On effective action from holography

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Second Holography and Dense Matter Workshop

APC Paris

September 19, 2025

- Motivation
- Soundbites of first-order phase transitions (FOPT)
- Critical bubbles from holography
  - Construct two-derivative effective action
  - Directly solving bubble in bulk (probe)
- Effective action w/ backreaction, application to GW production
- Outlook / Summary

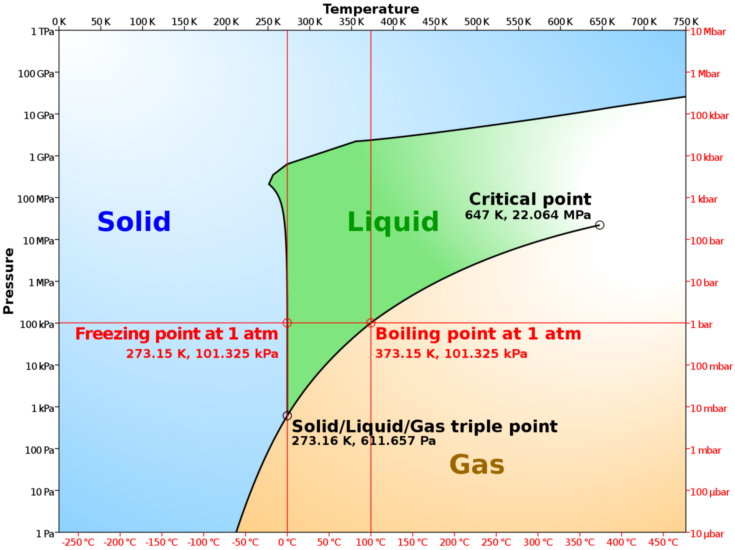
Talk based on papers w/

- Fëanor Ares, Oscar Henriksson, Mark Hindmarsh, Carlos Hoyos
  - constructing effective action in holography: 2109.13784
  - applied to GW production in early universe: 2110.14442

    ⋮
  - (holography→ GW: 2011.12878)
- Oscar Henriksson, Xin Li
  - showing effective action approach = bubbles in the bulk: 2507.11622

1. Motivation

- We observe 1st order PT in everyday life: water, metals



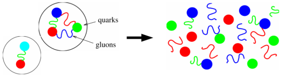
[Wikipedia]

- Pure water never boils..

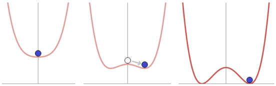
[Cho et al. PRL112, 157802(2014)]

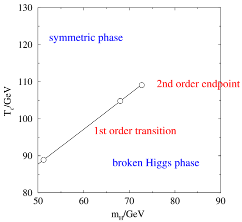
Motivation – Almost FOPT in SM

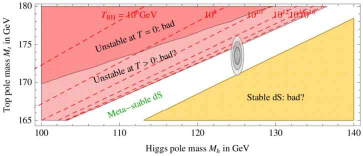
- QCD phase transition at $T \sim 170\text{MeV}$
  - Age of the universe $t \sim 10\mu s$
  - Hadrons $\leftrightarrow$ quark-gluon plasma
  - Smooth crossover (lattice QCD)
  - Pure Yang–Mills SU($N \geq 3$): FOPT



- EW phase transition at $T = T_c \sim 160\text{GeV}$
  - Age of the universe $t \sim 10^{-11}\text{s}$
  - Higgs acquires expectation value
  - Smooth crossover ($m_{\text{H}} > 72\text{GeV}$)



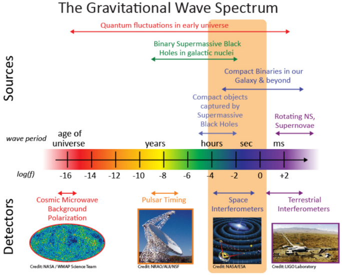


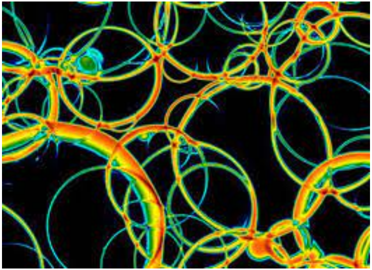


[Kajantie et al. hep-th/9605288 . . .]

