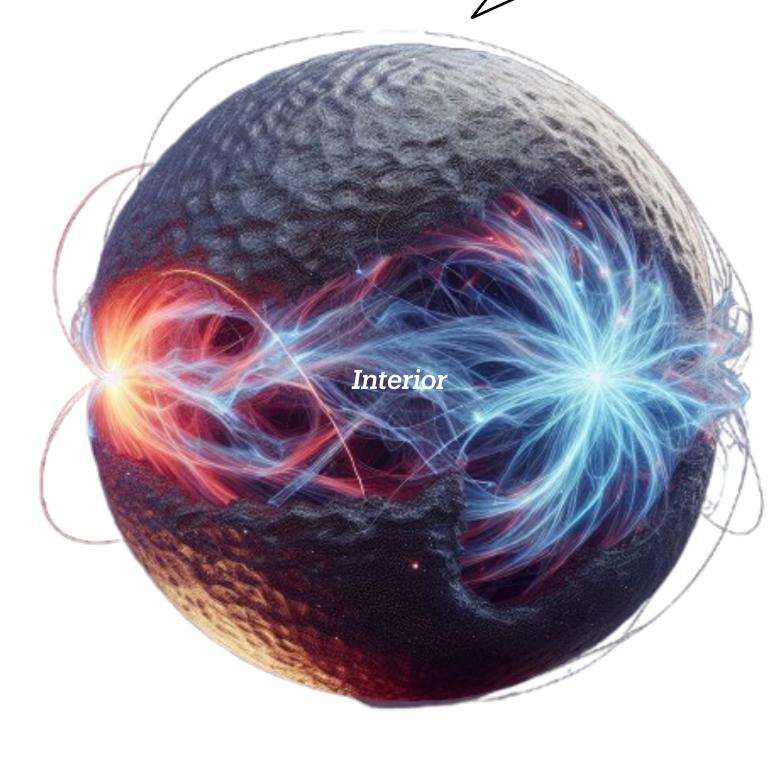
Hello world, I am a neutron star.

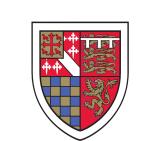




$$\mathcal{Z} = -\sqrt{-g} \left[\frac{M_p^2}{2} \mathcal{R} + \frac{M_p^2}{2} (\partial \varphi)^2 + \frac{M_p^2}{2} W^2(\varphi)(\partial \mathfrak{a})^2 + \mathcal{V}(\mathfrak{a}) \right] + \mathcal{L}_m(\psi, \varphi, \mathfrak{a}, \tilde{g}_{\mu\nu}).$$

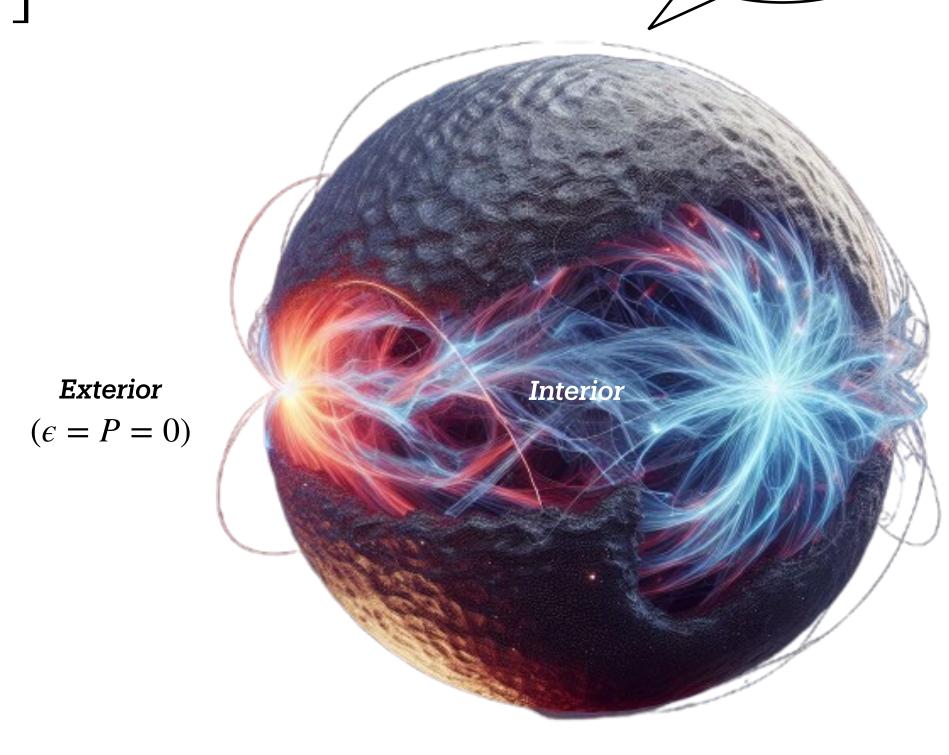
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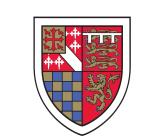


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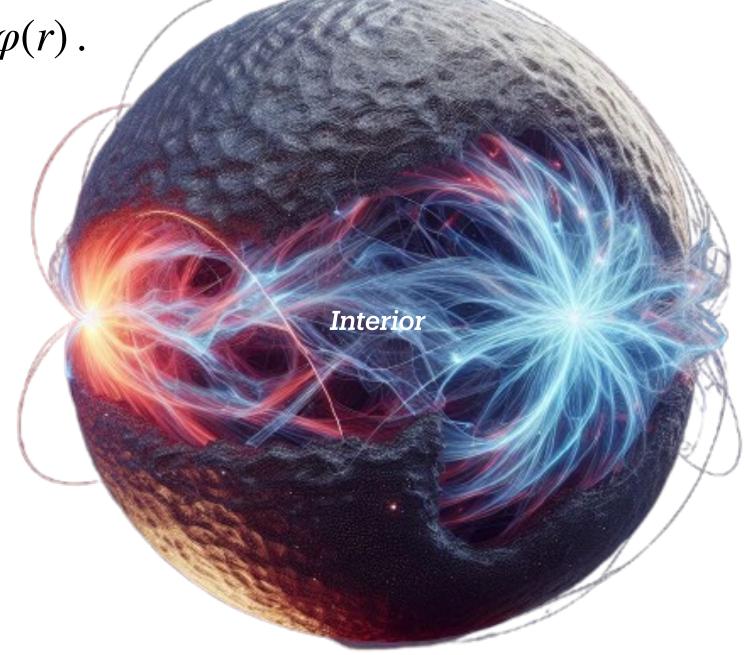




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Imposing spherical symmetry: $ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega^2$, and also $\mathfrak{a} = \mathfrak{a}(r)$ and $\varphi = \varphi(r)$.

