QCD axion at finite density

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16 September 2025

3rd Training School COST Action COSMIC WISPers (CA21106)

The QCD axion

Strong CP problem: $\left|\bar{\vartheta}\right| \lesssim 10^{-10}$ ➜ Peccei-Quinn mechanism

Additional $U(1)_{PQ}$ symmetry

- Anomalous under $SU(3)_C$ → Solves the Strong CP problem
- Spontaneously broken → pGB: QCD axion

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \frac{1}{4}g^0_{a\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a}\sum_q \bar{q}c^0_q\gamma^\mu\gamma_5 q$$

- Remnant of PQ solution to the strong CP problem
- Candidate for CDM ➜ Misalignment mechanism

SN1987A axion bound

SN1987A event: ~ 30 supernova neutrinos observed

New BSM weakly interacting particle → New cooling channel for the supernova → Reduced neutrino burst duration

For a 1 $M_\odot$ supernova core

$$L_a \lesssim 2 \times 10^{52}\ \mathrm{erg\ s^{-1}} \overset{\text{Benchmark models}}{\Longrightarrow} f_a \gtrsim 10^8\ \mathrm{GeV}$$

Is it possible to relax this bound?

Nucleophobic axion models

[L. Di Luzio *et al.*, 1712.04940]

$$\mathcal{L}_a \supset \frac{\partial_\mu a}{2f_a} c_N \bar{N}\gamma^\mu\gamma_5 N$$

Matching →

$$\begin{cases} c_p + c_n = 0.50(5)(c_u^0 + c_d^0 - 1) - 2\delta_s \\ c_p - c_n = 1.273(2)(c_u^0 - c_d^0 - f_{ud}) \end{cases}$$

$$f_{ud} = \frac{1-z}{1+z}, \qquad z = \frac{m_u}{m_d}$$

Nucleophobic axion models ⇒ Suppression of axion-nucleon couplings

- Require nonuniversal Peccei-Quinn charges for the SM quarks
- Yield flavour violating couplings

$$\begin{cases} c_p + c_n \approx 0 \implies c_u^0 + c_d^0 \approx 1 \\ c_p - c_n \approx 0 \implies c_u^0 - c_d^0 \approx f_{ud} \approx 1/3 \end{cases}$$

Finite density effects

[R. Balkin *et al.*, 2003.04903]

Density in the core of the supernova ~ $n_0 = 0.16\ \text{fm}^{-3}$

$$\begin{cases} c_p = g_A \frac{c_u - c_d}{2} + g_0^{ud} \frac{c_u + c_d}{2} \\ c_n = -g_A \frac{c_u - c_d}{2} + g_0^{ud} \frac{c_u + c_d}{2} \end{cases}$$

- Axion-quark couplings change due to modification of chiral condensate

$$\zeta_{qq}(n) \equiv \frac{\langle \bar{q}q \rangle_n}{\langle \bar{q}q \rangle_0} = 1 + \frac{1}{\langle \bar{q}q \rangle_0} \frac{\partial \Delta E(n)}{\partial m_q}, \quad q = u, d, s$$

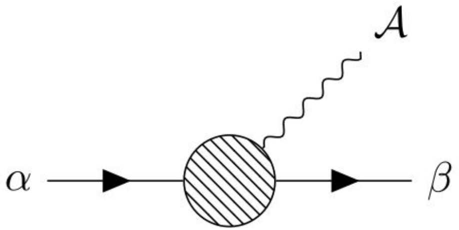
Hellmann-Feynman theorem

- Matrix elements g_A and g_0^{ud} are corrected too

Corrections to matrix elements

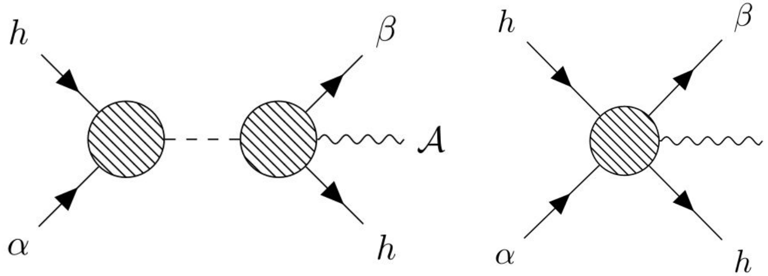
[T. S. Park *et al.*, 1997]

