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Hall droplet baryons in holographic QCD

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SEZIONE DI FIRENZE

- arXiv: 2506.16205, with Aldo L. Cotrone, Andrea Olzi, Jean-Loup Raymond
- JHEP 06 (2025) 018, with Aldo L. Cotrone, Andrea Olzi
- Phys.Rev.D 108 (2023) 2, with Aldo L. Cotrone, Andrea Olzi
- JHEP 02 (2023) 194, with Aldo L. Cotrone, Andrea Olzi

Apologies

- It’s going to be a talk mainly about one baryon
- No dense matter for the moment.

Plan

- Single-flavor baryons: which effective description?
- A proposal: baryons as quantum Hall droplets
- Testing the proposal: Hall droplet baryons in holographic QCD
- Other related configurations

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Effective low energy description of QCD

- Low energy effective action for QCD, for $N_f \geq 2$
- Pion matrix $U = e^{i \sum_{a=1}^{N_f^2-1} \frac{\pi^a(x) T^a}{f_\pi}}$
- Chiral Lagrangian (with postulated Skyrme term)

$$\mathcal{L}_{eff} = -\frac{f_\pi^2}{4} Tr\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2} Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

- At large N: meson interactions suppressed by 1/N powers
- Baryons mass scales like N: baryons as solitons?

Low energy description of baryons

- Solitons (Skyrmions) = finite energy collective excitations of the pion field
- Take e.g. $N_f=2$. Finite energy implies that, say

$$U(\boldsymbol{r},t) \underset{|\boldsymbol{r}|\to\infty}{\longrightarrow} 1$$

- A map from S^3 to the space of SU(2) matrices isomorphic to S^3
- Mappings of S^3 into S^3 fall into distinct equivalence classes.
- These are labeled by a integer winding number $(\Pi_3(SU(N_f=2))=\mathbb{Z})$

$$n_B = -\frac{1}{24\pi^2}\int_{\mathbb{R}^3} \mathrm{tr}(\mathrm{U}^{-1}\mathrm{dU})^3$$

- This can be seen as the charge associated to a conserved current

$$B_\mu = \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{tr}\left[\left(U^\dagger\partial^\nu U\right)\left(U^\dagger\partial^\rho U\right)\left(U^\dagger\partial^\sigma U\right)\right]$$

- This turns out to correspond to the $U(1)_B$ baryon current.
- Hence winding number n_B = baryon number.
- Skyrmion = baryon

Low energy description of baryons

$$n_B = -\frac{1}{24\pi^2}\int_{\mathbb{R}^3} \mathrm{tr}(\mathrm{U}^{-1}\mathrm{dU})^3$$

- Static spherically simmetric solution (hedgehog)

$$U_0(\boldsymbol{r}) = \exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)\right)$$

- Imposing $F(\infty)=0$, get $F(0) = n_B\,\pi$,
- Mass ~N, size ~ N^0 (and stabilized adding ω meson)
- This description holds for $N_f \geq 2$.

What about $N_f = 1$ baryons?

- Problem: no pions to construct the baryon from.
- In other words: topological current $J^\mu = 0$
- In other words: $\Pi_3(U(N_f = 1)) = 0$
- Hence: no Skyrmions with one flavor !

What about $N_f = 1$ baryons?

- Let us consider their microscopic description
- Single flavor baryon in SU(N) QCD:

$$(\Psi_{N_f=1\,\text{baryon}})_{s_1\ldots s_N} = \epsilon_{a_1\ldots a_N}\, q^{a_1}_{s_1} \cdots q^{a_N}_{s_N} \quad (s_i \text{ spin indices})$$

- Due to antisymm. and fermionic nature, need symm. spin indices
- Hence total spin is J=N/2
- Small quark mass, large N: spin-orbit and spin-spin effects deform it
- Expect a pancake-like structure

- Is there a low-energy description of these single flavor baryons?
- At large N, effective theory is nearly free theory of mesons and glueballs.
- What if the baryon charge is related to the entire space of these fields?

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A proposal for the solution

- In single flavor low energy SU(N≫1) QCD, baryons are charged sheets
  [Komargodski 2018 + others before]

- η' (phase of quark condensate) light at N≫1. LO effective Lagrangian is

$$\mathcal{L}_{eff} = \frac{1}{2}\left(\partial\eta'\right)^2 - \frac{1}{2}m\Lambda_{QCD}\cos(\eta')$$

  [m quark mass, $\Lambda_{\rm QCD}$ dynamical scale]

- Theory has unique gapped ground state at $\eta'=0$

A proposal for the solution

- This effective theory has a conserved current with 3 indices

$$J_{\mu\nu\rho} = \frac{1}{2\pi}\epsilon_{\mu\nu\rho\sigma}\partial^{\sigma}\eta'$$

- A two-form U(1) symmetry. Of course not a symmetry of full theory (QCD): this symmetry has to be broken in the full theory.
- Charged objects under this symmetry are 2+1 dimensional sheets
- η' has non-trivial monodromy through the sheet

- Infinite sheet similar to domain-wall but no related charge: unstable
- Still at large N expect it to be metastable

A proposal for the solution

- Actually it is well known that adding 1/N corrections

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2}m\Lambda_{QCD}\cos(\eta') - \frac{1}{2}m_{WV}^2 \mathrm{Min}_{k\in\mathbb{Z}}(\eta' + \theta + 2\pi k)^2$$

[m_{WV} Witten-Veneziano mass, set θ=0]

