

# Bayesian Inference of Neutron Star Properties: When Astro and Nuclear physics go hand in hand

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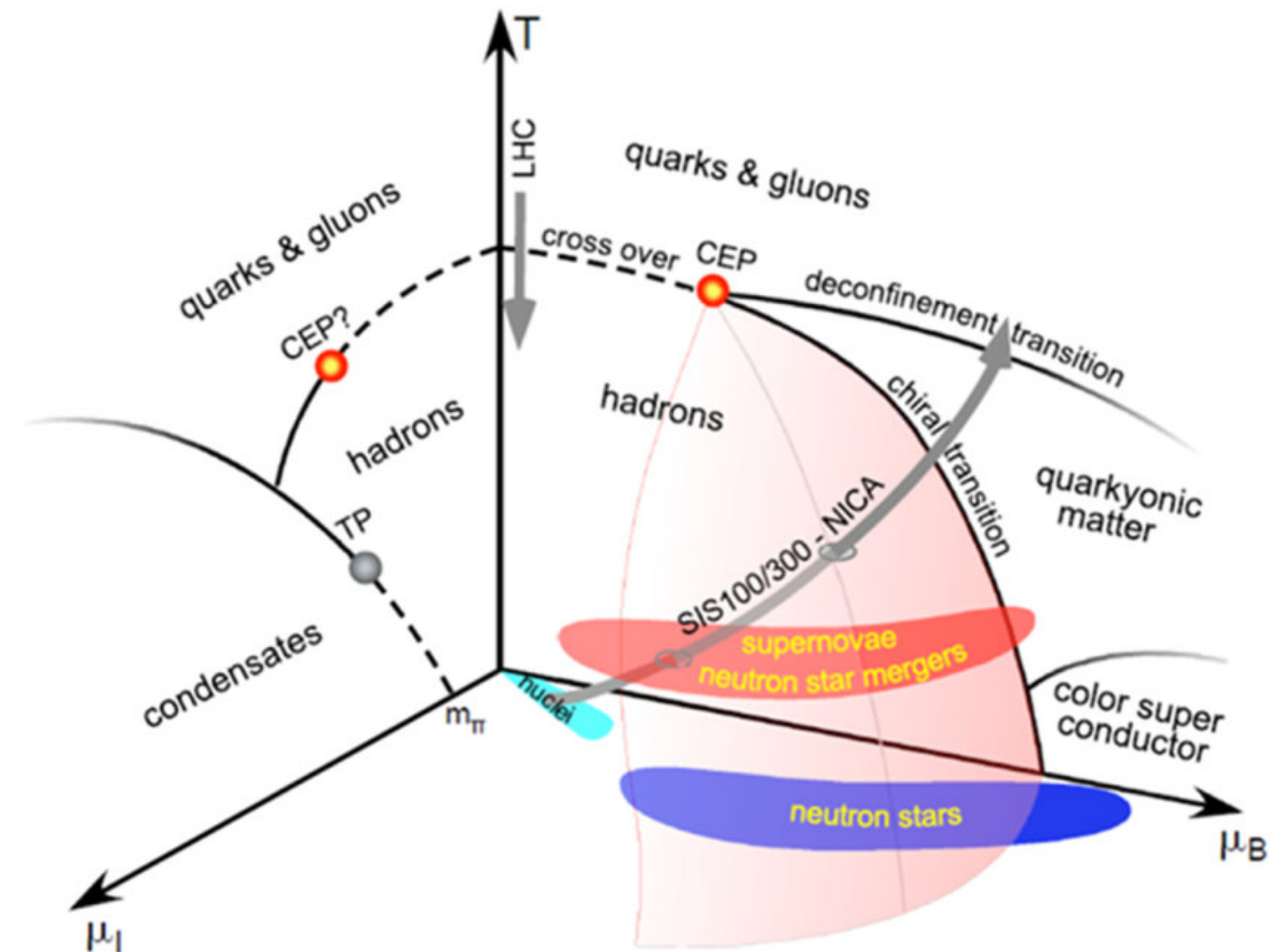
05/09/2025

# The QCD Phase Diagram & Neutron Star Matter

Neutron stars probe high-density,  
low-temperature QCD

$n_b$  up to  $\sim 10 n_0$   
**Ultradense**

$T \lesssim \text{MeV}$  and  $\epsilon_f \gtrsim 10 \text{ MeV}$   
**Cold**



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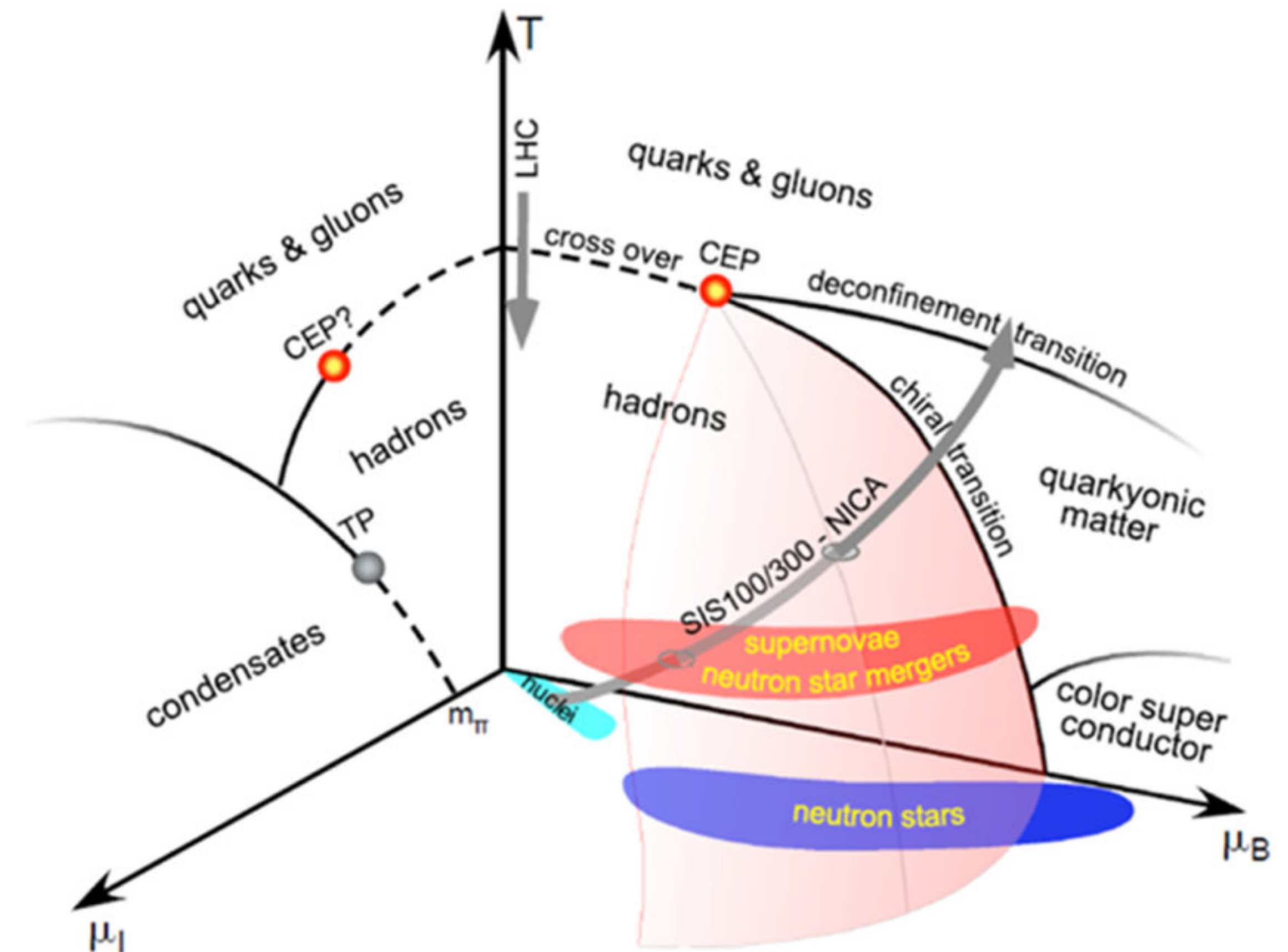
No direct  
experiments

$T \lesssim \text{MeV}$  and  $\epsilon_f \gtrsim 10 \text{ MeV}$

**Cold**

No theoretical  
derivation

Nucleonic  $\longrightarrow$  Hyperonic  $\longrightarrow$  Quark matter?