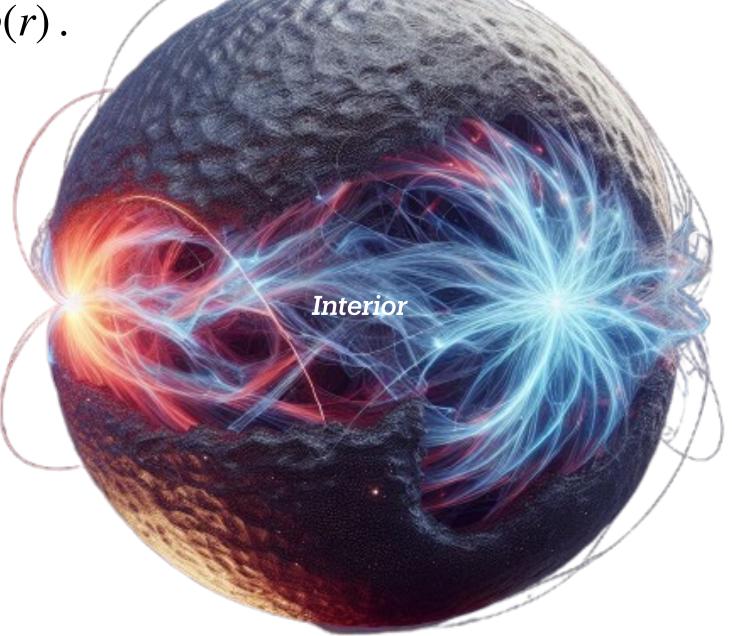


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The potential $\mathcal{V}(\mathfrak{a})$ can be considered as the contribution of two regions



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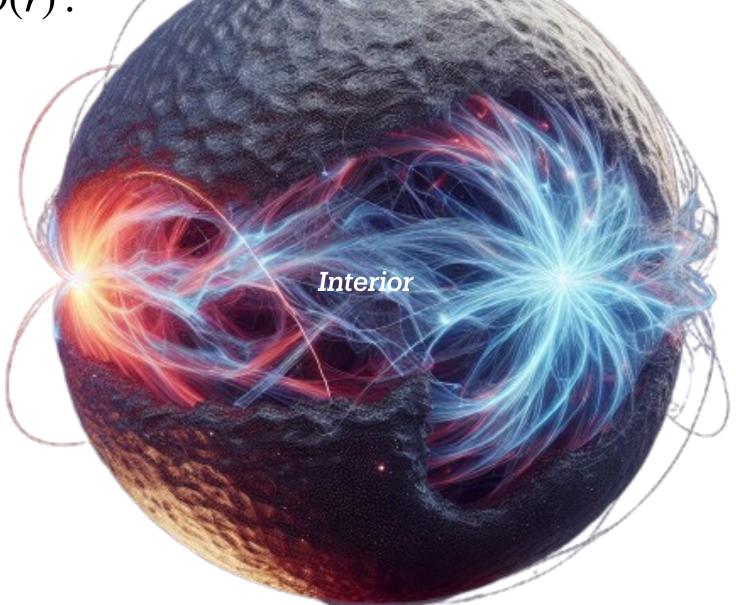
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, for the exterior,

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$$U = \frac{1}{2} \mu_{in}^2 M_p^2 [\mathfrak{a}(r) - \mathfrak{a}_0(r)]^2(r) \epsilon(r)$$
, for the interior,





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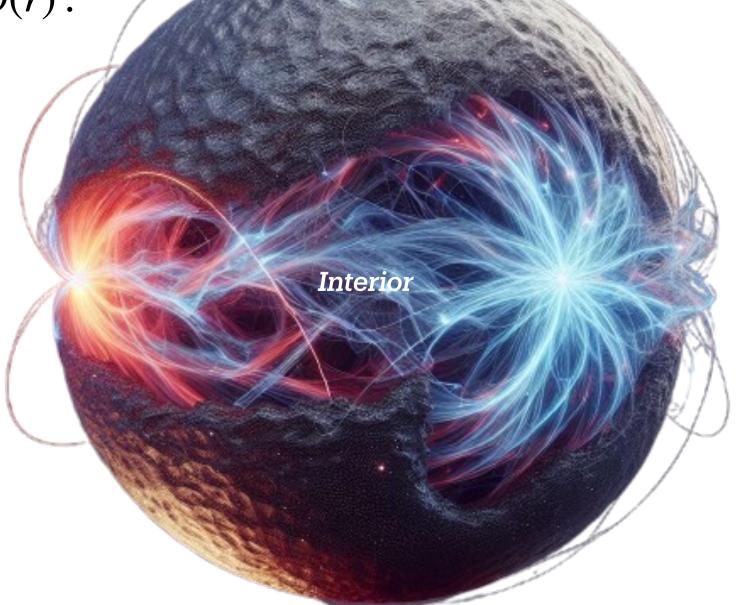
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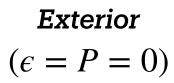
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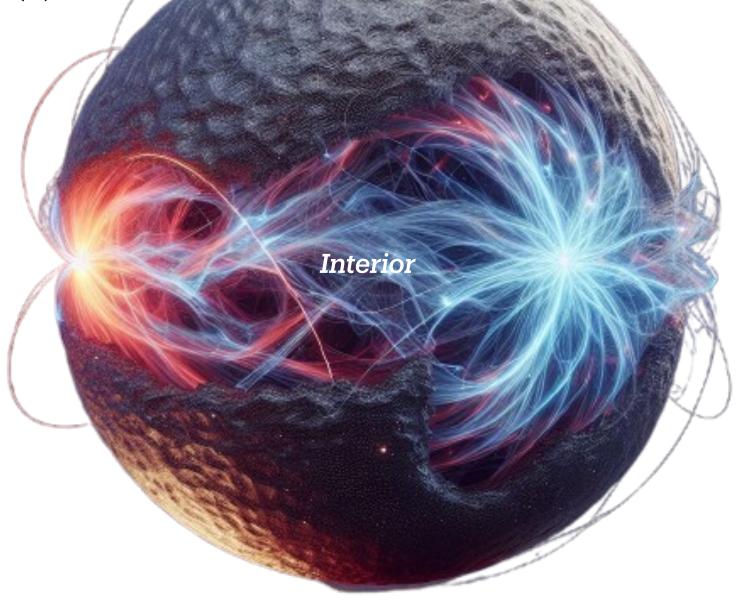
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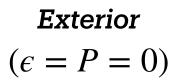
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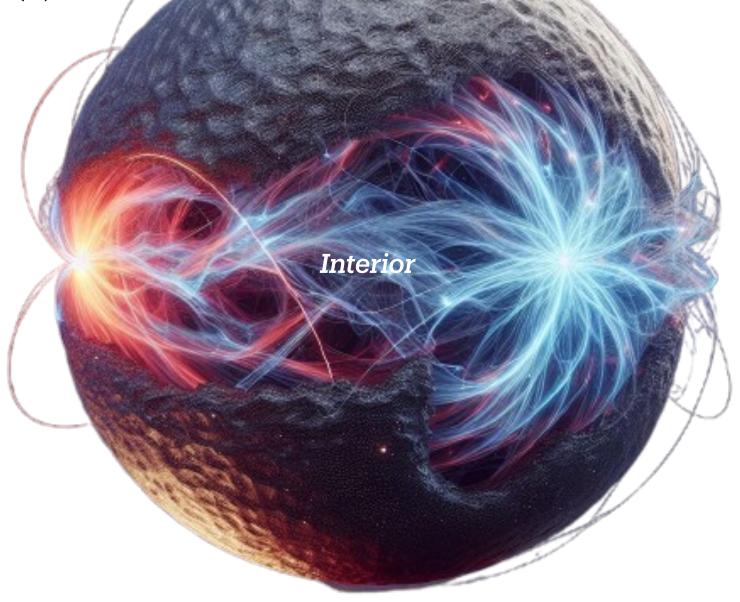
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Exterior $(\epsilon = P = 0)$