[Espinosa et al. 1505.04825]

- Might happen at finite density in SM, laboratories: heavy-ion collision experiments, neutron stars?
- Might have happened in the early universe
  - Prospects for BSM physics
  - Collisions of bubbles sources GW $\rightarrow$ detectable by LISA?
  - Nucleation theory requires important parameters: nucleation temperature, transition strength and rate, wall speed
  - Typically studied using perturbation theory at weak coupling





[Image credits: NASA and David Weir]

Most tantalizing questions?

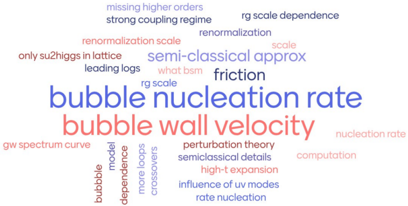
- Workshop at CERN?!

[https://indico.cern.ch/event/1511688/overview]

Advancing gravitational wave predictions from cosmological first-order phase transitions

25–29 Aug 2025
CERN
Europe/Zurich timezone

What are the largest uncertainties in predictions of phase transition thermodynamics?



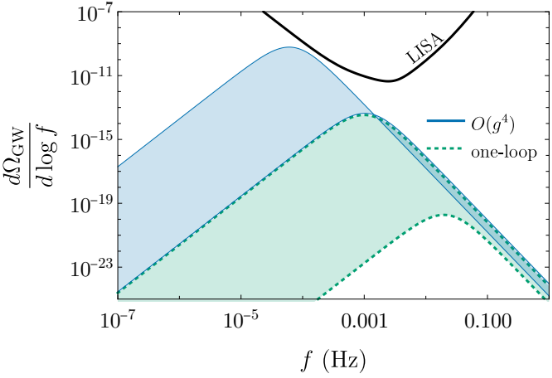
- Playground for holographists?

Motivation

- In perturbation theory, the nucleation rate
  [Langer '67-'69; Affleck–Linde '81; textbooks (Laine–Vuorinen); Gould–Hirvonen 2108.04377]

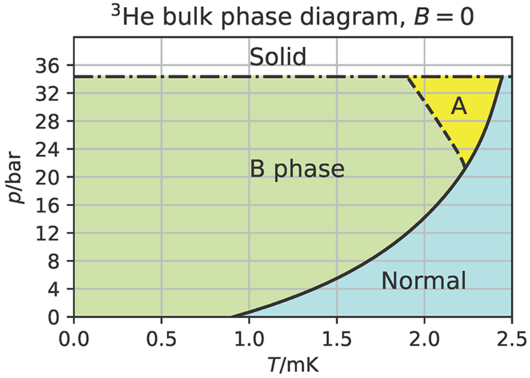
$$\sqrt{|\lambda_-|}\Gamma_{\mathrm{B}}^{3/2}\sqrt{\det(\text{fluct.})}e^{-\Gamma_{\mathrm{B}}}$$

- Poor convergence for eg. GW spectrum of SM+singlet scalar



[Gould–Tenkanen 2104.04399]

- Helium-3: A-B nucleation puzzle

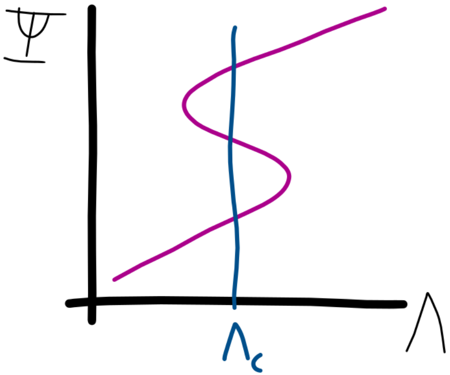


[Hindmarsh et al. 2401.07878]

- Essentially the same nucleation theory as for GWs and fails miserably:
  - incorrect prediction for frequency
  - not even GWs?

