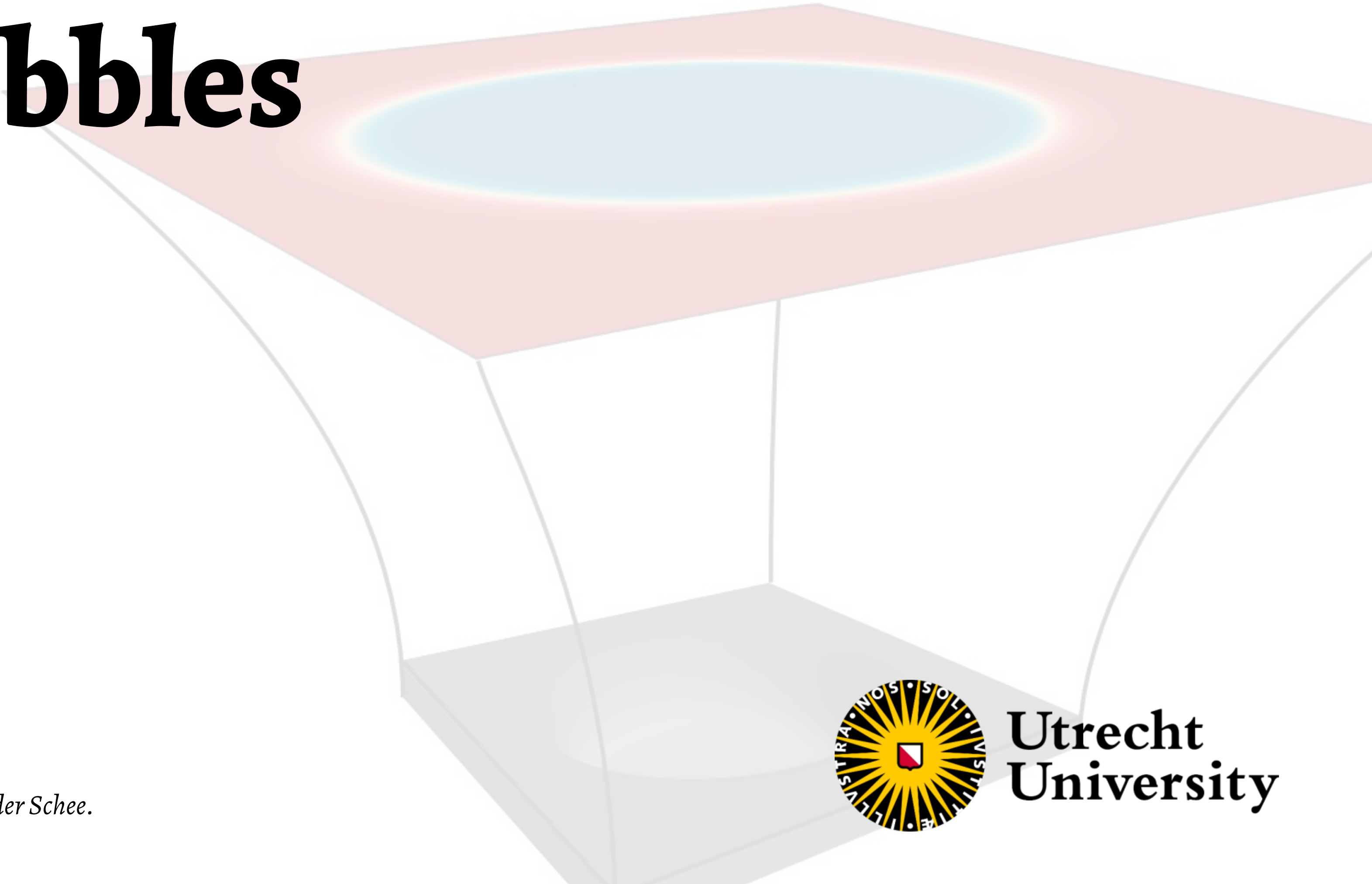
Microscopic Description of Critical Bubbles

Javier Subils
Second Holography and Dense Matter Workshop
September 18, APC (Paris)



Based on **work in progress**, with *D. Mateos*, and *W. van der Schee*.

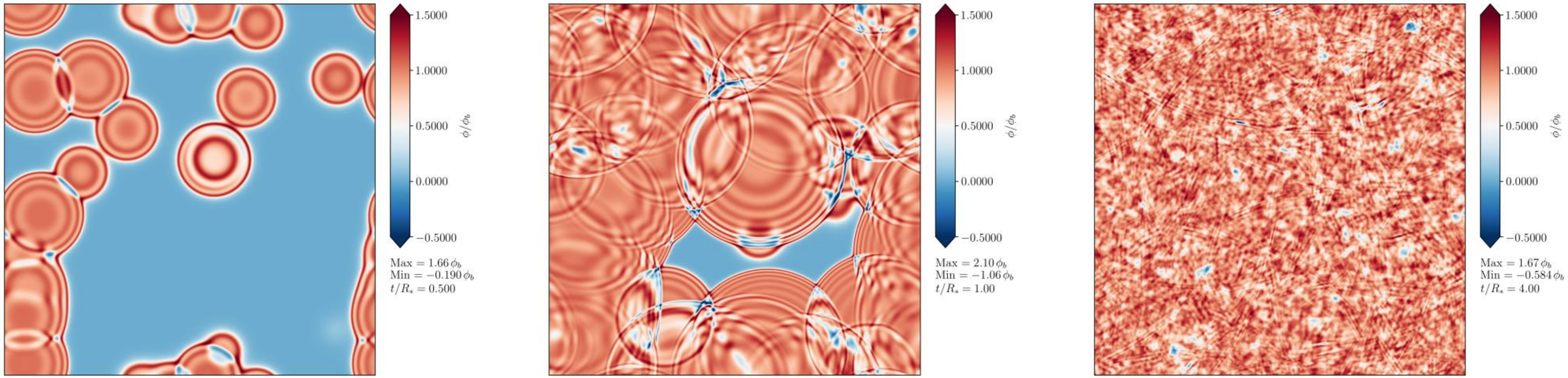
Utrecht University

Phase transitions

Phase transitions are rather ubiquitous phenomena;

Phase transitions in Cosmology and Astrophysics

A first order-phase transition in the early Universe could lead to a detectable gravitational wave signal



[*Cutting, Granados Escartin, Hindmarsh, Weir,* **2005.13537**]

Phase transitions in Cosmology and Astrophysics

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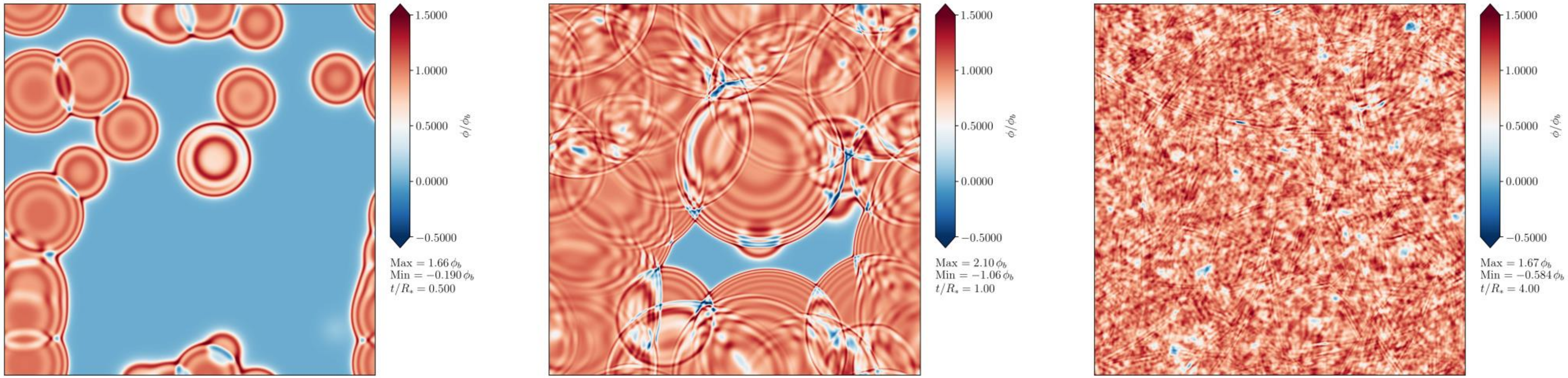
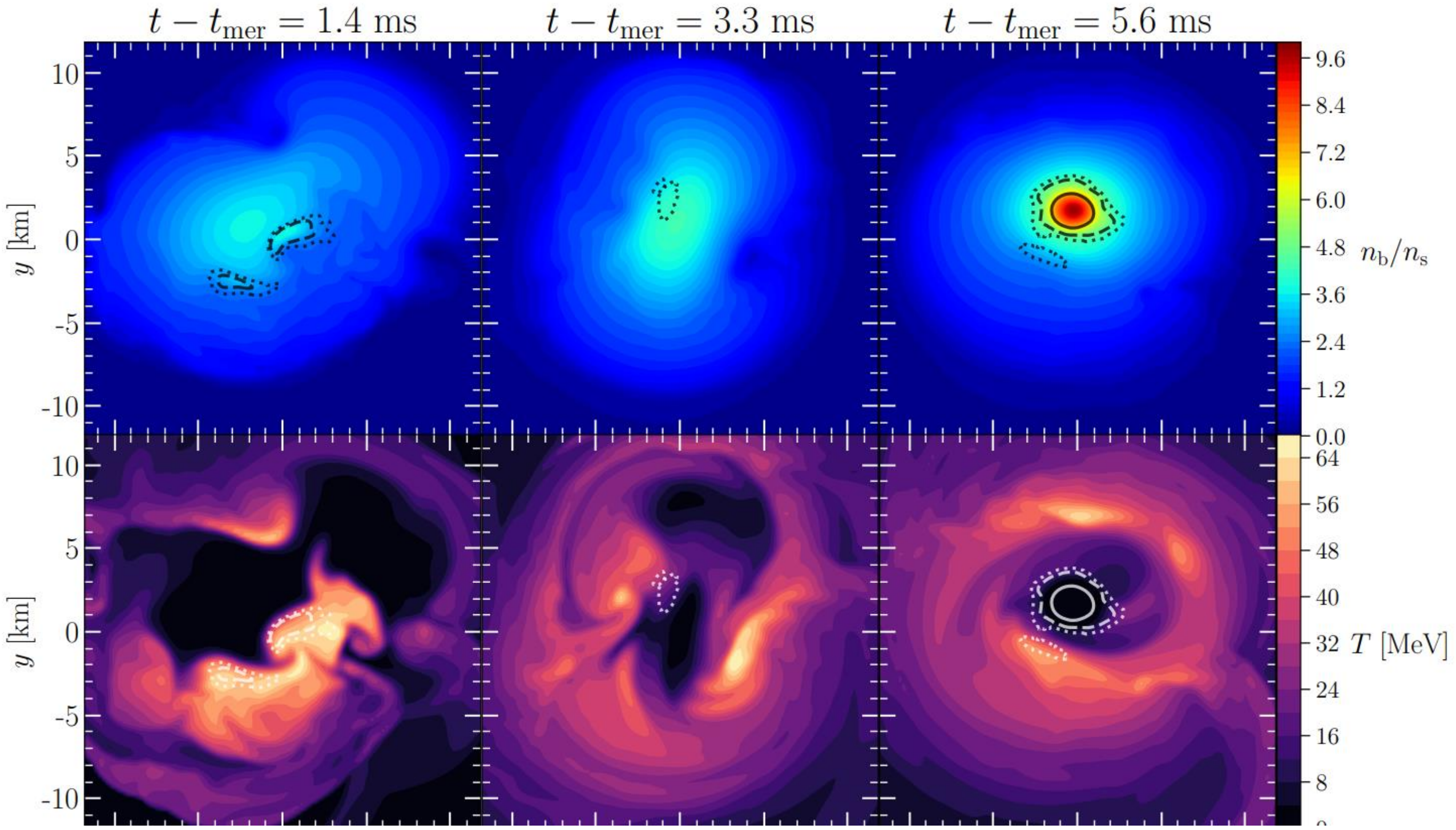


Window to new physics: within the SM, there is no (first-order) phase transition in the thermal history of the Universe.

[*Cutting, Granados Escartin, Hindmarsh, Weir,* ***2005.13537***]

Phase transitions in Cosmology and Astrophysics

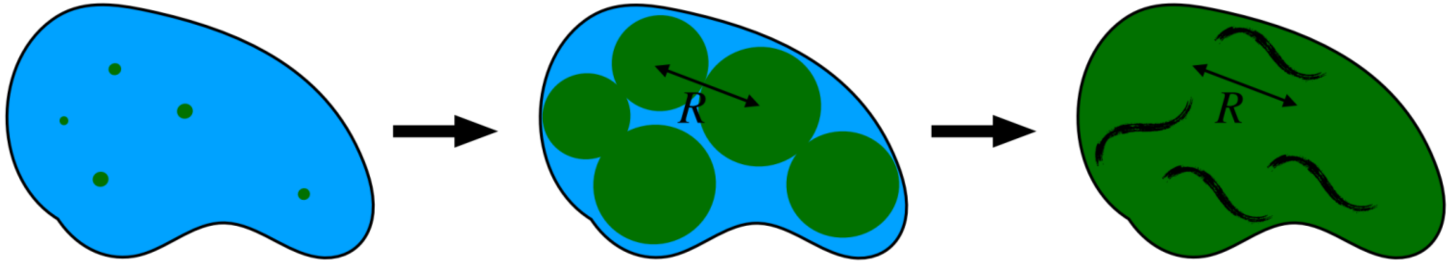
Evidence for the presence of a phase transition in a neutron star collisions



[*Tootle, Ecker, Topolski, Demircik, Jarvinen, Rezzolla*, ***2205.05691***]

Phase transitions in Cosmology and Astrophysics

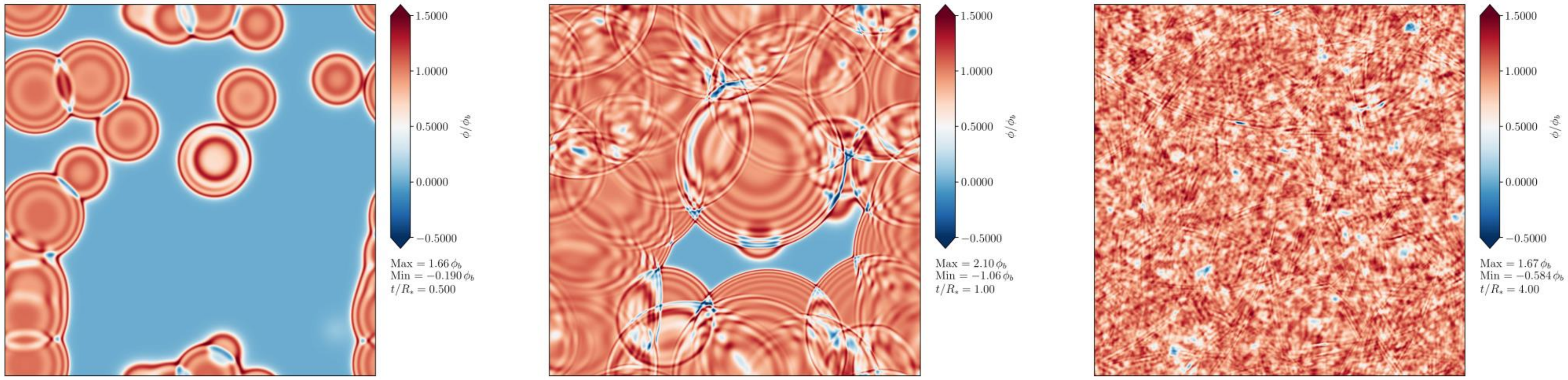
Possible distinctive signal in neutron star mergers

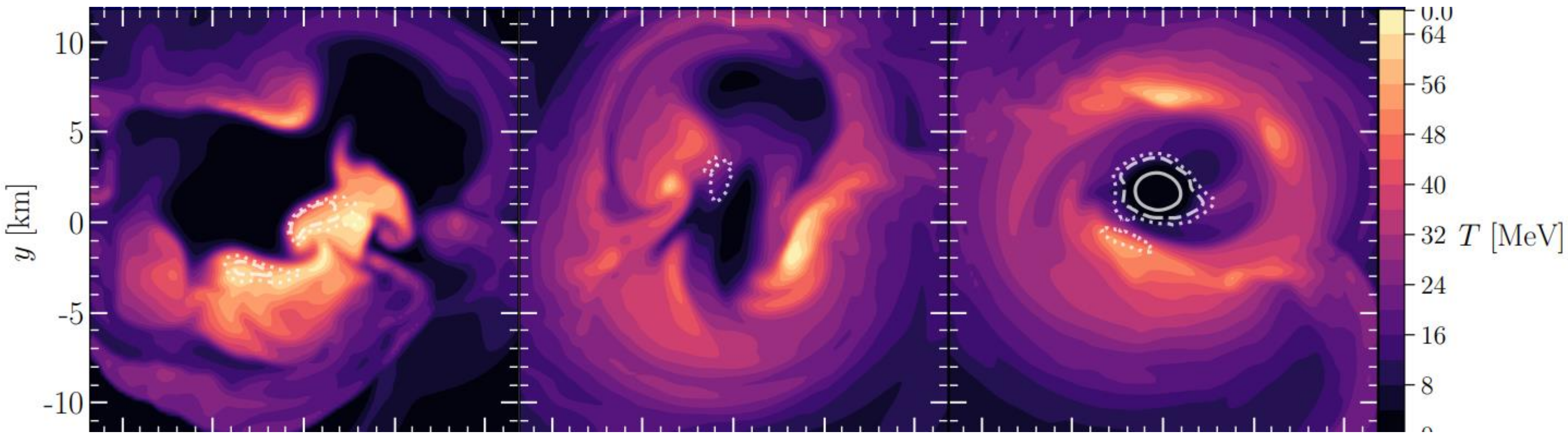


$$f \simeq 0.6\,\mathrm{MHz}$$

[*Casalderrey-Solana, Mateos, Sanchez-Garitaonandia,* **2210.03171**]

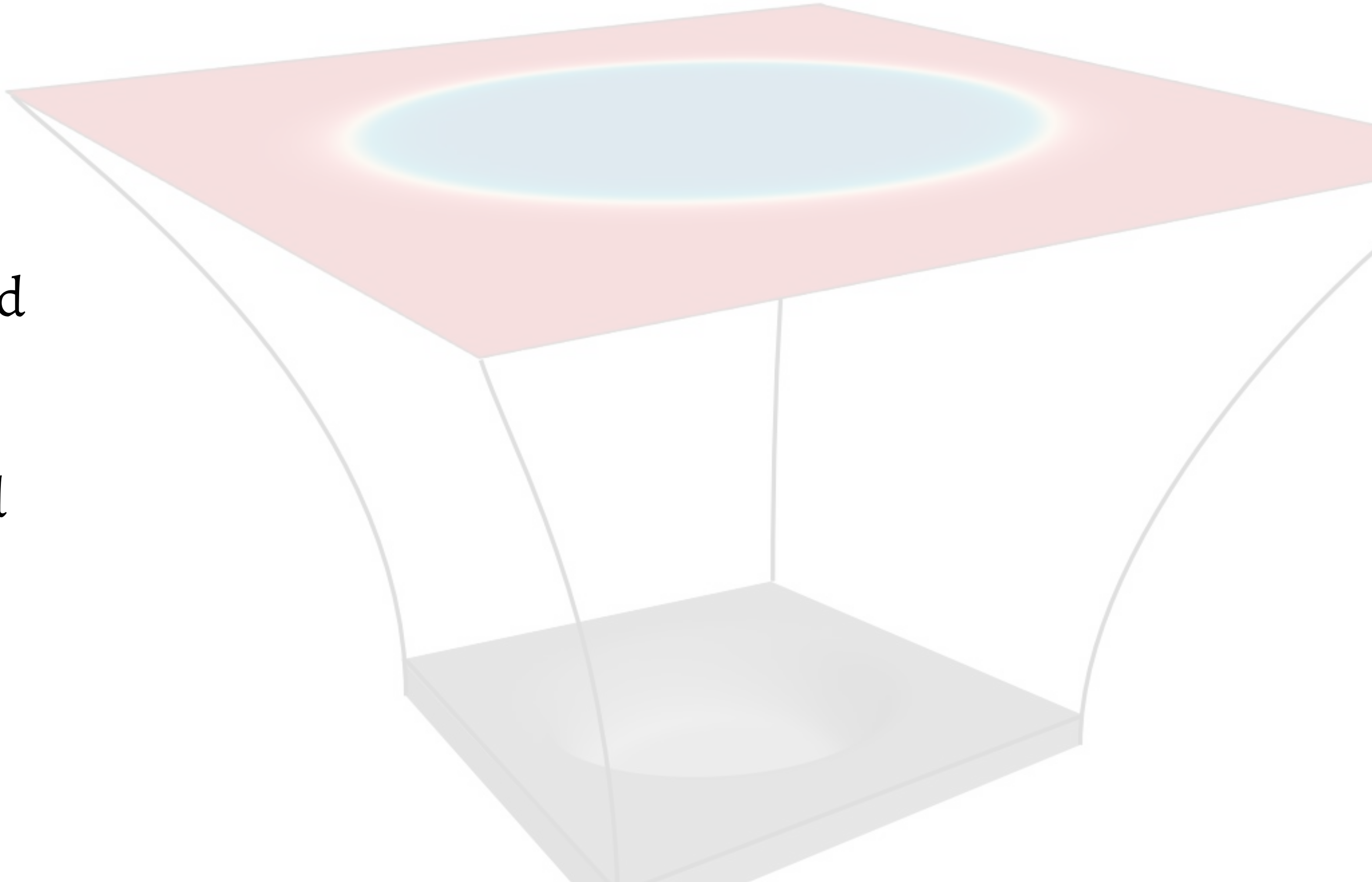
How do bubbles nucleate?





