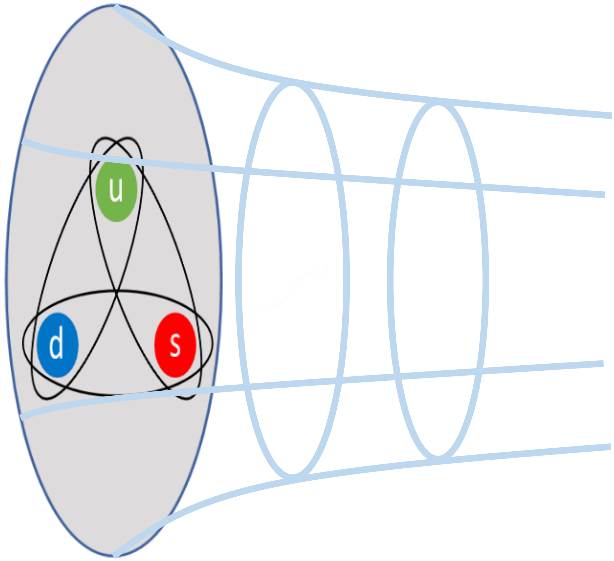
From top-down to bottom-up: a new construction for holographic color-superconductivity



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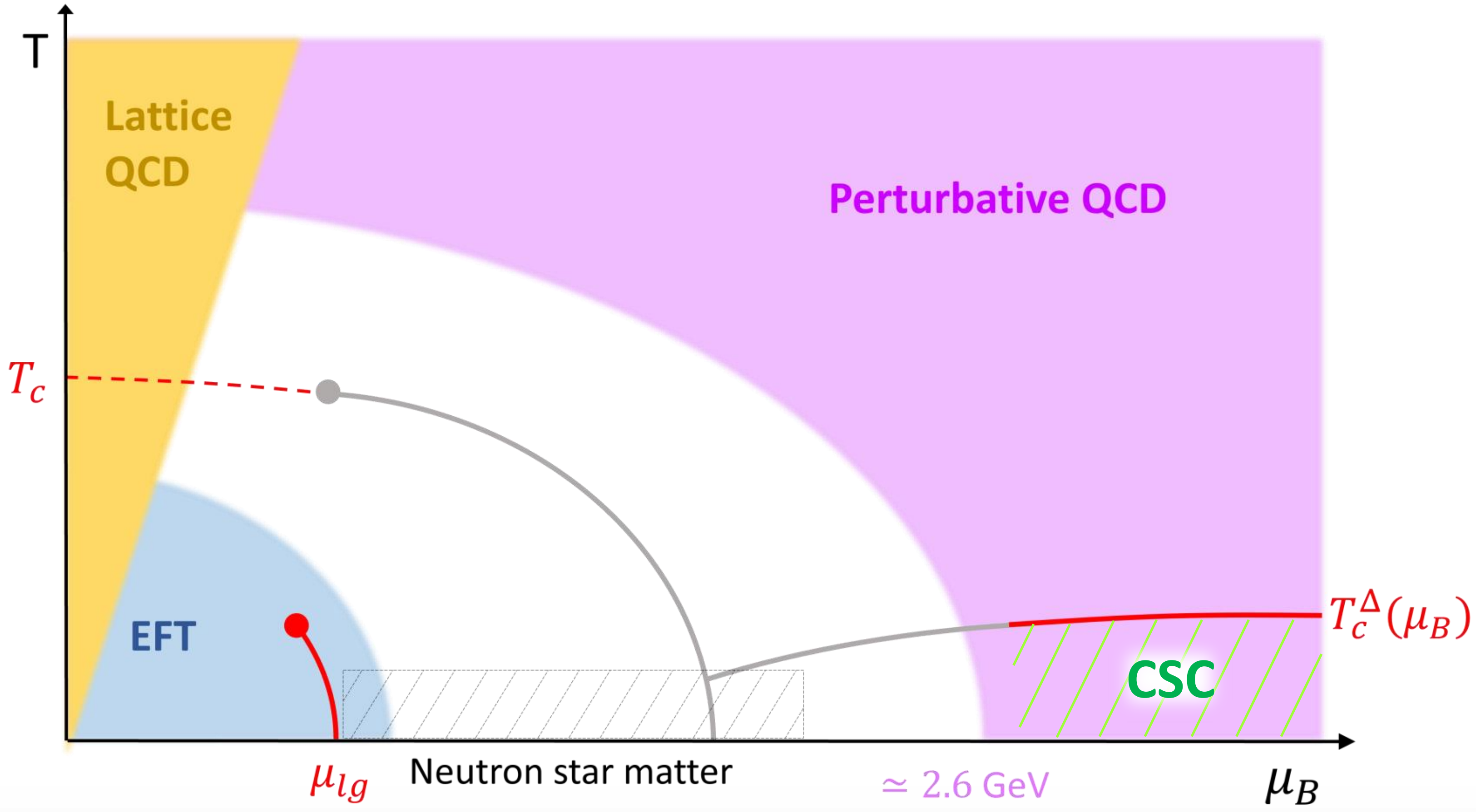
Outline

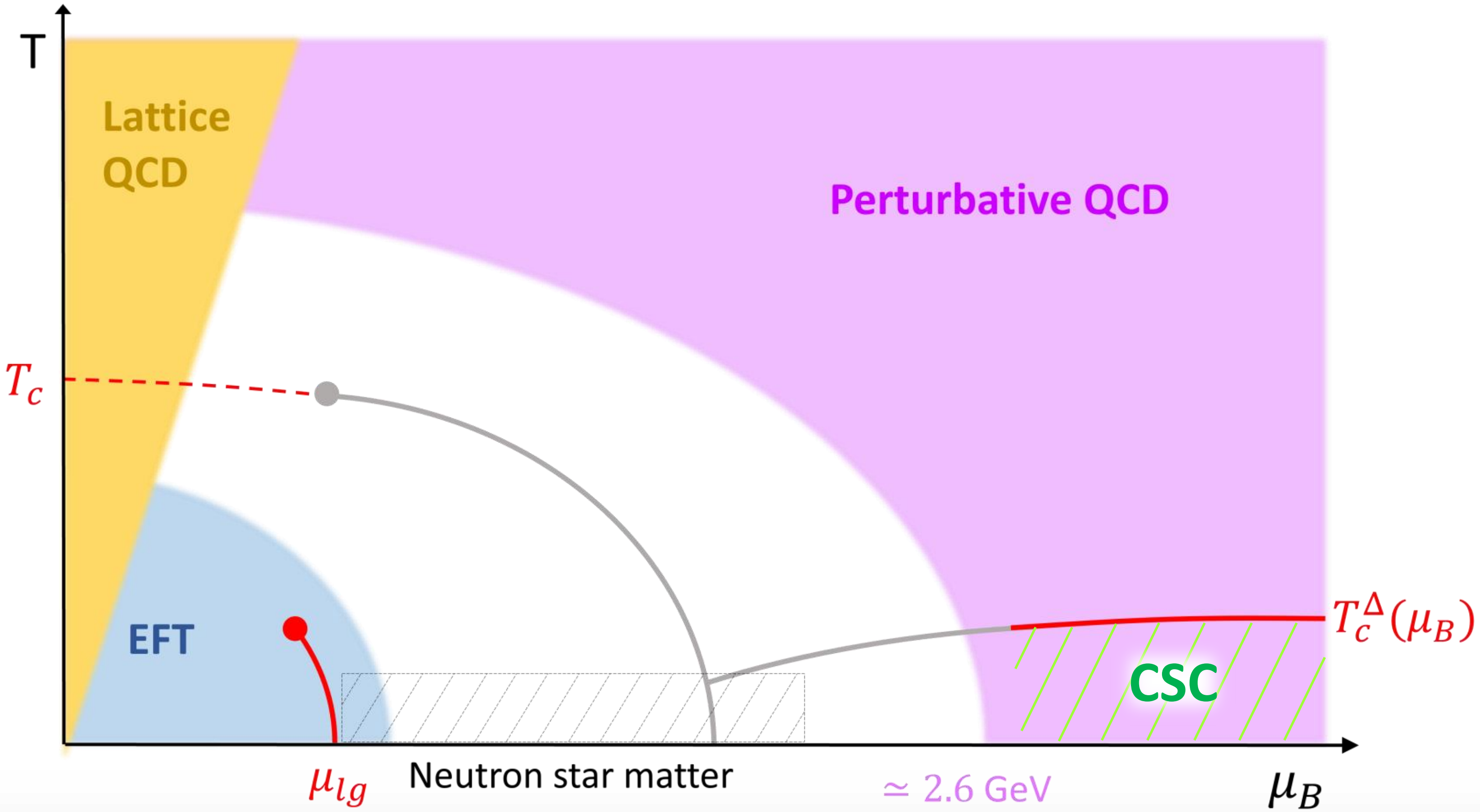
1) Context
2) Lessons from a top-down example
3) The bottom-up model
4) Stability analysis and phase diagram
5) Properties of the Higgsed phase
6) Summary

Context: QCD phases



At $\mu_B \gg \Lambda_{QCD}$, pQCD predicts **color-superconductivity (CSC)**: quarks form **pairs** that induce **Higgsing** of the color group

