

Towards an improved holographic equation of state for hot and dense QCD

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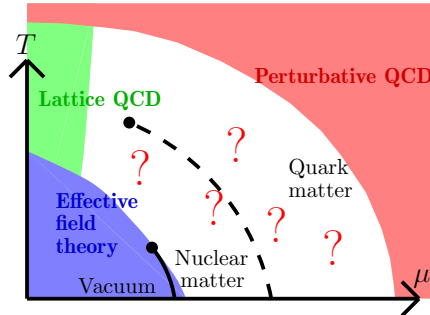
Outline

1. Holographic equation of state in V-QCD:
starting point
2. Quark flavors and masses
[MJ, Mitra 2507.08087]
3. Modeling color superconductivity
[Cruz Rojas, Demircik, Ecker, MJ 2505.06338]
4. Isospin asymmetric nuclear matter
[Bartolini, Gudnason, MJ 2504.01758]
5. Conclusion

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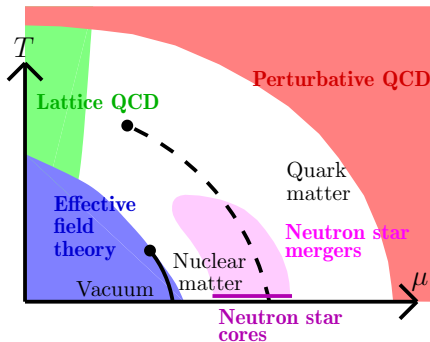
QCD phase diagram: theory status

- ▶ **Lattice data** only available at zero/small chemical potentials
- ▶ **Effective field theory** works at small densities
- ▶ **Perturbative QCD**: only at high densities and temperatures
- ▶ At intermediate densities no first-principles methods available
 - lots of open questions



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- ▶ This region is highly relevant for neutron star physics!
- ▶ Improving theoretical predictions important!
- ▶ Strongly coupled physics – use the gauge/gravity duality?

Why use holography for dense matter?

Already various models available in the literature – perhaps the gauge/gravity duality is just another uncontrolled approximation?

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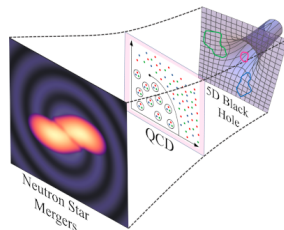
There is however **strong motivation** for this approach:

- ▶ Strongly coupled physics: holography may work better than many other approaches
- ▶ Different phases (quark, nuclear, color superconducting, quarkyonic . . .) in the same footing or even in a single model
 - ▶ Typically not achieved in the literature
 - ▶ Gives rise to predictions for phase transitions
- ▶ As it turns out, predictions do make sense!
 - ▶ Highly nontrivial – as the precise holographic dual for QCD is not known, these model cannot be derived
 - ▶ In particular, many details work out of the box at finite density

The starting point: our earlier EOS

A state-of-the-art EOS, to be used

1. to describe (isolated) neutron stars
2. in simulations of neutron star mergers
3. in simulations of core collapse supernovae
4. when analyzing heavy-ion collisions (?)

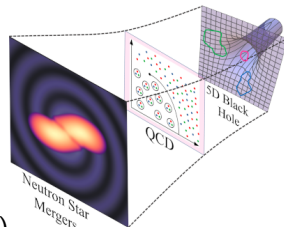


[Based on Demircik, Ecker, MJ 2112.12157 (PRX) + earlier work]

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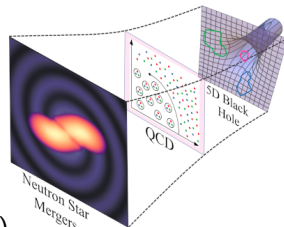
- Some parts could also be covered using simpler models (e.g. quark matter using Einstein-Maxwell-dilaton)

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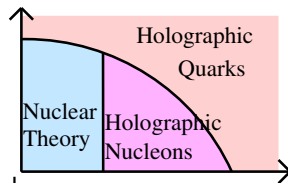


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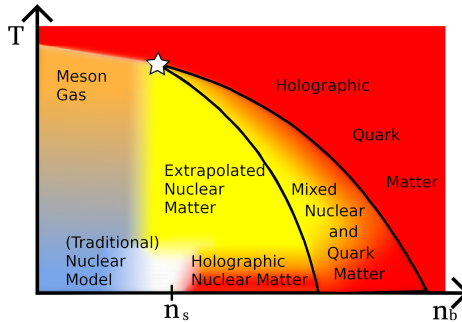
Main ingredients are

1. Holographic model for quark matter
2. (Slightly adjusted) holographic model for nuclear matter
3. Nuclear theory model for hadronic phase
– at low density holography not very useful

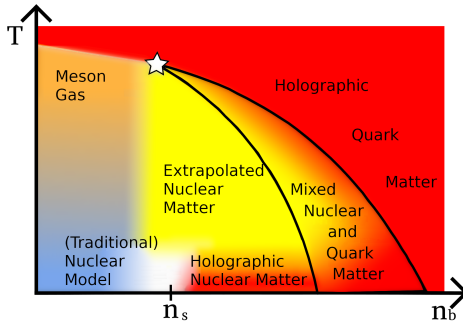


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Combining the building blocks: the hybrid model

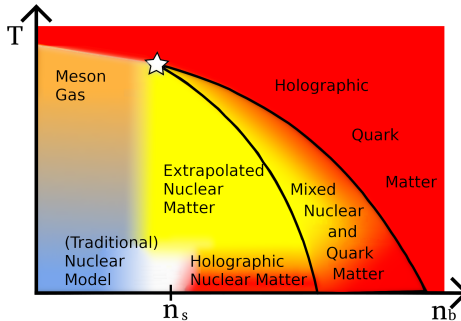


Combining the building blocks: the hybrid model



Covers regions relevant for neutron stars and heavy-ion collisions!