Outline

- Effective field theory approach
- Holographic approach and results
- Conclusions, Outlook and future directions



Effective field theory approach

Effective approach

We need to compute: $\mathcal{Z} = e^{-\beta F} \simeq e^{-S[\phi_{(0)}]}$

Effective action, $S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \left(\frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\nabla \varphi)^2 + V_{\text{eff}}(\varphi, T) \right)$

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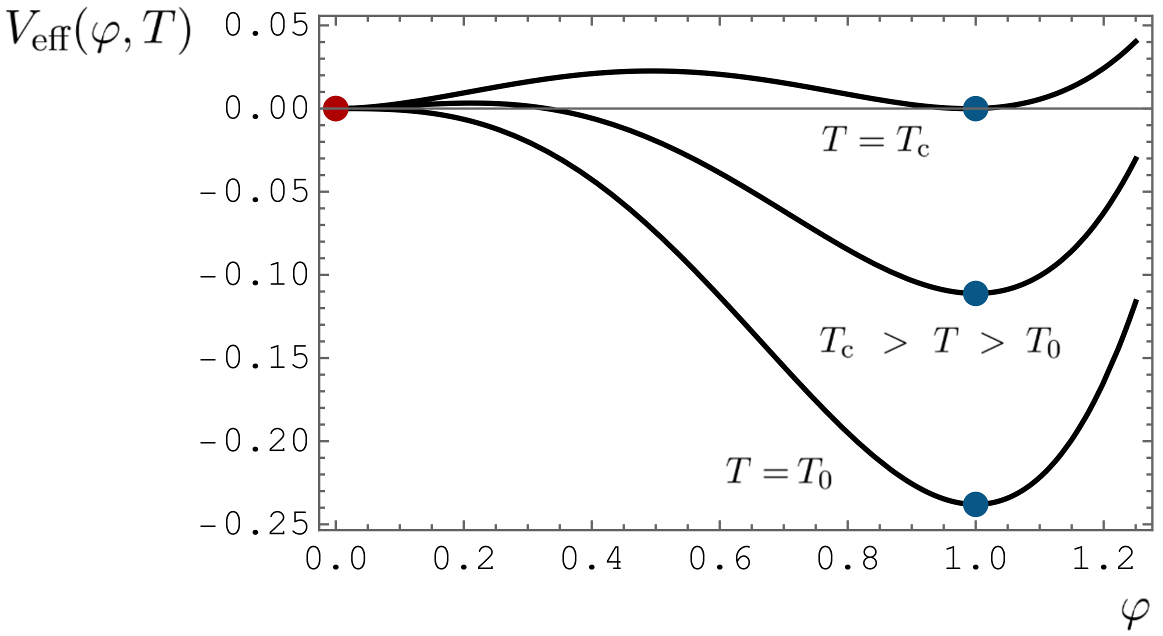

[*Enqvist, Ignatius, Kajantie, Rummukainen*, ***Phys. Rev. D* 45 (1992) 3415**]

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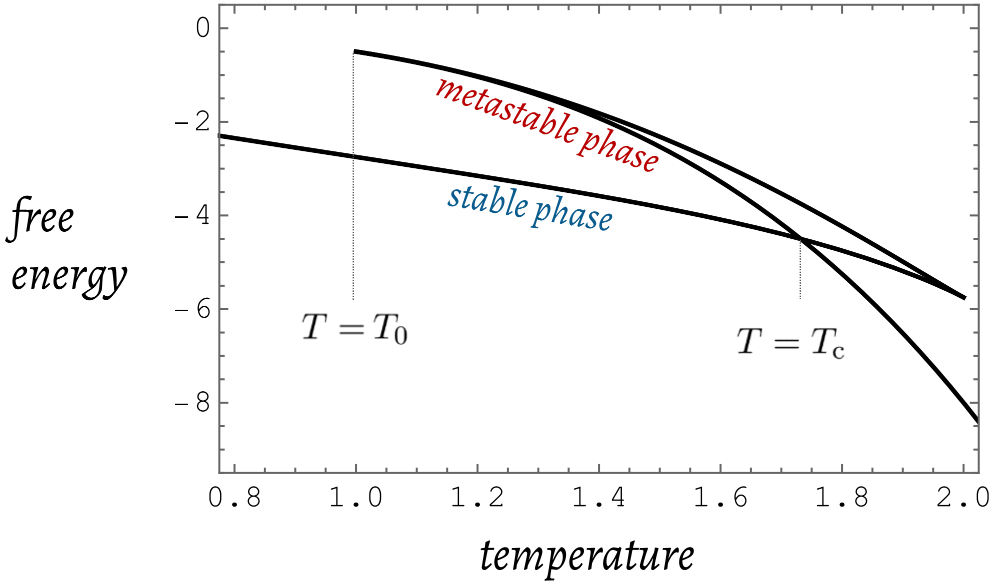

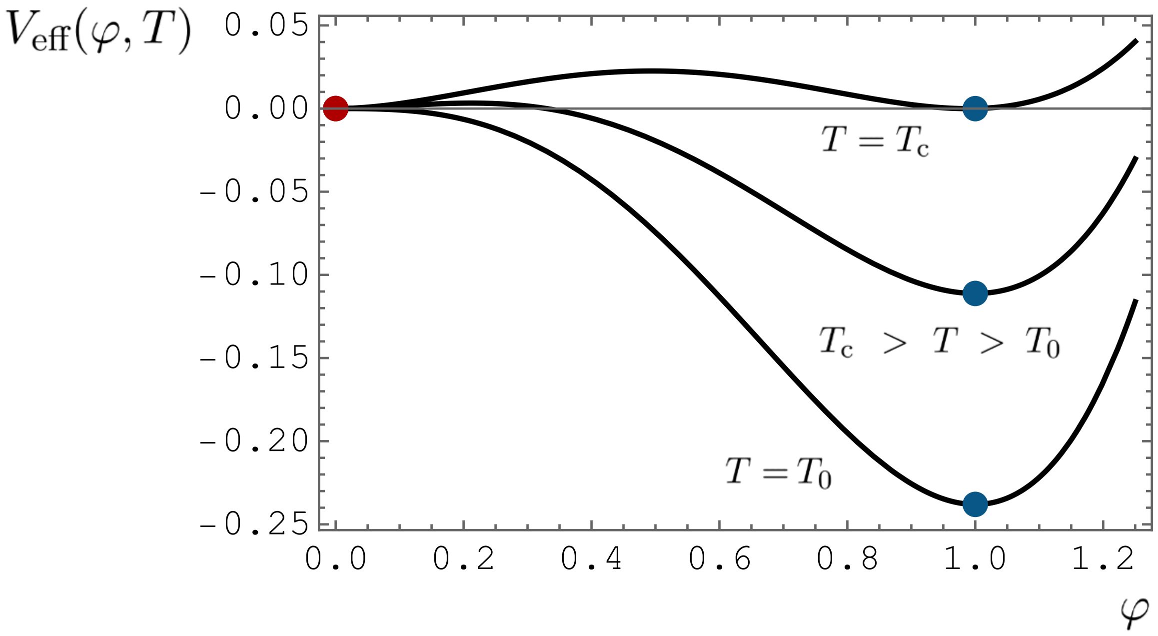

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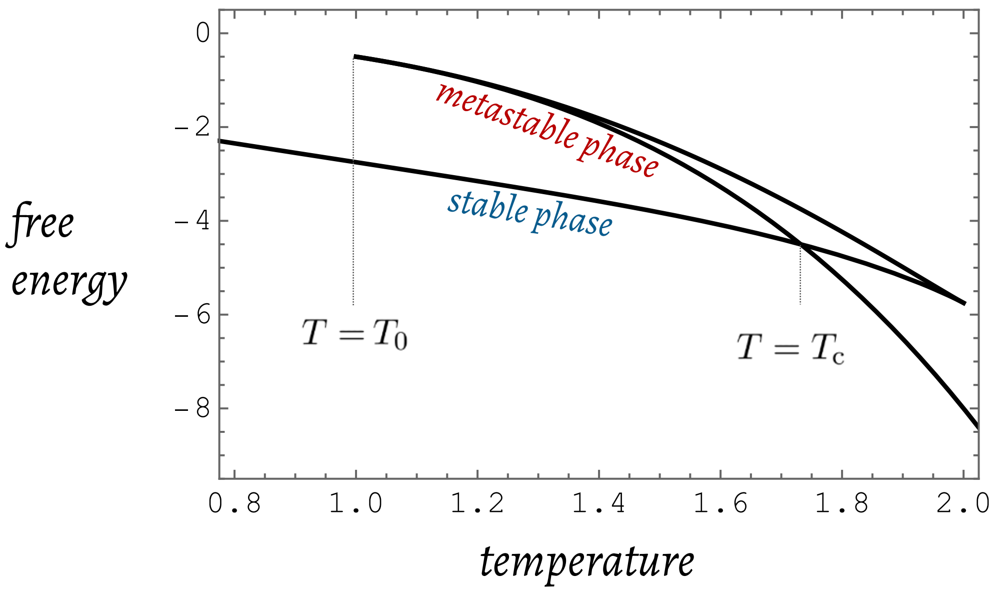

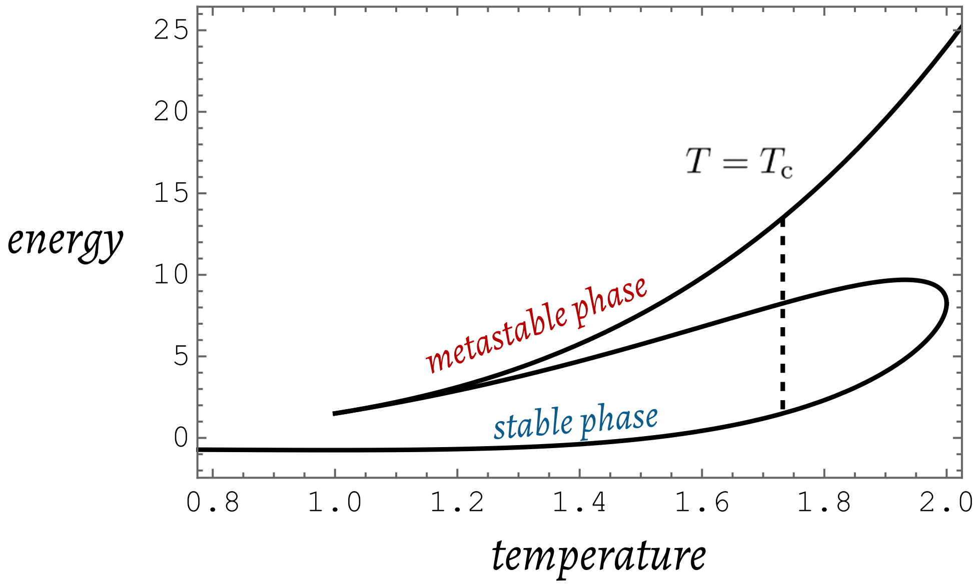

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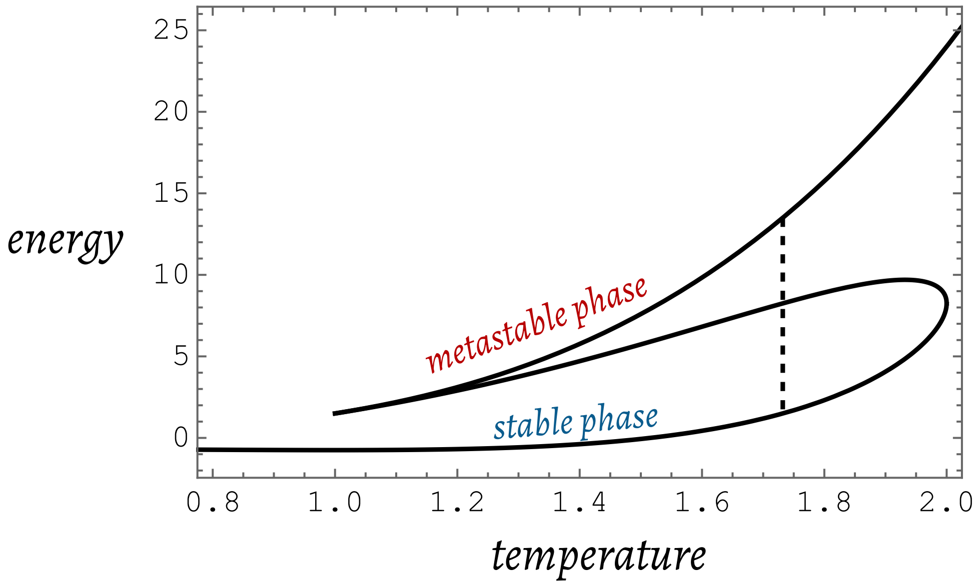
How does the phase transition occur?

- Below T_{c}, bubbles are nucleated.

$$\mathrm{P}(T) = \mathrm{P}_0 e^{-(S_{\text{bubble}} - S_{\text{hom.}})} = \mathrm{P}_0 e^{-\beta \Delta F}$$



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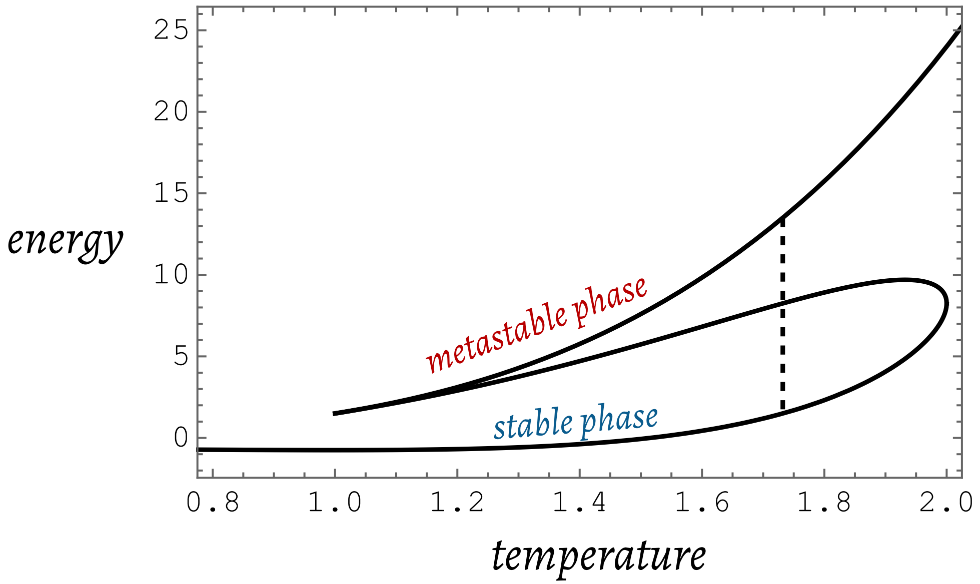
$$\Delta F \simeq 4\pi\sigma R^2 - \frac{4\pi}{3} R^3 \Delta p$$



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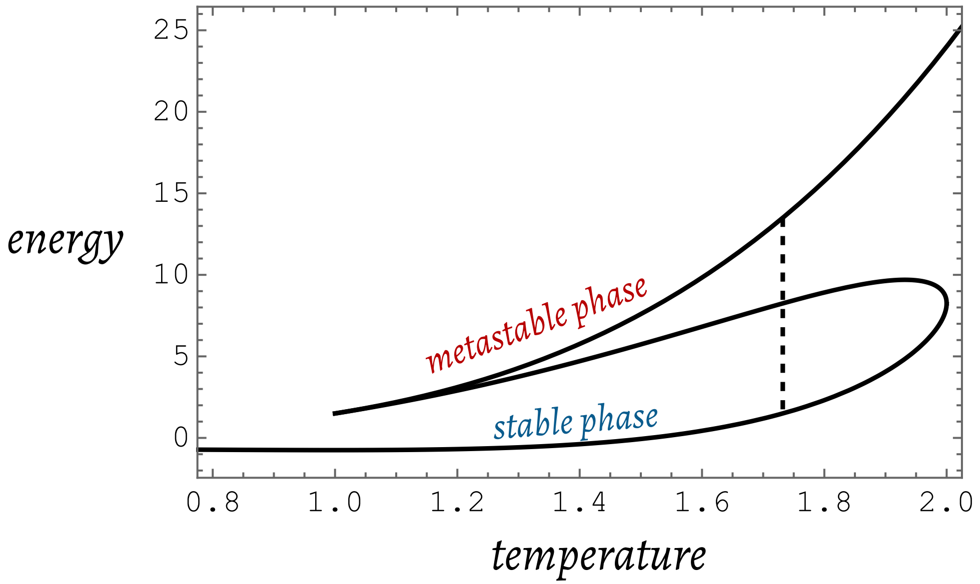

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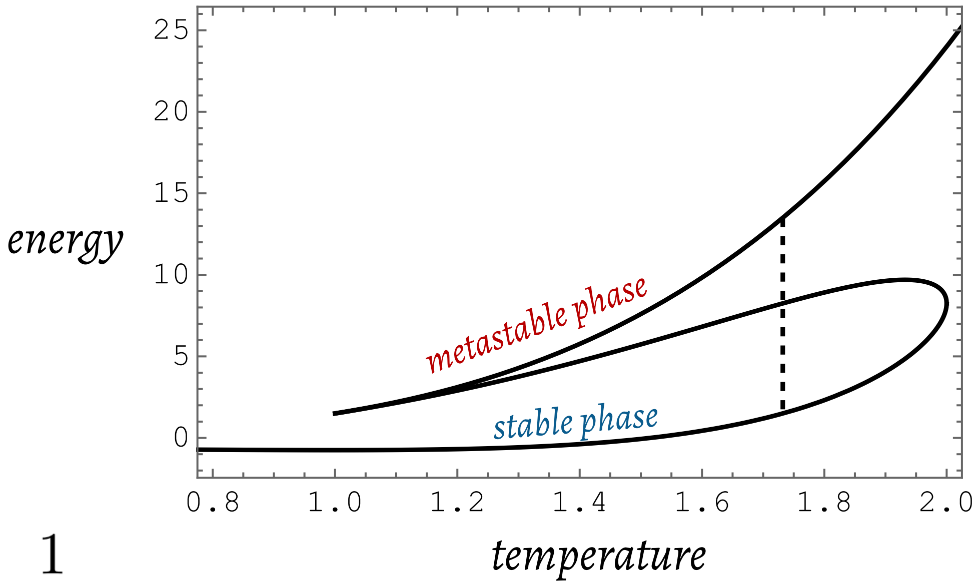
$$R_c = \frac{2\sigma}{\Delta p} \quad \Rightarrow \quad \Delta F = \frac{16\pi}{3}\frac{\sigma^3}{(\Delta p)^2} \propto \frac{1}{(T - T_c)^2}$$



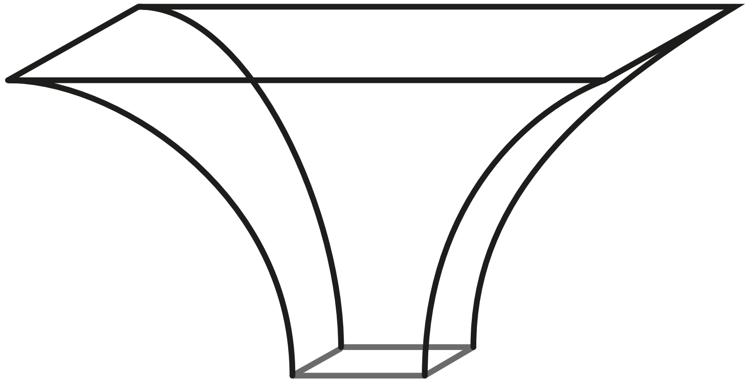
Holographic approach

Holography

Gravity theory in 5 dimensions

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(R - 2\Lambda + \dots\right)$$

Negative cosmological constant,
this is gravity in AdS:



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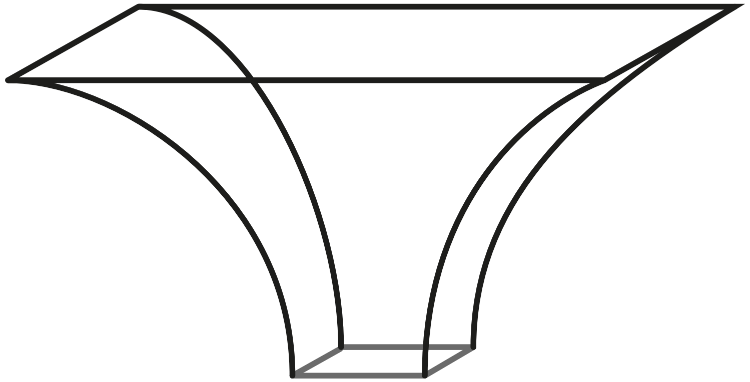
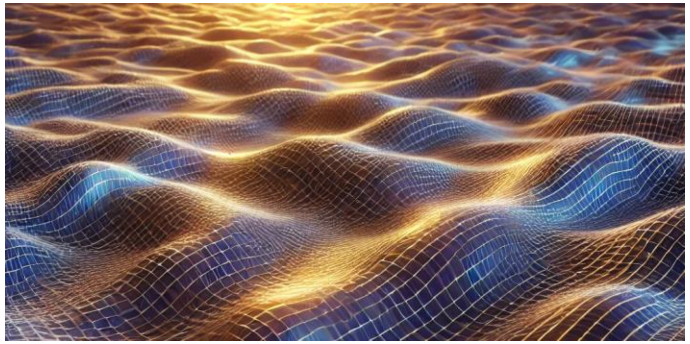


Yang – Mills theory in 4 dimensions

$$S = -\frac{1}{4}\int d^4x\sqrt{-\gamma}\left(F_{\mu\nu}{}^{a}F^{a\mu\nu} + \dots\right)$$

This is a QFT



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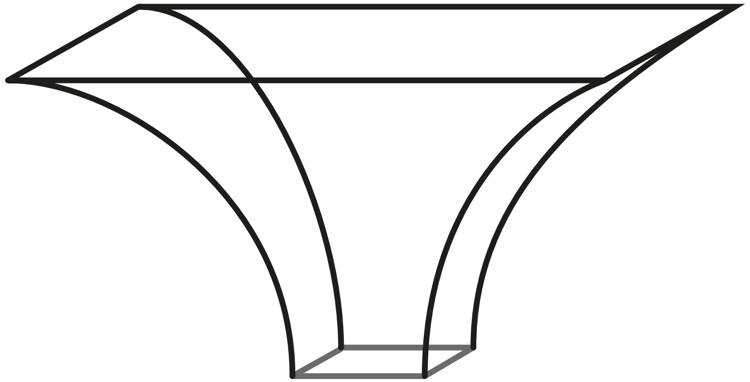

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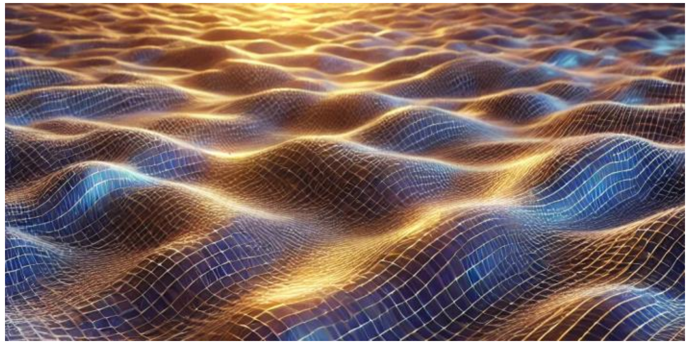

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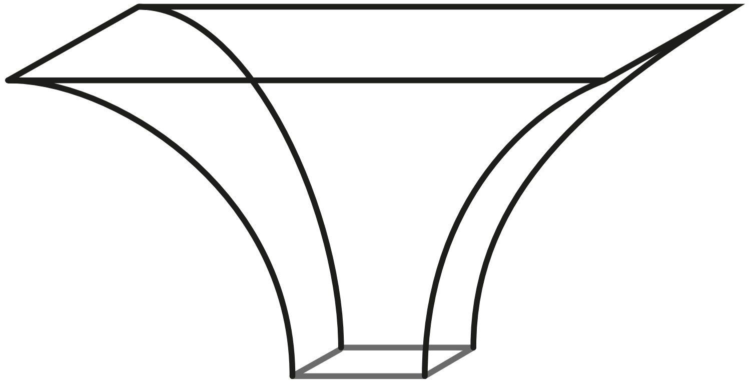

$\longleftrightarrow$

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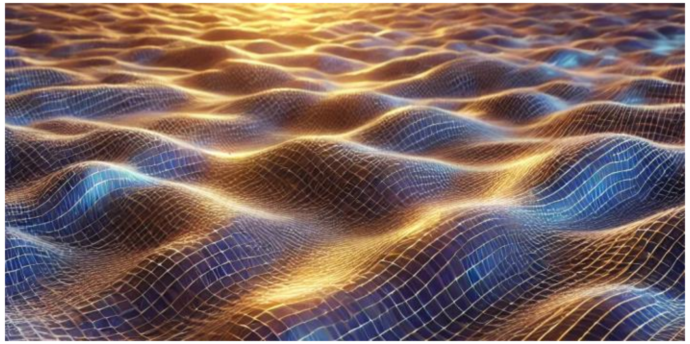
This is a QFT with two meaningful parameters:

- The rank of the group: N
- The ‘t Hooft coupling: $\lambda = g^2 N$



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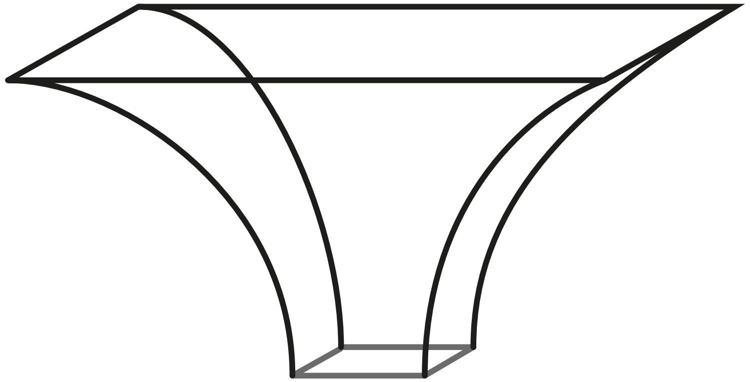

$$\frac{L}{\ell_p} \sim N$$

$$\frac{L}{\ell_s} \sim \lambda$$

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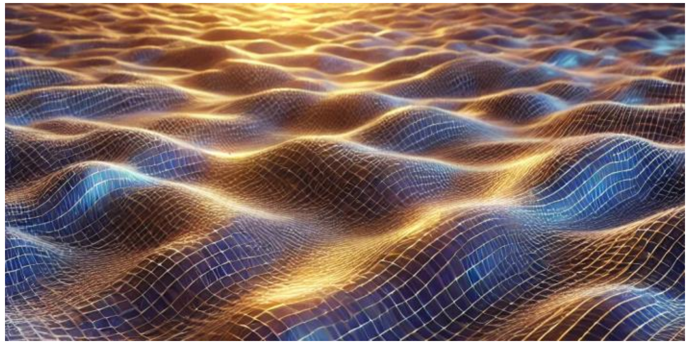

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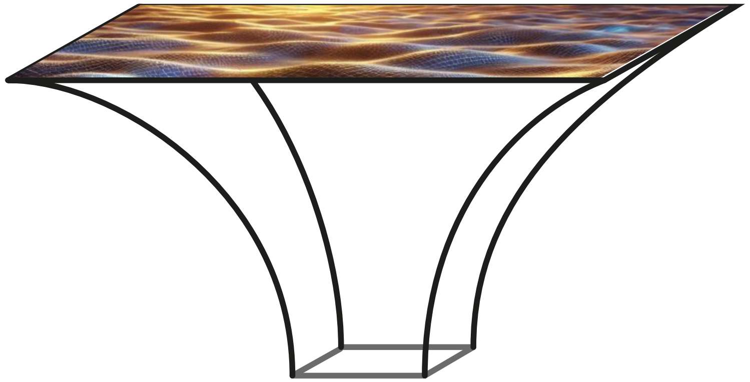

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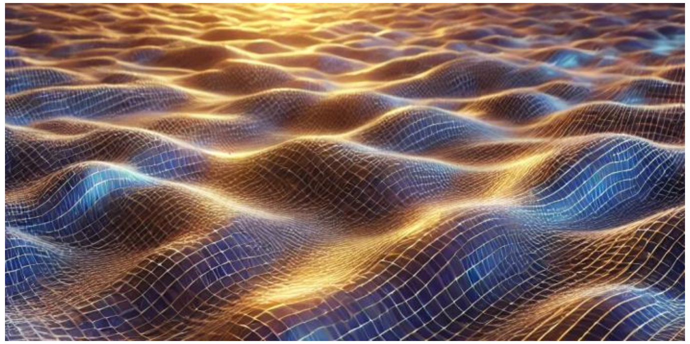
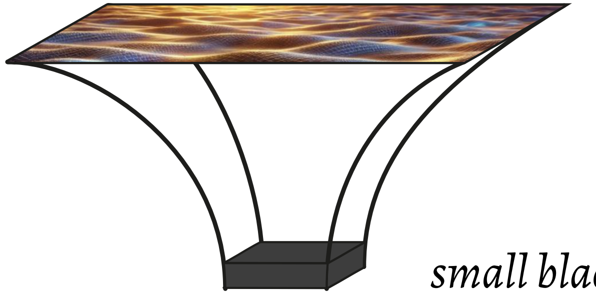


Holography

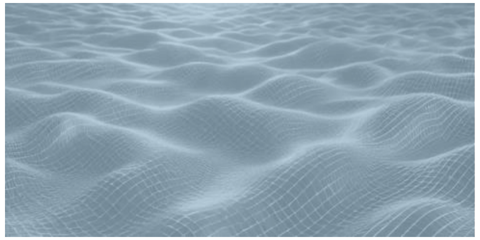
Area
Mass
Surface gravity

Entropy density (**s**)
Entropy density (**ρ**)
Temperature (**T**)

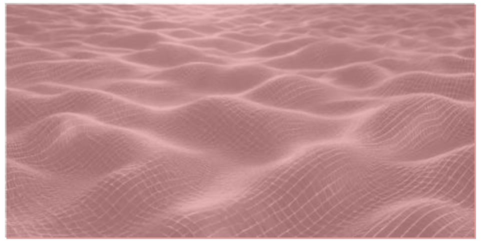


small black hole

big black hole



low energy phase

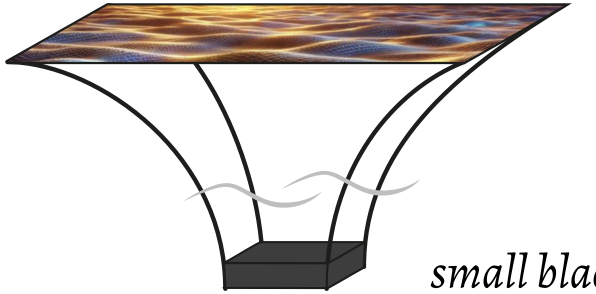


high energy phase

Holography

Area
Mass
Surface gravity
Perturbations of the metric

Entropy density (**s**)
Entropy density (**ρ**)
Temperature (**T**)
Transport properties (**ζ**, **η**)



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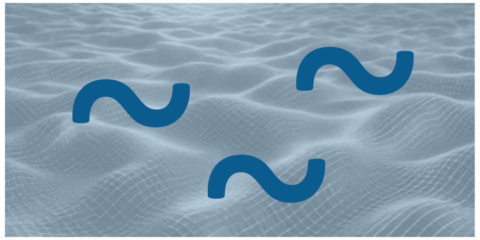

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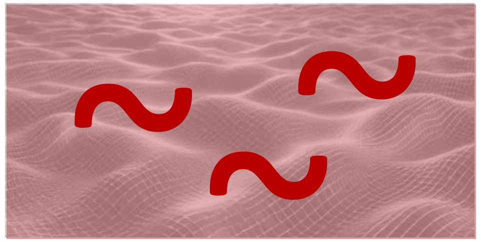
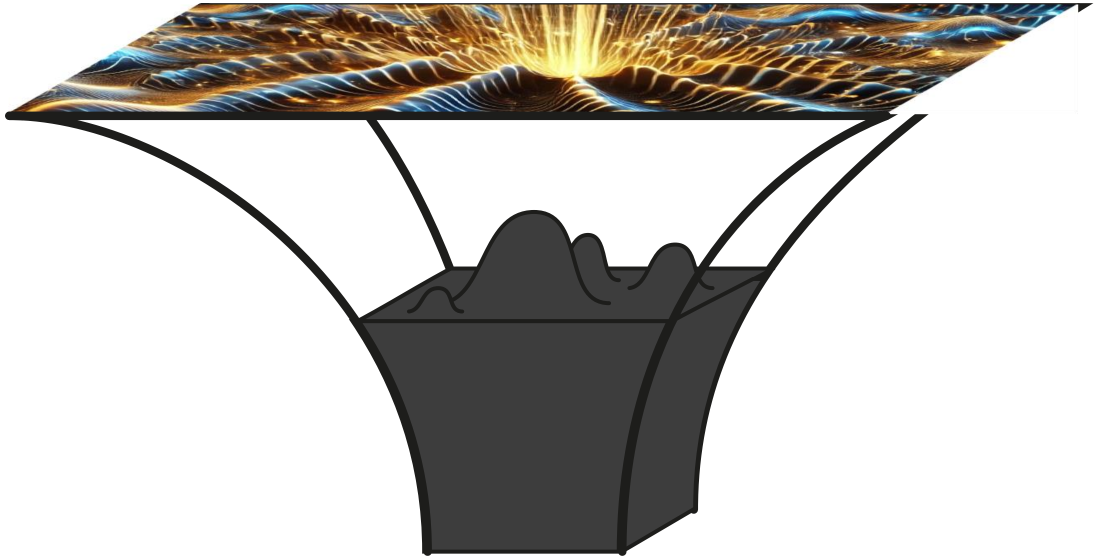


high energy phase

Holography

non-trivial horizon configurations

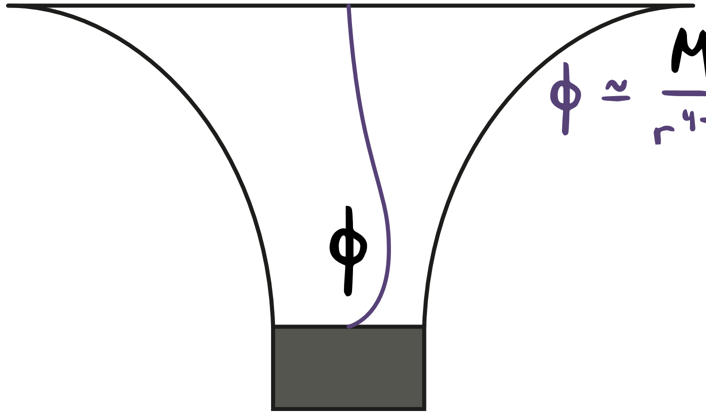
non-trivial field configurations (possibly far-from-equilibrium)



Holography

We can add extra fields in the bulk so that we have richer phase structure:

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(R - 2\Lambda + \ldots + (\partial\phi)^2 + V(\phi)\right) \longleftrightarrow S = -\frac{1}{4}\int d^4x\sqrt{-\gamma}\left(F_{\mu\nu}^{a}F^{a\mu\nu} + \ldots + M\,\mathcal{O}_\Delta\right)$$



$$\phi \simeq \frac{M}{r^{4-\Delta}} + \frac{\langle \mathcal{O}_\Delta \rangle}{r^\Delta} + \ldots$$

Holography

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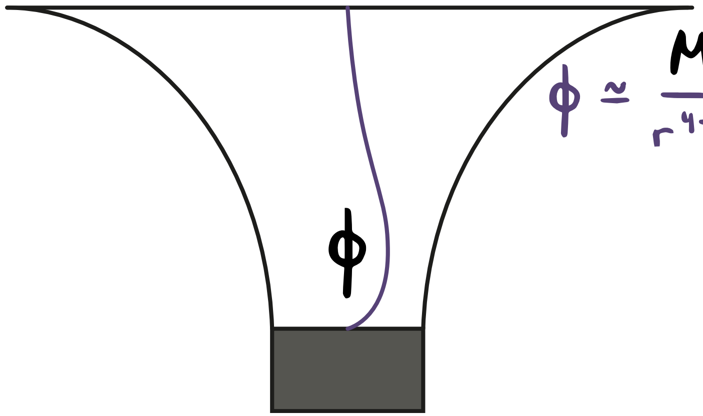
$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G}\left(R - 2\Lambda + \dots + (\partial\phi)^2 + V(\phi)\right)$$

With an appropriate choice of the potential, the system will undergo a phase transition.



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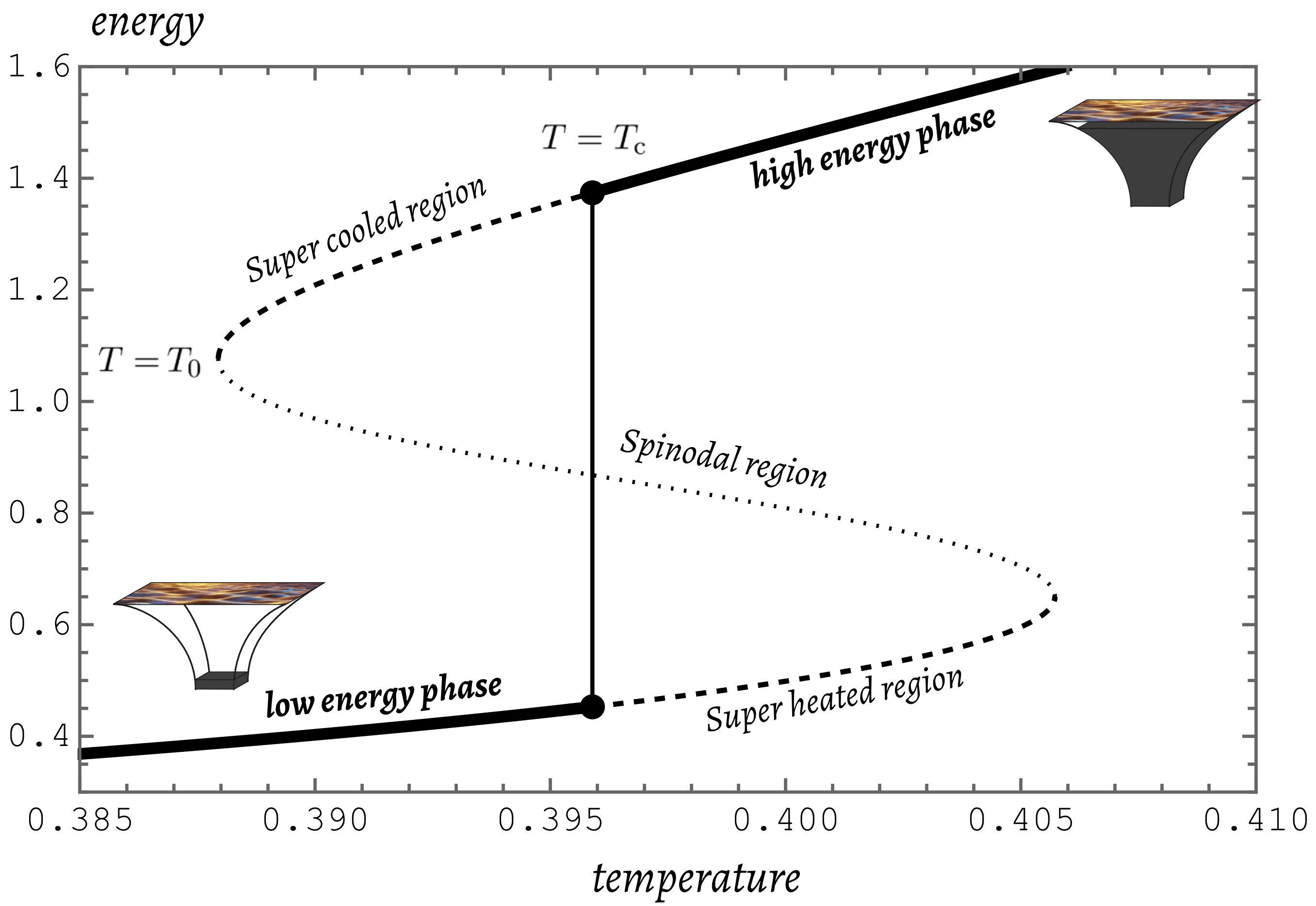

[*Bea, Mateos,* ***1805.01806***]

Inhomogeneous solutions

To write a **well-posed elliptical problem**, we used the *DeTurck trick*.

