On probing self interacting dark matter models through the absorption of gravitational waves

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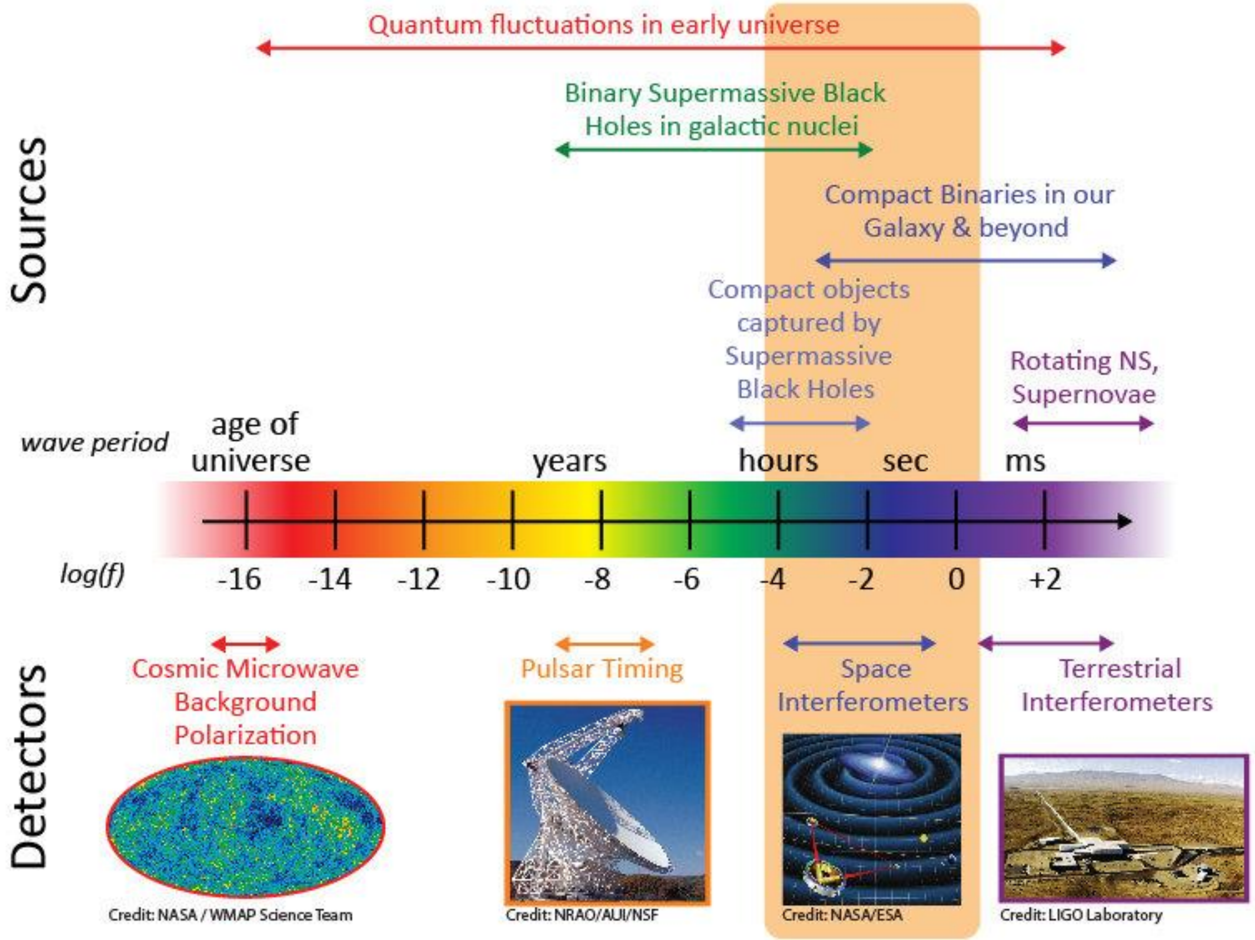
1. Introduction