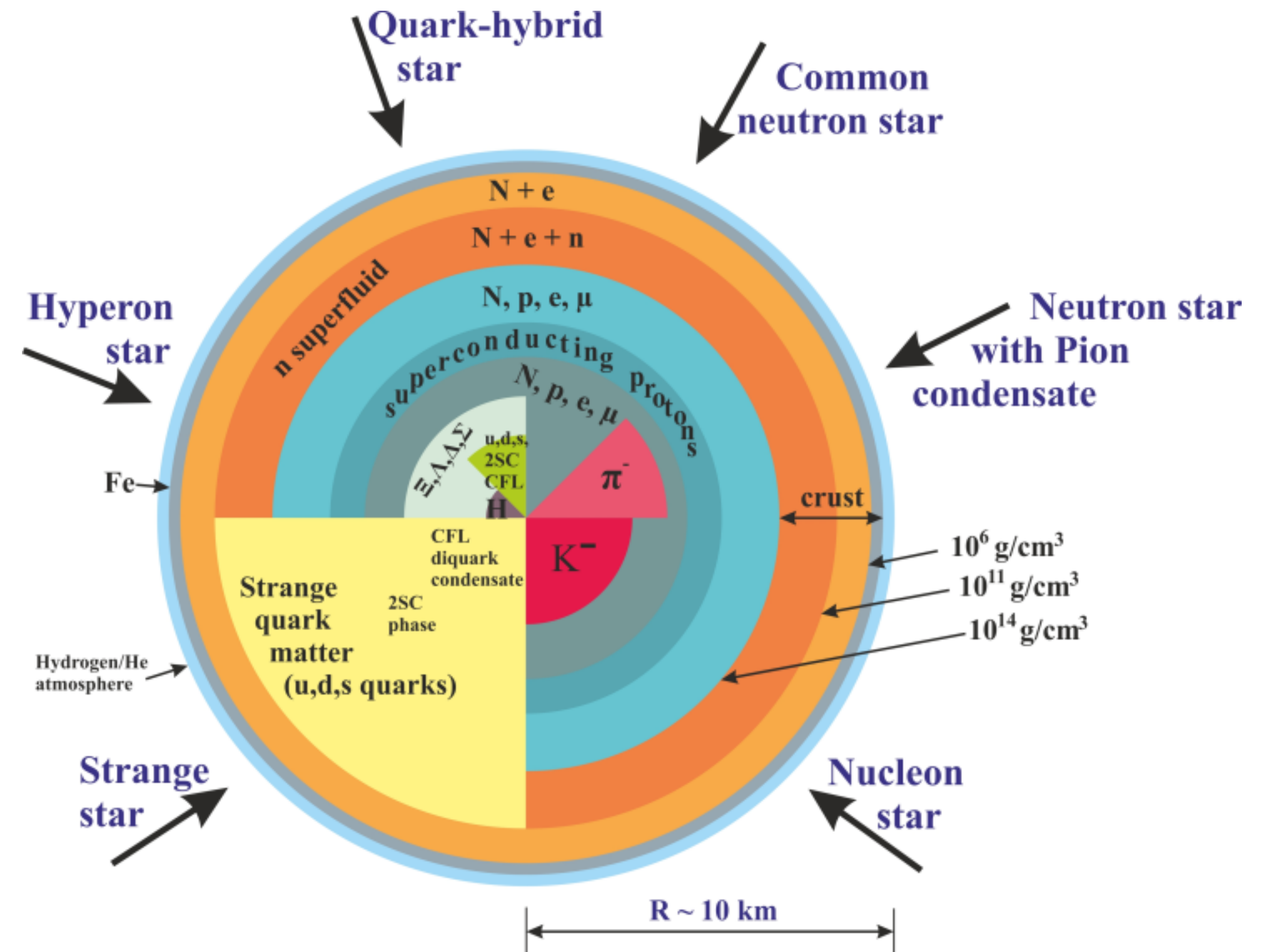


# Uncertain Composition of Neutron Star Interiors

We can't observe the  
**internal structure**

Compressing matter liberates  
degrees of freedom

We need **astro-observables** to  
disentangle the different  
possibilities





# Neutron star static observables: Mass - Radius

The TOV impose an **univocal relation**

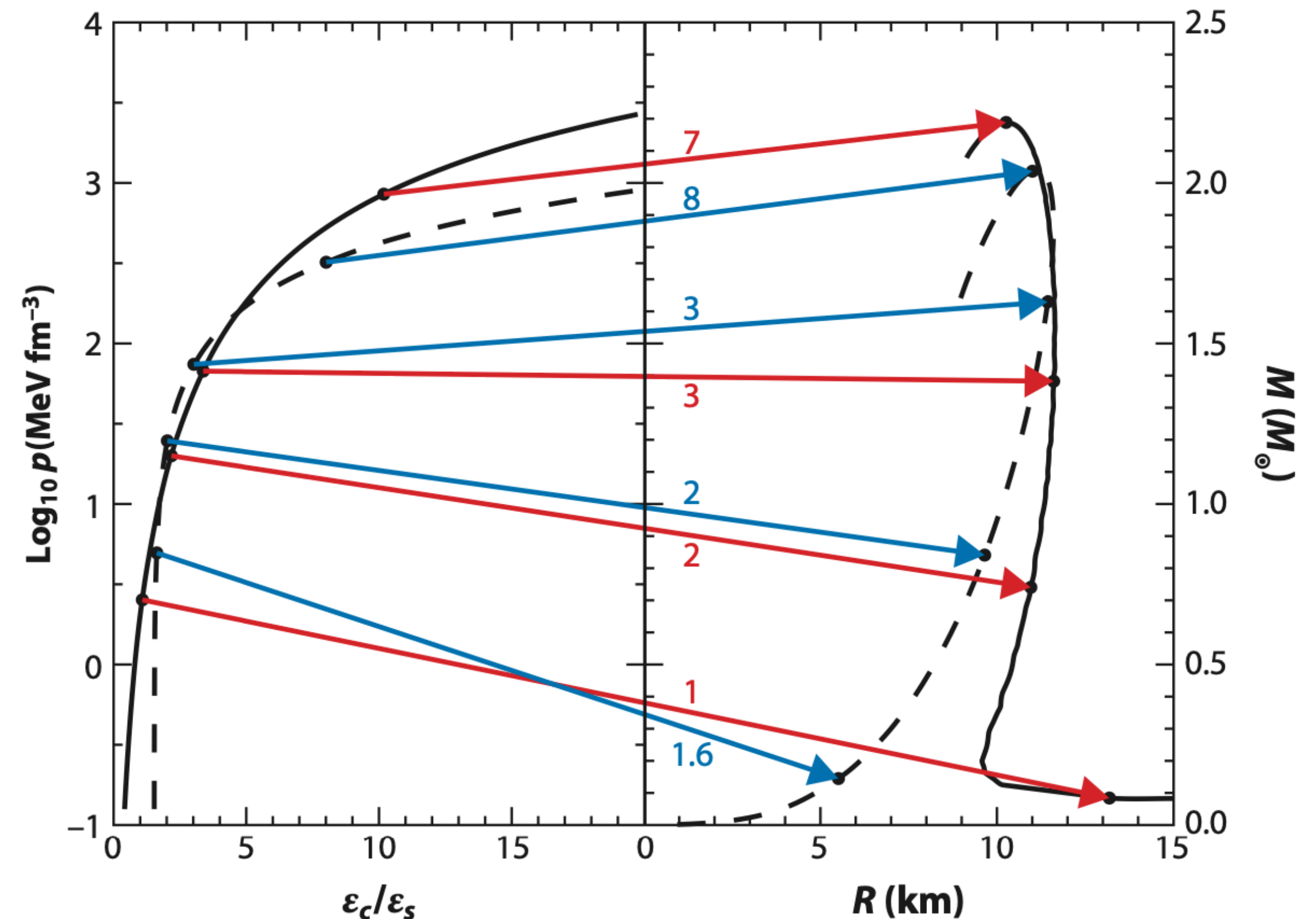
$$P(\rho) \Longleftrightarrow M(R)$$

By solving them for different central pressure (or densities) we obtain the  $M(R)$

$$\frac{dP}{dr} = - \frac{(\rho + P)(M + 4\pi r^3 P)}{r(r - 2M)}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

J.Lattimer Ann.Rev.Nucl.Part.Sci 2012



# Neutron star static observables: Mass - Radius

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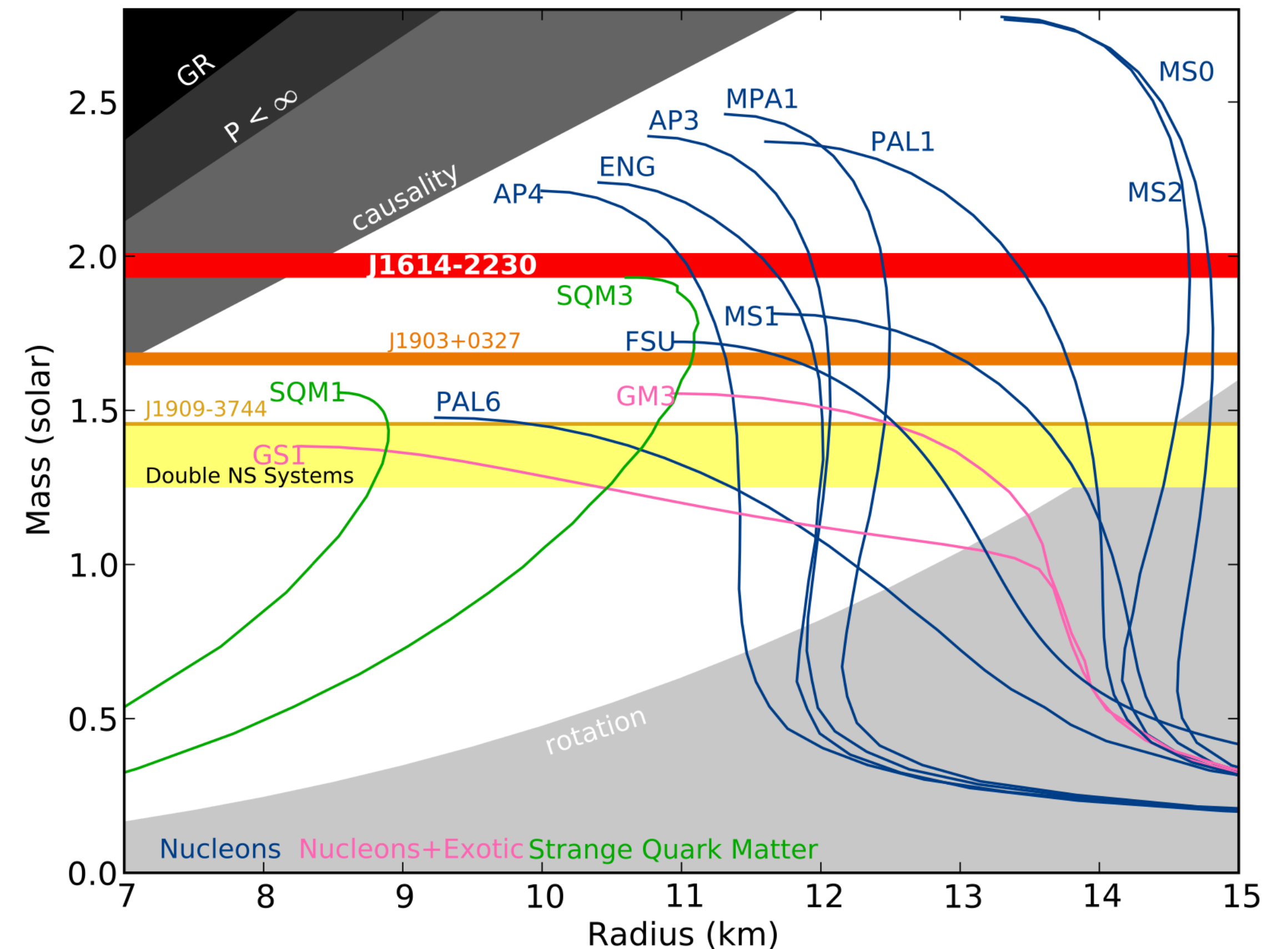
By solving them for different central pressure (or densities) we obtain the  $M(R)$

**Direct probe of the equation of state**

Stiffer EoS  $\longrightarrow$  Larger radius

Softer EoS  $\longrightarrow$  Smaller radius

Demorest et al, Nature 467 (2010)



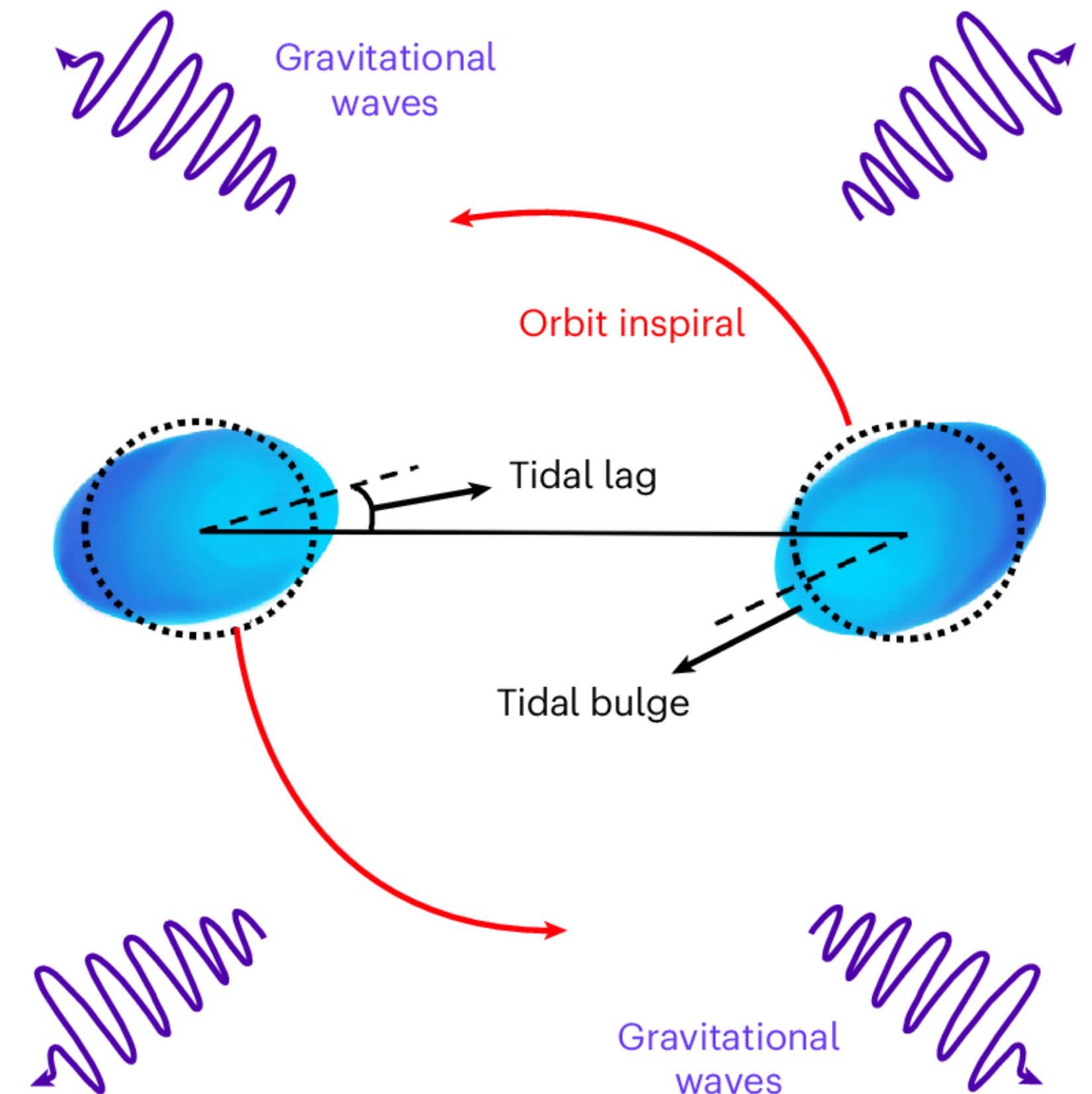
# Neutron star static observables: Tidal deformability

**Response of a neutron star's** to its  
companion's tidal field in the static limit

Ripley, Hegade, Chandramouli,  
Nature Astronomy (2024)

Quantified by the **dimensionless tidal  
deformability**

$$\Lambda = \frac{2}{3} \frac{k_2(P(\rho))}{C^5} = \frac{2}{3} k_2(P(\rho)) \left( \frac{R}{M} \right)^5$$





