

Neutrino Physics

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1. *history of neutrinos, ancient and modern*
2. *oscillations in quantum mechanics*
(why can one use a Schrodinger Eqn?)
3. from quantum mechanics to physics
Beyond-the-Standard-Model
4. the scale of neutrino masses
5. leptogenesis?

Why are neutrinos interesting?

1. they are *Beyond the Standard Model!*
the SM must be extended to include their small masses
2. they interact (only) weakly
= probe otherwise-unattainable places (nuclear reactors, star interiors, waay back in cosmology...)
3. can calculate with quantum mechanics!
(not need QuantumFieldTheory)

References (old)

other version of these lectures (2017 CERN school) :

https://physicschool.web.cern.ch/ESHEP/previous_eshep.html

Giunti website “neutrino unbound” : <http://www.nu.to.infn.it/>

fits : <http://www.nu-fit.org/>

Raffelt talks (astropart) : <http://wwwth.mpp.mpg.de/members/raffelt/>

Plots thanks to Strumia + Vissani : hep-ph/0606054

simple 3-gen probabilities for LBL : Cervera et al 0002108 (+ later versions)

current state of oscillation measurements : Gonzalez-Garcia @ CERN ν platform kickoff : <https://indico.cern.ch/event/572831/>

neutrino cosmology : Lesgourgues at CERN ν platform kickoff : <https://indico.cern.ch/event/572831/>

(hypothetical/ /known) **history of neutrinos** (shy in the lab, relevant in cosmo)

- ▶ ...
- ▶ inflation (gives large scale CMB fluctuations) (?driven by sneutrino?)
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- ▶ Big Bang Nucleosynthesis ($H, D, {}^3\text{He}, {}^4\text{He}, {}^7\text{Li}$ at $T \sim \text{MeV}$)
⇔ 3 species of relativistic ν in the thermal soup
- ▶ decoupling of photons — $e + p \rightarrow H$ (CMB spectrum today)
cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- ▶ for 10^{10} yrs — stars are born, radiate (γ, ν), and die
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- ▶ 1930 : Pauli hypothesises the “neutrino”, to conserve E in $n \rightarrow p + e(+\nu)$
- ▶ 1953 Reines and Cowan : neutrino CC interactions in detector near a reactor
- ▶ invention of the Standard Model (SM) : massless ν
- ▶
- ▶ **neutrinos have mass! There is more in the Lagrangian than the SM...**

Recent history of neutrinos($\equiv \nu$) and people

\sim 1930 :predicting the neutrino :

observe β -decay : $(A, Z) \rightarrow (A, Z - 1) + e^+ (+\nu)$

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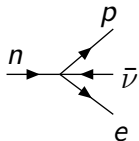
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\sim 1956 :confirming the neutrino

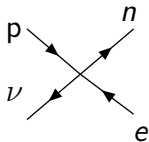
near a nuclear reactor (produces $\bar{\nu}$ flux : $n \rightarrow p + e + \bar{\nu}$)

Reines+Cowan detect $\bar{\nu} + p \rightarrow n + e^+$, $e^+ + e^- \rightarrow \gamma\gamma$

$\Rightarrow \nu$ exist, and have only weak interactions (and gravity)



($t \rightarrow$ in diagrams)



antiparticles

antiparticles

$$E^2 - |\vec{p}|^2 = m^2 \Rightarrow E = \pm \sqrt{m^2 + |\vec{p}|^2}$$

NR limit : $E \simeq m + |\vec{p}|^2/2m + \dots$?where went -ve E solns?

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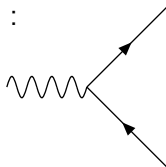
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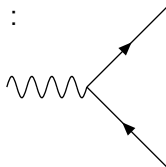
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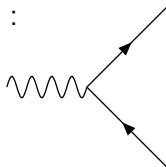
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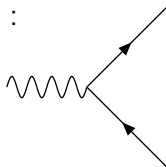
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How to tell particle from antiparticle?

charges reversed : electron = $e = e^- \leftrightarrow e^+ = \bar{e}$ = positron

no conserved charge? Maybe part = $\overline{\text{part}}$ (like photon = γ)

Is neutrino its own antiparticle?

