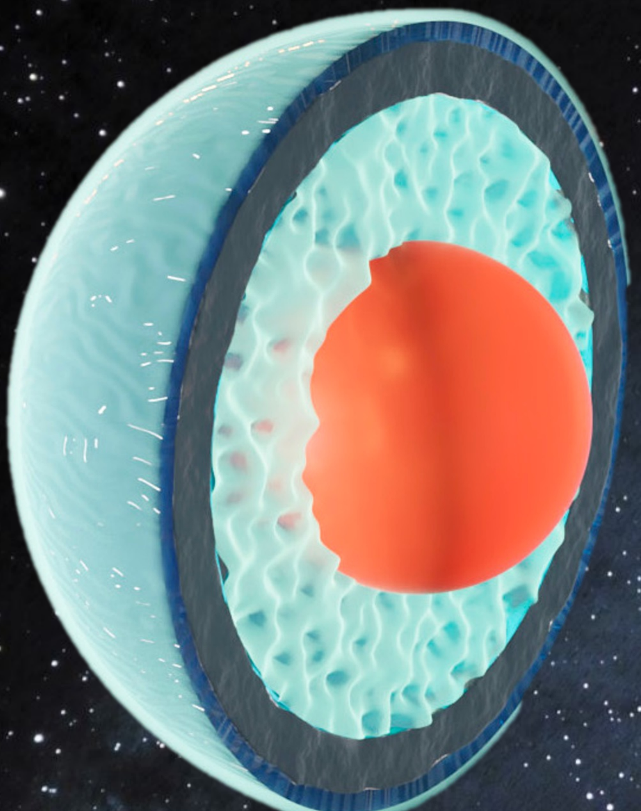


Neutron stars, quark cores and QCD

AB-INITIO QCD CALCULATIONS IMPACT THE INFERENCE OF NEUTRON-STAR EQUATION OF STATE

GORDA, KOMOLTSEV & AK, ASTROPHYS.J. 950 (2023), 2204.118

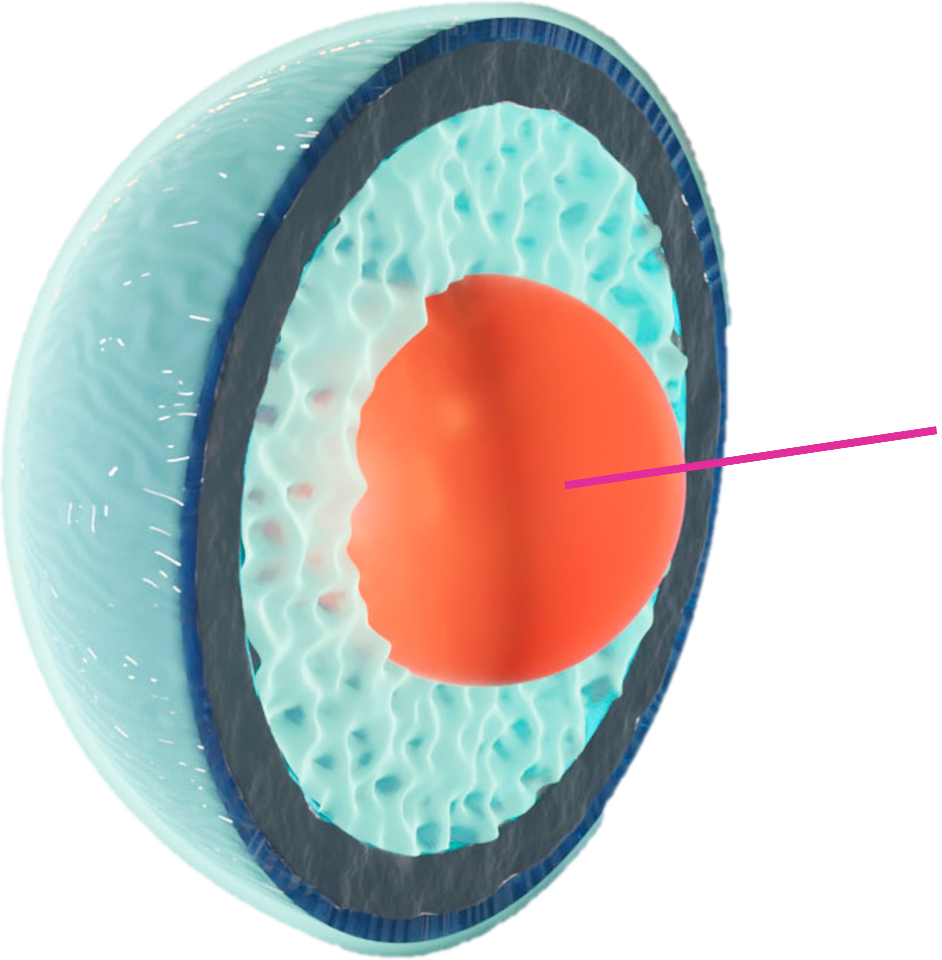
STRONGLY INTERACTING MATTER EXHIBITS DECONFINED BEHAVIOR IN MASSIVE NEUTRON STARS

ANNALA, AK, ET AL, NATURE COMMUN. 14 (2023) 2303.11356

CONSTRAINING THE EQUATION OF STATE IN NEUTRON-STAR CORES VIA THE LONG-RINGDOWN SIGNAL

ECKER, GORDA, AK & REZZOLLA NATURE COMMUNICATIONS 16, 2025

Neutron stars:



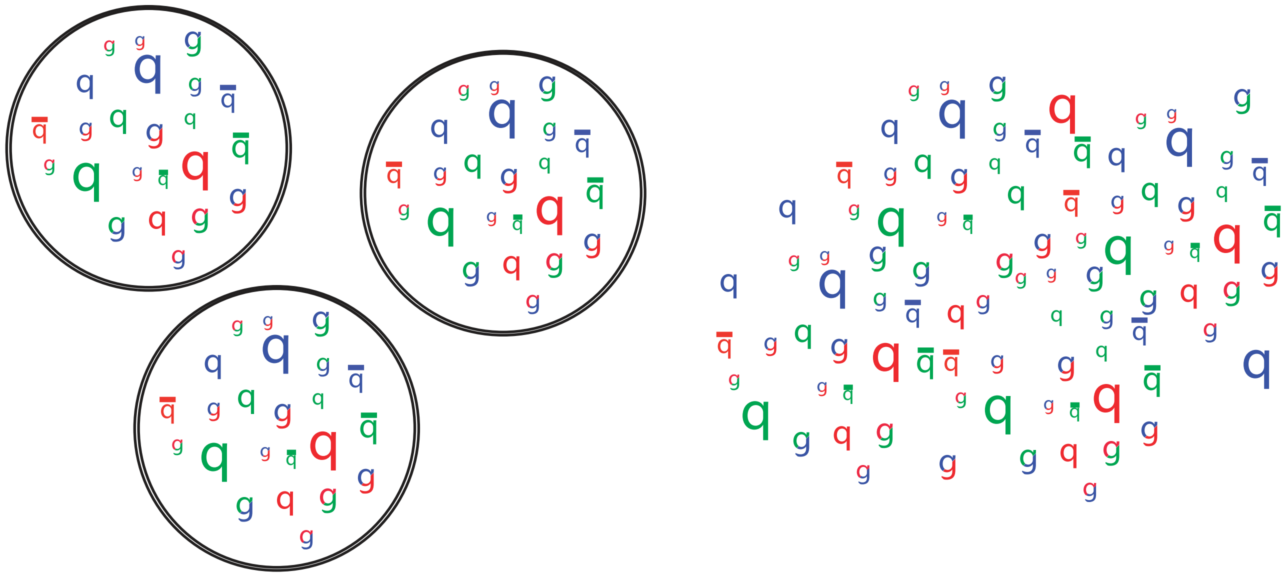
Strong **gravity**:

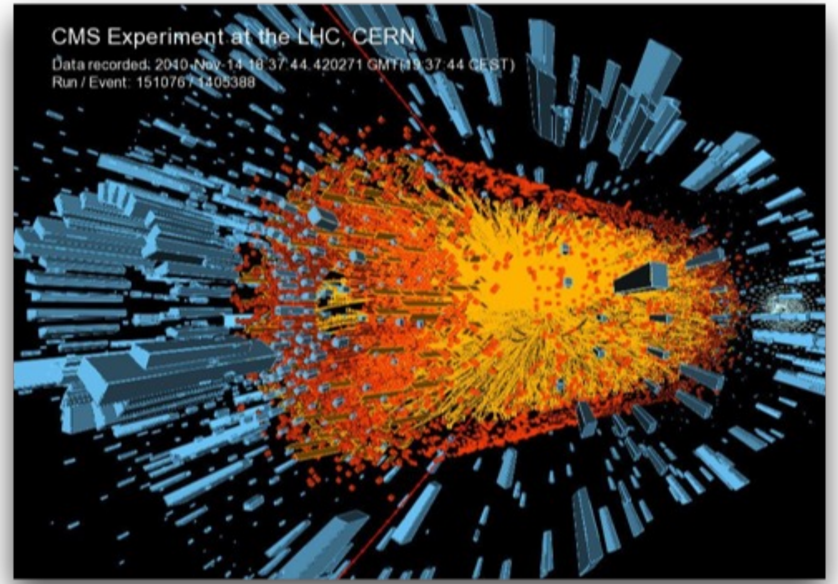
Radius:
$\sim 12\text{km}$
$\sim 2 \times$ Schwarzschild radius

Extreme **QCD**:

Density
$\sim 5 - 8 \times$ nuclear density
$\sim 2 \times$ density of neutrons

Matter in the extreme:

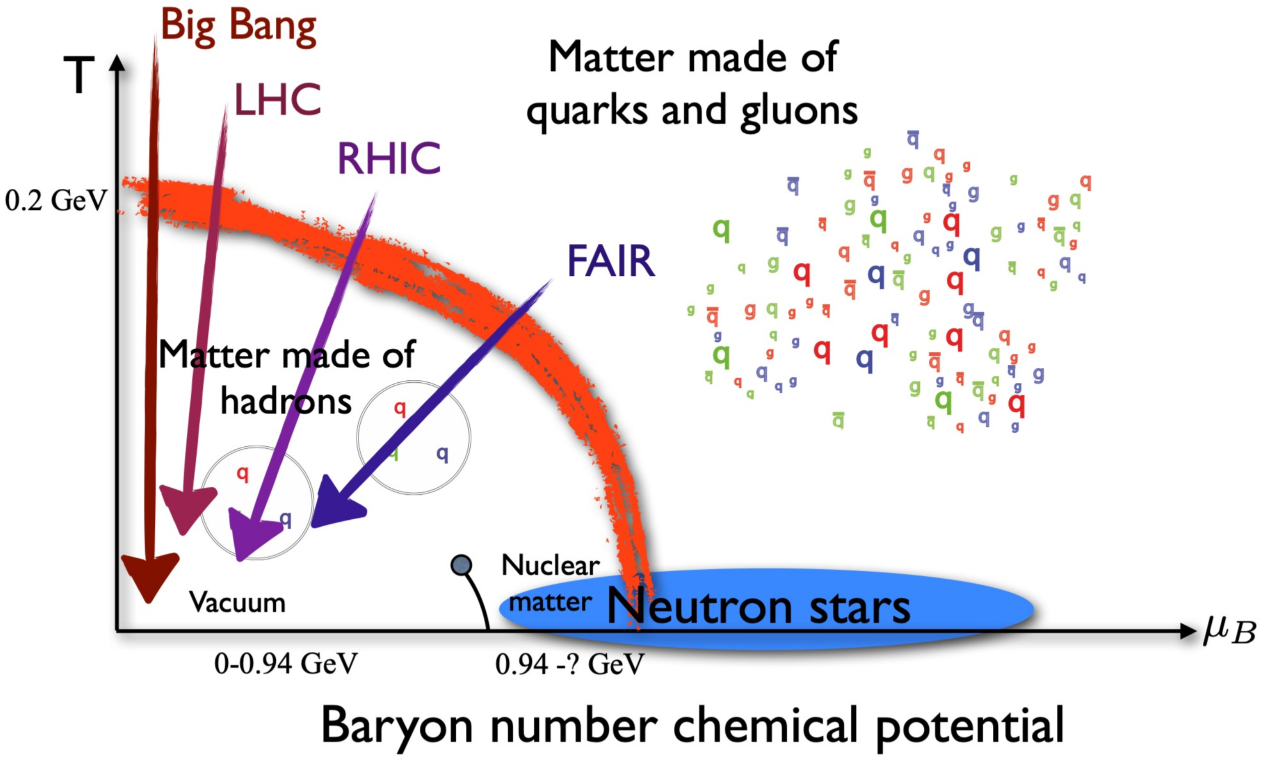


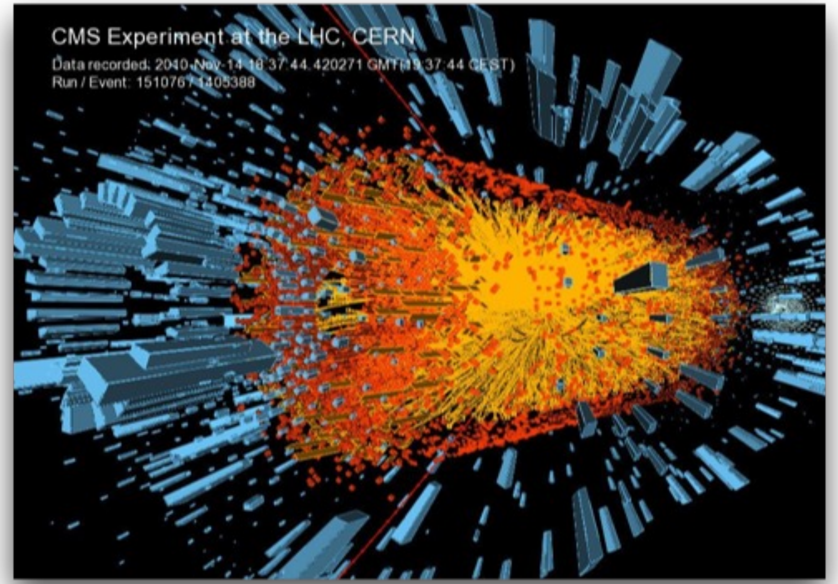


LHC, RHIC, FAIR, EIC,…

LIGO+Virgo+Kagra, NICER, eXTP,…

Matter in the extreme:



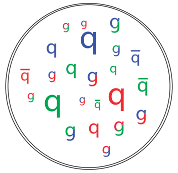


LHC, RHIC, FAIR, EIC,…

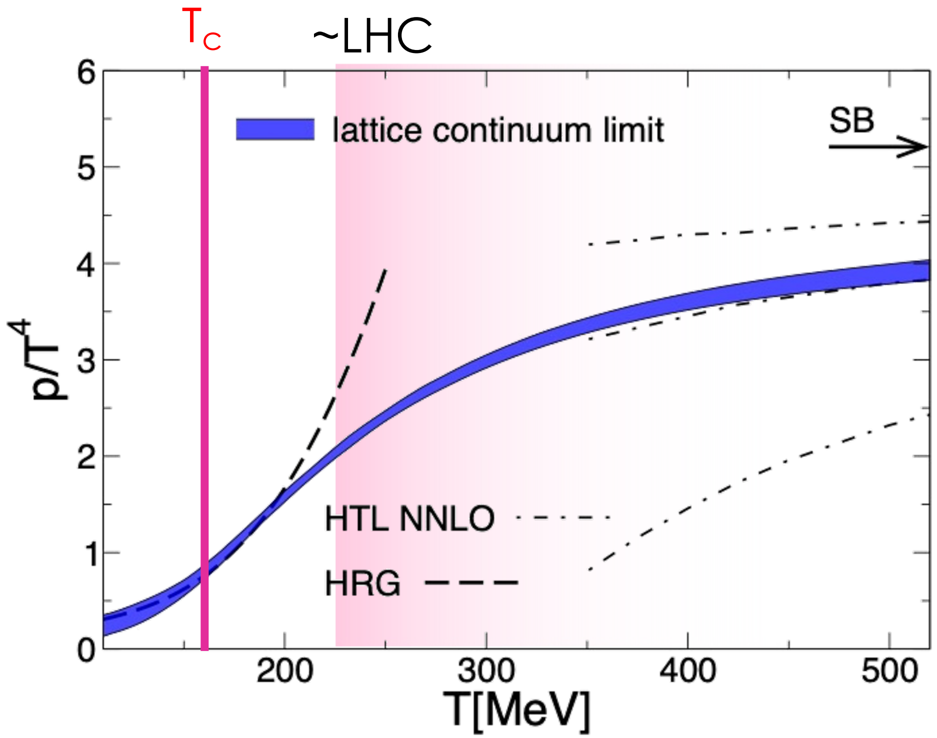
LIGO+Virgo+Kagra, NICER, eXTP,…

EoS tells about phases of matter: HIC

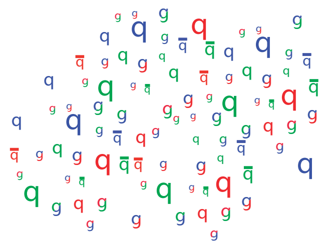
Hadrons



Matter described by hadronic mass scale



Quarks and gluons



Approximately conformal matter, with large number of d.o.f.

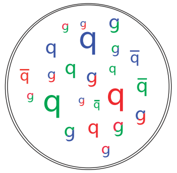
A clear **non-coformal** to **conformal** transition in heavy-ion collisions

In addition, grouping of v_2 between baryons and mesons + large energy-loss of particles, R_{AA}

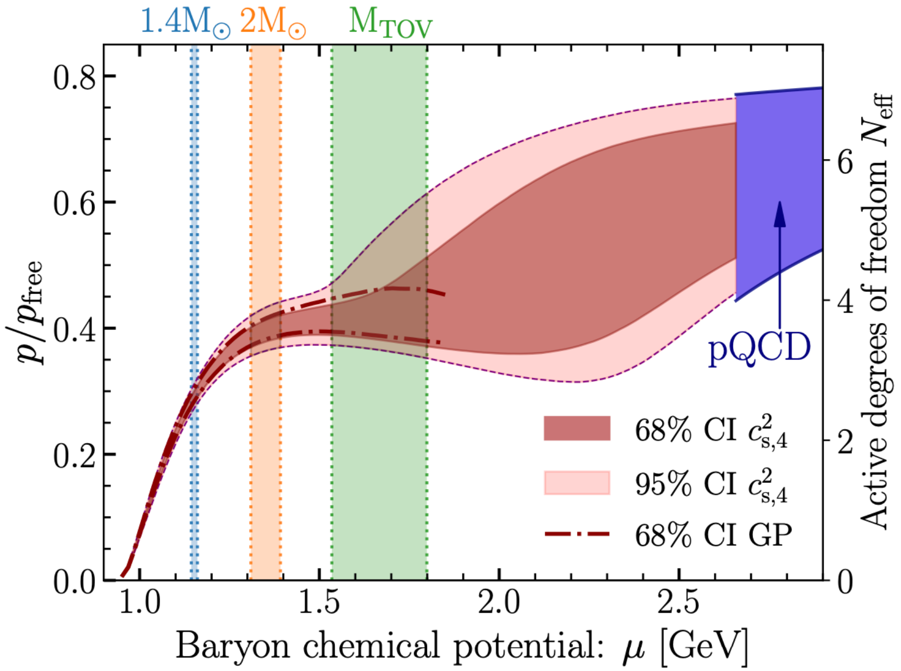
Gardim et al Nature Physics 16, 615-619 (2020)
Borsanyi et al PLB 730, 99 (2014)

EoS tells about phases of matter: Stars

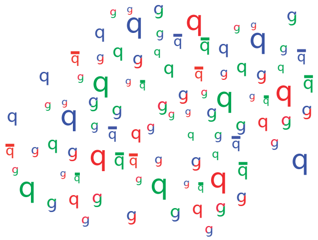
Hadrons



Matter described by Hadronic mass scale



Quarks and gluons



Approximately conformal matter, with large number of d.o.f.

A clear **non-coformal** to **conformal** transition within cores of neutron stars

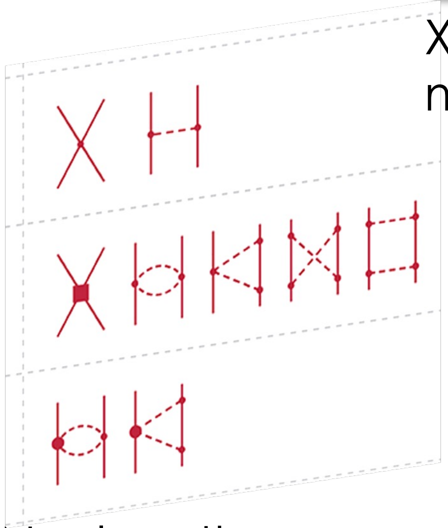
Annala, Gorda, **AK**, Nättilä, Vuorinen Nature Physics 16 (2020) 9
Annala, Gorda, **AK**, Nättillä, Vuorinen Nature Comms. 14 (2023)

Also: Fujimoto, Fukushima, McLerran, Praszalowicz 2207.06753, Kojo PRD 104, …

X-ray radius, measurements

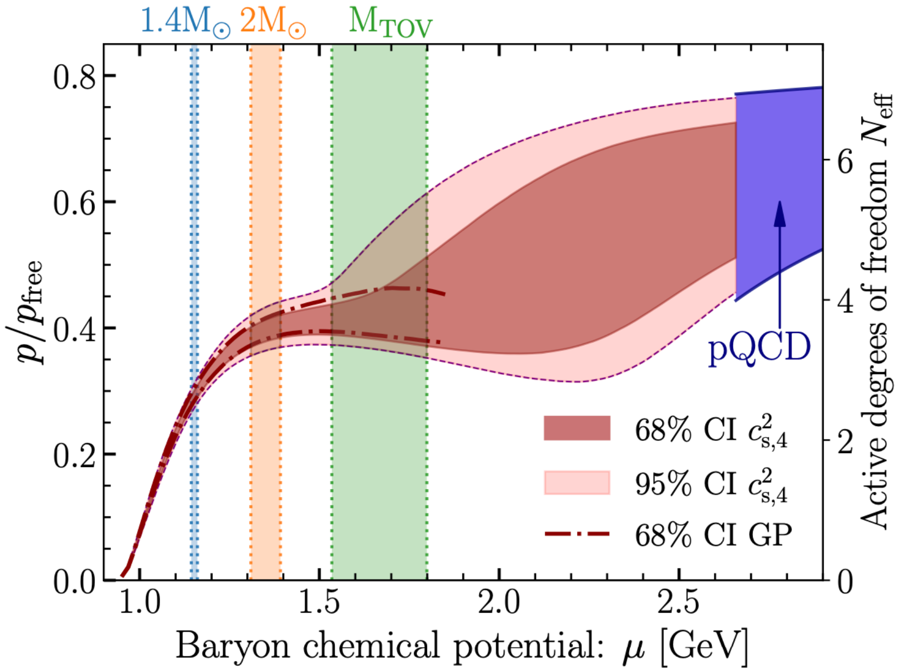
Gravitational waves

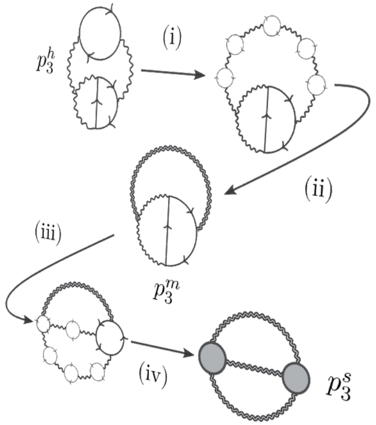
Radio, mass measurements



Nuclear theory

low-energy nuclear experiments



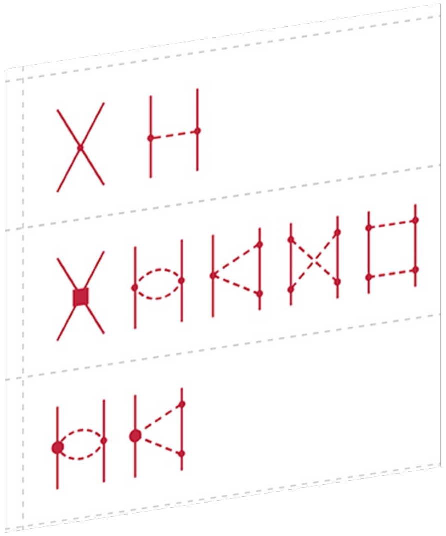


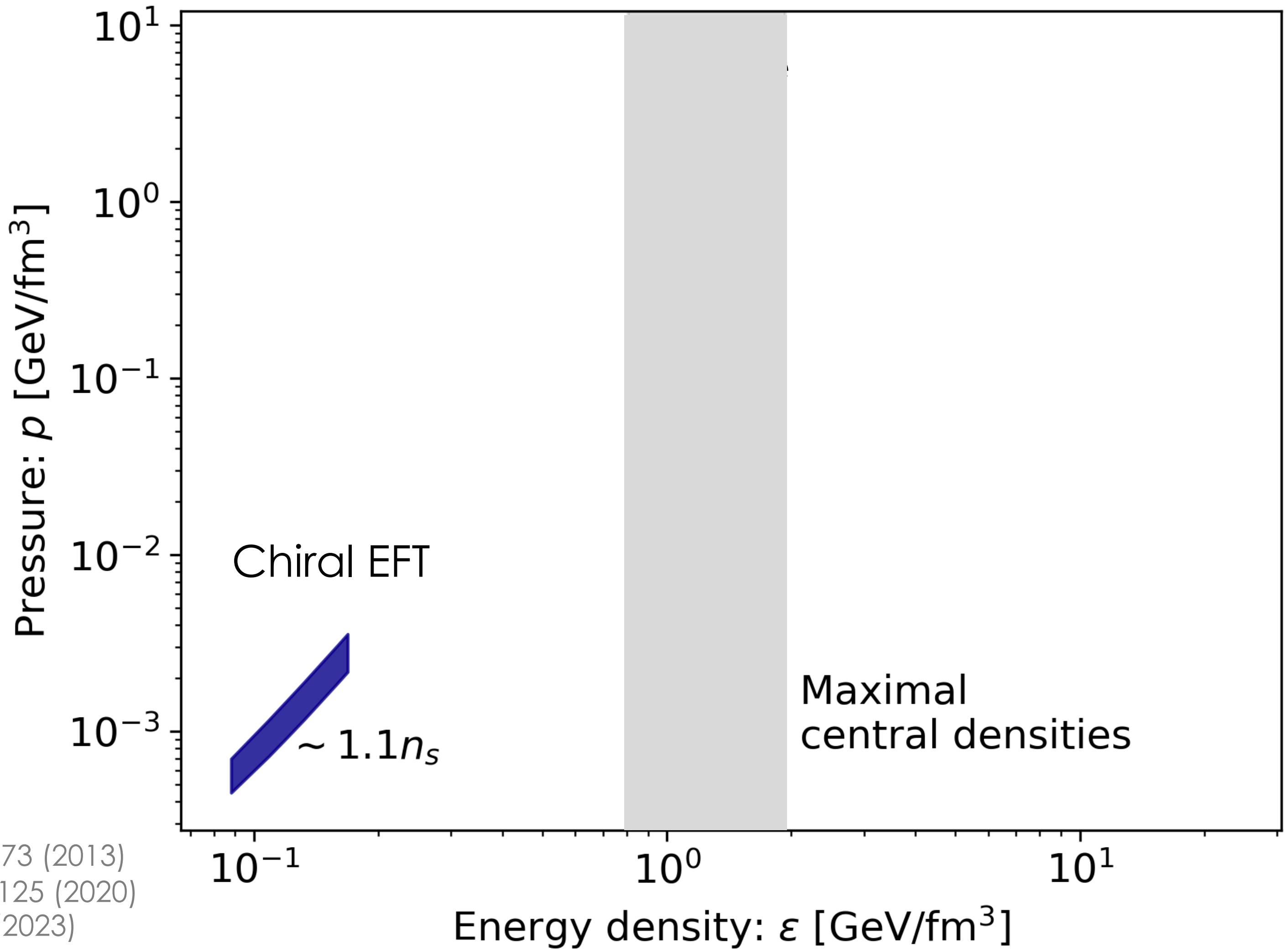
Particle theory

Particle/HIC experiment

What do we know about the neutron star **equation of state**?

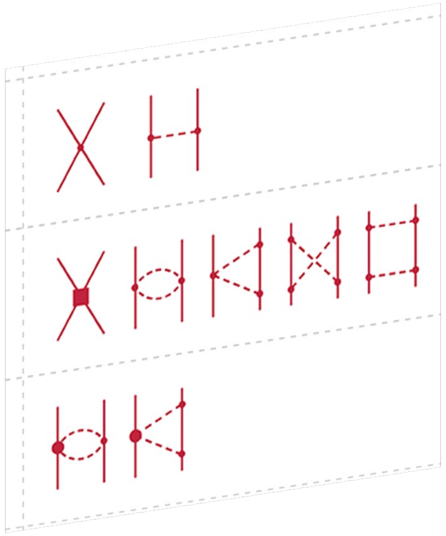
Equation of state, theoretically:

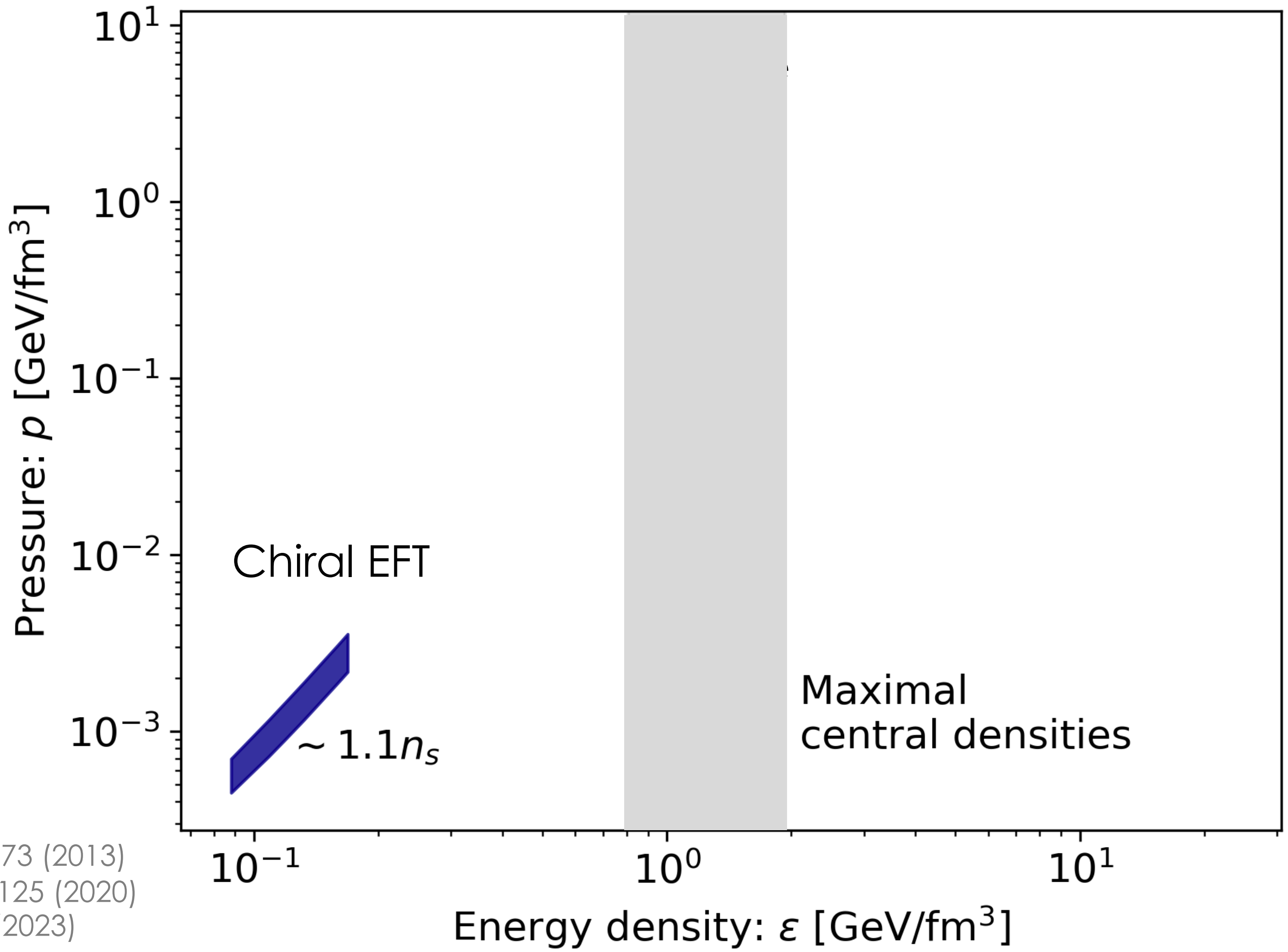




Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)
Keller, et al, PRL 130, 072701 (2023)

Equation of state, theoretically:





Perturbative QCD at high densities:

$$\alpha_s \sim \frac{1}{\log \mu^2}$$

Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)
Keller, et al, PRL 130, 072701 (2023)

Freedman, McLerran, PRD 16 (1977)
Quark masses: Kurkela et al. PRD 81 (2010)

Equation of state, pQCD:

EoS $P(\mu)$ = effective potential with: $\mathcal{L}_{QCD} + \mu\bar{\psi}\gamma_0\psi$

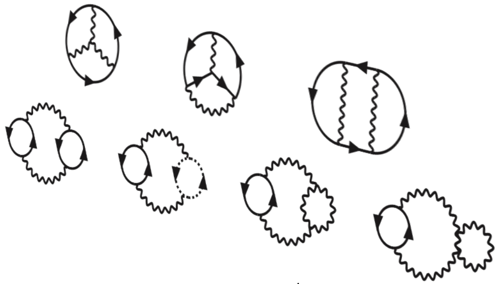
$$\frac{1}{p_0^2 + \vec{p}^2} \rightarrow \frac{1}{(p_0 + i\mu)^2 + \vec{p}^2}$$

$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

Freedman, McLerran, PRD 16 (1977)
Quark masses: Kurkela et al. PRD 81 (2010)

Equation of state, pQCD:

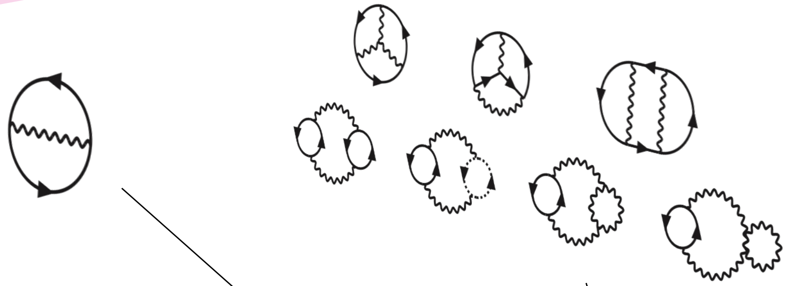
Free quark matter



$$\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3\, \alpha_s^3 + \cdots$$

Freedman, McLerran, PRD 16 (1977)
Quark masses: Kurkela et al. PRD 81 (2010)

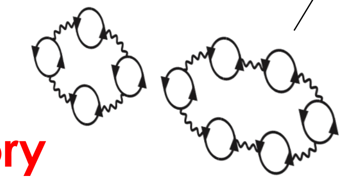
Equation of state, pQCD:



$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

$$+\, s_2\alpha_s^2 + s_3\,\alpha_s^3 + \cdots$$

Resummations going
beyond perturbation theory

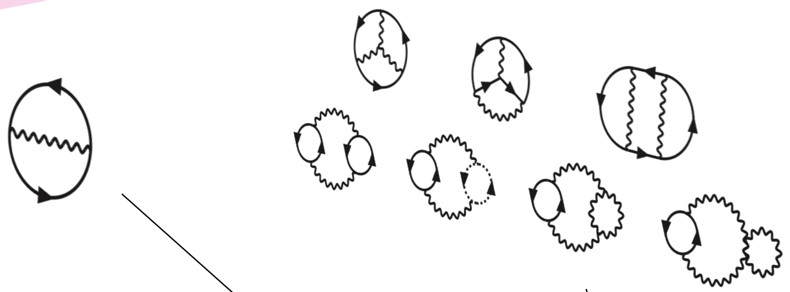


+ …

AK, Romatschke, Vuorinen, PRD 81 (2010)

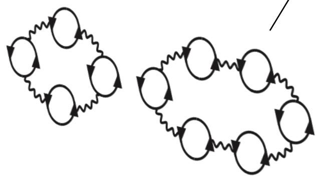
Kurkela, Vuorinen PRL 117 (2016)

Equation of state, pQCD:



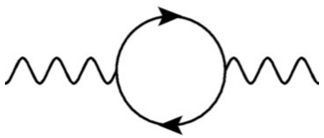
$$\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3\, \alpha_s^3 + \cdots$$

$$+ s_2 \alpha_s^2 + s_3\, \alpha_s^3 + \cdots$$



+ …

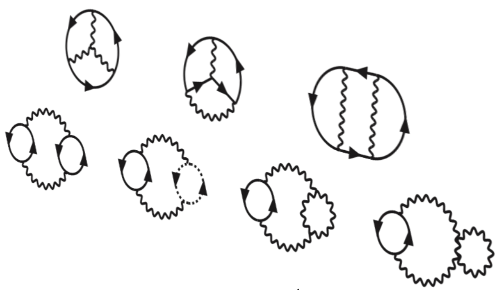
Medium polarization:



$$\Pi \sim \alpha_s \int^{\mu} \frac{d^3 p}{p} \sim \alpha_s \mu^2$$

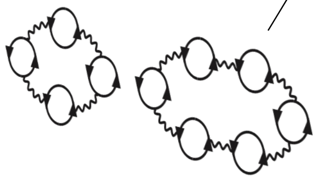
Kurkela, Vuorinen PRL 117 (2016)

Equation of state, pQCD:



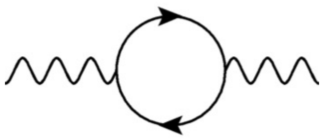
$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

$$+\, s_2\alpha_s^2 + s_3\,\alpha_s^3 + \cdots$$



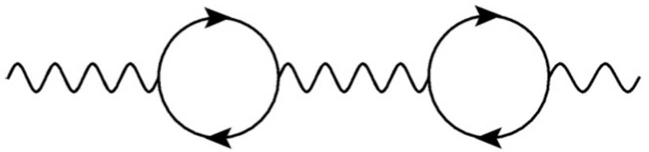
+ …

Medium polarization:



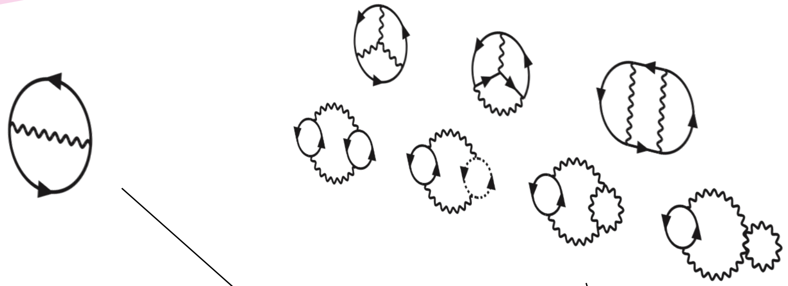
$$\Pi \sim \alpha_s \int^{\mu} \frac{d^3p}{p} \sim \alpha_s\mu^2$$

If $k^2 \sim \alpha_s\,\mu^2$ correction not small:



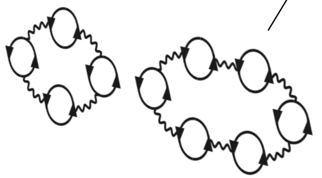
…

Equation of state, pQCD:



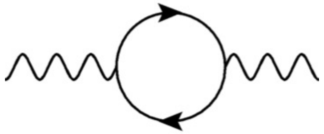
$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

$$+ s_2\alpha_s^2 + s_3\,\alpha_s^3 + \cdots$$



+ ...

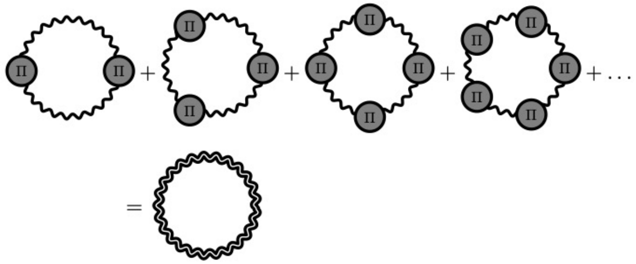
Medium polarization:



$$\Pi \sim \alpha_s \int^{\mu} \frac{d^3p}{p} \sim \alpha_s\mu^2$$

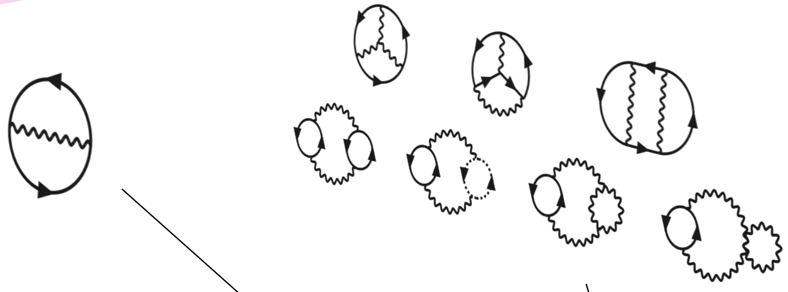
If $k^2 \sim \alpha_s\,\mu^2$ correction not small:

If $k^2 \ll \mu^2$: **Hard-thermal loop EFT**



Equation of state, pQCD:

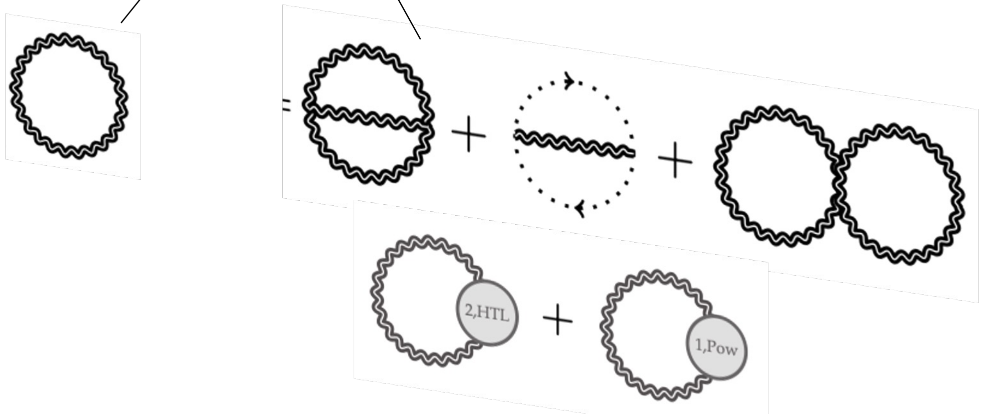
Gorda, Kurkela et al. PRL 121 (2018)
Gorda, Kurkela et al. PRL 127 (2021)
Gorda, Kurkela et al. PRD 107 (2023)
Gorda, Paatelainen et al. PRL131, (2023)



$$\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3\, \alpha_s^3 + \cdots$$

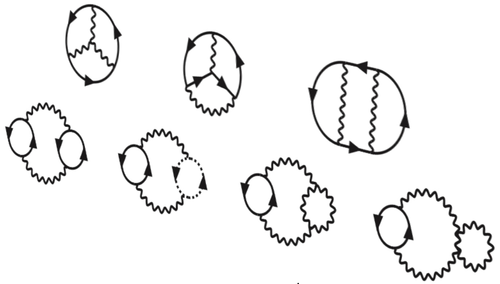
$$+ s_2 \alpha_s^2 + s_3\, \alpha_s^3 + \cdots$$

$$\sim \int^{g\mu} d^3p p \sim \alpha_s^2$$



Gorda, Kurkela et al. PRL 121 (2018)
Gorda, Kurkela et al. PRL 127 (2021)
Gorda, Kurkela et al. PRD 107 (2023)
Gorda, Paatelainen et al. PRL131, (2023)

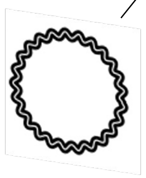
Equation of state, pQCD:

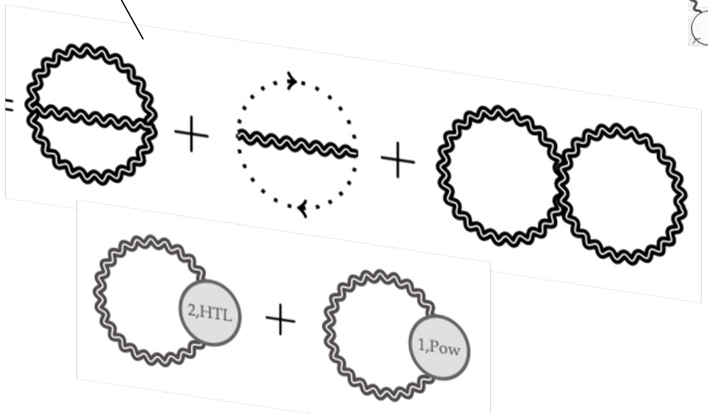


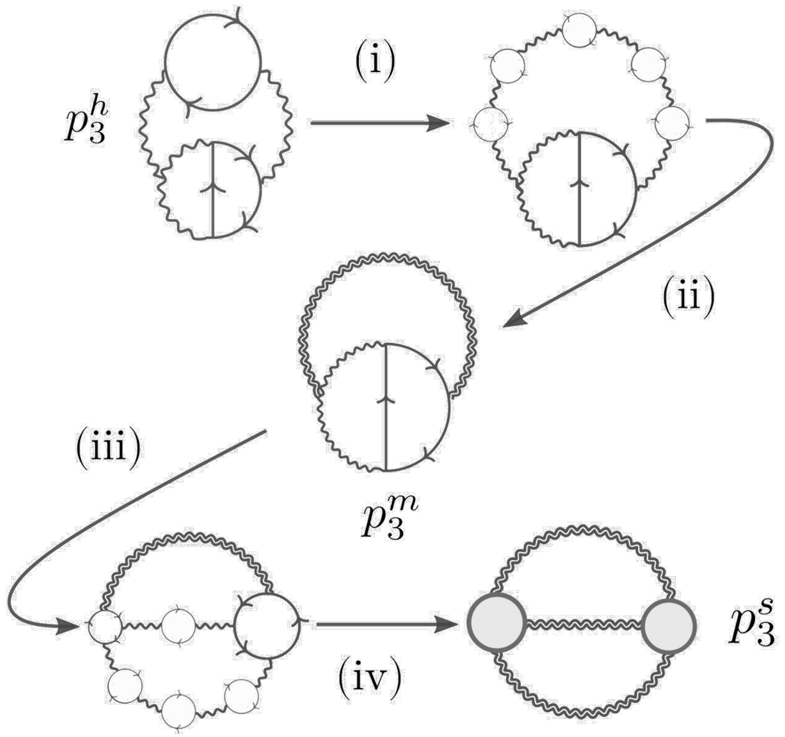
$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

$$+ s_2\alpha_s^2 + s_3\,\alpha_s^3 + \cdots$$

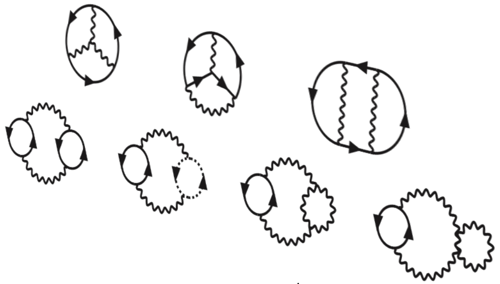
$$\sim \int^{g\mu} d^3pp \sim \alpha_s^2$$





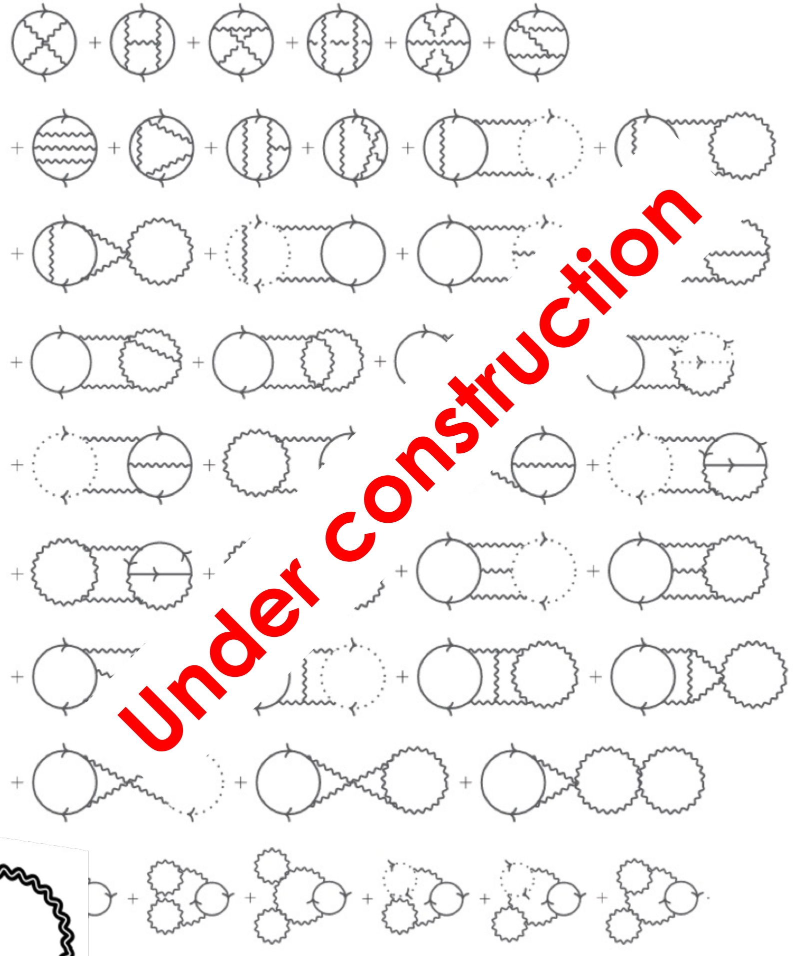


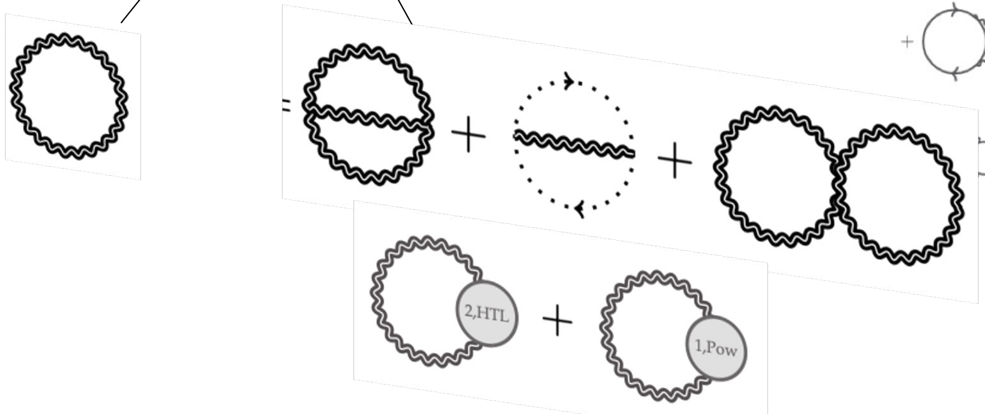
Equation of state, pQCD:



$$\frac{p}{p_0} = 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \cdots$$

$$+ s_2\alpha_s^2 + s_3\,\alpha_s^3 + \cdots$$

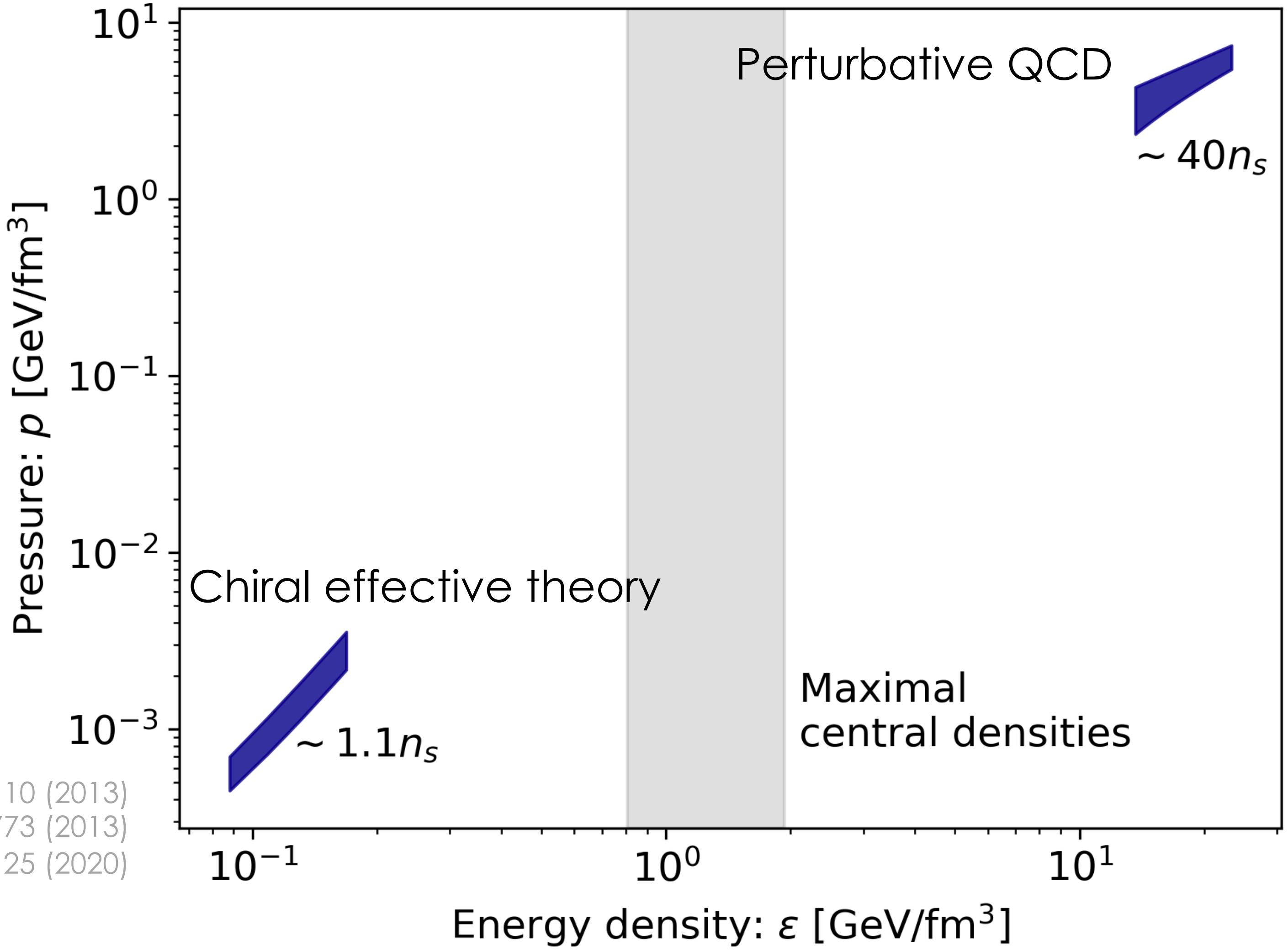


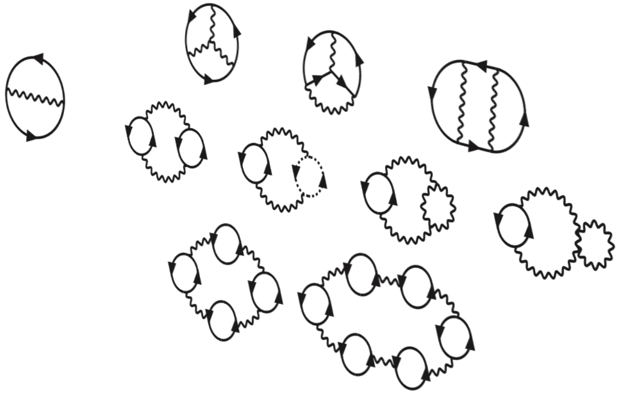


Novel loop-tree duality methods from collider physics

Navarrete, Paatelainen, Seppänen 2403.02180

Equation of state, theoretically:

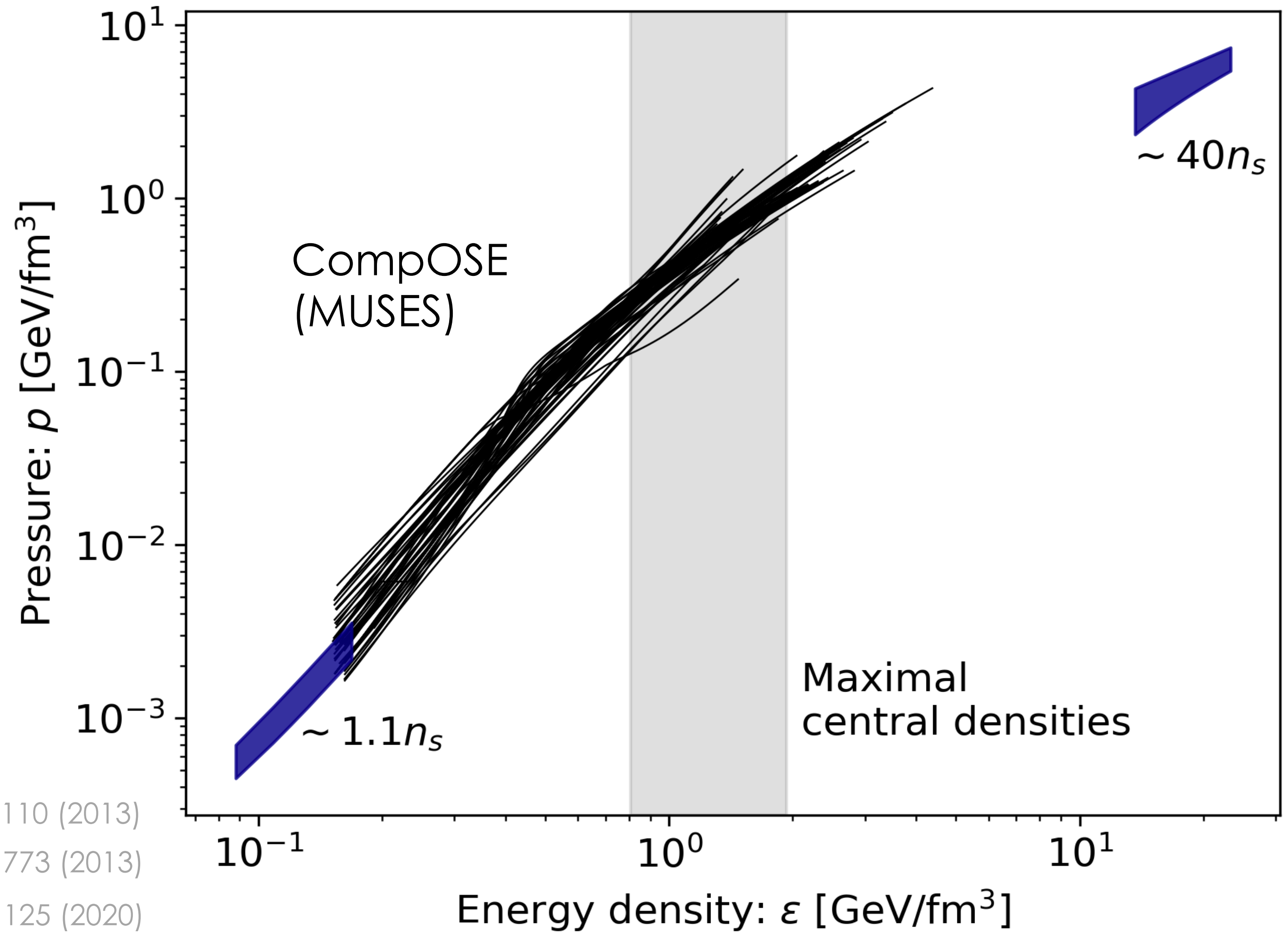


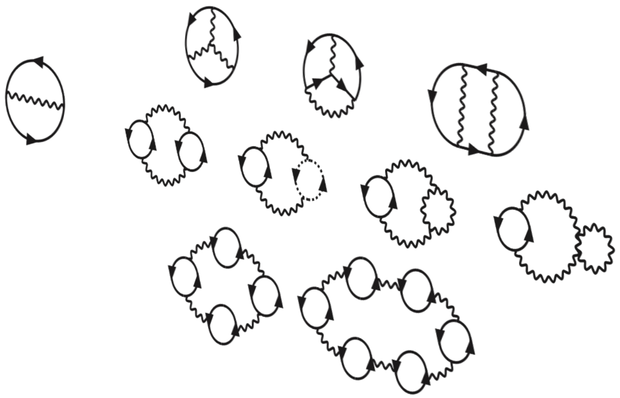


Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)

Freedman, McLerran, PRD 16 (1977)
AK et al. PRD 81 (2010)
AK, Vuorinen PRL 117 (2016)
Gorda, **AK** et al. PRL 121 (2018)
Gorda, **AK** et al. PRD 104 (2021)
Gorda, **AK** et al. PRL 127 (2021)
Gorda, **AK** et al. PRD 107 (2023)

Equation of state, models:





Tews et al. PRL 110 (2013)
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Drischler, Furnstahl et.al. PRL 125 (2020)

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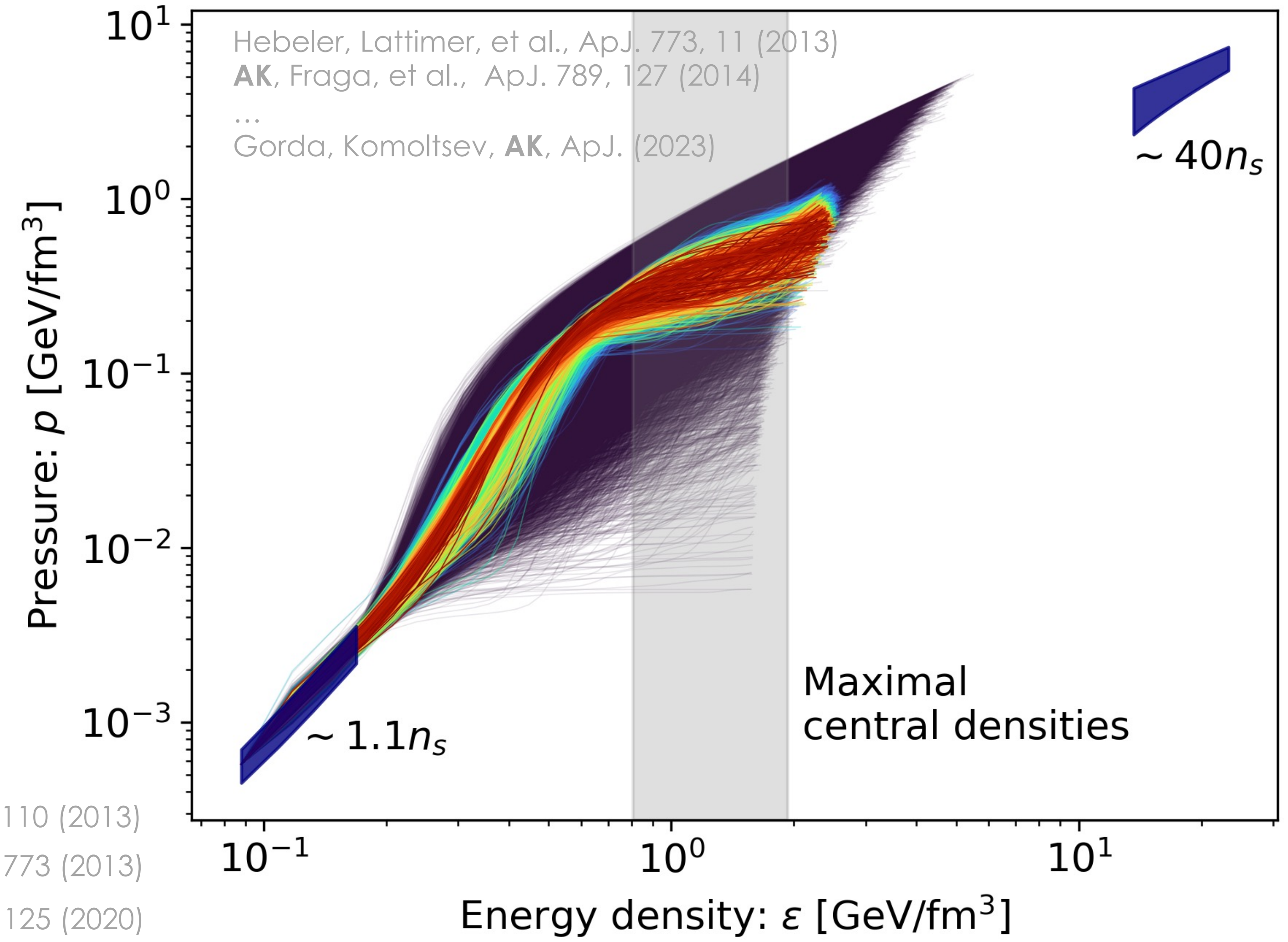
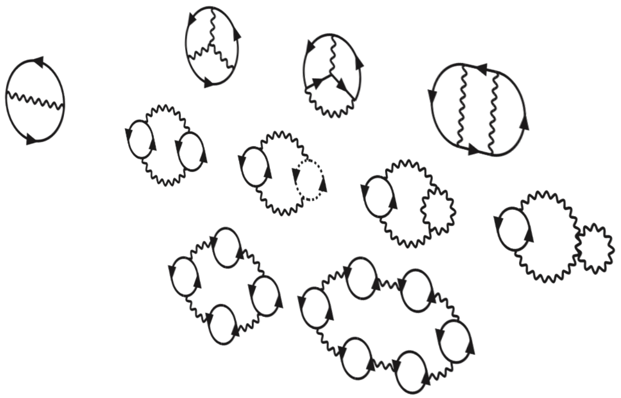
Gorda, **AK** et al. PRL 121 (2018)
Gorda, **AK** et al. PRD 104 (2021)
Gorda, **AK** et al. PRL 127 (2021)
Gorda, **AK** et al. PRD 107 (2023)

Equation of state, infrerence:





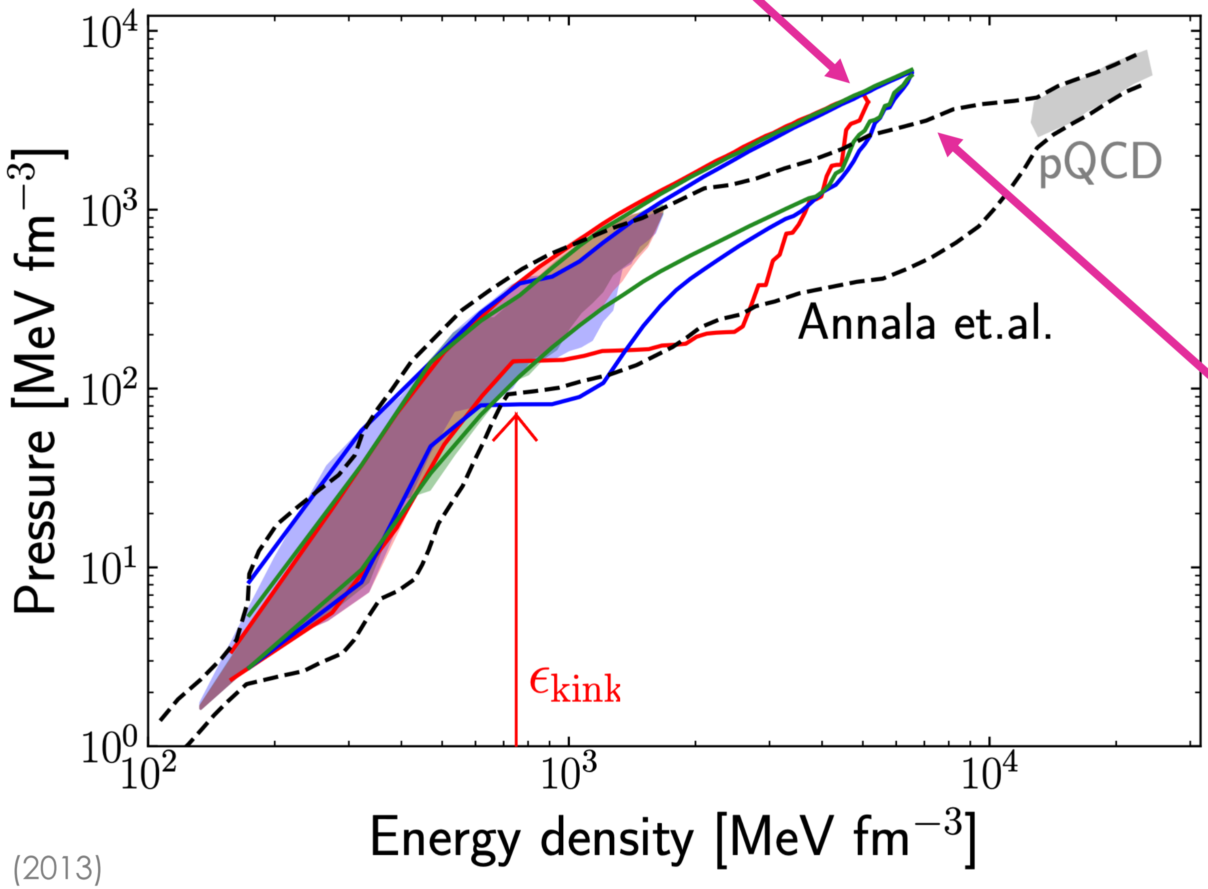
Tews et al. PRL 110 (2013)
Hebeler, Lattimer et.al. APJ 773 (2013)
Drischler, Furnstahl et.al. PRL 125 (2020)

Freedman, McLerran, PRD 16 (1977)
AK et al. PRD 81 (2010)
AK, Vuorinen PRL 117 (2016)
Gorda, **AK** et al. PRL 121 (2018)
Gorda, **AK** et al. PRD 104 (2021)
Gorda, **AK** et al. PRL 127 (2021)
Gorda, **AK** et al. PRD 107 (2023)

Is **pQCD** of any use in Neutron Stars?