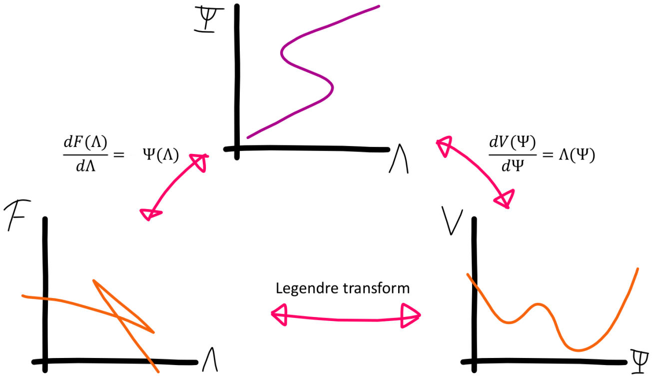
2. First-order phase transition

Multiple solutions: jump



- First-order phase transition: "order parameter" Ψ jumps

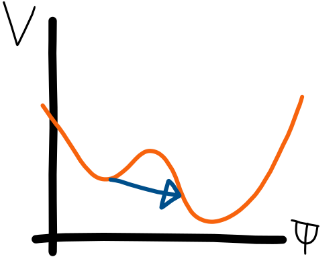
Underlying effective framework

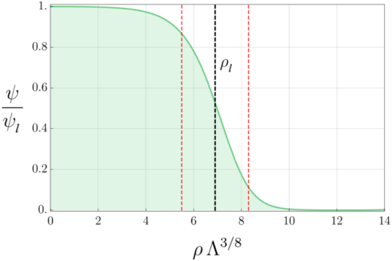


$$Z[\Lambda] = e^{-F[\Lambda]} = \int e^{-\int d^d x(\mathcal{L}+\Lambda\mathcal{O})} \;,\; \langle\mathcal{O}\rangle_\Lambda \equiv \Psi = \frac{\delta F}{\delta\Lambda}$$

$$\Gamma[\Psi] = F[\Lambda] - \int d^d x \Lambda\Psi \;,\; \Lambda = -\frac{\delta\Gamma}{\delta\Psi}$$

$$\Gamma[\Psi]\Big|_{\text{static homogeneous configs.}} = -\beta \mathrm{Vol}_{d-1} V[\Psi]$$

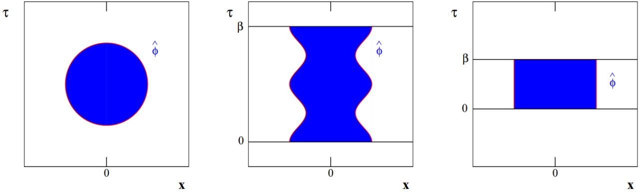




- Typically proceeds via homogeneous bubble nucleation [Cahn–Hilliard '59;Langer'69]
- Energy density aka "latent heat" jumps: $\Delta V = V(\psi_{\text{min1}}) - V(\psi_{\text{min2}}) \neq 0$
- Dynamics nontrivial: mechanism for energy transfer and dissipation needed

- Nucleation w/ combustion theory adapted to QFT[1]

[Landau–Lifshitz books;Steinhardt'82;Coleman'77; $T > 0$ by Linde'81]



[Laine-Vuorinen 1701.01554]

- Transition mediated by Euclidean bubble solution
- Nucleation rate $\sim e^{-\Gamma_{\mathrm{B}}}$
  - recent perturbative results

    [Gould–Hirvonen 2108.04377]
  - SM-like Higgs system non-perturbatively from lattice

    [Moore–Rummukainen hep-ph/0009132;Gould–Güyer–Rummukainen 2205.07238]

  ⋮

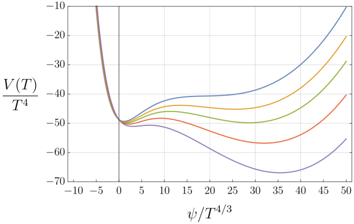
[1] Caution: nucleation theory fails in He-3 (ie. time-dependent Ginzburg–Landau theory) [Leggett'84]

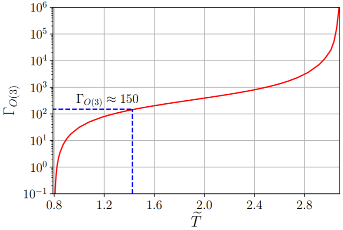
Protocol

Step 1: For each T, find effective action

Step 2: Solve for $O(3)$ bubble

Step 3: Get nucleation rate as a function of T





3. Ideas at strong coupling

Two possibilities:

A) Compute QFT effective action for "order parameter"

$$\Gamma[\psi] = F[J] - \int \mathrm{d}^d x J(x)\psi(x)$$

- Solve for spherically symmetric bubble solutions – only ODE

B) Can look for bubble solutions directly in dual gravity theory

- Need to solve for PDEs ($\because$ holographic direction)

[see Javier's talk]

I will discuss both and show that they agree.