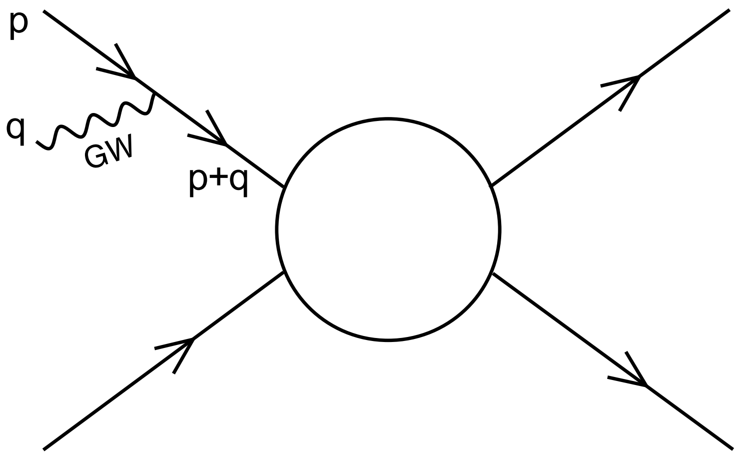
The dawn of GWs cosmology

- Gravitational waves are **messengers** of remote times

- Gravitational waves are **sensors of their medium of propagation**



Absorption of GWs in scattering processes

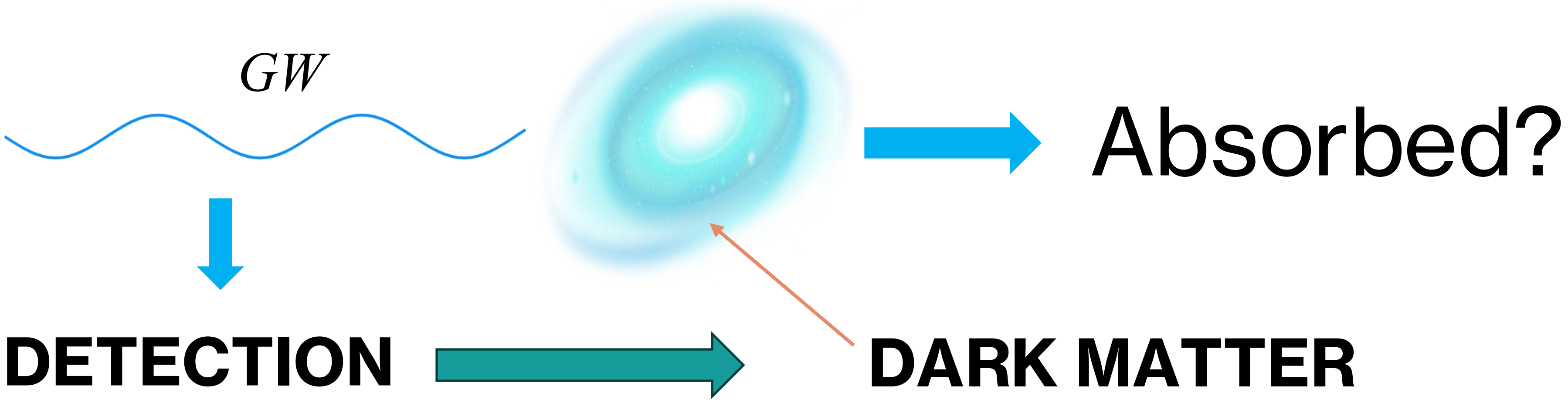


Net rate of absorption of a graviton in a non-relativistic 2-2 scattering:

$$\Gamma_{\text{net abs.}} = \frac{G\mu^2}{5\pi^2\hbar c^2 f^3} n_1 n_2 \overline{v^5 \sigma_D} \left[\frac{2\pi\hbar f}{kT}\right]$$

as long as $f >> f_\text{p}$

Direct detection: Probing dark matter



Optical depth of the line of sight: $$\tau = \int_0^D \frac{\Gamma_{\text{net abs}}(s)}{c} \mathrm{d}s$$

$\tau \gg 1$ → Opaque medium of propagation. High absorption

$\tau \ll 1$ → Transparent medium of propagation. Low absorption

2. Self-interacting dark matter

The flaws of the cold dark matter paradigm

- CDM fails to explain observational data on scales of 1–10 kpc. An interesting possibility is that DM carries self-interactions:

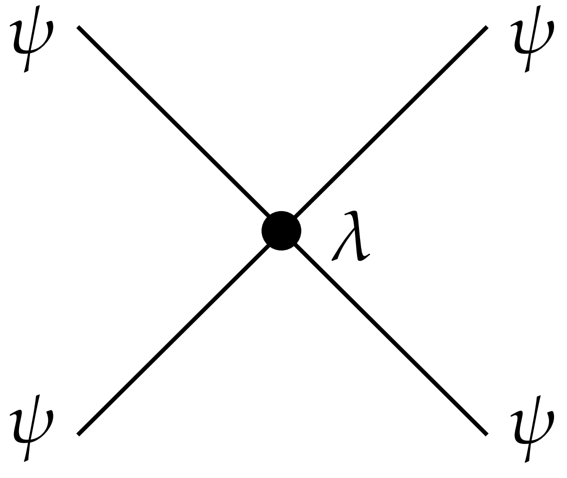
  - On the scale of ~1–10 kpc $(v \sim 10 - 10^2 \text{ km/s}),\ \sigma/m_{DM} \sim 1\text{cm}^2/\text{g}$