Studies with pQCD see **softening** of EoS

No evidence of **softening** without QCD



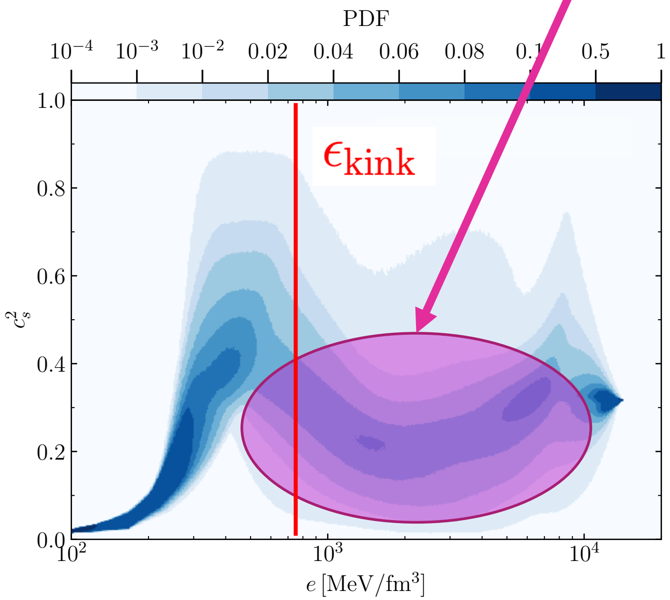
softening with pQCD

Hebeler, Lattimer, et al., ApJ. 773, 11 (2013)
AK, Fraga, et al., ApJ. 789, 127 (2014)
...
Annala, Gorda, **AK**, Nätttilä, Vuorinen Nature Physics 16 (2020) 9
Somasundaram, Tews, Margueron PRC 107 (2023)

Studies with pQCD see **softening** of EoS

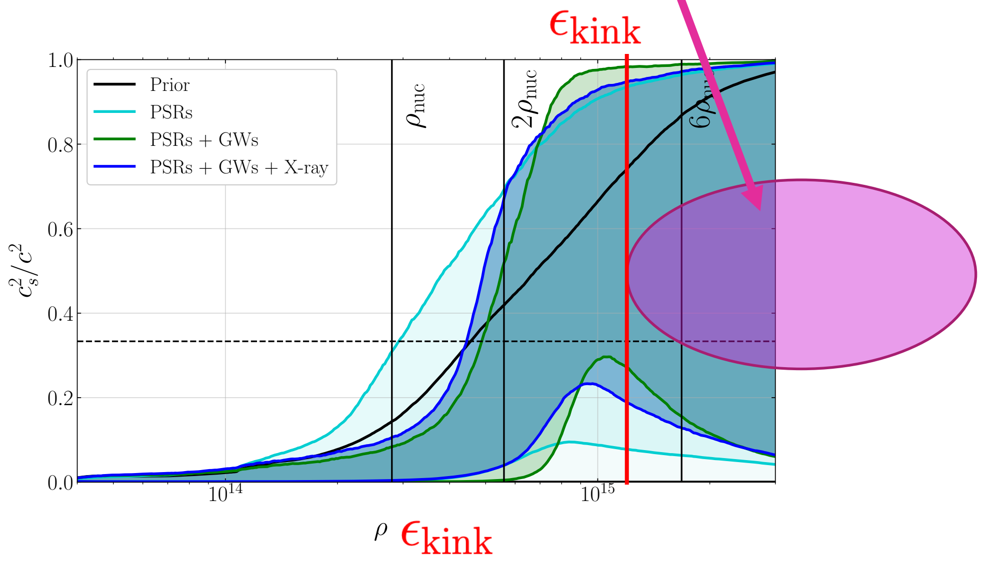
Softening with pQCD

Speed of sound



No evidence of **softening** without pQCD

Speed of sound



Annala, Gorda, Kurkela, Nätttilä, Vuorinen Nature Physics 16 (2020) 9
Altiparmak, Ecker, Rezzolla 2112.08157
Also: Han, Huang, Tang & Yi-Zhong Fan 2207.13613,
Marczenko, McLerran, Redlich & Sasaki 2207.13059
…

Landry, Essick, Chatziioannou PRD 101 (2020)
Somasundaram, Tews, Margueron 2112.08157
…

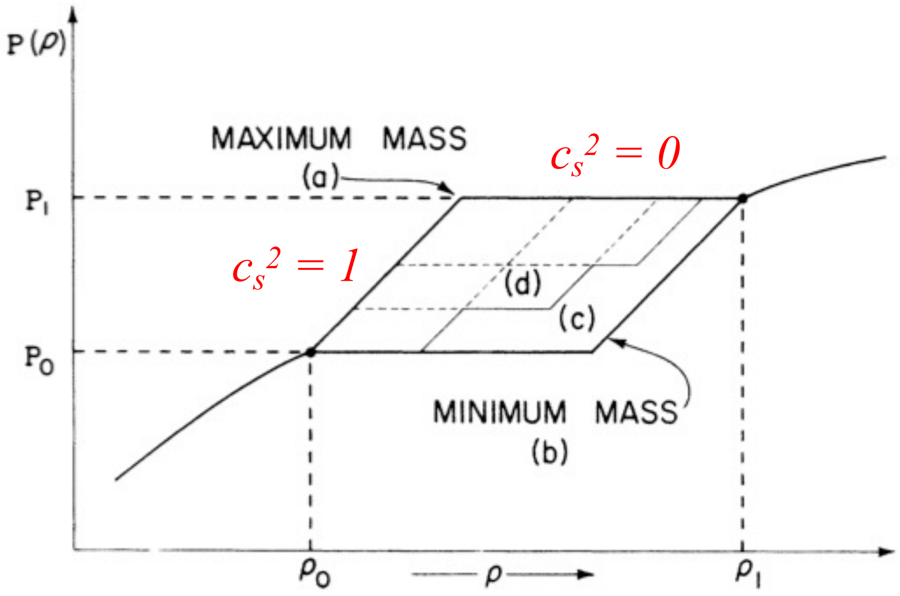
Is the **softening** an aftefact of the interpolation?

…or a robust prediction of QCD?

Robust EoS constraints:

General considerations:

- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$



Rhoades & Ruffini, Phys.Rev.Lett. 32 (1974)
Lope-Oter, Windisch, Llanes-Estrada, Alford, *J. Phys. G (2019)*
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)

Robust EoS constraints:

General considerations:

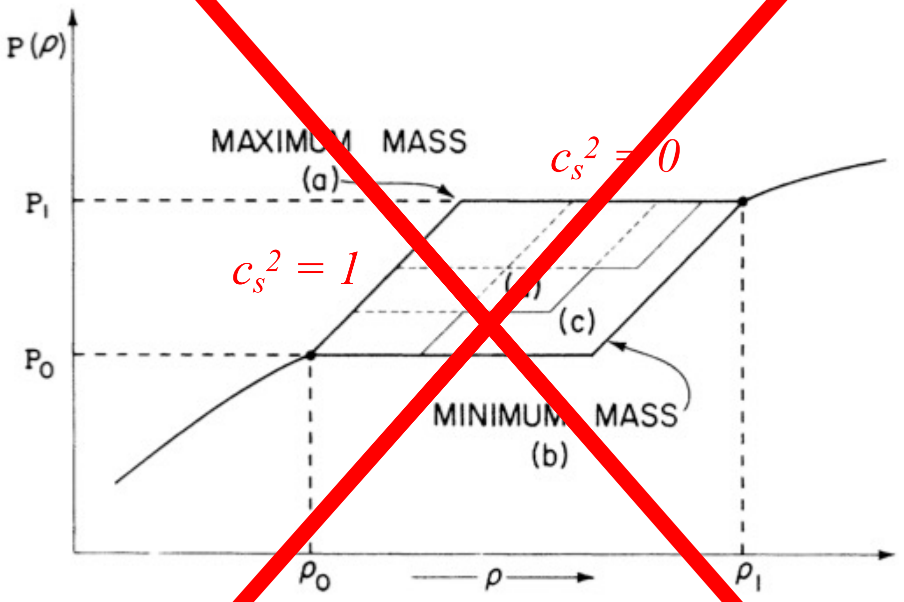
- Mechanical stability: $c_s{}^2 > 0$
- Causality: $c_s{}^2 < 1$
- Consistency:

$$P(\epsilon) \;\; vs. \;\; \Omega(\mu)$$

Reduced EoS

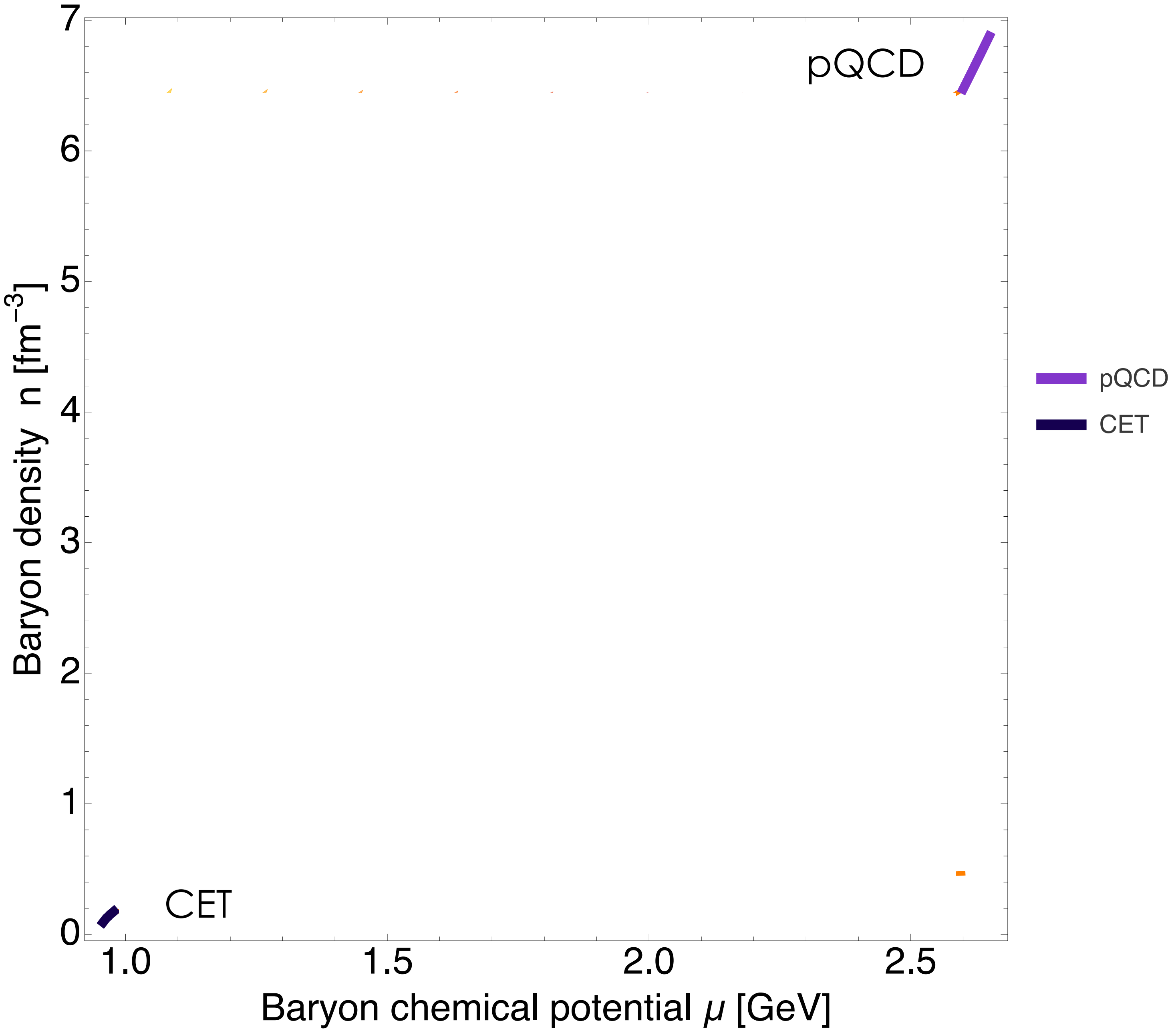
Full EoS

Information of $\{P, \epsilon, n\}$



Rhoades & Ruffini, Phys.Rev.Lett. 32 (1974)
Lope-Oter, Windisch, Llanes-Estrada, Alford, *J. Phys. G (2019)*
Lope-Oter, Llanes-Estrada, EPJA 58 (2022)

EoS constraints:

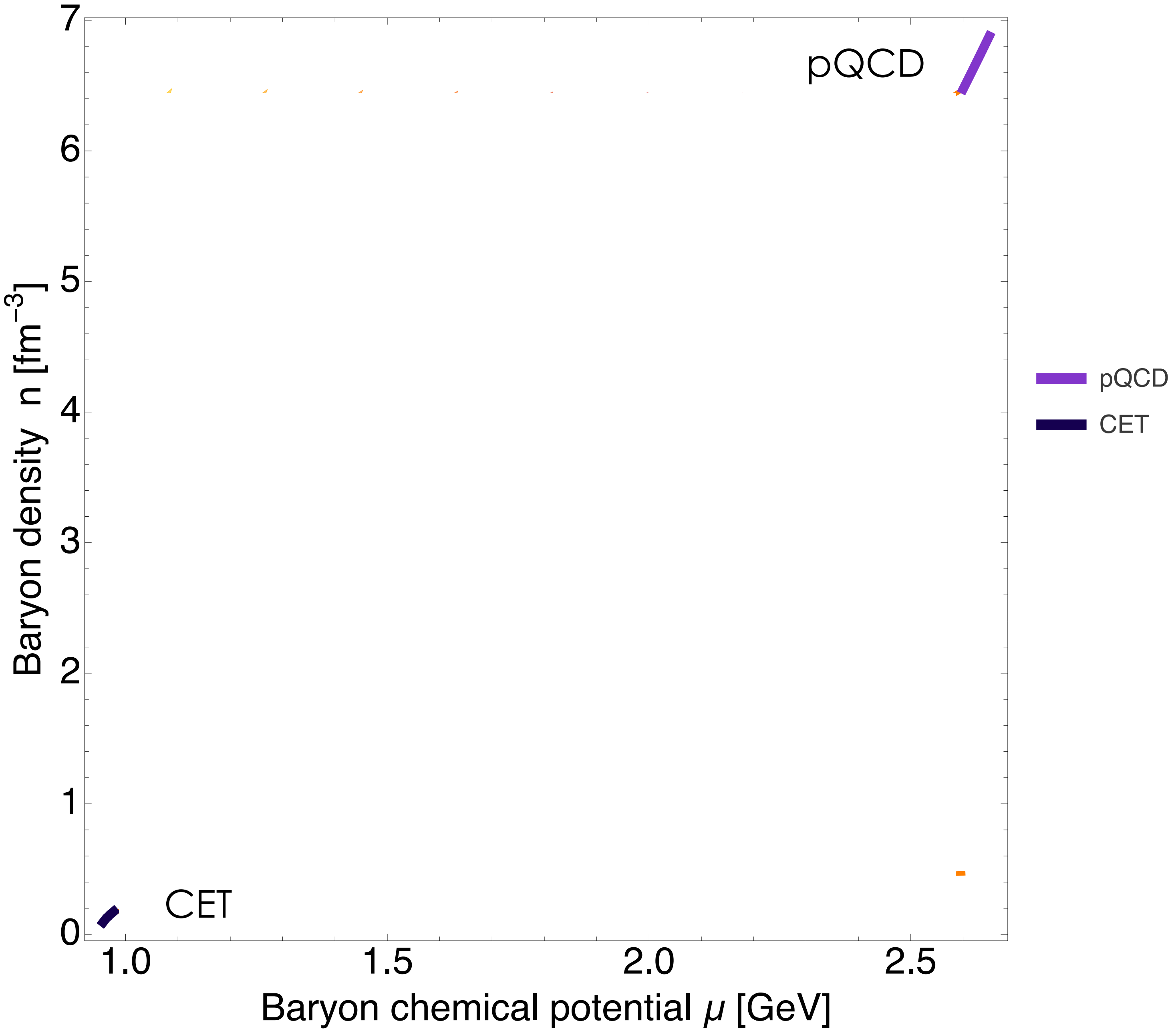


Komoltsev & **AK**, PRL128 (2022)

EoS constraints:

- Stability

$n(\mu)$ single valued, monotonic



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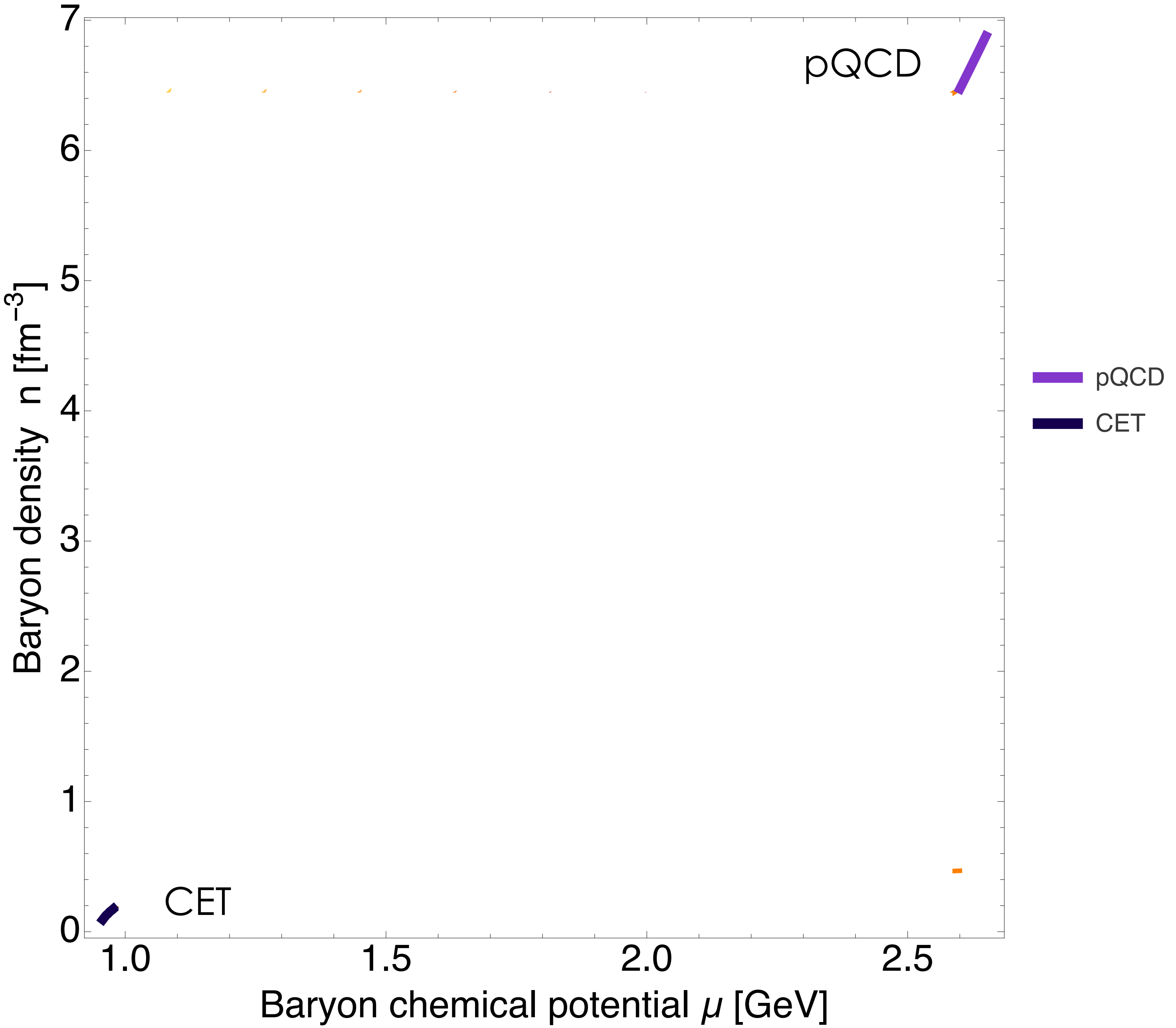
$$c_s^{-2} = \frac{\mu}{n}\frac{\partial n}{\partial \mu} \geq 1 \quad \Rightarrow \quad \partial_\mu n(\mu) \geq \frac{n}{\mu}$$



Komoltsev & **AK**, PRL128 (2022)

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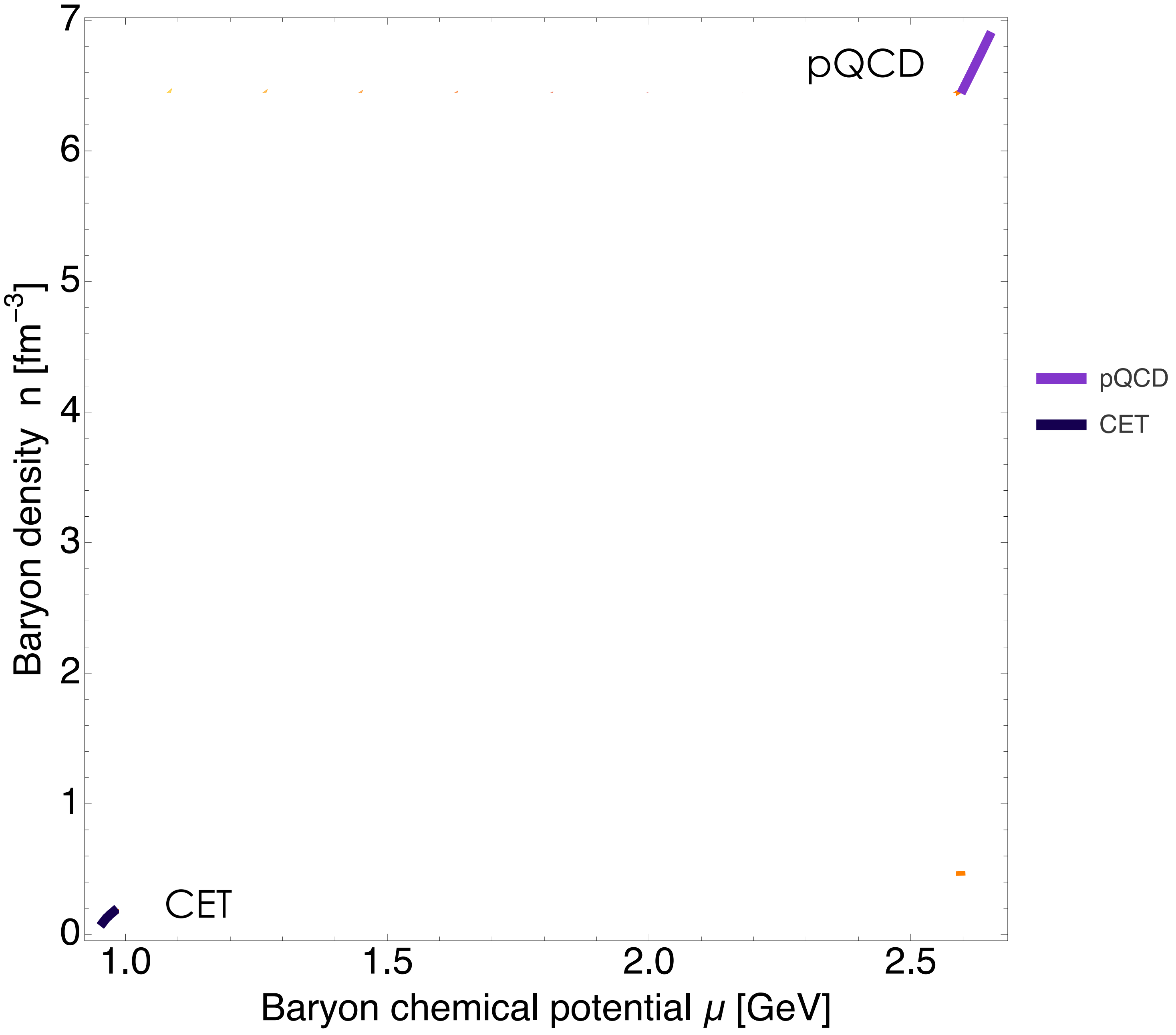
- Consistency

$$\int_{\mu_{CET}}^{\mu_{QCD}} n(\mu)\, d\mu = p_{QCD} - p_{CET} = \Delta p$$



Komoltsev & **AK**, PRL128 (2022)

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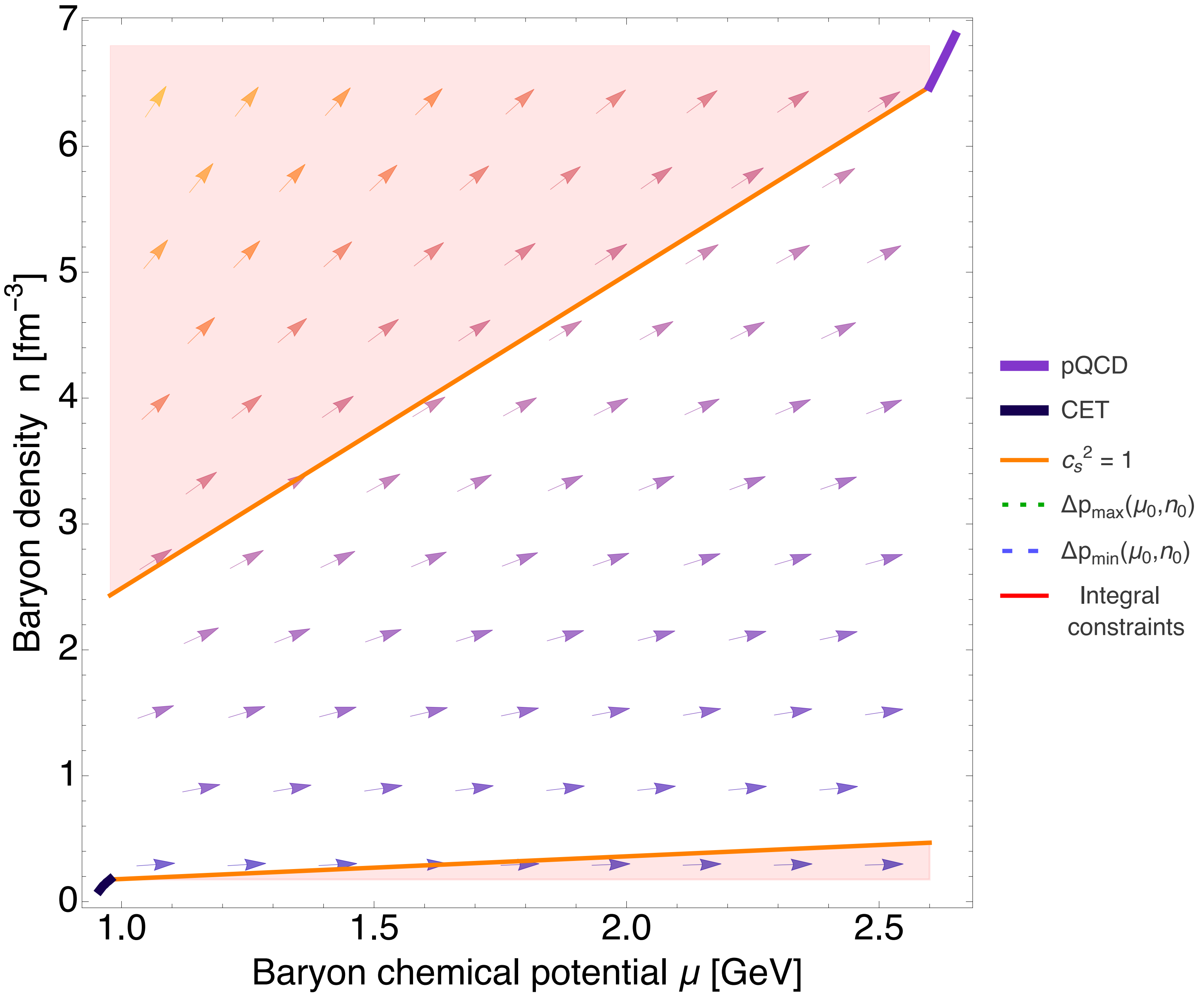

Komoltsev & **AK**, PRL128 (2022)

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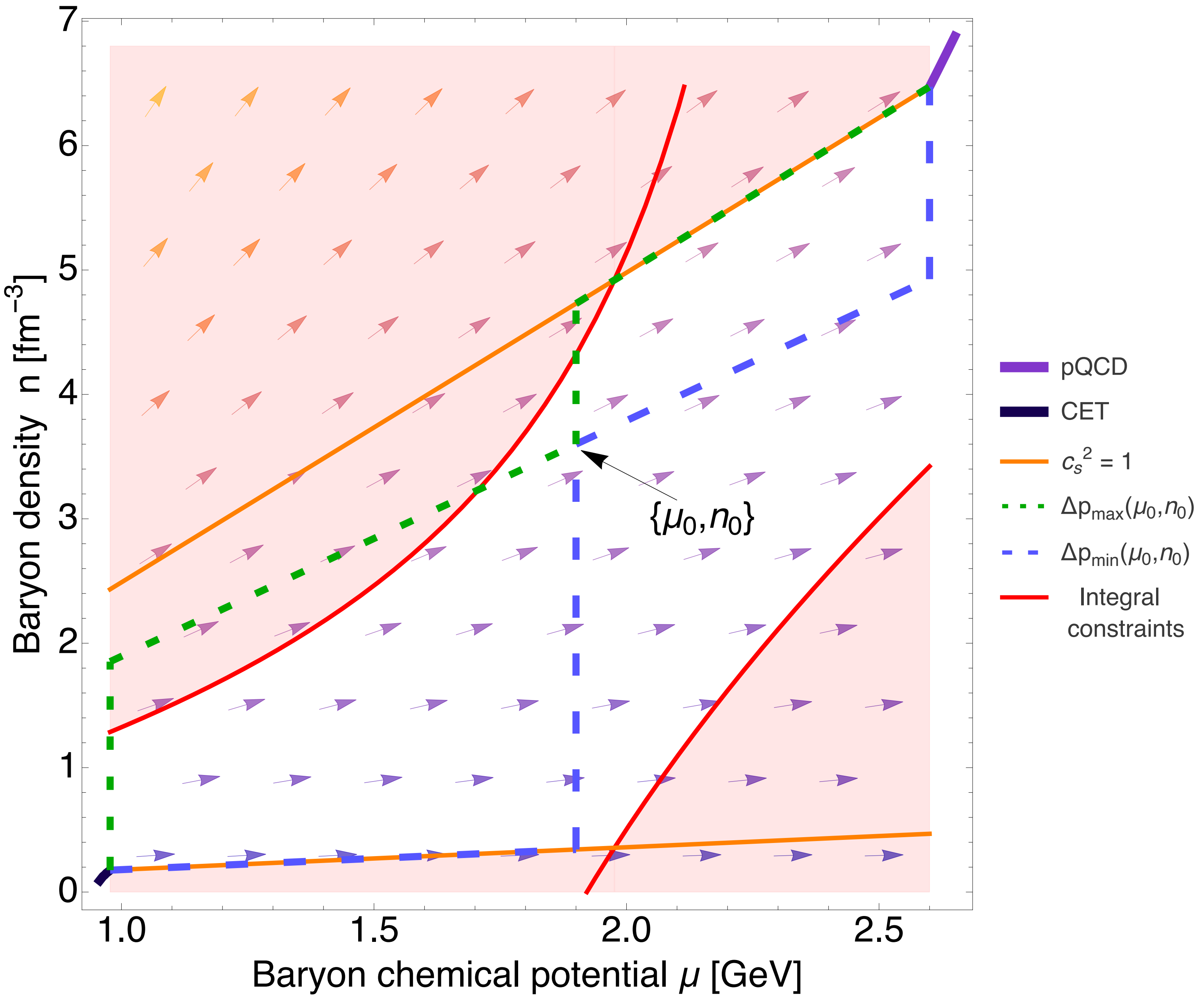

Komoltsev & **AK**, PRL128 (2022)