A) Effective action from holography

- We want to compute effective action, in a derivative expansion

$$\Gamma[\psi] = -N^2 \int d^d x \left\{ V(\psi) + \frac{1}{2} Z(\psi)(\nabla\psi)^2 + \dots \right\}$$

  using holography, here classical field $\psi = \langle \mathcal{O} \rangle$
- Standard is a real tour de force, Legendre trafo of the renormalized on-shell bulk action

  [see eg. Kiritsis–Li–Nitti 1401.0888]
- Alternate: $V(\psi)$ obtained from static and homogeneous black brane solutions ("designer gravity")

  [Horowitz–Hertog hep-th/0412169]
  - Extract S-curve $\Lambda(\psi)$; integrate to get $V(\psi)$
- Kinetic term $Z(\psi)$ obtained by fluctuations about homogeneous solutions

  [Ares–Henriksson–Hindmarsh–Hoyos–NJ 2109.13784]
  - $\Gamma[\psi]$ generates 1PI n-pt functions
  - 2-pt function to order k^2 gives $Z(\psi)$

A+B) Probe action in four dimensions

- Aim: Solve for the bubble directly in dual gravity side and compare with effective action method.
- Focus first in gravity in four dimensions:

$$S_{\text{gravity}} = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left\{R + \frac{6}{L^2} - \frac{1}{2}(\partial_\mu\phi)^2 - \underbrace{m^2}_{=-2}\phi^2 - \frac{1}{4}\phi^4\right\}$$

- Work in the probe limit, metric = AdS_4-Schwarschild

$$ds^2 = \frac{1}{u^2}\left(-fdt^2 - 2dtdu + d\vec{x}^2\right) \quad , \; f = 1 - \left(\frac{u}{u_H}\right)^3$$

$$\nabla_M\nabla^M\phi + 2\phi - \frac{1}{2}\phi^3 = 0 \; , \; \phi = \phi^- u + \phi^+ u^2 + \dots \; .$$

A+B) Probe action in four dimensions

$$\nabla_M \nabla^M \phi + 2\phi - \frac{1}{2}\phi^3 = 0 \ , \ \phi = \phi^- u + \phi^+ u^2 + \dots \ .$$

- Standard BC preserving AdS isometries (..if backreacted..)

  [Henneaux–Martinez–Troncoso–Zanelli hep-th/0404236, Hertog–Maeda hep-th/0404261]

$$\phi^- = 0 \ , \quad \text{Dirichlet}$$

$$\phi^+ = 0 \ , \quad \text{Neumann}$$

$$m^2_{\text{BF}} \leq m^2 \leq m^2_{\text{BF}} + 1 \ , \ \phi^+ \propto (\phi^-)^2$$

- Such more general BC

$$\phi^+ = \frac{\partial f(\phi^-)}{\partial \phi^-} \ , \ f = \text{arbitrary function}$$

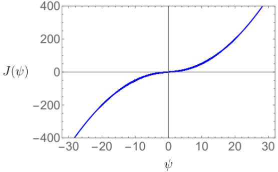
  maps into a multitrace deformation of the boundary action eg.

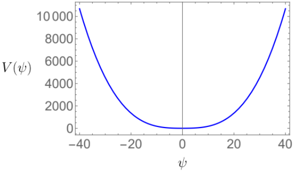
  [Witten hep-th/0112258, Papadimitriou hep-th/0703152, Vecchi 1005.4921]

$$S^{3D}_{\text{CFT}} \to S^{3D}_{\text{CFT}} + \int g_n \, \mathcal{O}^n \quad \Rightarrow \quad V(\psi) \to V(\psi) + g_n \psi^n$$

- Easy in bulk (change BC) and in boundary:
  multitrace deformations = knobs inducing FOPT

- Undeformed theory



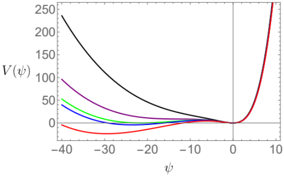


- Deform by double and triple trace operators

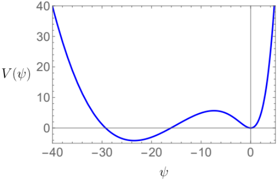
$$\phi^+ = J - g_2\phi^- + g_3(\phi^-)^2 \ , \ \langle \mathcal{O} \rangle \equiv \kappa_4^2 \frac{\delta S_{tot}}{\delta J} = -\phi^- \equiv \psi$$