- Potential is continous but with cusp singularity at $\eta' = \pi$ mod 2π k
- Signals that some degrees of freedom have been improperly integrated out
- Can think that it is due to extra (gluonic) degrees of freedom that rearrange

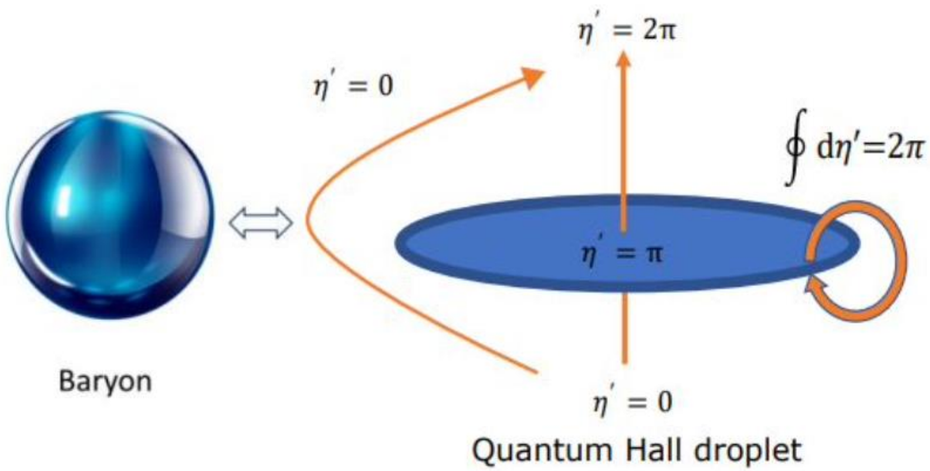
- Komargodski proposal: effect of singularity is to induce a topological field theory on the sheet

A proposal for the solution

- Proposal: sheet hosts a $U(1)_N$ Chern-Simons theory on its world-volume
- If it has circular boundary it can have chiral edge modes: baryonic charge!

$$S_{sheet} = T_{sheet}\int_{sheet} d^3x\sqrt{\det \gamma_{ind}} + \int_{sheet}\left(\frac{N}{4\pi}ada + \frac{1}{2\pi}adA^B\right)$$

- Hence: single flavor baryon = quantum Hall droplet
- Charge forbids shrinking of the sheet. Size = $O(N^0)$. Mass = $O(N)$
- It has spin J=N/2 ⇒ pancake shaped.



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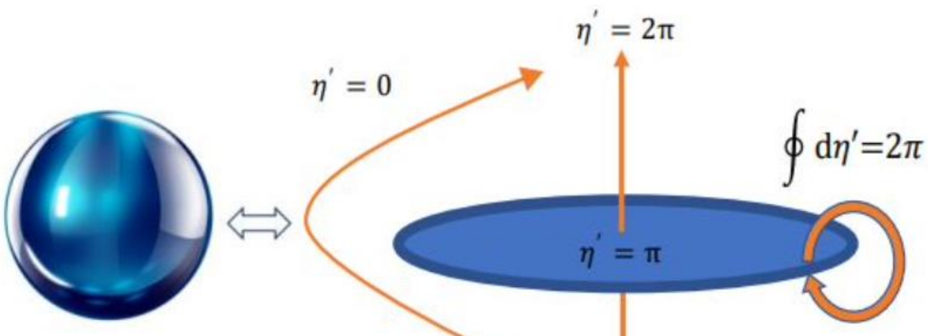

Effective field theory singular around the sheet:
How can we test the proposal and compute properties of this baryon?

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A Dp-brane tale

$$S_{\mathrm{D}p} = -\,T_p \int d^{p+1}x\, e^{-\phi}\sqrt{-\det(P[g+B]_{ab} + 2\pi\alpha' F)} \, +$$

$$+\,\mu_p \int \sum_{q=0}^{p+1} P[C_q e^B] \wedge e^{2\pi\alpha' F},$$

$$T_p \sim \frac{1}{g_s}$$

The holographic model

[Witten 1998, Sakai-Sugimoto 2004]

- Type IIA setup
- $N \gg 1$ D4-branes on circle S^1_{x4}, radius $R_4 = 1/M_{KK}$, antiperiodic fermions.
- Low energy: 4d non-susy SU(N) Yang-Mills + massive adjoint KK modes

- $N_f \ll N$ D8-anti D8 branes, placed at different points on S^1_{x4} circle
- Low energy: N_f chiral quarks in 4d
- Confinement, mass gap, chiral symmetry breaking

- Holographic description: gravity background sourced by D4s and probed by D8s (no backreaction = quenched approximation for flavors)

Holographic Yang-Mills [Witten 1998]

- Gravity action (closed string description)

$$S = \frac{1}{2k_0^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(\mathcal{R}+4(\partial\phi)^2\right)-\frac{1}{2}|F_4|^2-\frac{1}{2}|F_2|^2\right]$$

- Gauge theory action (open string description, IR limit of D4-brane action)

$$S = \frac{1}{8\pi g_s l_s M_{KK}}\int d^4x\ \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2 l_s}\int_{S_{x^4}} C_1 \int d^4x\ \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

- Theta term corresponds to C_1:

$$\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1$$

$$F_2 = d\, C_1$$

- **Classical gravity** picture **dual to gauge theory at** $\lambda_4 >> 1$, $N_c >> 1$