1. Write down a general ansatz for the metric.
2. Choose a *reference metric*.
3. Define the vector field $\xi^P = G^{MN}(\Gamma^P_{MN} - \overline{\Gamma}^P_{MN})$
4. Solve $R_{MN} - \frac{R}{2}G_{MN} - \left(\nabla_{(M}\xi_{N)} - \frac{1}{2}\nabla_A \xi^A G_{MN}\right) = \kappa_5^2 T_{MN}$
5. Check that $\xi^A = 0$
6. Extract the thermodynamic properties from the solution.

See [*Dias, Santos, Way*, ***1510.02804***]

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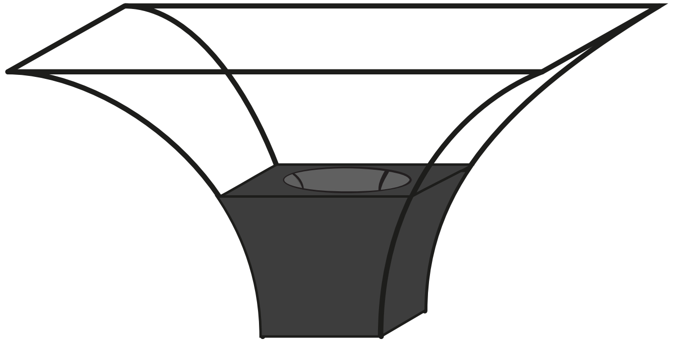

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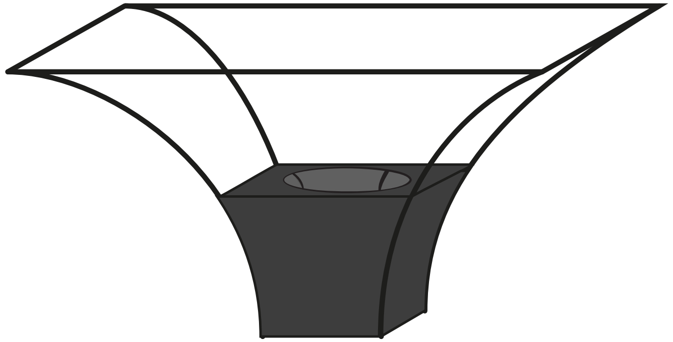

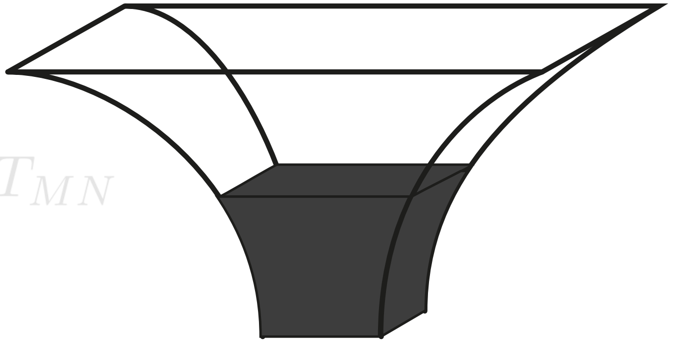

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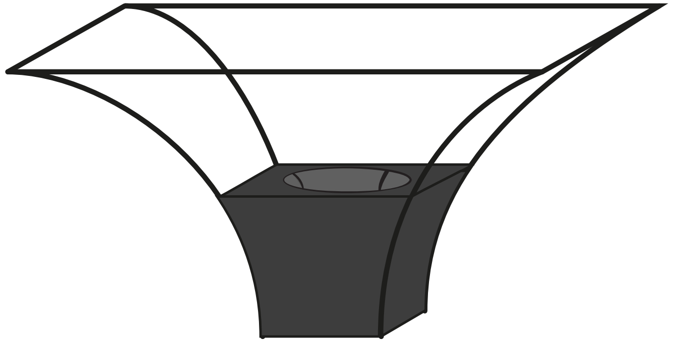
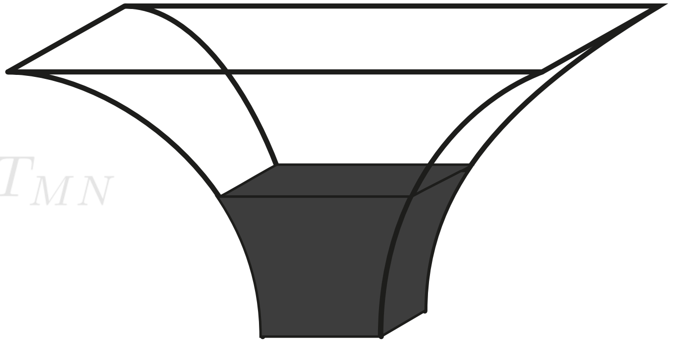


1. Write down a general ansatz for the metric.
2. Choose a *reference metric*.
3. Define the vector field $\xi^P = G^{MN}(\Gamma^P_{MN} - \overline{\Gamma}^P_{MN})$



4. Solve $R_{MN} - \frac{R}{2}G_{MN} - \left(\nabla_{(M}\xi_{N)} - \frac{1}{2}\nabla_A\xi^A G_{MN}\right) = \kappa_5^2 T_{MN}$
5. Check that $\xi^A = 0$
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See [*Dias, Santos, Way*, ***1510.02804***]

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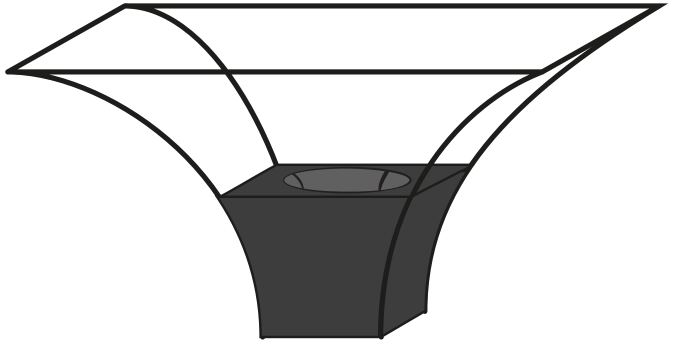

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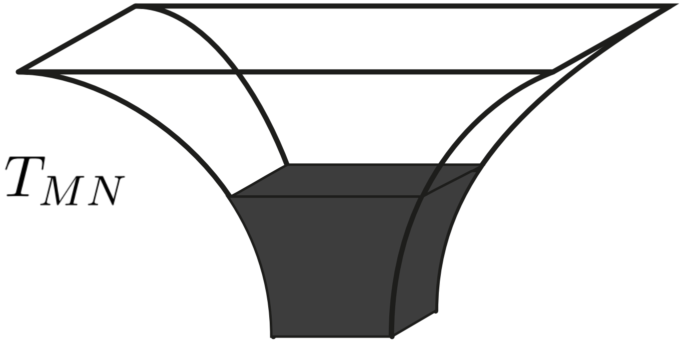

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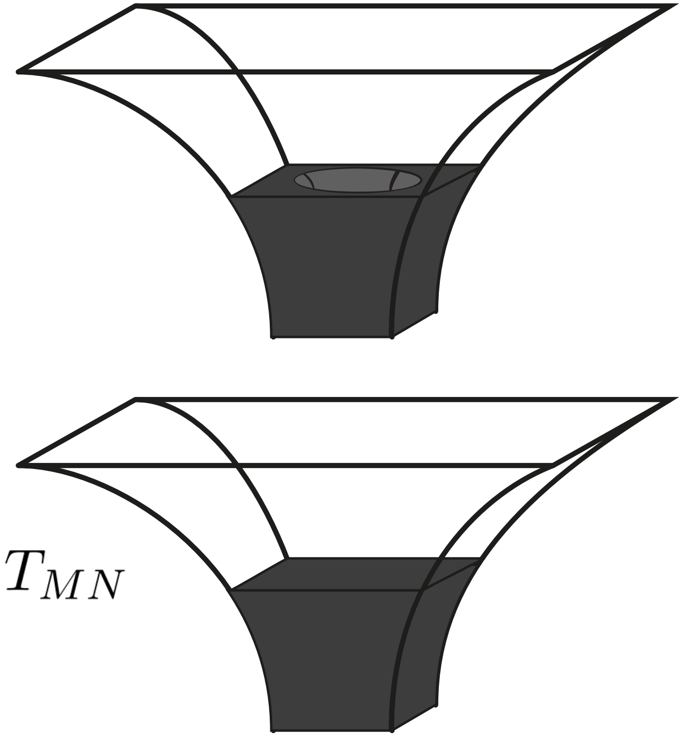

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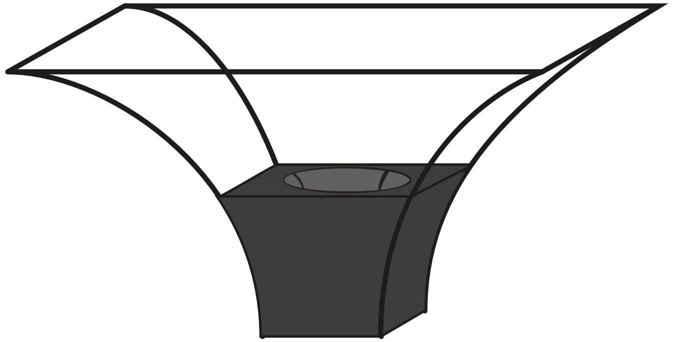

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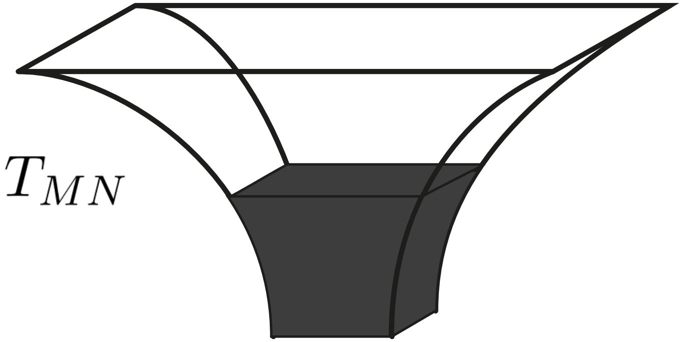

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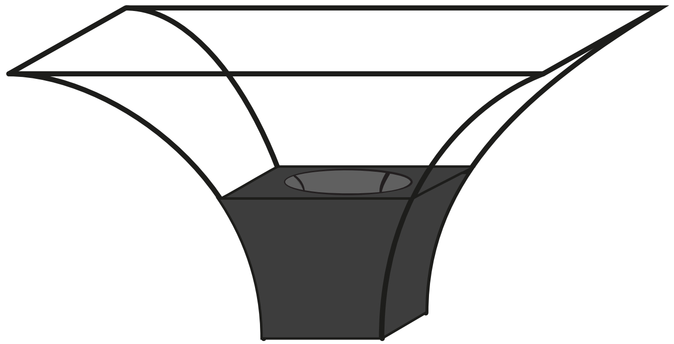

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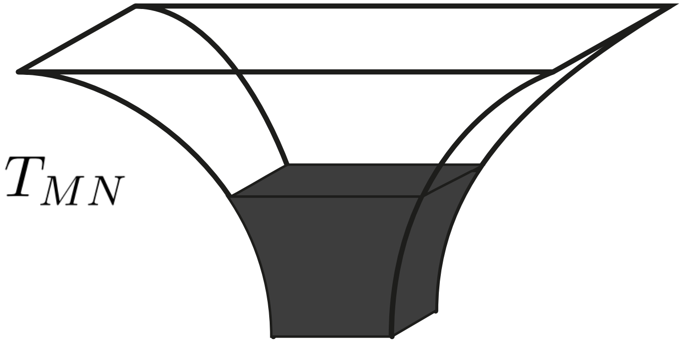

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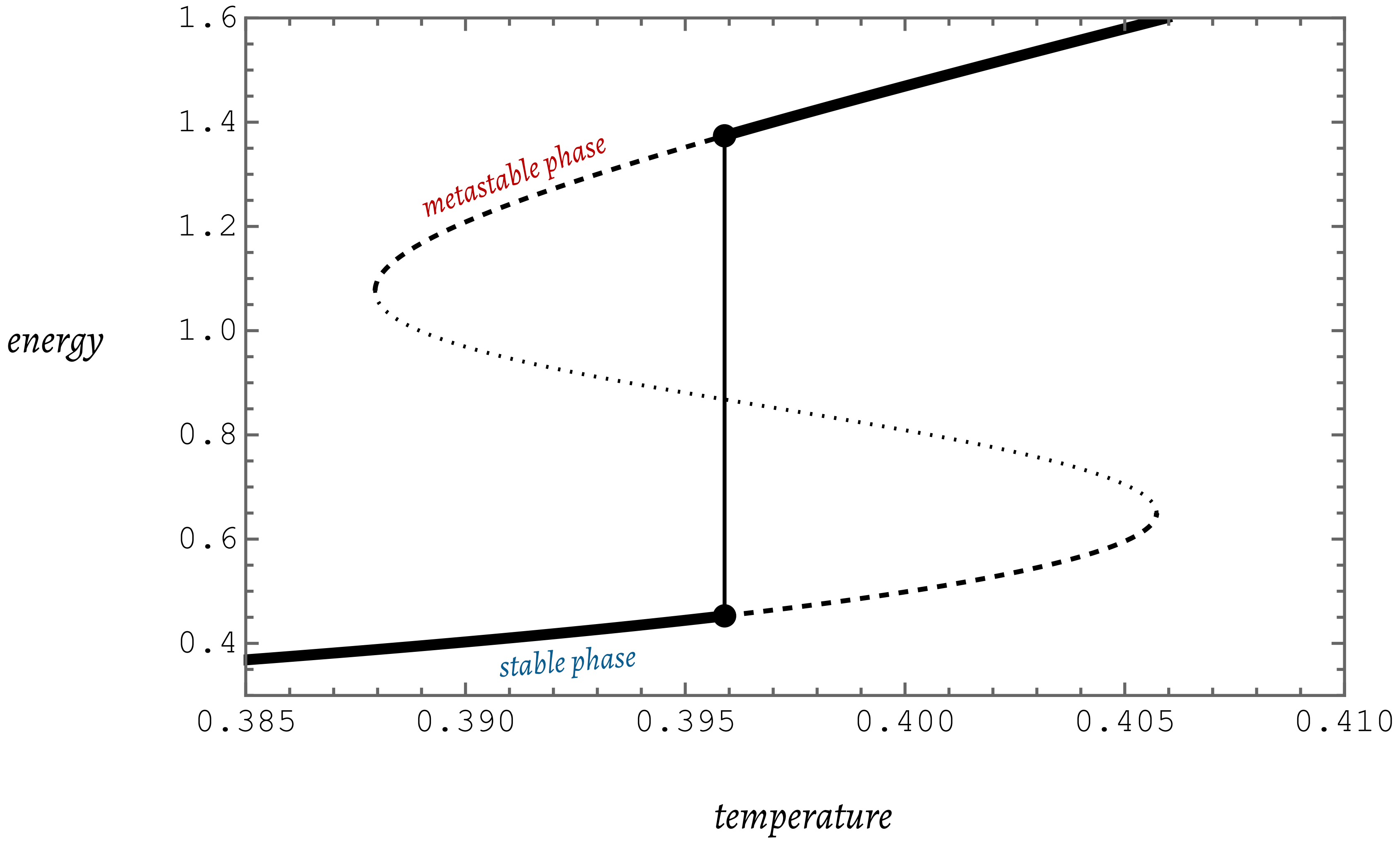
(comment logarithms)

See [*Dias, Santos, Way*, **1510.02804**]

Microscopical description of critical bubbles



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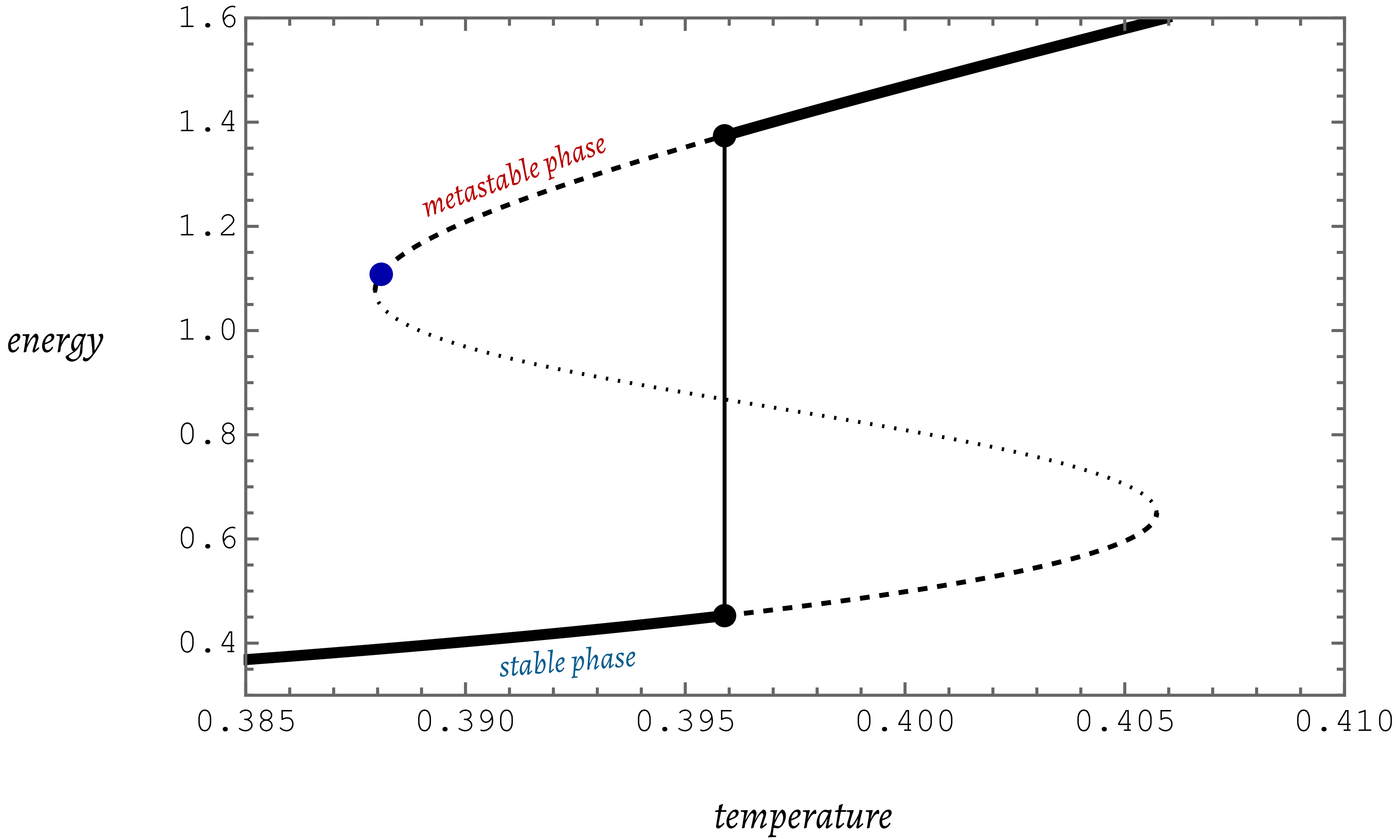