$$\varphi'' + \frac{2}{r}\varphi' - WW_{\varphi}e^{-\nu}\mathfrak{a}'^2 = -\frac{\mathfrak{g}T}{M_p^2} \quad \text{and} \quad \mathfrak{a}'' + \frac{2}{r}\mathfrak{a}' + 2\frac{W_{\varphi}}{W}e^{-\nu}\mathfrak{a}'\varphi' = \frac{1}{W^2M_p^2}(V_a + U_a),$$





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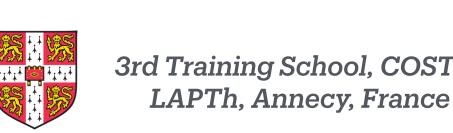
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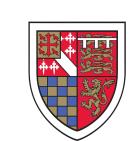
Exterior $(\epsilon = P = 0)$

with $\mu_{in/out}M_p=m_{in/out}f$, with f the axion decay constant. Therefore,

$$\varphi'' + \frac{2}{r}\varphi' - WW_{\varphi}e^{-\nu}\mathfrak{a}'^2 = -\frac{\mathfrak{g}T}{M_p^2} \quad \text{and} \quad \mathfrak{a}'' + \frac{2}{r}\mathfrak{a}' + 2\frac{W_{\varphi}}{W}e^{-\nu}\mathfrak{a}'\varphi' = \frac{1}{W^2M_p^2}(V_a + U_a),$$

where $T\simeq T_{\mu\nu}g^{\mu\nu}$ which in the non-relativistic limit can be approximated as $T\simeq -\epsilon$.





$$\frac{d}{dr} \begin{bmatrix} y_{0}(r) \\ y_{1}(r) \\ y_{2}(r) \\ y_{3}(r) \\ y_{4}(r) \\ y_{5}(r) \\ y_{6}(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \nu(r) \\ \varphi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \nu(r) \\ \varphi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \nu(r) \\ \varphi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \nu(r) \\ \varphi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \nu(r) \\ \varphi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ m(r) \\ \psi(r) \\ a(r) \\ \chi(r) \end{bmatrix} = \frac{d}{dr} \begin{bmatrix} \tilde{P}(r) \\ \tilde{$$



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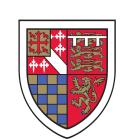
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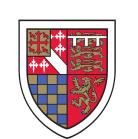
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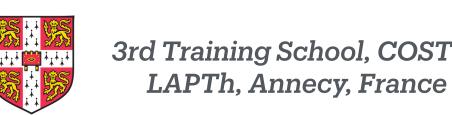
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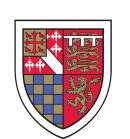
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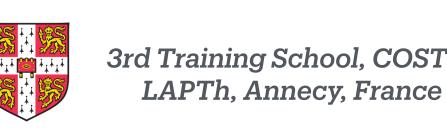
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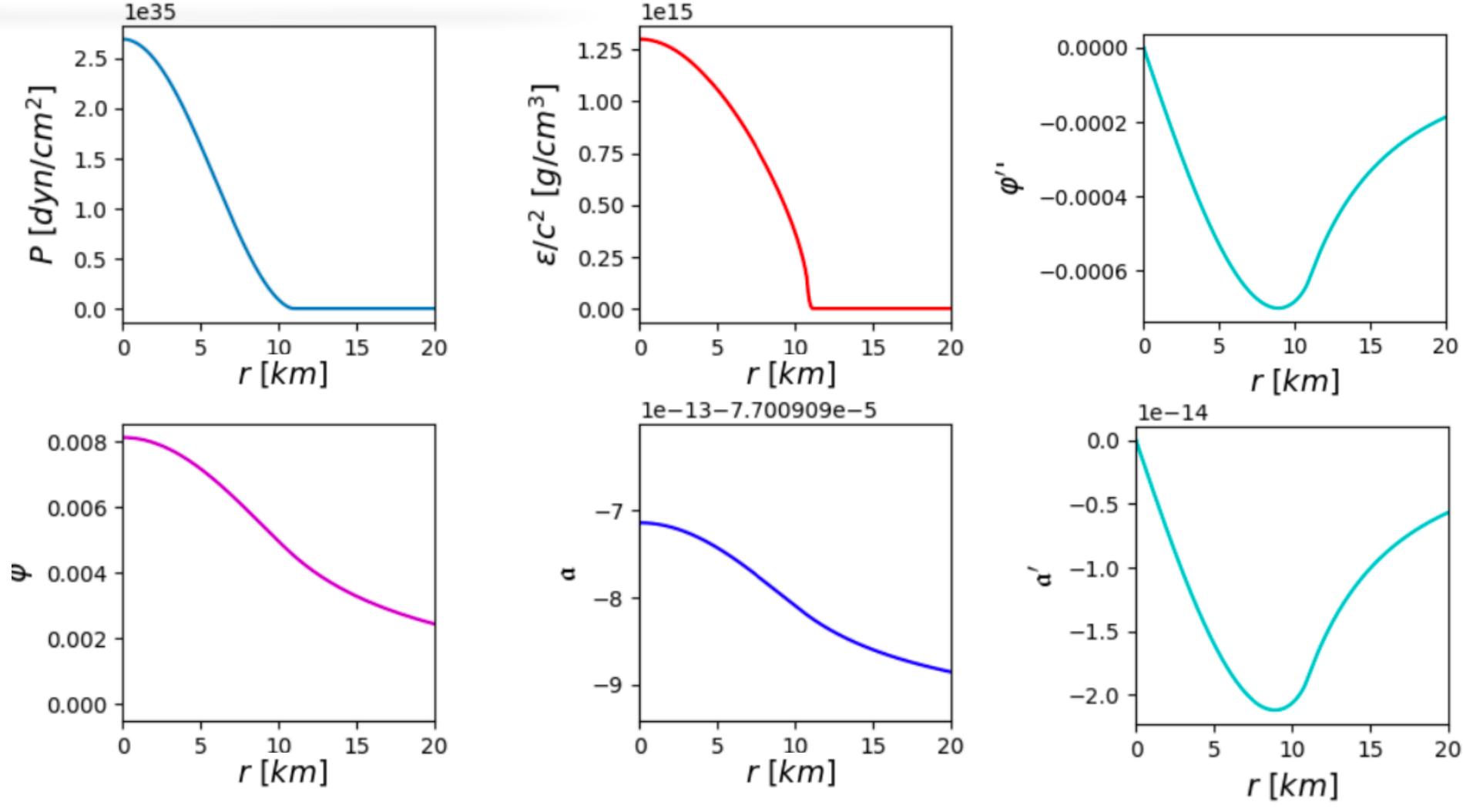
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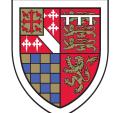
Numerical solutions