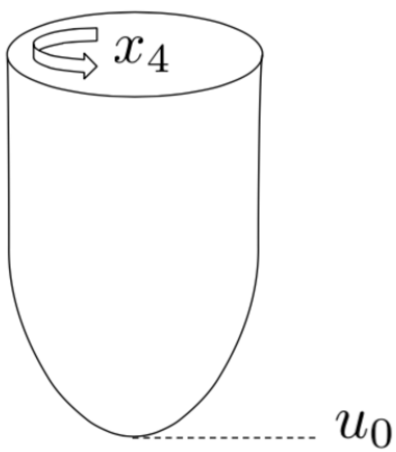
$$\lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c$$

The type IIA gravity background

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[dt^2 + dx^i dx^i + f(u)dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$f(u) = 1 - \frac{u_0^3}{u^3}\,, \qquad u_0 = \frac{4}{9}R^3 M_{KK}^2\,, \quad R^3 = \pi g_s N l_s^3$$

Moreover, importantly, RR 4-form $\int_{S^4} F_4 \sim N$ and running dilaton



- $x_4 \sim x_4 + 2\pi/M_{KK}$
- (u, x_4) subspace is a cigar
- $g_{00}(u_0) \neq 0$, regular metric
- ✓ IR: Confinement and mass gap
- ✓ Lowest glueball mass ~ M_{KK}

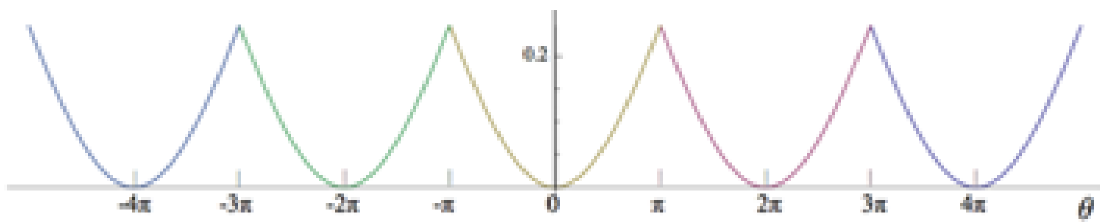
Theta dependence

- To leading order in θ/Nc treat C_1 as a probe [Witten 1998]:

$$F_2 = dC_1 = \frac{4\lambda^3 M_{KK}^4 l_s^6}{3^5\pi}\frac{\theta + 2\pi k}{u^4} du \wedge dx_4$$

- Theta dependence of free energy from holography:

$$\varepsilon(\theta) - \varepsilon(0) = \min_{k\in\mathbb{Z}} \frac{1}{2\kappa_0^2}\frac{l_s^2}{2}\int dC_1 \wedge \star dC_1\Big|_{\text{on-shell}} = \min_{k\in\mathbb{Z}} \frac{\lambda^3 M_{KK}^4}{8(3\pi)^6}(\theta + 2\pi k)^2.$$



- Topological susceptibility: $\chi_g = \frac{d^2\varepsilon(\theta)}{d\theta^2}\Big|_{\theta=0} = \frac{\lambda^3 M_{KK}^4}{4(3\pi)^6}$

- Physics periodic under $\theta \longrightarrow \theta + 2\pi k$ (since instanton charge is quantized)

- Cusp singularities at $\theta = (2k+1)\,\pi$: here CP spontaneously broken

Domain walls

- At $\theta = \pi$ two branches (k=0, k=-1) join. They correspond to two different (though degenerate) vacua. There is a domain wall separating these.
- Domain wall = D6-brane wrapped on S^4 [Witten 1998] at the cigar tip
- D6 electrically charged under C_7 , magnetically under Hodge dual C_1
- The value of k jumps by ±1 crossing it. It is infinitely extended and stable.
- Moreover, D6-brane action features a CS term (a= gauge field on D6)

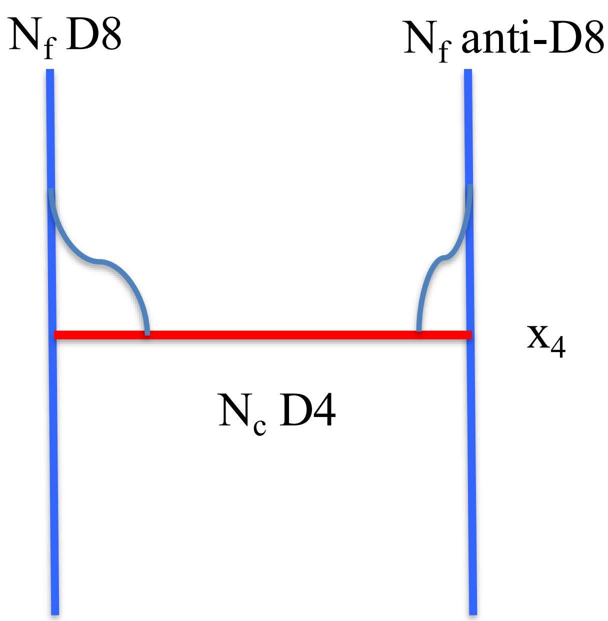
$$\frac{1}{8\pi^2}\int C_3 \wedge f \wedge f = \frac{1}{8\pi^2}\int F_4 \wedge a \wedge f = \frac{N}{4\pi}\int a \wedge f$$

- Effective theory on the DW is a $U(1)_N$ Chern-Simons theory in 2+1 dims! [Acharya-Vafa 2001, Argurio, Bertolini, FB, Cotrone, Niro 2018]
- DW tension (from D6 DBI action) scales like N

Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks

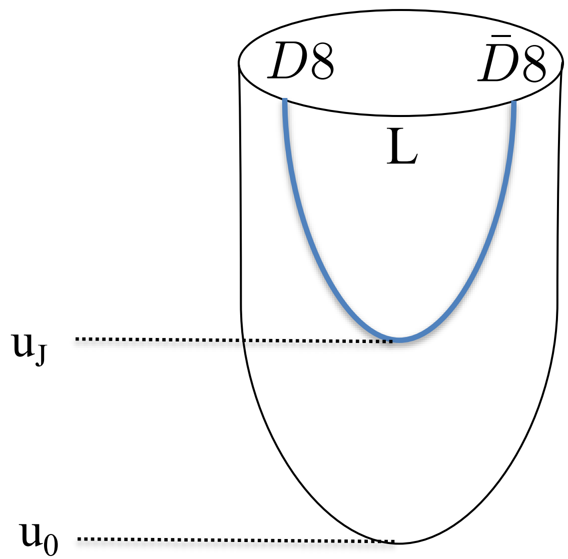
- Nf massless flavors from extra D4-D8 open strings
- $U(Nf)_L$ x $U(Nf)_R$ gauge symmetry on D8 dual to classical QFT chiral symm.



Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks

- At strong coupling, replace D4s by dual background.
- If $N_f << N_c$ treat D8-branes as probes.



- $U(N_f)$ gauge theory on D8
- Gauge field fluctuations = **mesons**
- Instanton solutions = **baryons** [Hata et al 2007]

- Chiral symmetry breaking = joining of the two branches
- Pion coupling $f_\pi \sim u_J$
- Extra parameter L corresponds to NJL-type coupling

Mesons

- D8-brane action reduced on S^4 gives effective action:

$$S_f = -\kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{h(z)}{2} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + k(z)\mathcal{F}_{\mu z}\mathcal{F}^{\mu}_{z} \right) + \frac{N_c}{24\pi^2} \int \mathrm{Tr} \left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5 \right)$$

$$\kappa = \frac{N_c \lambda}{216\pi^3}, \quad h(z) = (1+z^2)^{-1/3}, \quad k(z) = (1+z^2) \qquad (U, x_4) \to (z, y) \, , \, U^3 = 1 + y^2 + z^2$$

- $\mathcal{A} = \widehat{A}\frac{\mathbb{1}}{\sqrt{2N_f}} + A^a T^a$ U(N_f) gauge field in 5d. Eff. Action: 5d Yang-Mills CS theory

- Fluctuations correspond to pseudoscalars and whole tower of massive mesons:

$$\mathcal{U}(x^\mu) = \mathcal{P}\exp\left(-i\int_{-\infty}^{\infty} dz \, \mathcal{A}_z(x^\mu, z)\right)$$ Pion+ η’ $\quad f_\pi = 2\sqrt{\frac{\kappa}{\pi}}, \qquad e \sim -\frac{1}{2.5\kappa}$

$$\mathcal{A}_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z)$$ $B_\mu^{(n)}$: massive (axial) vectors, n (even) odd

- Chiral Lagrangian + Skyrme term + further mesonic contributions is **derived**

- Now C_7 action coupled to the abelian component of the gauge field on the D8

$$-\frac{(2\pi l_s)^6}{4\pi}\int dC_7 \wedge \star dC_7 + \frac{1}{2\pi}\int C_7 \wedge \mathrm{Tr}\, \mathcal{F} \wedge \omega_1$$

- Hence:

$$d \star dC_7 = \frac{1}{(2\pi l_s)^6} \mathrm{Tr}\, \mathcal{F} \wedge \omega_1$$

The η' sector

- Due to flavors modified field strength: $\tilde{F}_2 = dC_1 + \mathrm{Tr}\mathcal{A} \wedge \delta(y)dy$
- From $S_{\tilde{F}_2} = -\frac{1}{4\pi(2\pi l_s)^6}\int d^{10}x|\tilde{F}_2|^2$ and $\theta \sim \int C_1$
- Get, on-shell $S_{\tilde{F}_2} = -\frac{\chi_g}{2}\int d^4x \left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta'\right)^2$ $\int dz \widehat{A}_z = \frac{2\eta'}{f_\pi}$
- Veneziano-Witten relation. $m^2_{\eta'} = m^2_{WV} \equiv \frac{2N_f}{f^2_\pi}\chi_g$
- Focusing on Nf=1 case and reinserting the various branches we get the expected effective action

$$\mathcal{L} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2}m^2_{WV} Min_{k\in\mathbb{Z}}(\eta' + \theta + 2\pi k)^2$$

- [In the model we can add small quark masses as well]
- Note: after having integrated out closed string d.o.f. (glueballs), i.e. C_7 we get the expected effective action with cusps in the potential

Baryons

- Baryon vertex = D4-brane wrapped on S^4 [Witten 98]
- In fact on D4-brane action

$$\int C_3 \wedge f = \int F_4 \wedge a \sim N \int dt\, a_t$$

- Correspondingly N fundamental strings attached to D4-brane
- Other end-point on D8-brane: hence bound state of N fermions
- On the D8-brane worldvolume

$$\frac{1}{8\pi^2} \int_{\mathrm{D8}} C_5 \wedge \mathrm{Tr}\, (\mathcal{F} \wedge \mathcal{F})$$

- Hence, if we assume that the SU(Nf) gauge field is an instanton with

$$\frac{1}{8\pi^2} \int_{M_4} \mathrm{Tr}\, (\mathcal{F} \wedge \mathcal{F}) = n_B \in \mathbb{Z}$$

- Wrapped D4-brane = instanton on D8, with charge n_B (baryon charge)
- Baryon mass=D4-brane energy, scales with N. Size scales like $\lambda^{-1/2}$
- Realizes Skyrme picture; o.k. for Nf $\geq$ 2. What about Nf =1 ?

