Domain wall evolution beyond quartic potentials

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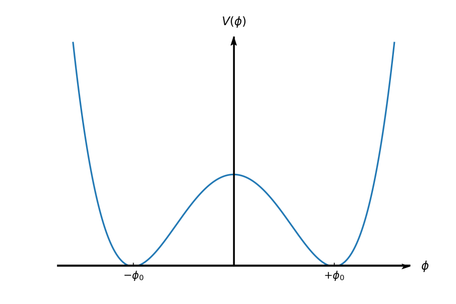
If we observe Topological Defects:
We obtain a direct link to the early Universe
What kind of defects form: depends on the underlying theory

The Standard Case: The Quartic Potential

$$V(\phi) = V_0 \left(1 - \frac{\phi^2}{\phi_0^2}\right)^2$$

- numerically simple
- but: not fully representative of the range of plausible realistic particle physics models
- for $\phi_0 = 1$ and $\omega_o = 5$: $V(\phi) = \frac{\pi^2}{50}\left(1 - \phi^2\right)^2$



Numerical Implementation of Domain Wall Evolution

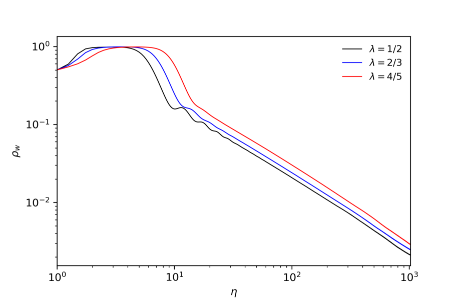
We consider a model with a real scalar field, with a Lagrangian

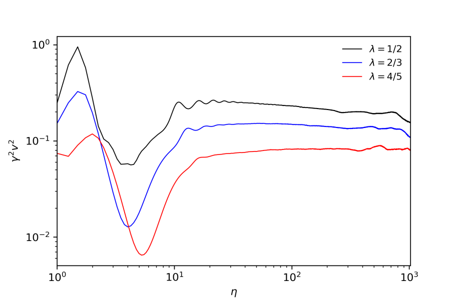
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{1}$$

The simplest field theory numerical implementation relies on the method developed by Press, Ryden & Spergel (1989) with the general evolution equation:

$$\frac{\partial^2\phi}{\partial\eta^2} + \alpha\left(\frac{d\ln a}{d\ln\eta}\right)\frac{\partial\phi}{\partial\eta} - \nabla^2\phi = -a^\beta\frac{\partial V}{\partial\phi} \tag{2}$$

where in the standard case $\alpha = \beta = 2$, but $\alpha = 3$ and $\beta = 0$ is used here since it leads to the same dynamics (in good approximation), with the numerical benefit of maintaining a constant comoving wall thickness.





$$\rho_\omega = \frac{A}{V}$$

$$\rho_\omega \propto \eta^\mu$$

expectation: $\mu = -1$

$$\langle \gamma^2 v^2 \rangle = \sum \frac{\dot{\phi}^2}{2V(\phi)}$$

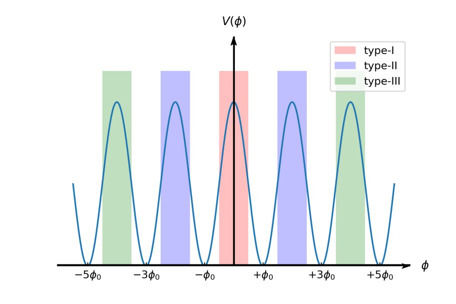
$$\gamma v \propto \eta^\nu$$

expectation: $\nu = 0$

$$V(\phi) = V_{0,\mathrm{SG}}\left[1 + \cos\left(\pi\frac{\phi}{\phi_0}\right)\right]$$

Plugging in values for ϕ_0 and ω_0:

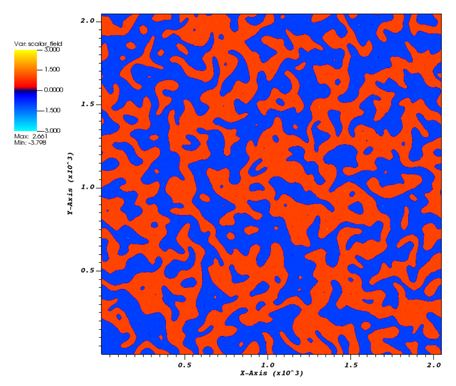
$$V(\phi) = \frac{4}{25}\left[1 + \cos\left(\pi\phi\right)\right]$$



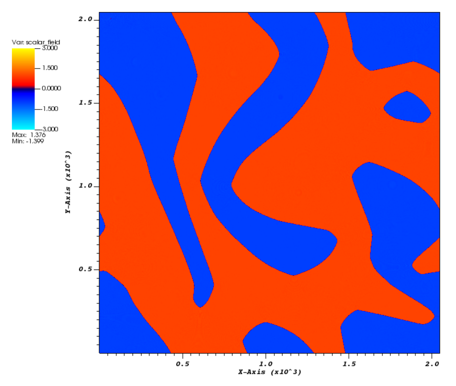
Snapshots in a 2048^2 grid for $\lambda = 1/2$

with initial distribution of ϕ uniformly distributed between $\pm\phi_0$

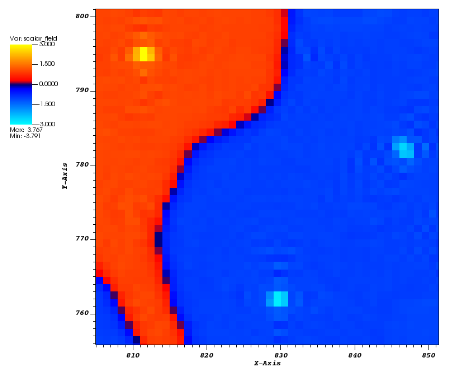
$\eta = 101$



$\eta = 751$



Zoom-in snapshot at $\eta = 26$



Scaling for Quartic Potential in 2048^2 grid

λ	μ	ν	$(\rho_w \eta)^{-1}$	γv
$1/2$	-0.938 ± 0.030	-0.058 ± 0.020	0.49 ± 0.10	0.39 ± 0.03
$2/3$	-0.942 ± 0.025	-0.034 ± 0.029	0.41 ± 0.08	0.33 ± 0.02
$4/5$	-0.949 ± 0.020	-0.004 ± 0.040	0.34 ± 0.05	0.28 ± 0.04

Scaling for Sine-Gordon Potential

box size	λ	μ	ν	$(\rho_w \eta)^{-1}$	γv
2048^2	$1/2$	-0.842 ± 0.018	-0.217 ± 0.025	0.42 ± 0.04	0.29 ± 0.03
2048^2	$2/3$	-0.853 ± 0.016	-0.184 ± 0.016	0.38 ± 0.03	0.27 ± 0.01
2048^2	$4/5$	-0.873 ± 0.017	-0.143 ± 0.021	0.33 ± 0.03	0.24 ± 0.02
8192^2	$1/2$	-0.856 ± 0.017	-0.190 ± 0.021	0.43 ± 0.02	0.25 ± 0.02
8192^2	$2/3$	-0.872 ± 0.016	-0.177 ± 0.022	0.40 ± 0.04	0.23 ± 0.01
8192^2	$4/5$	-0.891 ± 0.014	-0.139 ± 0.017	0.36 ± 0.03	0.22 ± 0.01
32768^2	$1/2$	-0.851 ± 0.002	-0.164 ± 0.008	0.37 ± 0.04	0.21 ± 0.01
32768^2	$2/3$	-0.852 ± 0.002	-0.139 ± 0.008	0.36 ± 0.03	0.20 ± 0.01
32768^2	$4/5$	-0.854 ± 0.002	-0.115 ± 0.010	0.32 ± 0.02	0.18 ± 0.01

- μ larger and ν smaller in Type-I walls of Sine-Gordon case compared to quartic potential

Conclusions

- canonical quartic potential analysis of domain wall dynamics can be extended
- potential symmetry, initial conditions, and cosmological expansion rates can have a significant impact on formation and evolution of networks
- these mechanisms can be incorporated in current analytic models for the evolution of such networks