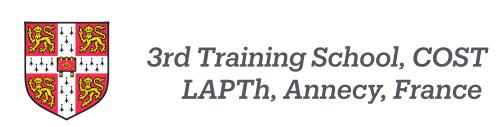


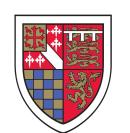
In this figure, we use the following values for the parameters:

$$g = 0.01, \quad \xi = 1, \quad m_{in} \gg m_{\text{Out}} = 10^{-9} eV \longrightarrow R = 11.4 km.$$

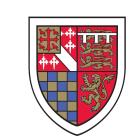




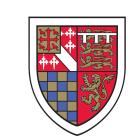




In the axio-dilaton scenario:

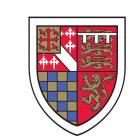


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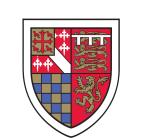
$$\mathscr{L} = -\sqrt{-g} \left[\frac{M_p^2}{2} \mathscr{R} + \frac{M_p^2}{2} (\partial \varphi)^2 + \frac{M_p^2}{2} W^2(\varphi)(\partial \mathfrak{a})^2 + \mathscr{V}(\mathfrak{a}) \right] + \mathscr{L}_m(\psi, \varphi, \mathfrak{a}, \tilde{g}_{\mu\nu}).$$



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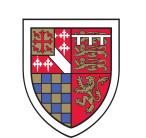
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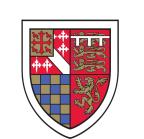
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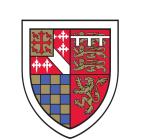
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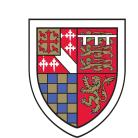


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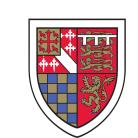


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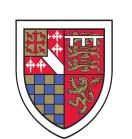
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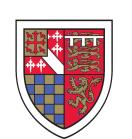
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Where the matter interaction is $\mathcal{L}_m \propto -\epsilon(r)a^2$, therefore, the axion experiences a different minimum inside and outside matter.

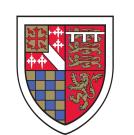
Notice that in the absence of an axion gradient, the scalar charge can be approximated at the surface by