  - On cluster scales ~ Mpc $(v \sim 10^3 \text{ km/s}),\ \sigma/m_{DM} \sim 0.1 \text{ cm}^2/\text{g}$

Absorption of GWs in DM self-scattering

Ultra-light scalar field with quartic interaction

$$\mathcal{L}_{\rm int} = -\frac{\lambda}{4!}\psi^4 \to \frac{d\sigma}{d\Omega} = \frac{\lambda^2}{256\pi^2 m_{ul}^2}$$



Thermal average:

$$\overline{v^5\sigma_D} = \frac{\hbar^2 c^2}{\sqrt{2}\pi^{3/2} m_{ul}^2 c^4}\left(\frac{kT}{m_{ul}c^2}\right)^{5/2} c^5$$

Gives us:

$$\Gamma^{ul}_{\rm n.a.} = 1.13\cdot 10^{-27}\frac{1}{\rm s}\frac{\lambda^2[n_a({\rm cm}^{-3})]^2}{[f({\rm Hz})]^3}\left(\frac{kT}{m_{ul}c^2}\right)^{5/2}\left[\frac{2\pi\hbar f}{kT}\right]$$

Sources of absorption in DM self-scattering

Optical depth of a DM halo

$$\tau = \int_0^D \frac{\Gamma_{\text{net abs}}(s)}{c} \mathrm{d}s$$

Number density of DM: $n_{\mathrm{DM}}(z_{vir}) = \frac{\rho_h(z_{vir})}{m_{\mathrm{DM}}} = \frac{200\rho_c(z_{vir})}{m_{\mathrm{DM}}}$

Temperature of DM: $T_{vir,i}(z_{vir}, M) = 4.91\ \Omega_{m,0}^{1/3}[m_i\ (\mathrm{GeV})] \left(\frac{M_h}{10^4 h^{-1} M_\odot}\right)^{2/3} (1 + z_{vir})\ \mathrm{K}$

Radius of DM halo: $R_h(z_{vir}, M) = \frac{3}{4\pi} \left(\frac{M}{\rho_h(z_{vir})}\right)^{1/3}$

Yields, for a **single halo**:

$$\tau_h(z_{vir}, M) = \frac{\Gamma_{\text{net abs}}(z_{vir}, M) \cdot D_h(z_{vir}, M)}{c}$$

Sources of absorption in DM self-scattering

Optical depth with structure formation

Optical depth of a **single halo:**

$$\tau_h(z_{vir}, M) = \frac{\Gamma_{\text{net abs}}(z_{vir}, M) \cdot D_h(z_{vir}, M)}{c}$$

How many halos will a graviton come across after its emission?

- **Press-Schechter formalism:** The halo mass function gives the differential number **density** of halos → $\frac{\mathrm{d}n_h}{\mathrm{d}M}$

- **Differential *number* of halos:**

$$\frac{dN_h(z,M)}{dM} = \int \frac{dn_h(z,M)}{dM} \mathrm{d}V(z,M) = \int \frac{dn_h(z,M)}{dM} S_h(z,M) \left| \frac{c\ dt}{dz} \right| \mathrm{d}z = \int \frac{dn_h(z,M)}{dM} \frac{c\ S_h(z,M)}{H(z)(1+z)} \mathrm{d}z$$

Sources of absorption in DM self-scattering

Optical depth with structure formation

- Differential **number of halos:** $$\frac{\mathrm{d}N_h}{\mathrm{d}M} = \int \frac{dn_h(z,M)}{dM} \frac{c\, S_h(z,M)}{H(z)(1+z)} \mathrm{d}z$$

- Optical depth of a **single halo:** $$\tau_h(z_{vir}, M) = \frac{\Gamma_{\text{net abs}}(z_{vir}, M) \cdot D_h(z_{vir}, M)}{c}^{*}$$

- The **total optical depth** will be: $$\tau_{z_i} = \int \tau_h \mathrm{d}N_h = \int \tau_h \frac{\mathrm{d}N_h}{\mathrm{d}M} \mathrm{d}M$$

$$\tau_{z_i} = \int_0^{z_i} \int_{M_{\min}}^{M_{\max}} \frac{\Gamma_{\text{net abs.}}(z,M) D_h(z,M) S_h(z,M)}{H(z)(1+z)} \frac{dn_h}{dM}(z,M)(1+z)^3 \, \mathrm{d}M \mathrm{d}z$$

*we approximate $z_{vir} \approx z$

4. Results

Prospects for constraints on self-interacting dark matter

Absorption in DM halos

- We fix $T = T_{vir}(z)$, $f = f_o(1+z)$ and *number density*, we study the **mass-self coupling** parameter space.