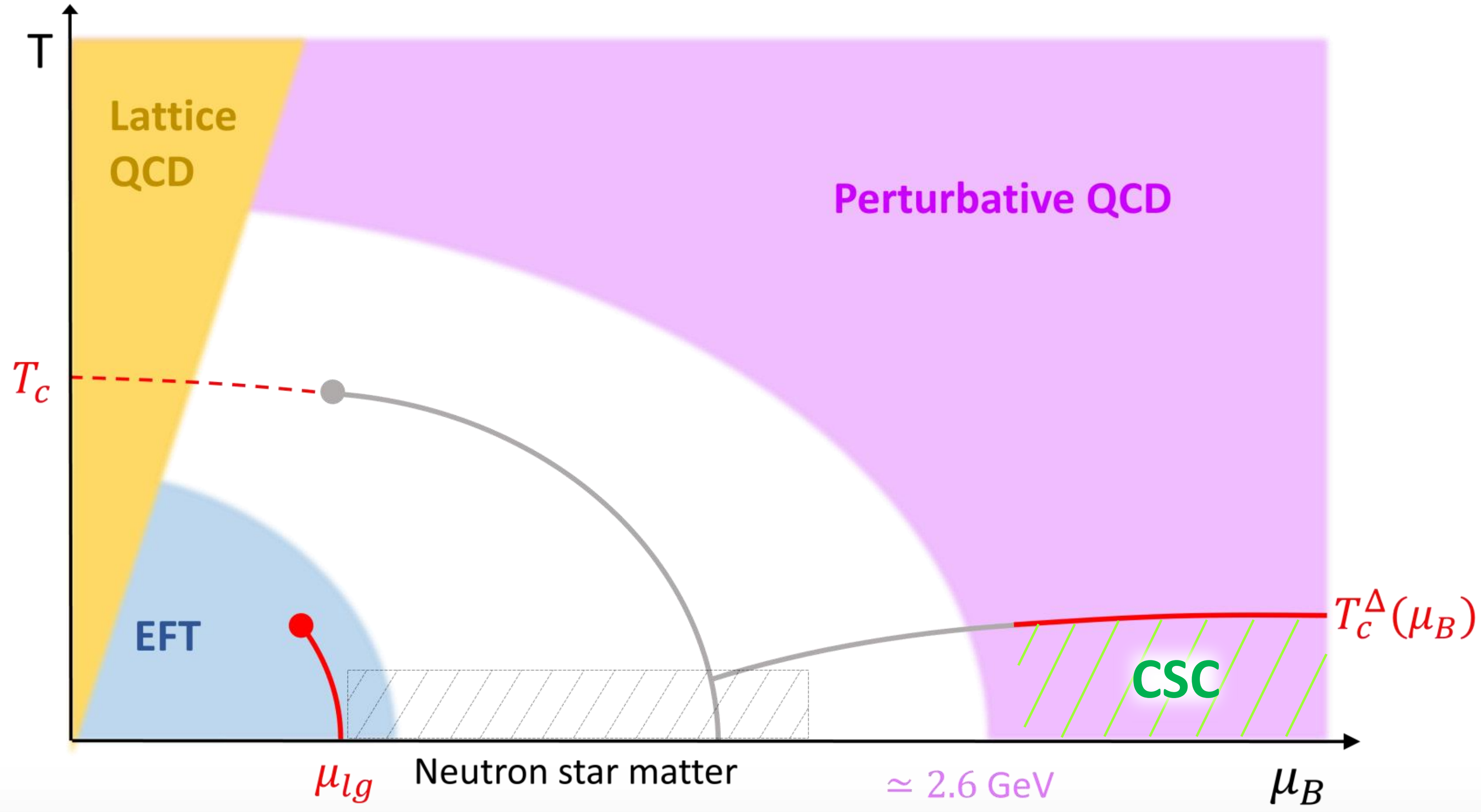
Context: QCD phases



Example: for 3 massless flavors, **color-flavor locking (CFL)** $\langle \psi_i^a \psi_j^b \rangle \propto \delta_i^a \delta_j^b$

With **symmetry breaking**: $[SU(3)_c] \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{c+L+R} \times \mathbb{Z}_2$

Context: QCD phases



The CSC phase should **connect to nuclear matter** at lower densities, but QCD matter is **strongly-coupled** in this regime

Context: holographic models

- This work: consider the **holographic** approach to this problem

→ Construct a **model** with **bulk solutions** dual to a **strongly-coupled CSC**

- Several models of **holographic QCD** (top-down **WSS**, bottom-up **V-QCD**)

→ **Many successes**: confinement, chiral symmetry breaking, flavor anomalies, realistic hadronic spectra, etc.

- However, **CSC phases are hard to realize**

→ No single-trace, gauge-invariant **order parameter**

Context: holographic models

- Some **top-down** examples [Apreda,Erdmenger,Evans,Guralnik’05] [Chen,Hashimoto,Matsuura’09], [Faedo,Mateos,Pantelidou,Tarrío’18]

→ But limited to the **quenched limit** $(N_f, N_{c,broken} \ll N_c)$ and $\boldsymbol{n_B \to 0}$

- Here: **bottom-up** setting

→ Indeterminacy in the model, but allows to access **large Higgsing and $\boldsymbol{n_B}$**

- Strategy: inspired from **dimensional reduction of the top-down setup** [Faedo,Mateos,Pantelidou,Tarrío’18]

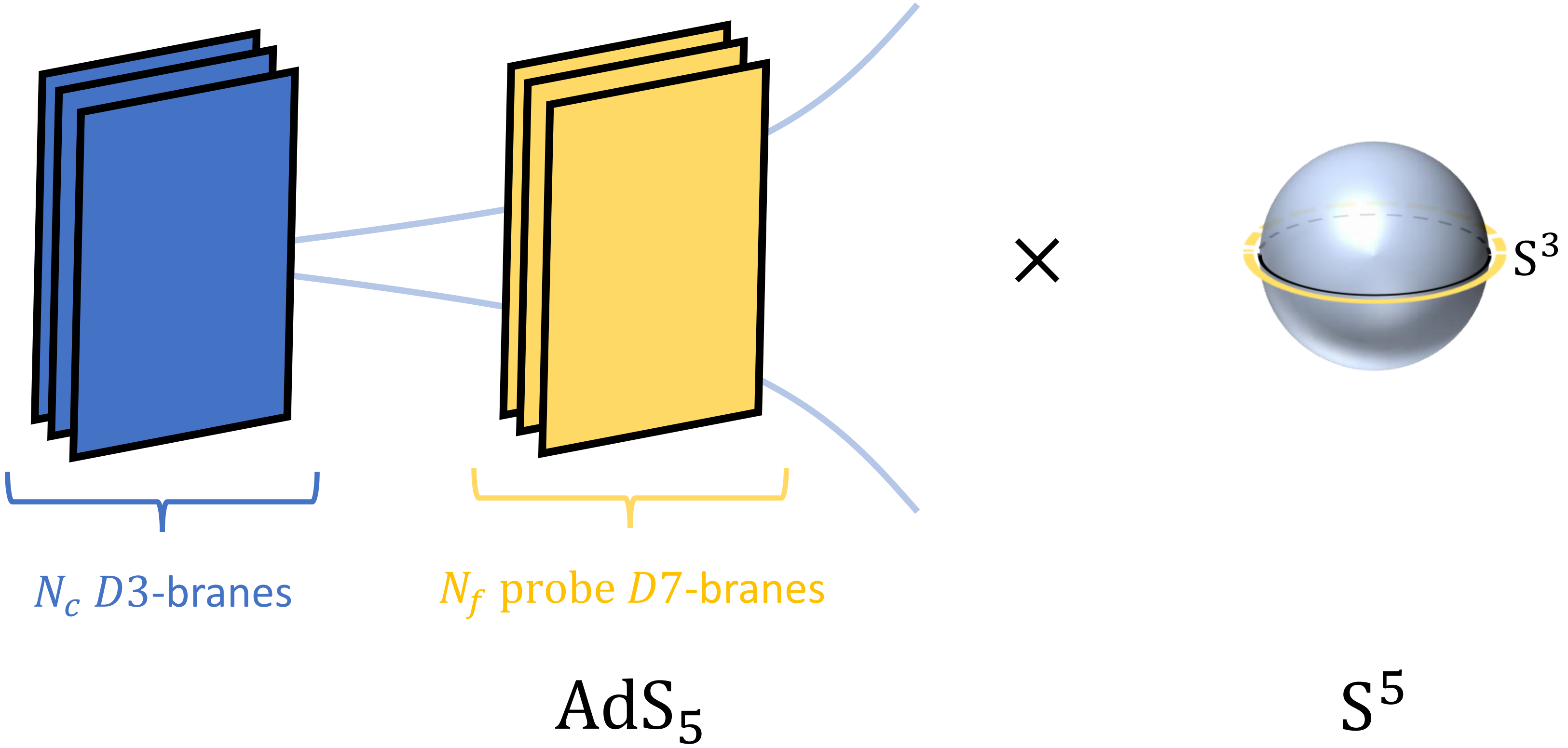
- Spoiler: χ**SB** at high density + color **Higgsing** with $N_{c,broken}/N_c = \mathcal{O}(1)$

2. Lessons from a top-down example

D3/D7 setup

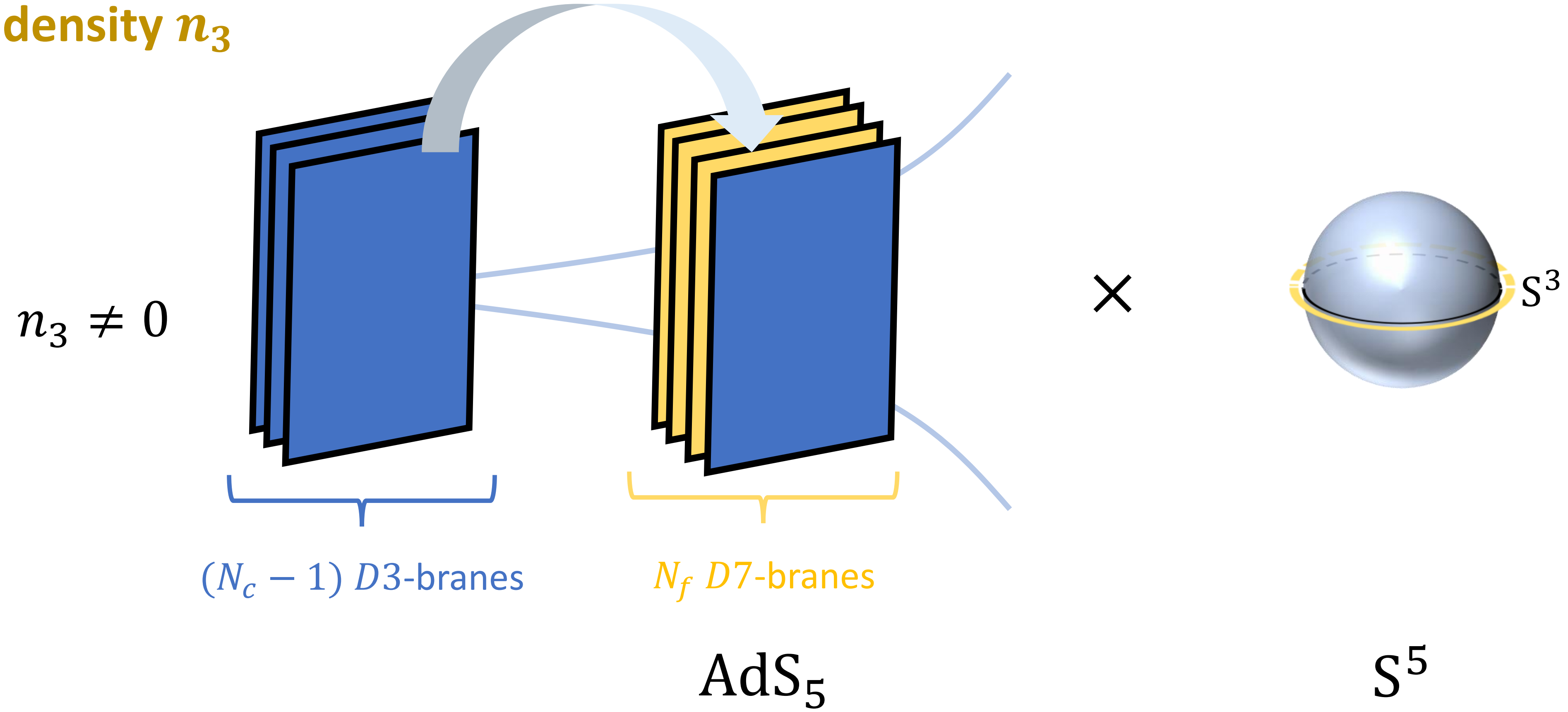
[Faedo,Mateos,Pantelidou,Tarrío'18]: **D3/D7 setup**, exhibiting a **CSC solution** at finite **isospin density n_3**