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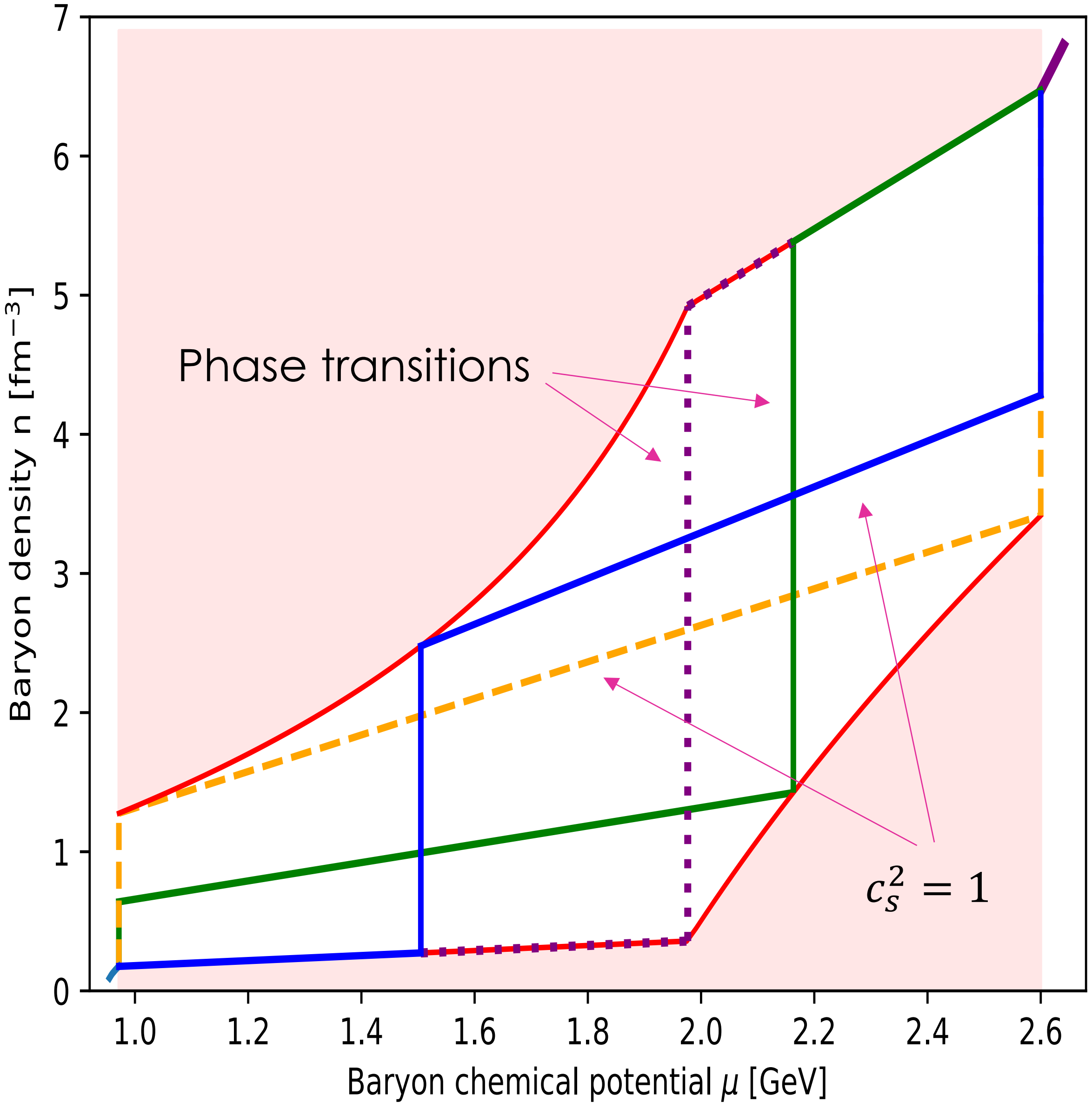
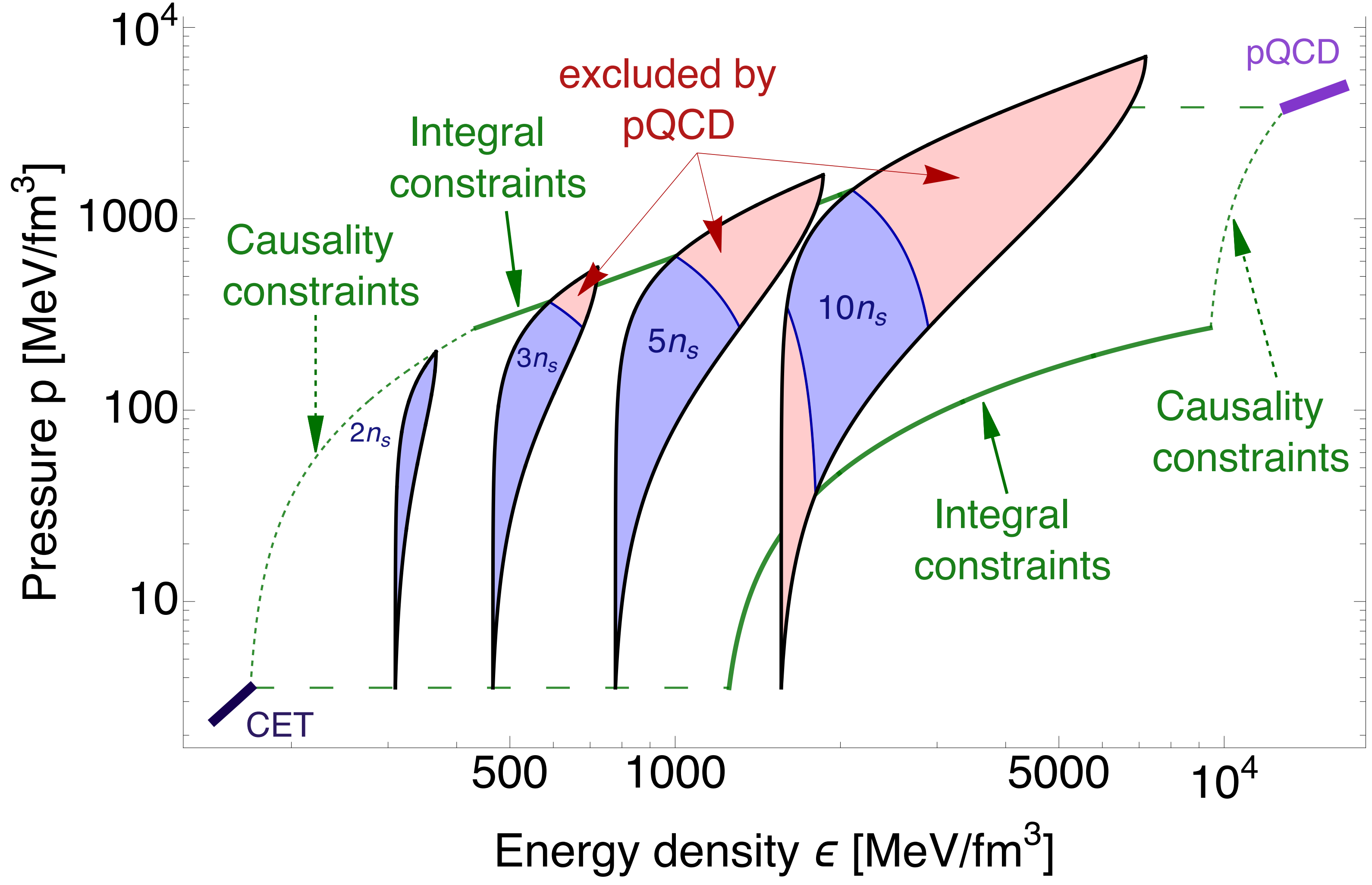


Komoltsev & **AK**, PRL128 (2022)

Constraints for fixed n on $\epsilon - p$ -plane

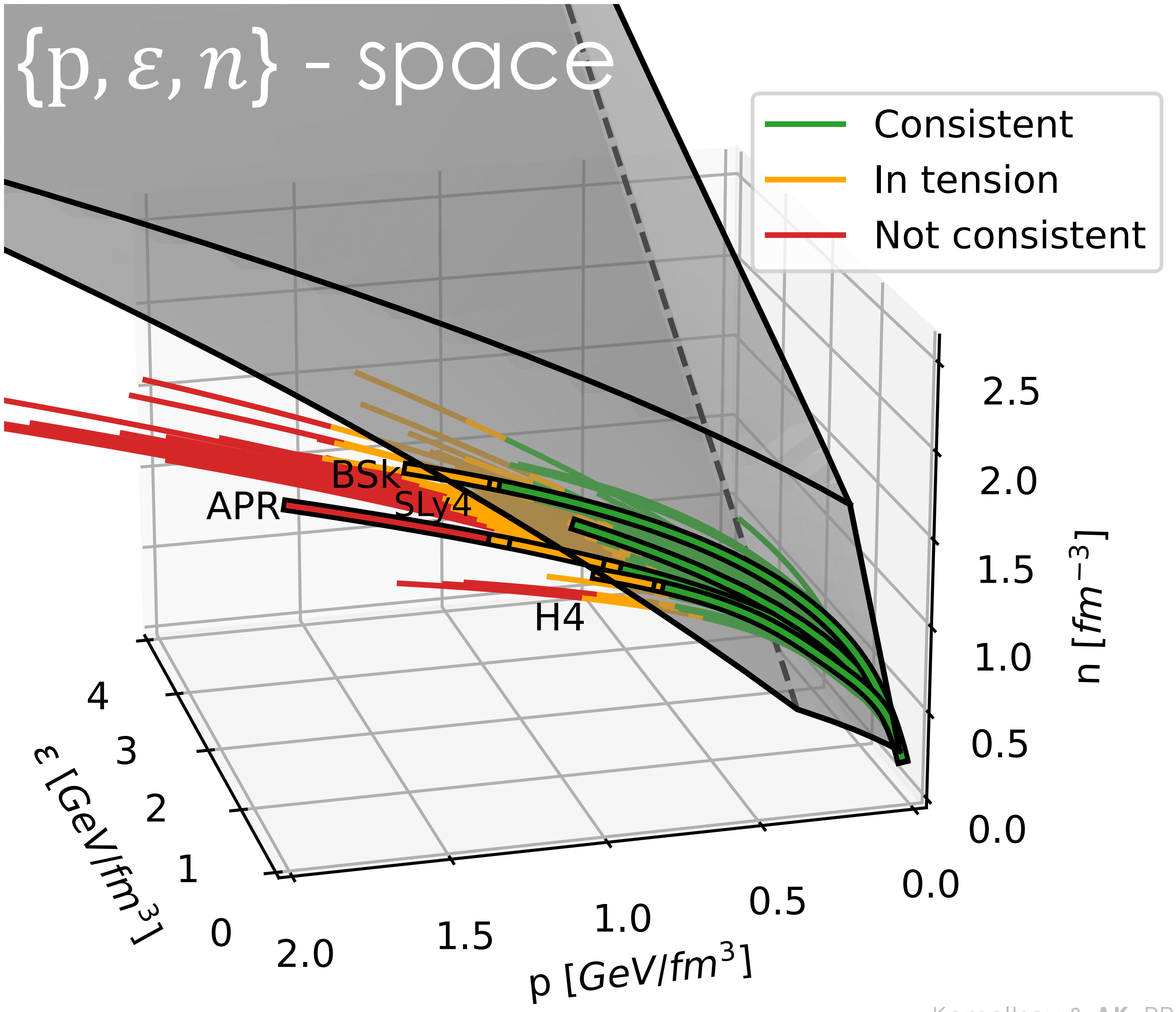


Komoltsev & **AK**, PRL128 (2022)

Does any of this matter?

Constraints in $\{p, \varepsilon, n\}$ - space

Models from CompOSE database



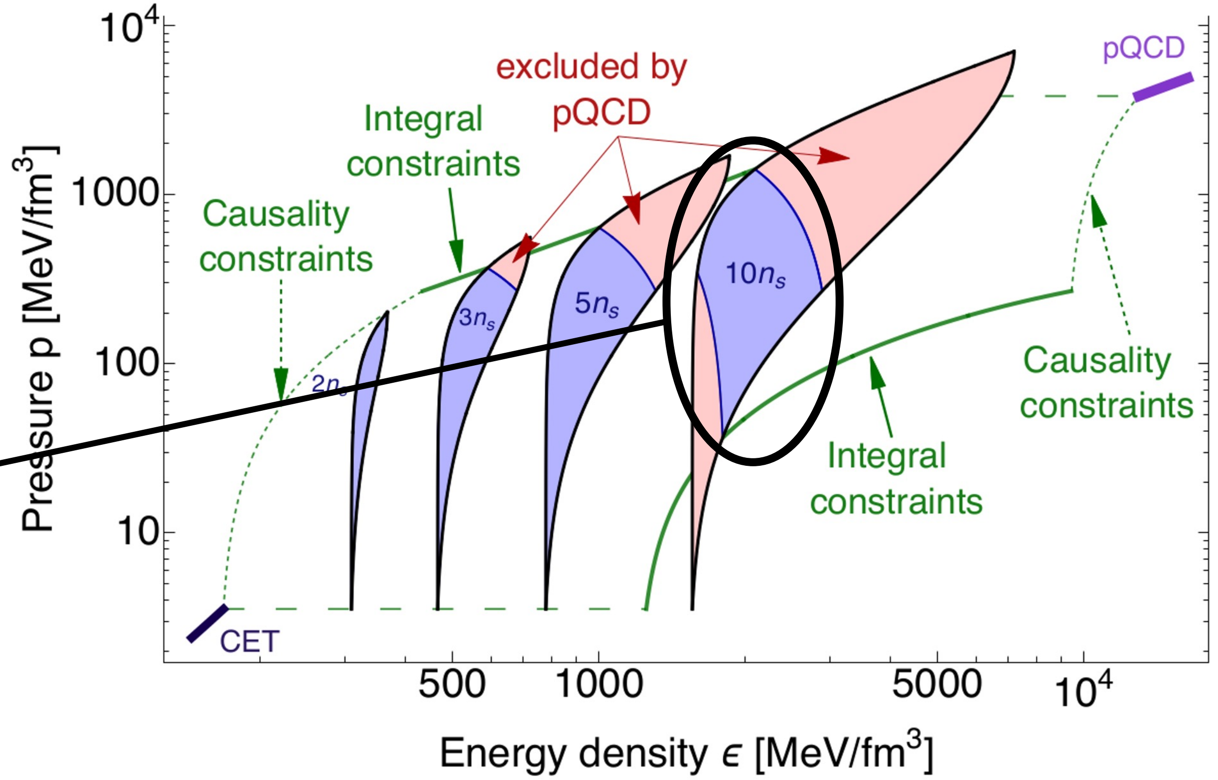
Komoltsev & **AK**, PRL128 (2022)

pQCD and Equation-of-State inference

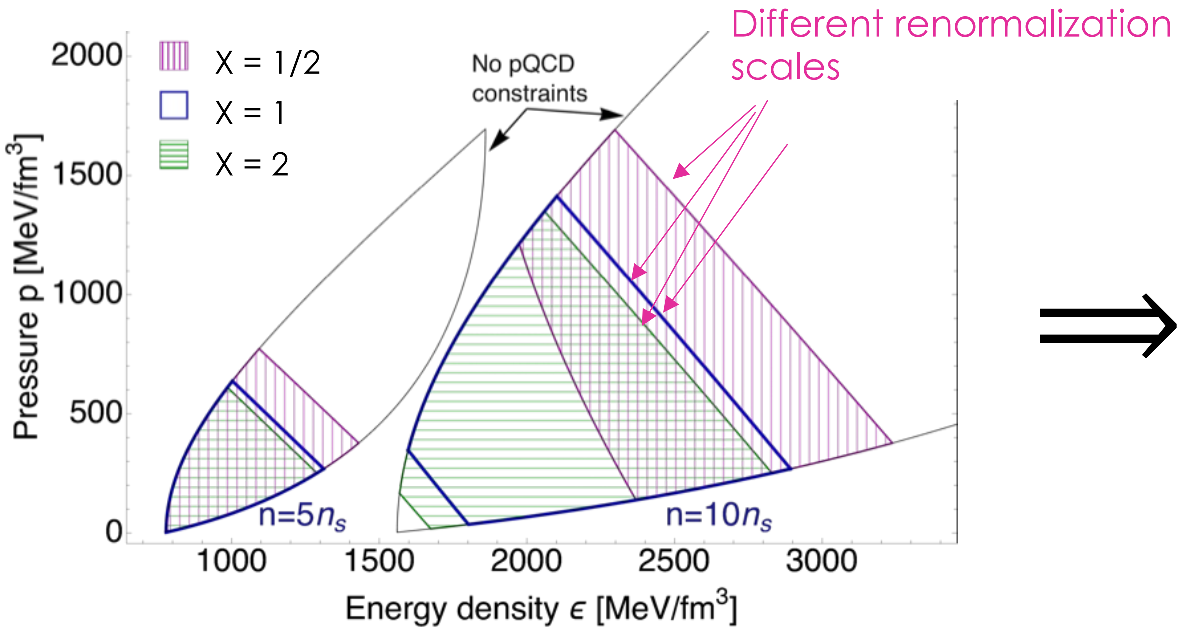
Implementing pQCD to EoS inference:

- Inference setup where QCD can be turned on/off
- Easily implmemented to any other extrapolation setup

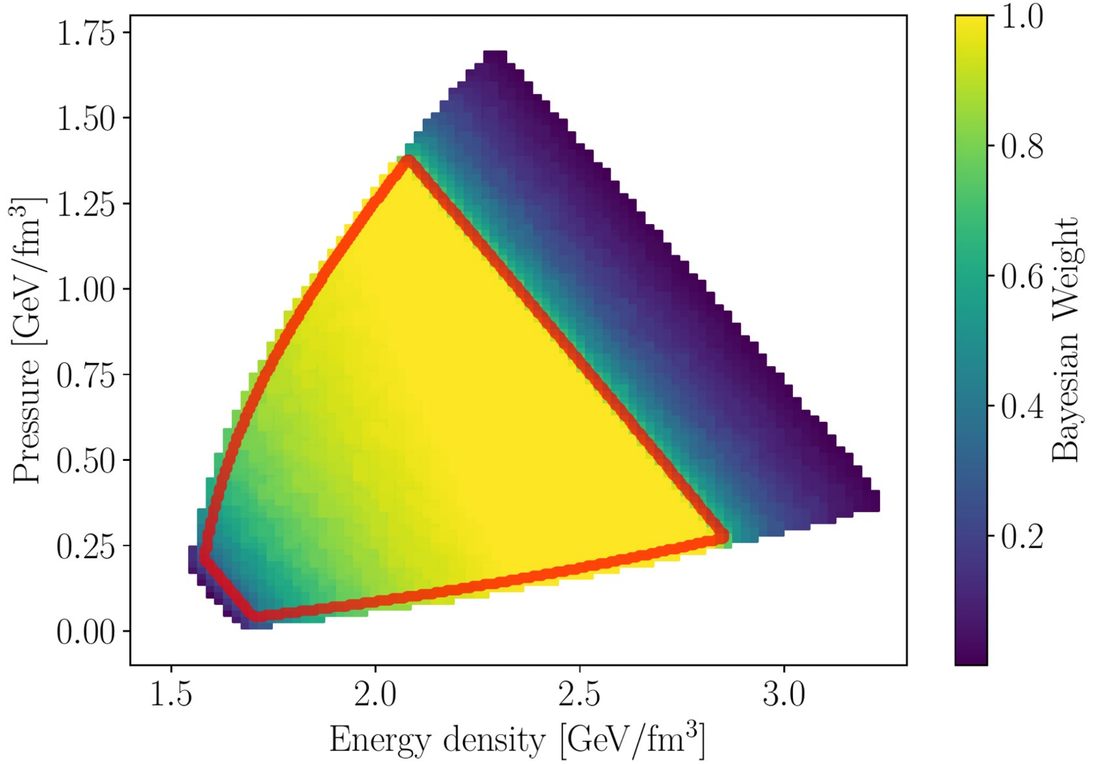
Use this area to condition an extrapolation



QCD likelihood function:



$\Longrightarrow$



- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors

  Cacciari & Houdeau, JHEP 09, (2011), Duhr et al. JHEP 122, (2021), Gorda, Komoltsev, AK, Mazeliauskas, 2303.02175

  - Perturbative series modelled as draw from statistical model that is trained with the available terms

Using **QCD likelihood function** in EoS inference

Implementing pQCD to EoS inference:

- Bayesian inference setup:

Prior

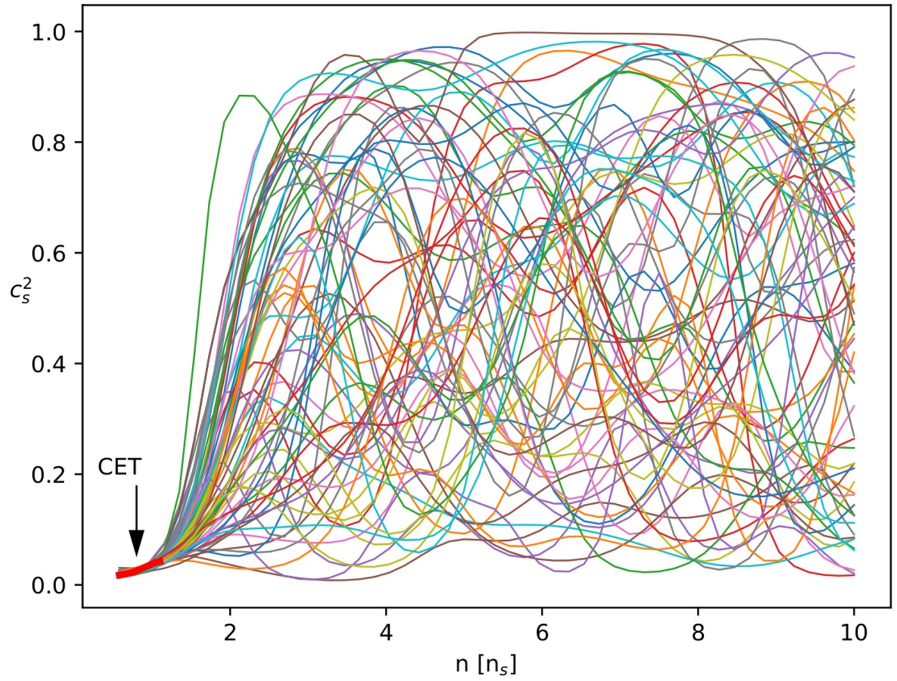
Likelihood of EoS
given data

$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})\,P(\mathrm{data}\,|\,\mathrm{EoS}\,)}{P(\mathrm{data})}$$

Implementing pQCD to EoS inference:

- **Gaussian-process** based inference:

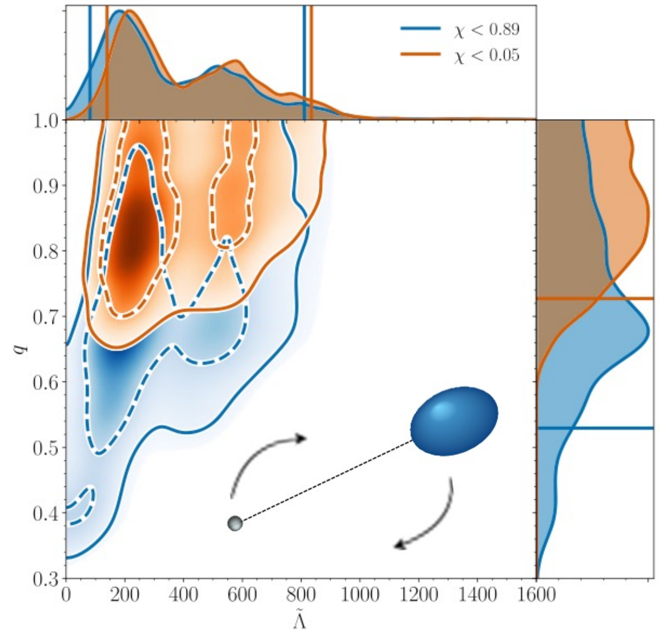
  Similar to Landry & Essick PRD 99 (2019), but for function of n instead of ε



$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})\,P(\mathrm{data}\,|\,\mathrm{EoS})}{P(\mathrm{data})}$$

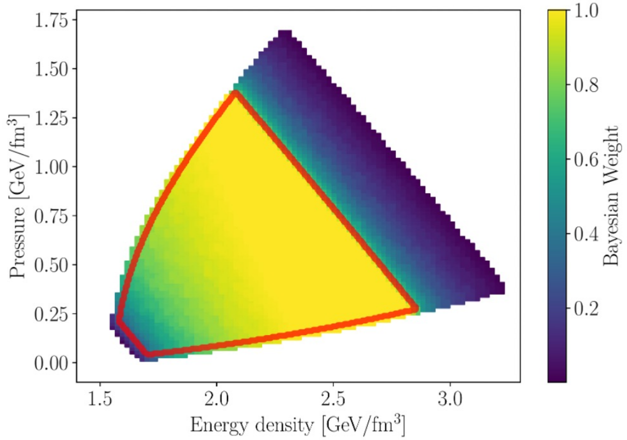
Setup:

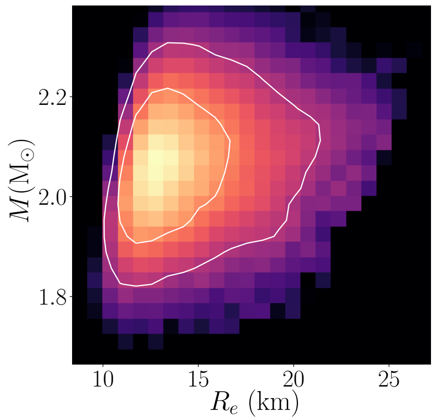
$$P(\text{EoS}\,|\,\text{data}) = \frac{P(\text{EoS})\,P(\text{data}\,|\,\text{EoS})}{P(\text{data})}$$



$M_{J1614\text{-}2230}$=1.908(16)
$M_{J048+0432}$ = 2.01(4)
$M_{J0740+6620}$ = 2.14(10)

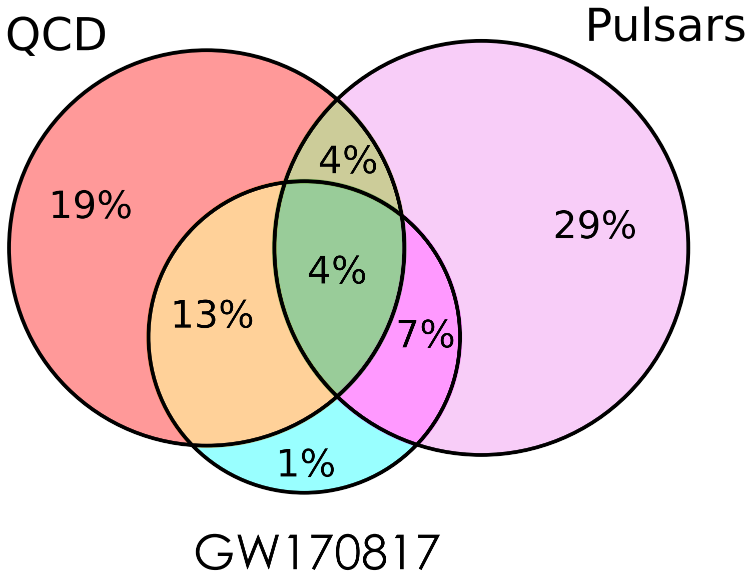
$$P(\text{data}\,|\,\text{EoS}) = P(\text{QCD}\,|\,\text{EoS})P(\text{Mass}\,|\,\text{EoS})P(\text{NICER}\,|\,\text{EoS})P(\widetilde{\Lambda},\text{BH}\,|\,\text{EoS}).$$

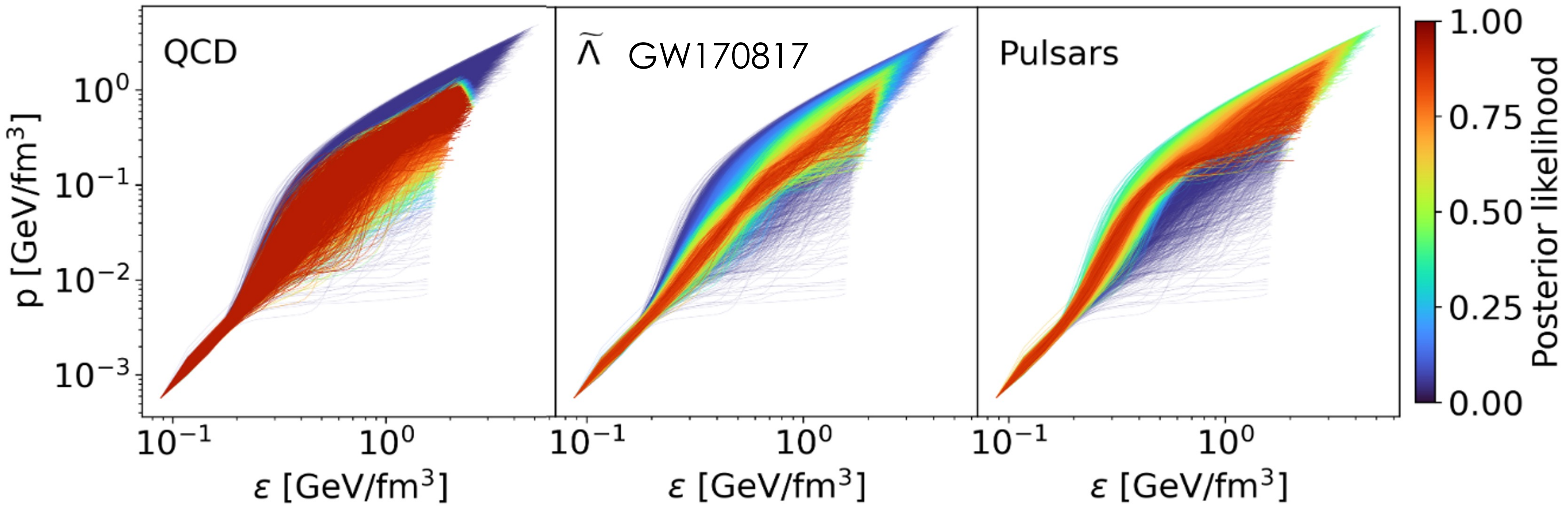




Effect of QCD:

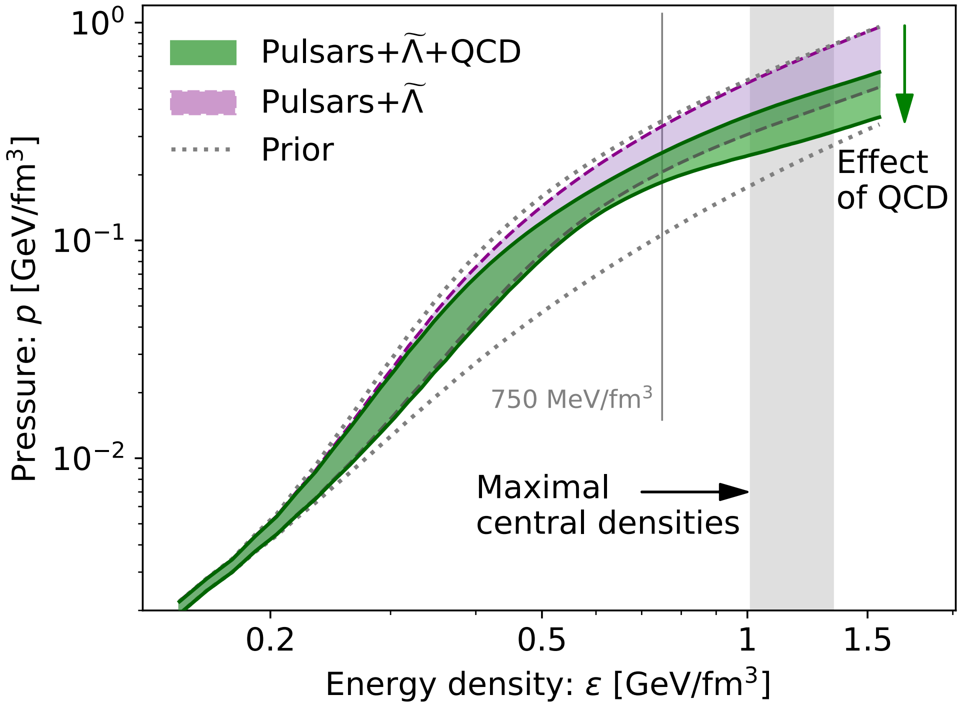
QCD input complements NS observations

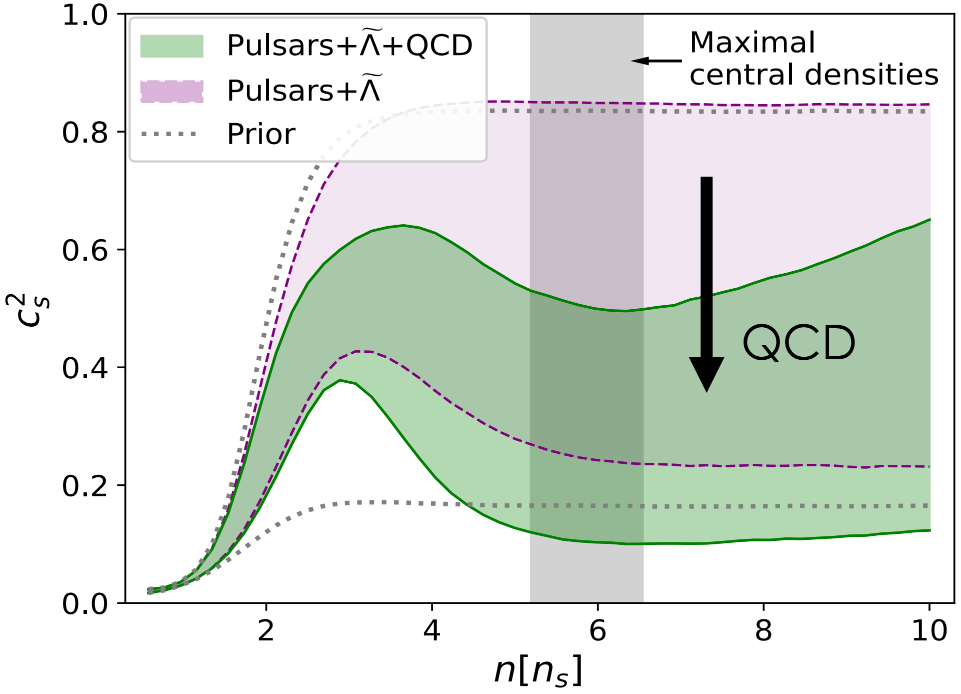




Gorda, Komoltsev & AK APJ (2023)