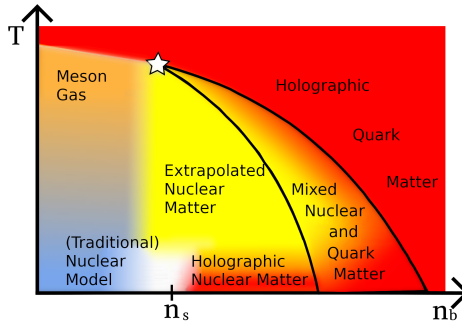
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- ▶ One of the most ambitious attempts to describe the QCD EOS to date
- ▶ Consistent with theoretical and observational constraints

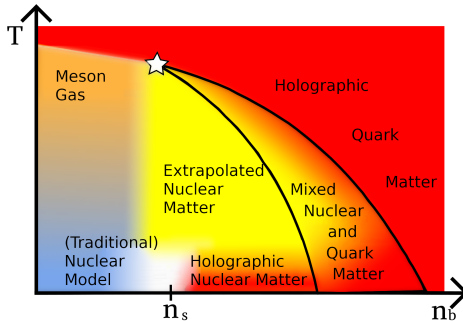
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- ▶ Probabilistic approach using these EOSs: see Christian's talk

Shortcomings of the current EOSs

However our model has some obvious shortcomings:

- I Deviation from β -equilibrated or isospin symmetric matter not implemented through holography
 - ▶ Neutron star merger EOSs depend on temperature T , baryon number density n_b , and charge fraction Y_q
 - ▶ Y_q dependence not coming from holography so far in any phase
- II No dependence on quark masses
 - ▶ So far matching $N_f = 2 + 1$ data with a massless model
- III No model for “exotic” color superconducting or color-flavor locked phases
- IV Holographic nuclear matter model overly simple (homogeneous field instead of instantons)

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This talk: developments towards fixing these problems

1. Quark flavor dependence in V-QCD (issues I and II)
2. Simple model for color superconductor (issue III)
3. Simple model for isospin asymmetric nuclear matter (issue I)

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V-QCD model

A holographic bottom-up model for QCD in the Veneziano limit (large N_f , N_c with $x = N_f/N_c$ fixed): V-QCD

- ▶ Bottom-up, but trying to follow principles from string theory as closely as possible
- ▶ Many parameters: effective description of QCD
- ▶ Comparison with QCD data essential

The model is obtained through a fusion of two building blocks:

[MJ, Kiritsis [arXiv:1112.1261](#)]

1. IHQCD: model for glue inspired by string theory (dilaton gravity)

[Gürsoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

[Bigazzi, Casero, Cotrone, Iatrakis, Kiritsis, Paredes]

Full backreaction between the two sectors in the Veneziano limit!

Quark flavors in V-QCD

- ▶ Goal: construct a model for $N_f = 2 + 1$
- ▶ The tachyonic brane action readily implements N_f flavors and full chiral symmetry
- ▶ Optimally, one would just take the “old” model and turn on quark masses or chemical potentials for each flavor
- ▶ We actually tried this in order to analyze the bulk viscosity
[Cruz Rojas, Gorda, Hoyos, Jokela, MJ, Kurkela, Paatelainen, Säppi, Vuorinen 2402.00621 (PRL)]
- ▶ However, the details of the flavor dependence turn out to be in conflict with lattice data \Rightarrow model needs adjusting
[MJ, Mitra 2507.08087]

Quark flavors in V-QCD

Dictionary (with $i, j = 1, \dots, N_f$; only vectorial gauge fields)

- ▶ Dilaton $\lambda \leftrightarrow \text{tr} G^2$
- ▶ Tachyon $T_{ij} \leftrightarrow \bar{q}_i q_j$
- ▶ Gauge fields $A_{ij}^\mu \leftrightarrow \bar{q}_i \gamma^\mu q_j$
- ▶ source = 't Hooft coupling
- ▶ sources = quark masses
- ▶ sources = chemical potentials

$$S_{\text{V-QCD}} = S_{\text{IHQCD}} + S_{\text{DBI}}$$

$$= N_c^2 M^3 \int d^5 x \sqrt{-g} \left[R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ - N_c M^3 \int d^5 x \text{Tr} \left[V_f(\lambda, T) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a T \partial_b T^\dagger + w(\lambda, T) F_{ab})} \right]$$