# Neutron star static observables: Tidal deformability

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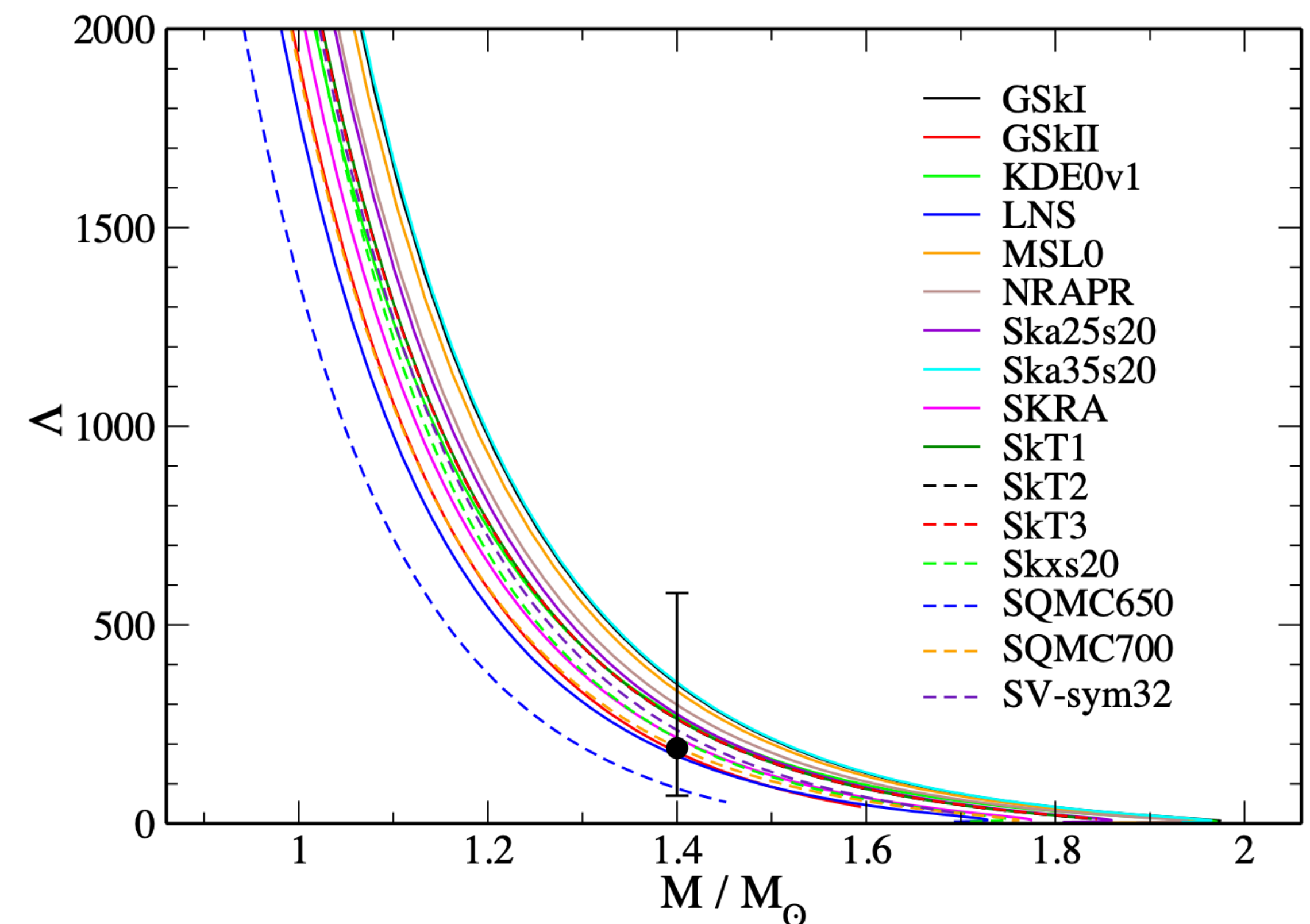
$$\Lambda = \frac{2}{3} \frac{k_2(P(\rho))}{C^5} = \frac{2}{3} k_2(P(\rho)) \left( \frac{R}{M} \right)^5$$

Encodes information about the **EoS**

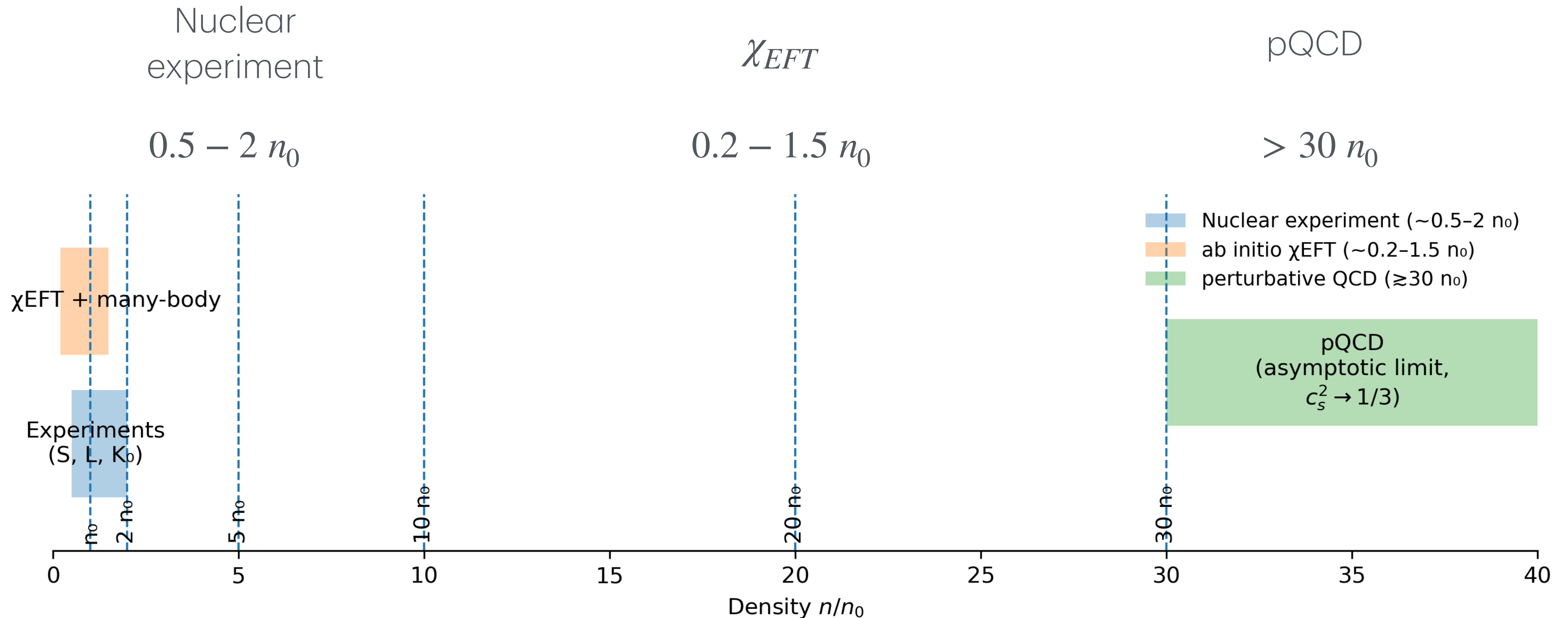
Stiffer EoS  $\longrightarrow$  Larger radius  $\longrightarrow$  Larger  $\Lambda$

Softer EoS  $\longrightarrow$  Smaller radius  $\longrightarrow$  Smaller  $\Lambda$

Leaves imprints in gravitational wave  
signals during inspiral



# The quest for the Nuclear EoS: Information across scales



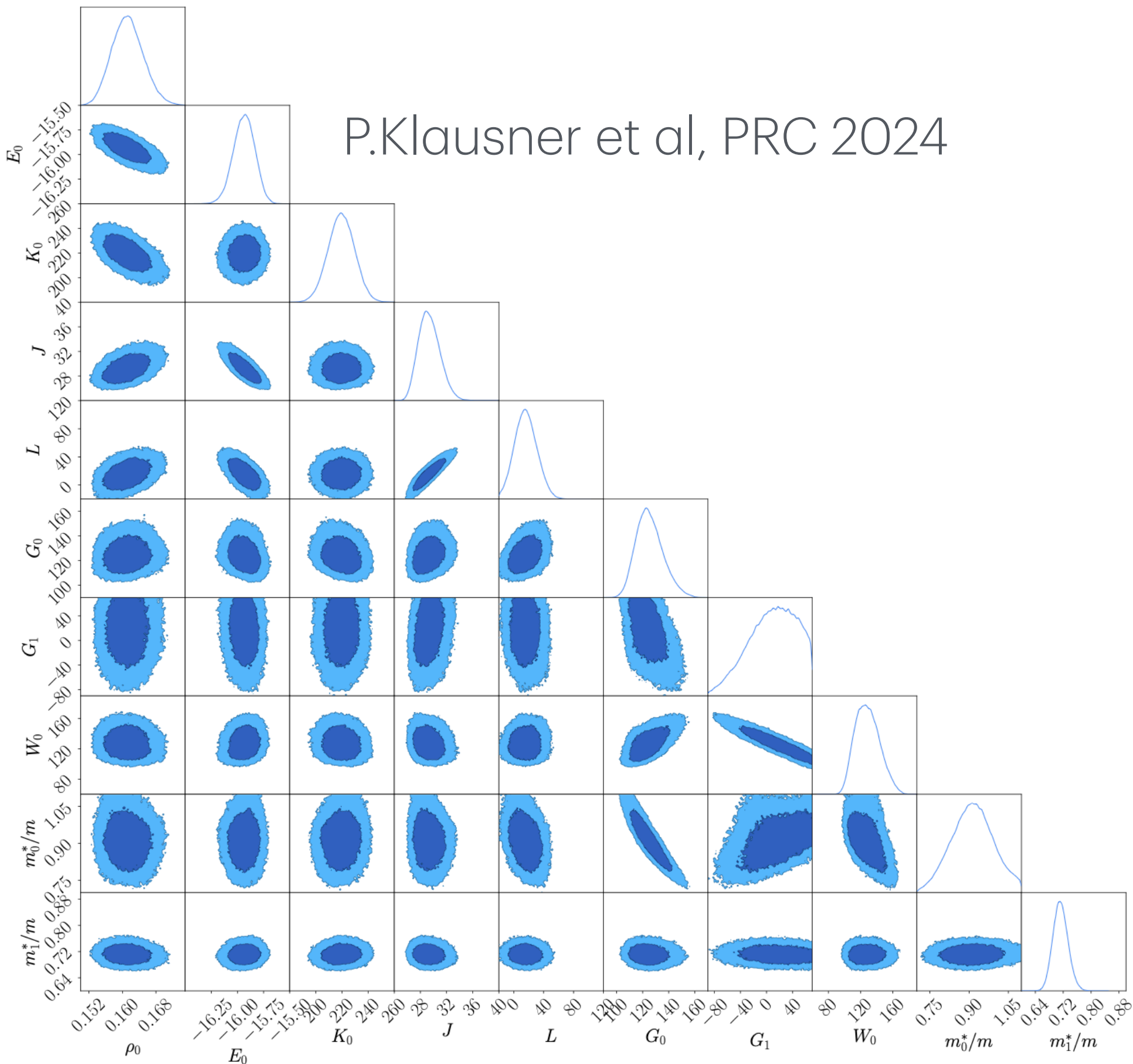
# The quest for the Nuclear EoS: Nuclear experiment

Nuclear structure constrain the EOS near  
saturation

Ground-state properties			
	$B.E.$ [MeV]	$R_{\text{ch}}$ [fm]	$\Delta E_{\text{SO}}$ [MeV]
$^{208}\text{Pb}$	$1636 \pm 1.8$	$5.49 \pm 0.03$	$2.34 \pm 0.16$
$^{48}\text{Ca}$	$417 \pm 1.2$	$3.51 \pm 0.02$	$1.92 \pm 0.20$
$^{40}\text{Ca}$	$342 \pm 1.6$	$3.50 \pm 0.02$	-
$^{56}\text{Ni}$	$482 \pm 1.4$	-	-
$^{68}\text{Ni}$	$590 \pm 1.0$	-	-
$^{100}\text{Sn}$	$826 \pm 1.6$	-	-
$^{132}\text{Sn}$	$1103 \pm 1.7$	$4.71 \pm 0.03$	-
$^{90}\text{Zr}$	$784 \pm 1.3$	$4.27 \pm 0.02$	-

Isoscalar resonances		
	$E_{\text{GMR}}^{\text{IS}}$ [MeV]	$E_{\text{GQR}}^{\text{IS}}$ [MeV]
$^{208}\text{Pb}$	$13.5 \pm 0.3$	$10.8 \pm 0.4$
$^{90}\text{Zr}$	$17.8 \pm 0.4$	-

Isovector properties			
	$\alpha_D$ [fm <sup>3</sup> ]	$m(1)$ [MeV fm <sup>2</sup> ]	$A_{PV}$ [p.p.b.]
$^{208}\text{Pb}$	$19.5 \pm 0.5$	$958 \pm 22$	$589 \pm 5$
$^{48}\text{Ca}$	$2.30 \pm 0.08$	-	$2591 \pm 54$





# The quest for the Nuclear EoS: Ab initio Nuclear theory

Interaction from chiral effective field theory  $\chi_{EFT}$

Diagrammatic  
expansion



Controlled truncation  
errors

Momentum  
expansion

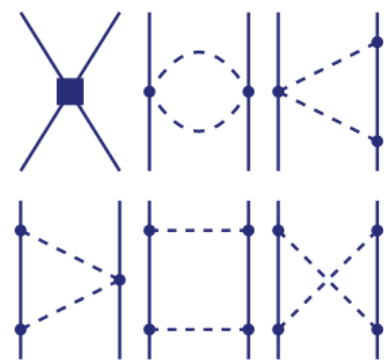


Only valid at low density

**LO**  
 $(Q/\Lambda_\chi)^0$



**NLO**  
 $(Q/\Lambda_\chi)^2$

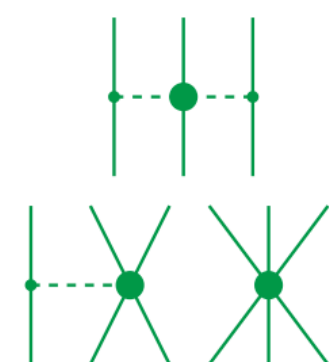


**NNLO**  
 $(Q/\Lambda_\chi)^3$



**3N Force**

Machleidt R., Int J Mod Phys.  
(2017)