$n_3 = 0$



D3/D7 setup

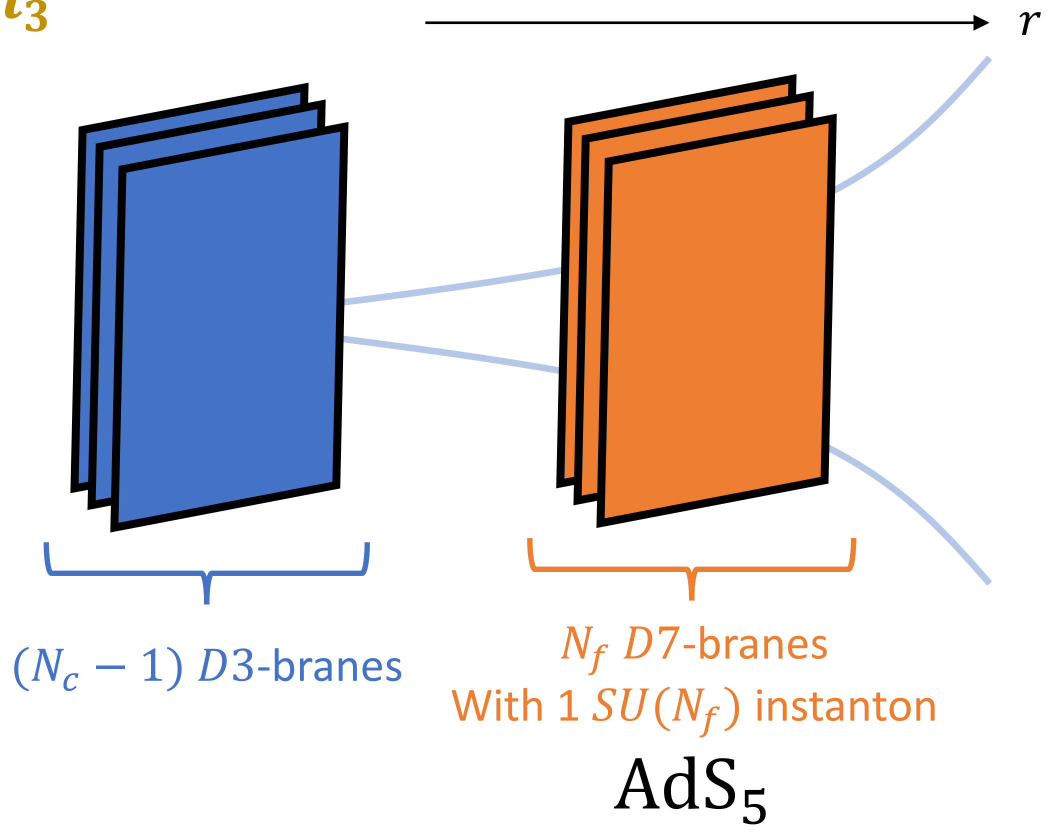
[Faedo,Mateos,Pantelidou,Tarrío'18]: **D3/D7 setup**, exhibiting a **CSC solution** at finite **isospin density n_3**



D3/D7 setup

[Faedo,Mateos,Pantelidou,Tarrío'18]: **D3/D7 setup**, exhibiting a **CSC solution** at finite **isospin density n_3**

$n_3 \neq 0$



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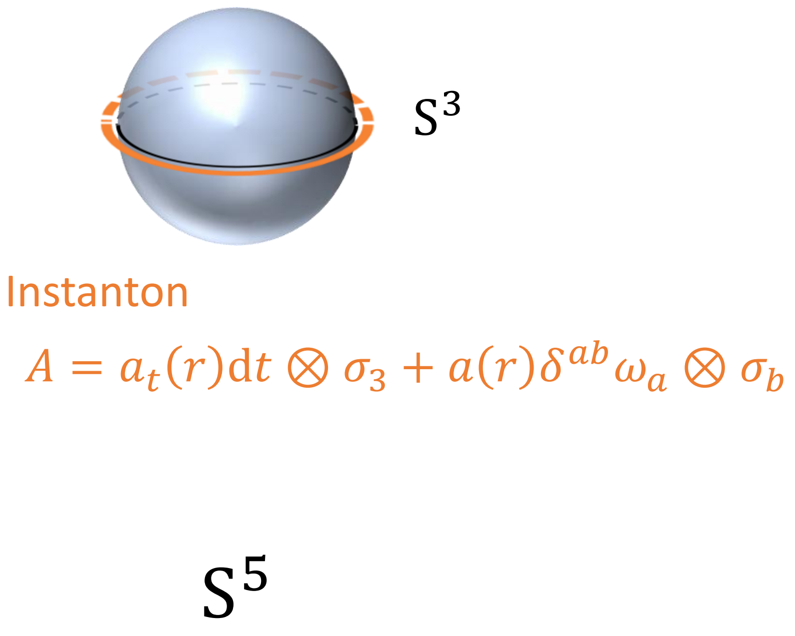


Dimensional reduction

- **Flavor brane action** $S_f = S_{DBI} + S_{WZ}$

- $\boldsymbol{S_{WZ}}$ **couples** the background **RR 4-form $\boldsymbol{C_4}$** to the **flavor gauge fields**

$$S_{WZ} = \frac{1}{2}\int C_4 \wedge \mathrm{Tr}(F \wedge F)$$

→ The **instanton** carries **color-brane charge**

- **Dimensional reduction** to 5d:

$$S_{WZ,5} = 24\pi^2 \int C_4 \wedge \mathrm{d}V(a) \qquad A = a_t(r)\mathrm{d}t \otimes \sigma_3 + a(r)\delta^{ab}\omega_a \otimes \sigma_b$$

- Lessons: CSC **in a bottom-up 5d setting** could be realized by a **similar mechanism** (with $D7 \to$ space$-$filling $D4$)

- The **top-down example** suggests that this can be obtained by including the **appropriate WZ term**

→ **Bottom-up point of view**: only term with C_4 compatible with **RR gauge-invariance**

3. The bottom-up model

Holographic dictionary

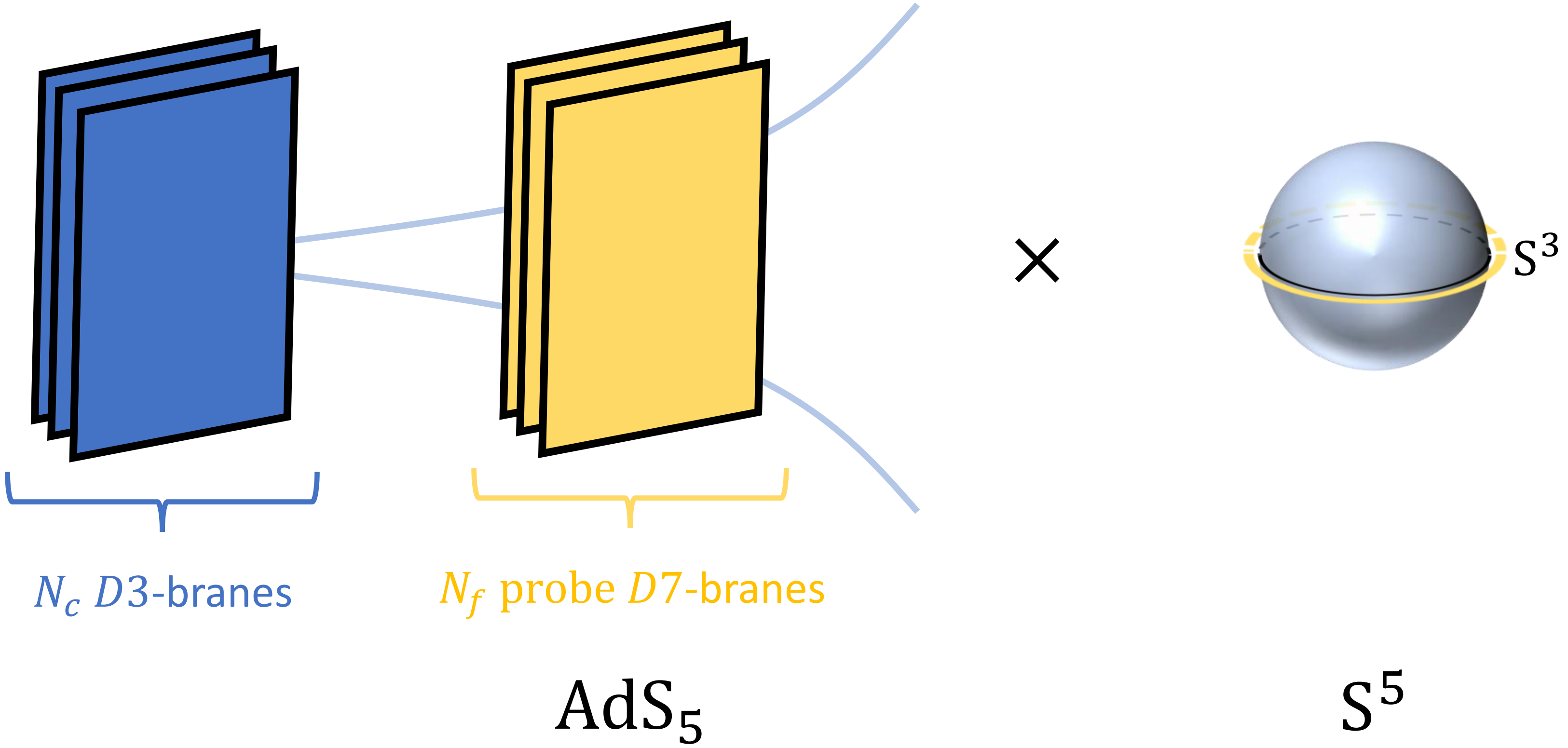
$T_{\mu\nu}$	$\leftrightarrow$	g_{MN}
$U(N_f)_L \times U(N_f)_R : J^{\mu}_{L/R} = 0$	$\leftrightarrow$	$A_M^{L/R}$ ($D4$ and $\overline{D4}$)
$\bar{\psi}^i \psi_j$ or $\phi^{i\dagger}\phi_j$	$\leftrightarrow$	"tachyon" $\mathcal{T}_j^{\bar{\imath}}$ ($D4$ •-∘ $\overline{D4}$)