QCD responsible for **softening**:



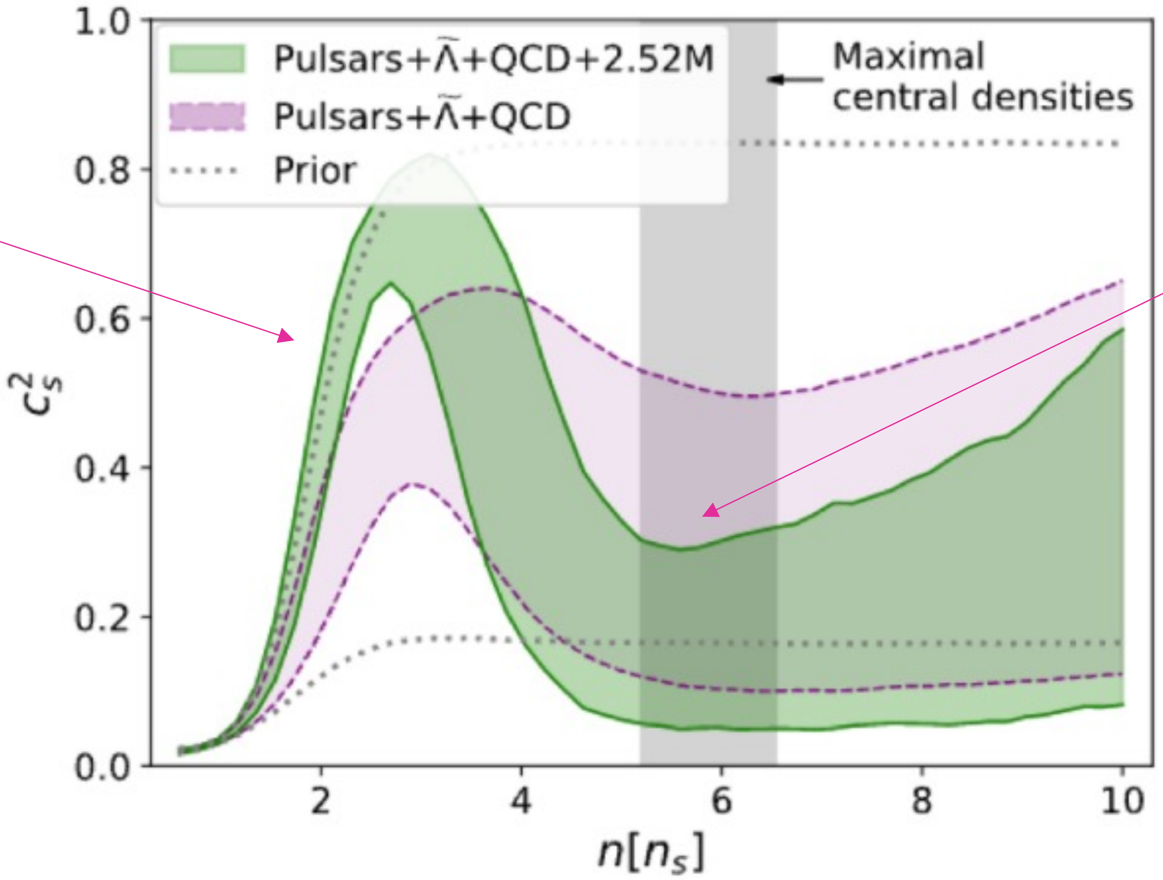


Gorda, Komoltsev, **AK**, Astrophys.J. 950 (2023)
Komoltsev, Somasundaram, Gorda, **AK**, Margueron, Tews, PRD 109 (2024)

QCD responsible for **softening**:

- **Hypothetical** observation of a very massive star $M = 2.52M_{\odot}$:

Stiff to reach high mass

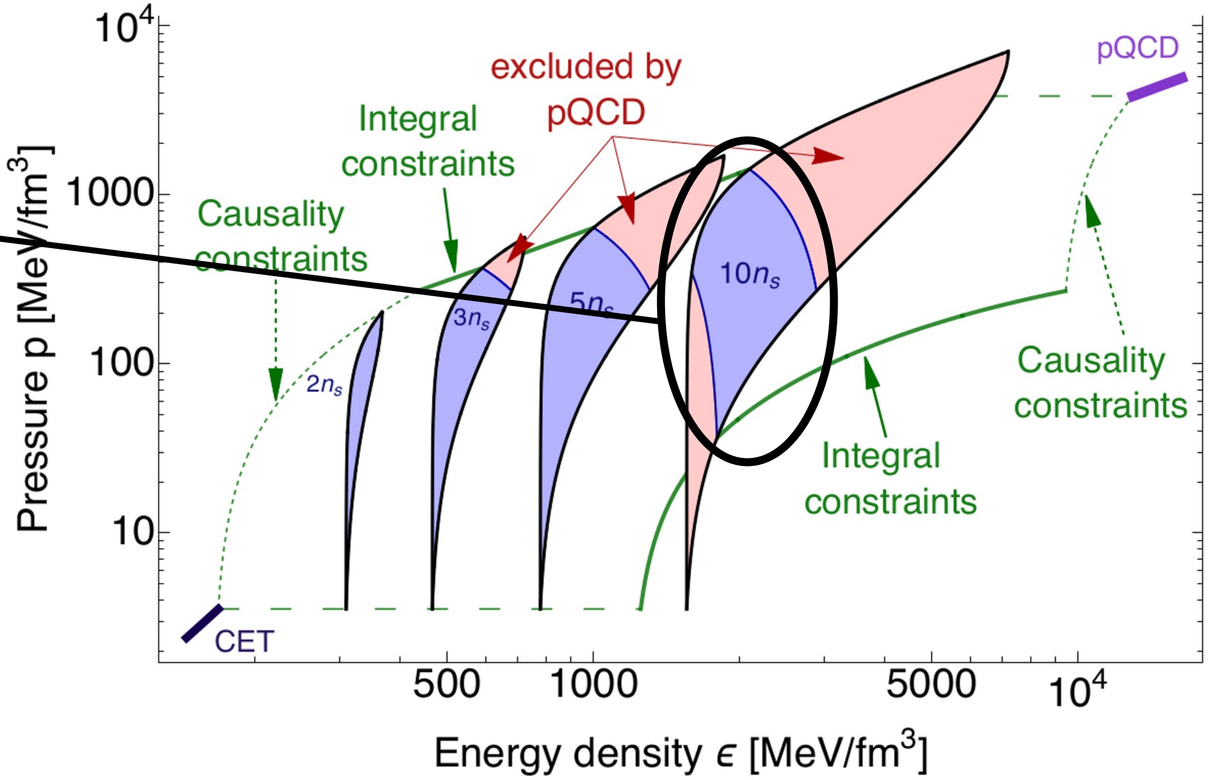


Very soft to have enough pressure to reach pQCD

Caveats?

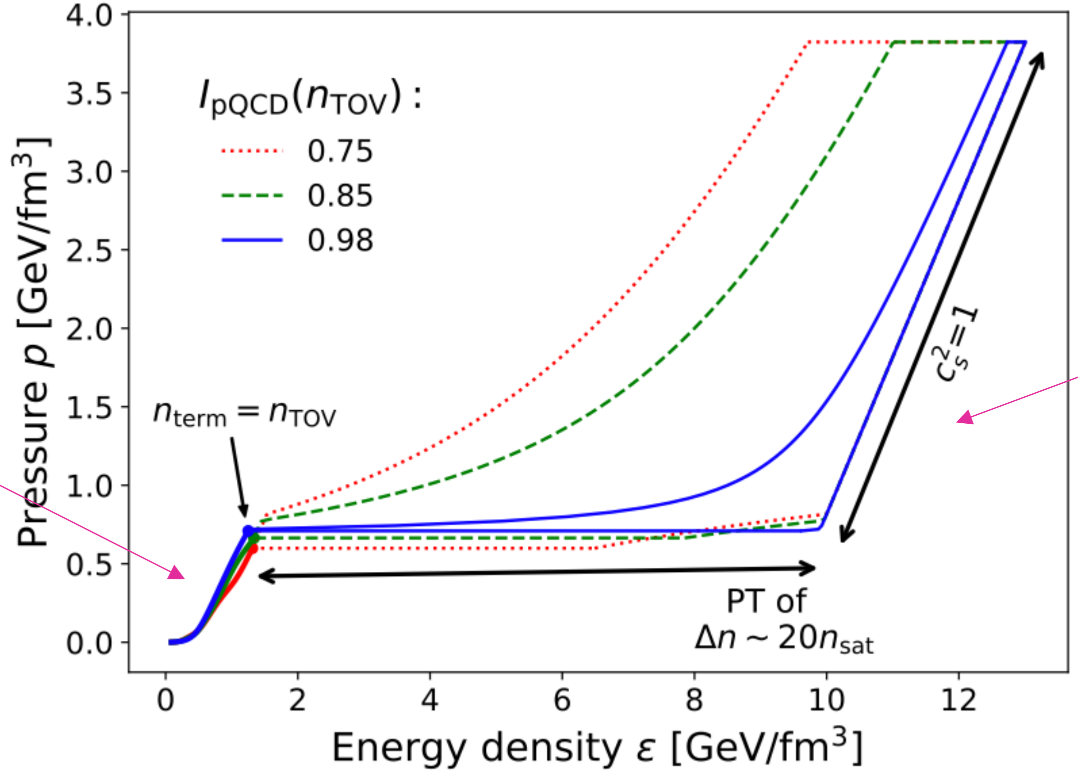
Termination density:

- The termination point n_{term} of Gaussian process is modelling choice



Termination density:

EOS inference sensitive to bias of prior below n_{term}

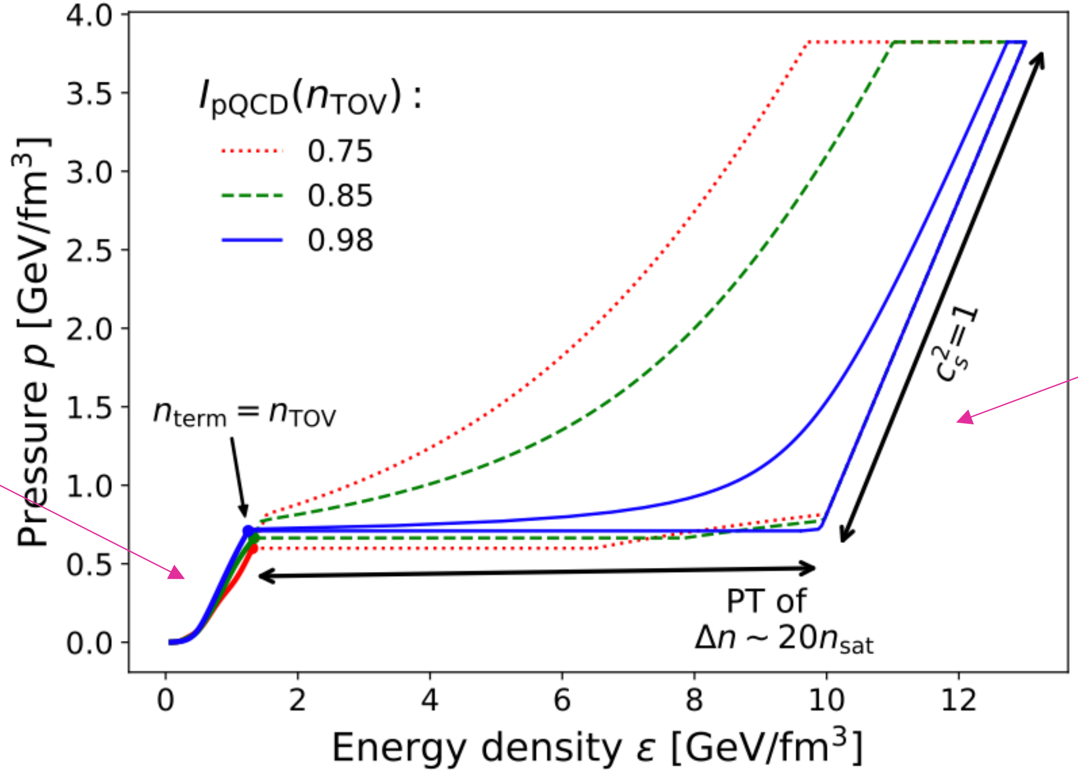


The pQCD construction between $n_{term} < n < n_{QCD}$ is **maximally conservative**

- **Asymmetric** treatment above and below n_{term} leads to sensitivity

Termination density:

EOS inference sensitive to bias of prior below n_{term}

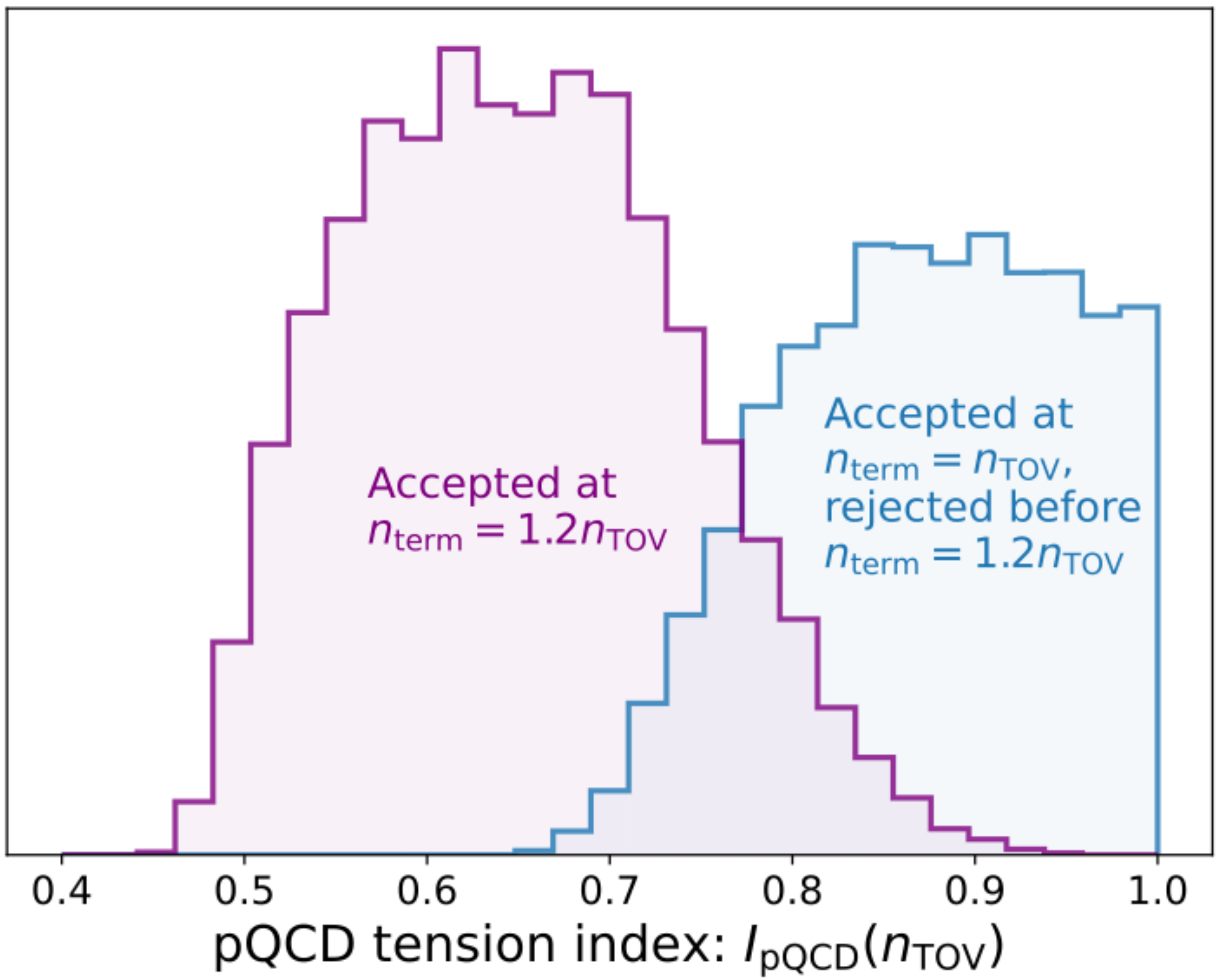


The pQCD construction between $n_{term} < n < n_{QCD}$ is **maximally conservative**

- **Asymmetric** treatment above and below n_{term} leads to sensitivity
- For $n_{term} = n_{\mathrm{TOV}}$ QCD quantitatively less impactful

Somasundaram, Margueron, Tews, PRC107 (2023)
Komoltsev, Somasundaram, Gorda, AK, Margueron, PRD 109 (2024)

Termination density:



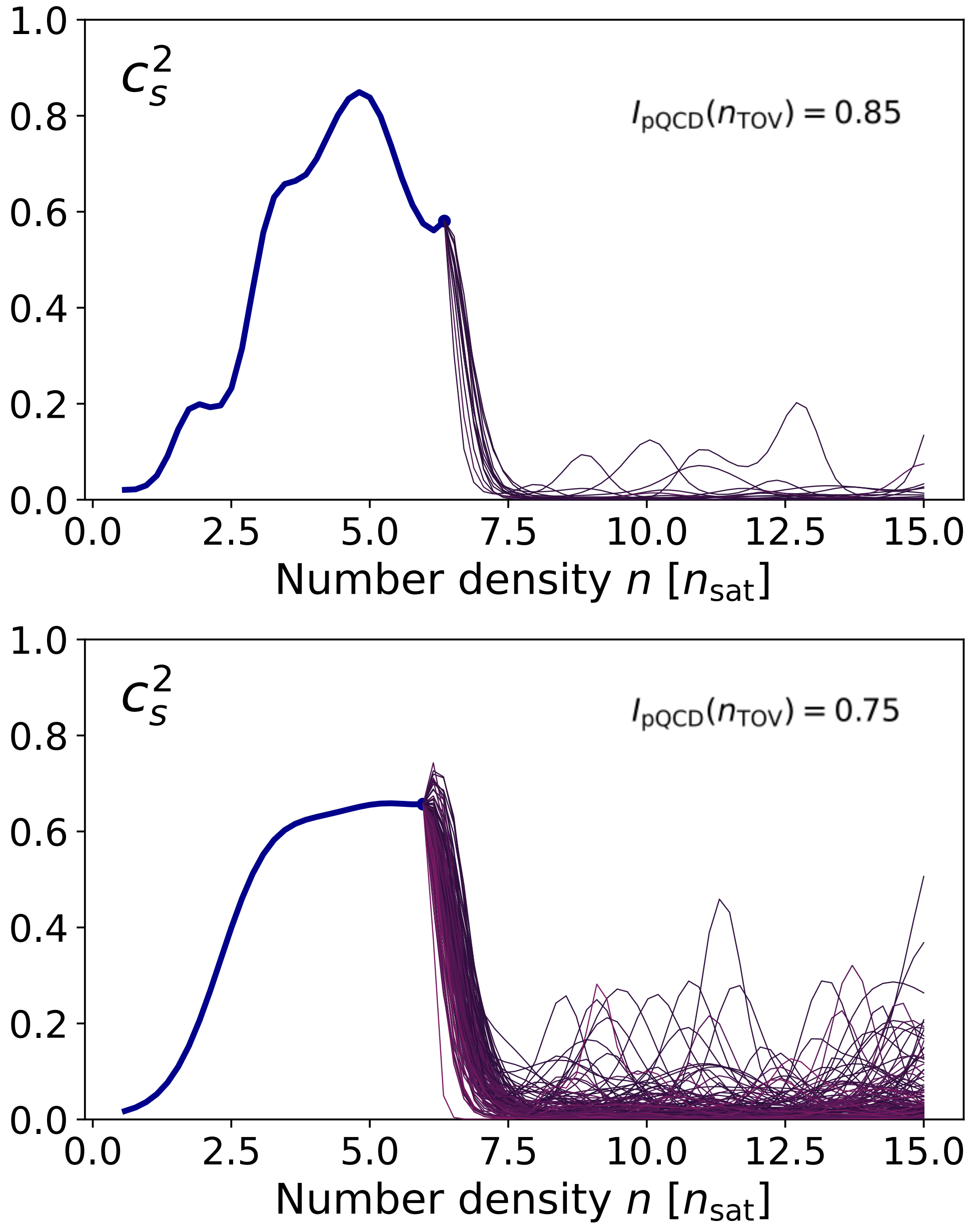
- The EOSs close to the boundary need to have extreme form beyond n_{term}
- Closeness to boundary can be measured

$$I_{pQCD} = \frac{\Delta p - \Delta p_{min}}{\Delta p_{max} - \Delta p_{min}}$$

- The sensitivity arises exclusively from EOSs near boundary

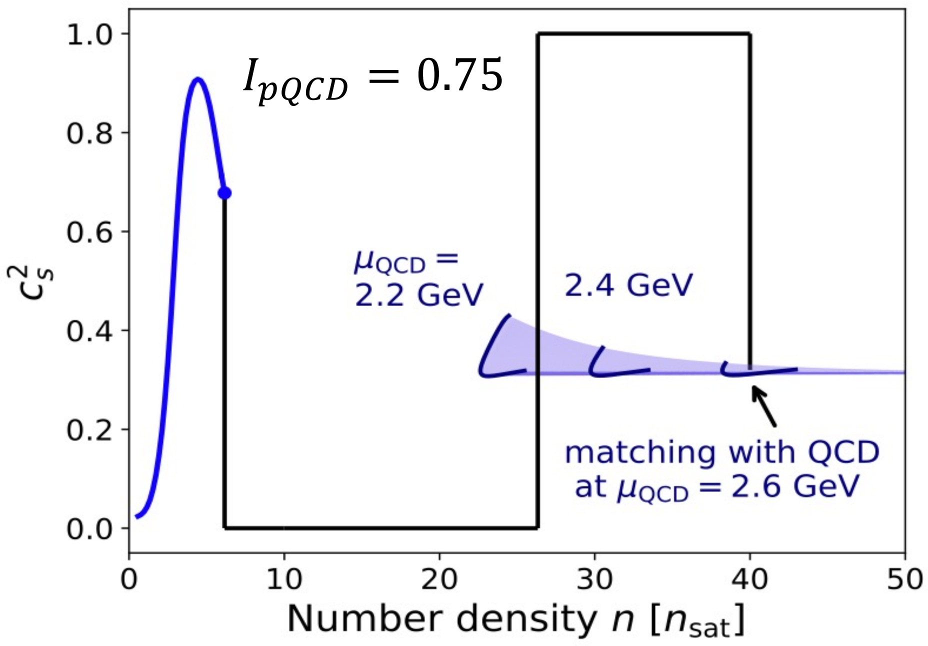
$$I_{pQCD} \gtrsim 0.75$$

Komoltsev, Somasundaram, Gorda, AK, Margueron, PRD 109 (2024)



Termination density:

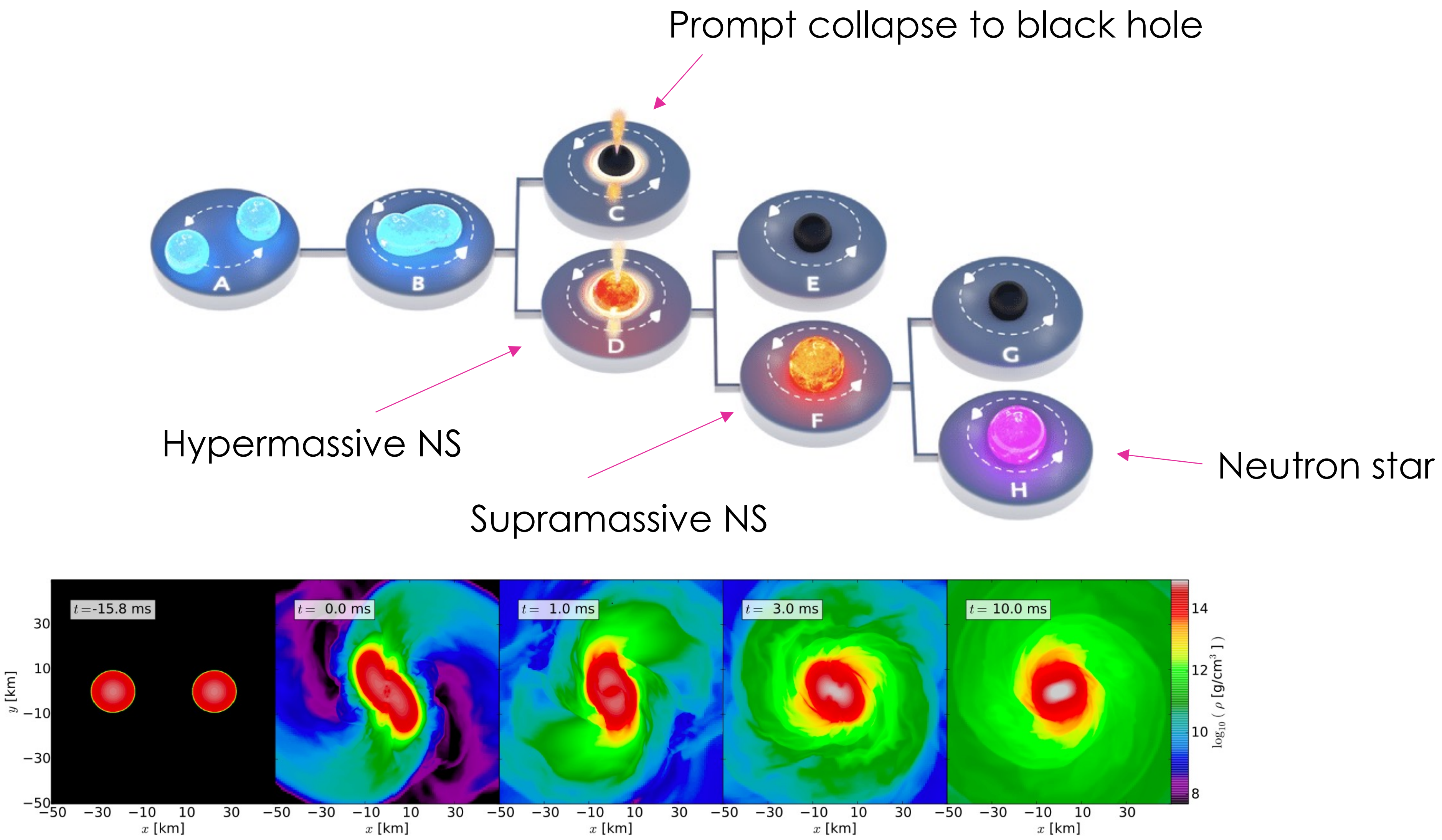
- If the EOS does not soften by $n_{\rm TOV}$, the EOS must be soften **dramatically** above $n_{\rm TOV}$



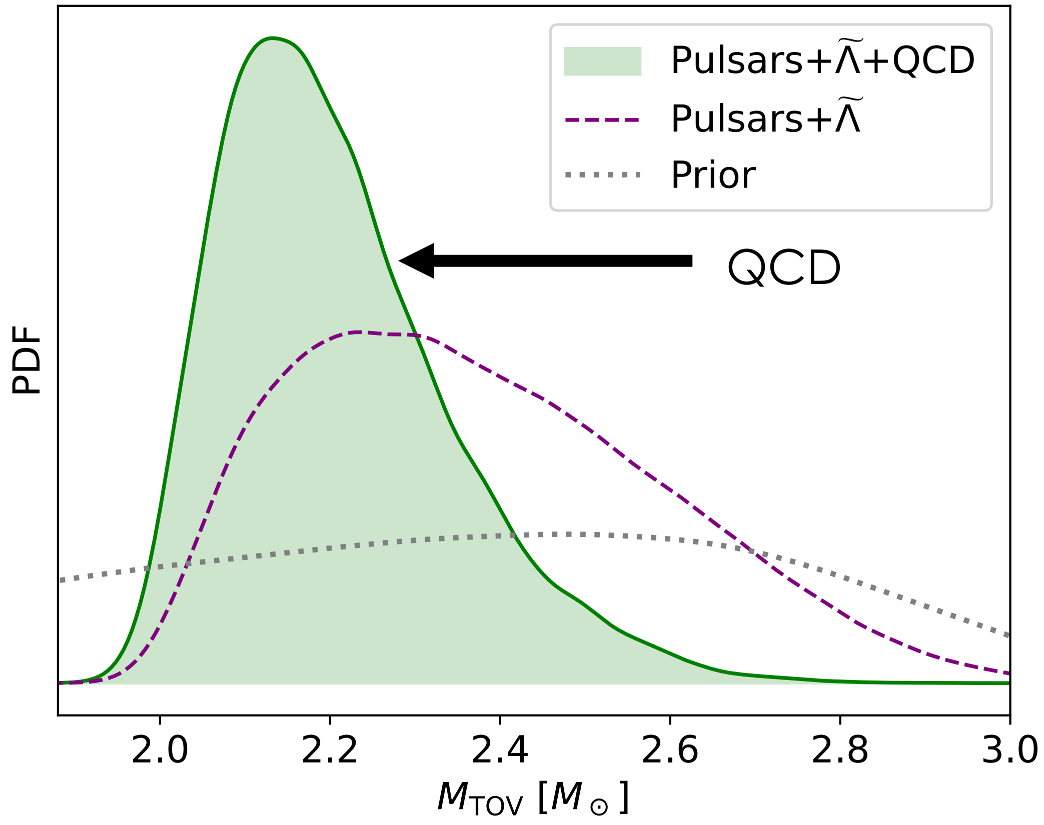
Komoltsev, Somasundaram, Gorda, AK, Margueron, PRD 109 (2024)

Can the **softening** be observationally verified?