Interactions of neutrino($\equiv \nu$)

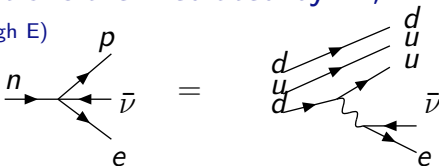
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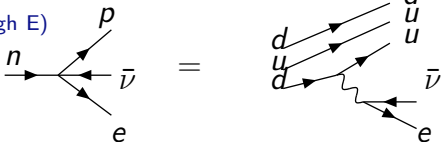


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$\mathcal{A} \sim$ current \times propagator \times current , $p_n = p_p + p_{\bar{\nu}} + p_e$

$$\sim g^2 (\bar{u} \gamma^\alpha d_L) \frac{1}{p_W^2 - m_W^2} (\bar{e} \gamma_\alpha \nu_L) \quad p_W = p_n - p_p = p_{\bar{\nu}} + p_e$$

$$p_W^2 \sim (m_n - m_p)^2 \sim (0.1 \text{ GeV})^2 \ll m_W^2 \sim (80 \text{ GeV})^2$$

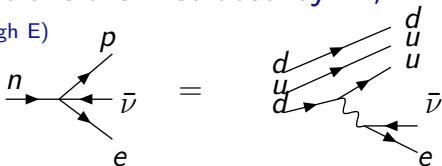
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- at least three neutrinos

3 charged leptons = $\{e, \mu, \tau\}$. Observe each has own ν

(fermion w/o strong int. ; (.5, 105, 1770 MeV)

$$\left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\}$$

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To calculate in a theory, evaluate PI : \sim perturb in cplg ctes.
Can read particle properties/interactions from \mathcal{L} .

Historical problems : neutrinos disappear...

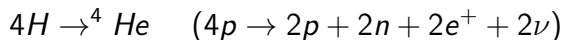
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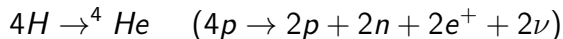
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ν escape, γ diffuse to surface ($10^3 \rightarrow 10^6$ yrs)

ν_e flux $\sim .3 \rightarrow .5$ expected from solar energy output

Flux in \sum flavours \sim expected (SNO).

Nobel-winning plot # 2 : SNO

solar ν_e deficit, but expected $\sum \nu_\alpha$ flux(PRL 89 (2002) 011301)

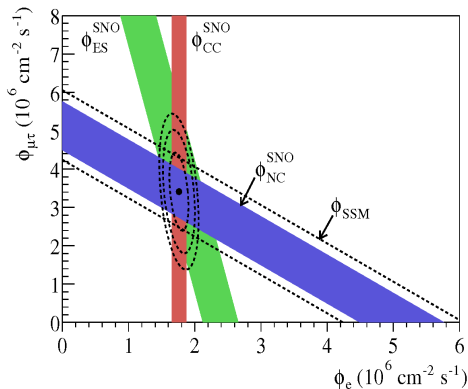
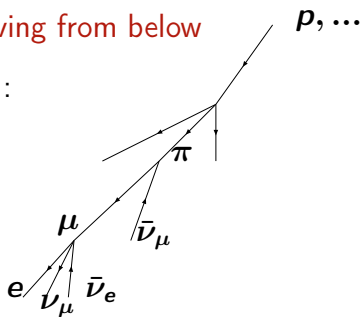


FIG. 3: Flux of ^8B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^8B flux as predicted by the SSM [11] (dashed lines) and that measured

Atmospheric ν problem : deficit of ν_μ arriving from below

ν produced in cosmic ray interactions :
expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$

height atmosphere ~ 10 - 100 km,
 $R_{earth} \sim 6000$ km



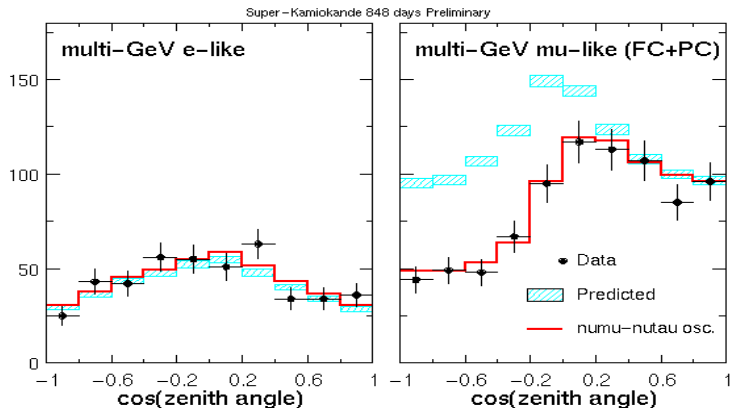
...see deficit of $\nu_\mu, \bar{\nu}_\mu$ from below



(photo courtesy of SK)

Nobel plot #1 : SK-98 :

$\nu_\mu + H_2O \rightarrow \mu + ..$, deficit in ν_μ from below (PRL 81 (1998) 1562-1567)



upwards \leftrightarrow $\cos = -1$; down \leftrightarrow $\cos = +1$.

L : 20 km \leftrightarrow 10 000 km.

Lets calculate!

oscillations of massive ν

a relativistic muon decays at the top of the atmosphere,
produces a ν .

Suppose massive ν_2, ν_3 , but not reconstruct (E_ν, \vec{k}_ν) well
enough to identify if ν is ν_3 or ν_2 ...