+ **4-form C_4** sourced by the **color branes**

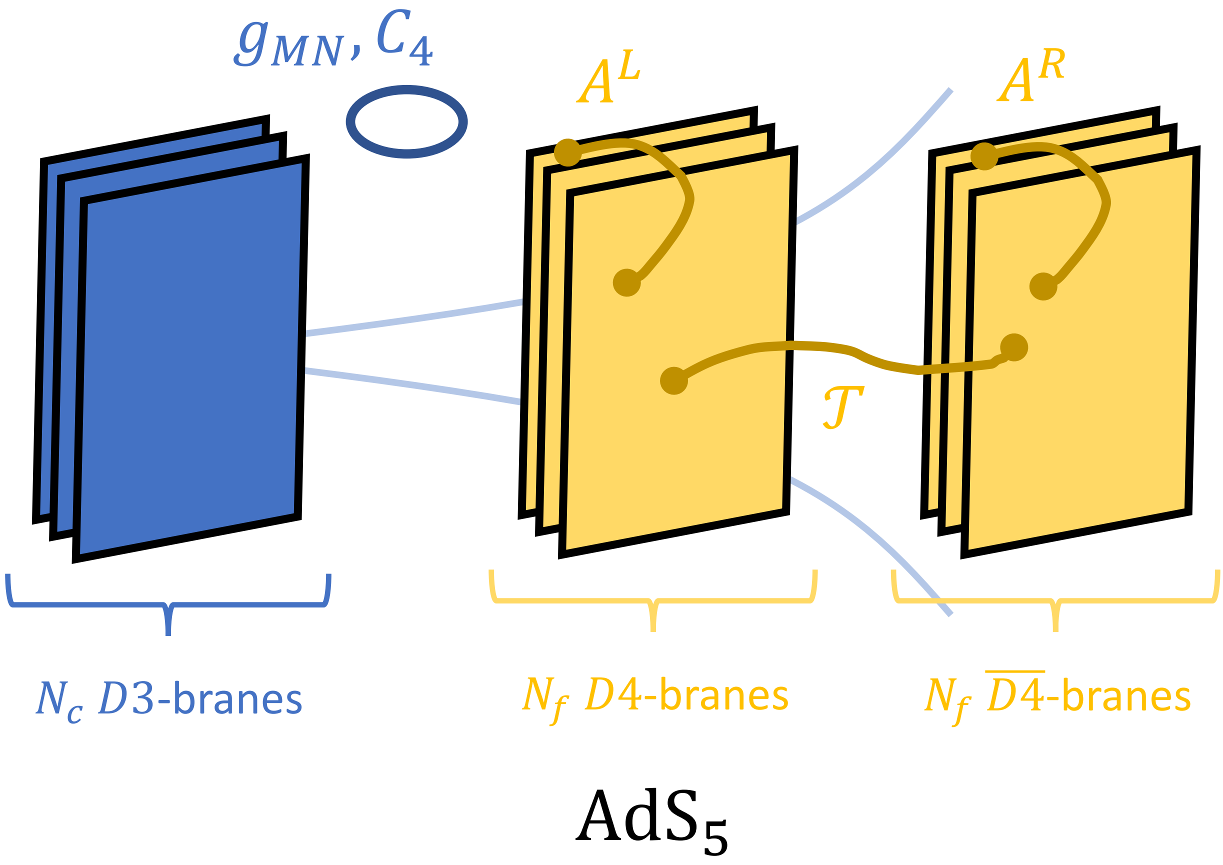
Order parameter: $\langle\bar{\psi}\psi\rangle$ (not $\langle\bar{\psi}\psi\bar{\psi}\psi\rangle$) → generated by **instanton** effects in **pQCD**

[Rajagopal & Wilczek'01]