Fermi gas approximation $\Rightarrow$ $|F\rangle = \prod_{h\in F} a_h^\dagger |0\rangle$ $\qquad \langle\beta|J_5^{\pm,i}|\alpha\rangle \to \langle\beta;F|J_5^{\pm,i}|\alpha;F\rangle$



~ $(g_A)_0$

In medium $\longrightarrow$



~ δg_A

$$\frac{(g_A)_n}{(g_A)_0} = 1 + \frac{n}{\Lambda_\chi f_\pi^2}\left[\frac{c_D}{4(g_A)_0} - \frac{I(m_\pi/k_F)}{3}\left(2\hat{c}_4 - \hat{c}_3 + \frac{\Lambda_\chi}{2m_N}\right)\right]$$

$$\frac{(g_0^{ud})_n}{(g_0^{ud})_0} = 1 + \kappa\frac{n}{n_0}$$

Missing LECs

Is nucleophobia compatible with finite density effects?

PHYSICAL REVIEW D **111,** 015018 (2025)

Do finite density effects jeopardize axion nucleophobia in supernovae?

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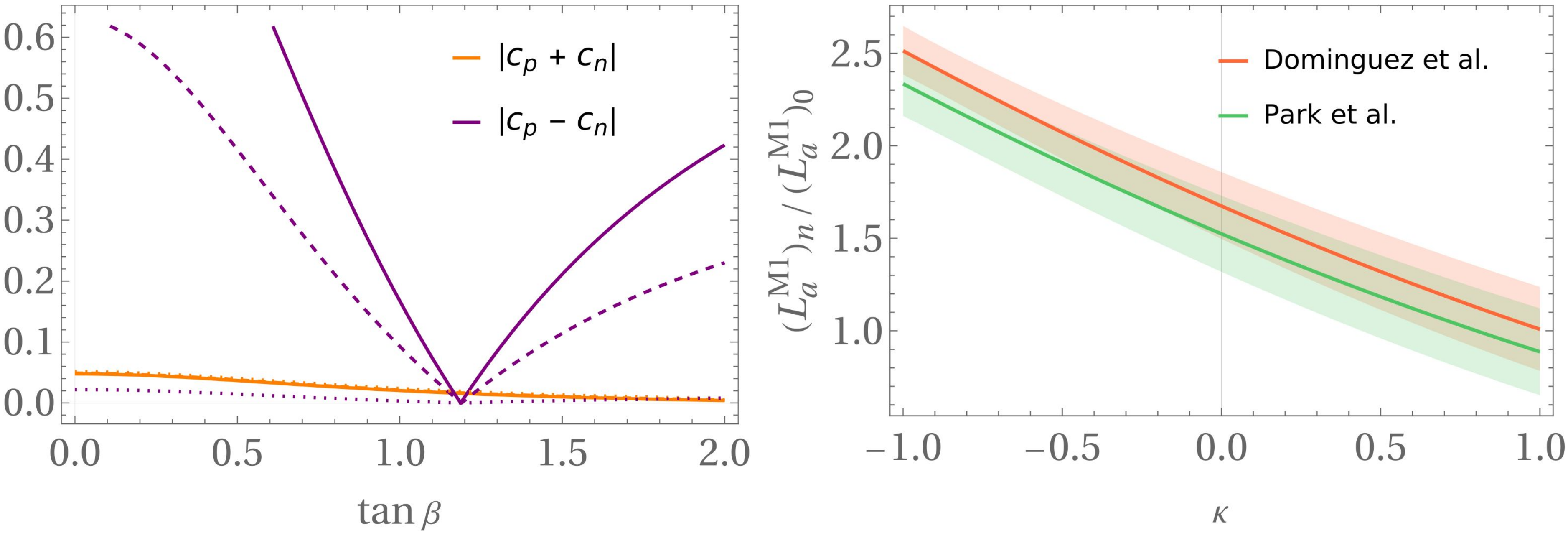
(Received 10 October 2024; accepted 5 January 2025; published 22 January 2025)