- For a GW emitted at z=30, and considering halos of masses 10^5 M$_\odot$ < M < 10^{15} M$_\odot$:

$$\tau_{\rm halos}^{ul} = 3.3 \cdot 10^{-25} \frac{\lambda^2}{m_{ul}^3\, f_0^2},$$

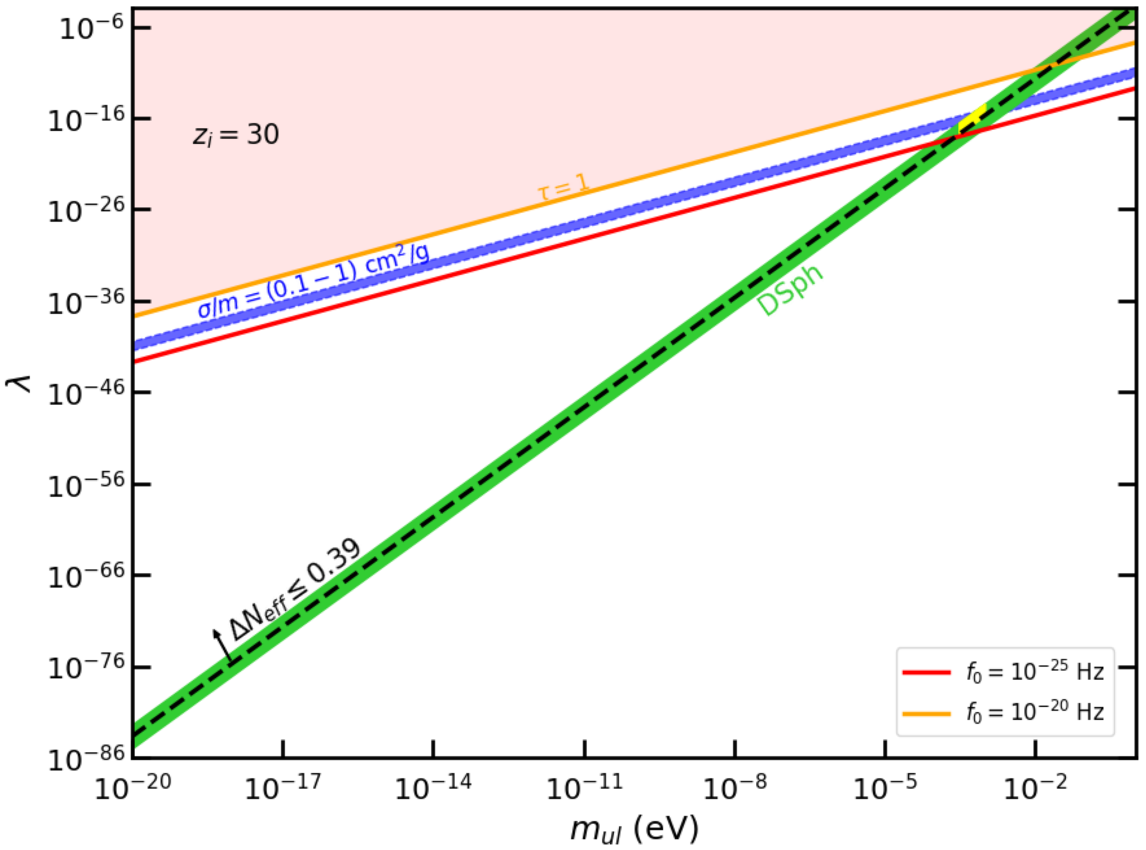


Fig 2. Mass and coupling parameter space for an ultra-light scalar boson that self-interacts with a quartic coupling. The red shaded region sets τ >1. Existing constraints are plotted for comparison.

5. Conclusions

- The study of the absorption of GWs provides a new path to study the characteristics of the medium in which GWs propagate – embodied in the optical depth of the line of sight.
- We have developed simple expressions to compute the optical depth of the line of sight, taking into account structure formation. These expressions are generic for any scattering process.
- We have developed new prospective constraints for the mass-coupling space and for the temperature of SIDM. The arising constraints are less stringent than existing ones.
- These results and expressions are still preliminary. A more thorough derivation and a less simplistic model could give more interesting results.