$N_f = 1$ baryons

- Dual of sheet identified as follows ($N_f = 1$):
- From potential

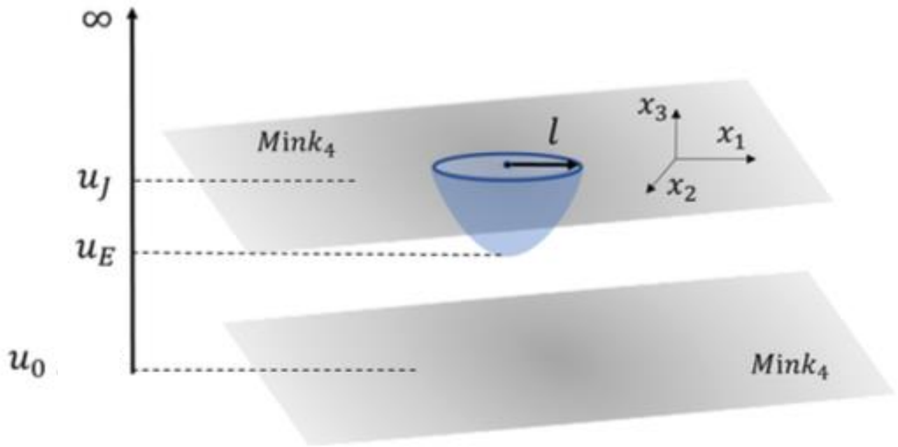
$$\frac{1}{2} m_{WV}^2 \mathrm{Min}_{k \in \mathbb{Z}} \left(\eta' + \theta + 2\pi k\right)^2$$

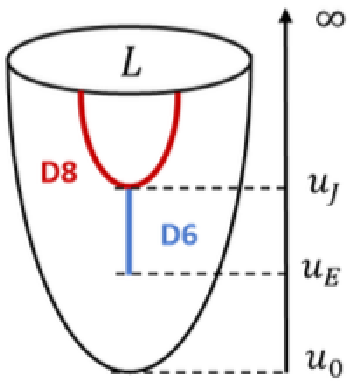
- η' transforms as θ
- At $\eta' = \pi$ we can envisage the occurrence of a DW-like sheet
- η' has non-trivial monodromy through the sheet if
- the sheet is a D6-brane (wrapped on S^4) [Witten 98, Dubovsky et al 2011, FB, Cotrone, Olzi 2022]
- As for the DW, the sheet hosts a $U(1)_N$ CS theory on its worldvolume

$$\frac{1}{8\pi^2} \int_{S^4 \times M_3} C_3 \wedge f \wedge f = \frac{1}{8\pi^2} \int_{S^4 \times M_3} F_4 \wedge a \wedge f = \frac{N}{4\pi} \int_{M_3} a \wedge f$$

- D6-brane can end on flavor D8 with circular boundary

  [FB, Cotrone, Olzi 2024; FB, Cotrone, Olzi, Raymond 2025]





- D6-brane wants to shrink due to its tension, but theory has $U(1)_B$
- Hence D6-brane can be stabilized by baryonic charge on its boundary
- Charged D6-brane = gluonic core of the baryon

- Circular boundary parameterized by angle ψ
- D6 embedding ρ(u). Pancake radius: l= $\rho(u_J)$
- Stability condition (orthogonality at D6-D8 intersection) $\rho'(u_J) = 0$
- At the D6-brane tip $\rho(u_E) = 0$, $\rho'(u_E) = \infty$ (regularity)
- Static, axisymmetric ansatz for the gauge field, in radial gauge $a_u = 0$

$$a_t(u)\,, \quad a_\psi(u)$$

- Then effective D6-brane action (after integration over S^4 and ψ)

$$S_{\rm D6} = -N \int dt \int_{u_*}^{u_J} du \left\{ \frac{(a_t\,\partial_u a_\psi - a_\psi\,\partial_u a_t)}{2} + \frac{u\,\rho(u) D(u)}{3(2\pi l_s^2)^2} \right\}$$

$$D(u) = \sqrt{\rho'(u)^2 \left(\frac{u}{R}\right)^3 + \frac{1}{f(u)} + (2\pi l_s^2)^2 \left(\frac{(\partial_u a_\psi)^2}{\rho^2} - (\partial_u a_t)^2\right)}$$

- Equations of motion

$$\partial_u \left(\frac{u^4 \rho'(u) \rho(u)}{D(u)} \right) = \frac{R^3 u}{D(u)} \left(\rho'(u)^2 \left(\frac{u}{R} \right)^3 + \frac{1}{f(u)} - (2\pi l_s^2)^2 (\partial_u a_t)^2 \right)$$

$$\frac{u\rho(u)}{D(u)} \partial_u a_t + 3a_\psi = \mathbf{k}_t$$

$$\frac{u}{\rho(u)D(u)} \partial_u a_\psi + 3a_t = \mathbf{k}_\psi$$

Baryon charge

- We saw that baryon vertex is a D4-brane wrapped on S^4
- D4 charged under C_5 RR potential
- On D6-brane worldvolume

$$\frac{1}{2\pi}\int_{\mathrm{D6}} C_5 \wedge f = \frac{1}{2\pi}\int_{\Sigma} f \cdot \int_{S^4\times\mathbb{R}_t} C_5$$