Nucleophobic axion models, wherein axion couplings to both protons and neutrons are simultaneously suppressed, can relax the stringent constraints from SN 1987A. However, it remains uncertain whether these models maintain their nucleophobic property under the influence of finite baryon density effects. These are especially relevant in astrophysical environments near saturation density, such as supernovae (SNe). In this study, we demonstrate that the nucleophobic solution remains viable also at finite density. Furthermore, we show that the SN axion bound relaxes significantly in nucleophobic models, even when accounting for the integration over the nonhomogeneous environment of the SN core.

DOI: 10.1103/PhysRevD.111.015018

What about nucleophobia?



[L. Di Luzio, VF, F. Mescia, M. Giannotti, E. Nardi, 2025]

Conclusions

Next steps:

- Update the SN1987A bound with
  - Finite density corrections ➜ New formalism [K. Springmann *et al.*, 2410.10945]
  - New production channels ➜ Pion production, model-independent channel
- Estimate the relevance of the new model-independent axion production channel [K. Springmann *et al.*, 2410.10945]
  - Implications on the SN1987A bound
  - New detection channel?
- Extend the finite density formalism to other BSM particles
  - Dark photon
  - Dark scalars

Thank you!

BACKUP SLIDES

Hellmann–Feynman theorem

Theorem (Hellmann–Feynman)

Let $\hat{H}_\lambda$ be a Hamiltonian operator depending upon a continuous parameter λ and let $|\psi_\lambda\rangle$ be an eigenstate of $\hat{H}_\lambda$ depending implicitly on λ, with eigenvalue E_λ. Then

$$\frac{\mathrm{d}E_\lambda}{\mathrm{d}\lambda} = \left\langle \psi_\lambda \middle| \frac{\mathrm{d}\hat{H}_\lambda}{\mathrm{d}\lambda} \middle| \psi_\lambda \right\rangle$$

In particular, the QCD vacuum energy density satisfies

$$\frac{\mathrm{d}\mathcal{E}_0}{\mathrm{d}m_q} = \left\langle 0 \middle| \frac{\mathrm{d}\mathcal{H}_{\mathrm{QCD}}}{\mathrm{d}m_q} \middle| 0 \right\rangle = \langle 0|\bar{q}q|0\rangle \quad \text{since} \quad \mathcal{H}_{\mathrm{QCD}} \supset \sum_q m_q \bar{q}q$$

Linear approximation

Neglecting interactions between nucleon and relativistic corrections, the in-medium shift of the QCD vacuum energy is $\Delta E(n) = \sum_{x=n,p,...} m_x n_x$, so that, from the Hellmann-Feynman theorem,

$$\zeta_{\bar{q}q}(n) \equiv \frac{\langle \bar{q}q \rangle_n}{\langle \bar{q}q \rangle_0} = 1 + \frac{1}{\langle \bar{q}q \rangle_0} \frac{\partial \Delta E(n)}{\partial m_q}$$

$$= 1 + \frac{1}{\langle \bar{q}q \rangle_0} \sum_x n_x \frac{\partial m_x}{\partial m_q}$$

Corrections to axion-quark couplings

$$(Q_a^*)_0 = \frac{\mathrm{diag}(1, z)}{1+z}, \qquad z = \frac{m_u}{m_d} \quad \xrightarrow{\; m_q \to \frac{\langle \bar{q}q \rangle_n}{\langle \bar{q}q \rangle_0} \times m_q \;} \quad (Q_a^*)_n = \frac{\mathrm{diag}(1, zZ)}{1+zZ}, \qquad Z = \frac{\langle \bar{u}u \rangle_n}{\langle \bar{d}d \rangle_n}$$

$$\boxed{(c_q)_0 \to (c_q)_n = c_q^0 - [(Q_a^*)_n]_q}$$

Density dependence of couplings in SN