Different binary merger products:

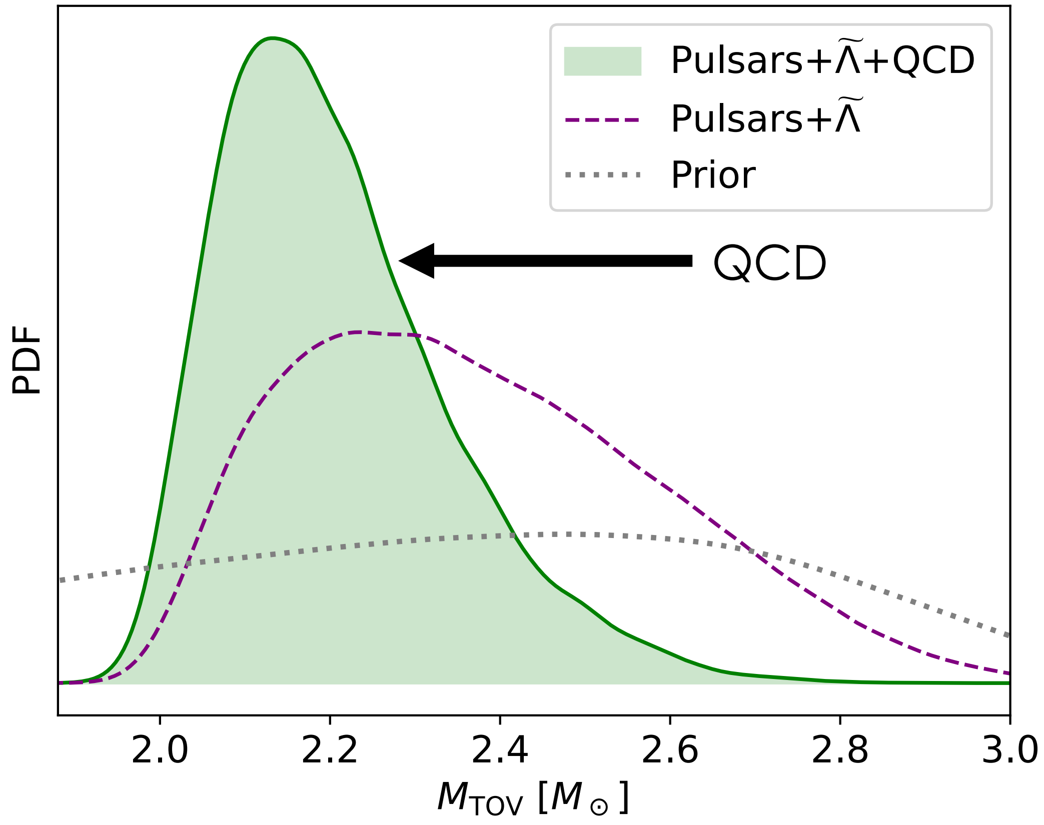


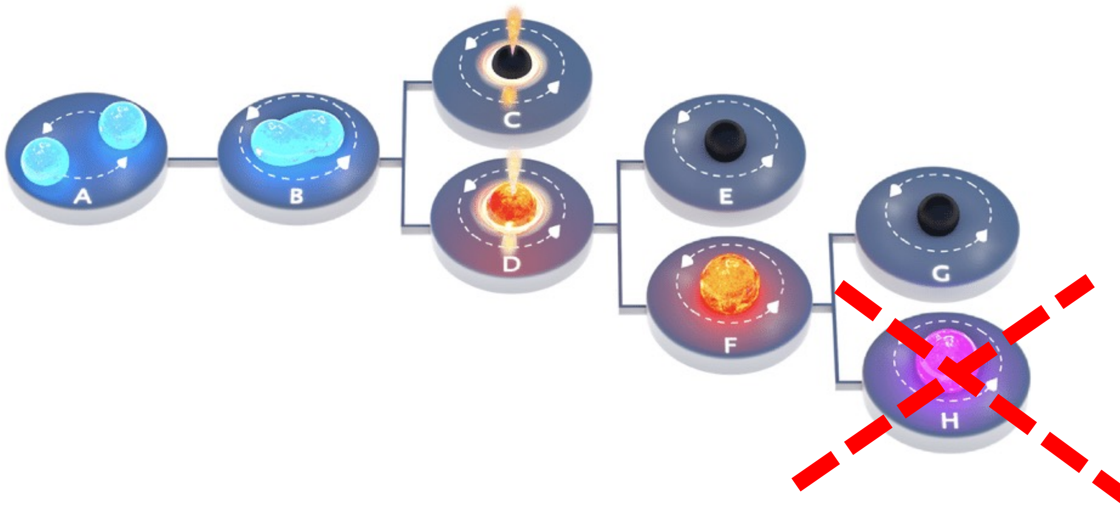
Effect of QCD:



- **Softening** implies an upper limit for the maximal mass

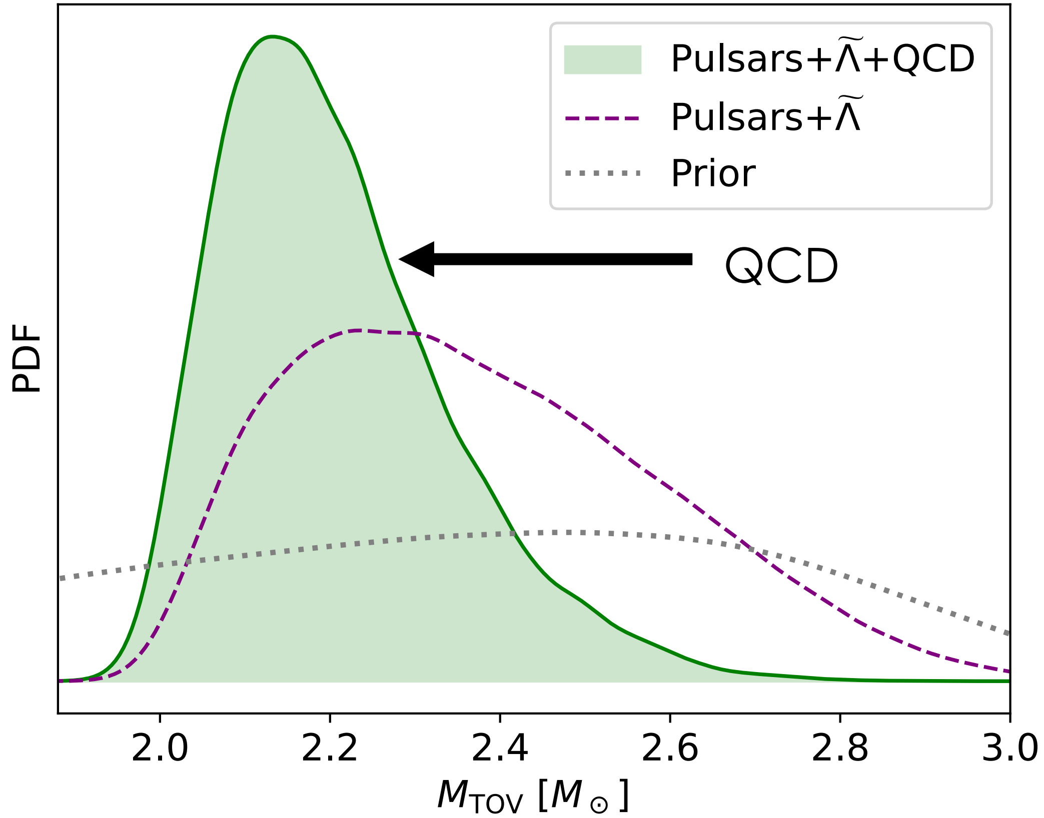
Effect of QCD:

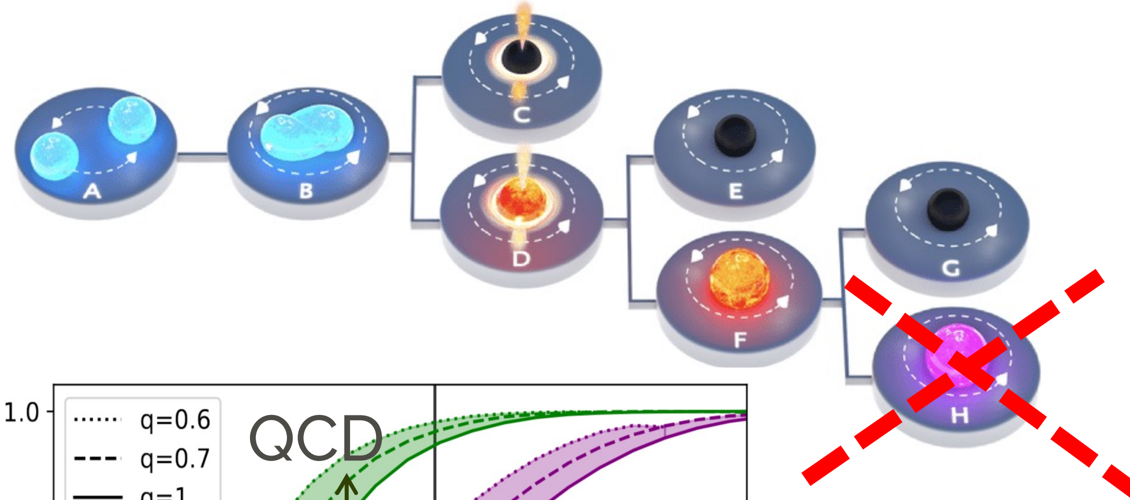


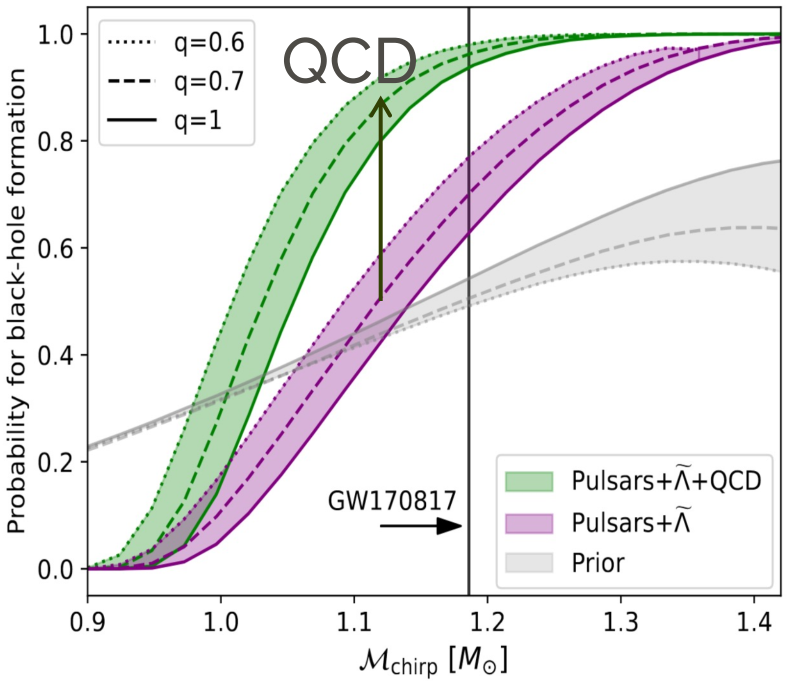


- **Softening** implies an upper limit for the maximal mass
- **Softening** implies BH formation in most mergers

Effect of QCD:

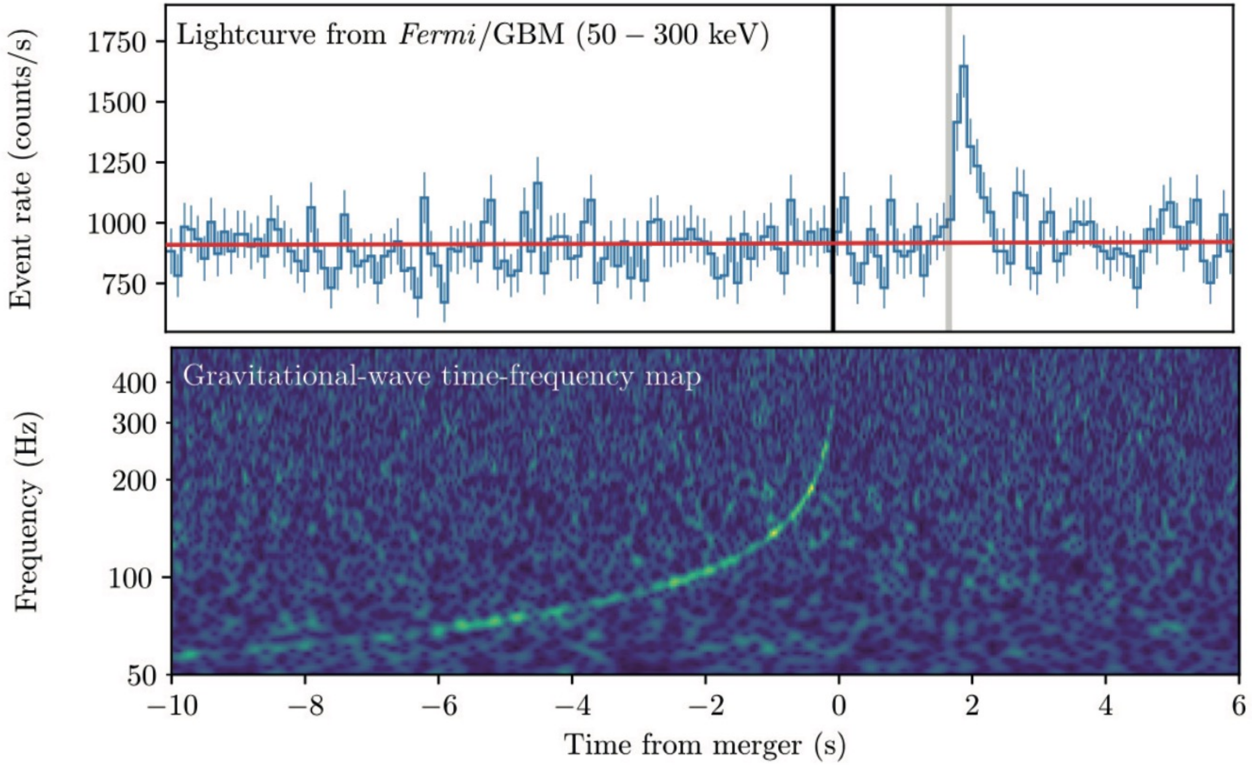






- **Softening** implies an upper limit for the maximal mass
- **Softening** implies BH formation in most mergers

Binary merger product of GW170817

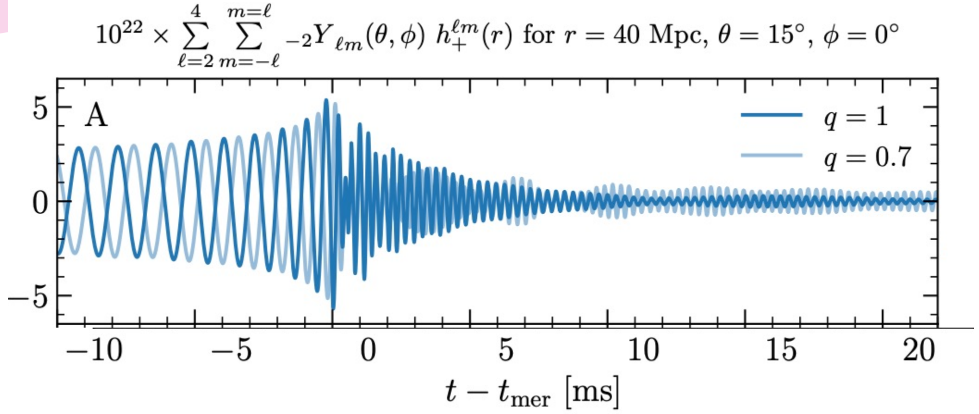


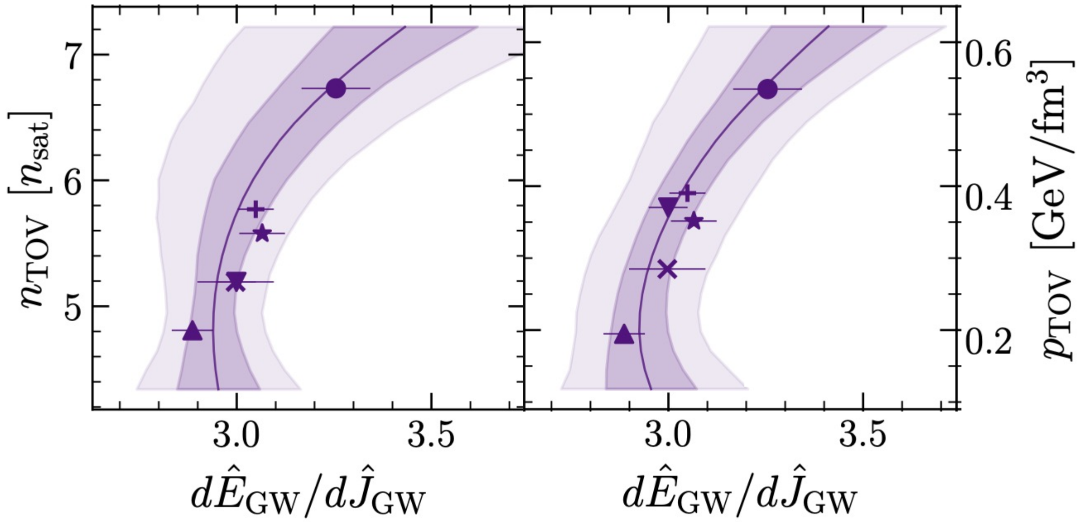
- Astrophysical modelling: GW170817 is consistent with collapse to BH

Margalit & Metzger *ApJL* 850 (2017)
Rezzolla, Most & Weih ApJL 852 (2018)

GRB ⇒ Jet ⇒ Ergoregion ⇒ Black Hole

Future observations and constraints:





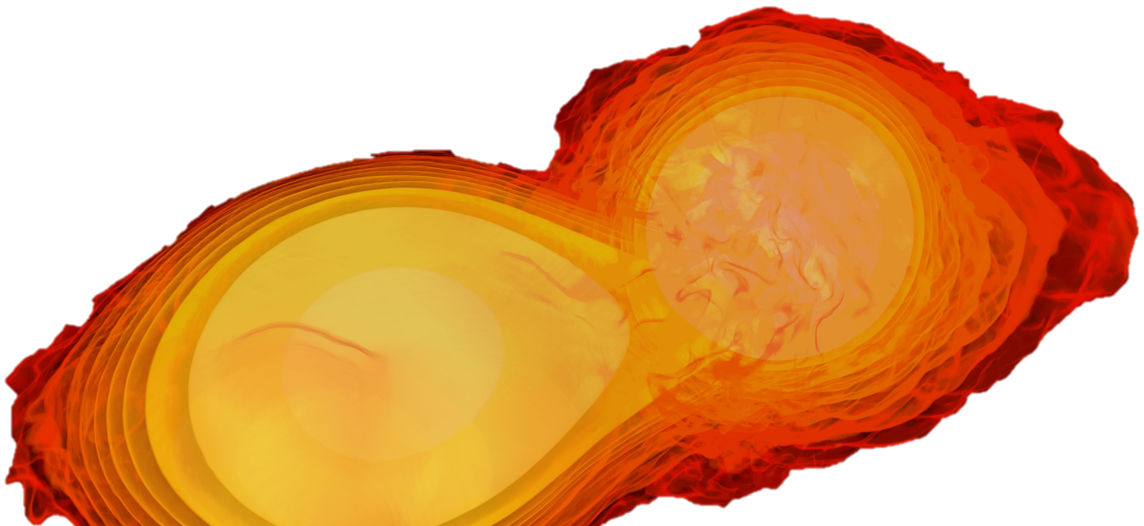
Ringdown frequency

- **3rd gen. gravitational-wave observatories**

  Einstein Telescope, (Cosmic Explorer)

- Merger simulations reveal the correlation of the **long ringdown** and the Equation of State at highest densities in NSs

Ecker, Gorda, **AK**, Rezzolla
Nature Communications 16, 2025



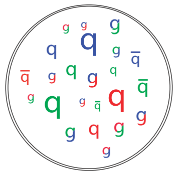
Ask not what your QCD can do the EOS inference,

ask what EOS inference can do to your QCD.

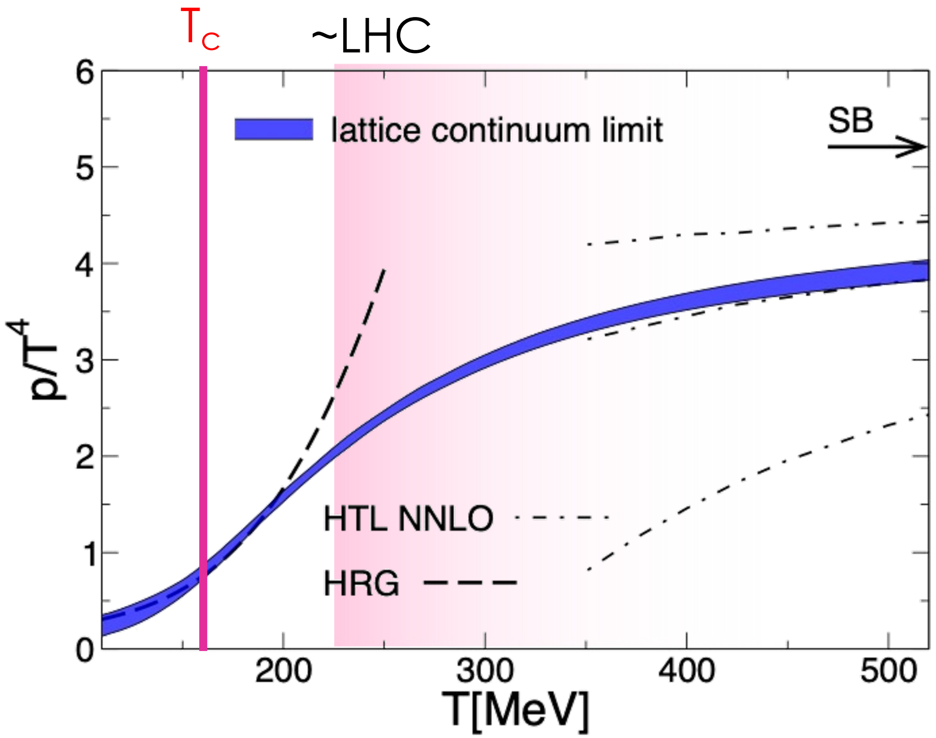
Is there **quark matter** in neutron stars?

EoS tells about phases of matter: HIC

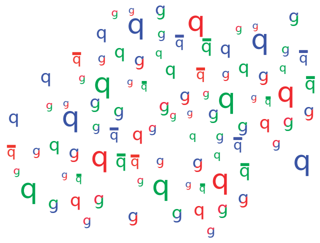
Hadrons



Matter described by hadronic mass scale



Quarks and gluons



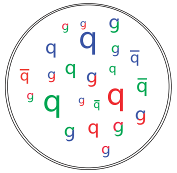
Approximately conformal matter, with large number of d.o.f.

A clear **non-coformal** to **conformal** transition in heavy-ion collisions

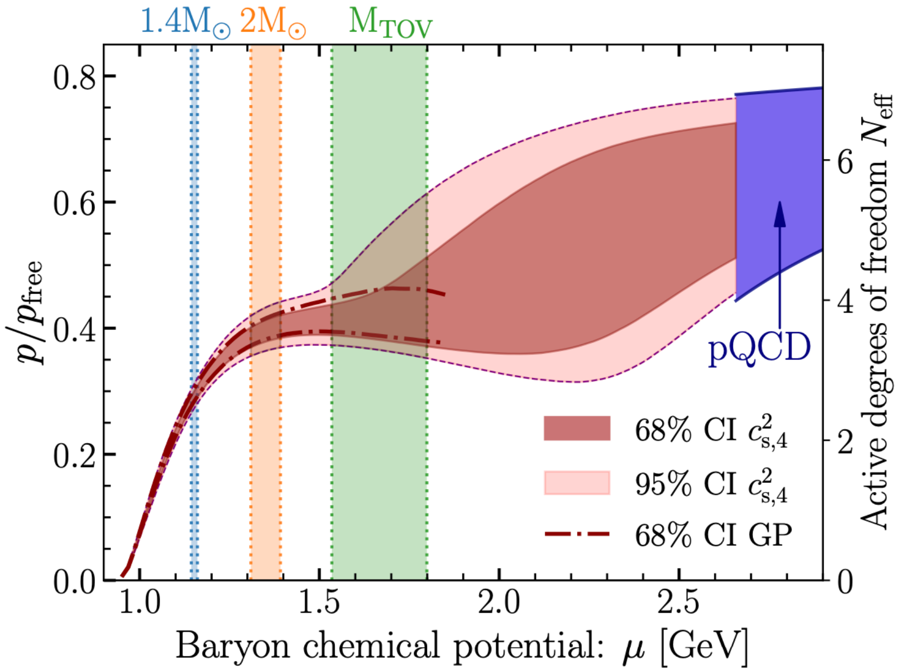
Gardim et al Nature Physics 16, 615-619 (2020)
Borsanyi et al PLB 730, 99 (2014)

EoS tells about phases of matter: Stars

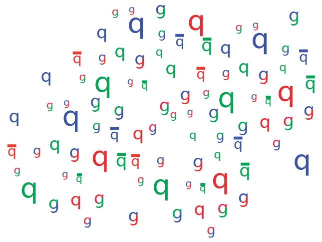
Hadrons



Matter described by
Hadronic mass scale



Quarks and gluons



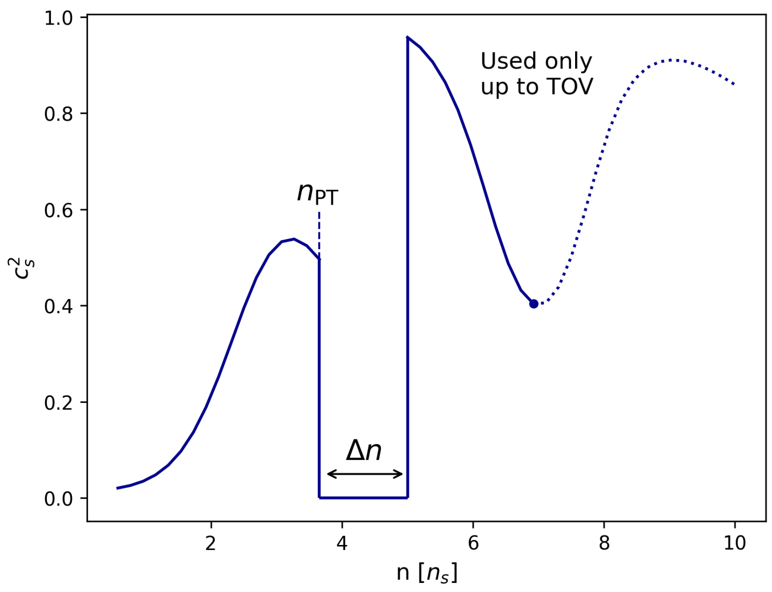
Approximately conformal matter,
with large number of d.o.f.

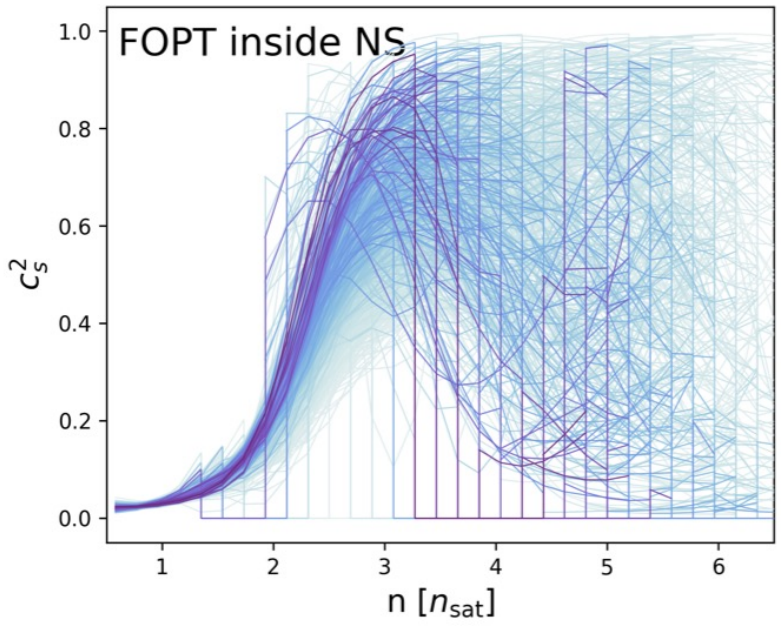
A clear **non-coformal** to **conformal** transition within cores of neutron stars

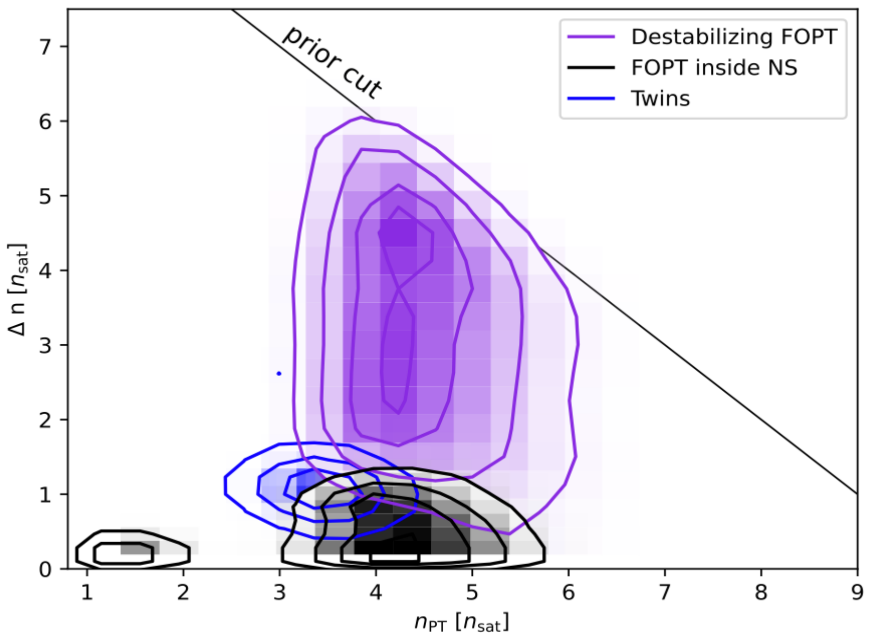
Annala, Gorda, **AK**, Nätttilä, Vuorinen Nature Physics 16 (2020) 9
Annala, Gorda, **AK**, Nättillä, Vuorinen Nature Comms. 2303.11356

Also: Fujimoto, Fukushima, McLerran, Praszalowicz 2207.06753,
Kojo PRD 104, …

First order transitions?







- Transition by discontinuously merging two Gaussian Processes
- Constraints to allowed range and strength of the transition
- Currently, no sensitivity to exclude/confirm existence

Komoltsev PRD 110 (2024) 7
Also: Gorda, Heber, **AK,** Schwenk, Vuorinen APJ 955 (2023) 2, many others

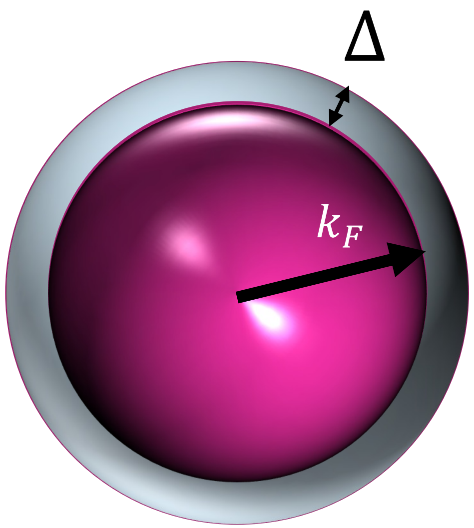
If there **quark matter,** what is it?

Condensed quark-matter physics?

- If there is quark matter, it is **color-superconducting**
  - At high enough densities, Color-Flavor-Locked pattern

Alford, Rajagopal, Wilczek, PLB 422 (1998)

$$\left\langle \psi_i^{\alpha} C\,\gamma_5\,\psi_j^{\beta} \right\rangle = \Delta\,\epsilon^{\alpha\beta A}\epsilon_{ijA}$$



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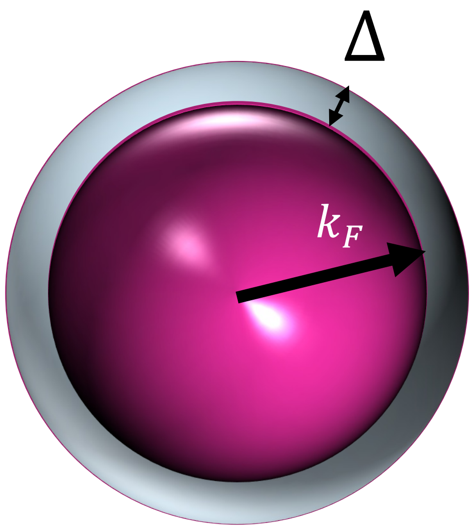


- The **gap** Δ difficult to compute:
  - Asymptotically, ***ab-initio***: $\Delta \approx 845\,\frac{\mu}{g^5}\,e^{-\frac{20.9}{g}} \ll \mu$

Son, Phys. Rev. D 59, 094019 (1999)
Schäfer, Wilczek, PRD 60 (1999)

Pisarski, Rischke, PRD 61 (2000)
Brown, Liu, Ren, PRD 61 (2000)

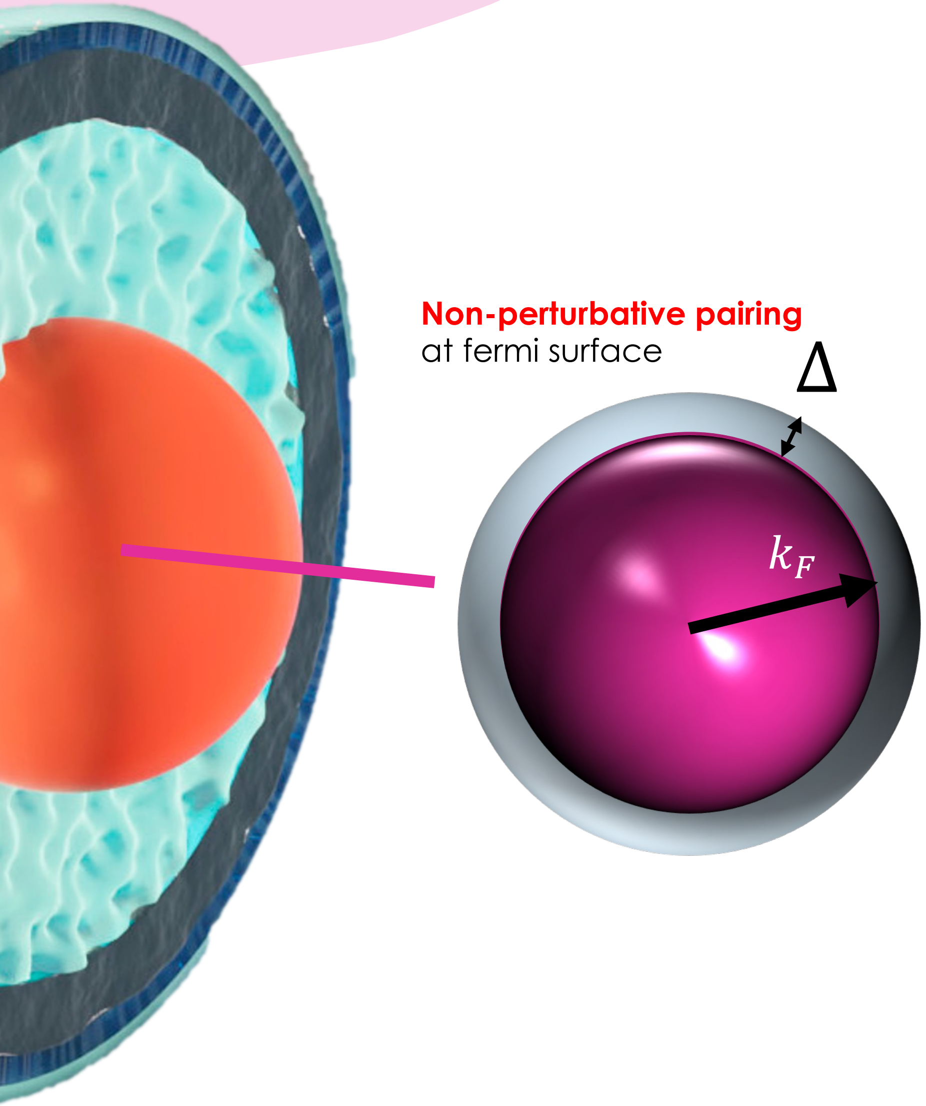
  - At NS densities, **models**: $\Delta \approx 20\ \mathrm{MeV} - 250\ \mathrm{MeV}$, even $\Delta \approx 300\mathrm{MeV}$

Rapp, Schäfer, Shuryak, Velkovsky, 81 (1998)
G. W. Carter and D. Diakonov, PRD 60, (1999)
Baym, Hatsuda, Kojo, et al. Rept. Prog. Phys. 81, (2018)
Leonhardt, Pospiech, Schallmo, et al. PRL 125 (2020), …

Berges and Rajagopal, NPB 538 (1999)
Wang and Rischke, PRD 65 (2002)
Brown, Liu, Ren, PRD 61 (2000),

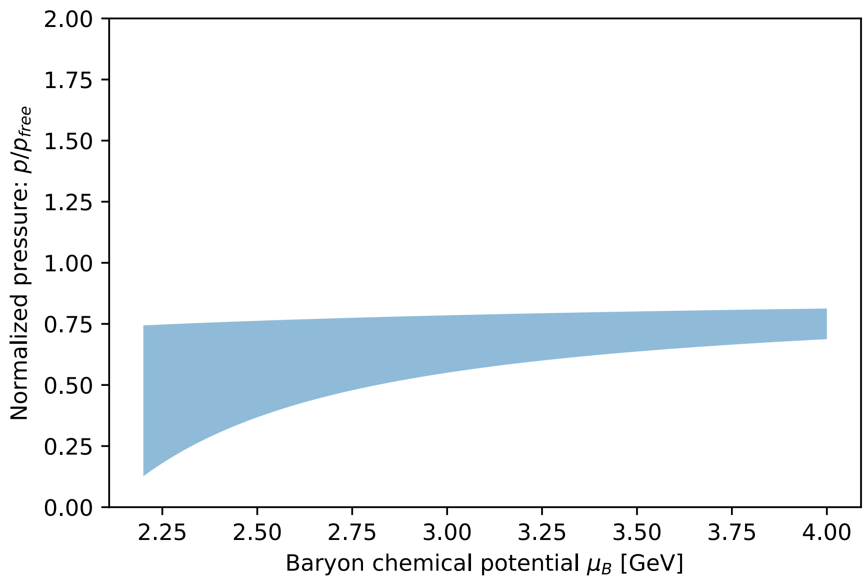
Braun, Schallmo, PRD 105 (2022)

Condensed quark-matter physics?