Trace over flavor indices

Plug in diagonal Ansatz in flavors $T_{ij} = \tau_i(r) \delta_{ij}$, $A_{ij} = A^{(i)}(r) \delta_{ij}$:

$$S_{\text{DBI}} \propto \sum_{i=1}^{N_f} \int d^5 x \, V_f(\lambda, \tau_i) \sqrt{-\det \left(g_{ab} + \kappa(\lambda) \delta_a^r \delta_b^r (\tau_i')^2 + w(\lambda, \tau_i) F_{ab}^{(i)} \right)}$$

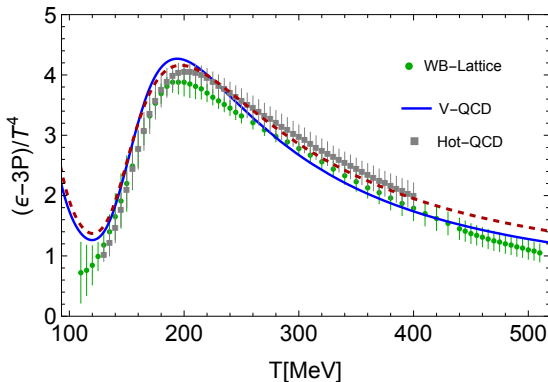
- ▶ Take $N_f = 2 + 1$; two massless and one massive quark
- ▶ Functions V_g , V_f , κ and w determined by comparing to QCD

Adjusting the potentials

Asymptotics unchanged wrt. the unflavored case

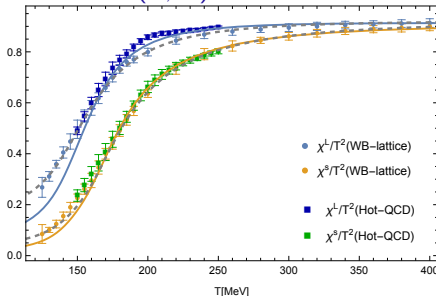
- ▶ Require confinement, discrete spectrum, linear meson trajectories ...
- ▶ Working potentials turn out to have power law asymptotics
- ▶ E.g. $V_g \sim \lambda^{4/3}$, $w \sim \lambda^{-4/3}$

Rest fitted to lattice data sequentially, e.g. V_f to EOS at $\mu = 0$



Fitting the susceptibilities

Fitting susceptibilities $\chi_{ij} = \frac{\partial^2 p}{\partial \mu_i \partial \mu_j}$ more tricky — constrains gauge field coupling function $w(\lambda, \tau_i)$

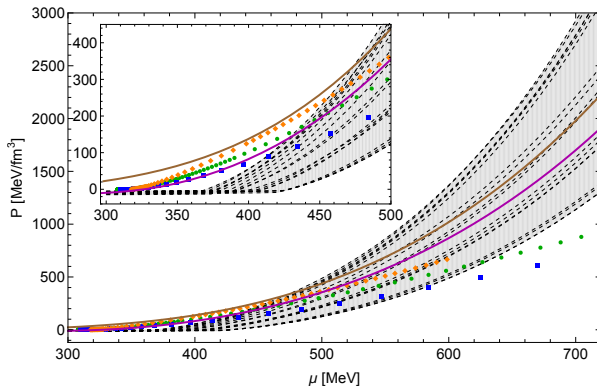


[WB 1112.4416]

[hotQCD 1203.0784]

- ▶ Lattice results for light/strange quarks significantly different
- ▶ Strange quark mass sources tachyons — tachyon dependence of potentials modifies χ 's
- ▶ Tachyon dependence in $V_f(\lambda, \tau_i)$ way too weak to reproduce the effect \Rightarrow strong tachyon dependence in $w(\lambda, \tau_i)$ **required**
- ▶ N.B. We fit only diagonal χ_{ij} and **NOT** χ_B : nondiagonal components vanish in our model

Implications for neutron star EOS



Comparison of

1. Our models (solid curves)
2. Nuclear theory models (colored dots)
3. Old unflavored V-QCD EOSs

[Jokela, MJ, Remes 1809.07770]

- ▶ New EOSs match more smoothly with nuclear theory than old
- ▶ May lead to **stable quark cores** inside neutron stars!

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Paired phases in QCD

Model computations, and analysis at large μ , indicate that at low T deconfined quarks become paired

- ▶ Color-flavor locking (CFL)
 - ▶ Known to be present at asymptotically high μ
 - ▶ $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R}$
- ▶ Two-flavor color superconductivity (2SC)

Symmetry breaking patterns involve the gauge group — tricky in holography, where observables are gauge singlets ...