$$\varphi'_{NG} \simeq \frac{2\mathfrak{g}GM}{R^2} \equiv \frac{L_0}{R^2},$$

and then, the effective screening is measured by the ratio

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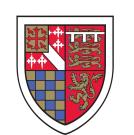
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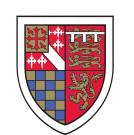
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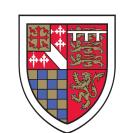
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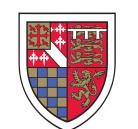
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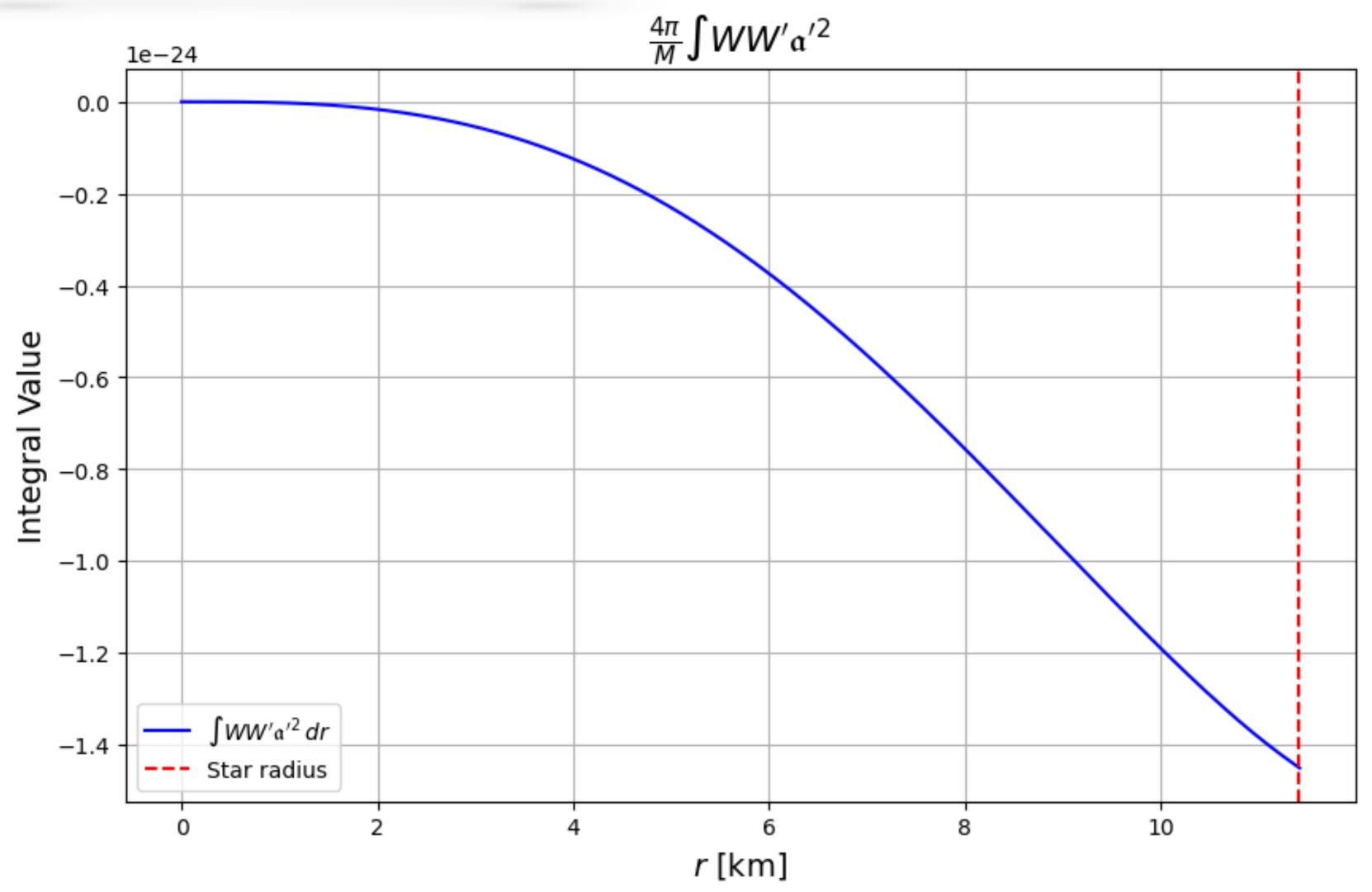
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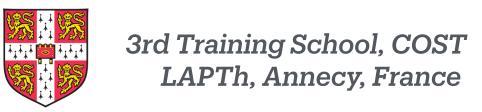
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Measuring the screening

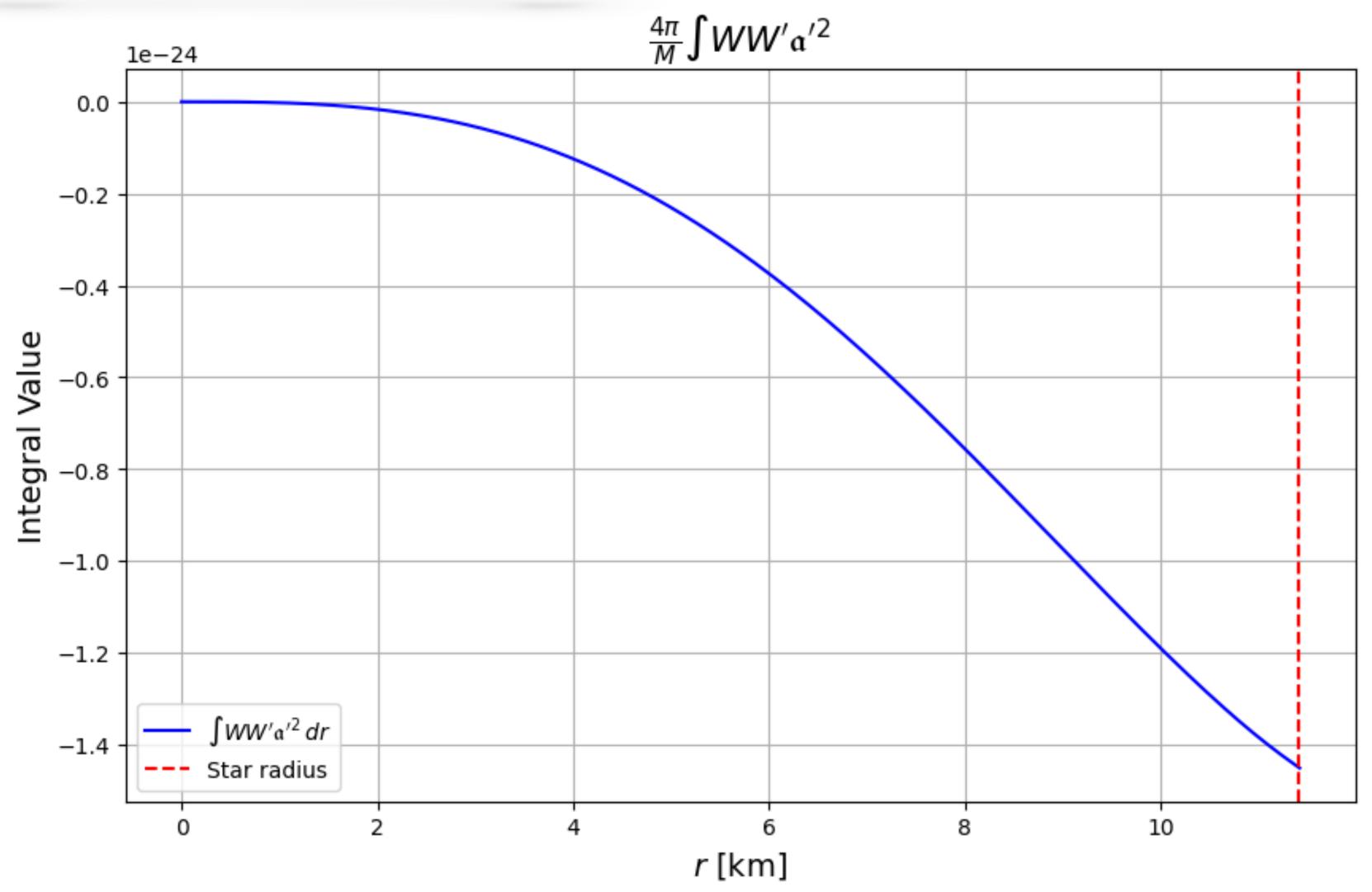


Contribution to the screening when $\xi \sim 1$.