The ν travels to the SK detector, where it produces another μ

\Rightarrow must sum in *amplitude* possibility to travel as ν_2 or ν_3

\Leftrightarrow neutrino propagation is a quantum process

neutrinos “oscillate”(QM version : easy to rederive)

A relativistic neutrino, with momentum \vec{k} , is produced in muon decay at $t = 0$ (at Tokai/edge atmosphere). Describe as a quantum mechanical state :

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It travels a distance L in time t to the detector (SuperK)

$$|\nu(t)\rangle$$

where it produces an μ in CC scattering. With what probability ?

$$\mathcal{P}_{\mu \rightarrow \mu}(t) = |\langle \nu_\mu | \nu(t) \rangle|^2 = ?$$

1. Suppose massive neutrinos (two generations for simplicity).
Flavour and mass eigenstates related by : $\nu_\alpha = U_{\alpha i} \nu_i$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

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2. Suppose time evolution in the mass basis described by

$$i \frac{d}{dt} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} = \begin{bmatrix} E_2 & 0 \\ 0 & E_3 \end{bmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \quad , \quad E_i^2 = k^2 + m_i^2$$

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3. If produce relativistic ν_μ at $t = 0$, then at t later :

$$|\nu(t)\rangle = \sum_j U_{\mu j} |\nu_j(t)\rangle = \sum_j U_{\mu j} e^{-iE_j t} |\nu_j\rangle$$

Amplitude for neutrino to produce charged lepton α in CC scattering in detector after t :

$$|\langle \nu_\alpha | \nu(t) \rangle| = \left| \sum_j U_{\mu j} e^{-iE_j t} U_{\alpha j}^* \right|$$

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So in 2 generation case, using $t = L$, $E_3 - E_2 \simeq \frac{m_3^2 - m_2^2}{2E} \equiv \frac{\Delta_{32}^2}{2E}$:

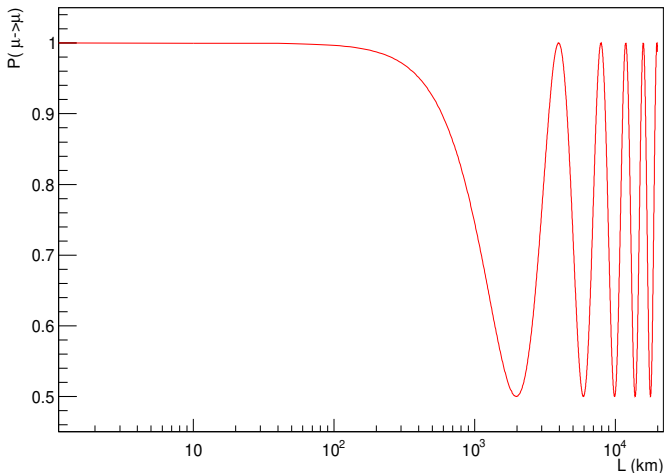
$$\begin{aligned} \mathcal{P}_{\mu \rightarrow \tau}(t) &= \left| \sin \theta \cos \theta \left(e^{i\Delta_{32}^2 L/4E} - e^{-i\Delta_{32}^2 L/4E} \right) \right|^2 \\ &= \sin^2(2\theta) \sin^2 \left(L \frac{\Delta_{32}^2}{4E} \right) \end{aligned}$$

$$\mathcal{P}_{\mu \rightarrow \mu}(t) = 1 - \sin^2(2\theta) \sin^2 \left(L \frac{\Delta_{32}^2}{4E} \right) = 1 - \sin^2(2\theta) \sin^2 \left(1.27 \frac{L \Delta_{32}^2 \text{ GeV}}{\text{km eV}^2} \frac{\text{GeV}}{4E} \right)$$

$E = \nu$ energy, L source-detector distance, $\Delta_{32}^2 \sim 10^{-3} \text{eV}^2$

$E \sim 10 \text{ GeV}$ for atmospheric ν s; $L : 20 \text{ km} \rightarrow 10000 \text{ km}$

2 generation survival probability $P(\mu \rightarrow \mu)$, $2\theta = 45$, Δm_{atm}^2 , $E = \text{GeV}$



$$\mathcal{P}_{\mu \rightarrow \mu}(L) = 1 - \sin^2(2\theta) \sin^2 \left(1.27 \frac{L}{\text{km}} \frac{\Delta^2}{\text{eV}^2} \frac{\text{GeV}}{4E} \right)$$

$$\Delta_{32}^2 = 2.5 \times 10^{-3} \text{eV}^2$$

$$E \sim 0.6 \text{GeV (T2K)}$$

$$\sim \text{MeV (reactors)}$$

$$\sim 10 \text{GeV (atmospheric)}$$

Schrodinger Eqn for relativistic particles ?

is ok : have Eqn for the number operator $\hat{n}_p \equiv \hat{a}_p^\dagger \hat{a}_p$:

$$i \frac{\partial}{\partial t} \hat{n} = [\hat{H}, \hat{n}]$$

...take expectation values and get QM version.

quantum coherence over km ?