Despite this, many studies already exist (see also talk by Edwan Préau) ... we did the first attempt in V-QCD

Simple model for paired phases in V-QCD

We model the quark condensate through an additional charged scalar field ψ

$$S_\psi = -M^3 N_c N_f \int d^5x \sqrt{-\det g} Z(\lambda) \left[|D_M \psi|^2 + M(\lambda)^2 |\psi|^2 \right]$$

$$D_M \psi = \partial_M \psi - i \hat{A}_M \psi, \quad \hat{A}_M = \text{Tr}[A_M]/N_f$$

- ▶ Trivial (massless) flavors so far
- ▶ Unit charge: allows generalization to a matrix field
- ▶ Allow for dilaton dependent potential functions Z , M
- ▶ Here restrict to probe field ψ : no backreaction to metric
- ▶ Similar to “holographic superconductor”

[Cruz Rojas, Demircik, Ecker, MJ 2505.06338]

The holographic mechanism

Mechanism leading to instabilities at low T

- ▶ Deconfined quark matter phase \leftrightarrow black hole geometry
- ▶ As $T \rightarrow 0$, black hole (nearly) extremal: IR geometry $\sim \text{AdS}_2 \times \mathbb{R}^3$
- ▶ This geometry easily leads to instabilities due to its low Breitenlohner-Freedman (BF) bound for fluctuations
 - ▶ Violation at zero momentum: instability towards a homogeneous condensate
 - ▶ Violation at nonzero momentum: instability towards an inhomogeneous condensate
- ▶ Both kind of instabilities possible in V-QCD
 - ▶ The charge scalar ψ may condense homogeneously (focus in this talk)
 - ▶ Gauge fields may condense inhomogeneously: Nakamura–Ooguri–Park instability driven by the Chern–Simons term
[Cruz Rojas, Demircik, MJ 2405.02399;
Demircik, Jokela, MJ, Piispa 2405.02392]
- ▶ $\text{AdS}_2 \times \mathbb{R}^3$ geometry implies nonzero entropy at zero T
 - ▶ Condensing charged ψ expected to absorb the charge supporting the AdS_2 , leading to zero entropy at zero T

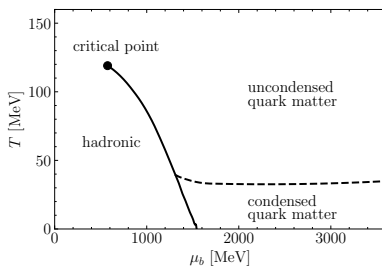
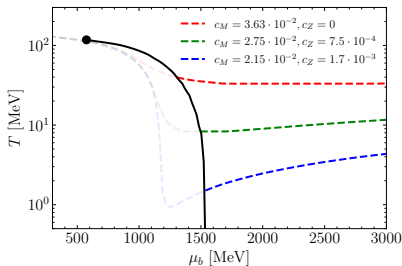
Choice of potentials and extent of instability

$$\mathcal{L}_\psi \propto Z(\lambda) \left[|D_M \psi|^2 + M(\lambda)^2 |\psi|^2 \right]$$

How to choose the potentials M and Z in the Lagrangian?

- ▶ Earlier observation: working potentials obey asymptotics such that they become flat in the **string frame**, $g_s = \lambda^{4/3} g$
- ▶ Require the same as $\lambda \rightarrow \infty$:

$$Z(\lambda) = 1 + c_Z \lambda^2, \quad M(\lambda) = -3(1 + c_M \lambda^{4/3})$$

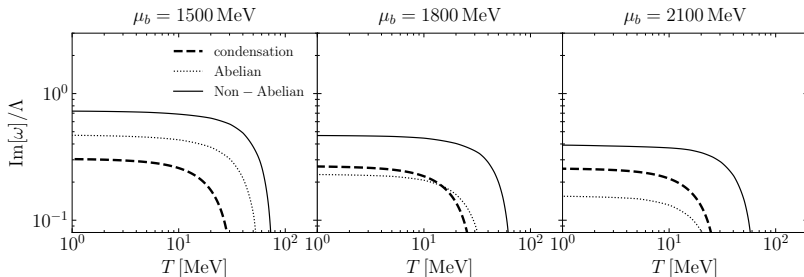


- ▶ Vary other parameters such that no instability in deconfined region at $\mu = 0 \Rightarrow$ maximal phenomenologically viable effect
- ▶ N.B. constant M does not “work”: essentially no instability

Competition with modulated instability

Compare the (maximal) frequency of the unstable mode to the frequency of the modulated Nakamura–Ooguri–Park instability

- ▶ Both singlet (Abelian) and non-singlet (non-Abelian) gauge fields unstable



The color superconducting instability (thick dashed)

- ▶ appears only at lower temperature
- ▶ is “weaker” than the modulated instability

But the final ground state could include **both** effects ... merits further study

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Model for nuclear matter

Standard method for baryons in holographic models: Each baryon maps to a solitonic 5D “instanton” of gauge fields

- ▶ Such instantons have been studied in many models, including V-QCD
[MJ, Kiritsis, Nitti, Préau 2209.05868; 2212.06747]
- ▶ Already constructing an isolated instanton solution is nontrivial
- ▶ Dense nuclear matter requires studying many-instanton solutions . . . extremely challenging!