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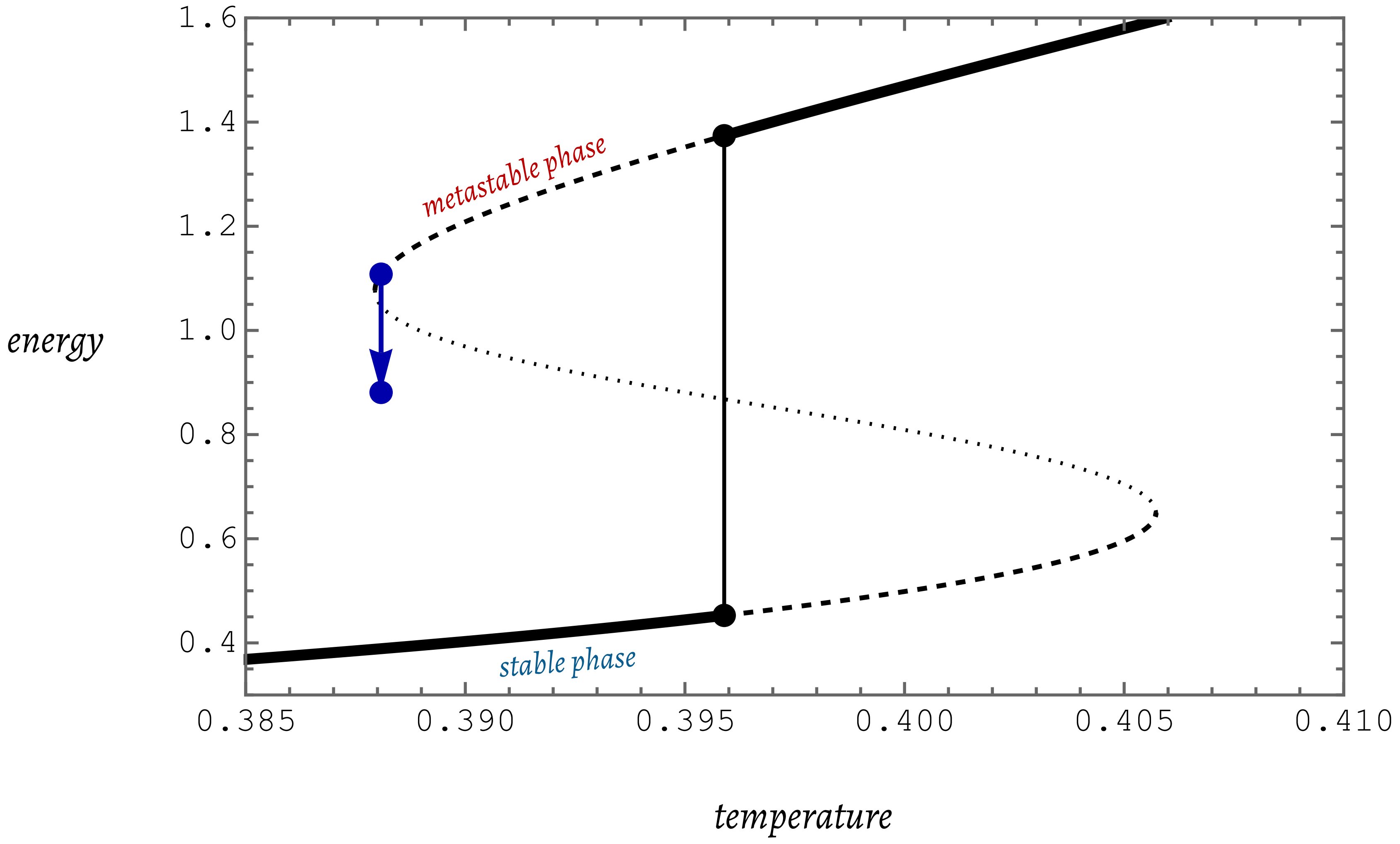

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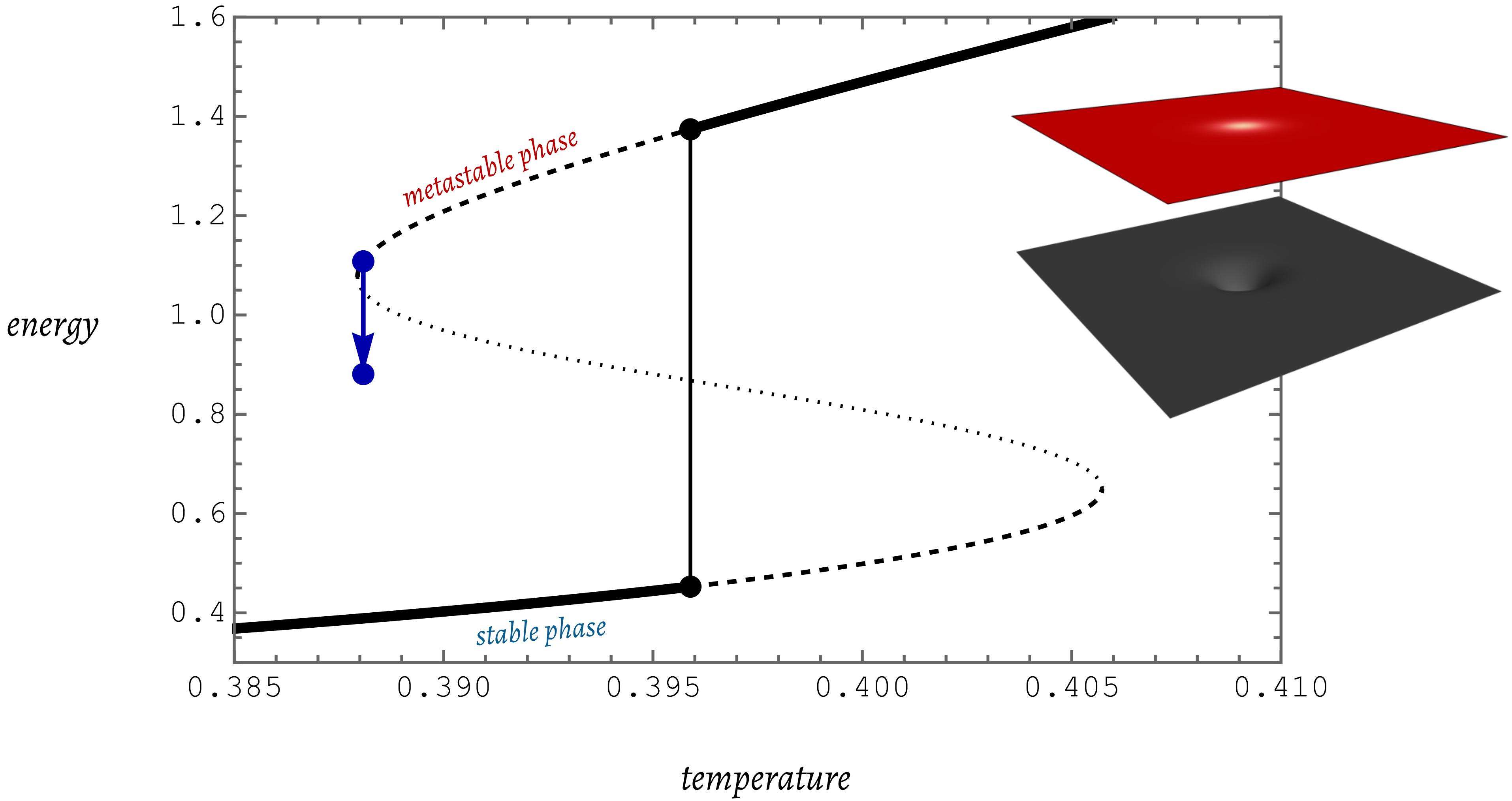

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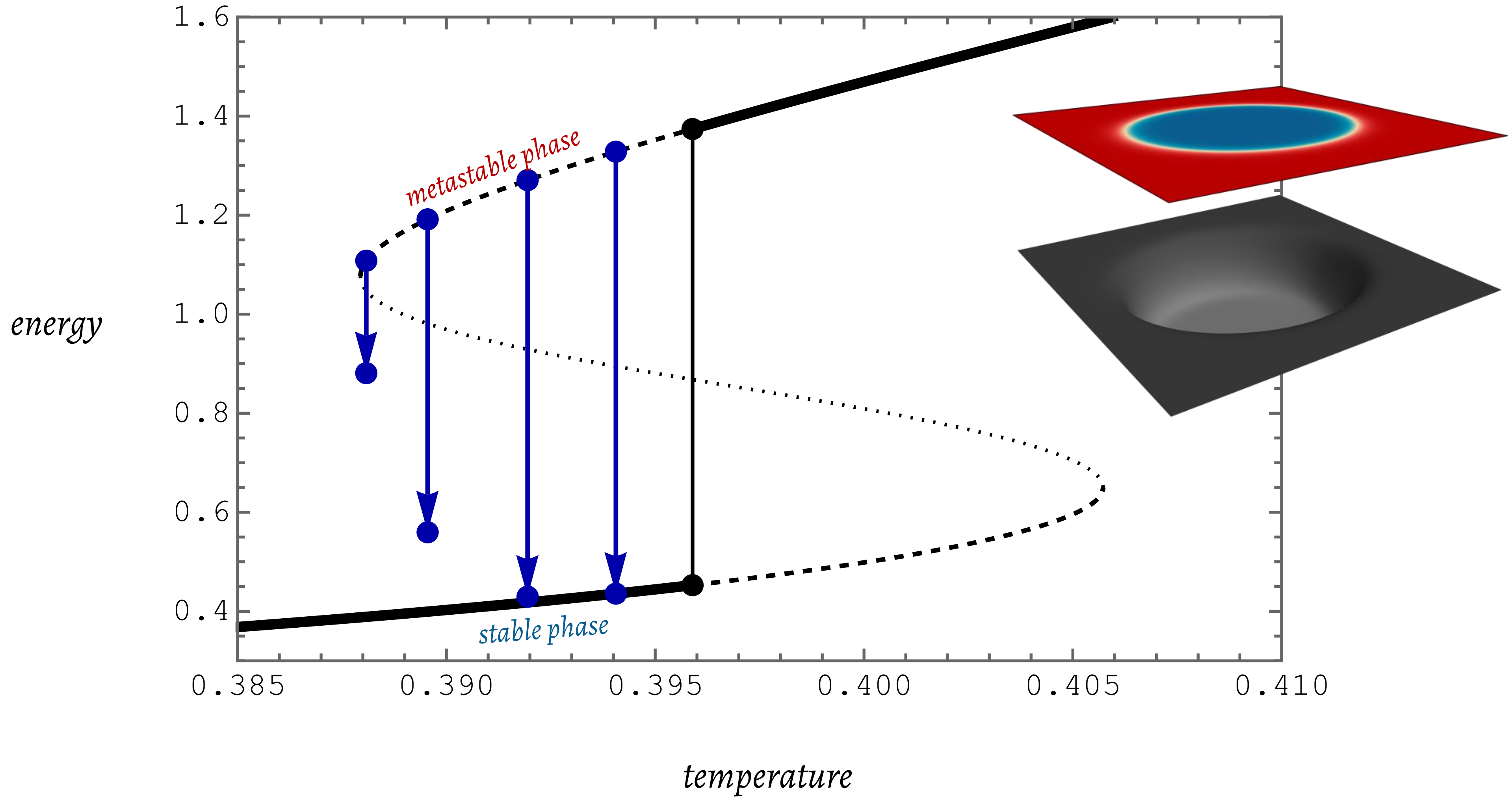

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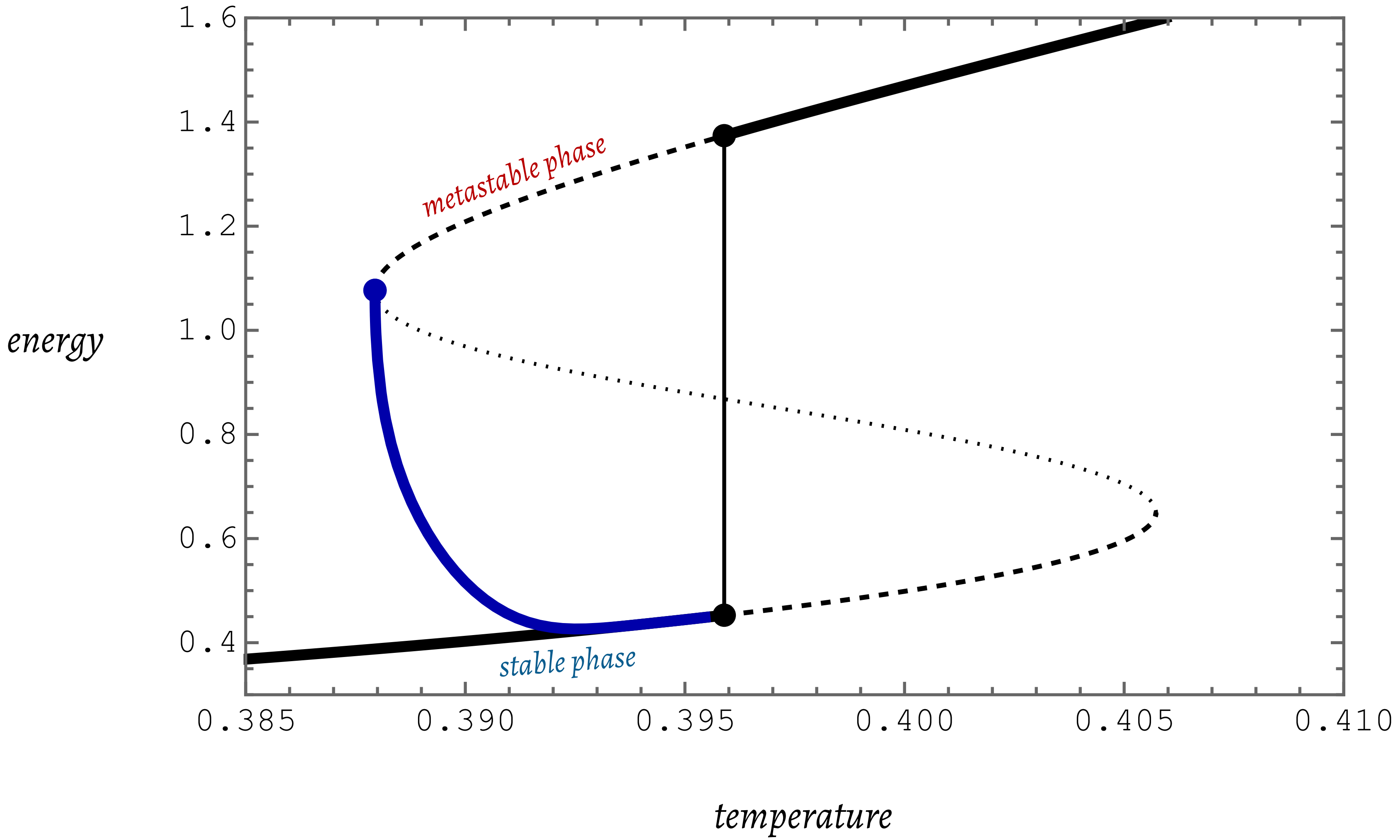

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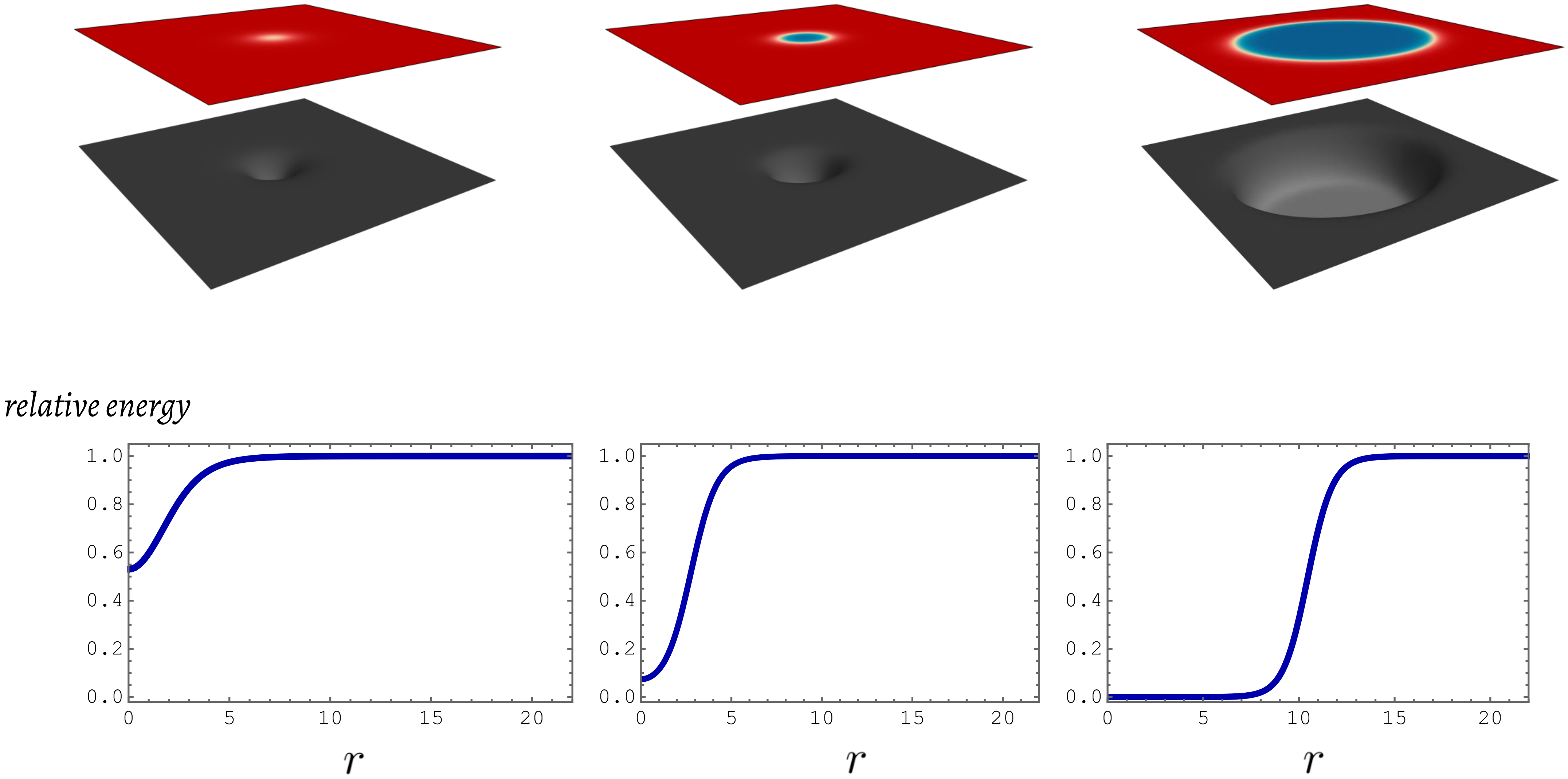


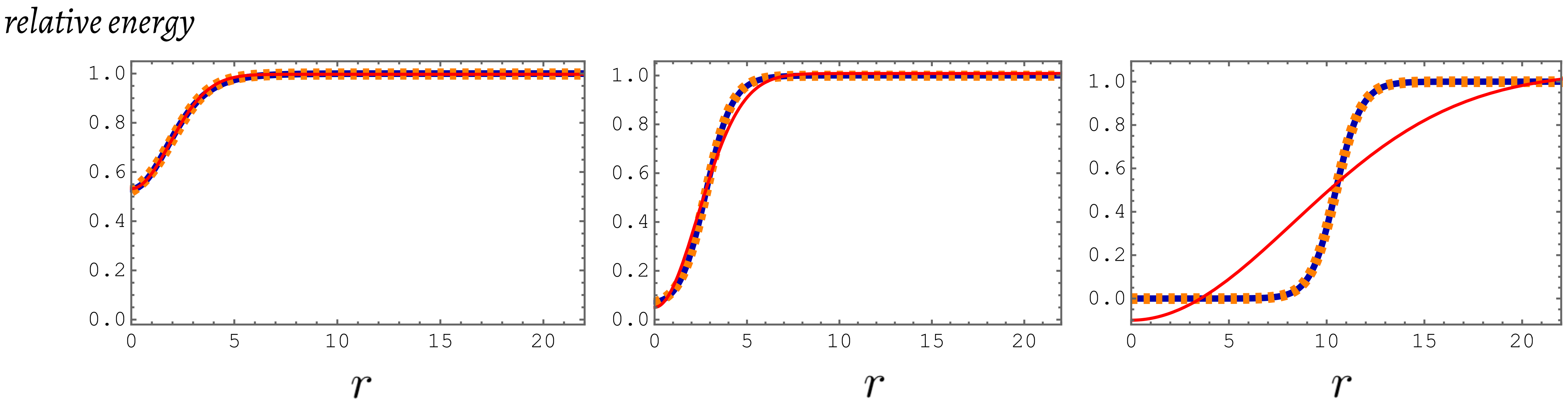
Microscopical description of critical bubbles