- $m_\nu \ll$, so $\Delta_{\text{expt}} \sqrt{E_\nu^2 - |\vec{p}_\nu|^2} \gg m_\nu$ (decoherence slide)
- recall ν only interact *weakly*, can cross earth without interaction (no “observations” to collapse wavefns)

But...there is forward scattering \Rightarrow effective contribution to m_ν from matter in sun, earth and supernovae (more later, maybe)

decoherence of neutrinos for large $L/E \gg 1/\Delta^2$

- at production, 2 superposed wavepackets of masses m_2, m_3 .
- group velocity of packets

$$v_i = \frac{\partial E}{\partial p} = \frac{p}{E} \simeq 1 - \frac{m_i^2}{2E^2}$$

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$$(v_2 - v_3)L \simeq \frac{\Delta_{23}^2}{E^2}L \simeq \frac{L}{\ell_{osc}} \frac{1}{E}$$

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$$v_i = \frac{\partial E}{\partial p} = \frac{p}{E} \simeq 1 - \frac{m_i^2}{2E^2}$$

- after distance L , packets have separated by

$$(v_2 - v_3)L \simeq \frac{\Delta_{23}^2}{E^2}L \simeq \frac{L}{\ell_{osc}} \frac{1}{E}$$

- no interference if larger than size of packets $\sim 1/(\delta E)$ where packet energy uncertain by δE . so no oscillations once

$$\frac{L}{\ell_{osc}} \gtrsim \frac{E}{\delta E}$$

can make similar estimate doing sum over paths, phases should sum coherently

Massive ν in the Standard Model

From antique 2-flavour QM calculation and astro problems to \geq three light ν in a lively exptal programme using reactors, accelerators and astro sources

What masses ?

oscillations say there are mass differences : (global fits of
www.nu-fit.org)

$$\begin{aligned} |\Delta_{atm}^2| = |\Delta_{3j}^2| &= |m_3^2 - m_j^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2 \\ &\gg \Delta m_{21}^2 \simeq 7.50 \pm 0.2 \times 10^{-5} \text{ eV}^2 \\ \sqrt{\Delta m_{31}^2} &\simeq 0.05 \text{ eV} \qquad \sqrt{\Delta m_{21}^2} \simeq 0.008 \text{ eV} \end{aligned}$$

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mass scale \lesssim eV from

- cosmology : massive ν are DM today, and affect CMB.
- spectrum of e in β decay : Katrin expt
- $0\nu 2\beta$... if ν own antiparticle

And there are mixing angles

In 2 flavour, wrote :

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \cdot \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}.$$

but there are three lepton flavours in SM, should write

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} U \end{bmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

U

Can write as :

$$\begin{aligned}
 U_{\alpha i} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & s_{13} e^{-i\delta} \\ 0 & 1 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} P \\
 &\text{atm. + LBL disa.} \quad \text{reac. disa. + LBL app.} \quad \text{sol + reac. disa.} \\
 &= \begin{bmatrix} c_{12} c_{13} & c_{13} s_{12} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - c_{12} s_{13} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & c_{13} s_{23} \\ s_{23} s_{12} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - c_{23} s_{12} s_{13} e^{i\delta} & c_{13} c_{23} \end{bmatrix} P
 \end{aligned}$$

$$\theta_{23} \simeq \pi/4 \pm \pi/40 \quad \theta_{12} \simeq \pi/6 \quad \theta_{13} \simeq 8^\circ$$

(global fits of www.nu-fit.org)

Where to put U in SM?

Previously wrote

$$\left\{ \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \right\}$$

write ν in mass eigenstates too (propagate eigenstates of Hamiltonian...)

Where to put U in SM?

$$\ell_L^e \equiv \begin{pmatrix} U_{ei} \nu_L^i \\ e_L \end{pmatrix}, \quad \ell_L^\mu \equiv \begin{pmatrix} U_{\mu j} \nu_L^j \\ \mu_L \end{pmatrix}, \quad \ell_L^\tau \equiv \begin{pmatrix} U_{\tau k} \nu_L^k \\ \tau_L \end{pmatrix}$$

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3×3 mixing matrix $U_{\alpha,i}$ appears at W^\pm vertices (like CKM)

$$\rightarrow -i \frac{g U_{ej}^*}{\sqrt{2}} \bar{\nu}_L^j \gamma^\mu W_\mu^+ e_L + \dots$$

but flavour-diagonal Z vertex :

$$\propto \sum_\alpha -i \frac{g}{2} U_{\alpha j}^* \bar{\nu}_L^j \gamma^\mu Z_\mu^+ U_{\alpha k} \nu_L^k = \delta_{jk} \frac{g}{2} \bar{\nu}_L^j \gamma^\mu Z_\mu^+ \nu_L^k$$

The drunken Unitarity triangle

Not hear much about “leptonic unitarity triangle”

1. not measure elements at tree in CC
2. Also, it drinks.