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- ▶ Already constructing an isolated instanton solution is nontrivial
- ▶ Dense nuclear matter requires studying many-instanton solutions . . . extremely challenging!
- ▶ Simple approach: V-QCD with two flavors and a homogeneous gauge field, mimicking dense solitons: $A^i = h(r)\sigma^i$
[Rozali, Shieh, Van Raamsdonk, Wu 0708.1322; Li, Schmitt, Wang 1505.04886]
[Ishii, MJ, Nijs, 1903.06169]
 - ▶ Note: spatial and flavor indices linked
- ▶ This is what we used so far . . . however by construction isospin symmetric: no distinction between protons and neutrons

Going away from symmetric case

Introduce asymmetric gauge field and isospin chemical potential

$$A_t(r) = \hat{a}_0(r)\mathbb{1} + a_0(r)\sigma^3, \quad a_0(0) = \mu_I$$

- ▶ Here r is the holographic coordinate, bdry at $r = 0$

Then consistency requires a more general Ansatz

$$\begin{aligned} A_i &= H(r)\sigma^i, & (i = 1, 2) \\ A_3 &= H_3(r)\sigma^3 + L_3(r)\mathbb{1} \end{aligned}$$

[Kovensky, Schmitt 2105.03218; Bartolini, Gudnason 2209.14309;
Kovensky, Poole, Schmitt 2302.10675]

- ▶ A discontinuity at some $r = r_c$ required for nonzero n_b
- ▶ Beyond the discontinuity, $r > r_c$, gauge fields vanish
- ▶ Solution found by minimizing the action, both wrt functions and r_c

See also talk by Andreas Schmitt

Model parameters and the Chern–Simons term

We follow the approach from isospin symmetric construction

[Ishii, MJ, Nijs, 1903.06169; Demircik, Ecker, MJ 2112.12157]

- ▶ Match with a nuclear theory model at $\sim 1.5n_s \Rightarrow$ determine remaining model parameters
- ▶ These include the parameters from the Chern–Simons action, in our case (including L/R components) [MJ, Kiritsis, Nitti, Préau]

$$S_{\text{TCS}} \sim iN_c \int F_1(\tau) \text{tr} \left[A \wedge A \wedge A \wedge \left(F^{(L)} + F^{(R)} \right) \right] \\ - iF_3(\tau) \text{tr} \left[A \wedge \left(F^{(L)} - F^{(R)} \right) \wedge \left(F^{(L)} - F^{(R)} \right) \right]$$

with $A = A^{(L)} - A^{(R)}$ and taking

$$F_1(\tau) = e^{-b_1\tau^2}, \quad F_3(\tau) = e^{-b_3\tau^2}$$

- ▶ b_3 does not appear in iso-symmetric setup, we set it to $b_3 = 1$
- ▶ b_1 and normalization of the action c_f determined by matching with nuclear models

Symmetry energy

A key characteristic of EOS away from isospin symmetry:
symmetry energy S_N

$$\frac{\mathcal{E}}{n_b} = \left. \frac{\mathcal{E}}{n_b} \right|_{\delta=0} + S_N(n_b)\delta^2 + \dots \quad \delta = \frac{n_p - n_n}{n_p + n_n} = \frac{2n_I}{n_b}$$

Two ways to compute in holography:

- ▶ “Directly” by turning on small μ_I and analyzing the energy
- ▶ Computing the isospin density by quantizing the iso-rotation of the homogeneous nuclear matter in analogy to what one does with instanton solutions

Both give the same answer

[Bartolini, Gudnason 2209.14309]

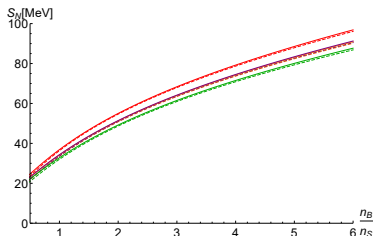
Computation done in the literature for WSS and hard-wall models, giving too large $S_N(n_b = n_s)$ as compared to experiments

Symmetry energy: results

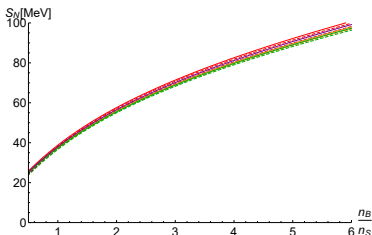
Our results for various nuclear models and matching points

- ▶ Value ~ 30 MeV at saturation, agreement with experiment!
- ▶ Comes at the expense of playing with parameters (we set $b_3 = 1$ and matched b_1)

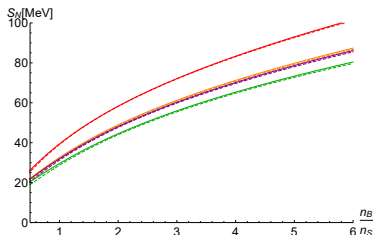
Matching at $1.5n_s$



Matching at $1.2n_s$



Matching at $1.8n_s$



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Summary

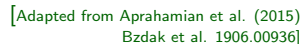
1. We analyzed V-QCD with flavors, and strange quark mass
 - ▶ Obtained good agreement with lattice data, including different quark susceptibilities
 - ▶ Weaker phase transition in the neutron star regime than earlier, leading to stable quark cores?
2. Considered a simple model for color superconducting phase
 - ▶ Identified the maximal extent of the phase
 - ▶ Compared to the inhomogeneous (Nakamura–Ooguri–Park) instability
3. Studied isospin asymmetric nuclear matter
 - ▶ Symmetry energy in agreement with experimental results

(Some) future directions

- ▶ Further analysis of the effect of strange quark mass
 - ▶ Map the EOS and the phases at all T and μ
 - ▶ What happens to the critical point??
 - ▶ Range of the modulated instability
 - ▶ Transport ... in particular bulk viscosity
- ▶ Backreacted color superconductivity, maybe with improved flavor structure
- ▶ Putting all improvements together to construct a new hybrid EOS
 - ▶ Use in merger simulations, e.g., effect of quark cores?