Close to the critical temperature: **Hyperbolic tangent**

$$\propto \tanh\left(\frac{r - R_c}{l_\mathrm{w}}\right)$$

Close to the turning point : **Gaussian profile**

$$\propto \exp\left(-r^2/(2\sigma^2)\right)$$



Microscopical description of critical bubbles

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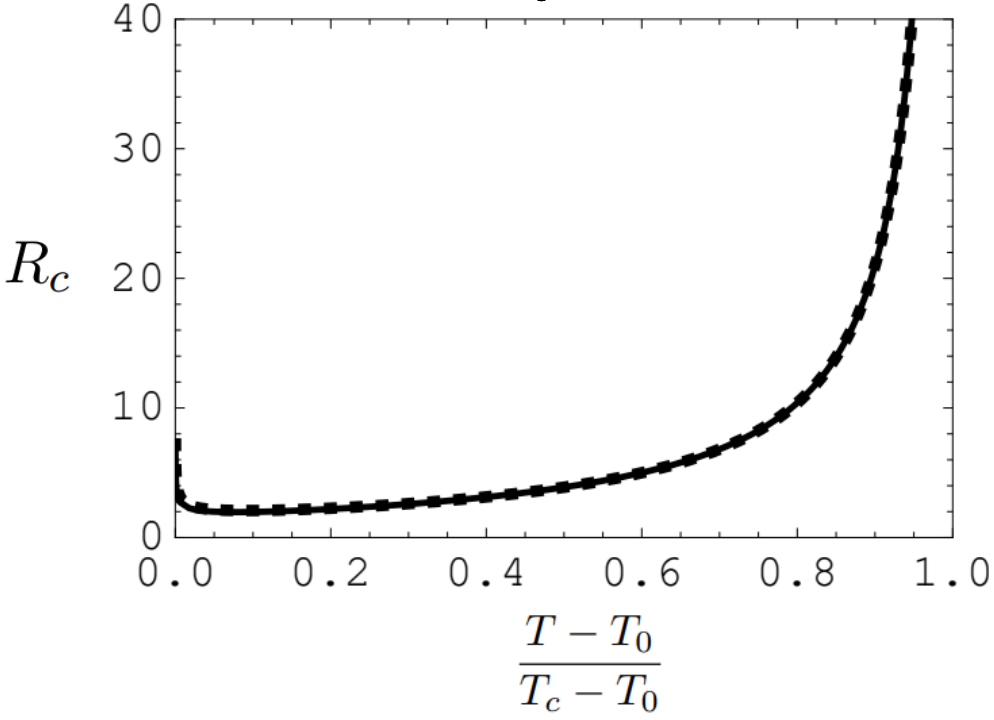
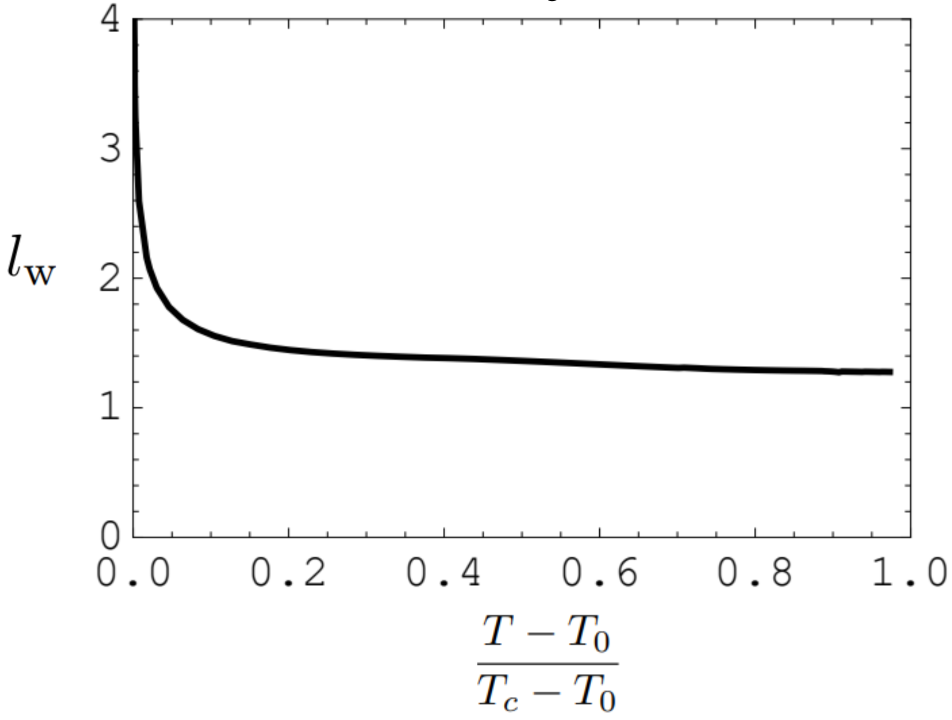
Radius of the bubble



Thickness of the wall



Pressures and energy “conservation”

Profile of the longitudinal and transverse pressures:

$$\nabla_\mu T^{\mu r} = 0 \quad \Rightarrow \quad p'_{||}(r) = -\frac{2}{r}\Big(p_{||}(r) - p_\perp(r)\Big)$$

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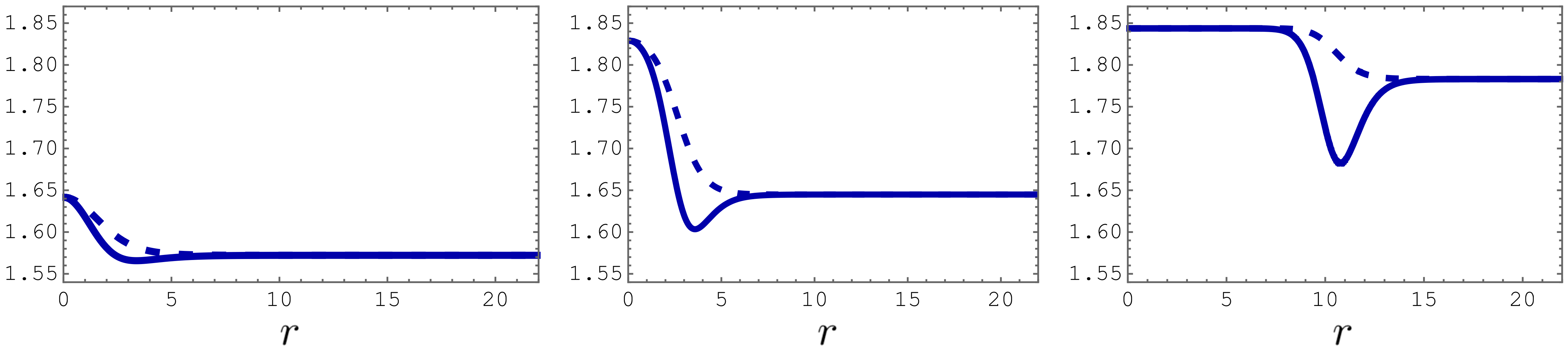
$p_\perp$ (solid) $p_{||}$ (dashed)



Nucleation probability

Profile of the longitudinal and transverse pressures:

$$\mathrm{P}(T) = \mathrm{P}_0 e^{-(S_{\mathrm{bubble}} - S_{\mathrm{hom.}})} = \mathrm{P}_0 e^{-\beta \Delta F}$$

We need to integrate the free energy density to obtain ΔF

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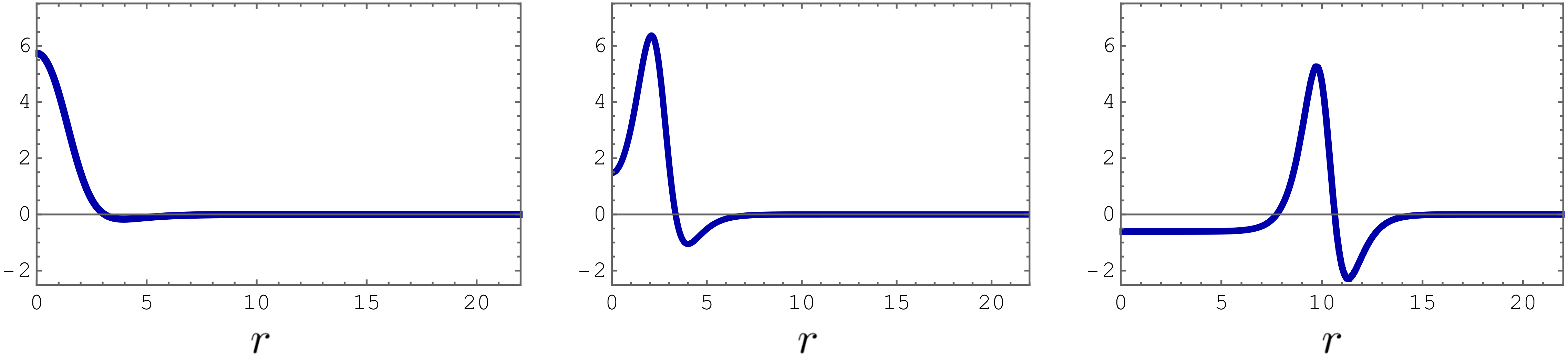
Free energy density



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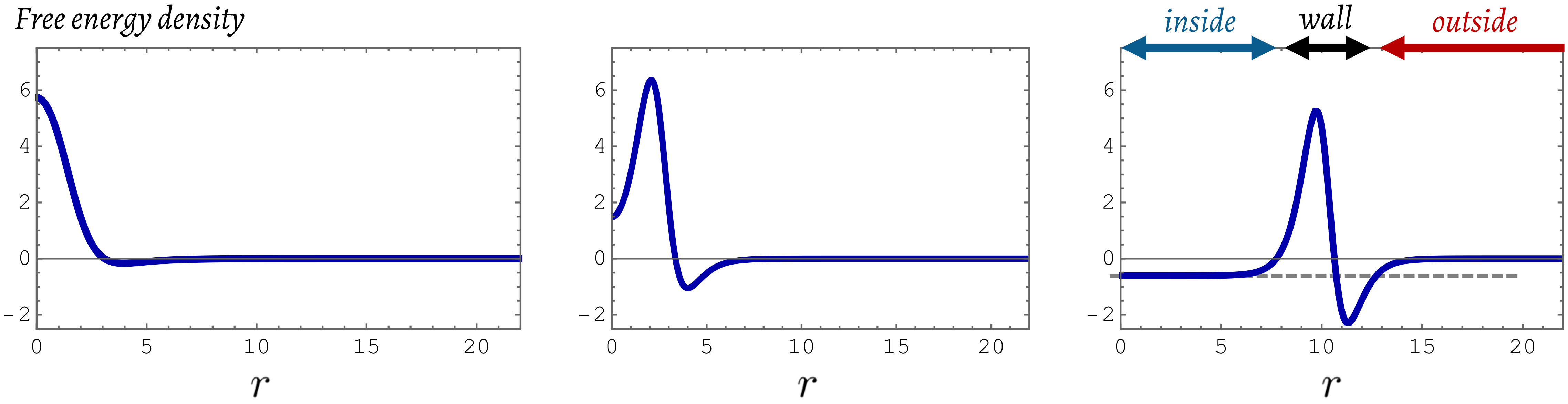

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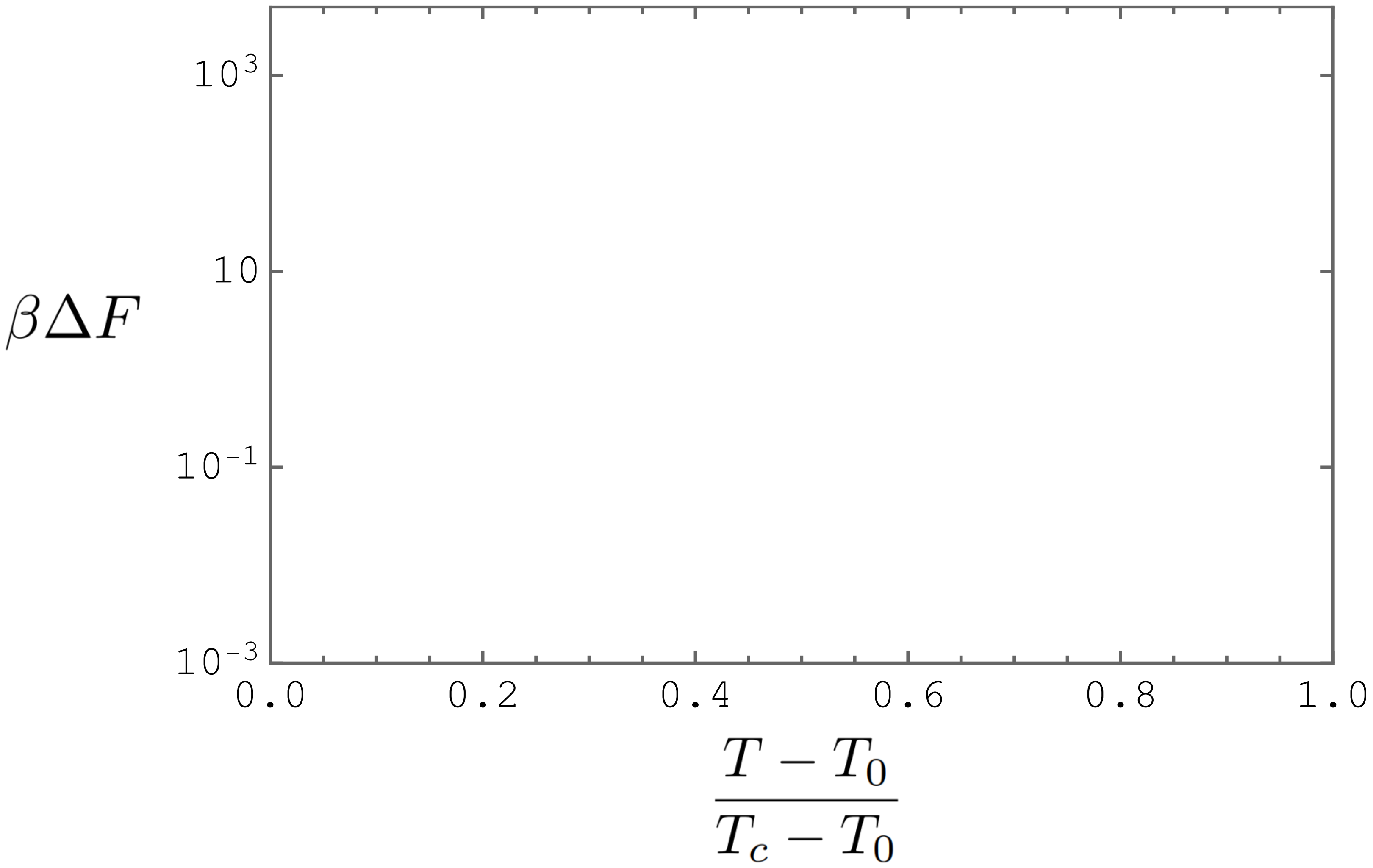

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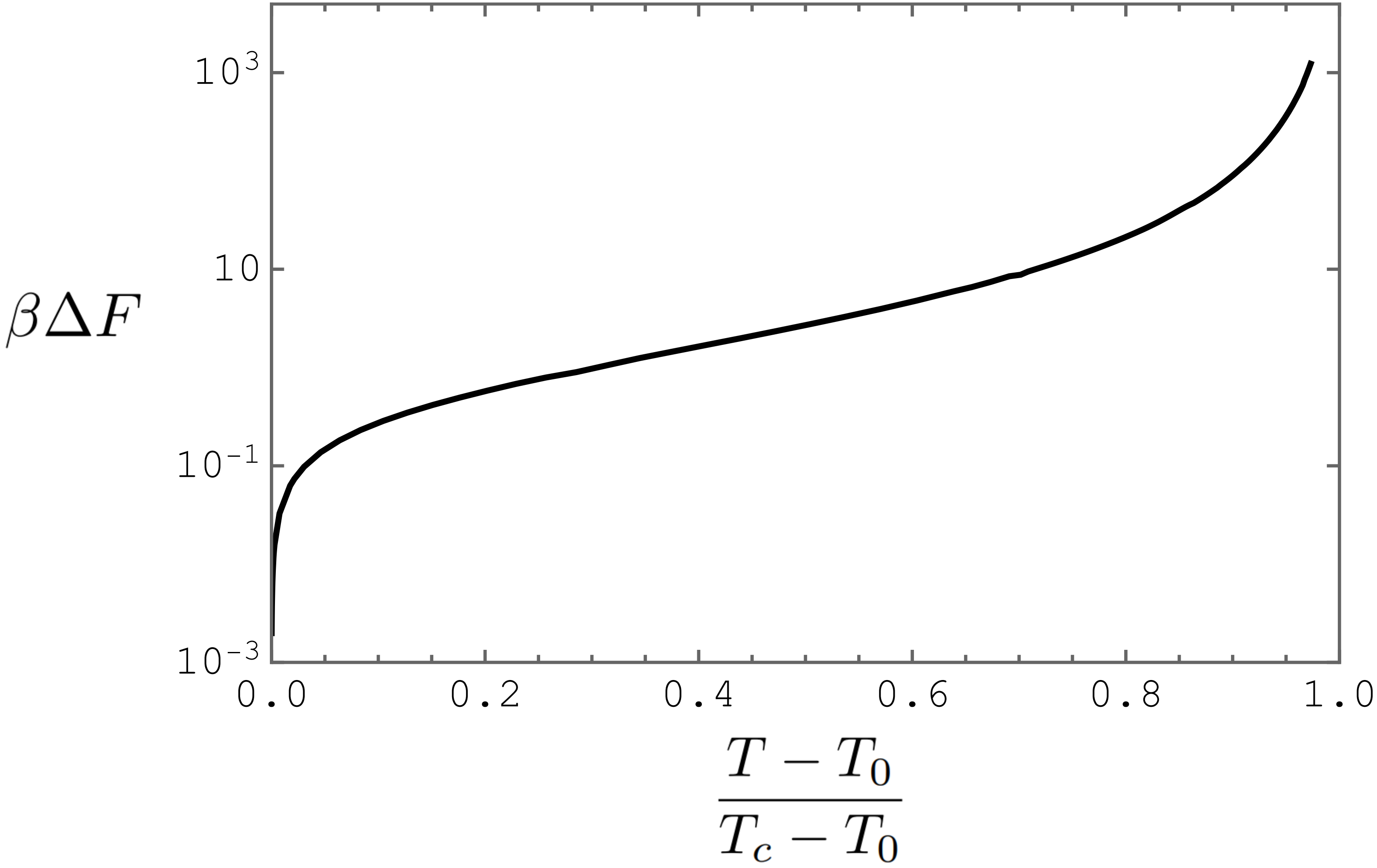

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$$P(T) = P_0 e^{-\beta \Delta F}$$

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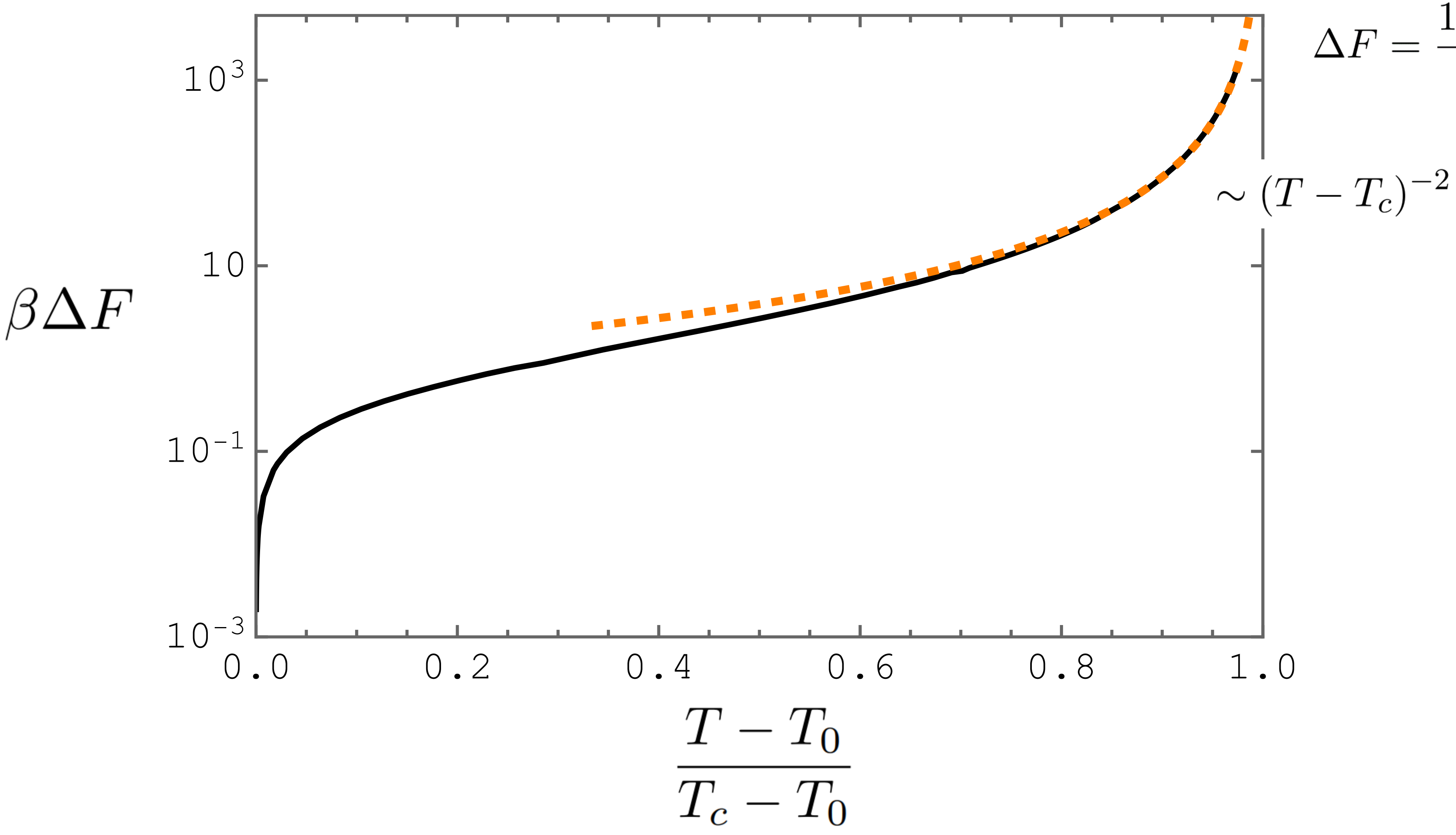