- Get an effective potential by design:

$$V(\psi) \ \rightarrow \ V(\psi) + \frac{g_2}{2}\psi^2 + \frac{g_3}{3}\psi^3$$



(a) $g_3 = 0.49$ and $g_2 \neq 0$



(b) $g_3 = 0.49$ and $g_2 = -0.245$

A) Effective action the easy way

- Consider black brane background and solve for static and homogeneous case:

$$\phi = \phi(u): \; f\partial_u^2(\phi/u) + f'\partial_u(\phi/u) - \frac{u}{u_H^3}\phi/u - \frac{1}{2}(\phi/u)^3 = 0 \,.$$

- Get the curve $J = J(\psi)$, integrate:

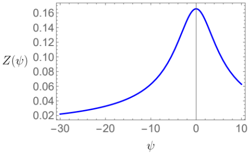
$$J(\psi) = \frac{dV(\psi)}{d\psi} \;\rightarrow\; V(\psi) = \underbrace{V(0)}_{\rightarrow 0} + \int_0^{\psi} d\psi' J(\psi')$$

- For non-canonical kinetic term, expand about homogeneous solution

$$\phi(u) \rightarrow \bar{\phi}(u) + \delta\phi(u,x) = \bar{\phi}(u) + e^{ikx}\left(\delta\phi_0(u) + k^2\delta\phi_2(u) + \ldots\right)$$

- Solve for coupled EoMs for $\delta\phi_0, \delta\phi_2$ using regularity and plug into

$$Z(\psi) = \frac{\delta\phi_0^+\delta\phi_2^- - \delta\phi_2^+\delta\phi_0^-}{(\delta\phi_0^-)^2}$$



A) Bubbles from effective action

- Summary, effective action for a classical field $\psi = \frac{\delta F[J]}{\delta J(x)}$:

$$\Gamma[\psi] = F[J] - \int d^3x \psi(x) J(x) \, , \; -J(x) = \frac{\delta \Gamma[\psi]}{\delta \psi}$$

- Local action (if no gapless dofs..) as an derivative expansion about a homogeneous state

$$\Gamma[\psi] = \int d^3x \left[-V(\psi) + \frac{1}{2} Y(\psi) (\underbrace{\partial_t \psi}_{\to 0})^2 - \frac{1}{2} Z(\psi) \nabla\psi \cdot \nabla\psi + \dots \right]$$

- Find bubbles using polar coordinates (ρ, θ), $J = 0$, $\psi = \psi(\rho)$:

$$\frac{d^2\psi}{d\rho^2} + \frac{1}{\rho}\frac{d\psi}{d\rho} + \frac{1}{2}\frac{Z'(\psi)}{Z(\psi)}\left(\frac{d\psi}{d\rho}\right)^2 - \frac{V'(\psi)}{Z(\psi)} = 0 \, , \psi'(0) = 0 = \psi(\rho \to \infty)$$

B) Bubbles from gravity

- We can solve for bubbles in gravity

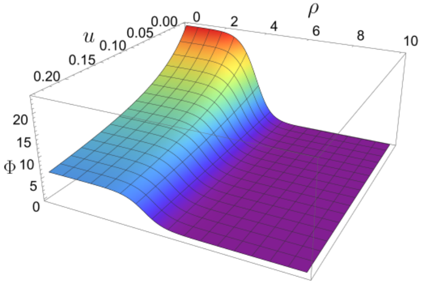
$$ds^2 = \frac{1}{u^2}(-fdt^2 - 2dtdu + d\rho^2 + \rho^2 d\theta^2)$$

- Ansatz for the bulk scalar field $\phi = u\Phi(u,\rho)$:

$$f\partial_u^2\Phi + f'\partial_u\Phi + \partial_\rho^2\Phi + \frac{1}{\rho}\partial_\rho\Phi - \frac{u}{u_H}\phi - \frac{1}{2}\Phi^3 = 0$$

- Boundary conditions (spectral method..)