Thank you!

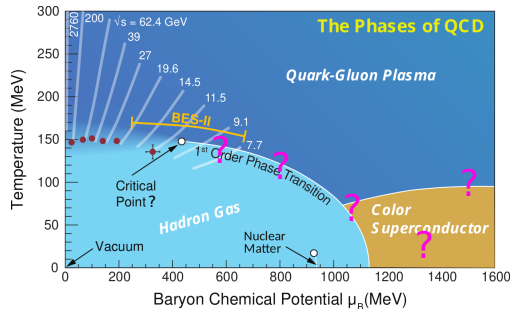
Search for the critical point: ongoing effort at RHIC (Beam Energy Scan)



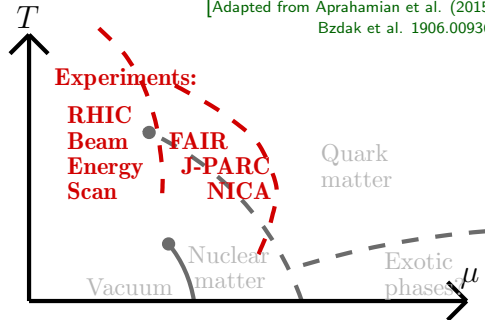
QCD phase diagram and the critical point

Search for the critical point: ongoing effort at RHIC (Beam Energy Scan)

Will be extended by future experiments at FAIR, J-PARC, NICA



[Adapted from Aprahamian et al. (2015)
Bzdak et al. 1906.00936]

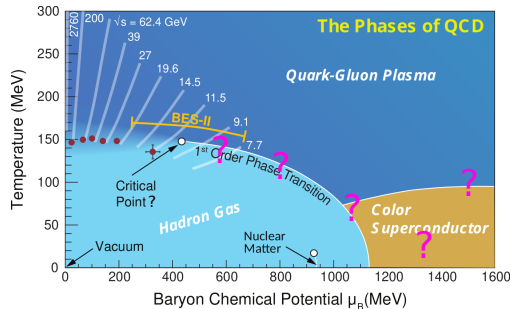


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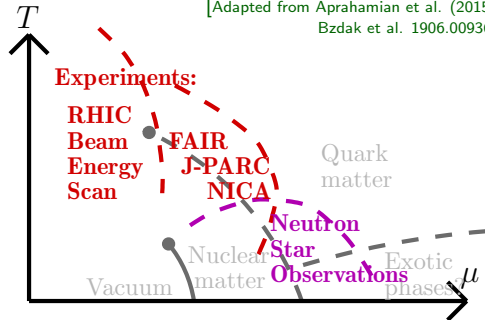
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Neutron star observations give complementary information at high density



[Adapted from Aprahamian et al. (2015)
Bzdak et al. 1906.00936]



Homogeneous nuclear matter in V-QCD

Nuclear matter in the probe limit: consider full brane action

$S = S_{\text{DBI}} + S_{\text{CS}}$ where

[Bigazzi, Casero, Cotrone, Kiritsis, Paredes; Casero, Kiritsis, Paredes]

$$S_{\text{DBI}} = -\frac{1}{2} M^3 N_c \text{Tr} \int d^5 x V_{f0}(\lambda) e^{-\tau^2} \left(\sqrt{-\det \mathbf{A}^{(L)}} + \sqrt{-\det \mathbf{A}^{(R)}} \right)$$

$$\mathbf{A}_{MN}^{(L/R)} = g_{MN} + \delta_M^r \delta_N^r \kappa(\lambda) \tau'(r)^2 + \delta_{MN}^{rt} w(\lambda) \Phi'(r) + w(\lambda) F_{MN}^{(L/R)}$$

gives the dynamics of the solitons (will be expanded in $F^{(L/R)}$) and

$$S_{\text{CS}} = \frac{N_c}{8\pi^2} \int \Phi(r) e^{-b\tau^2} dt \wedge \left(F^{(L)} \wedge F^{(L)} - F^{(R)} \wedge F^{(R)} + \dots \right)$$

sources the baryon number for the solitons

► Extra parameter, $b > 1$, to ensure regularity of solutions

Set $N_f = 2$ and consider the **homogeneous** SU(2) Ansatz

[Rozali, Shieh, Van Raamsdonk, Wu 0708.1322]

$$A_L^i = -A_R^i = h(r) \sigma^i$$

[Ishii, MJ, Nijs, 1903.06169]

Discontinuity and smeared instantons

With the homogeneous Ansatz $A_i^a(r) = h(r)\delta_i^a$ baryon number vanishes for any smooth $h(r)$:

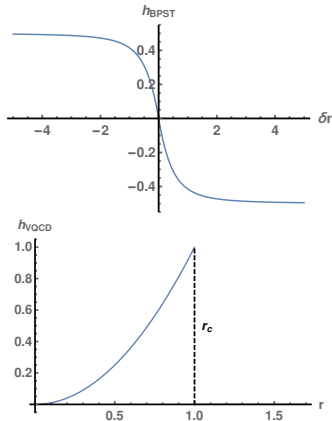
$$N_b \propto \int dr \frac{d}{dr} [\text{CS - term}] = 0$$

How can this issue be avoided?