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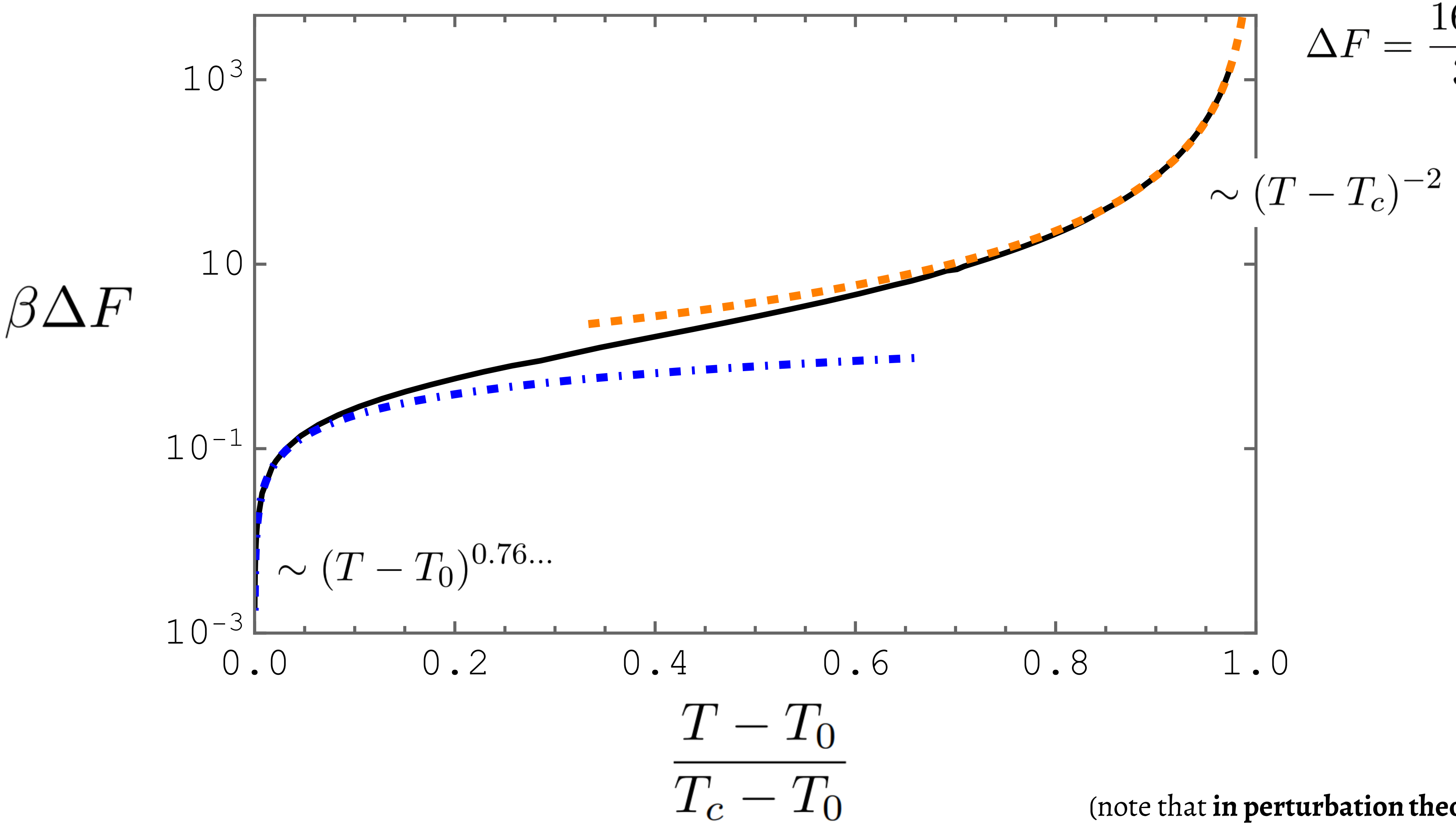

(note that **in perturbation theory one obtains 3/2**)

[*Enqvist, Ignatius, Kajantie, Rummukainen*, ***Phys.Rev.D 45 (1992)***]

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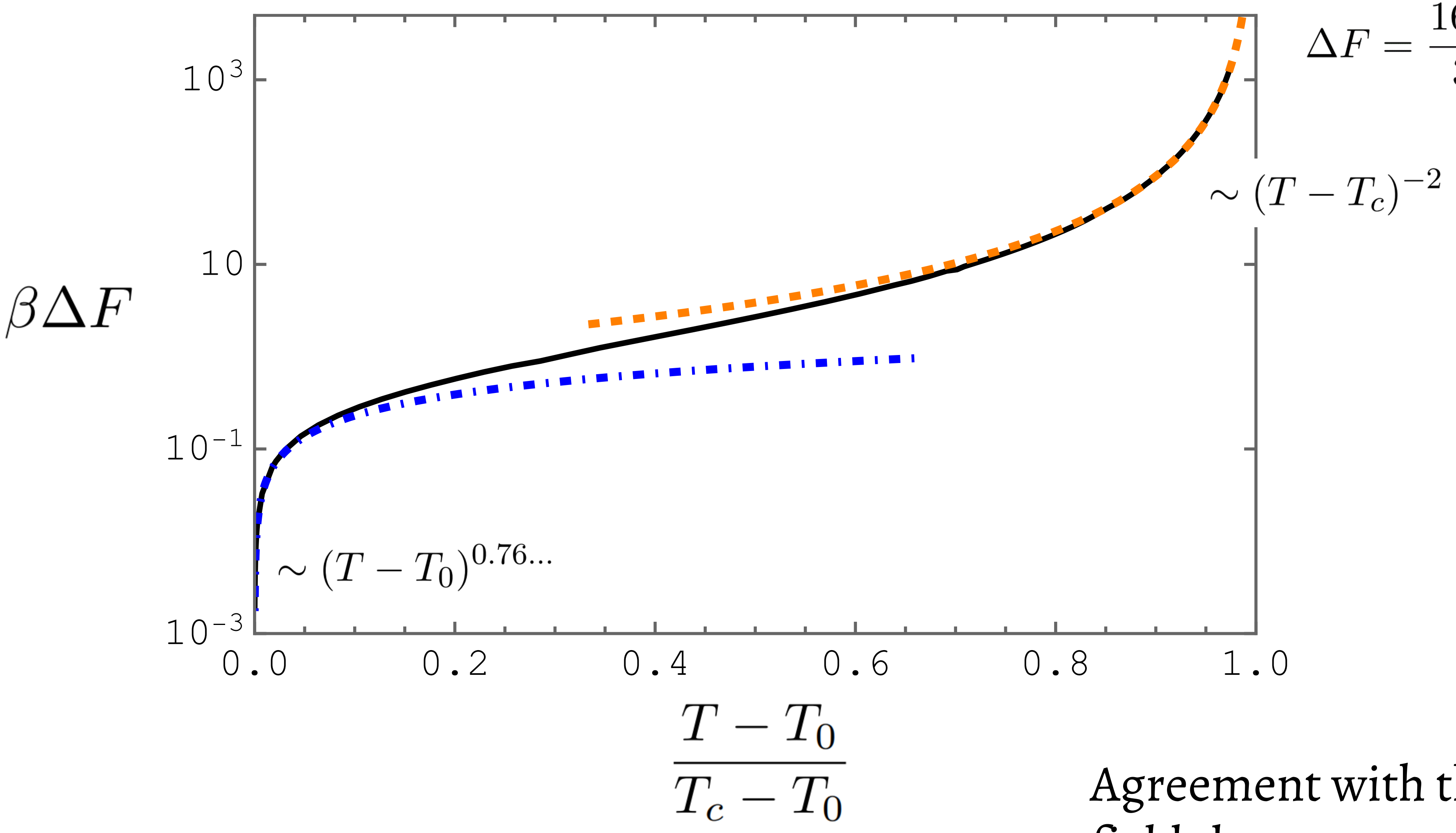

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Agreement with the effective field theory approach?

“Holographically informed” effective action

Can we construct an effective potential that captures the properties of microscopic bubbles we just obtained?

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \left(\frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\nabla\varphi)^2 + V_{\text{eff}}(\varphi, T) \right)$$

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Too simple, we still have freedom in the **kinetic term of the scalar**:

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3 x \left(Z(\varphi, T)(\nabla\varphi)^2 + V_{\mathrm{eff}}(\varphi, T) \right)$$

[See Niko’s talk] [*Ares, Henriksson, Hindmarsh, Hoyos, Jokela,* ***2109.13784***]

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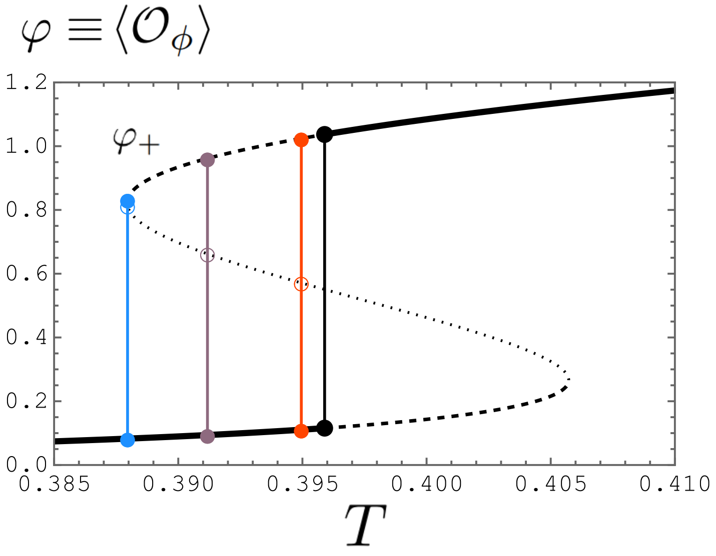
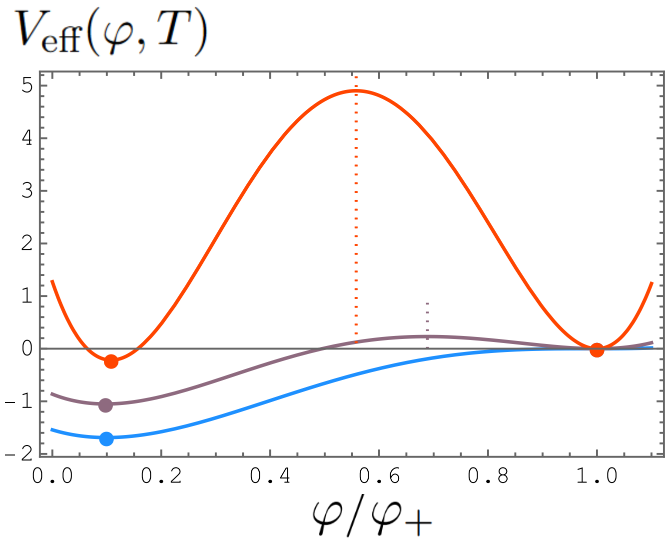
We take a simplified approach: $V_{\text{eff}}(\varphi, T)$ polynomial

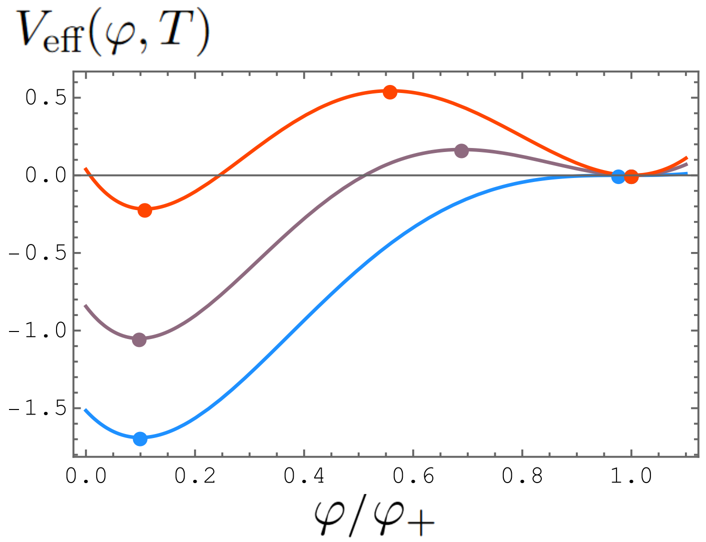
“Holographically informed” effective action

<u>Input for the **effective potential**</u>: homogeneous phases.





4th order polynomial



5th order polynomial

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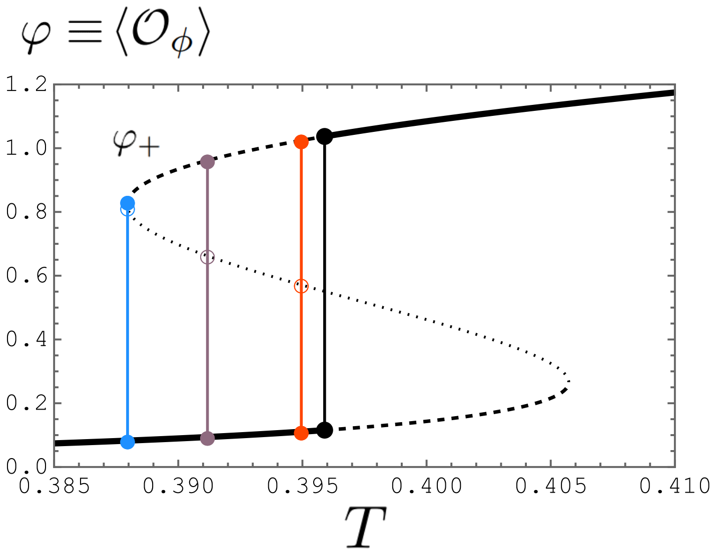

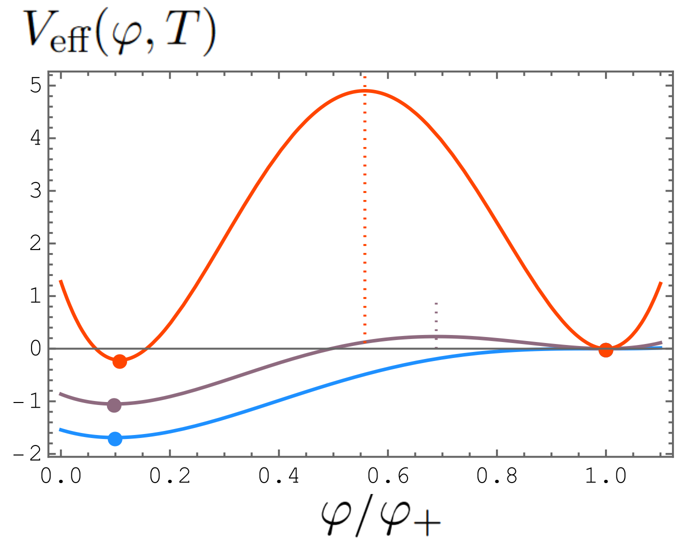

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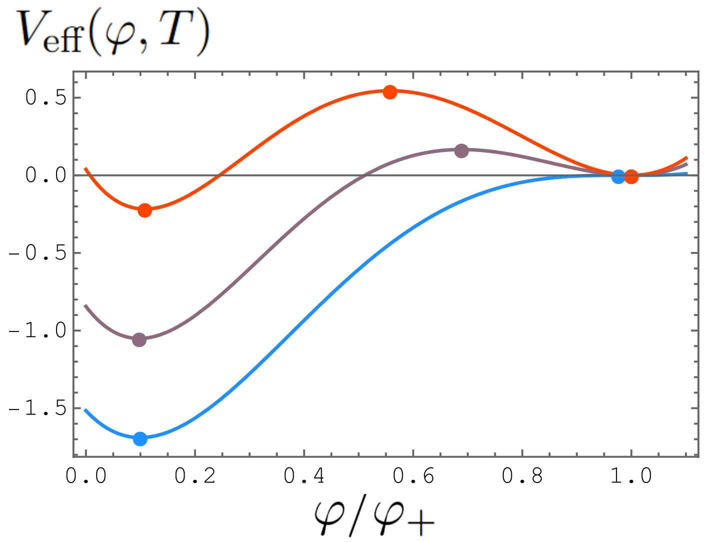


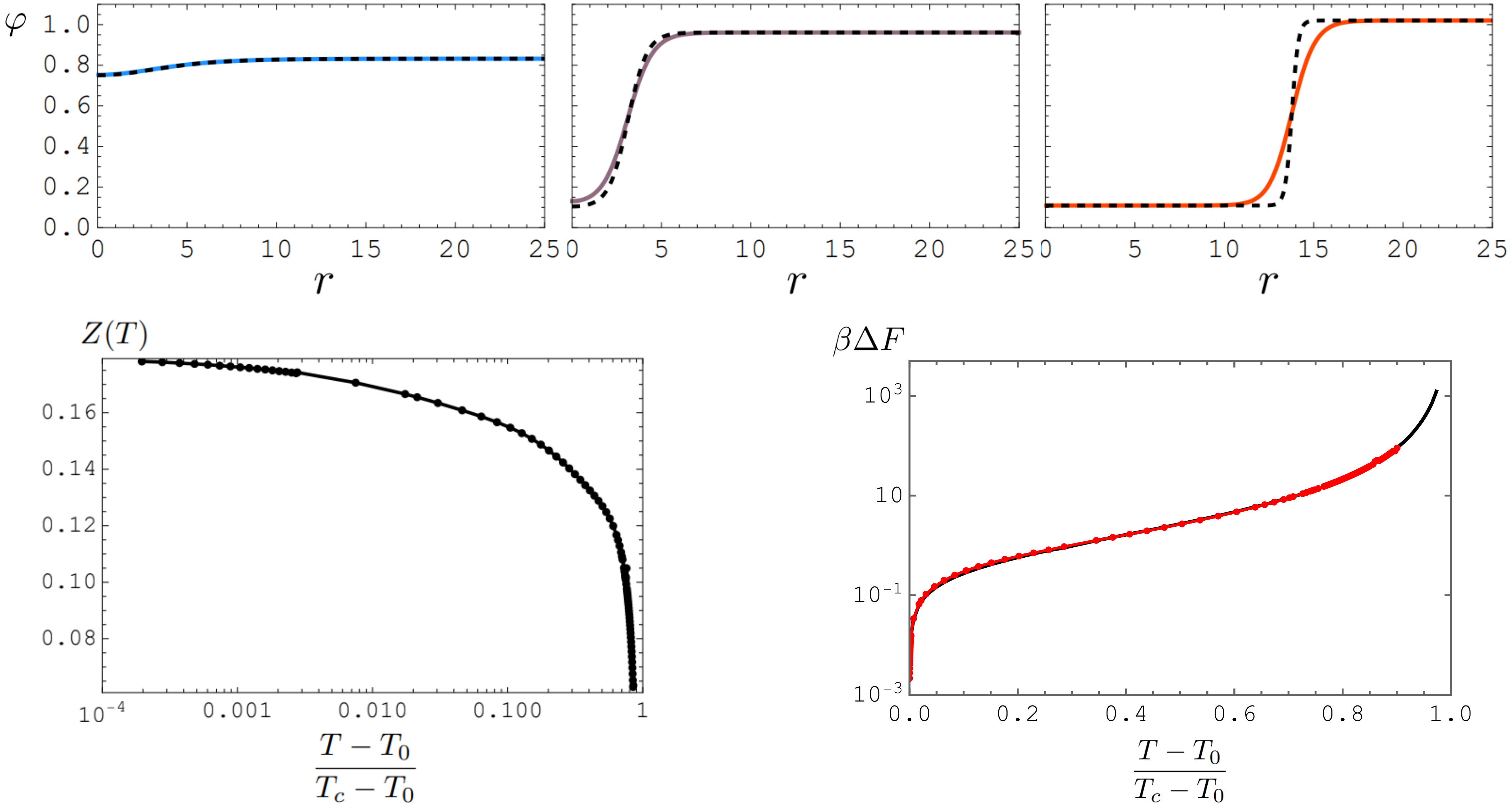
5th order polynomial

<u>Input for **Z(T)**</u>: radius of the bubbles.