# The quest for the Nuclear EoS: Ab initio Nuclear theory

Interaction from chiral effective field theory  $\chi_{EFT}$

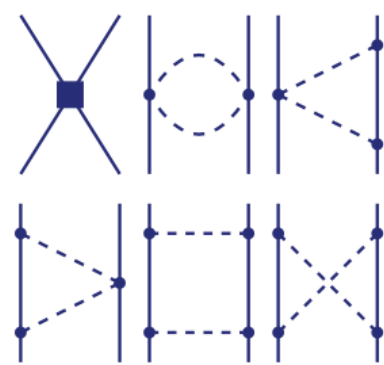
Diagrammatic expansion  $\longrightarrow$  Controlled truncation errors

Momentum expansion  $\longrightarrow$  Only valid at low density

**LO**  
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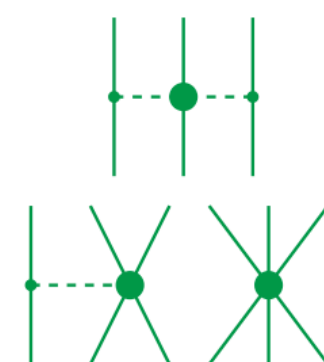
**NNLO**  
 $(Q/\Lambda_\chi)^3$



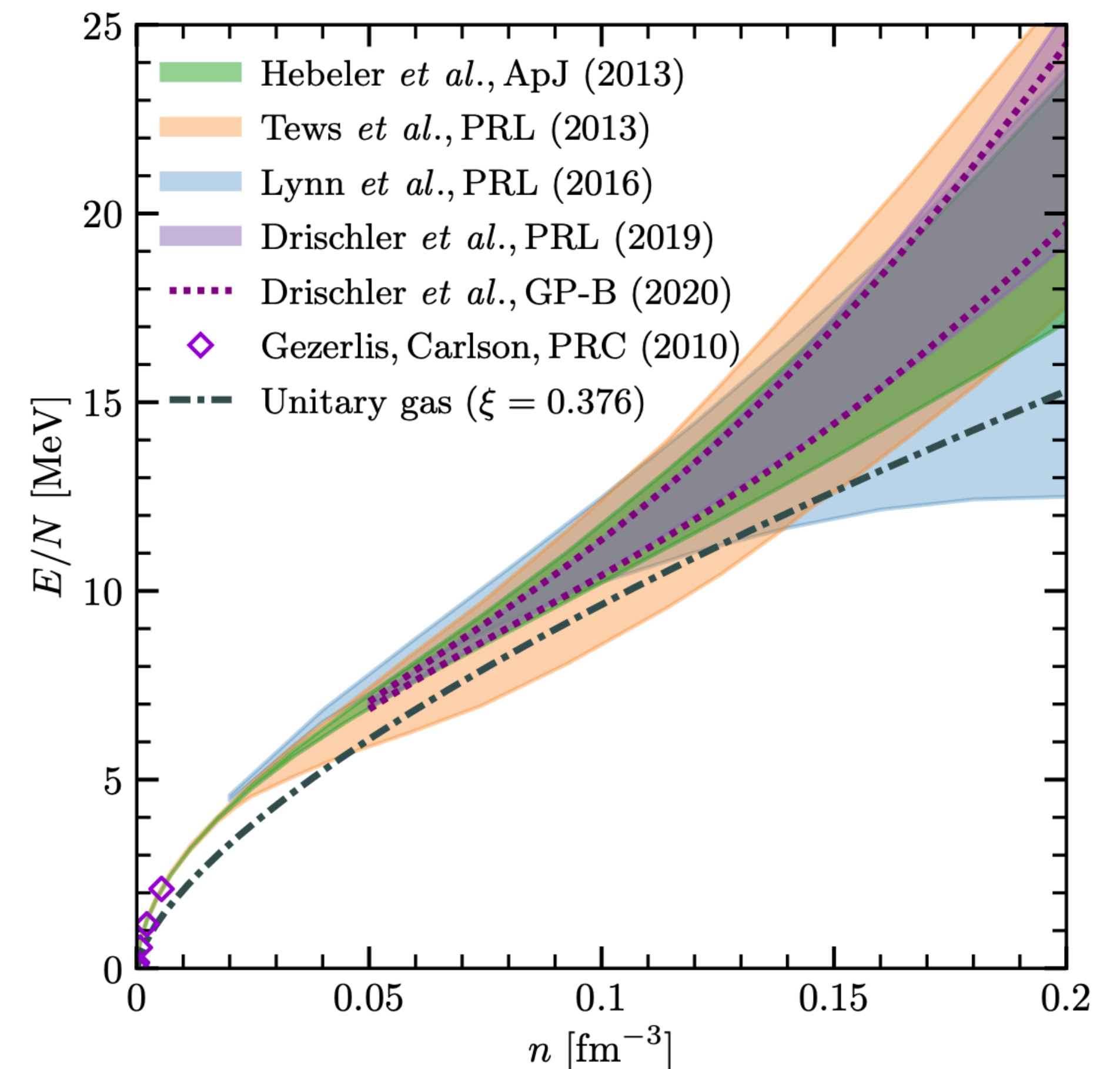
**2N Force**

**3N Force**

Machleidt R., Int J Mod Phys.  
(2017)



S. Huth et al, Phys. Rev. C 103, 025803  
(2021).

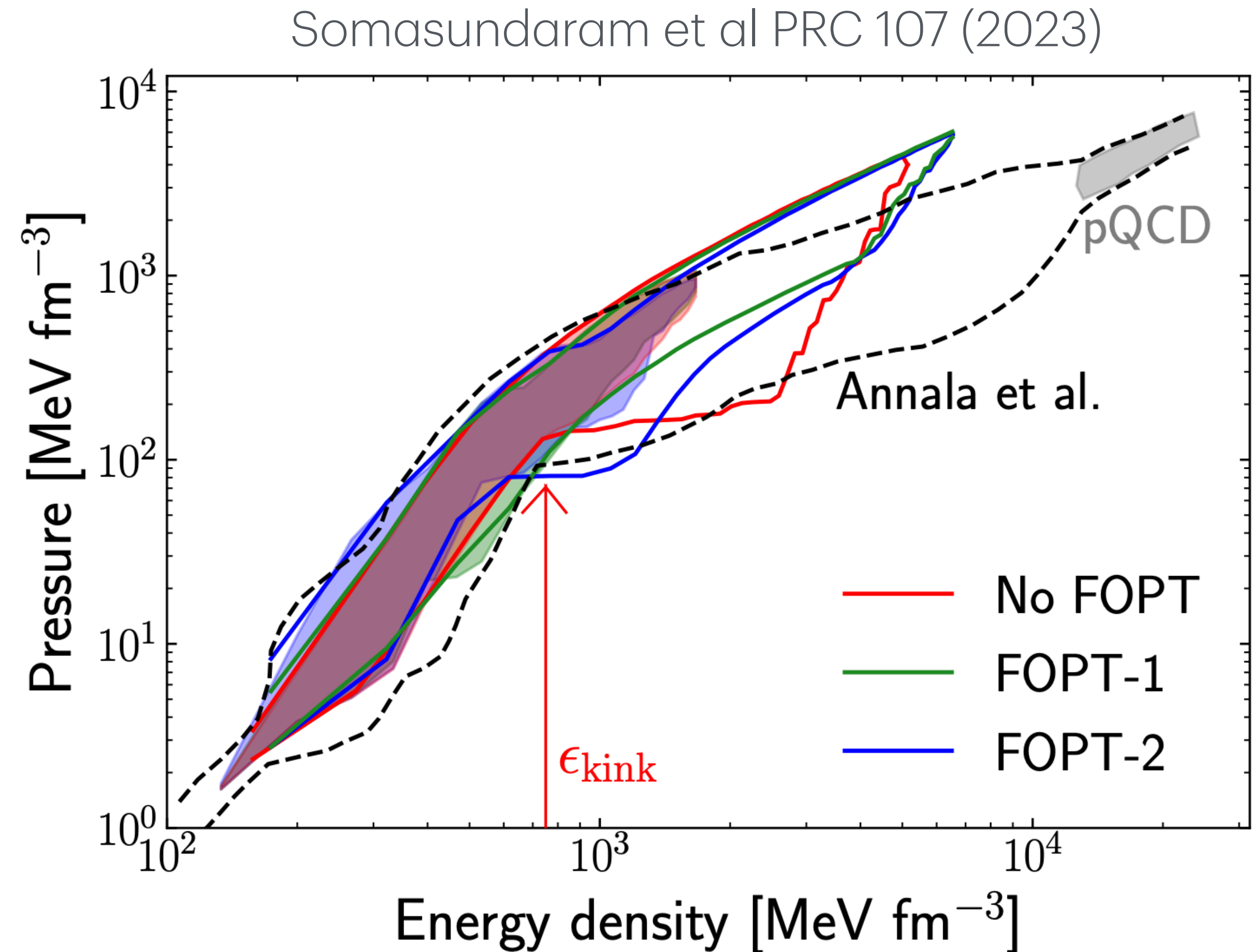


# The quest for the Nuclear EoS: Perturbative QCD

At very high  $\mu_b$  QCD is tractable



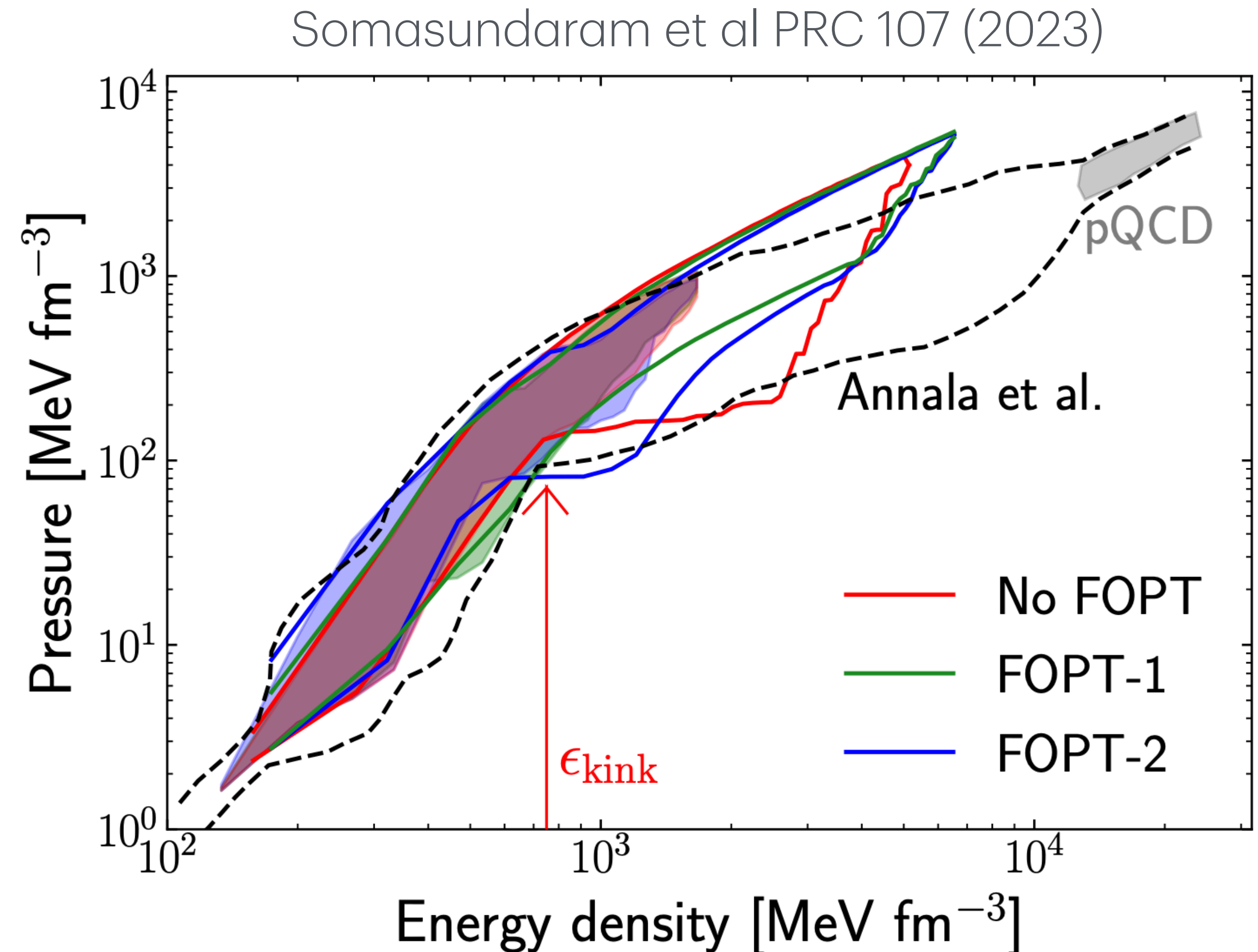
EoS must be able to approach it consistently  
with thermodynamic and causality