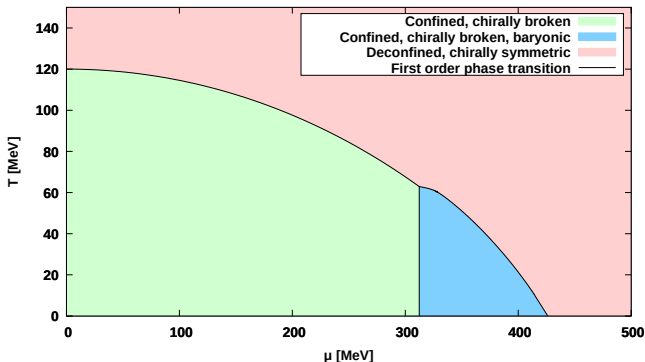
- Smearing the BPST soliton in **singular Landau gauge**:

$$\begin{aligned} \langle A_i^a \rangle &\sim \int \frac{d^3\mathbf{x} \ \eta_{i4}^a \ \delta r}{(\delta r^2 + \mathbf{x}^2 + \rho^2)(\delta r^2 + \mathbf{x}^2)} \\ &\sim - \frac{\delta_i^a \ \delta r}{\sqrt{\delta r^2 + \rho^2} + |\delta r|} \end{aligned}$$

- This suggests a solution: introduce a discontinuity in $h(r)$ at $r = r_c$
- The discontinuity sources nonzero baryon charge!



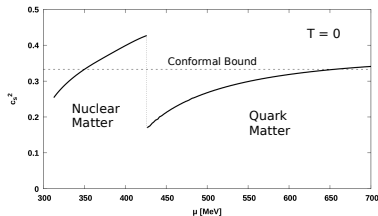
Phase diagram after including nuclear matter



Stiff EOS (high c_s^2) in the nuclear matter phase \Rightarrow helps to pass the bounds from neutron star observations!

Such a stiff EOS also found in WSS and hard wall models

[Kovensky, Poole, Schmitt 2111.03374;
Bartolini, Gudnason, Leutgeb, Rebhan 2202.12845]



Adjusting the nuclear matter model

The V-QCD nuclear matter EOS as such is however not fully satisfactory:

- ▶ Temperature dependence is absent in the confined phases, and therefore also for holographic nuclear matter
- ▶ This is likely to be a good first approximation, but not enough for a state-of-the-art model

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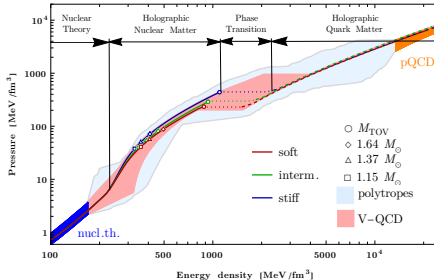
Our solution: we extrapolate the holographic nuclear matter EOS to nonzero T by using a van der Waals approach

- ▶ Gas of protons, neutrons and electrons with an excluded volume correction and a potential term

[Demircik, Ecker, MJ 2112.12157]

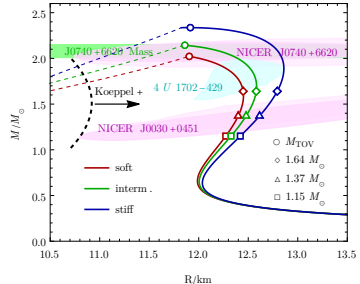
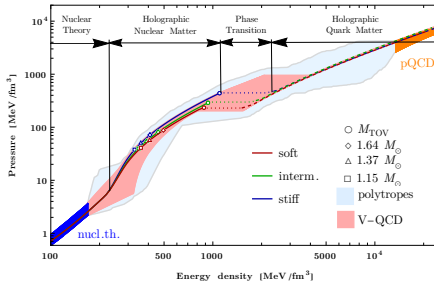
Cold EoS and known constraints

- ▶ Three choices of EoSs: **soft**, **intermediate**, and **stiff** \leftrightarrow the degrees of freedom of V-QCD left free by fit to lattice data
- ▶ Compared to bands of all feasible cold matter EoS: **Without** and **with** holography



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- ▶ Plug EoSs in TOV: neutron star $M(R)$ curves (left plot)
- ▶ Compares well with mass/radius observations
- ▶ No stable quark cores inside neutron stars

[Ecker, MJ, Nijs, van der Schee 1908.03213]

[Jokela, MJ, Nijs, Remes 2006:01141]

[Demircik, Ecker, MJ 2112.12157]