- Hence baryon number:

$$n_B = \frac{1}{2\pi}\int_{\Sigma} f = -\frac{1}{2\pi}\int du\, d\psi \partial_u a_\psi = a_\psi(u_E) - a_\psi(u_J)$$

- Charge carried by gauge field on D6-brane
- Actually from regularity at the tip it follows that $a_\psi(u_E) = \frac{\mathbf{k}_t}{3}$
- Fundamental string charge

$$q_s = \frac{\partial}{\partial(\partial_u a_t)}\int d\psi\, \mathcal{L}_{\mathrm{D6}} = N\,\frac{\mathbf{k}_t}{3}$$

$q_s = N\, n_B$ setting $a_\psi(u_J) = 0$

Spin

$$J = \int \sqrt{-g} g_{\psi\psi} T^{t\psi}$$

- From equations of motion and boundary conditions automatically get:

$$J = \frac{N}{2} n_B^2$$

- [Hence J =N/2 for baryon number one, as expected]

- [Curved-space analogs of «supertubes» [Mateos-Townsend 2001]]

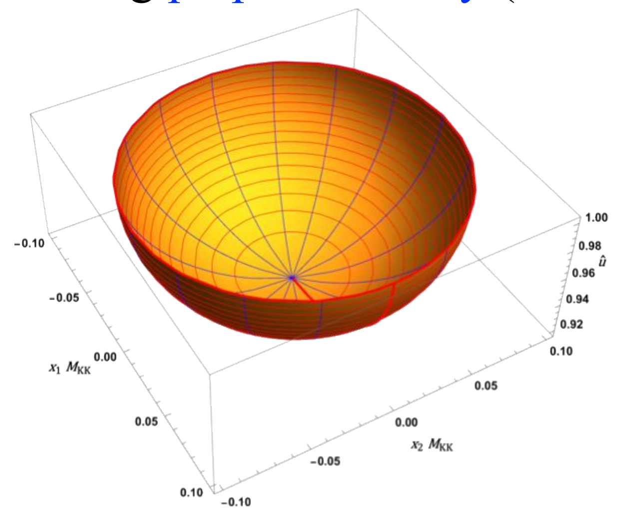
- Hallmark for fractional quantum Hall effect, anyonic system:

- anyons ⇒ quarks
- electrons ⇒ baryons
- solitonic chiral edge mode ⇒ χ such that $a|_{bdry} = d\,\chi$

Holographic baryon as quantum Hall droplet

[FB, Cotrone, Olzi, Raymond 2025]

- Solution of D6 eom ending perpendicularly (for stability) to D8



- Can compute exactly its properties, e.g.:
- Radius: $l \sim \lambda^{-2/3} N^0$
- Mass: $M \sim \lambda N$
- Total energy $\backsim$ energy of a wrapped D4 (baryon vertex) at $u=u_E$ + N fundamental strings from u_E to u_J

The core and the shell

- The D6-brane provides the description of the «hard» gluonic core of the baryon
- However there is also a «soft» mesonic shell related to the η' (D8)
- We still miss the latter description for the baryons
- Work is in progress in this direction

- Meanwhile we have studied simpler setups where the η' physics can be «easily» accounted for.
- D6-brane boundary plays the role of a magnetic source for the D8 gauge field.

Infinite sheet

- Hard gluonic core = D6-brane (e.g. extended along x_1, x_2)

$$S^{\rm DBI}_{\rm D6} = -T_6 \int d^7x\, e^{-\phi}\sqrt{-\det P[g]} \equiv -T_{\rm hard}\int dt dx_1 dx_2 \qquad T_{\rm hard} = \frac{1}{\pi^3 3^6}\lambda^2 N M^3_{KK}$$

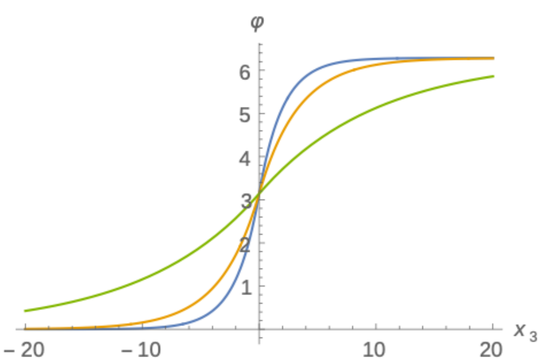
- Soft mesonic shell = eta' profile (on D8)

$$S_{C_7} = -\frac{1}{4\pi}(2\pi l_s)^6 \int dC_7 \wedge \star dC_7 + \frac{1}{2\pi}\int C_7 \wedge \frac{F}{\sqrt{2}} \wedge \omega_1 + \int C_7 \wedge \omega_3$$

$$d\widetilde{F}_2 = \frac{F}{\sqrt{2}} \wedge \delta(y)dy + 2\pi\delta(z)\delta(y)\delta(x_3)dz \wedge dy \wedge dx_3 \qquad \widetilde{F}_2 = \frac{A}{\sqrt{2}} \wedge \delta(y)dy + 2\pi\Theta(x_3)\delta(z)\delta(y)dz \wedge dy$$

$$\mathcal{L} = -\frac{\kappa}{\pi}(\partial_\mu\varphi)(\partial^\mu\varphi) + 2\,c\,m\cos\varphi - \frac{\chi_g}{2}\min_{k\in\mathbb{Z}}(\varphi - 2\pi\Theta(x_3) + 2\pi k)^2$$