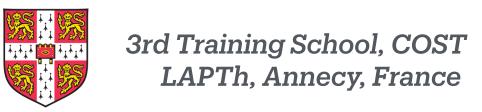




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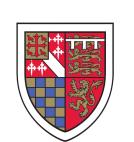
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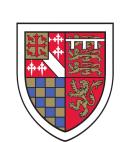




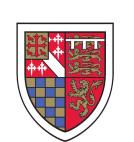
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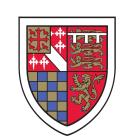


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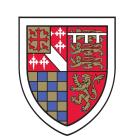
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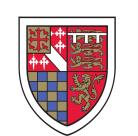
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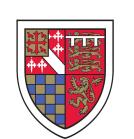


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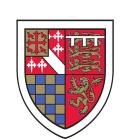
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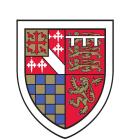
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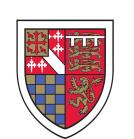
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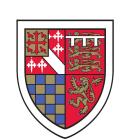
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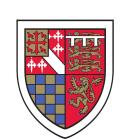
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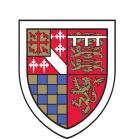
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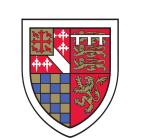


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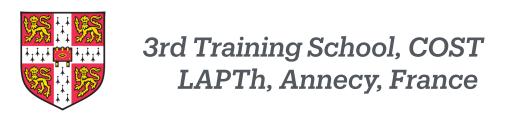
[P. Brax, , C. Burgess and F. Quevedo, 2023]

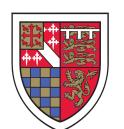


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[P. Brax, , C. Burgess and F. Quevedo, 2023]

• Further studies of multiple scalars interacting through two-derivative sigma-model couplings.







Compact objects

They are the endpoint of stellar evolution and an astrophysical laboratory. Different mechanisms support them against gravitational

collapse:

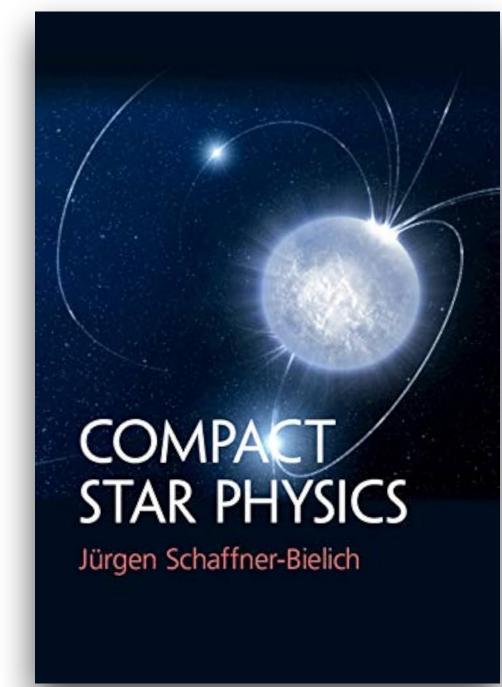
- White dwarfs: supported by the degeneracy pressure of electrons.
- Neutron stars (NS): interaction between nucleons.
- Black holes: completely collapsed, gravitational singularity.

More information on radii and masses available for NS:

- Original massive star containing 8-25 M_{\odot} before supernova explosion.
- Theoretically, remnant NS compresses $1.4 M_{\odot}$ into a $10-15 \mathrm{km}$ sphere.
- Experimentally, (Satellite mission NICER) measured PSR J0030 + 0451 radius:

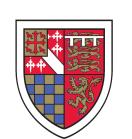
$$12.71 \pm 1.19 \,\mathrm{km}$$

 $13.02 \pm 1.24 \,\mathrm{km}$



[J. Schaffner-Bielich , 2020]

[Riley, T. E. et al , 2019] [Miller, M. C. et al , 2019]



Equation of state

We consider a piecewise polytropic equation of state:

$$P(\rho) = K_i \rho^{\Gamma_i},$$

[F. Douchin and P. Haensel,, 2001]

where ρ is the rest-mass density, K_i are some constants and Γ_i adiabatic indices. In addition, $\rho_1=10^{14.7}$ g/cm³ and $\rho_2=10^{15.0}$ g/cm³, such that for each region of the piecewise density

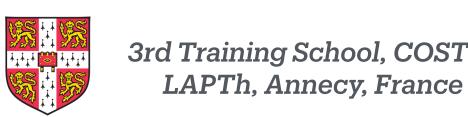
$$\rho_{i-1} \le \rho \le \rho_i.$$

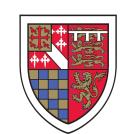
The energy density ϵ of this adiabatic ideal gas follows the first law of thermodynamics

$$d\left(\frac{\epsilon}{\rho}\right) = -P d\left(\frac{1}{\rho}\right),$$

which implies by continuity,

$$\epsilon = (1 + \alpha_i)\rho + \frac{K_i}{\Gamma_i - 1}\rho^{\Gamma_i}$$
, with $\alpha_i = \frac{\epsilon(\rho_{i-1})}{\rho_{i-1}} - 1 - \frac{\Gamma_i}{\Gamma_i - 1}K_i\rho^{\Gamma_i - 1}$.







Einstein equations

We can write variables in the JF, particularly pressure and density as

$$P = A^4(\tau)\tilde{P}$$
 and $\epsilon = A^4(\tau)\tilde{\epsilon}$.