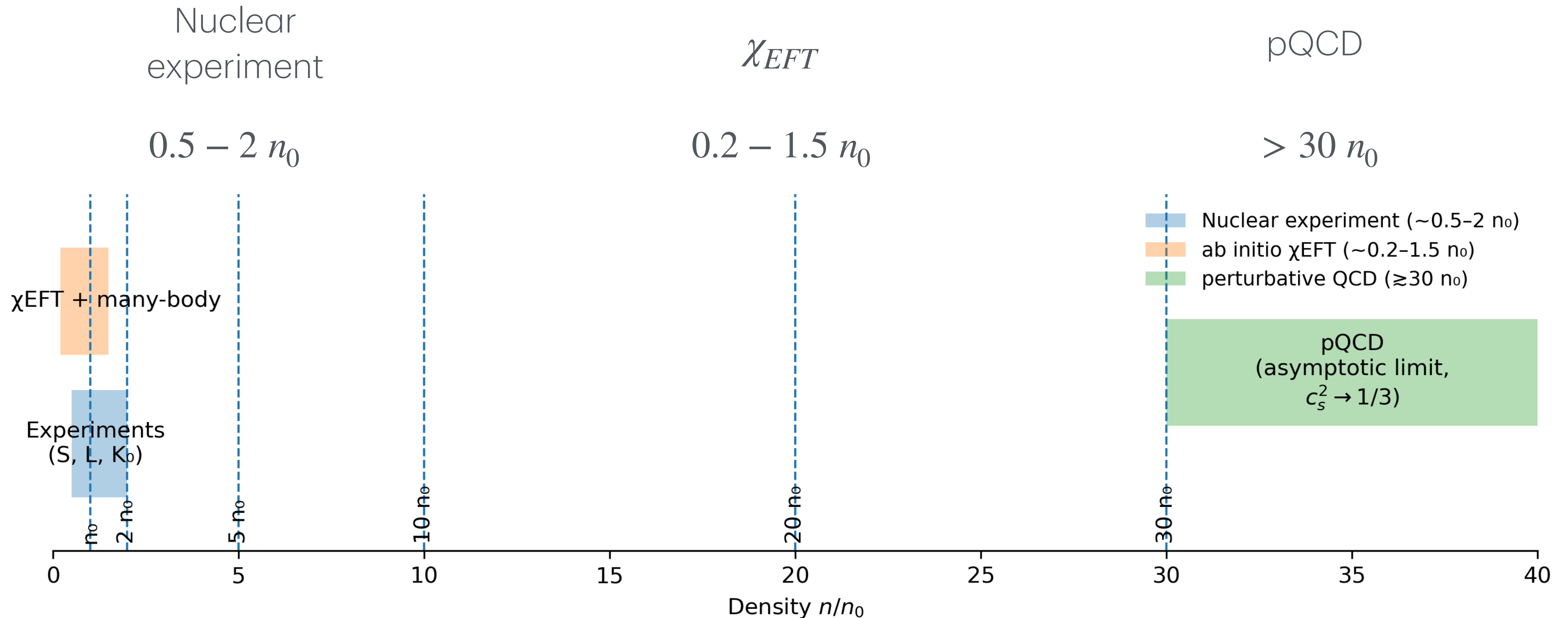


# The quest for the Nuclear EoS: Perturbative QCD

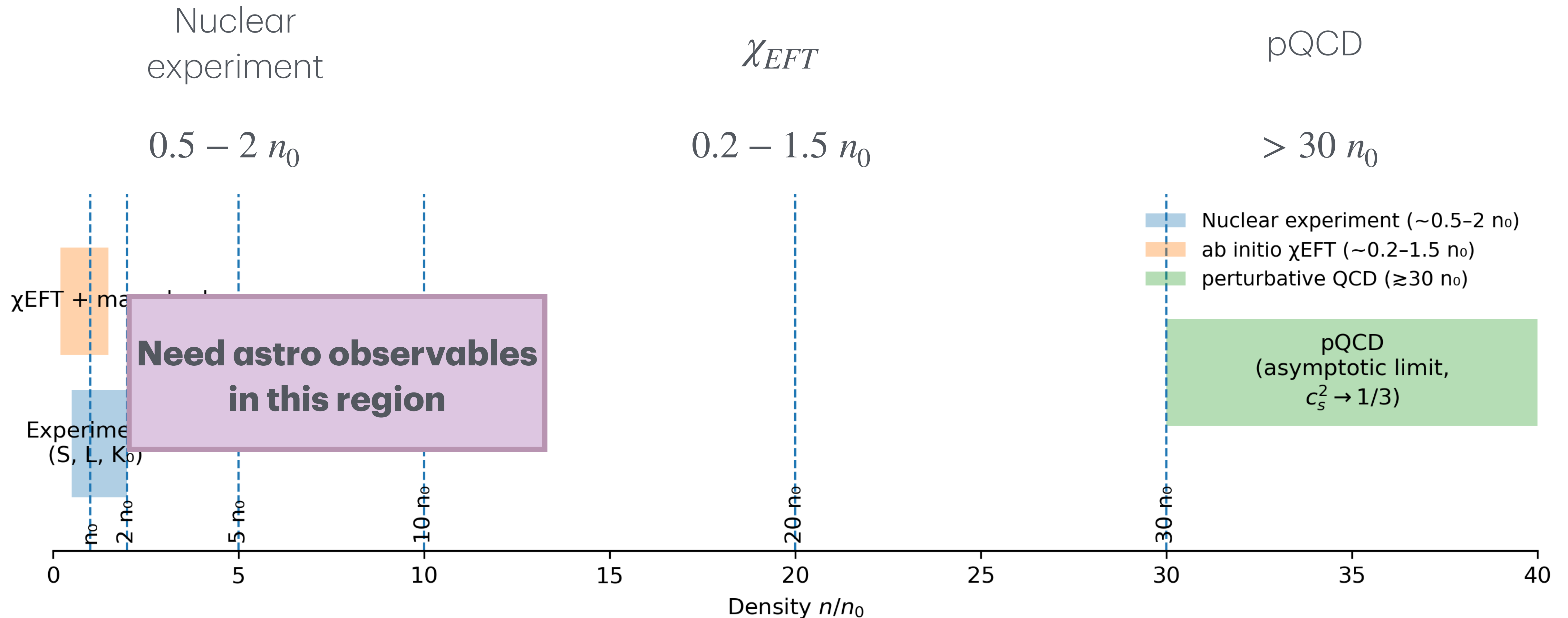
At very high  $\mu_b$  QCD is tractable  
↓  
EoS must be able to approach it consistently  
with thermodynamic and causality  
  
It has a softening effect



# The quest for Nuclear EoS: Complementing with astro-observables



# The quest for Nuclear EoS: Complementing with astro-observables





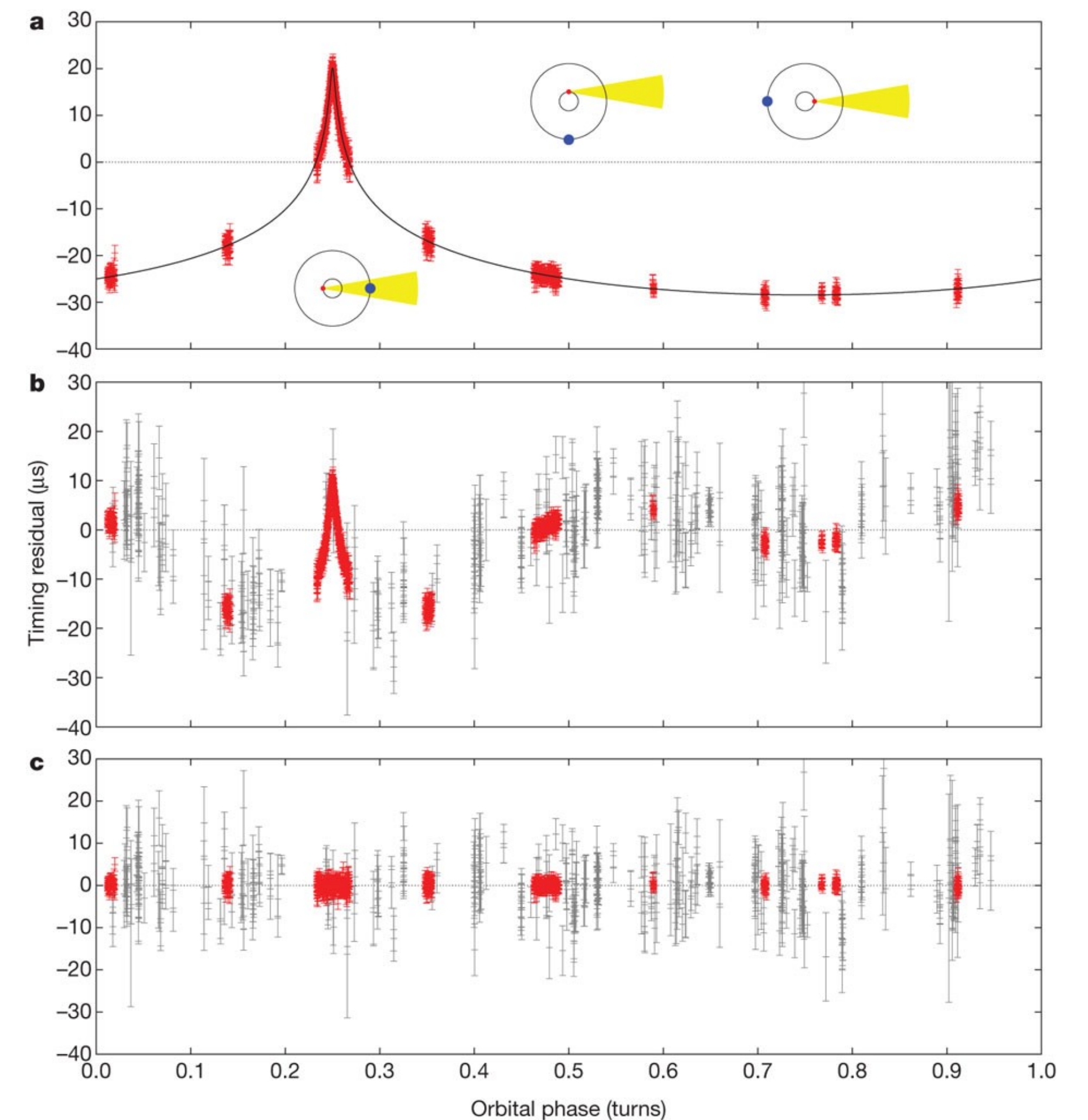
# Measuring Neutron Star Masses with the Shapiro Delay

Signal delay caused when **pulsar radio waves** pass near a massive companion's

$$\Delta t \propto -2 \frac{GM_c}{c^3} \ln(1 - \sin i \sin \phi)$$

$M_c$  and  $i$  **fitted on the measured curve**

Demorest et al, Nature 467 (2010)