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Amplitude to oscillate from flavour α to β over distance L :

$$\mathcal{A}_{\alpha\beta}(L) = U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{\alpha 3} U_{\beta 3}^* e^{-i(m_3^2 - m_1^2)L/(2E)}$$

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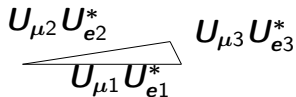
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at $L = 0$ unitarity : $\Rightarrow \mathcal{A}_{\alpha\beta} = 1$ for $\alpha = \beta$

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\Leftrightarrow unitarity triangle (in complex plane)



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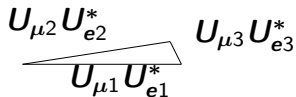
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At $L = t \neq 0$, two of the vectors rotate in the complex plane,
with frequencies $(m_j^2 - m_1^2)/2E$

oscillations \leftrightarrow time-dependent non-unitarity

About two- flavour analyses : atm/LBL ν_μ disappearance

Amplitude to oscillate from flavour μ to τ over distance L :

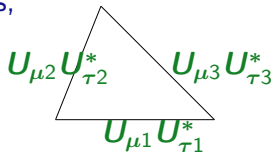
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At $L \sim E/(m_3^2 - m_1^2)$, vector "3" rotates,
frequency $(m_3^2 - m_1^2)/2E$

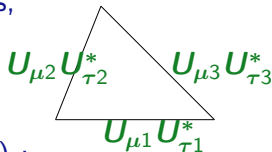


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At $L \sim E/(m_3^2 - m_1^2)$, vector "3" rotates,
frequency $(m_3^2 - m_1^2)/2E$



\Rightarrow "Atmospheric" neutrinos, also LBL
(ν_μ disappearance via Δm_{31}^2 oscillations) :

$$\mathcal{A}_{\mu\tau}(L) \simeq U_{\mu 1} U_{\tau 1}^* + U_{\mu 2} U_{\tau 2}^* + U_{\mu 3} U_{\tau 3}^* e^{-i(m_3^2 - m_1^2)L/(2E)}$$

$U_{\mu 3} U_{\tau 3}^*$ oscillates on timescale $t = L \sim (m_3^2 - m_1^2)/E$

$U_{\mu 2} U_{\tau 2}^* \sim$ stationary, measure θ_{23}

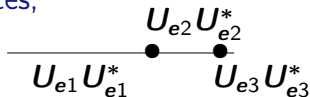
About two- flavour analyses : solar and Kamland

Amplitude to oscillate from flavour e to e over distance L :

$$\mathcal{A}_{ee}(L) = U_{e1}U_{e1}^* + U_{e2}U_{e2}^* e^{-i(m_2^2 - m_1^2)L/(2E)} + U_{e3}U_{e3}^* e^{-i(m_3^2 - m_1^2)L/(2E)}$$

At $L \sim 2E/(m_2^2 - m_1^2)$, vector 2 rotates,
frequency $(m_2^2 - m_1^2)/2E$

vec. 3 spins rapidly



\Rightarrow "Solar" + "KamLAND" (reactor $\bar{\nu}_e$ for $L \sim 100$ km)
neutrinos

$\Leftrightarrow \nu_e$ disappearance over long baselines $L \sim (m_2^2 - m_1^2)/2E$
two- ν approx works because θ_{13} is small ($U_{e3} = \sin\theta_{13}$) :

$$\mathcal{A}_{ee} \simeq |U_{e1}|^2 + |U_{e2}|^2 e^{-i(m_2^2 - m_1^2)L/(2E)}$$

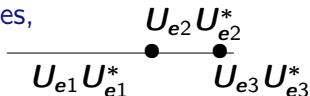
measure θ_{12}

About two- flavour analyses : θ_{13} at reactors

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At short enough L , only third vector rotates,
frequency $(m_3^2 - m_1^2)/2E$



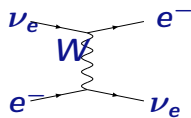
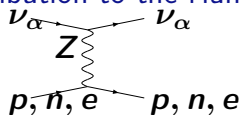
\Rightarrow reactor θ_{13} by $\bar{\nu}_e$ disappearance ; select short baseline such that only $|U_{e3}(t)|^2$ moves

$$\begin{aligned}\mathcal{A}_{ee} &\simeq (|U_{e1}|^2 + |U_{e2}|^2) + |U_{e3}|^2 e^{-i(m_3^2 - m_1^2)L/(2E)} \\ &= c_{13}^2(c_{12}^2 + s_{12}^2) + s_{13}^2 e^{-i(m_3^2 - m_1^2)L/(2E)}\end{aligned}$$

*Flavour transition in matter
oscillations and adiabatic*

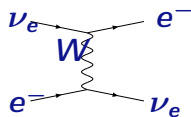
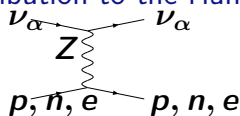
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Coherent forward scattering of ν in matter give extra contribution to the Hamiltonian :