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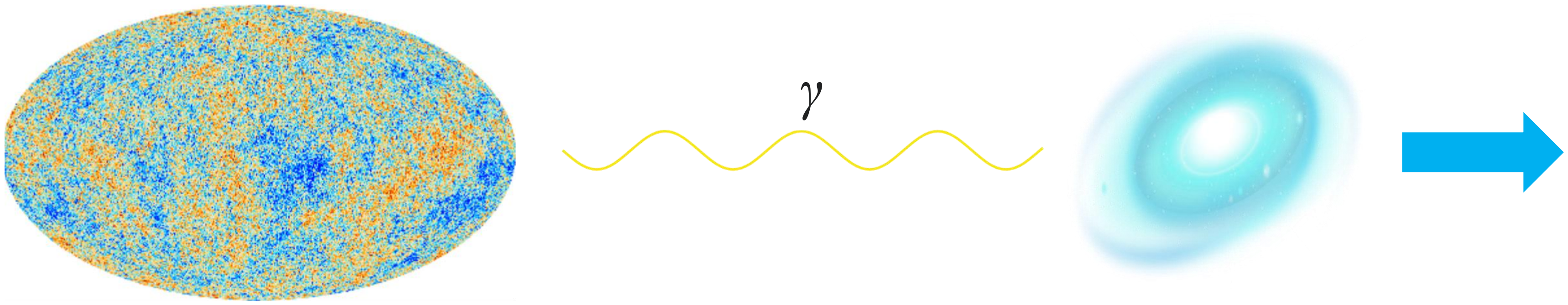
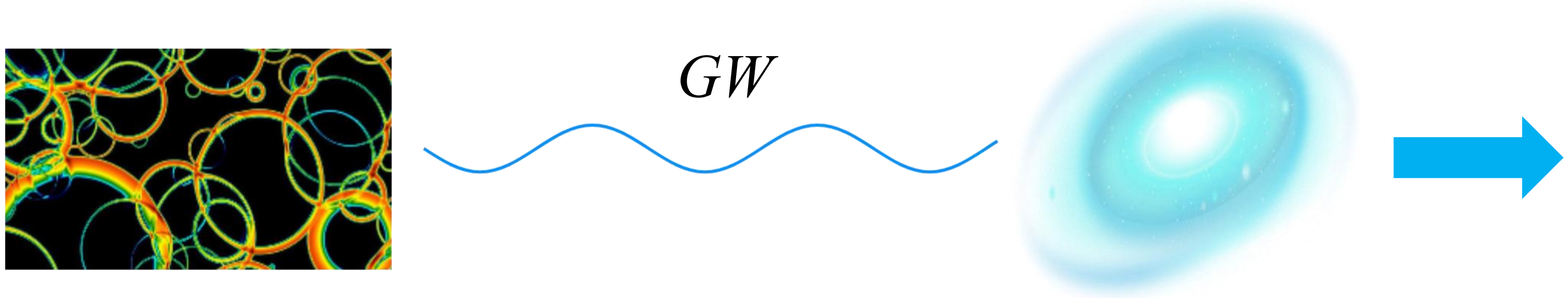
Backup slides

Introduction

GWs as sensors of the Universe



Thermal history of the Universe
(among others)



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GWs as sensors

Direct detection

Indirect detection

How should the medium be according to what we observe?

Bremsstrahlung emission and absorption of GWs

Feynman rules for gravitons

The interaction lagrangian with matter is represented by

$$\mathcal{L} = -\frac{1}{4}\left(\frac{\partial \gamma_{\mu\nu}}{\partial x_\lambda}\frac{\partial \gamma^{\mu\nu}}{\partial x_\lambda} - \frac{1}{2}\frac{\partial \gamma}{\partial x_\lambda}\frac{\partial \gamma}{\partial x_\lambda}\right) - \frac{\kappa}{2}(\gamma_{\mu\nu} - \tfrac{1}{2}\delta_{\mu\nu}\gamma)T^{\mu\nu}$$

Where:

$$\gamma_{\mu\nu} = \sum_k \frac{1}{2\sqrt{k}}\{a_{\mu\nu}(k)\exp\{\mathrm{i}(k\cdot r - \omega t)\} + a^+_{\mu\nu}(k)\exp\{-\mathrm{i}(k\cdot r - \omega t)\}\}$$

$$\gamma = \sum_k \frac{2}{2\sqrt{k}}\{a(k)\exp\{\mathrm{i}(k\cdot r - \omega t)\} + a^+(k)\exp\{-\mathrm{i}(k\cdot r - \omega t)\}\}.$$