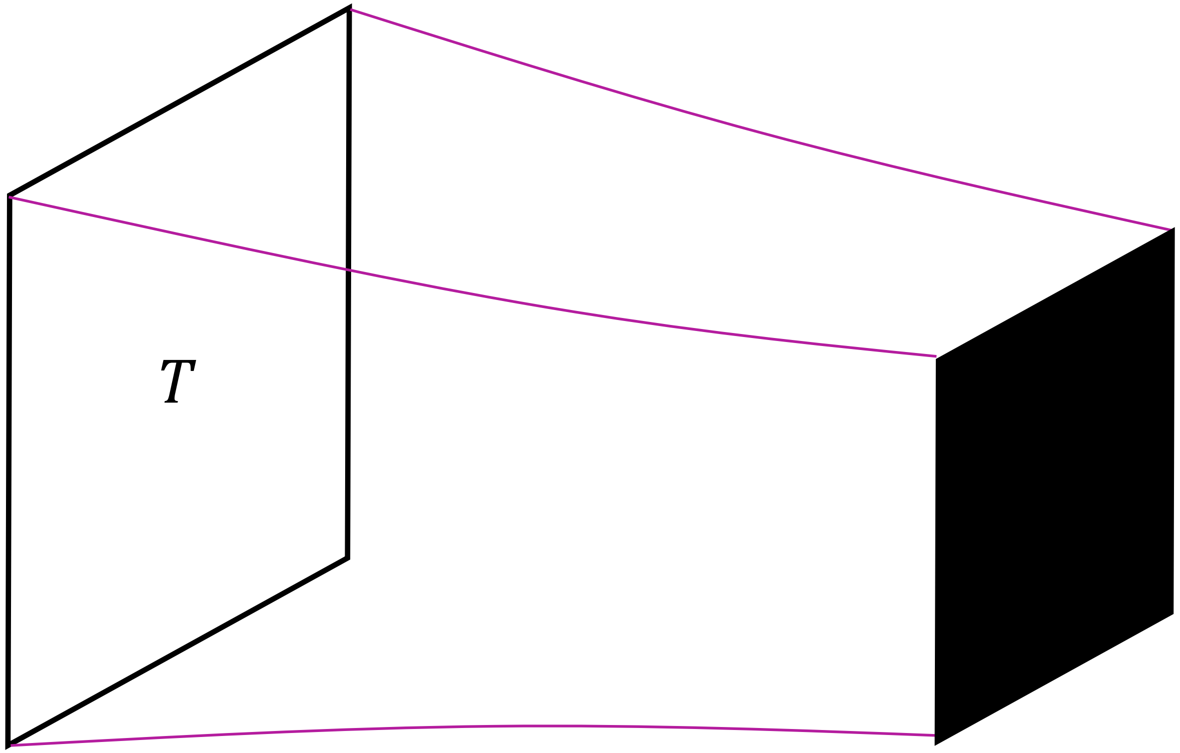
String-theoretic picture



String-theoretic picture



Ansatz



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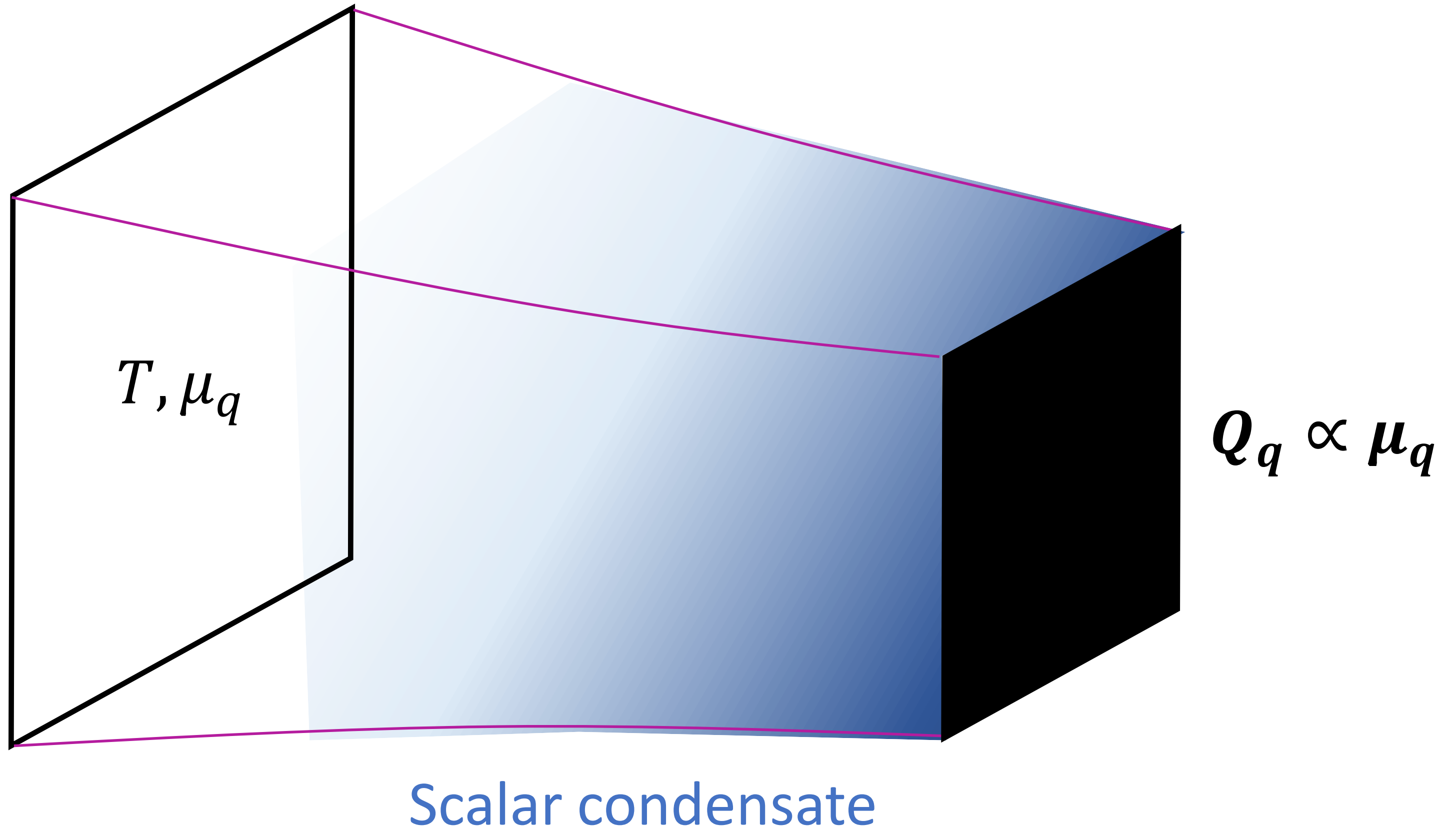
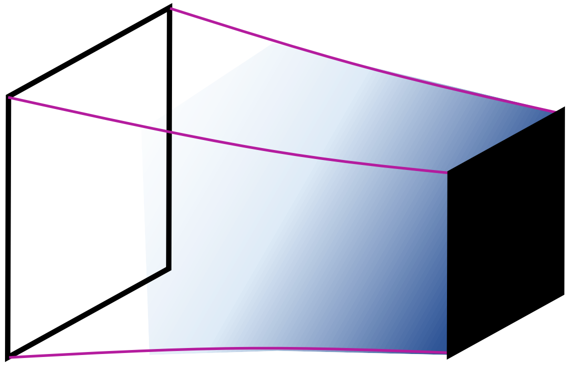


Ansatz



- States at finite **temperature T** quark number **chemical potential μ_q**.
  μ_q sources the $\boldsymbol{U(1)_{L+R}}$ gauge field
$$A^L = A^R = A_M \mathrm{d}x^M \otimes \mathbb{I}_{N_f}$$

- **Chiral symmetry breaking** $\quad \mathcal{T} = \chi \mathbb{I}_{N_f}$

→ χ is the equivalent of $\boldsymbol{a}$ in the **top-down** example

- **Metric** ansatz: $\mathrm{d}s^2 = \mathrm{e}^{2A(r)}(-f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + \mathrm{d}\vec{x}^2)$

Summary of bulk fields

$$A, f \qquad C_4 \qquad A_M \qquad \chi$$

Bulk action

$$S = S_c + S_f + S_{WZ}$$

- **Color** action

$$S_c = \int \mathrm{d}x^5 \sqrt{-g}\left(R - \Lambda - \frac{1}{2 \cdot 5!} F_5^2\right)$$

- **Flavor** part: Sen action [Sen'04] ($D4 - \overline{D4}$ including the **tachyon**)

$$S_f = -\frac{\boldsymbol{N_f}}{\boldsymbol{N_c}} \int \mathrm{d}x^5 \boldsymbol{V_f(\chi)} \sqrt{-\det(g_{MN} + \ell^2 F_{MN} + \ell^2 \partial_M \chi \partial_N \chi)}$$

- **WZ term**: of the form found in the **top-down example**

$$S_{WZ} = \frac{\boldsymbol{N_f}}{\boldsymbol{N_c}} \ell \int C_4 \wedge \mathrm{d}\boldsymbol{V_{WZ}(\chi)}$$

→ Responsible for color **Higgsing** + crucial role for **chiral condensation** (next section)

Parameters

$$x_f \equiv \frac{N_f}{N_c} \qquad V_f(\chi) \qquad V_{WZ}(\chi)$$

- Consider the simplest case of **quadratic potentials**

$$V_f(\chi) = c_0 \ell^{-2} + \frac{1}{2}(m_{UV}^2 - m_{WZ}^2)\chi^2, \qquad V_{WZ}(\chi) = \frac{1}{2} m_{WZ}^2 \chi^2$$

- $\boldsymbol{c_0}$ **is irrelevant** (can be absorbed), and m_{UV} is related to the **dimension Δ** of $\mathcal{O}$ dual to χ

$$m_{UV}^2 = \Delta(\Delta - 4)\ell^{-2}$$

- So we work with **2 parameters**

$$x_f \quad \text{and} \quad m_{WZ}^2$$

4. Stability analysis

Instability mechanism

- Consider **solutions to the EOMs** at finite temperature $\boldsymbol{T}$ **and** $\boldsymbol{\mu_q}$

→ Depend on $\boldsymbol{T/\mu_q}$

- The **uncondensed** solution ($\chi = 0$) always exists: **DBI black brane** [Alho,Järvinen,Kajantie,Kiritsis,Rosen,Tuominen'14]

- **Question:** when is the DBI BB unstable to **scalar condensation** ?

→ Look for **marginally stable modes** = normalizable solutions of

$$\delta\chi''(r) - \alpha(r)\delta\chi'(r) - \frac{\ell^2 m_{eff}^2(r)}{r^2 f(r)}\delta\chi(r) = 0$$

- **Qualitative mechanism**: instability when $\boldsymbol{m_{eff}^2}$ is **sufficiently negative** over a **large enough region** in the bulk

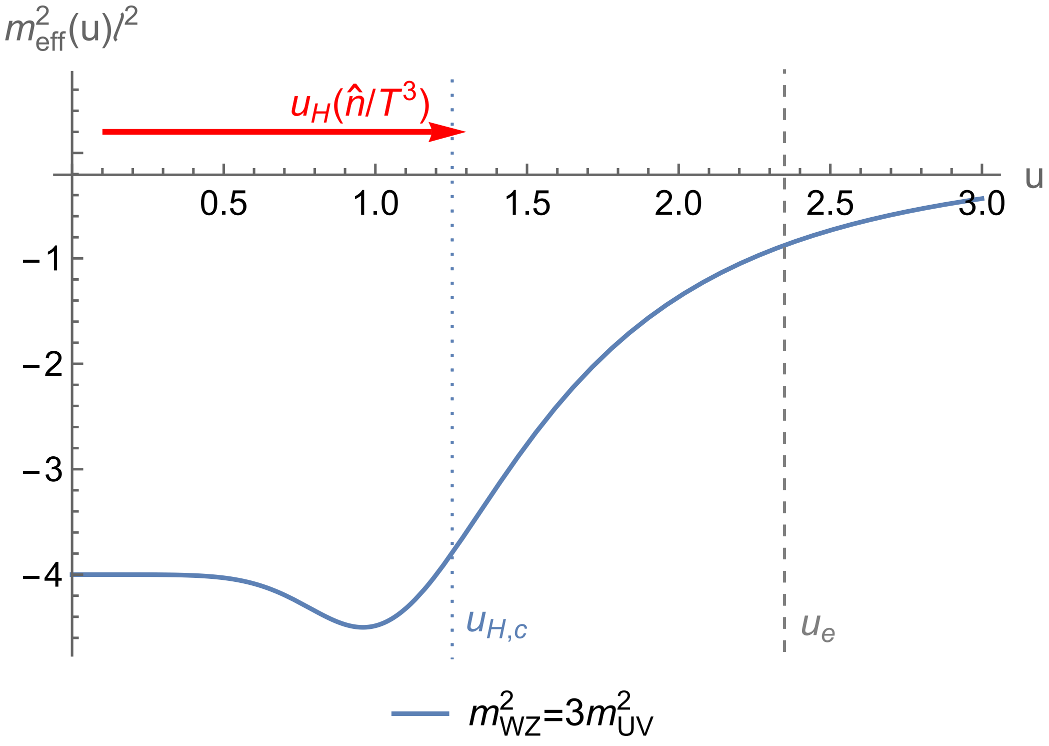
Instability mechanism

- $m_{eff}^2(r)$ can be expressed as a **function of $u \equiv \hat{n}^{1/3} r$**

→ **No other dependence on $\hat{n}/T^3$**

- $m_{eff}^2(u)$ has a **minimum**, which becomes **more negative with decreasing m_{WZ}^2**