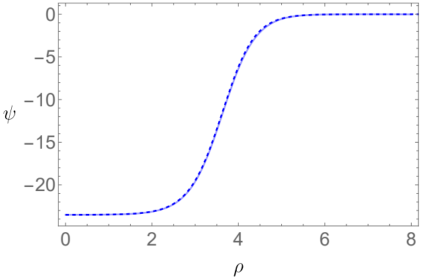
$$\partial_u\Phi - g_2\Phi + g_3\Phi^2\big|_{u=0} = 0\ ,\partial_\rho\Phi\big|_{\rho=0} = 0\ ,\partial_\rho\Phi\big|_{\rho=\infty} = 0\ .$$



Comparison of A) w/ B)

- The critical bubble solution by
  - A) Using effective action method
  - B) Solving directly bulk equations

  yield perfect match



g_3	g_2	$R_{\mathrm{EA}}/R_{\mathrm{G}}$	$\Delta E/\Delta V$
0.49	−0.234	1.004	0.209
0.49	−0.238	1.004	0.385
0.49	−0.242	1.004	0.576
0.49	−0.246	1.003	0.782
0.49	−0.250	1.003	1.003
0.49	−0.400	1.004	28.652

(a) Solutions with fixed g_3.

g_3	g_2	$R_{\mathrm{EA}}/R_{\mathrm{G}}$	$\Delta E/\Delta V$
0.49	−0.245	1.003	0.729
0.48	−0.380	1.003	1.096
0.47	−0.480	1.003	1.071
0.46	−0.560	1.004	0.860
0.45	−0.640	1.004	1.088
0.44	−0.700	1.004	0.804

(b) Solutions with variable g_2 and g_3.

- Independent of bubble wall being thin, thick, ...
- Nucleation rates $\sim e^{-\Gamma}$ agree within 1–3%

4. w/ backreaction + application to early universe FOPT

A) Simplest model possible

- Bottom-up 5D gravity-scalar theory

[Ares–Henriksson–Hindmarsh–Hoyos–NJ 2109.13784]

$$S_{\text{gravity}} = \int d^5x\sqrt{-g}\left\{R - (\partial_\mu\phi)^2 + \frac{12}{L^2} - m^2\phi^2\right\}$$

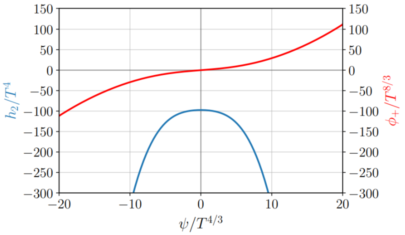
- Simplest imaginable action: free scalar, no bulk potential
- No probe approximation, full backreaction
- Pick m^2 s.t. (single-trace) dual operator ψ is dimension $4/3$

$$ds^2 = -e^{-2\chi(r)}h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\vec{x}^2$$

$$h(r_H) = 0$$

$$\phi = \frac{\phi_-}{r^{4/3}} + \frac{\phi_+}{r^{8/3}} + \dots$$

$$h = r^2 + \frac{4}{9}\frac{\phi_-^2}{r^{2/3}} + \frac{h_2}{r^2} + \dots$$



- source $J = \phi_+$, classical field $\psi = -\frac{4}{3}\phi_-$

A) CFT effective potential and kinetic term

- Undeformed "CFT" potential

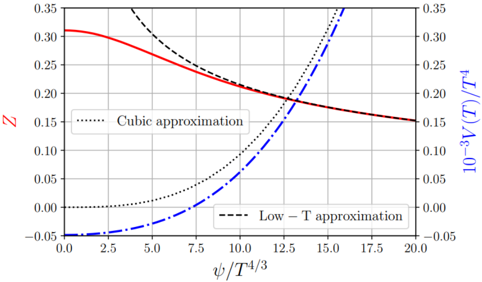
$$V_{\mathrm{CFT}}(\psi) = -F[J] + J\psi = \frac{h_2(\psi)}{2} + \frac{8}{27}\phi_+(\psi)\psi$$

- Small ψ approach high-T CFT:

$$V_{\mathrm{CFT}}(\psi) = T^4\left(V_0 + \frac{V_2}{2}\frac{\psi^2}{T^{8/3}} + \dots\right), \quad V_0 = -\frac{\pi^4}{2}, \quad V_2 = \frac{9\pi^{17/6}}{\Gamma(1/6)^3}$$

- Large ψ: bound on triple-trace deformation: $g < \gamma_3$:

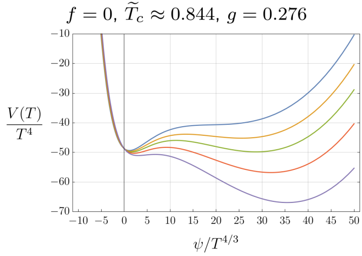
$$V_{\mathrm{CFT}}(\psi) = \frac{\gamma_3}{3}|\psi|^3 + \dots \ , \gamma \approx 0.278 \ , \ Z(\psi) \sim |\psi|^{-1/2}$$

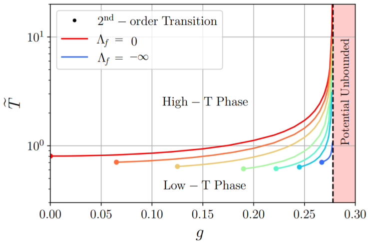


$$S_{CFT}^{4D} \to S_{CFT}^{4D} + \int d^4x \left\{ \Lambda\psi + \frac{f}{2}\psi^2 + \frac{1}{3}g\psi^3 \right\}$$

- Multi-trace deformations are implemented by BC
- Dimensionless quantities

$$\tilde{T} = \frac{T}{|\Lambda|^{3/8} + |f|^{3/4}} \ , \ \Lambda_f = \frac{\Lambda}{f^2} \ , \ g$$





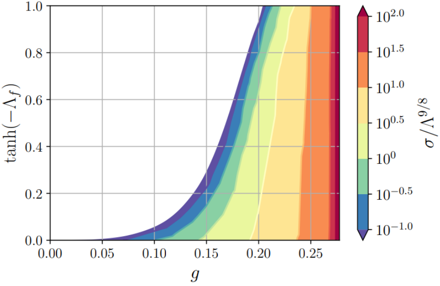
A) Domain wall solutions

- At $T = T_c$ the two minima $V(\psi_l) = V(\psi_h) = V_c$
- Mixed phases are possible
- Domain wall solution separating the two stable phases:

$$\lim_{x\to-\infty}\psi(x) = \psi_h \quad , \quad \lim_{x\to\infty}\psi(x) = \psi_l$$

- Surface tension

$$\sigma = \int_{-\infty}^{\infty} dx \left(\frac{1}{2}Z(\psi)(\partial_x\psi)^2 + V(\psi) - V_c\right)$$



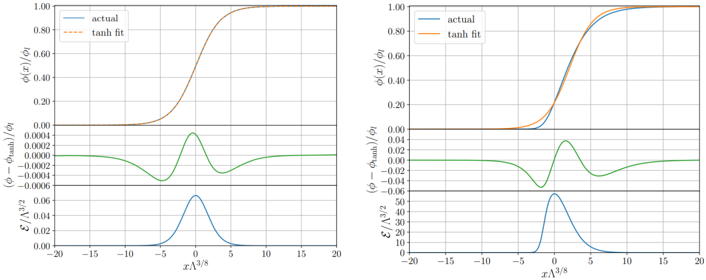
A) Domain wall solutions

- Tanh-approximation?

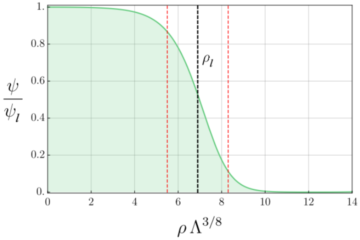
[Janik–Järvinen–Sonnenschein 2106.02642]

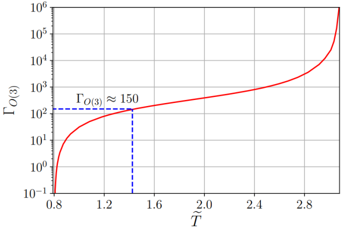
$$\mathcal{L}_{toy} = \frac{1}{2}(\partial_x \phi)^2 - V(\phi) \ , V(\phi) \sim \phi^4$$

$$\rightarrow \psi \approx \psi_h + \frac{\psi_l - \psi_h}{2}(1 + \tanh(x/\ell + \delta))$$



- If potential minima far apart, tanh-approximation breaks down





$$\Gamma_{O(3)} = \frac{4\pi N^2}{T} \int_0^\infty d\rho \rho^2 \left(\frac{1}{2} Z(\psi) \left(\frac{d\psi}{d\rho} \right)^2 + V(\psi, T) \right)$$

$$\Gamma \sim N^2 (T - T_0)^x \ , x \approx 1.4..1.5 \ , \ \Gamma \sim N^2 (T_c - T)^{-2}$$