Defining $\mu(r) = m(r)/r$, the independent Einstein equations are given by

$$r\mu' + \mu = 4\pi G r^2 (A^4 \tilde{\epsilon} - V) - \frac{r^2}{4} (1 - 2\mu)(\varphi'^2 + W^2 \mathfrak{a}'^2),$$

$$\nu' = \frac{8\pi G r^2 (A^4 \tilde{P} + V) + 2\mu}{r(1 - 2\mu)} - \frac{r}{2} (\varphi'^2 + W^2 \mathfrak{a}'^2),$$

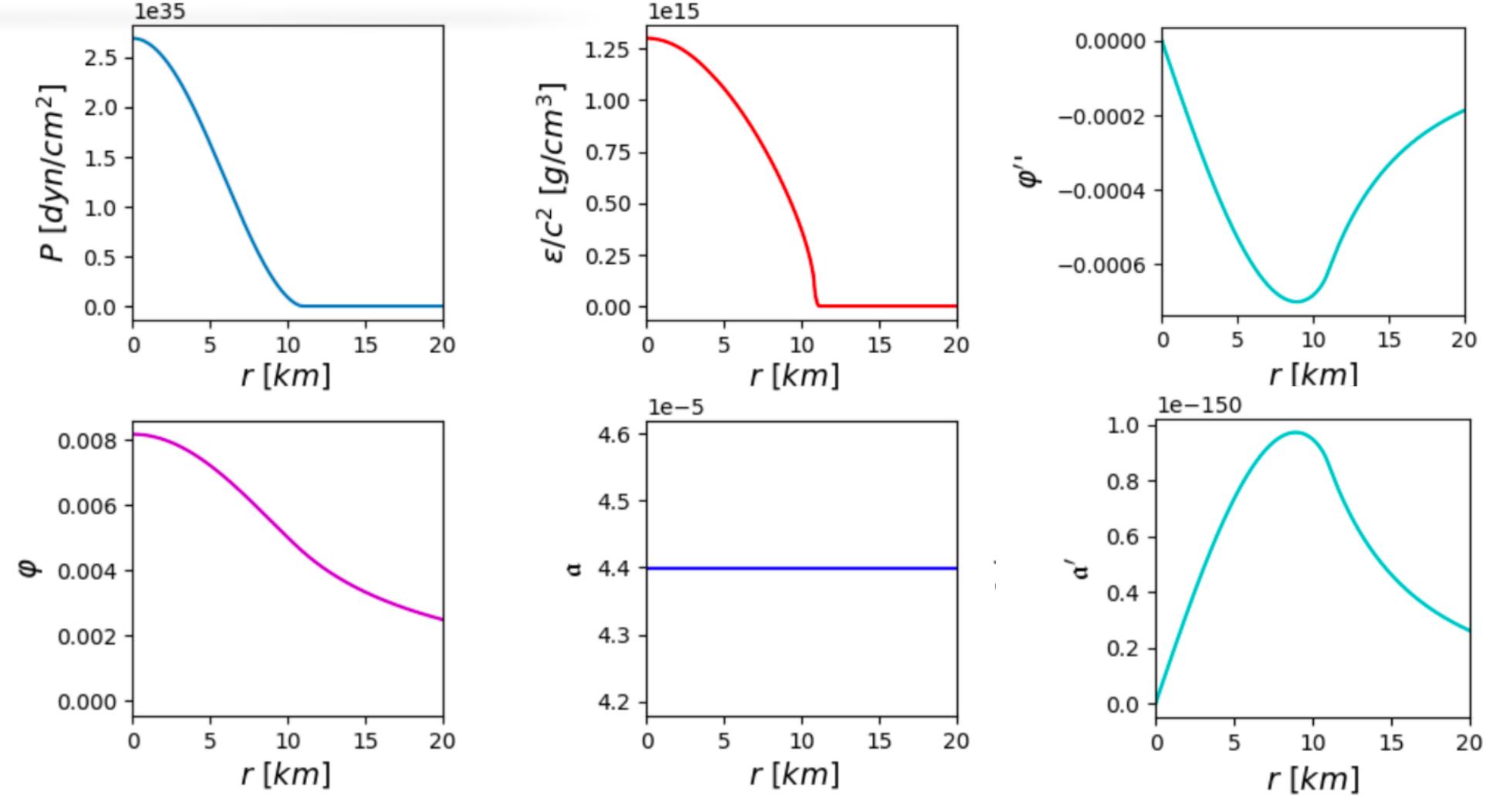
$$\frac{\nu''}{2} + \frac{(\nu')^2}{4} - \frac{\nu'\mu'}{2(1-2\mu)} + \frac{1}{2r} \left(\nu' - \frac{2\mu'}{1-2\mu} \right) = \frac{8\pi G(A^4\tilde{P} + \mathcal{V})}{(1-2\mu)} + \frac{1}{2} (\varphi'^2 + W^2 \alpha'^2),$$

And additionally, the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\tilde{P}' = -(\tilde{\epsilon} + \tilde{P}) \left[\frac{4\pi G r^2 (A^4 \tilde{P} + \mathcal{V}) + \mu}{r(1 - 2\mu)} - \frac{r}{4} (\varphi'^2 + W^2 \mathfrak{a}'^2) + \mathfrak{g} \varphi' \right].$$



Numerical solutions



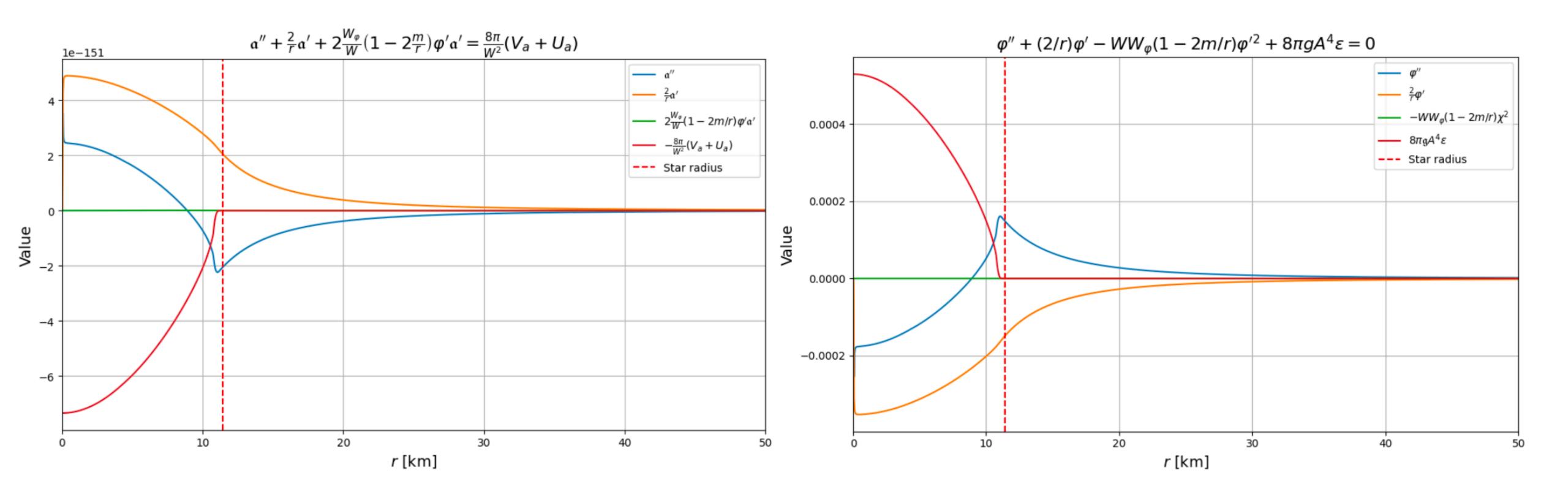
In this figure, we use the following values for the parameters:

$$\mathfrak{g}=0.01,\quad \xi=1,\quad m_{in}=10^{-6}eV\quad \text{and}\quad m_{\mathrm{Out}}=10^{-9}eV\quad \longrightarrow \quad R=11.4km\,.$$