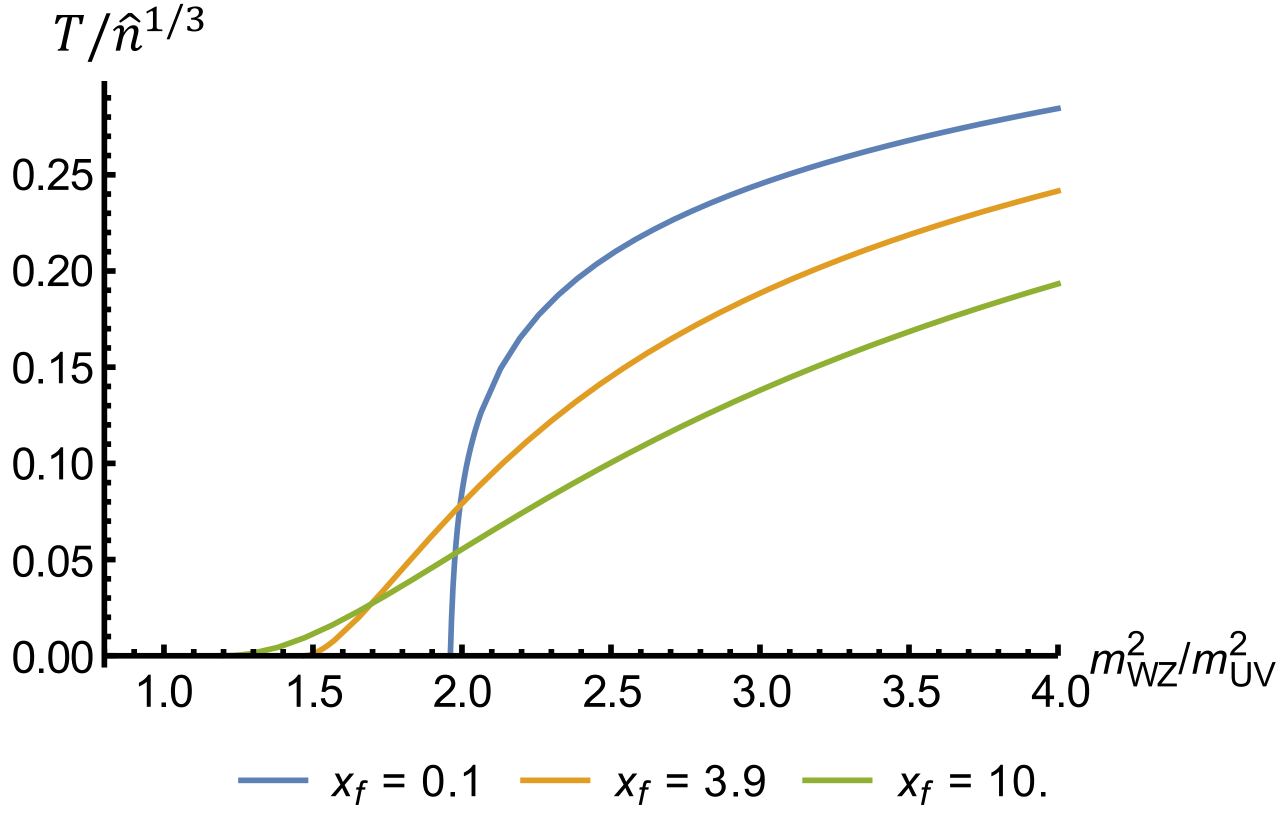
- The bulk extends from $u = 0$ to $u_H(\hat{n}/T^3)$



$\Delta = 2$

Phase diagram $(x_f, m_{WZ}^2; T/\hat{n}^{1/3})$

The model with **quadratic potentials** has a **second order** phase transition
→ The **instability surface** determines the **phase diagram**

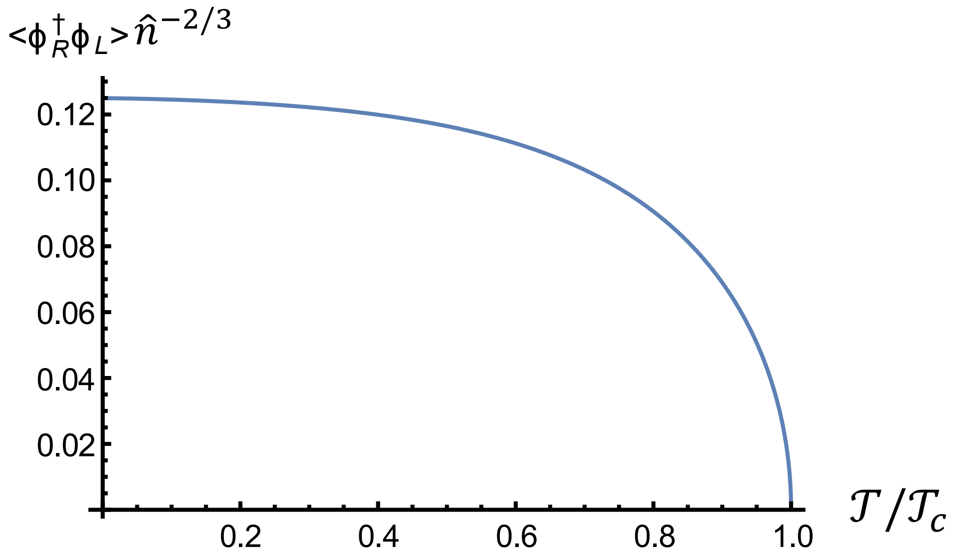


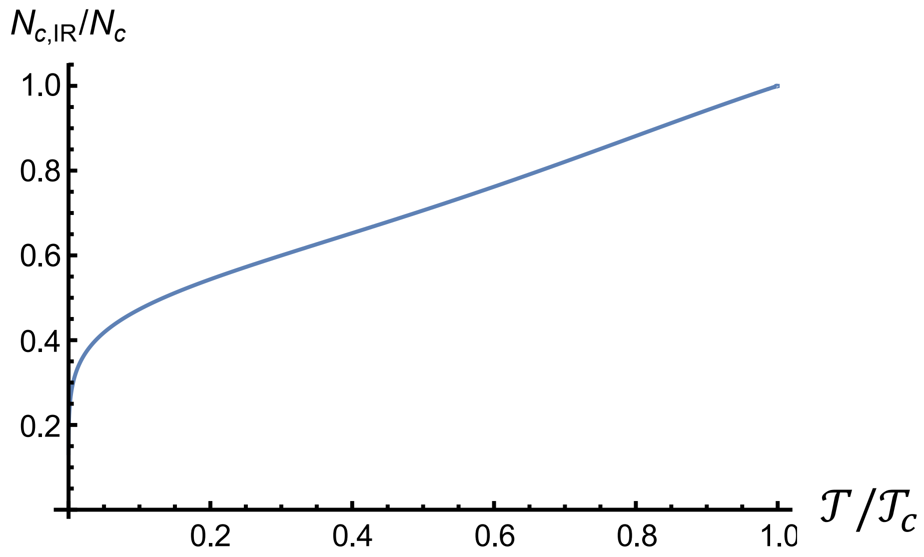
5. Properties of the condensed phase

The high-density phase features:

- Chiral condensate

- Finite Higgsing fraction



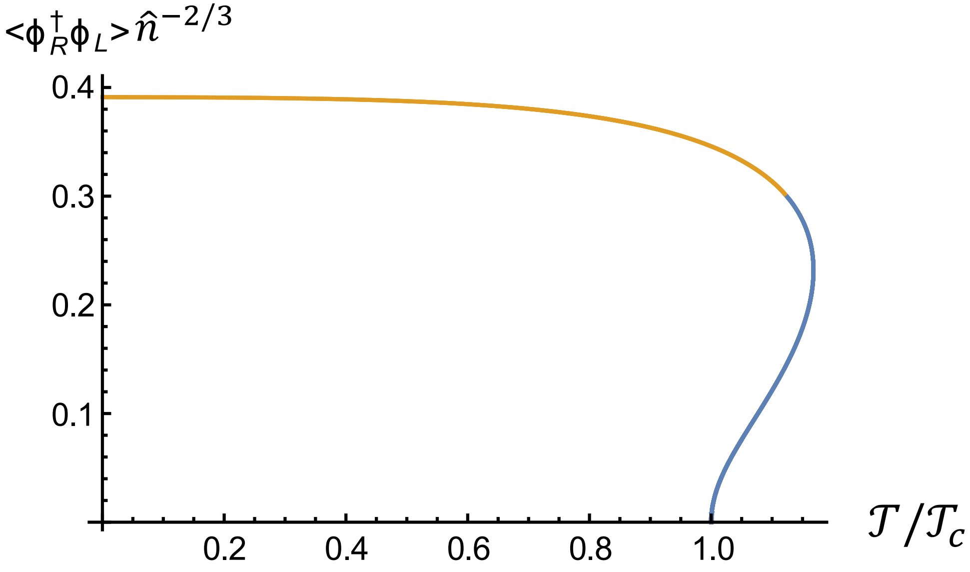


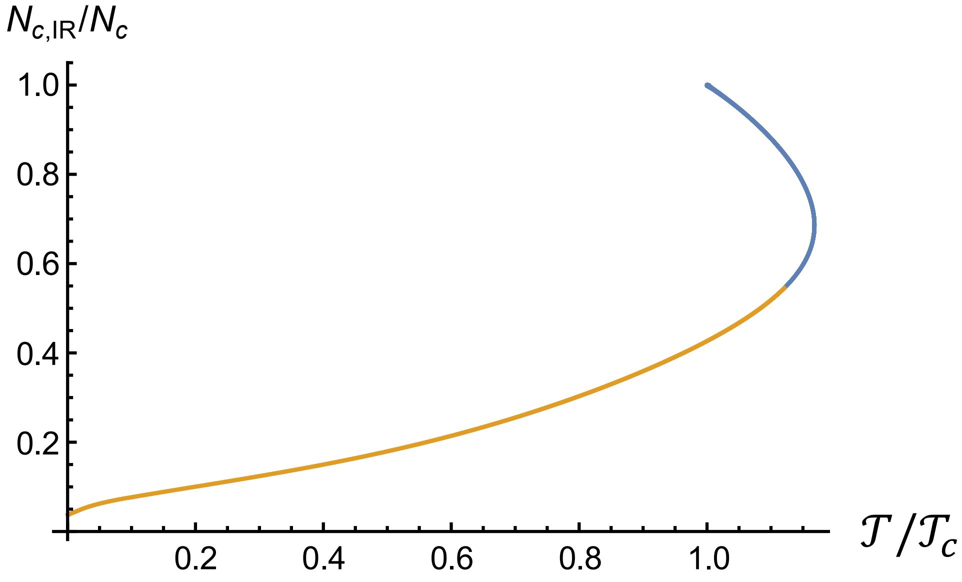
$\boldsymbol{\mathcal{T} \equiv T/\hat{n}^{1/3}}$; $m_{WZ}^2 = 4m_{UV}^2$ and $x_f = 1$

- Non-standard behavior: **Higgsing is continuous** at the transition, and **decreases** towards low temperatures

A first-order example

$$V_{WZ}(\chi) = \frac{1}{2} m_{WZ}^2 \chi^2 (1 + \alpha\chi^2), \qquad (\alpha > 5)$$





$\boxed{\alpha = 8}$

6. Summary

- Taking inspiration from a **D3/D7 setup**, we built a **bottom-up model** with a **CSC-like phase at high density**. The latter features:
  - **Chiral symmetry breaking**
  - Partial **Higgsing** of the color group

- **Main particularity**: new type of **WZ term**

$$S_{WZ} \propto \int C_4 \wedge \mathrm{d}V_{WZ}(\chi)$$

- Depending on the choice of **potentials**, the model can accomodate both **2nd-order and 1st-order** phase transitions

Outlook

- **Transport** properties (mostly determined by χ**SB**)

- Extend to **V-QCD** by including the **dilaton**

- Dynamics of the NSNS and RR **2-forms** (B and C_2) [Chen,Hashimoto,Matsuura'09]

→ Become **light** in the IR for **strong Higgsing**

- Explore the **dynamics of C_4** in other bottom-up constructions

→ Even with a **standard WZ term** in 5d $\supset C_{-1} \wedge \mathrm{CS}_5(A)$

→ **Baryons** carry **magnetic $D3$ charge**

Thank you !