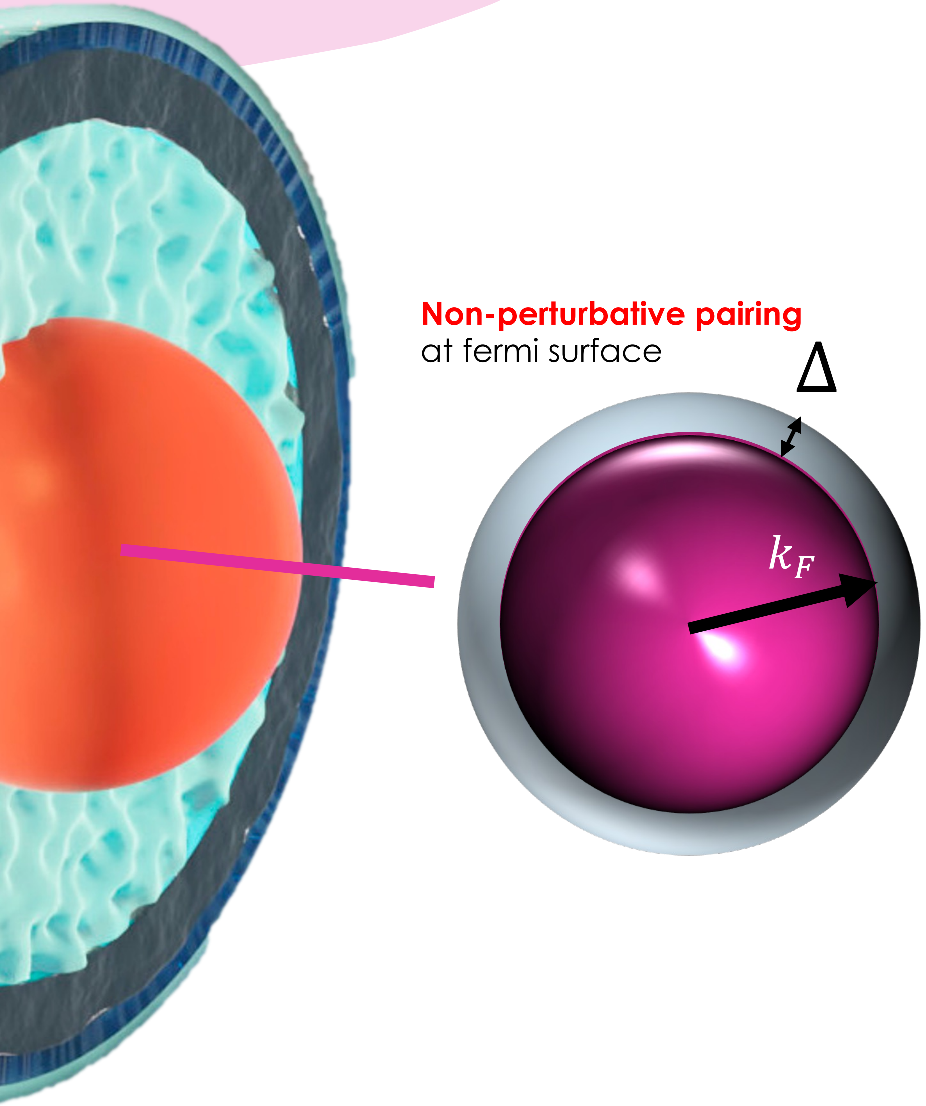


- Big effect for transport, small effect for the equation of state

$$p_H \approx 1 + h_1\alpha_s + h_2\alpha_s^2 + h_3\,\alpha_s^3 + \#\,\Delta^2\mu^2$$



Condensed quark-matter physics?



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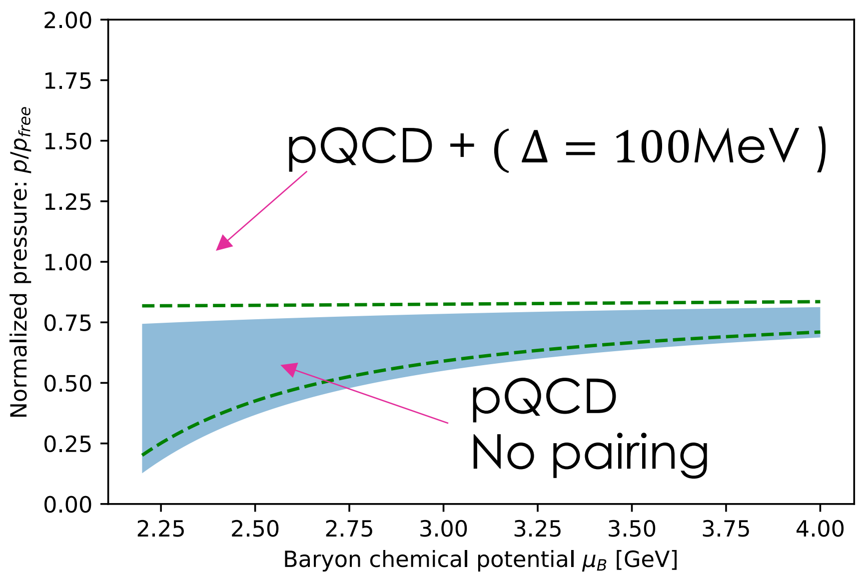
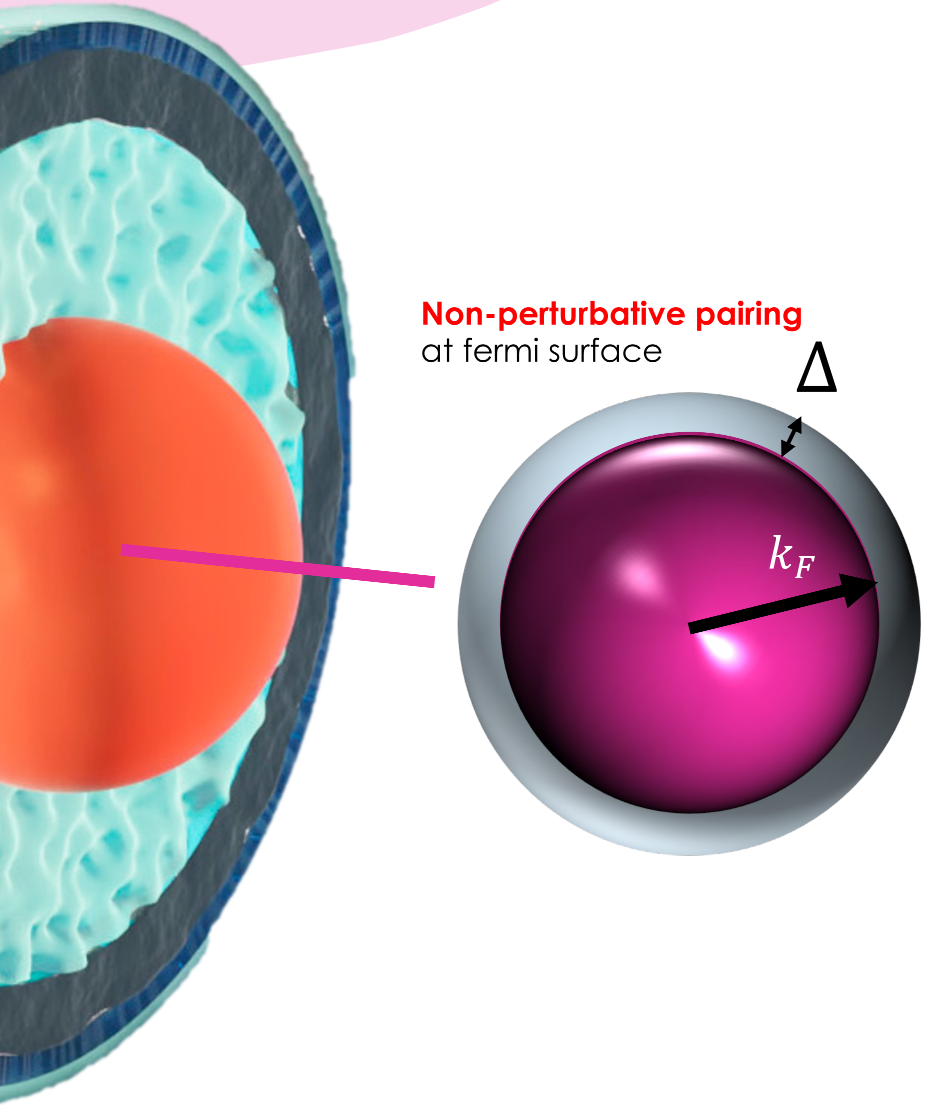
- Gap can only increase pressure → tighter EOS constraints



Condensed quark-matter physics?



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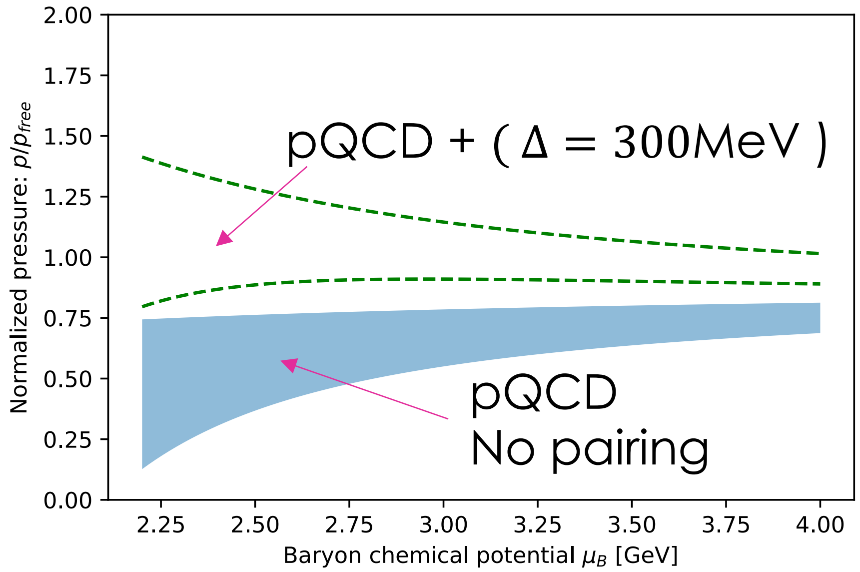

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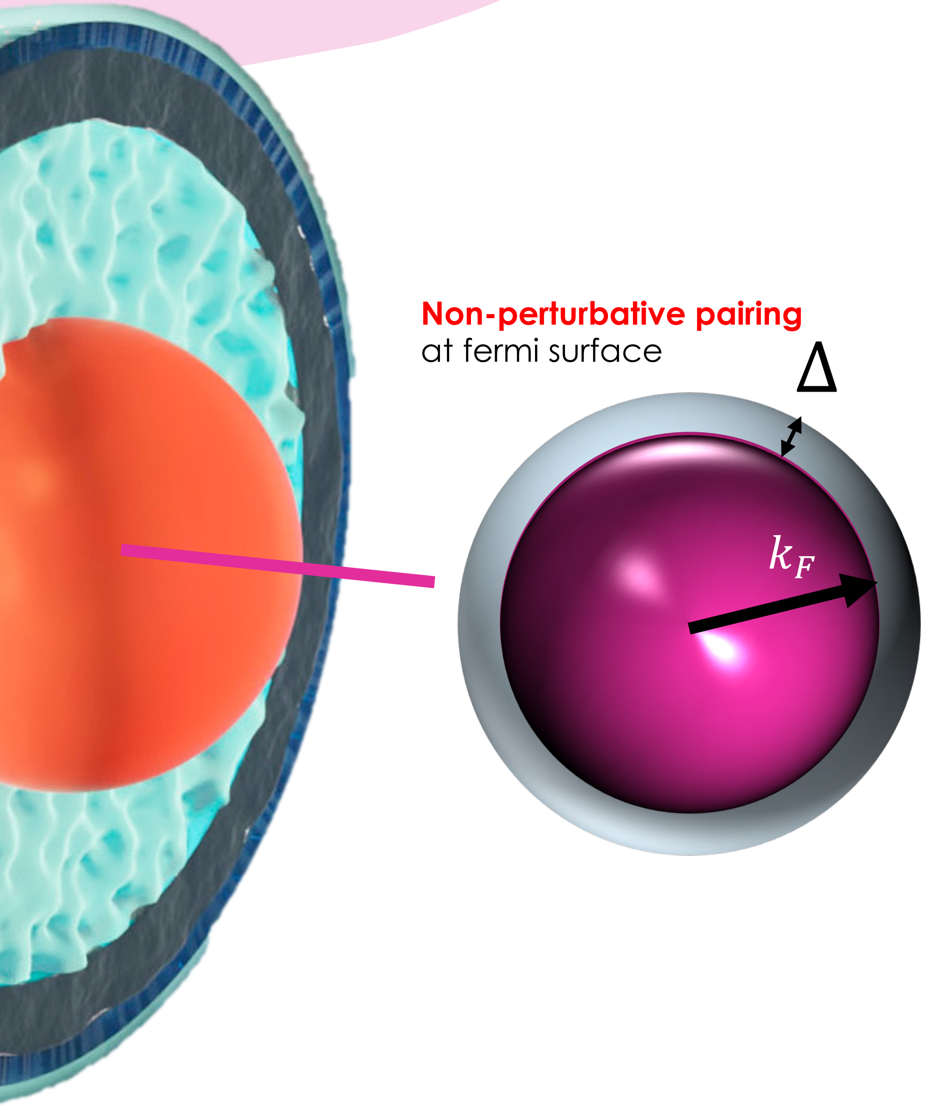

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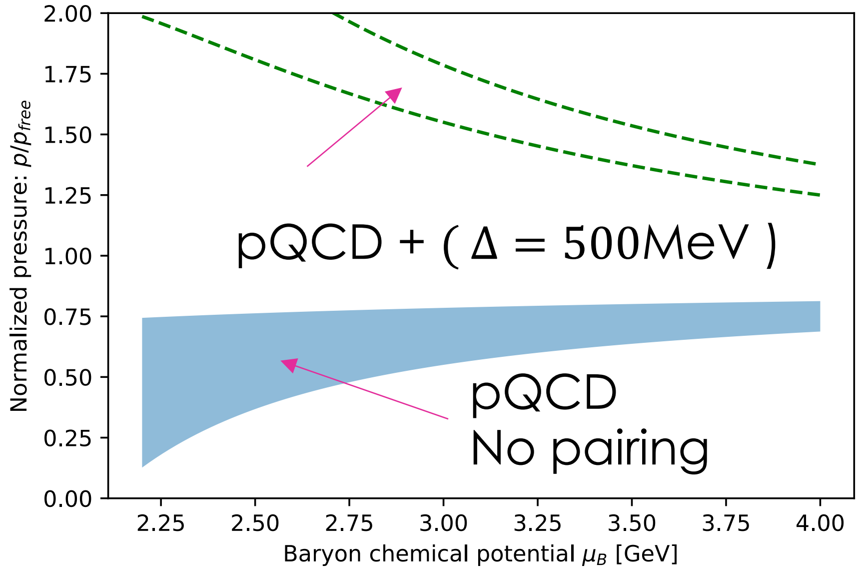

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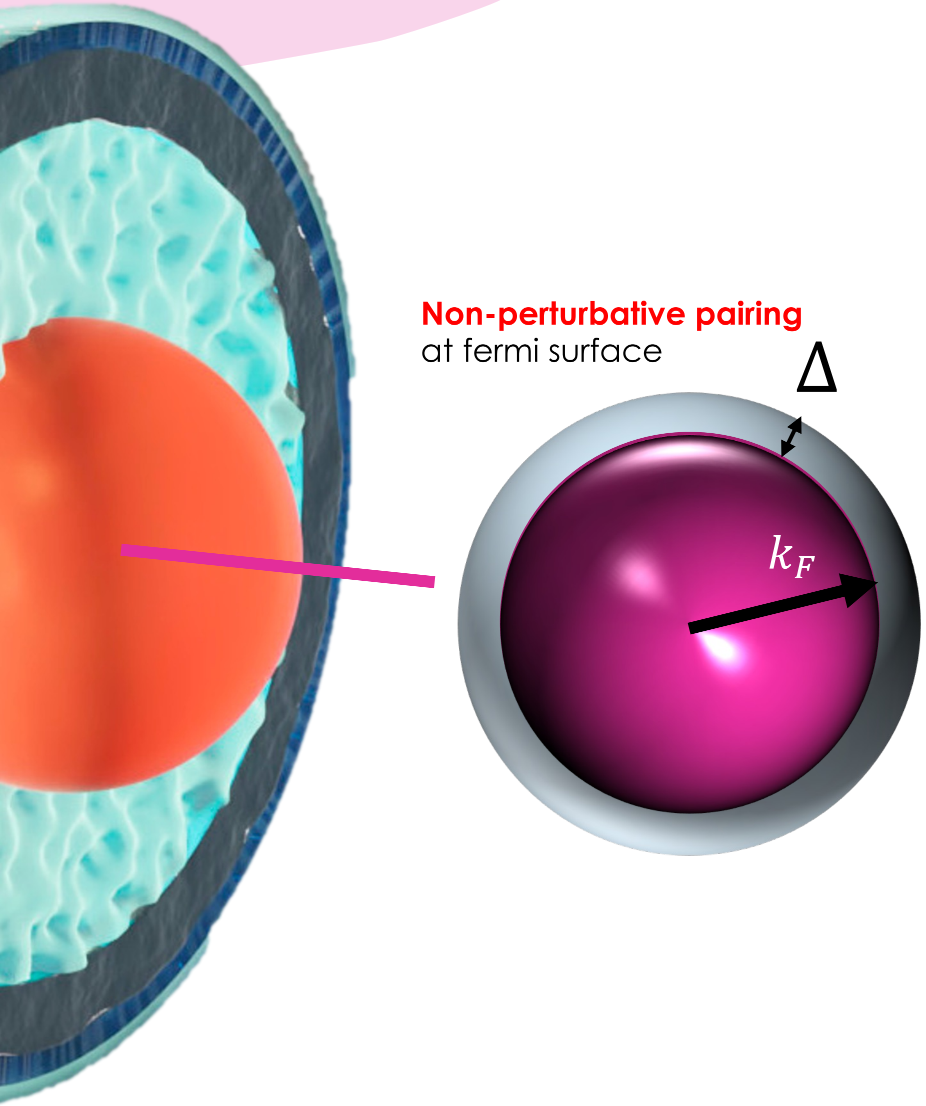

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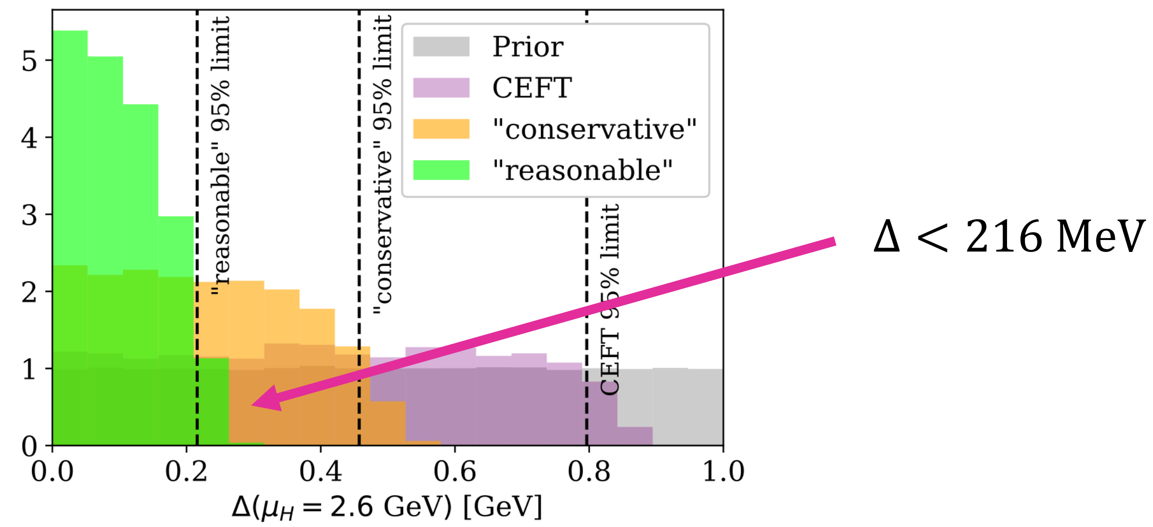
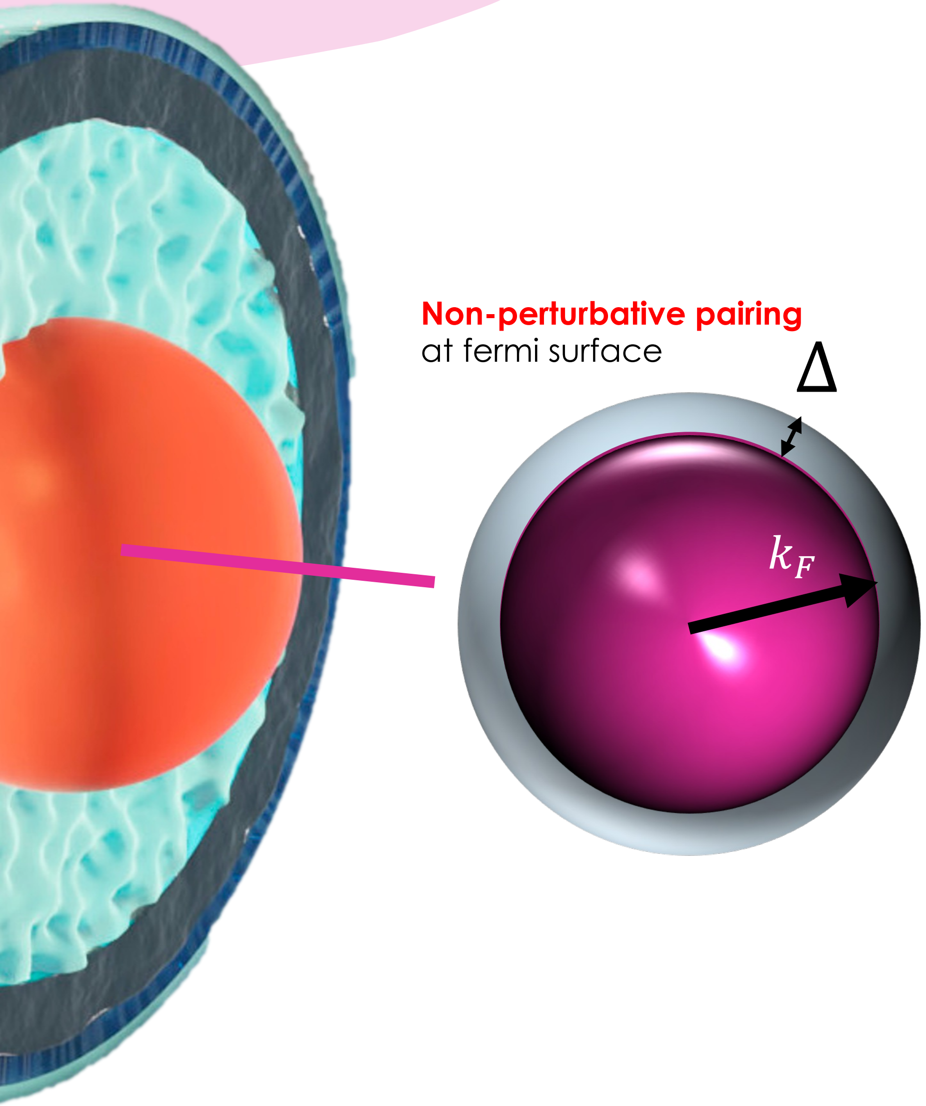
- **Large values of superconducting gap** inconsistent with pQCD + neutron-star observations

AK, Rajagopal, Steinhorst, PRL 132 (2024)



Condensed quark-matter physics?



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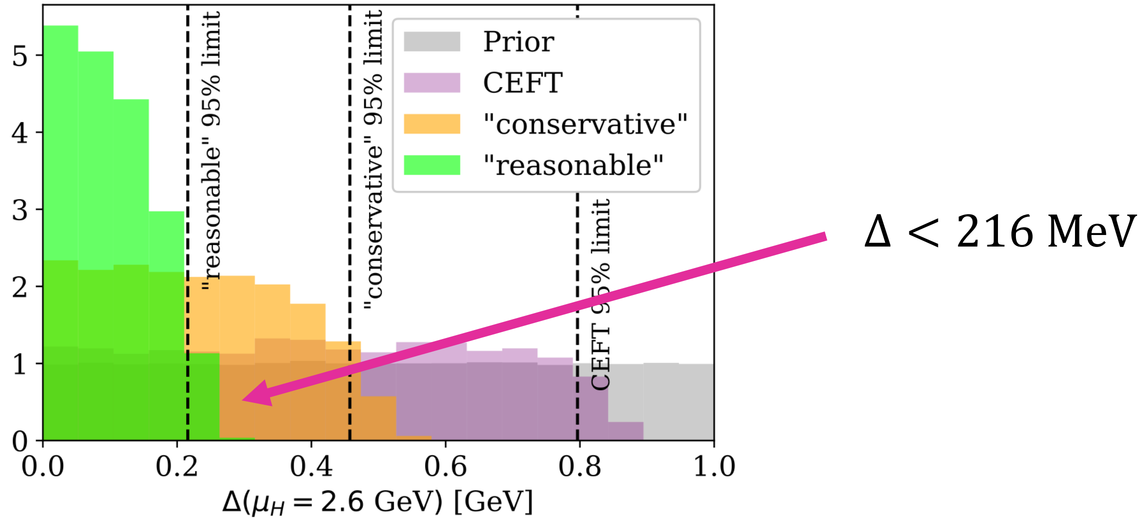


- Further observations/pQCD lead to **tighter constraint**

Geissel, Gorda, Braun, 2504.03834

Summary:

- pQCD computations inform about neutron-star cores because of **stability**, **causality**, and **consistency**
- QCD at $n = 40\ n_s$ offers a **robust** constraint down to $n = 2.3\ n_s$

- QCD input **softens** the inferred EoS at high densities.

Annala et al. Phys.Rev.X 12 (2022), Altiparmak, Ecker, Rezzolla APJL (2022)
Gorda, Komoltsev & AK APJ (2023)

- Softening has natural interpretation as onset of **Quark Matter**

Annala, Gorda, Komoltsev, **AK**, et al. *Nature Comm* 14 (2023)
Annala, Gorda, Kurkela, Nättilä, Vuorinen, *Nature Phys.* 16 (2020)

- QCD predicts that binary mergers lead to BHs. Softening detectable from long ringdown with future GW observatories

Ecker, Gorda, **AK**, Rezzolla, Nature Communications 16, (2025)

Discussions:

- pQCD constraints complementary to astro. **No modelling uncertainties.**

No trasnport models, no extrapolation in isospin, no stellar models, no assumption of general relativity

- Can be used to test hypothesis (BSM, beyond GR)

QCD and neutron stars offer an interesring intersection of theoretically solid calculations combined with rapidly evolving observational field.

Implementing pQCD to EoS inference:

- Bayesian inference setup:

Prior

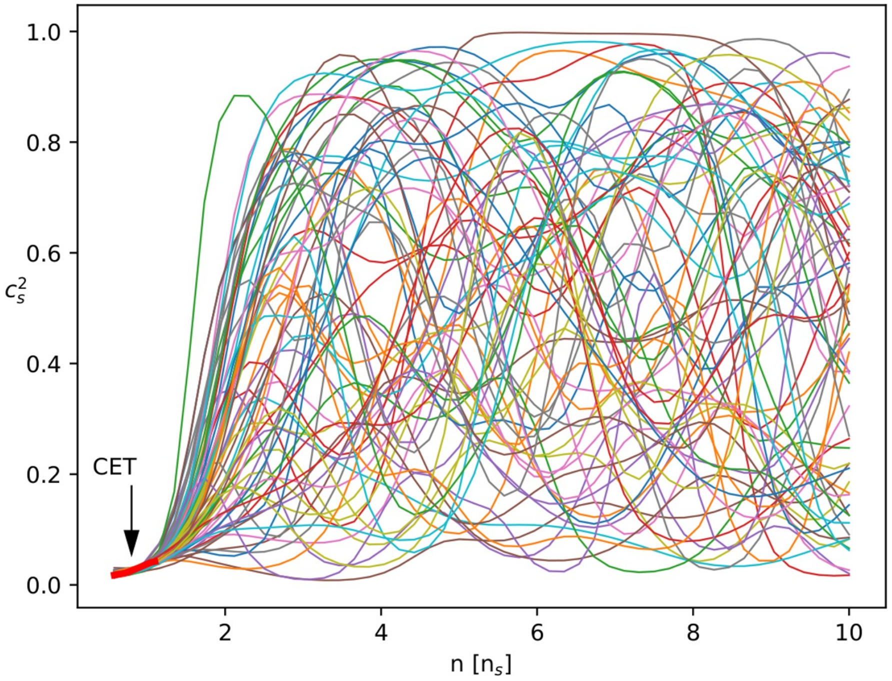
Likelihood of EoS given data

$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})\,P(\mathrm{data}\,|\,\mathrm{EoS})}{P(\mathrm{data})}$$

Implementing pQCD to EoS inference:

- **Gaussian-process** based inference:

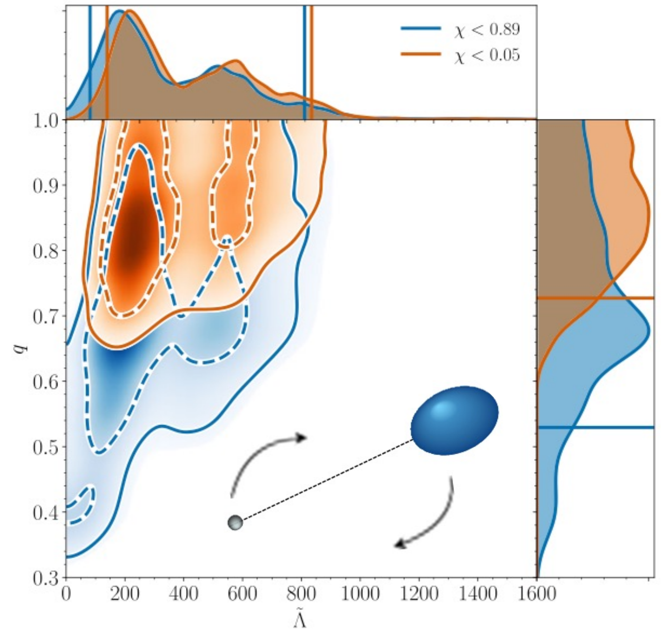
Similar to Landry & Essick PRD 99 (2019), but for function of n instead of ε



$$P(\mathrm{EoS}\,|\,\mathrm{data}) = \frac{P(\mathrm{EoS})\,P(\mathrm{data}\,|\,\mathrm{EoS})}{P(\mathrm{data})}$$

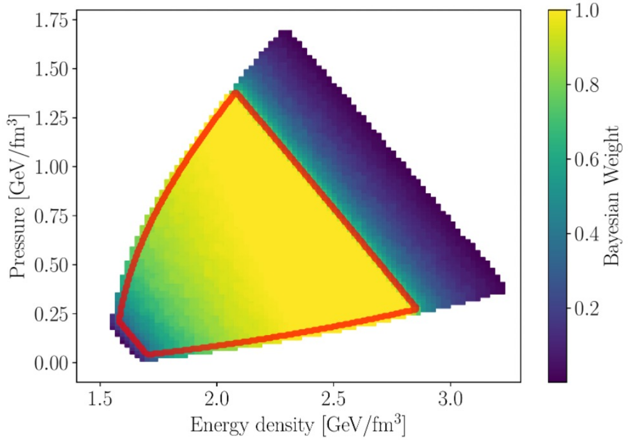
Setup:

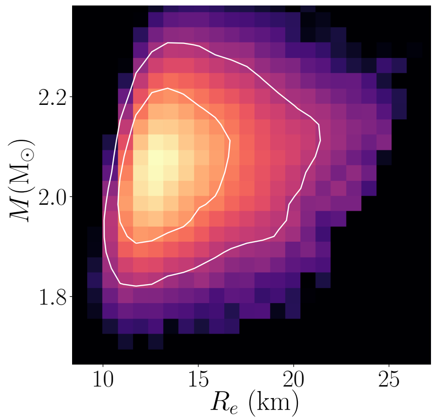
$$P(\text{EoS}\,|\,\text{data}) = \frac{P(\text{EoS})\,P(\text{data}\,|\,\text{EoS})}{P(\text{data})}$$



$M_{J1614\text{-}2230}$=1.908(16)
$M_{J048+0432}$ = 2.01(4)
$M_{J0740+6620}$ = 2.14(10)

$$P(\text{data}\,|\,\text{EoS}) = P(\text{QCD}\,|\,\text{EoS})P(\text{Mass}\,|\,\text{EoS})P(\text{NICER}\,|\,\text{EoS})P(\widetilde{\Lambda},\text{BH}\,|\,\text{EoS}).$$