# Measuring Neutron Star Masses with the Shapiro Delay

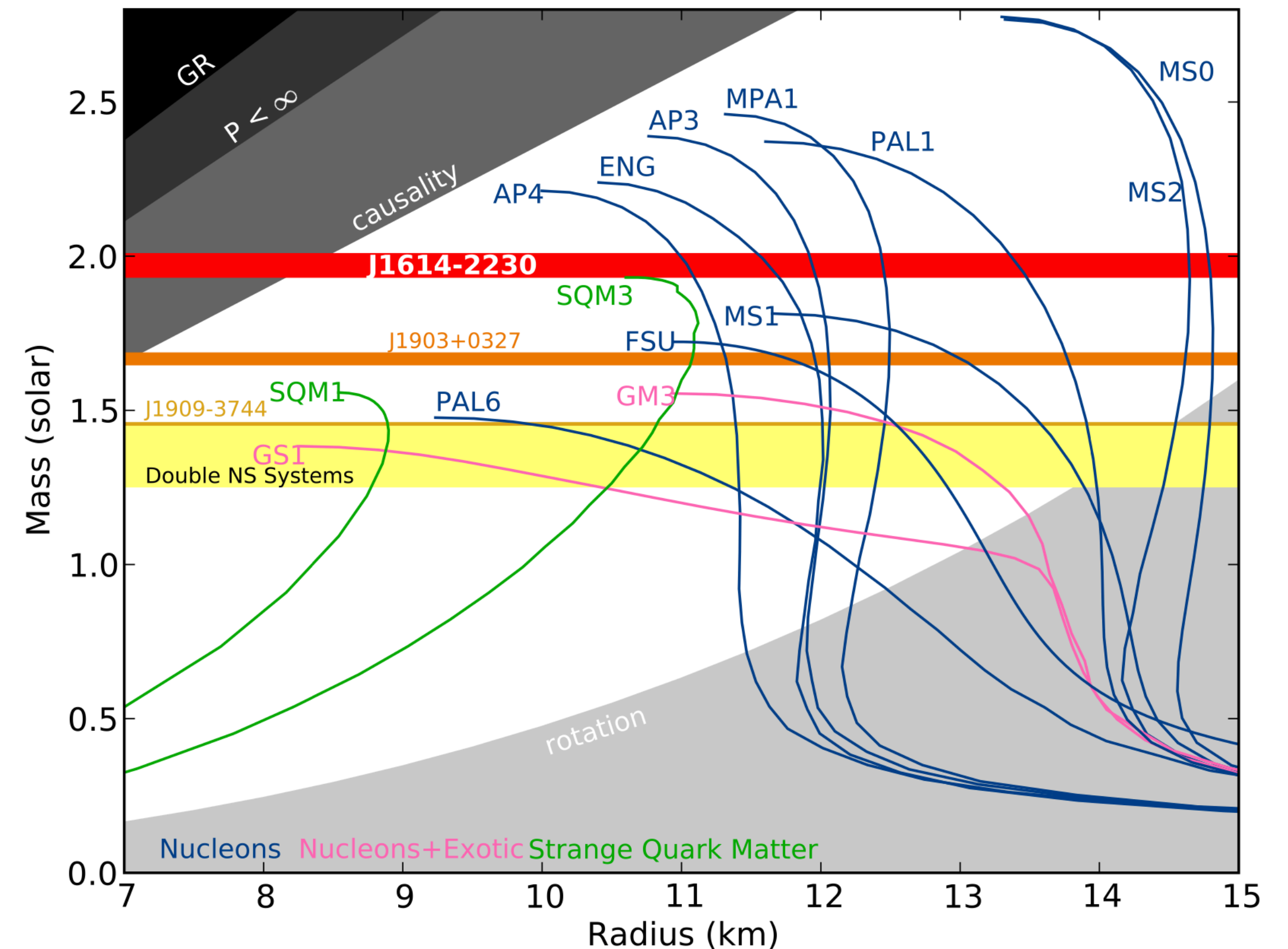
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$M_c$  and  $i$  **fitted on the measured curve**

Pulsar mass  $M_p$  **extracted from the period**  
once the rest is known

Demorest et al, Nature 467 (2010)





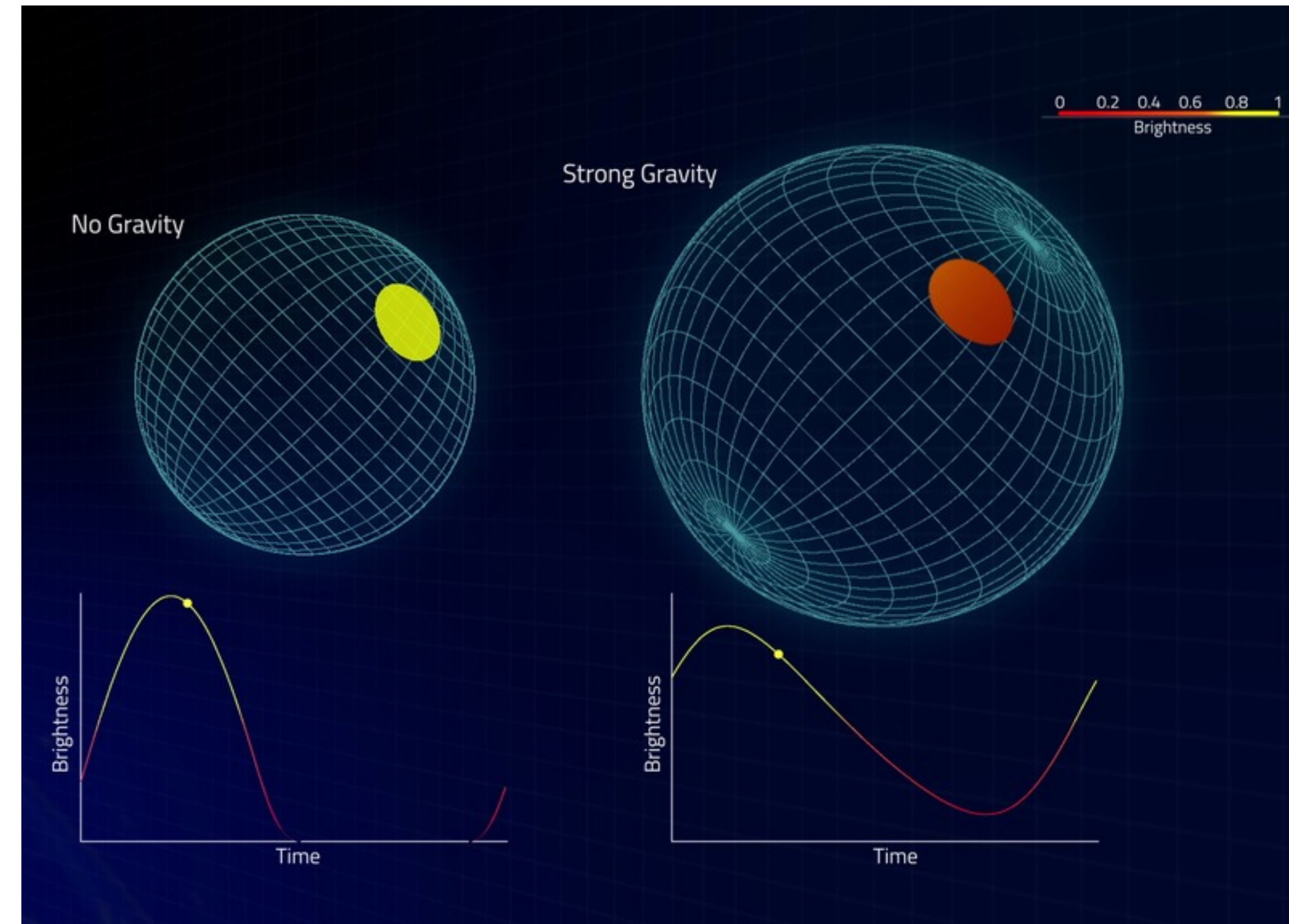
# NICER Constraints on the Dense Matter Equation of State

NICER (Neutron Star Interior Composition ExploreR)

Records X-rays from **rotating hot spots**  
with high time and energy resolution

The shape encodes **light bending**  
and other effects

Stronger bending  $\longrightarrow$  Higher compactness  
 $M/R$



# NICER Constraints on the Dense Matter Equation of State

NICER (Neutron Star Interior Composition ExploreR)

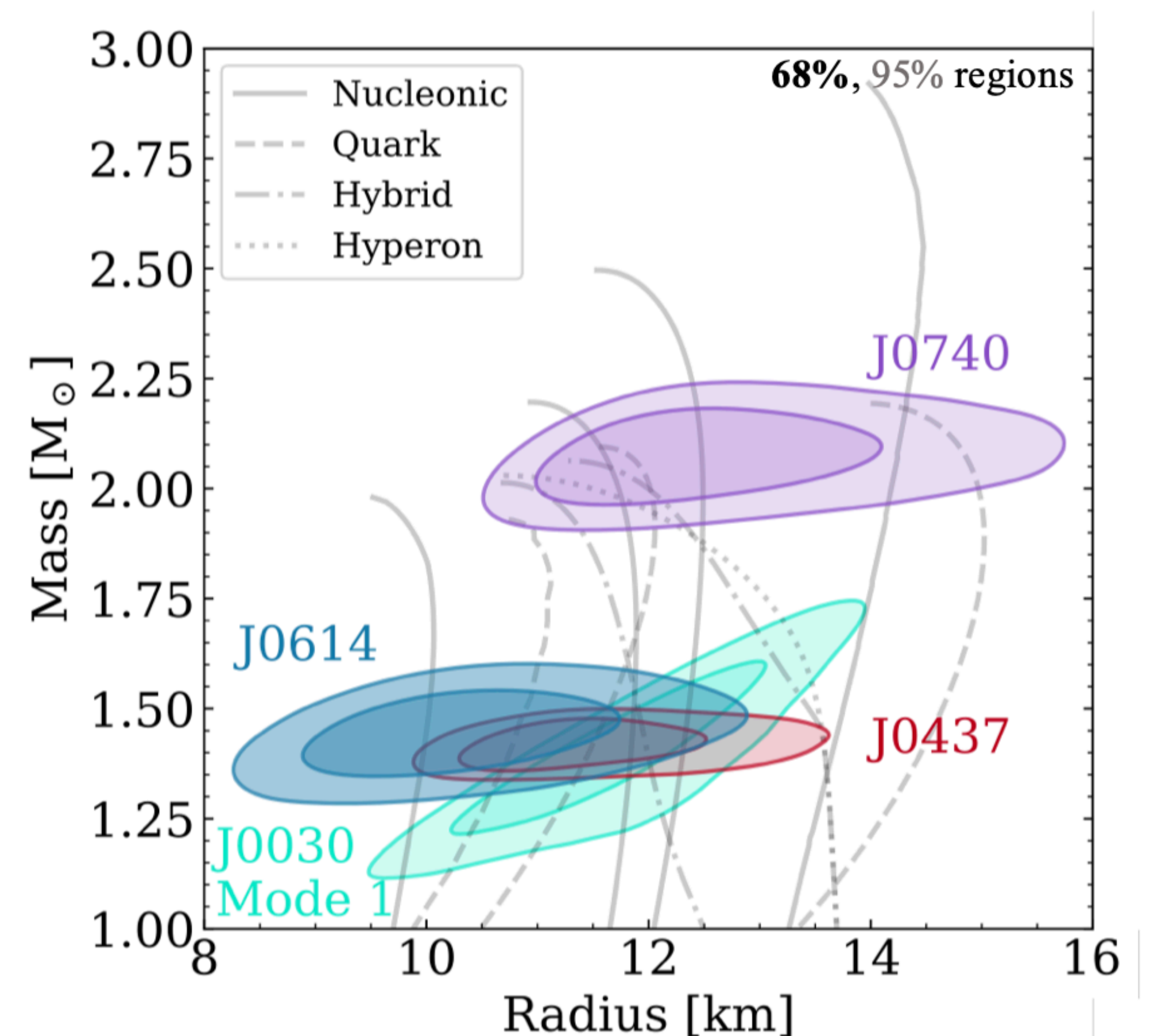
Records X-rays from **rotating hot spots**  
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The shape encodes **light bending**  
and other effects

Stronger bending  $\longrightarrow$  Higher compactness  
M/R

Combine with **mass estimates from  
other sources** to break degeneracy

Joint M-R probability





# Neutron Star Mergers & Gravitational Waves

E. Burns, Living Reviews in Relativity (2020)

## Late inspiral

Static properties:

- TOV + tidal
- NSs still cold

## Merger

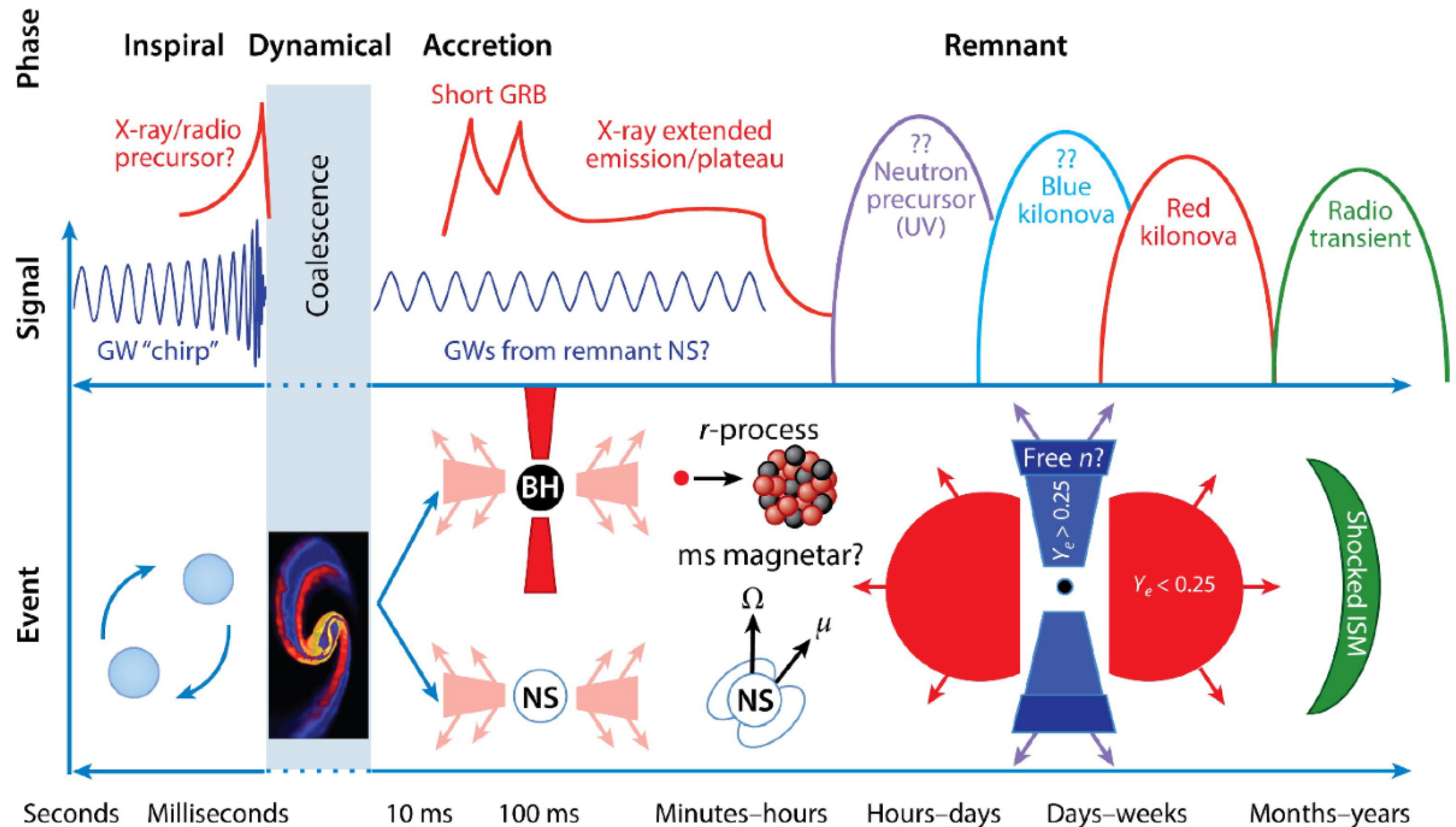
Expensive NR simulations

- Big limitation

## Post-merger

Hot and dense

- Harder to detect
- Carries a lot of EoSs information



# Extraction of the tidal effects

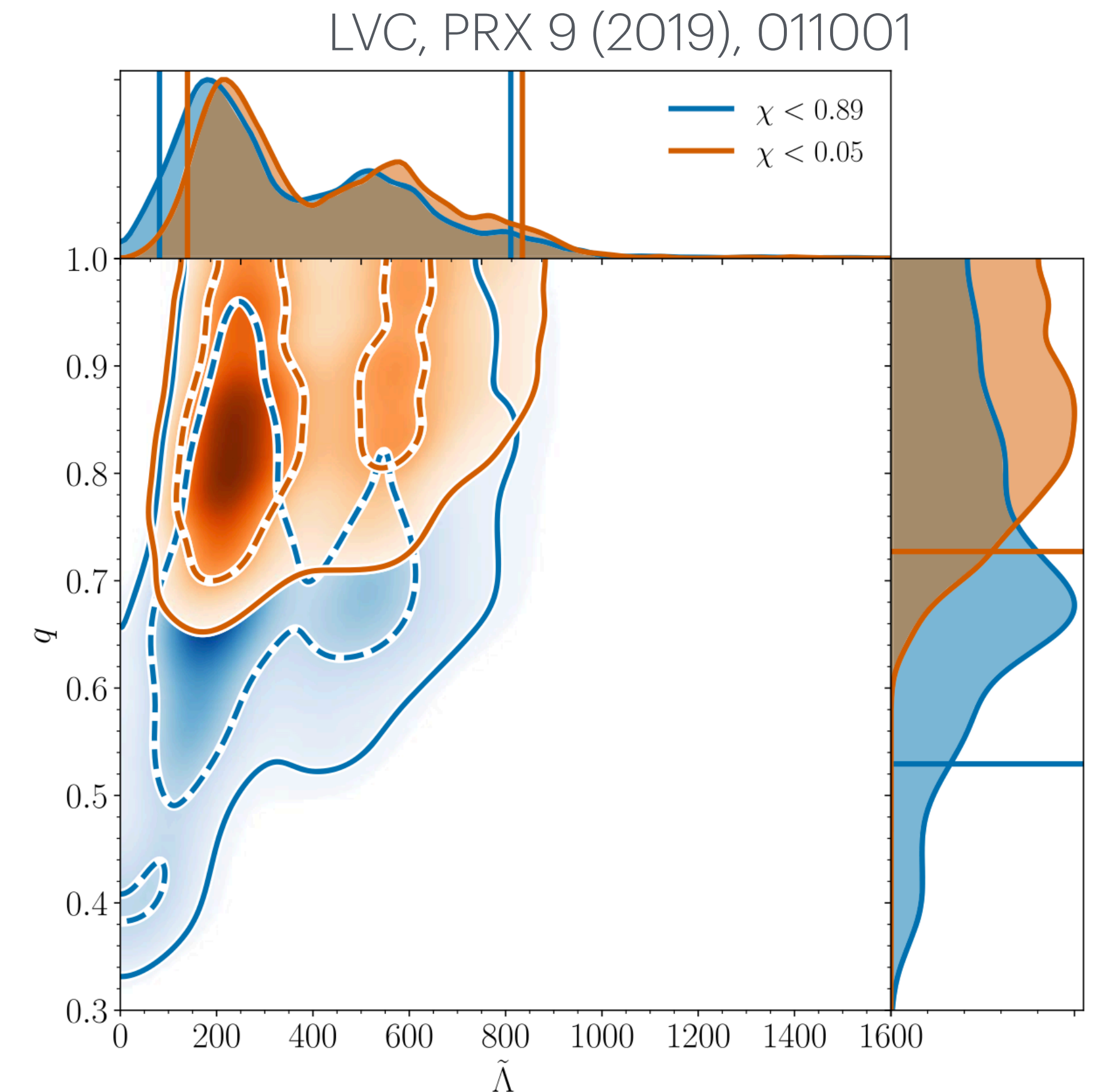
## Information from the inspiral:

→ The joint lambda  $\tilde{\Lambda}$  not  $\Lambda_1, \Lambda_2$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}$$

→ The chirp mass  $M_c$  and  $q$  not  $m_1, m_2$

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \quad q = \frac{m_2}{m_1}$$



# Modelling of the EoS: Agnostic frameworks

**Agnostic models** aim to describe the behavior of matter at extremely high densities, as found in NS, **without specific microphysics assumptions** (e.g., baryonic composition or phases like quark matter)

Microphysics at supranuclear densities  
is uncertain



Avoids bias from specific nuclear or  
particle physics models

Certain physical condition must be fulfilled,  
e.g. causality  $v_s < c$  and thermodynamic  
stability  $dP/d\epsilon > 0$



Designed to automatically enforce  
those

Can accommodate a wide variety of  
behaviors



Bayesian frameworks provide natural  
ways to estimate uncertainties and  
posterior distributions.

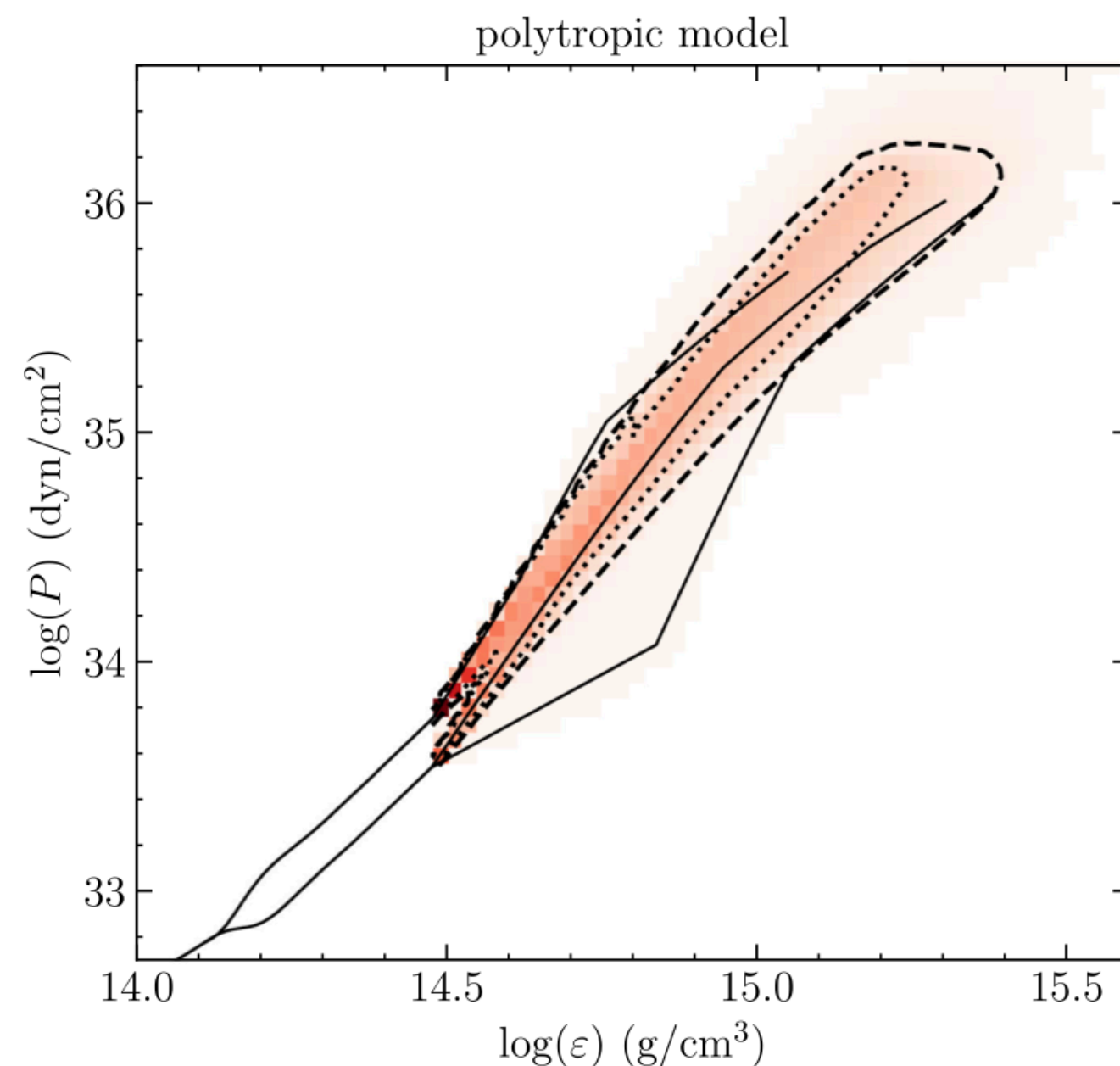


# Modelling of the EoS: Agnostic frameworks example

## Piecewise polytrope

Divide density range into segments, apply simple polytropic relation  $P = k\rho^\gamma$