- Solution interpolating from 0 to 2π

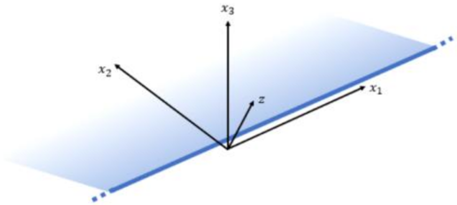


$$\int dx_3\, dz\, F_{x_3 z} = -2\sqrt{2}\pi$$ Induces anti-D6-brane charge

$$T_{\rm soft} = \frac{1}{3^3 2^3 \pi^2}\lambda N m_{\rm WV} M^2_{KK} = \frac{1}{3^{9/2}2^3\pi^3}\lambda^2 N^{1/2} M^3_{KK}$$

Semi-infinite sheet

- Hard gluonic core = D6-brane (e.g. extended along $x_2 \geq 0$)



- Soft mesonic shell = eta' profile (D8)

$$d\widetilde{F}_2 = \frac{F}{\sqrt{2}} \wedge \delta(y)dy + 2\pi\Theta(x_2)\delta(x_3)dx_3 \wedge \delta(z)dz \wedge \delta(y)dy\,.$$

$$dF = -2\sqrt{2}\,\pi\,\delta(x_2)\delta(x_3)\delta(z)dx_2 \wedge dx_3 \wedge dz$$

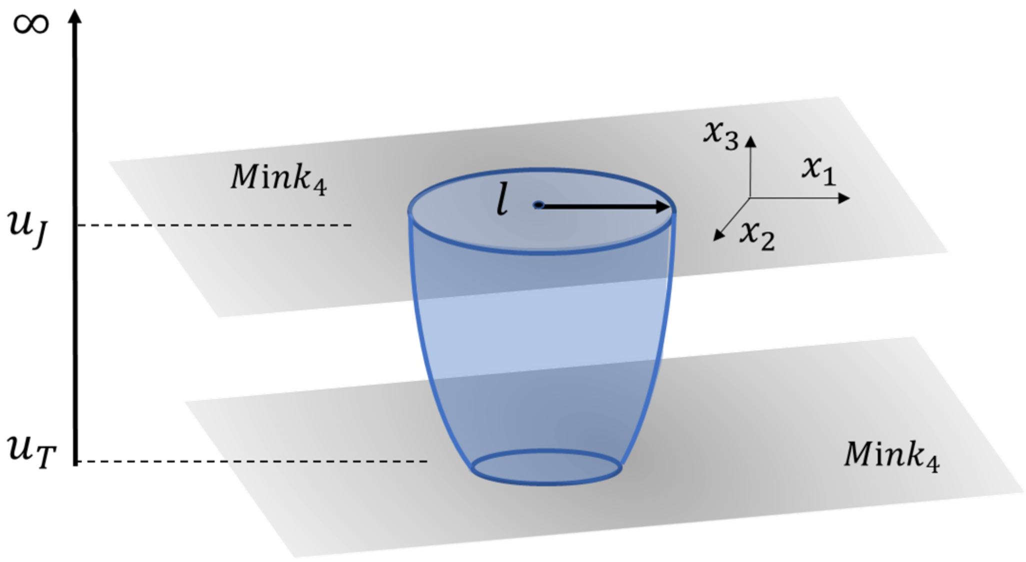
- The boundary is a magnetic source for the gauge field on the D8
- Now need to solve Maxwell-CS 5d eom with a magnetic cource
- Can be done: get the tension of the mesonic shell and tension of the string
- Can turn on electric charge, find chiral mode
- Find effective theory on the string

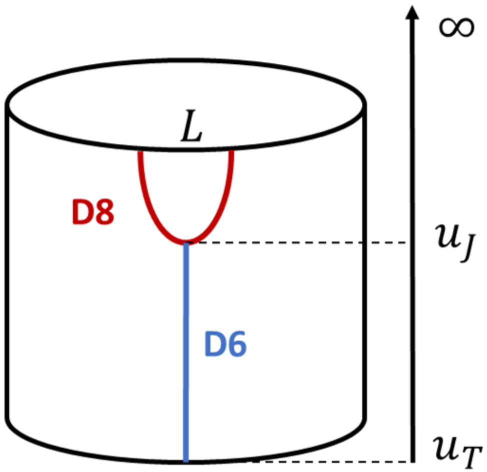
Plan

- Single-flavor baryons: which effective description?
- A proposal: baryons as quantum Hall droplets
- Testing the proposal: Hall droplet baryons in holographic QCD
- Other related configurations

Vortons

- Cylinder-like D6 configurations in the deconfined phase
- The latter is dual to a black-brane gravity solution where the x_4 circle does not shrink.
- Vortons are charged D6-brane solutions with circular boundaries on the flavor D8-brane and at the horizon. If uncharged (string loops) unstable.