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3 x \left(Z(T)(\nabla\varphi)^2 + V_{\text{eff}}(\varphi, T) \right)$$

4th order degree polynomial:

Solid: *holographic*
Dashed: *effective*



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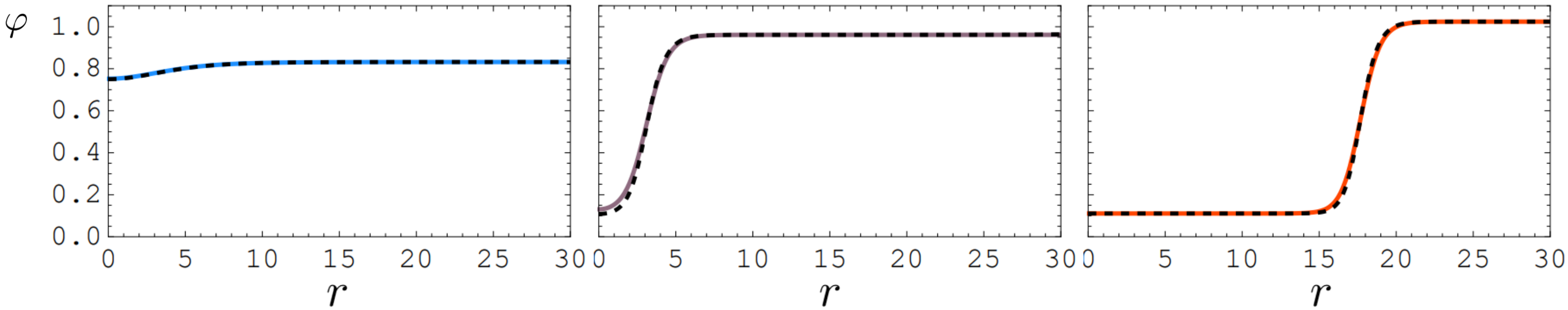
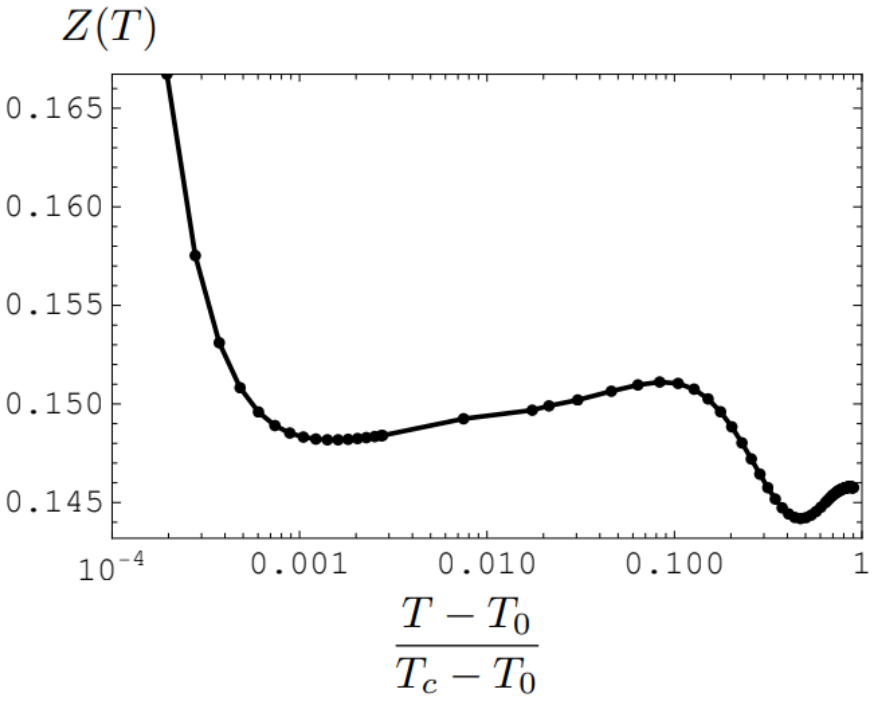
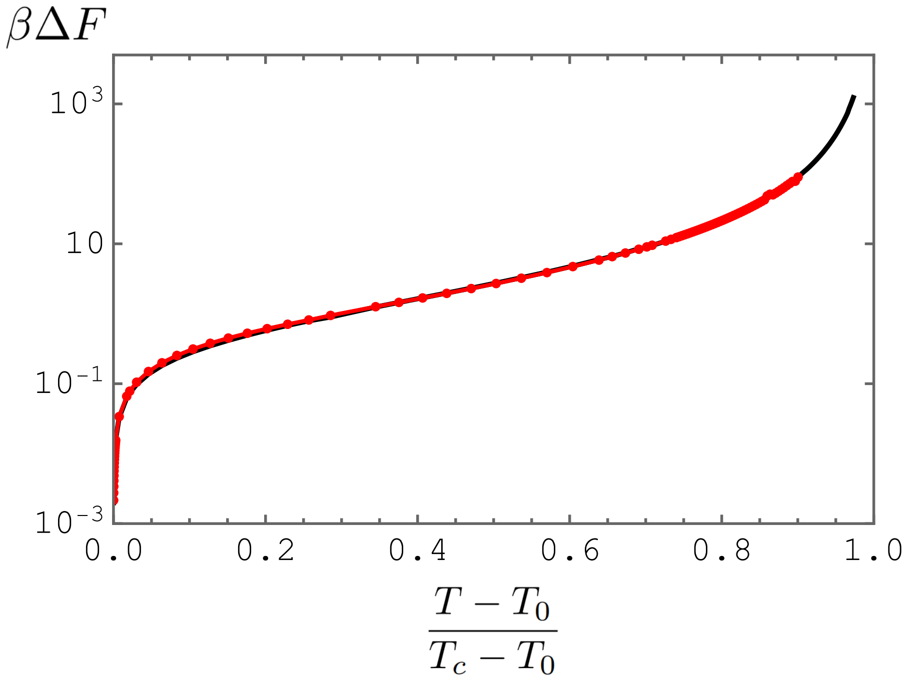




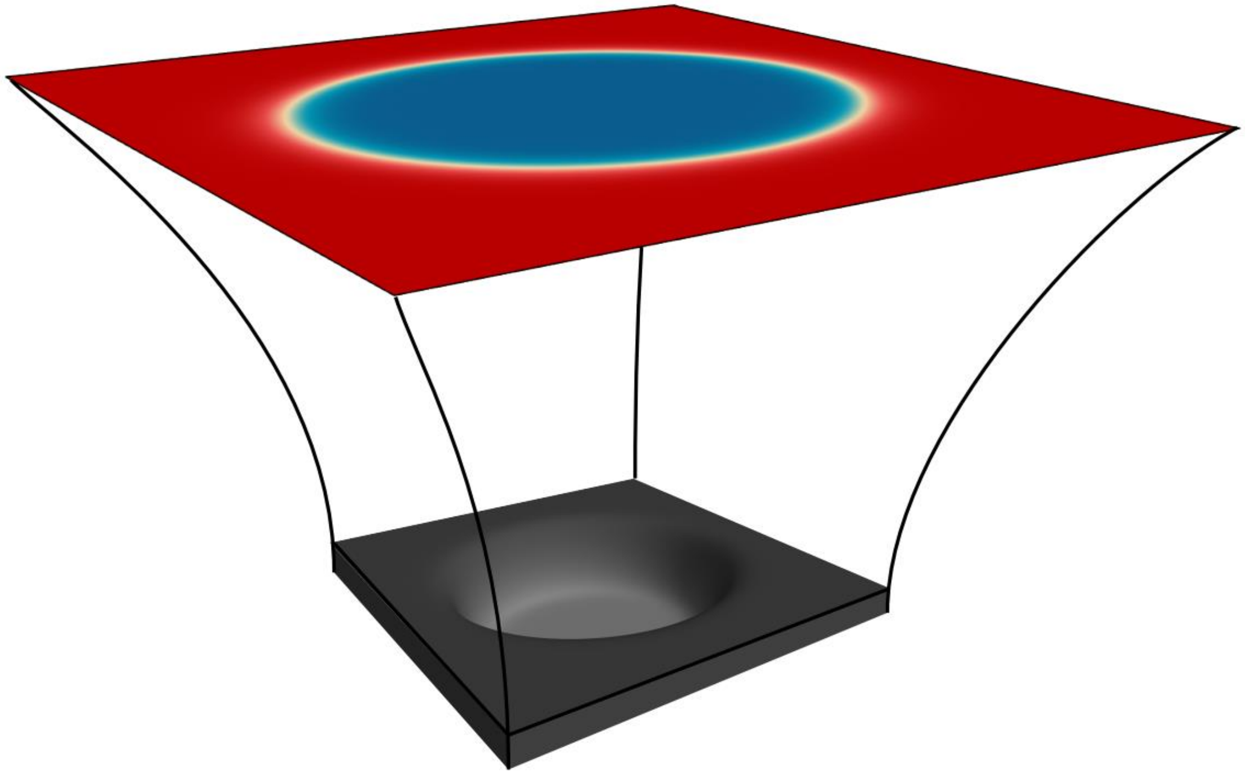


Conclusions and Outlook

Conclusions

- We provided a **microscopic description of critical bubbles** in terms of (*fully-backreacted*) inhomogeneous black hole geometries.

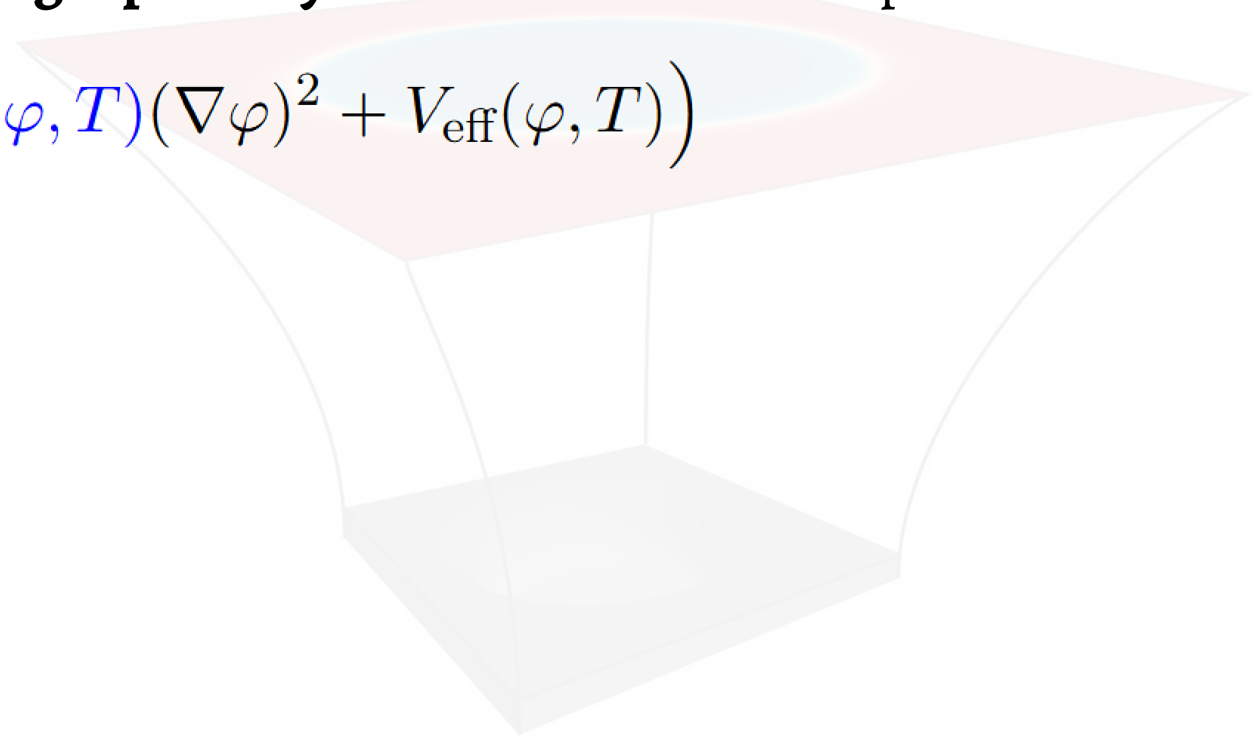
- *So far*, a simple effective action consisting of a 5^{th} order degree polynomial and constant kinetic term reproduces the results nicely.



Outlook

- Can we do better in providing a **“holographically informed”** effective potential?

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \left(Z(\varphi, T)(\nabla\varphi)^2 + V_{\text{eff}}(\varphi, T) \right)$$



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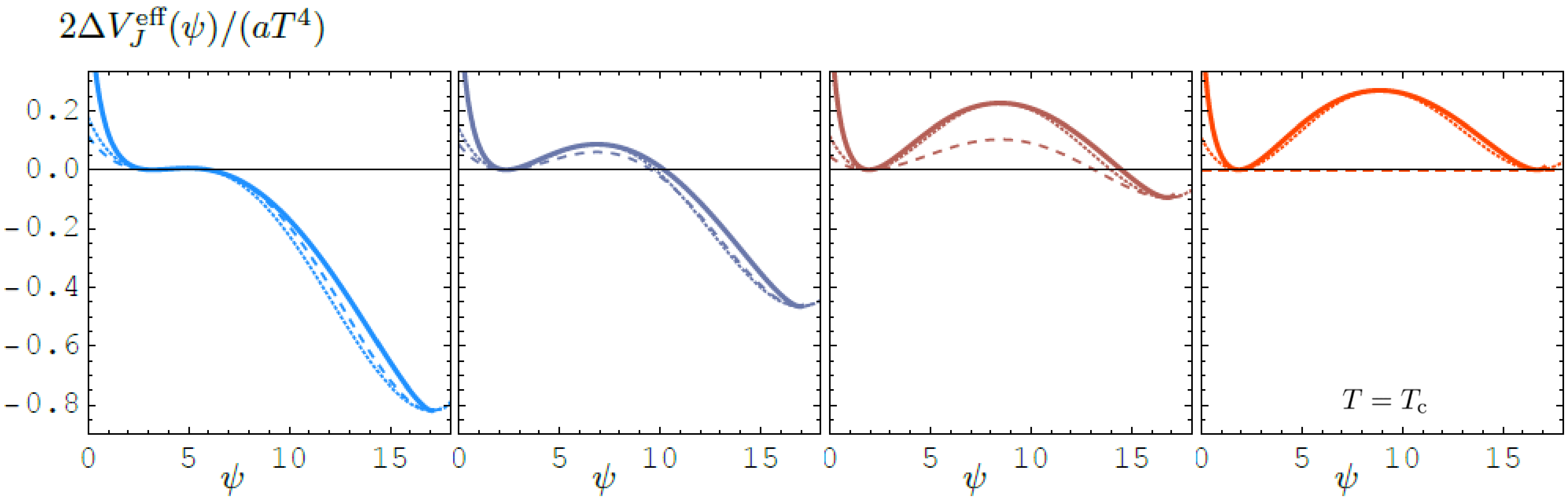

[See Niko’s talk] [*Ares, Henriksson, Hindmarsh, Hoyos, Jokela,* ***2109.13784***]

Outlook

- Understand the transition from *quantum* to *thermal* fluctuations:

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \left(\frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\nabla \varphi)^2 + V_{\mathrm{eff}}(\varphi, T) \right)$$

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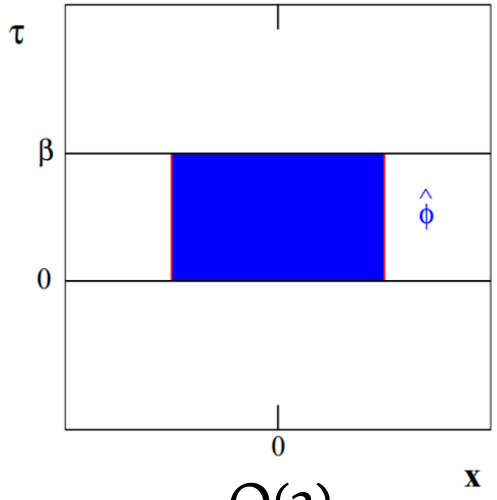
Thermal



O(3)

[*Laine, Vuorinen*, ***1701.01554***]

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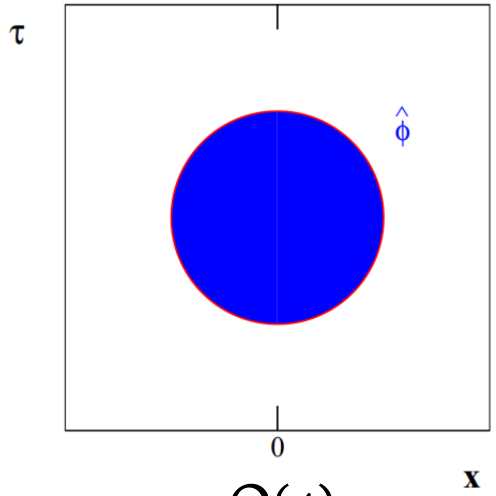
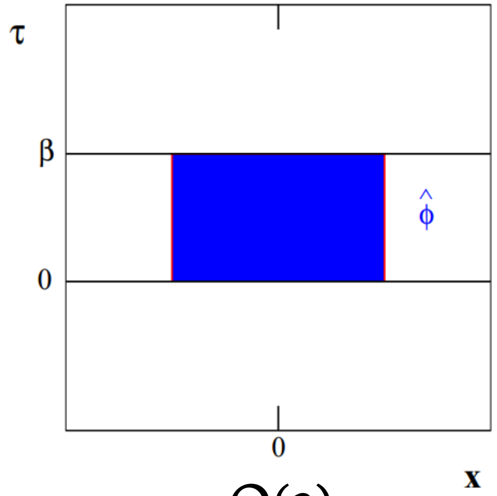
Quantum



O(4)

Thermal



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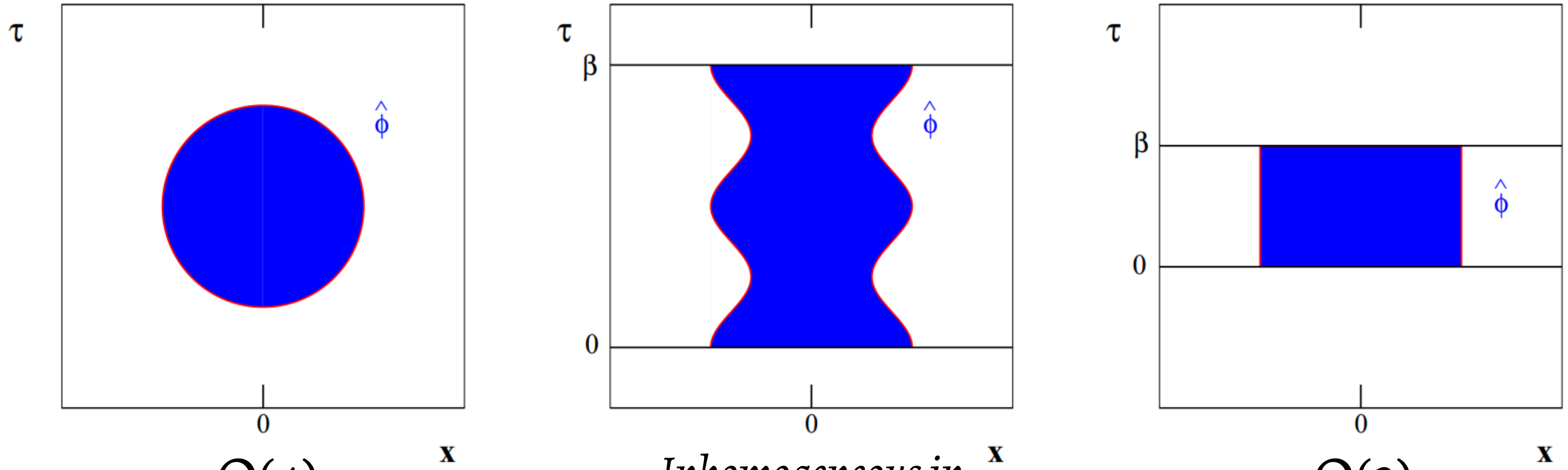
Quantum *Thermal*



O(4) *Inhomogeneous in Euclidean time* O(3)

[*Laine, Vuorinen*, ***1701.01554***]

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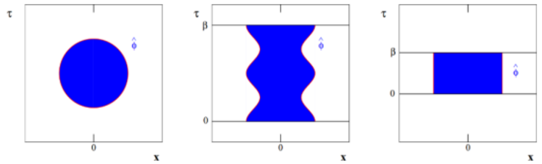
- Understand the transition from *quantum* (O(4)) to *thermal* (O(3)) fluctuations:



- **Applications to QCD and neutron stars:**
  (surface tension and nucleation rate in V-QCD).

Outlook

- Can we do better in providing a **"holographically informed"** effective potential?

$$S(T) = \int_0^\beta \mathrm{d}\tau \mathrm{d}^3x \left(Z(\varphi, T)(\nabla\varphi)^2 + V_{\text{eff}}(\varphi, T) \right)$$

- Understand the transition from *quantum* (O(4)) to *thermal* (O(3)) fluctuations.

- **Applications to QCD and neutron stars:**
  (surface tension and nucleation rate in V-QCD).

- **Beyond static configurations:** Quasinormal-modes of the critical bubbles.

$$\Gamma \sim \frac{T}{\mathcal{Z}_0} \exp\left\{ -S_E[\hat{\phi}] \right\} \left| \det\left(\delta^2 S_E[\hat{\phi}]/\delta\phi^2 \right) \right|^{-\frac{1}{2}}$$

[*Laine, Vuorinen*, ***1701.01554***]

Thanks.