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To see : use $\mathcal{H}_{\text{mat}} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}$ in QFT oscillation derivation,

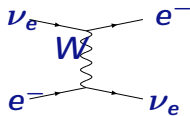
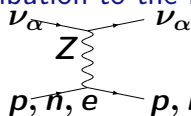
$$\mathcal{H}_{\text{int}} \simeq 2\sqrt{2}G_F \int d^4x (\bar{\hat{\nu}}_e(x)\gamma^\alpha P_L \hat{\nu}_e)(\bar{\hat{e}}\gamma_\alpha P_L \hat{e}(x))$$

evaluated in a medium with electrons (NC irrelevant ; same for all ν generations = add unit matrix to H . And no μ or τ in the matter.)

$$\langle \text{medium} | \bar{\hat{e}}\gamma_\alpha P_L \hat{e}(x) | \text{medium} \rangle \rightarrow \delta_{\alpha 0} \frac{n_e}{2}$$

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H_{mat} in flavour basis $(\nu_e, (\nu_\tau - \nu_\mu)/\sqrt{2})$, $V_e = \sqrt{2}G_F n_e$:

$$H_{\text{mat}} = \dots + \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & \Delta^2/(2E) \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} V_e & 0 \\ 0 & 0 \end{bmatrix}$$

Oscillations in matter — ctd

H_{mat} in flavour basis $(\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2})$:

$$H_{\text{mat}} = \dots + \begin{bmatrix} -\frac{\Delta^2}{4E} \cos 2\theta + V_e & \frac{\Delta^2}{4E} \sin 2\theta \\ \frac{\Delta^2}{4E} \sin 2\theta & \frac{\Delta^2}{4E} \cos 2\theta \end{bmatrix}$$

With $U_{\text{mat}}^T H_{\text{mat}} U_{\text{mat}}^* = \text{diagonal}$:

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- ▶ for $V_e \ll \frac{\Delta^2}{2E} \cos(2\theta_{21})$, matter effects negligible
- ▶ $\theta_{\text{mat}} \rightarrow \pi/4$ ("resonance") at $V_e = \frac{\Delta^2}{2E} \cos(2\theta_{21})$
- ▶ $V \gg \frac{\Delta^2}{2E} \cos(2\theta_{21})$: $\nu_e \sim$ mass eigenstate

What is V_e ?

$$H_{\text{mat}} = \dots + \begin{bmatrix} -\frac{\Delta^2}{4E} \cos 2\theta + V_e & \frac{\Delta^2}{4E} \sin 2\theta \\ \frac{\Delta^2}{4E} \sin 2\theta & \frac{\Delta^2}{4E} \cos 2\theta \end{bmatrix}$$

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$$\Delta m_{21}^2 \simeq 7.5 \pm \times 10^{-5} \text{ eV}^2$$

$$V_e = \sqrt{2} G_F n_e \simeq 8 \text{ eV} \frac{\rho Y_e}{10^{14} \text{ g/cm}^3}$$

$$Y_e = \frac{n_e}{n_n + n_p}, \quad \rho = \begin{cases} 10 \text{ g/cm}^3 & \text{earth} \\ 100 \text{ g/cm}^3 & \text{sun} \\ 10^{14} \text{ g/cm}^3 & \text{SN} \end{cases}$$

For $\bar{\nu}$ V_e of opposite sign! (because

$$\langle \text{out} | \bar{\nu} \hat{\nu} | \text{in} \rangle \sim \langle \text{out} | \hat{a}^\dagger \hat{a} + \hat{b} \hat{b}^\dagger | \text{in} \rangle$$

\Rightarrow solar matter effect for ν_e , not $\bar{\nu}_e$, fixes sign of $m_2^2 - m_1^2 > 0$.

Mass scale

First of 3 probes of the mass scale :cosmology

- a late contribution to DM in cosmology :
relic ν free-stream til they become non-rel. (after recomb. for $\Sigma \lesssim \text{eV}$), then contribute to DM $\propto \sum_i |m_i| \equiv \Sigma$.

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- Σ has effects on CMB :

Relativistic \rightarrow non-rel transition affects CMB propagation...parameter in cosmological fits :

Lesgourgues book

$$\begin{aligned} \Sigma &\lesssim 0.1 \rightarrow .6 \text{ eV} && \text{now : PLANCK, +LSS/Ly}\alpha \text{ (in } \Lambda\text{CDM)} \\ &\lesssim 0.6 \text{ eV} && \text{now : PLANCK + BAO (in 12 param } \Lambda\text{CDM)} \\ \rightarrow &\lesssim 2m_{\text{atm}} && \text{cosmo.indep. (Planck + EUCLID...)} \\ &\sim m_{\text{atm}} && \Lambda\text{CDM} \end{aligned}$$

DiValentino etal
1507.06646

beta decay

m_ν^2 distorts e spectrum in $n \rightarrow p + e + \bar{\nu} \Leftrightarrow$ bound