Substitution of the above two equations into (2.24) and (2.25) gives:

$$[a_{\mu\nu}(k), a^+_{\lambda\rho}(k)] = \delta_{\mu\nu}\delta_{\nu\rho} + \delta_{\mu\rho}\delta_{\nu\lambda}$$

$$[a(k), a^+(k)] = -1 .$$

And also:

$$a_{\mu\nu}(k) = e_{\mu\nu,+}(k)\,a_+(k) + e_{\mu\nu,-}(k)a_-(k)$$

$$e_{\mu\nu,\pm}(k) = \frac{1}{\sqrt{2}}[(e^1_\mu(k)\,e^1_\nu(k) - e^2_\nu(k)e^2_\nu(k)) + \mathrm{i}(e^1_\mu(k)\,e^2_\nu(k) + e^2_\mu(k)\,e^1_\nu(k))] \qquad e_\pm \equiv e_1 \mp ie_2$$

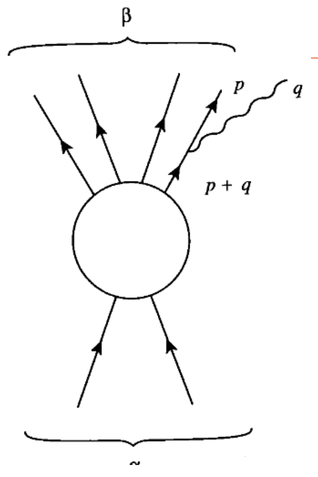
Feynman rules for gravitons

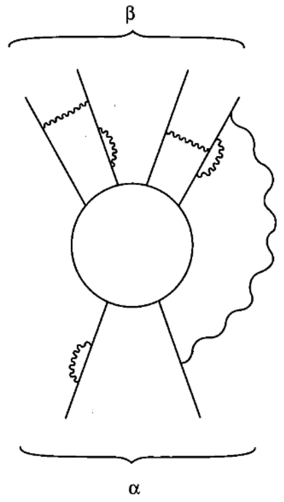
Graviton vertices

Particles	Vertex
Photon–photon–graviton	$-\frac{1}{2}\sqrt{16\pi G}\,\{(\epsilon_{1\mu}p_{1\alpha}-\epsilon_{1\alpha}p_{1\mu}) \times (\epsilon_{2\nu}p_2^{\alpha}-\epsilon_2^{\alpha}p_{2\nu}) -\frac{1}{4}\delta_{\mu\nu}(\epsilon_{1\alpha}p_{1\beta}-\epsilon_{1\beta}p_{1\alpha}) \times (\epsilon_2^{\alpha}p_2^{\beta}-\epsilon_2^{\beta}p_2^{\alpha})\}$
Electron–electron–graviton	$-\frac{1}{2}\sqrt{16\pi G}(\gamma_\mu p_\nu^e+\gamma_\nu p_\mu^e)\delta^4$
Electron–electron–photon	$-e(2\pi)^4\gamma_\mu\delta^4$
Scalar–scalar–graviton	$-\frac{1}{2}(p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu}-m^2\eta_{\mu\nu})\sqrt{16\pi G}$
Neutrino–neutrino–graviton	$-\frac{1}{2}\sqrt{16\pi G}(1+i\gamma_5)(\gamma_\mu p_\nu+\gamma_\nu p_\mu)\delta^4$

An incoming graviton	$(2\pi)^{-3/2}\frac{1}{(2\lvert\boldsymbol{p}\rvert)^{1/2}}\epsilon_\pm^\mu(\boldsymbol{p})\,\epsilon_\pm^\nu(\boldsymbol{p})$
An outgoing graviton	$(2\pi)^{-3/2}\frac{1}{(2\lvert\boldsymbol{p}\rvert)^{1/2}}\epsilon_\pm^\mu(\boldsymbol{p})\,\epsilon_\pm^\nu(\boldsymbol{p})$
Graviton propagator ($\mu\nu\to\lambda\eta$)	$\frac{-i(2\pi)^{-4}}{2(q^2-i\epsilon)}[\eta^{\mu\lambda}\eta^{\nu\eta}+\eta^{\mu\eta}\eta^{\nu\lambda}-\eta^{\mu\nu}\eta^{\lambda\eta}]$

Bremsstrahlung emission of gravitons





$$\sum_n \frac{(8\pi G)^{1/2}\eta_n p_n^\mu p_n^\nu}{p_n \cdot q - i\eta_n \epsilon}$$

Contracted with: $\dfrac{\epsilon_\mu(q)\epsilon_\nu(q)}{(2\pi)^{3/2}(2q)^{1/2}}$

$$\xrightarrow[\text{N grav.}]{} S_{\alpha\beta} = S^0_{\alpha\beta} \prod_{r=1}^{N} \frac{(8\pi G)^{1/2}}{(2\pi)^{3/2}(2q_r)^{1/2}} \sum_n \frac{\eta_n [p_n \cdot \epsilon(q_r)]^2}{[p_n \cdot q_r]}$$