[arXiv:2501.16472]

Appendix

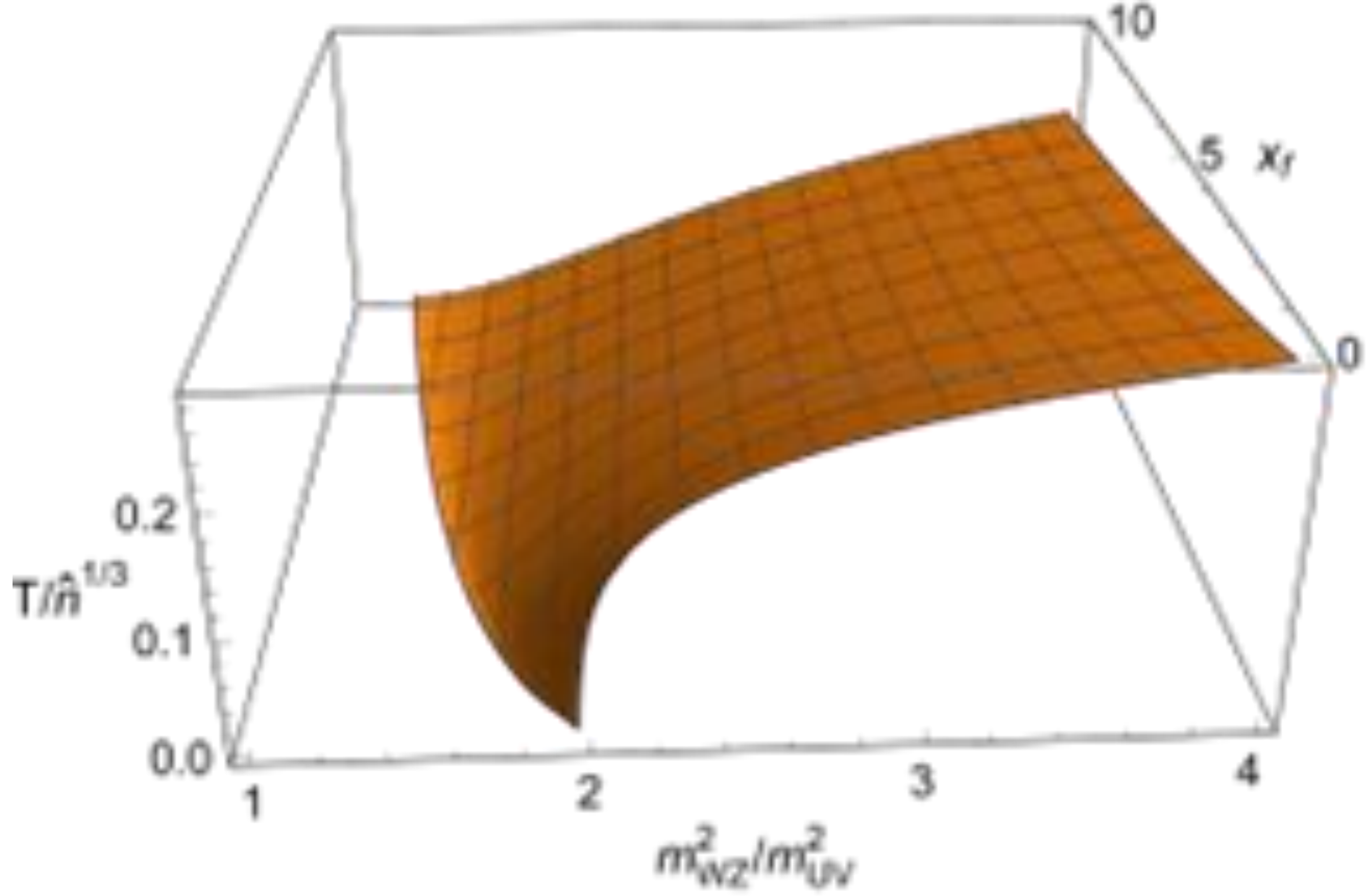
Definition of $N_{c,eff}(\mathrm{r})$

From the 5-form field strength:

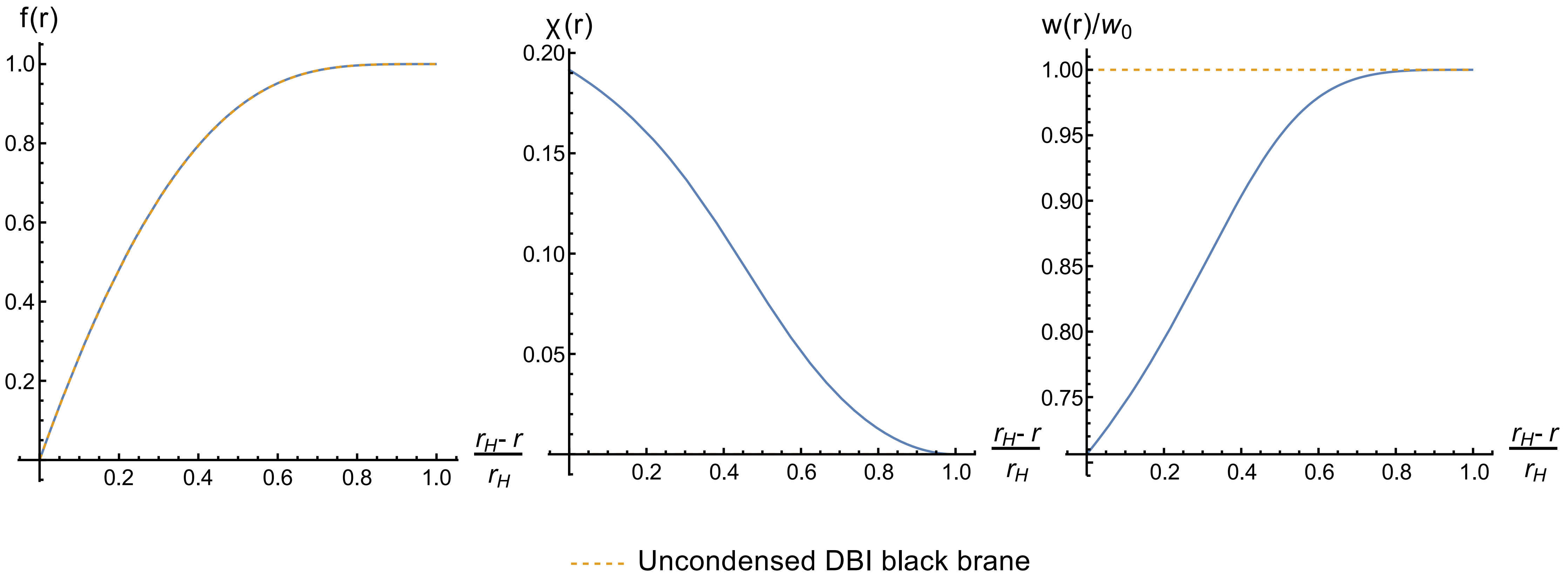
$$F_5 = \sqrt{-g}\,\frac{w(r)}{\ell}\,\mathrm{d}t \wedge \mathrm{d}r \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