Consider Tritium β decay :

$${}^3\text{H} \rightarrow {}^3\text{He} + e + \bar{\nu}_e, \quad Q = E_e + E_\nu = 18.6\text{eV}$$

where $E_e = Q - E_\nu \leq Q - "m_{e\nu}"$

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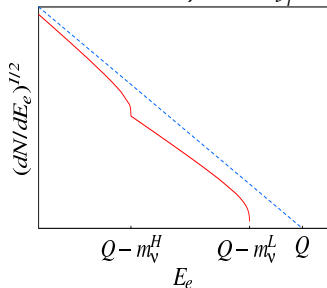
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Endpoint of e spectrum :

$$\frac{dN_e}{dE_e} \propto \sum_i |U_{ei}|^2 \sqrt{(18.6 \text{ keV} - E_e)^2 - m_{\nu_i}^2}$$



Current Katrin bound $\gtrsim 0.3 \text{ eV}$.

beta decay

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Neutrinoless double beta decay : looking for lepton *number* violation

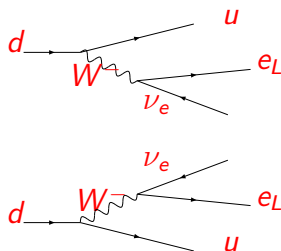
Single β decay kinematically forbidden for some nuclei

(eg ${}_{32}^{76}\text{Ge}$ lighter than ${}_{33}^{76}\text{As}$, so ${}_{32}^{76}\text{Ge} \rightarrow {}_{34}^{76}\text{Se} + ee\bar{\nu}_e\bar{\nu}_e$. $\tau \sim 10^{21}$ yrs)

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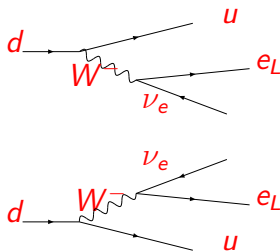
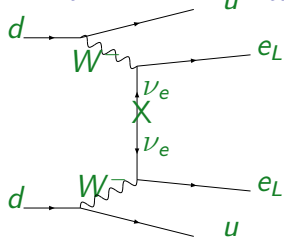
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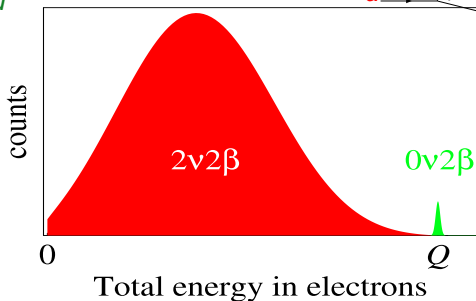
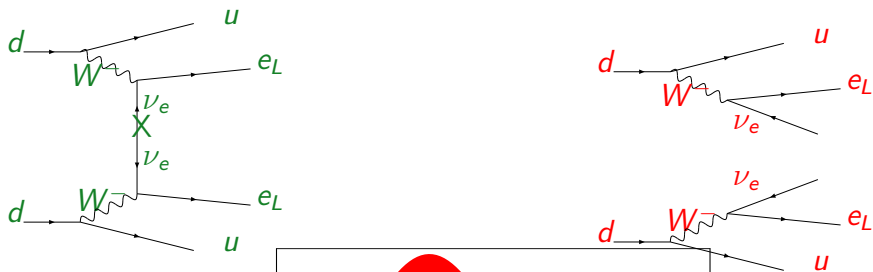
Single β decay kinematically forbidden for some nuclei

(eg ${}^{76}_{32}\text{Ge}$ lighter than ${}^{76}_{33}\text{As}$, so ${}^{76}_{32}\text{Ge} \rightarrow {}^{76}_{34}\text{Se} + ee\bar{\nu}_e\bar{\nu}_e$. $\tau \sim 10^{21}$ yrs)



for majorana neutrinos, or other LNV, but not Dirac neutrinos.

Neutrinoless double beta decay : $(Z, A) \rightarrow (Z + 2, A) + 2e$



Summary

1. neutrinos are crucial astrophysical and cosmological participants in the history of our Universe...much yet to learn about what they do
2. neutrinos are massive — we see oscillations— but we don't know how many light neutrinos, whether $\nu = \bar{\nu}$, whether there is CP violation, ...
3. neutrinos share a weak doublet with charged leptons : maybe we can learn about neutrino mass mechanism by studying flavour-change among charged leptons?
4. although at colliders, neutrinos are just missing energy :(

Can neutrinos make the Universe we see? Leptogenesis

a class of recipes, that use majorana neutrino mass models to generate the matter excess

- ▶ what matter excess?
- ▶ required ingredients?
- ▶ a simple seesaw model
- ▶ how it works...

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\Rightarrow Question : where did that excess come from ?