Contracted with: $\dfrac{-i(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma})}{2(2\pi)^4(q^2 - i\epsilon)}$

$$\xrightarrow[\text{N grav.}]{} S_{\alpha\beta} = S^0_{\alpha\beta} \frac{1}{N!}\left[\frac{1}{2}\int \mathrm{d}^4 q \mathcal{B}(q)\right]^N =$$

$$= S^0_{\alpha\beta} \exp\left\{\frac{1}{2}\int_\lambda^\Lambda \mathrm{d}^4 q \mathcal{B}(q)\right\}$$

$$\mathcal{B}(q) = \frac{-i8\pi G}{(2\pi)^4[q^2 - i\epsilon]} \sum_{nm} \frac{\eta_n\eta_m\{(p_n \cdot p_m)^2 - 1/2\, m_m^2 \cdot m_n^2\}}{[p_n \cdot q - i\eta_n\epsilon][-p_m \cdot q - i\eta_m\epsilon]}.$$

Virtual emission of gravitons

$$S_{\alpha\beta} = S^0_{\alpha\beta} \exp\left\{ \frac{1}{2} \int_\lambda^\Lambda \mathrm{d}^4 q \mathcal{B}(q) \right\} \longrightarrow \Gamma_{\alpha\beta} = \Gamma^0_{\alpha\beta} \exp\left\{ \mathrm{Re} \int_\lambda^\Lambda \mathrm{d}^4 q \, \mathcal{B}(q) \right\}$$

From $$\mathcal{B}(q) = \frac{-i8\pi G}{(2\pi)^4 [q^2 - i\epsilon]} \sum_{nm} \frac{\eta_n \eta_m \{(p_n \cdot p_m)^2 - 1/2 \, m_m^2 \cdot m_n^2\}}{[p_n \cdot q - i\eta_n \epsilon][-p_m \cdot q - i\eta_m \epsilon]}.$$

$$\mathrm{Re} \int_\lambda^\Lambda \mathrm{d}^4 q \mathcal{B}(q) = \frac{-8\pi G}{2(2\pi)^3} \int_\lambda^\Lambda \mathrm{d}^4 q \, \delta(q^2) \sum_{nm} \frac{\eta_n \eta_m [(p_n \cdot p_m)^2 - \frac{1}{2} m_n^2 m_m^2]}{[p_n \cdot q][p_m \cdot q]}$$

We can express the denominators as $$\frac{1}{p_n \cdot q} = \frac{1}{|\mathbf{q}|[E_n - \mathbf{p}_n \cdot \hat{q}]}$$

$$\int_\lambda^\Lambda \mathrm{d}^2 q \frac{\delta(q^2)}{|\mathbf{q}|^2} = \int_\lambda^\Lambda \mathrm{d}|\mathbf{q}| \int \mathrm{d}q^0 \, \frac{1}{2|\mathbf{q}|} [\delta(q^0 + |\mathbf{q}|) + \delta(q^0 - |\mathbf{q}|)] = \int_\lambda^\Lambda \mathrm{d}|\mathbf{q}| \frac{1}{|\mathbf{q}|} = \ln(\Lambda/\lambda)$$

In the end: $$\Gamma_{\alpha\beta} = \Gamma^0_{\alpha\beta} \left(\frac{\lambda}{\Lambda} \right)^B$$

$$B \equiv \frac{8\pi G}{2(2\pi)^3} \sum_{nm} \int \mathrm{d}^2\Omega \, \frac{\eta_n \eta_m [\{p_n \cdot p_m)^2 - \frac{1}{2} m_n^2 m_m^2\}}{[E_n - \mathbf{p}_n \cdot \hat{q}][E_m - \mathbf{p}_m \cdot \hat{q}]} = \frac{G}{2\pi} \sum_{nm} \eta_n \eta_m m_n m_m \frac{1 + \beta_{nm}^2}{\beta_{nm}(1 - \beta_{nm}^2)^{1/2}} \ln\left(\frac{1 + \beta_{nm}}{1 - \beta_{nm}} \right)$$

Cancellation of divergences

A similar process for the real gravitons yields: $$\Gamma_{\alpha\beta} = \Gamma^0_{\alpha\beta} b(B) \left(\frac{E}{\lambda}\right)^B \qquad b(B) \approx 1 - \pi^2 B^2/12$$