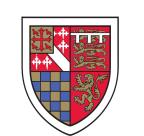




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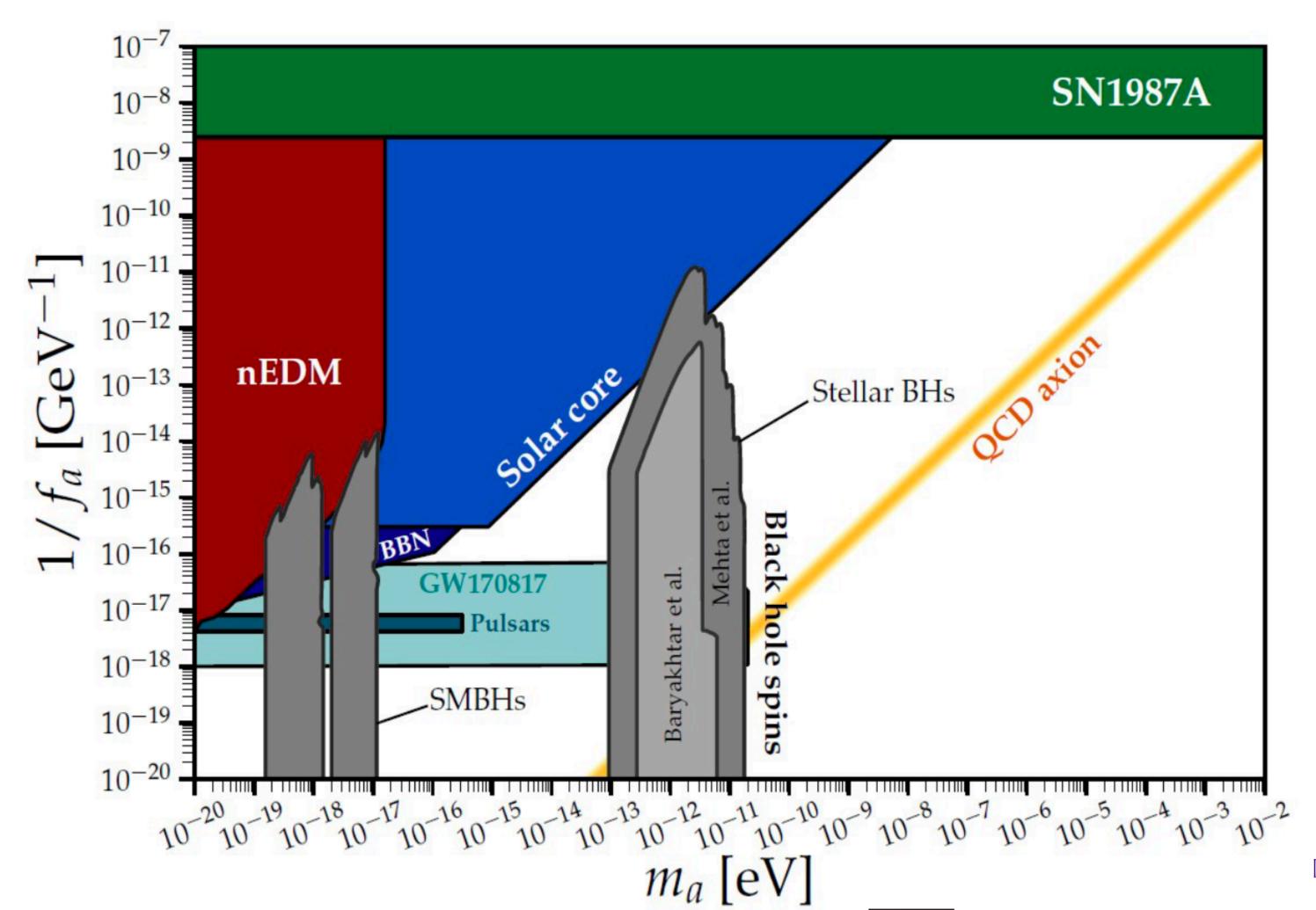
Evolution of the axio-dilaton system. The equations are satisfied along the whole radial coordinate.



Phenomenology

Phenomenological values would require $\xi > 10^9$ which requires $f \leq 10^9$ GeV.

[P. Brax, , C. Burgess and F. Quevedo, 2023]



3rd Training School, COST

LAPTh, Annecy, France

[P.A. Zyla et al. (Particle Data Group), 2020]

Adiabatic approximation

Consider the following potential for the axion field:

$$V(\mathfrak{a}) = \frac{1}{2} \mu_{\text{out}}^2 M_p^2 (\mathfrak{a} - \mathfrak{a}_+)^2 \quad \text{and} \quad U(a) = \frac{1}{2} \mu_{\text{in}}^2 M_p^2 (\mathfrak{a} - \mathfrak{a}_-)^2 \epsilon(r),$$

where $\mu_{in/out}M_p=m_{in/out}f=m_{\mathfrak{a}}M_PW(\varphi)$. Extremizing the effective potential with respect to the axion field,

$$\frac{d(V(a) + U(a))}{da} = 0,$$

turns out into the axion solution

$$\alpha(r) = \frac{\mu_{\text{Out}}^2 \alpha_+ + \mu_{\text{in}}^2 \epsilon(r) \alpha_-}{\mu_{\text{Out}}^2 + \mu_{\text{in}}^2 \epsilon(r)}.$$

The **adiabatic limit** inside the source implies, in particular, $m_{in}R \gg 1$, that is, the Compton wavelength is way smaller than the star's radius.

The axion profile adiabatically follows the minimum of the local potential



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