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- ▶ (only theory explaining coherent temperature fluctuations in microwave background that arrive from causally disconnected regions today...)
- ▶ "60 e-folds" inflation $\equiv V_U \rightarrow > 10^{90} V_U$

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3. created/generated/cooked after inflation...

Three ingredients to prepare in the early U (old russian recipe)

1. B violation : if U_{universe} starts in state of $n_B - n_{\bar{B}} = 0$, need \mathcal{B} to evolve to $n_B - n_{\bar{B}} \neq 0$

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From end inflation \rightarrow BBN, Universe is an expanding,
cooling thermal bath, so non-equilibrium from :

- ▶ slow interactions : $\tau_{int} \gg \tau_U = \text{age of Universe}$
($\Gamma_{int} \ll H$)
- ▶ phase transitions :

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Yes? proton appears stable : $\tau_p \gtrsim 10^{33}$ yrs ($\tau_U \sim 10^{10}$ yrs).

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Electroweak field configurations "of non-zero winding number" are sources of a doublet lepton and three (for colour) doublet quarks for each generation.

SM B+L violation : rates

't Hooft
Kuzmin Rubakov+
Shaposhnikov

At $T = 0$ is tunneling process (from winding # to next, "instanton") : $\Gamma \propto e^{-8\pi/g^2}$

At $0 < T < m_W$, can climb over the barrier :

$$\Gamma_{B+L} \sim \begin{cases} e^{-m_W/T} & T < m_W \\ \alpha^5 T & T > m_W \end{cases}$$

\Rightarrow fast SM B+L at $T > m_W$

SM B+L called "sphalerons"

\Rightarrow if produce a lepton asym, "sphalerons" partially transform to a baryon asym. !!

*** SM B+L is $\Delta B = \Delta L = 3 (= N_f)$. No proton decay! ***

Summary of preliminaries : A Baryon excess today :

- Want to make a baryon excess $\equiv Y_B$ after inflation, that corresponds today to ~ 1 baryon per 10^{10} γ s.
- Three required ingredients : \mathcal{B} , \mathcal{CP} , \mathcal{TE} .
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\Rightarrow *evidence for physics Beyond the Standard Model (BSM)*

One observation to fit, many new parameters...

\Rightarrow *prefer BSM motivated by other data* $\Leftrightarrow m_\nu \Leftrightarrow$
seesaw! (uses non-pert. SM $B \nleftrightarrow L$)

Type 1 seesaw, one generation

Add to SM a massive N (right-handed neutrino), without weak interactions, but mass-mixing to ν_L :

$$+m_D\overline{\nu}_L N \qquad +\frac{M}{2}\overline{N^c} N + h.c.$$

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\Rightarrow neutrino mass matrix :

$$\left(\overline{\nu}_L \quad \overline{N^c} \right) \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \end{pmatrix} \qquad (\nu_L^c \equiv (\nu_L)^c)$$

\Rightarrow eigenvectors \simeq : ν_L with $m_\nu \sim \frac{m_D^2}{M}$, N with mass $\sim M$

The type I seesaw, 3 generations

Minkowski, Yanagida
Gell-Mann Ramond Slansky

- add 3 singlet N to the SM in charged lepton and N mass bases :

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{\alpha J} \overline{N}_J \ell_\alpha \cdot H - \frac{1}{2} \overline{N}_J M_J N_J^c$$

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- at low scale, for $M \gg m_D = \lambda v$, light ν mass diagram



9 parameters :
 m_1, m_2, m_3
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$$[m_\nu] = \lambda M^{-1} \lambda^T v^2$$

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for $\lambda \sim h_t$, $M \sim 10^{15}$ GeV $\sim .05$ eV
 $\lambda \sim 10^{-6}$, $M \sim$ TeV

“natural” $m_\nu \ll m_f$, but N hard to detect ?

Leptogenesis in the type 1 seesaw : usually a Fairy Tale

Fukugita Yanagida
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Covi et al
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If this asymmetry can escape the big bad wolf of thermal equilibrium...

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 - 2 The temperature drops below M , N population decays away.
 - 3 In the \cancel{CP} and \cancel{L} interactions of the N , an asymmetry in SM leptons is created.
 - 4 If asymmetry escapes the wolf of thermal equilibrium...
 - 5 the lepton asym gets partially reprocessed to a baryon asym by non-perturbative $B + L$ -violating SM processes ("sphalerons")
- And the Universe lived happily ever after, containing many photons. And for every 10^{10} photons, there were 6 extra baryons (wrt anti-baryons).

Summary

Leptogenesis is a class of recipes, that use (majorana) neutrino mass models to generate the matter excess.

These scenarios generate a lepton asymmetry (before the Electroweak Phase Transition), and the non-perturbative SM $B+L$ violn reprocesses it to a baryon excess.

- ★ efficient, to use the BSM for m_ν to generate the Baryon Asym.
- ★ using SM $B+L$ violn ($\Delta B = \Delta L = 3$) avoids proton lifetime bound
- ★ seems to work ...rather well, for a wide range of parameters