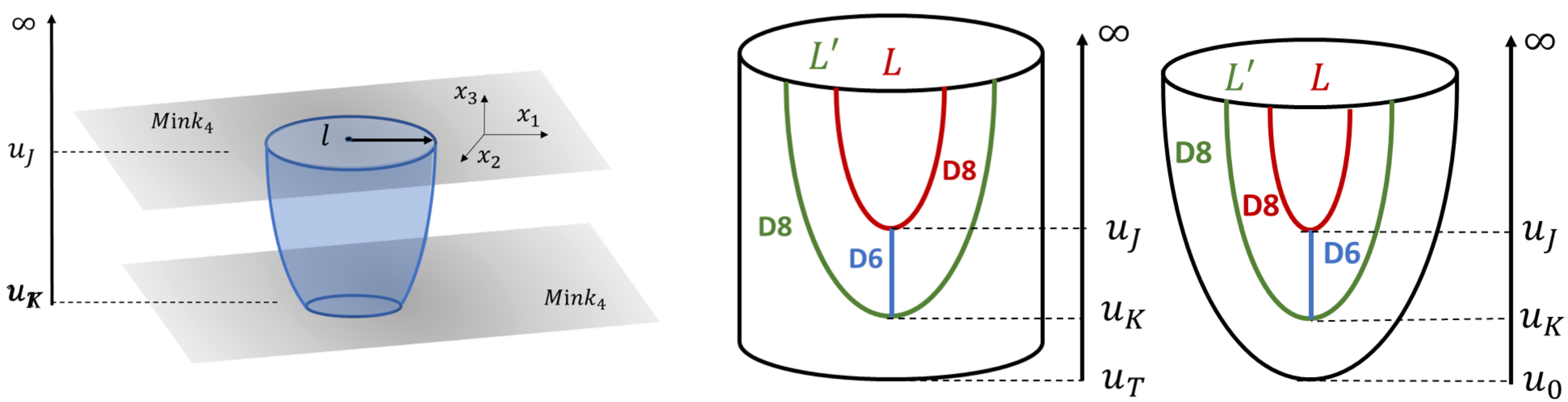




- Can be metastable if large charge $n_B = O(\lambda)$

Sandwich vortons

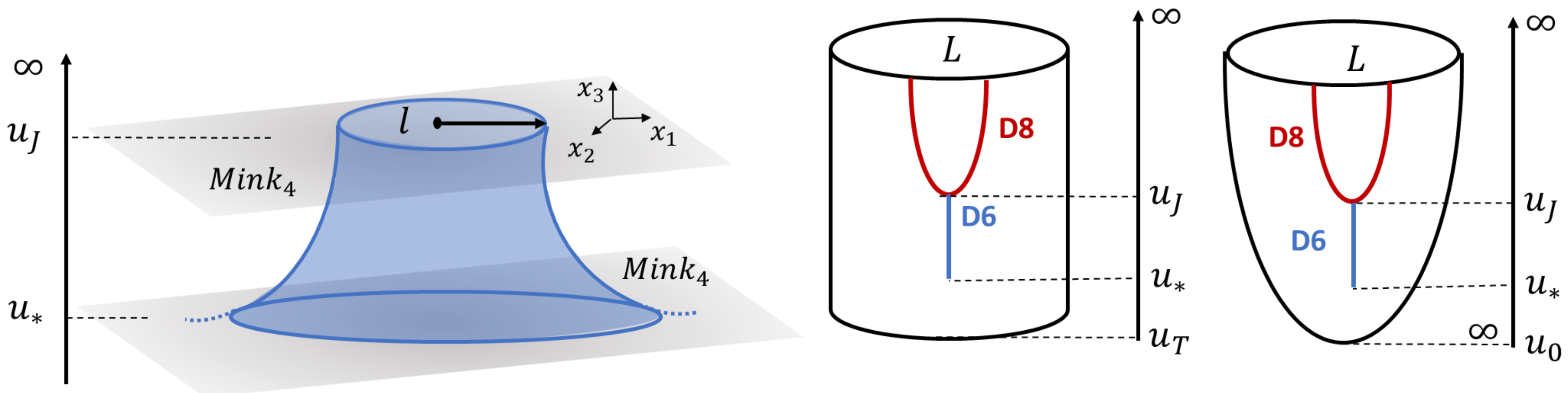
- Cylinder-like solutions which can exist in conf and deconf phases
- Charged D6-brane solutions with circular boundaries on one flavor D8-brane and on another flavor D8-brane



- E.g green D8 related to η' and red D8 related to axion
- Red D8: extra massless flavor undergoing chiral symmetry breaking at scale f_a : let's call axion the wouldbe η' here
- Picture realized axion- η' sandwich structures in [Gabadadze, Schwimmer]

Punctured domain walls

- Infinitely extended domain -wall-like structures
- Charged D6-brane solutions with circular boundary on one flavor D8-brane and infinitely extending on the other «boundary»



- Metastable also without charge: increasing radius l lowers their tension, but as the same time it increases string tension on the circular boundary

Decay channels

- Comparing free energies (on-shell D6-actions) of the various configurations we can identify possible decay channels

- For instance, we envisage the possibility of a first order phase transition between the sandwich and the baryon, as we vary the distance between the two D8-brane tips.
- Large distance, sandwich preferred. Short distance, baryon.

- In the deconfined phase baryon can melt into the horizon as we increase the temperature, leaving q_s fundamental strings terminating at the horizon.

- A first order phase transition is also envisaged bewteen string loop (uncharged vorton) and uncharged baryon-like D6

Conclusions

- Single-flavor baryons are quantum Hall droplets in Holographic QCD
- Holographic dual allows for precise investigation beyond effective theory
- These baryons have a «gluonic core» described by D6-brane, equivalent to baryon vertex=string junction in standard baryons
- Baryons have «mesonic shell» described by D8-brane gauge field (ongoing work)
- Other related configurations: vortons, sandwich vortons, punctured domain-walls.
- The model can have rich cosmological history [FB, Cotrone Olzi 2022, 2024; FB, Cotrone, Olzi, Raymond 2025].

Thank you for your time