A) Bubbles

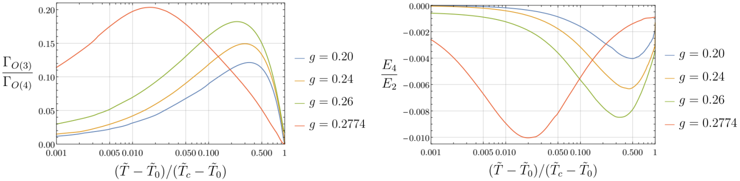
- Dominant saddle?

$$\Gamma_{O(4)} = 2\pi^2 N^2 \int_0^\infty d\tilde{\rho}\tilde{\rho}^3 \left(\frac{1}{2} Z(\psi) \left(\frac{d\psi}{d\tilde{\rho}} \right)^2 + V(\psi) \right)$$

vs.

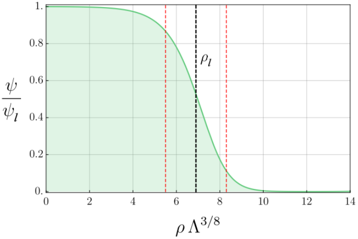
$$\Gamma_{O(3)} = \frac{4\pi N^2}{T} \int_0^\infty d\rho\rho^2 \left(\frac{1}{2} Z(\psi) \left(\frac{d\psi}{d\rho} \right)^2 + V(\psi, T) \right)$$

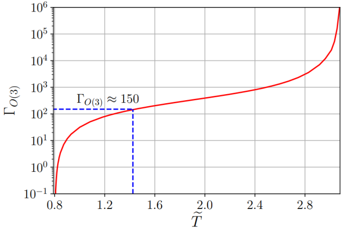
- Higher derivative terms $\partial^2\psi\partial^2\psi \leftrightarrow k^4$?



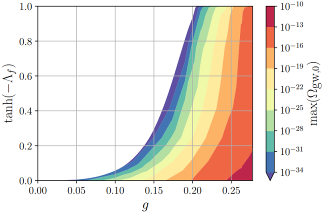
- Can focus on O(3) and two-derivative effective action

A) Simple model meets early universe





- Compute all quasi-equilibrium GW parameters $(T_n/T_c, \beta/H_n, \alpha(T_n))$
  [Ares–Henriksson–Hindmarsh–Hoyos–NJ 2110.14442]

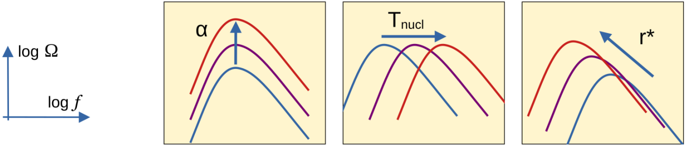


- Estimate low-wall speed using
  [Bea et al. 2104.05708; Bigazzi et al. 2104.12817;
  Henriksson 2106.13254; Janik et al. 2205.06274]

$$\text{wall speed} = \mathcal{O}(1) \times \left.\frac{P_{low} - P_{high}}{\epsilon_{high}}\right|_{T_n}$$

- Use (improved) LISA cosmo WG model to find GW power spectrum
  - Detectable signal $f_{\rm peak} \sim 10^{-3}..10^{-2}$Hz when close to EW scale ($0.3 < T_c < 1.8$TeV) for small Λ_f and "large" g

A) Simple model meets early universe



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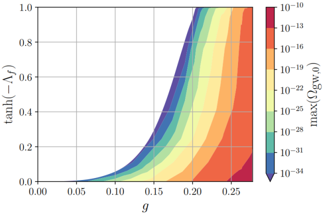

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Summary/outlook

At strong coupling

- Obtained an effective action for an order parameter
- Crosschecked that critical bubble obtained using direct computation in the bulk matches w/ effective action approach
- Noted that all quasi-equilibrium nucleation theory parameters are computable
- Application to "dark sector" GW production in early universe

Outlook

- Do this in a top-down example
- Allow time-dependence
- Spatial dependence, either explicit or spontaneous: inhomogeneous phases
- Higher-derivative terms to effective action
- Extend to effective action to finite density

[Kiritsis–Niarchos 1205.6205]