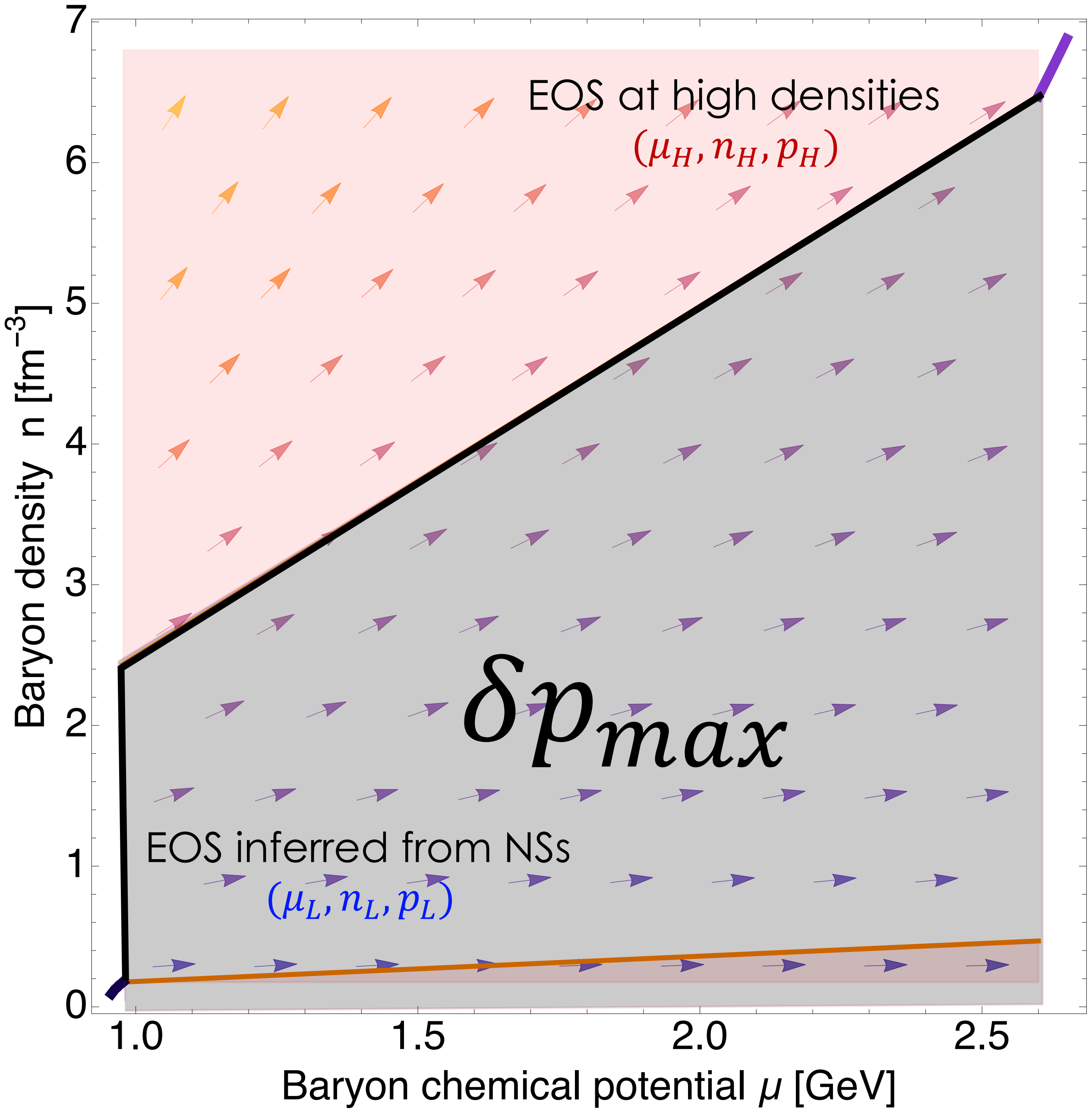




Consistency:

- **Inconsistency** between low and high-density EOS if:

$$\delta p_{max} < p_H - p_L$$



Komoltsev & AK, PRL128 (2022)

Constraint on the gap:

- High-density EOS marginally consistent with low density if

$$\delta P_{max} = p_H - p_L$$

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$$n_{pQCD}(\mu_H) + \frac{2}{3\pi^2}\Delta^2\mu_H$$

$$\frac{n_H}{2}\left(\mu_H - \frac{\mu_L^2}{\mu_H}\right)$$

$$p_{pQCD}(\mu_H) + \frac{1}{3\pi^2}\Delta^2\mu_H^2$$

Constraint on the gap:

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$$n_{pQCD}(\mu_H) + \frac{2}{3\pi^2}\Delta^2\mu_H \qquad \delta P_{max} = p_H - p_L$$

$$\frac{n_H}{2}\left(\mu_H - \frac{\mu_L^2}{\mu_H}\right) \qquad p_{pQCD}(\mu_H) + \frac{1}{3\pi^2}\Delta^2\mu_H^2$$

- If $\Delta > \Delta_{max}$, high-density EOS inconsistent with low-density EOS

$$\Delta_{max}(\mu_H)^2 = \frac{3\pi^2}{\mu_L^2}\left[\frac{n_{pQCD}(\mu_H)}{2\,\mu_H}(\mu_H^2 - \mu_L^2) - \left(p_{pQCD}(\mu_H) - p_L\right)\right]$$

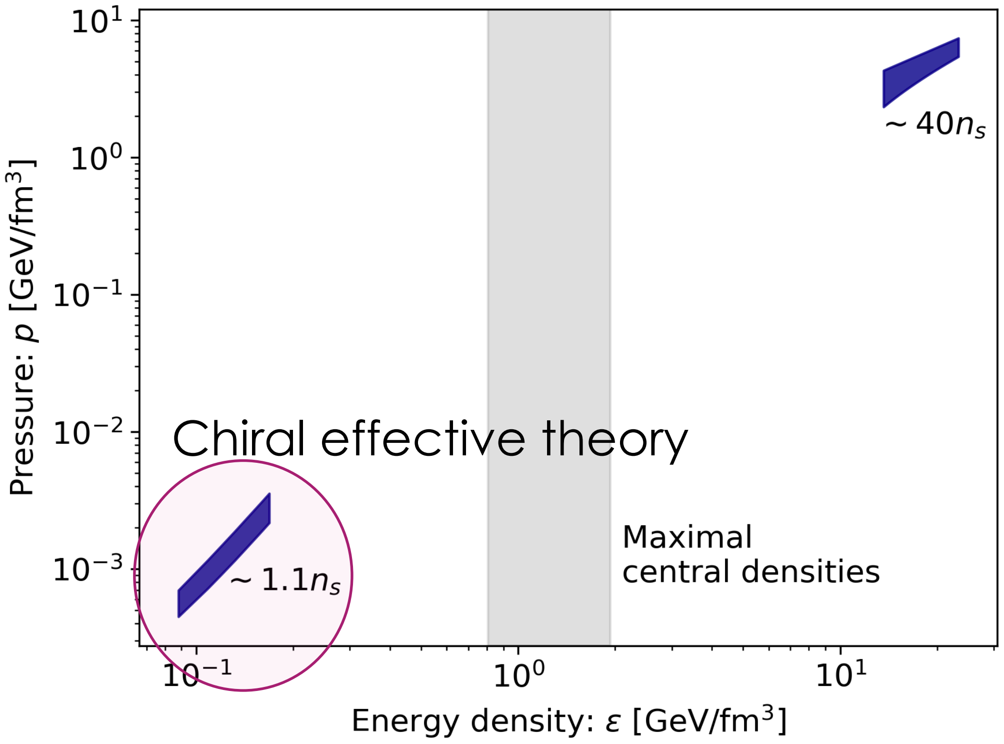
Back of the envelope:

- High densities: $(\mu_H, n_{pQCD},\ p_{pQCD}) = \left(2.6\ \text{GeV}, 40\ n_s, 3.6\,\frac{\text{GeV}}{\text{fm}^3}\right)$
- Low densities:
  - No information from NSs
  - From CEFT:

$$(\mu_L, n_L, p_L) = \left(0.97\ \text{GeV}, 1.1\ n_s, 3\,\frac{\text{MeV}}{\text{fm}^3}\right)$$

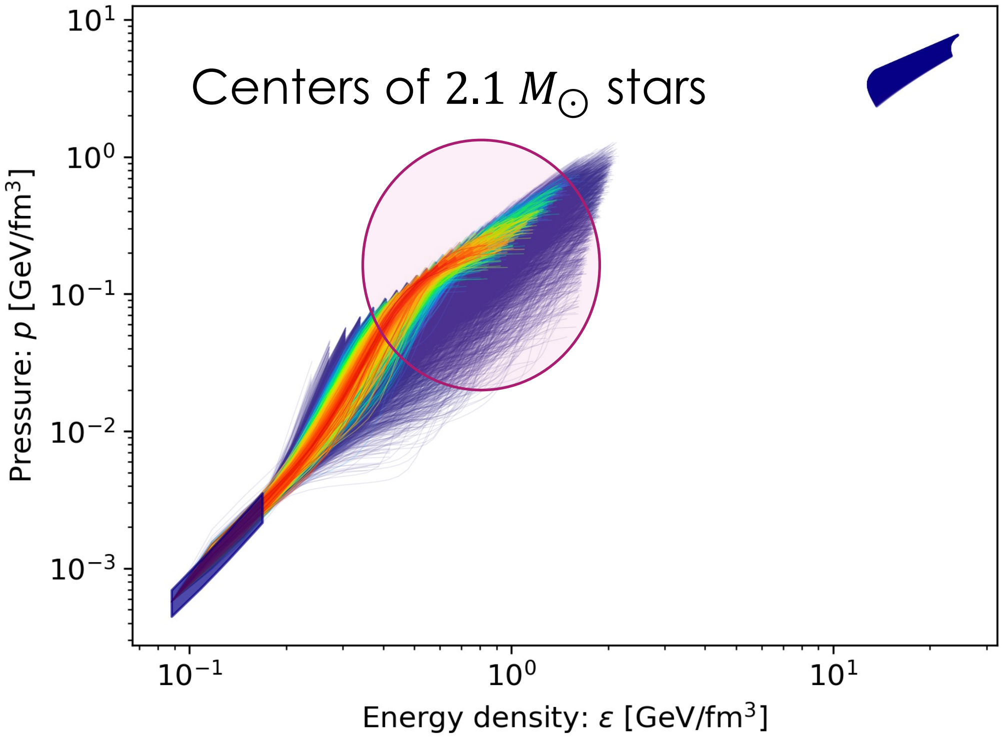
Hebeler, Lattimer, et al., ApJ. 773, 11 (2013)

$$\Delta_{max}(2.6\text{GeV}) \approx 880\ \text{MeV}$$



Back of the envelope:

- High densities: $(\mu_H, n_{pQCD},\ p_{pQCD}) = \left(2.6\ \text{GeV}, 40\ n_s, 3.6\,\frac{\text{GeV}}{\text{fm}^3}\right)$
- Low densities:
    - Using neutron stars up to 2.1 $M_\odot$
    - From EoS inference:

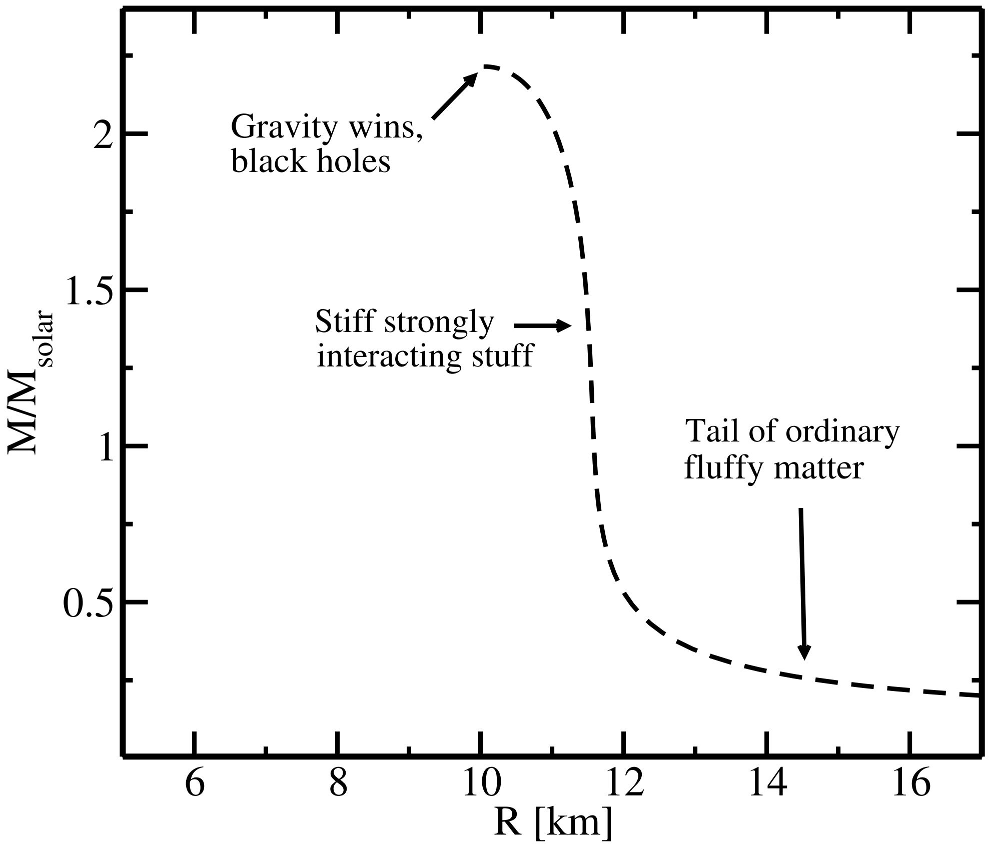


$$(\mu_L, n_L, p_L) = \left(1.45\ \text{GeV}, 3.5\ n_s, 160\ \frac{\text{MeV}}{\text{fm}^3}\right)$$

Gorda, Komoltsev, AK, ApJ. (2023)
Annala, Gorda, Kurkela, Nättilä, et al. Nature Commun. 14 (2023)

$$\Delta_{max}(2.6\text{GeV}) \approx 440\ \text{MeV}$$

Map from micro to macro:



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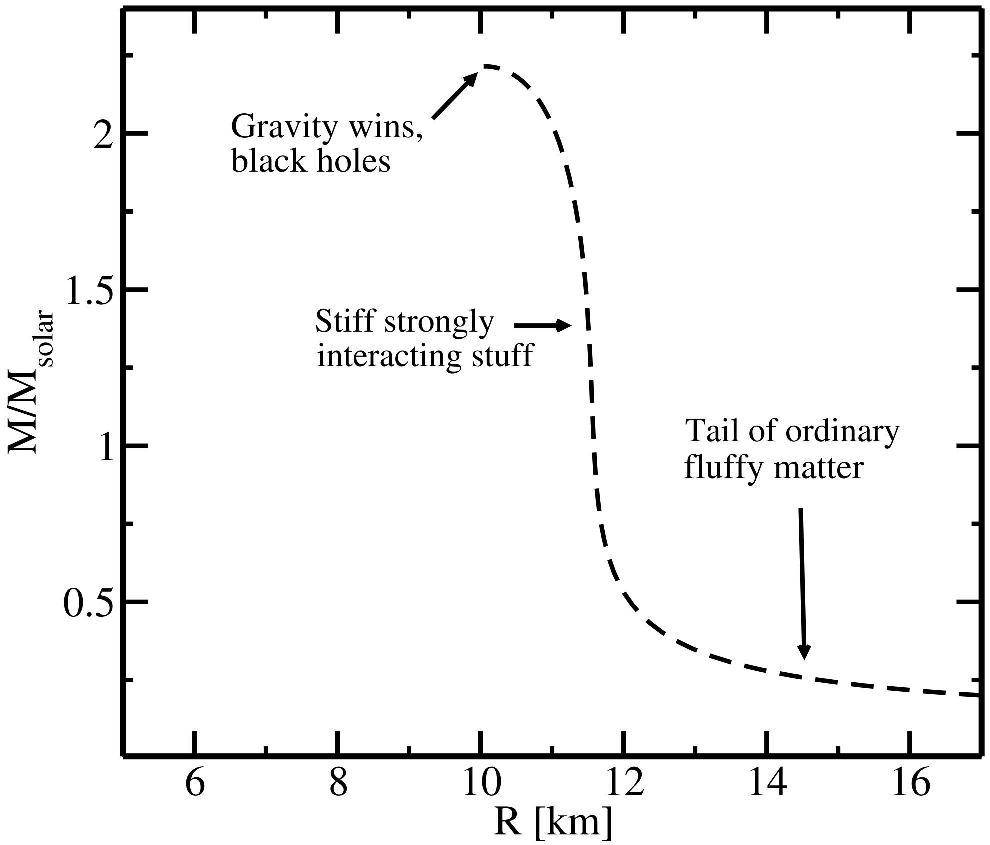
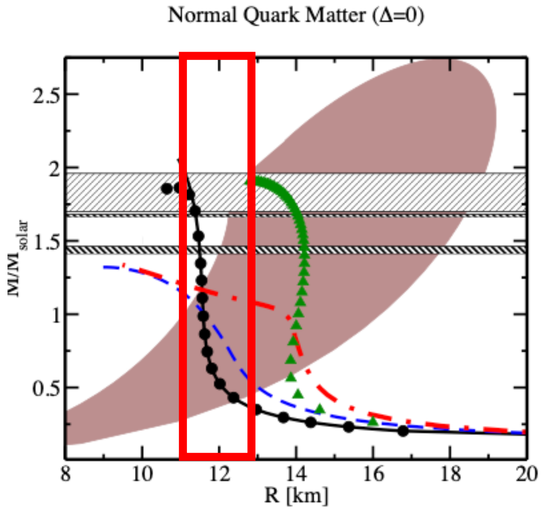
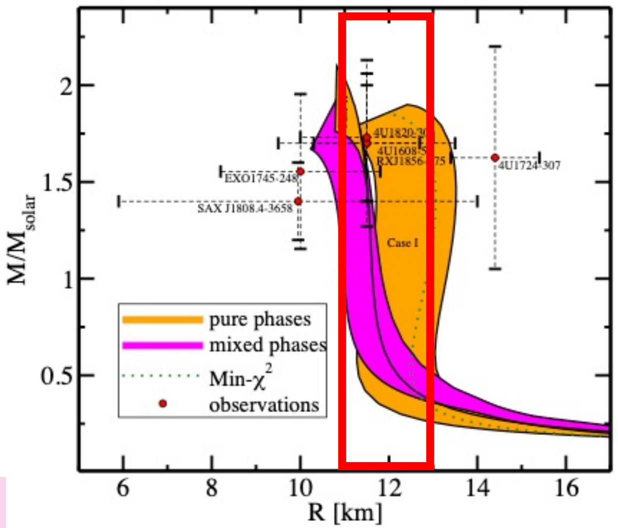


Neutron stars are *femtoscopes*

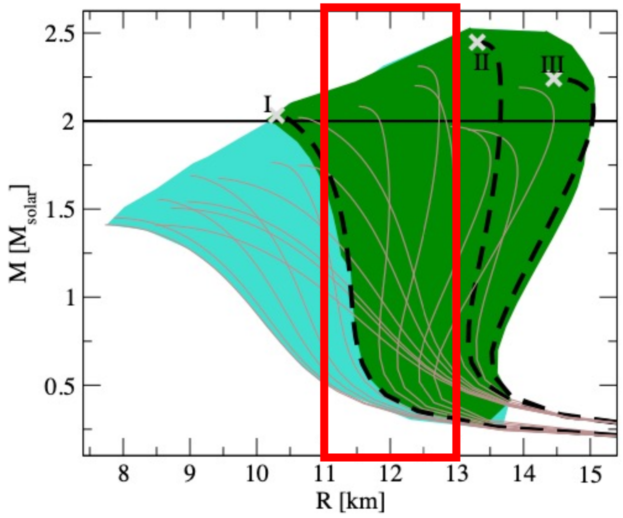
$$10^{-15}\text{m} \rightarrow 10^{4}\text{m}$$



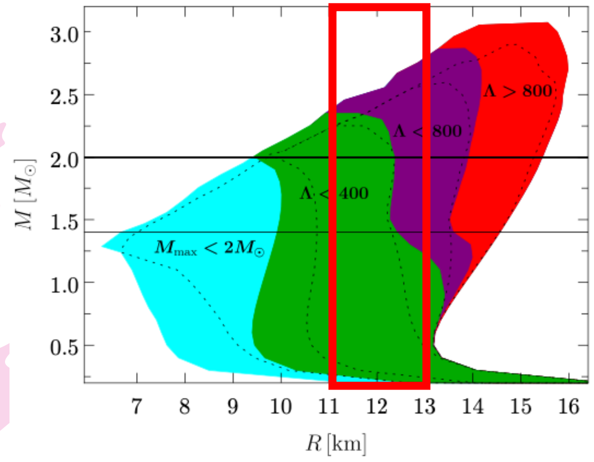
0912.1856



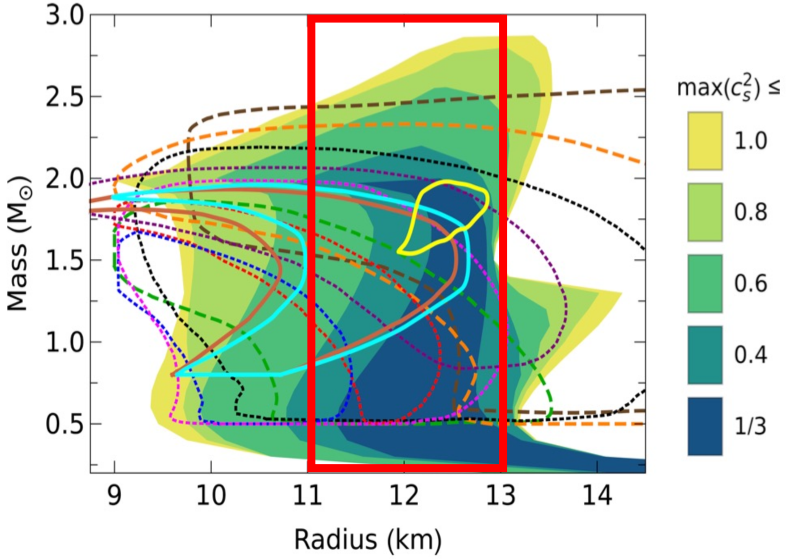
1006.4062



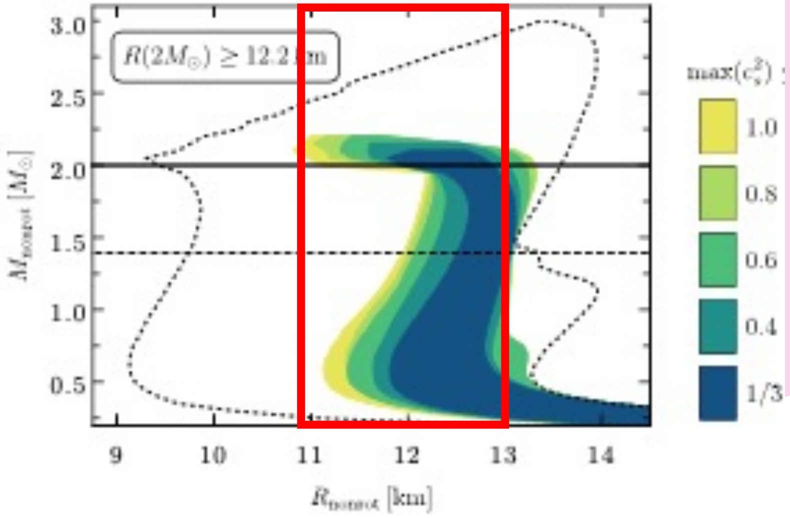
1402.6618



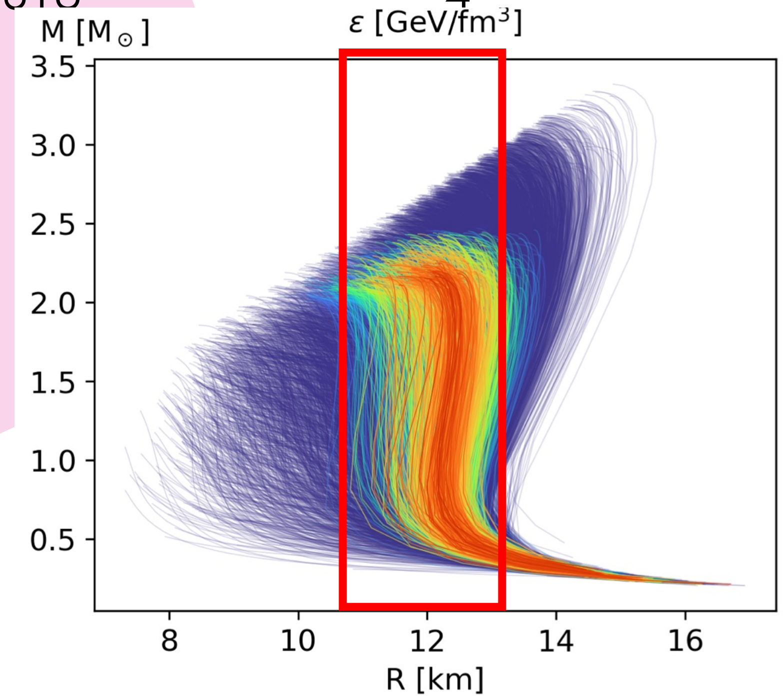
1711.0264
4



Nature Phys. 2020

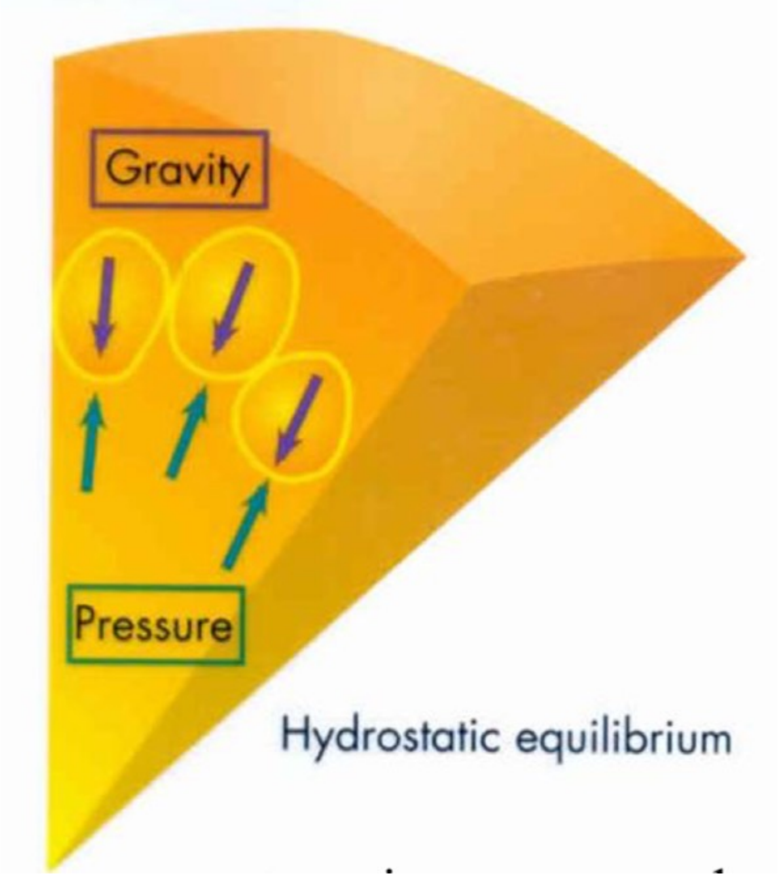


PRX. 2021



APJ 2022

Map from micro to macro:



macroscopic properties of neutron stars reflect
microscopic properties of ultradense matter

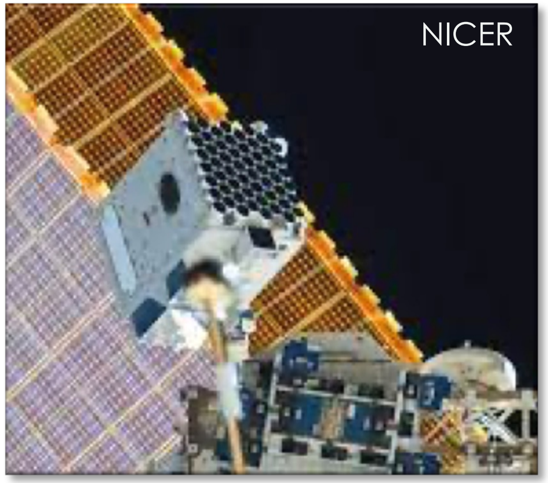
$$\frac{dP}{dr} = -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r)$$

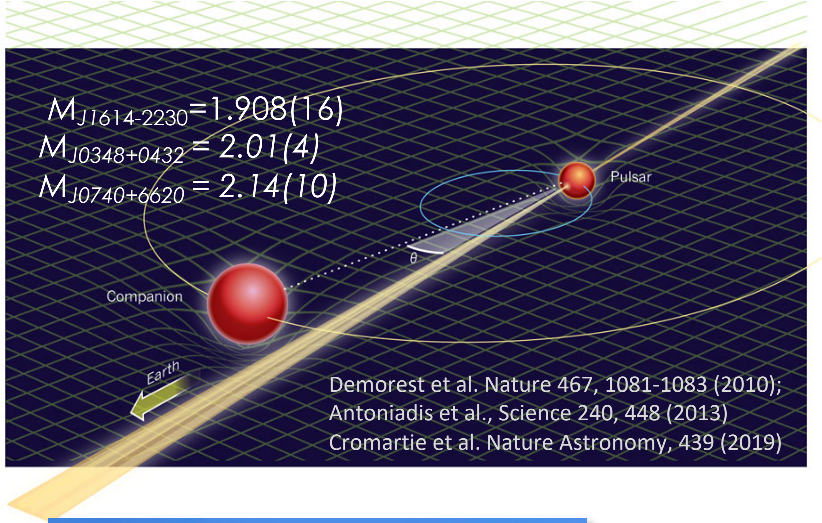
$$\epsilon(P) \Leftrightarrow R(M)$$

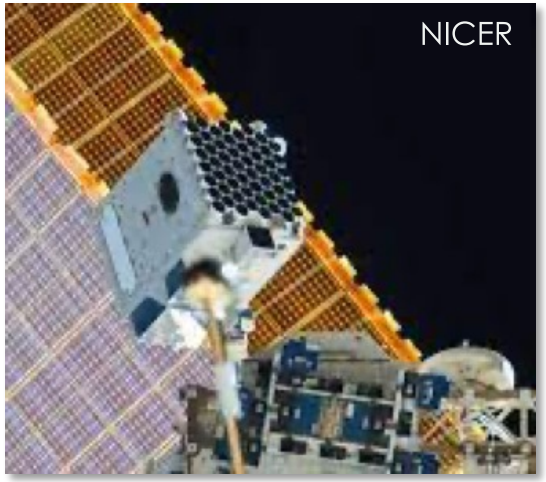
EOS reflects the microscopic structure,
phase diagram

Properties of neutron stars reflect properties of dense matter



Properties of neutron stars reflect properties of dense matter





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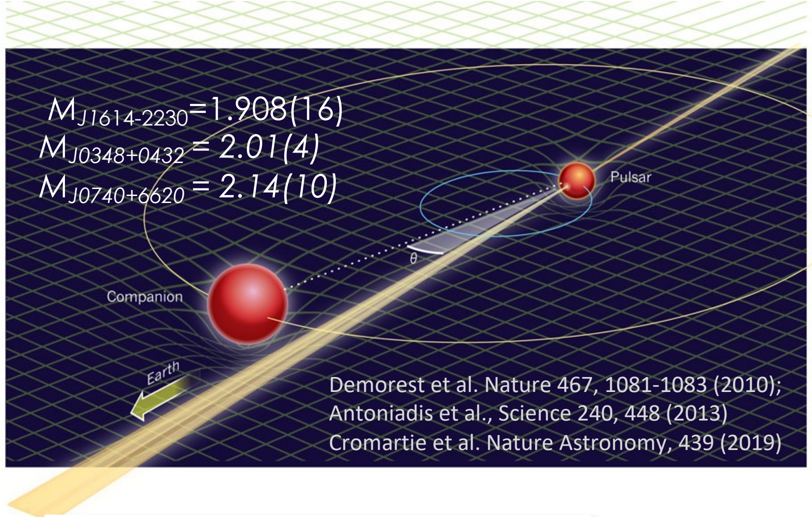

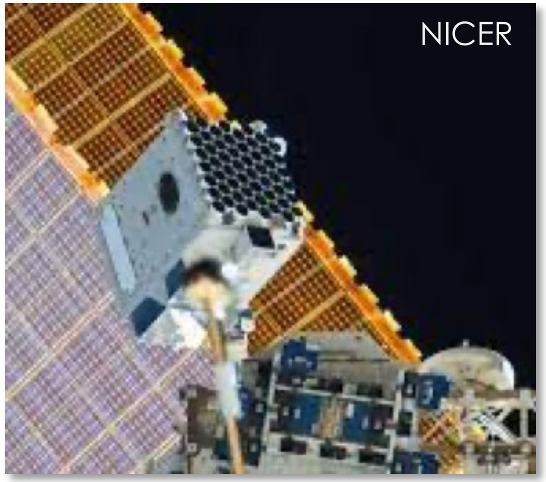

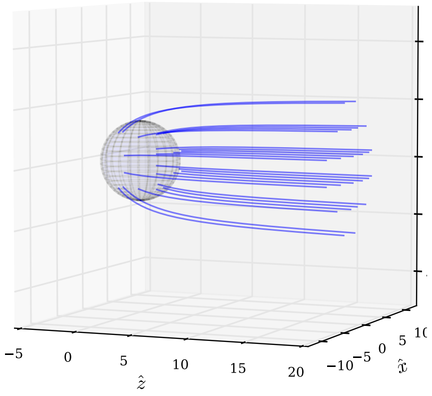

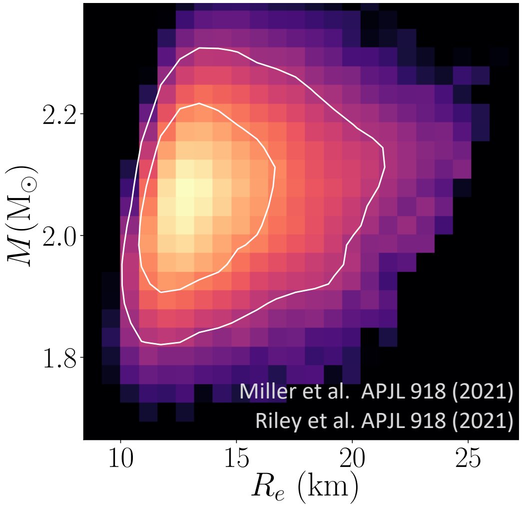

Properties of neutron stars reflect properties of dense matter