$$N_{c,eff}(r) = N_c w(r)\ (\mathrm{Gauss's\ law})$$

3D phase diagram

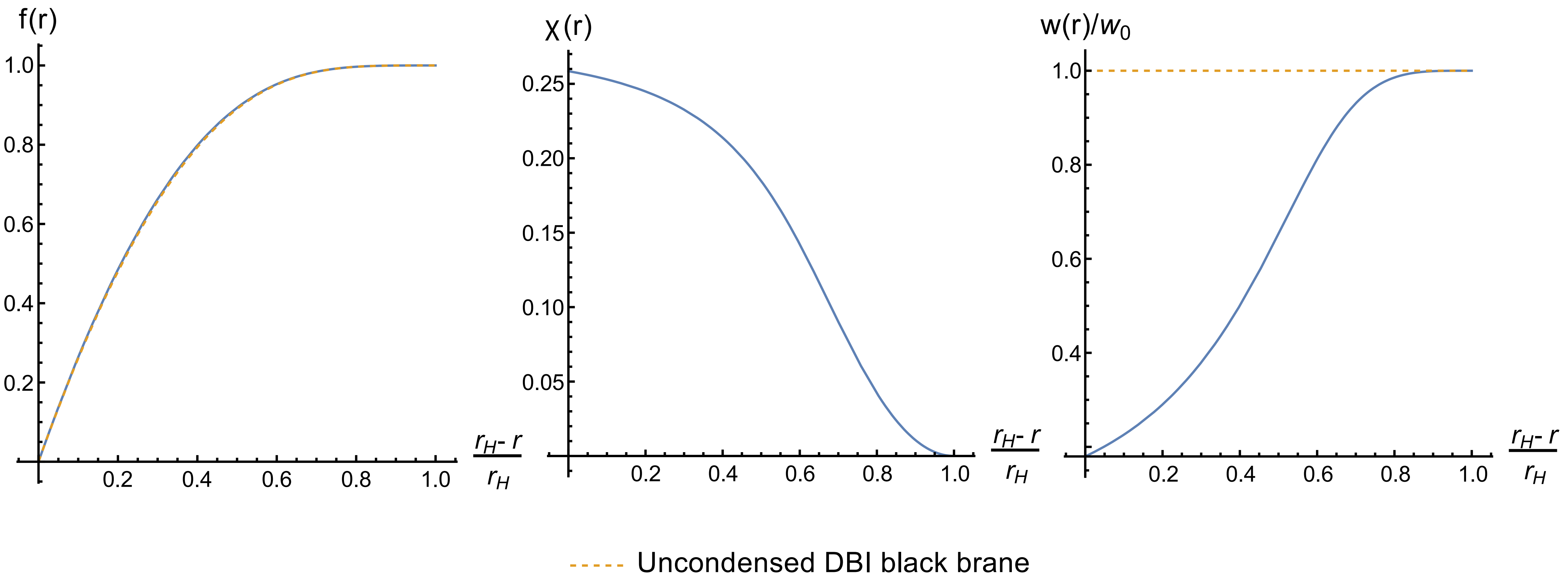


Background – 2nd order



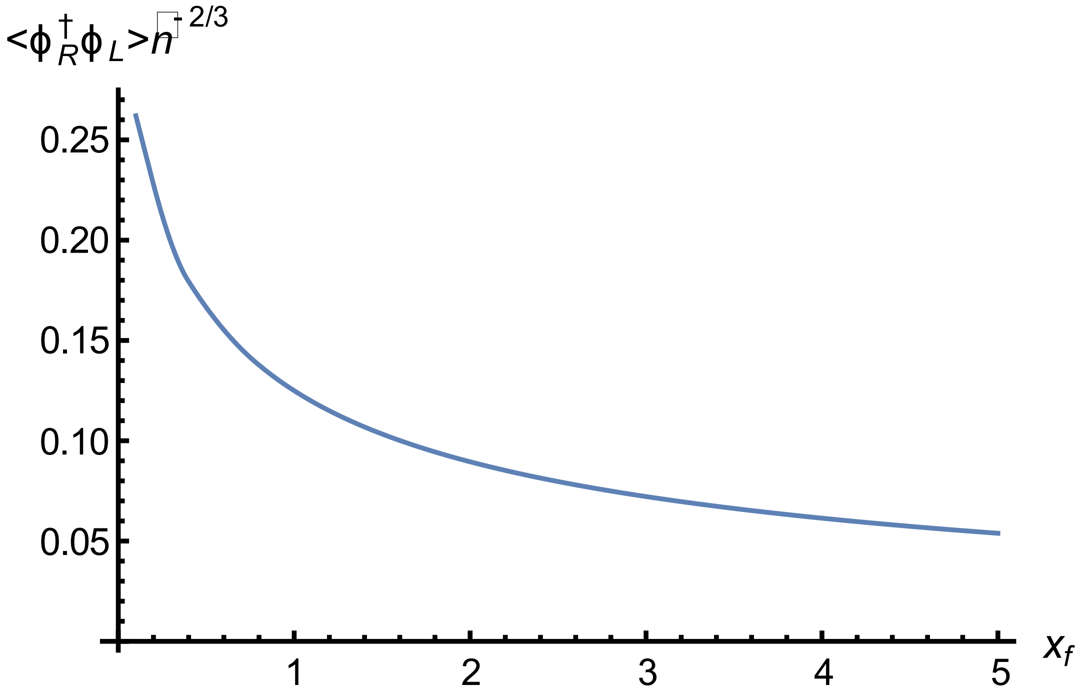
$\mathcal{T} = \mathcal{T}_c/2; m_{WZ}^2 = 4m_{UV}^2$ and $x_f = 1$

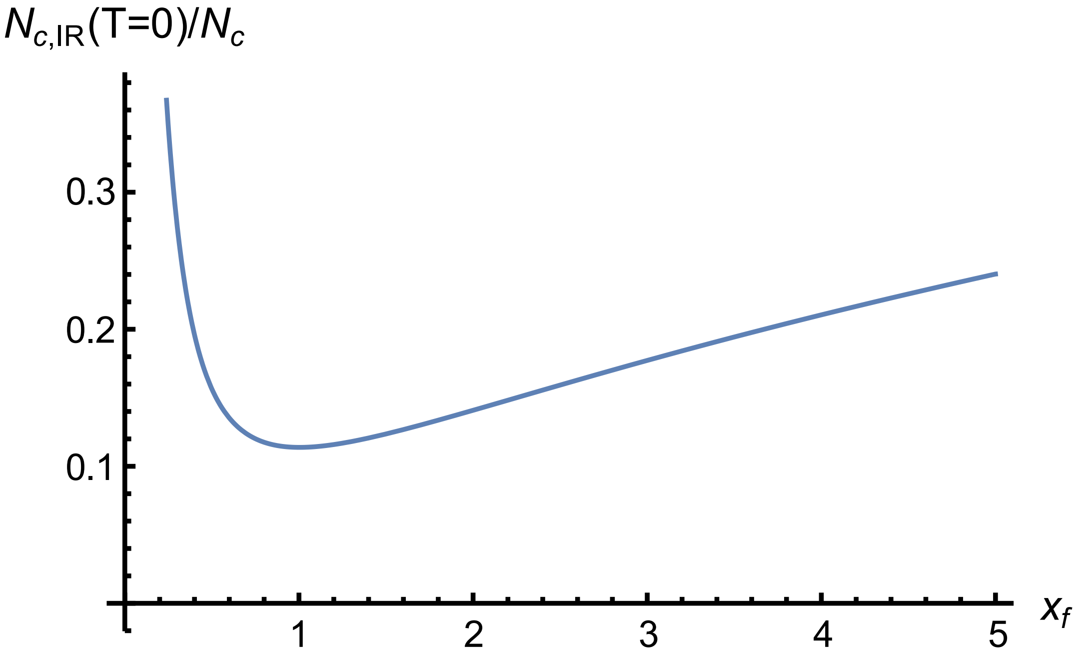
Background – 1st order



$\mathcal{T} = \mathcal{T}_c/2; m_{WZ}^2 = 4m_{UV}^2$ and $x_f = 1$

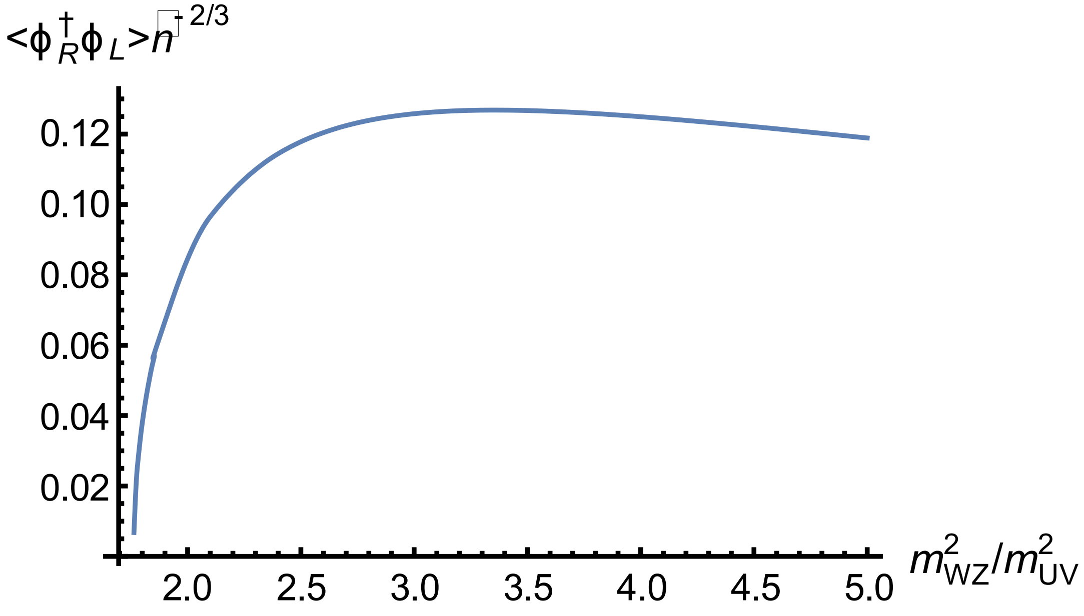
Parameter dependence - x_f

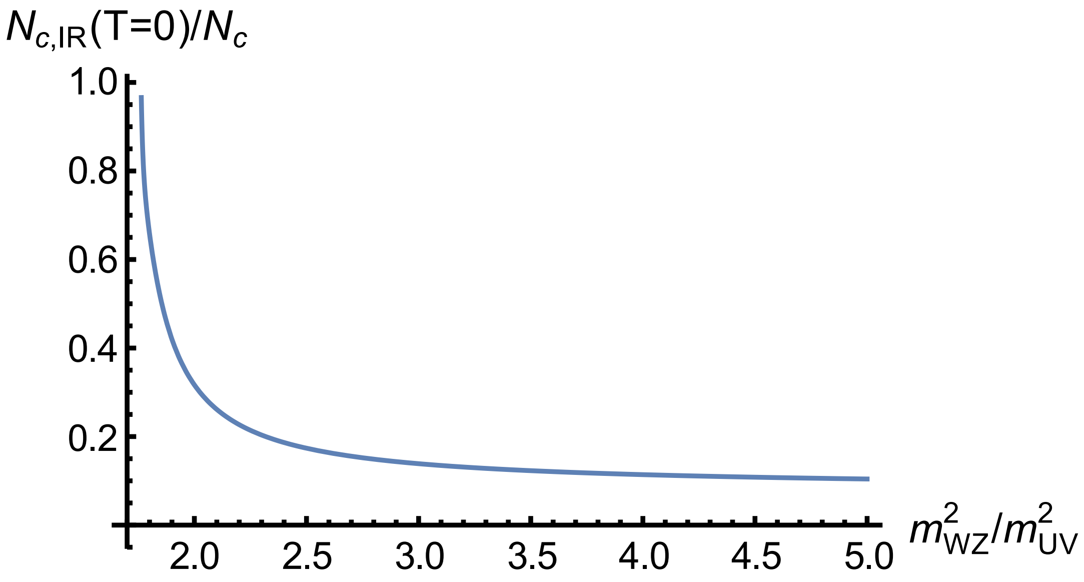




$$\mathcal{T} = 0; m_{WZ}^2 = 4m_{UV}^2$$

Parameter dependence - m_{WZ}^2





$$\boxed{\mathcal{T} = 0;\, x_f = 1}$$