For the virtual case we found: $$\Gamma_{\alpha\beta} = \Gamma^0_{\alpha\beta} \left(\frac{\lambda}{\Lambda}\right)^B$$

Taking both contributions into account, one finds $$\boxed{\Gamma_{\alpha\to\beta}(\leq E) = \left(\frac{E}{\Lambda}\right)^B b(B)\Gamma^0_{\alpha\to\beta},}$$

Absorption of GWs in scattering processes

Crossing symmetry implies that

$$\Gamma_{em} = V|\mathcal{M}|^3 d^3 q \xrightarrow[Cross.]{} \Gamma_{abs} = \frac{1}{2}(2\pi\hbar)^3|\mathcal{M}|^2$$

In terms of the emissivity: $\mathrm{d}^3 q = q^2 \mathrm{d}q \mathrm{d}\Omega = (2\pi\hbar/c)^3 \nu^2 \mathrm{d}\nu \mathrm{d}\Omega$

$$j_\nu = 2\pi\hbar\nu \left(\frac{2\pi\hbar}{c}\right)^3 \nu^2 |\mathcal{M}|^2 \rightarrow \Gamma_{abs}(\nu) = \left(\frac{c^3}{4\pi\hbar\nu^3}\right) j_\nu.$$

For a medium in thermal equilibrium,

$$j_\nu = \Gamma_{\text{net abs}} B(\nu)/4\pi \xrightarrow[2\pi\hbar\nu<<kT]{} \Gamma_{\text{net abs}}(\nu) \approx \left(\frac{c^3}{4\pi\hbar\nu^3}\right) j_\nu \left[\frac{2\pi\hbar\nu}{kT}\right]$$

$$B(\nu) = \frac{16\pi^2\hbar\nu^3}{c^3}\left[e^{2\pi\hbar\nu/kT} - 1\right]^{-1}$$

Absorption of GWs in scattering processes

Emission rate of a graviton: $$\Gamma_{\alpha\to\beta}(\leq E) = \left(\frac{E}{\Lambda}\right)^B b(B)\Gamma^0_{\alpha\to\beta},$$ $$b(B) \approx 1 - \pi^2 B^2/12$$

In the non-relativistic limit, for a 2-2 scattering: $$B = \frac{8G}{5\pi\hbar c^5}\mu^2 v^4 sin^2\theta_c$$

For typical values of B, B << 1:

$$d\Gamma_{\rm em}(\leq 2\pi\hbar\nu) \approx [1 + B\ln(2\pi\hbar\nu/\Lambda)]d\Gamma^0 \to j_\nu = \frac{2\pi\hbar}{4\pi}\frac{8G\mu^2}{5\pi\hbar c^5}n_1 n_2 \overline{v^5\sigma_D},$$

Therefore:

$$\Gamma_{\text{net abs.}} = \frac{G\mu^2}{5\pi^2\hbar c^2\nu^3}n_1 n_2 \overline{v^5\sigma_D}\left[\frac{2\pi\hbar\nu}{kT}\right]$$

Plasma frequency IR cut-off

Net rate of absorption of a graviton in a non-relativistic 2-2 scattering:

$$\Gamma_{\text{net abs.}} = \frac{G\mu^2}{5\pi^2 \hbar c^2 f^3} n_1 n_2 \overline{v^5 \sigma_D} \left[\frac{2\pi\hbar f}{kT} \right]$$

diverges for $f \longrightarrow 0$

The expression holds if the absorption occurs in **independent collisions**

$f >> f_p$

where $f_p = \overline{n \cdot v \cdot \sigma_D}$ is the *plasma frequency of collisions*

For our model: $$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!}\psi^4 \rightarrow \frac{d\sigma}{d\Omega} = \frac{\lambda^2}{256\pi^2 m_{ul}^2}$$

Yields a plasma frequency: $$f_p = \overline{n \cdot v \cdot \sigma_D} = \frac{\lambda^2 (\hbar \cdot c)^2 n \sqrt{k_B T}}{24\sqrt{2\pi^3} \cdot (m \cdot c^2)^{5/2}} \cdot c$$

For a DM halo, using $n_{\text{DM}}(z_{vir}), T_h(M_h, z_{vir})$

$$f_p = 4.78 \cdot 10^{-4} \frac{\lambda^2 M_h^{1/3} \cdot (1+z)^{7/2}}{m_{\text{DM}}^3} \frac{1}{\text{s}}$$

$$f_p \ll 10^{-20} \frac{1}{\text{s}} \quad \forall (\lambda, m_{DM}) \text{ of interest}$$

taking $M_h = 10^{14}\ \text{M}_\odot,\ z_{vir} = 30$