Advantages of the model

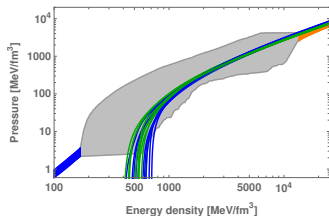
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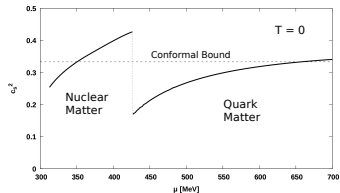
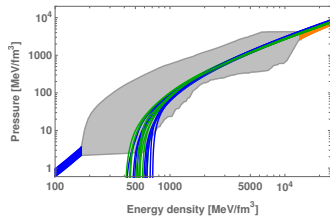
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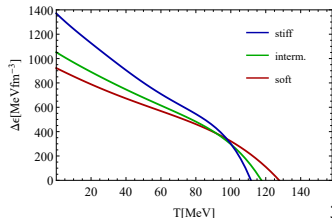
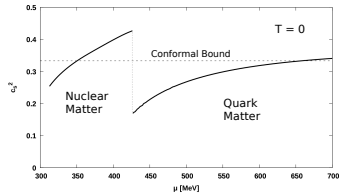
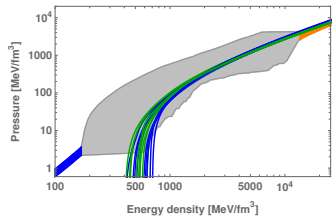
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4. Simultaneous modeling of nuclear and quark matter phases
 - ▶ Predictions for the phase transition

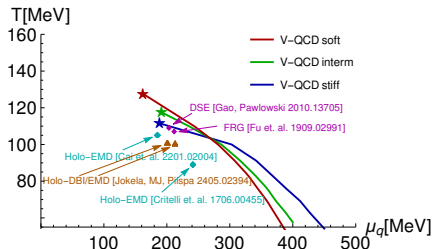


Agreement with FRG

Close agreement with functional renormalization group (FRG)

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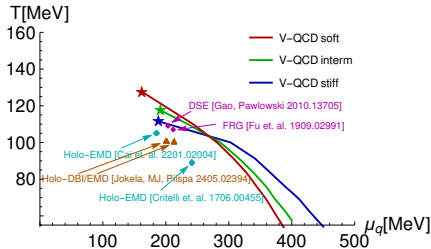
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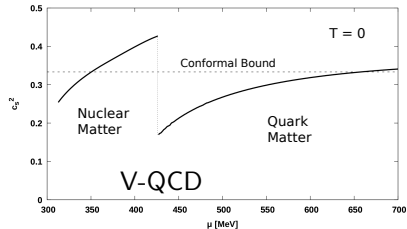
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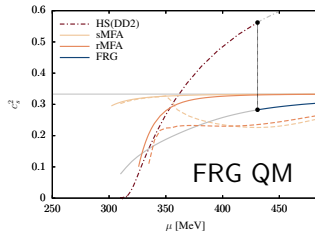
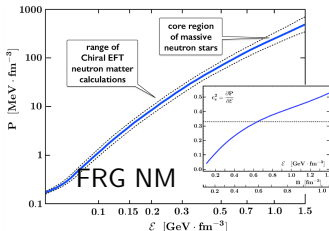
Close agreement with functional renormalization group (FRG)



► Critical point



► Speed of sound at $T = 0$



Ansatz for potentials

$$V_g(\lambda) = 12 \left[1 + V_{g,1}\lambda + \frac{V_{g,2}\lambda^2}{1 + c_\lambda\lambda/\lambda_0} + V_{\text{IR}}e^{-\lambda_0/(c_\lambda\lambda)} \left(\frac{c_\lambda\lambda}{\lambda_0} \right)^{4/3} \sqrt{\log(1 + c_\lambda\lambda/\lambda_0)} \right]$$

$$V_f(\lambda, \tau_i) = V_{f\lambda}(\lambda) (1 + \tau_i^4)^{\tau_p} \exp(-\tau_i^2)$$

$$V_{f\lambda}(\lambda) = W_0 + W_1\lambda + \frac{W_2\lambda^2}{1 + c_\lambda\lambda/\lambda_0} + W_{\text{IR}}e^{-\lambda_0/(c_\lambda\lambda)}(c_\lambda\lambda/\lambda_0)^2$$

$$\kappa(\lambda) = \kappa_0 \left[1 + \bar{\kappa}_0 \left(1 + \frac{\bar{\kappa}_1\lambda_0}{c_\lambda\lambda} \right) e^{-\lambda_0/(c_\lambda\lambda)} \frac{(c_\lambda\lambda/\lambda_0)^{4/3}}{\sqrt{\log(1 + c_\lambda\lambda/\lambda_0)}} \right]^{-1}$$

$$w(\lambda, \tau) = \frac{w_0}{1 + \beta_s \tanh(\gamma_s^2 \tau_i^2)} \left[1 + \frac{w_1 c_\lambda \lambda / \lambda_0}{1 + c_\lambda \lambda / \lambda_0} + \bar{w}_0 e^{-\lambda_0/(c_w \lambda)} \frac{(c_w \lambda / \lambda_0)^{4/3}}{\log(1 + c_w \lambda / \lambda_0)} \right]^{-1}.$$