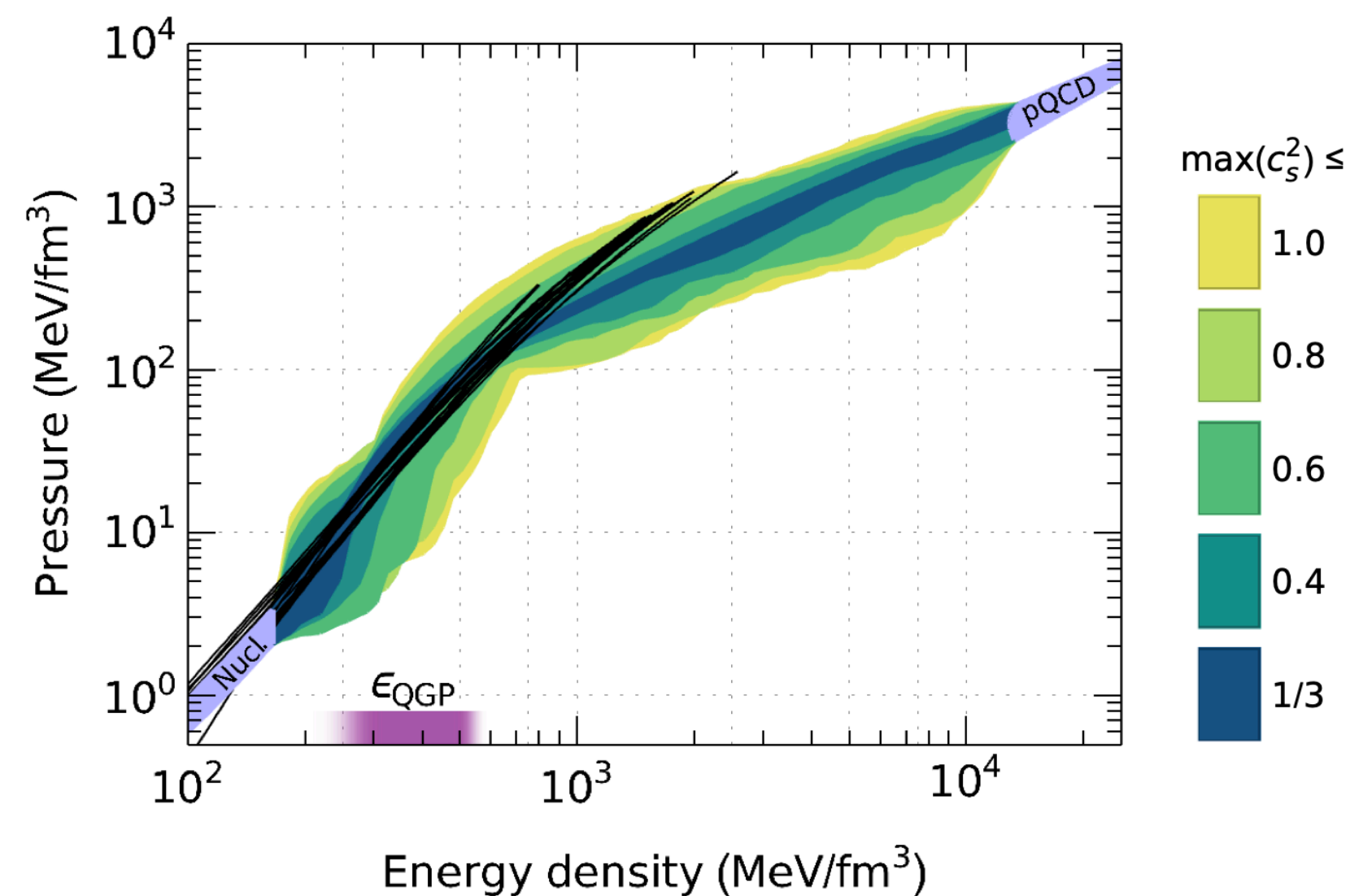
Greif et al 2019 MNRAS.485.5363G



## Speed of sound models

Parametrize directly the speed of sound

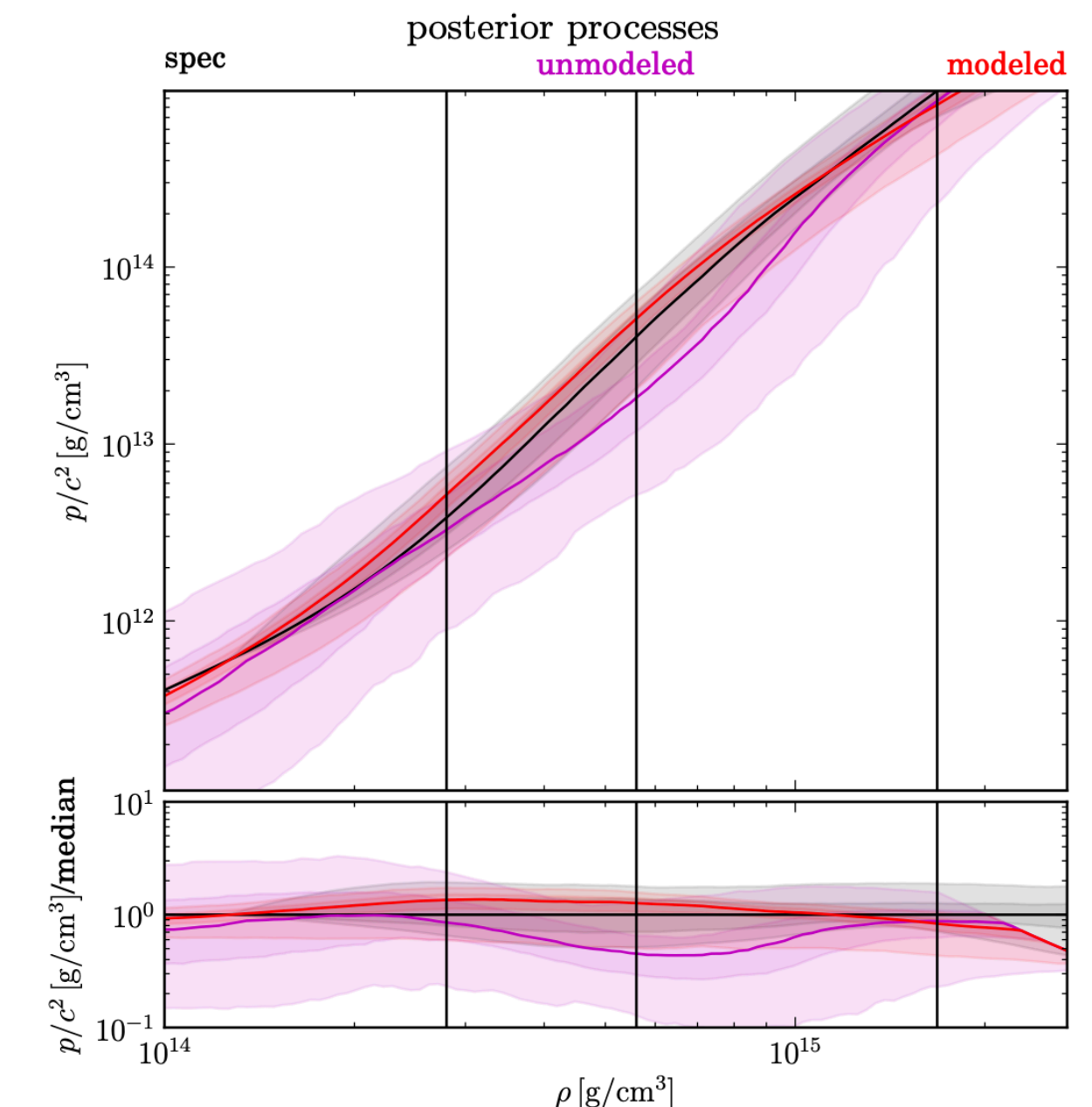
Annala et al Nature Physics (2020)



## Gaussian processes

Models EoS as a random function constrained by data

Landry et al PhysRevD.99.084049





# Modelling of the EoS: Density functional approach

An **Energy Density Functional** is a tool from nuclear many-body theory used to describe the energy of a nuclear system as a functional of the local densities

Experimental data above nuclear saturation density are limited



Computationally tractable way to model such systems

$e(\rho_b, \rho_l, \rho_s)$  directly modeled, giving access to the composition



Cooling, nucleosynthesis, out-of- $\beta$ -equilibrium effect and transport properties

They are mostly tuned on finite nuclei and low density ab-initio



Natural extension to non-homogeneous phase in NS, such as crustal lattice

# Modelling of the EoS: Density functional approach

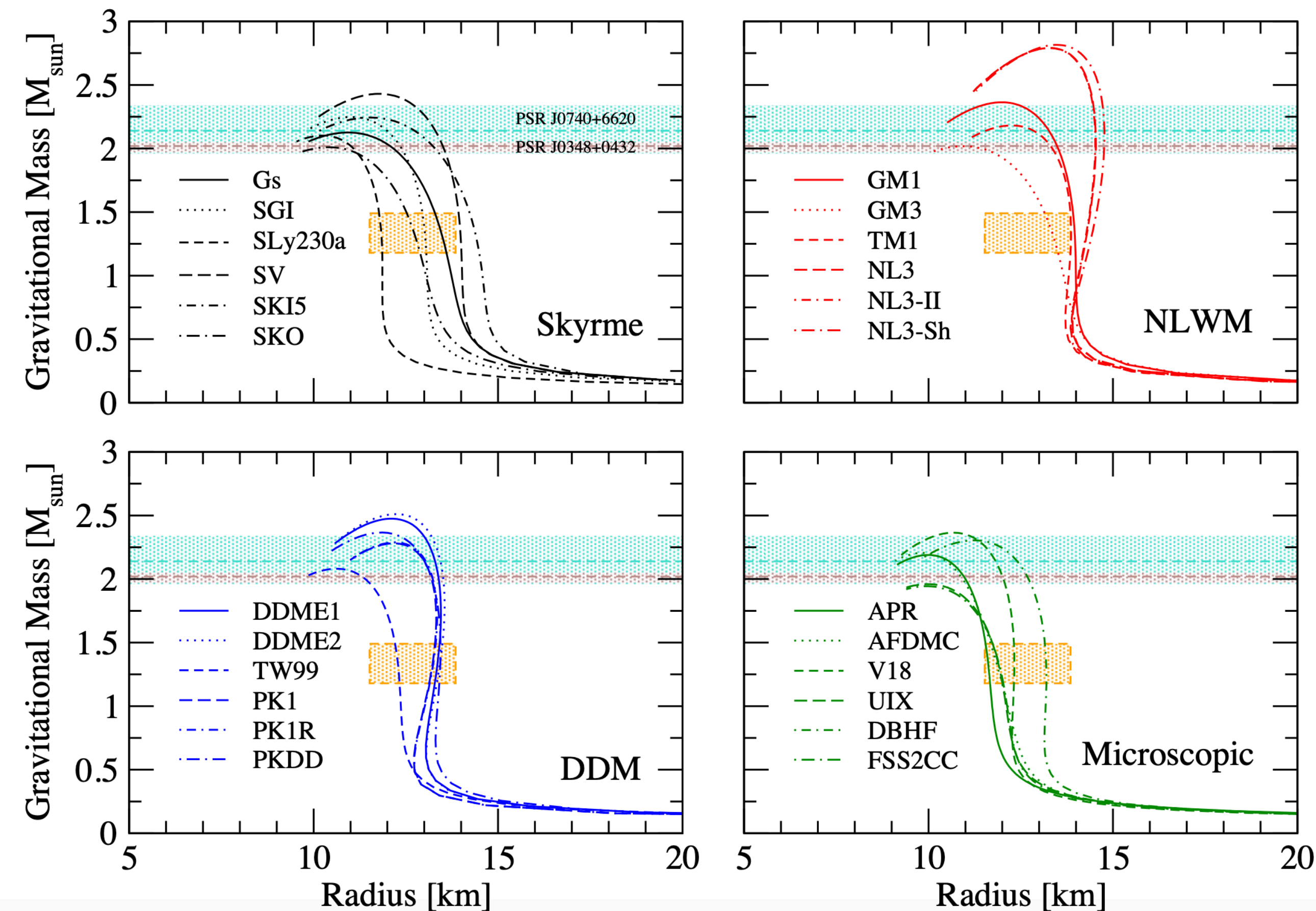
## Choice of functional form

Model dependent phenomenological approach  
with limited connection to fundamental theory

## Extrapolation Uncertainty at High Densities

Typically fitted to data at or below nuclear  
saturation density  $n_0$  but NS exceed it by several  
times

Uncertainty grows rapidly with density,  
predictions diverge among different EDF



# Modelling of the EoS: Summary of the strategies

Two possible directions

From nuclear to astro

$$\epsilon(n_p, n_n) \Rightarrow P(\rho) \Rightarrow M, R, \Lambda$$

Controlled dof, hypotheses and approximations, exp info included

The predictive power for astro observables is limited, lack of description for exotic phases

From astro to “nuclear”

$$M, R, \Lambda \Rightarrow P(\rho) \nRightarrow \epsilon(n_p, n_n)$$

Agnostic and parametric modeling of the equation of state

Not enough constraints for the EoS, some observables not accessible (e.g. cooling or g-modes)

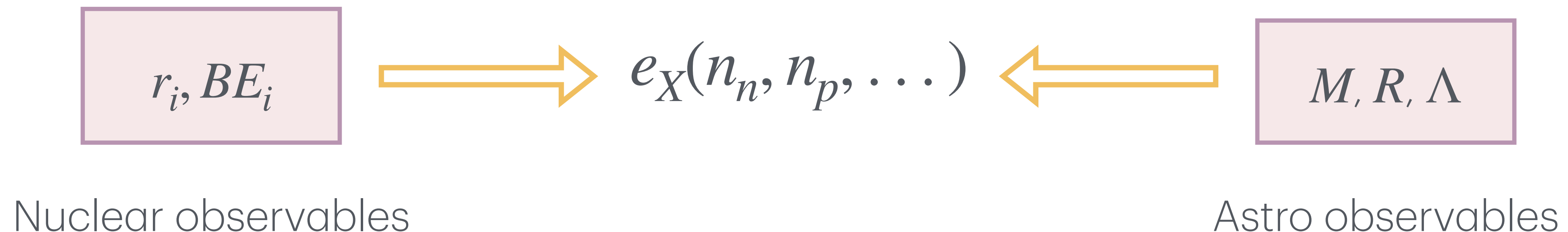
# A point in between: Meta-modelling of the EoS

Originally presented in [PRC 97, 025805 (2018)]

Parametric representation of the energy density  $\epsilon_X(n_n, n_p, \dots)$  as a function of the different species

The variation of the parameters set  $X$  makes possible to explore the EoS space compatible with the hypothesis of a matter with the chosen species

Both nuclear and Astro observables are accessible





# Almost causal meta-model: A possible choice of the energy density

Montefusco et al [in prep]

## Causality asymptotically implemented

Starting ansatz:

$$\epsilon(n, x_e, x_\mu) = \boxed{\epsilon_k(n, x_e, x_\mu)} + n \left[ \boxed{e_0(n)} + \delta^2 \boxed{e_2(n)} + \delta^4 \boxed{e_4(n)} \right]$$

free fermi gas energy density  
for  $npe\mu$  matter

Nuclear asymmetry  
 $\delta = 1 - 2(x_e + x_\mu)$

Quartic correction

$$e_4(n) = A \frac{n/n_0}{1 + (n/n_0)^B}$$

Nucleonic Potential  
(per baryon)

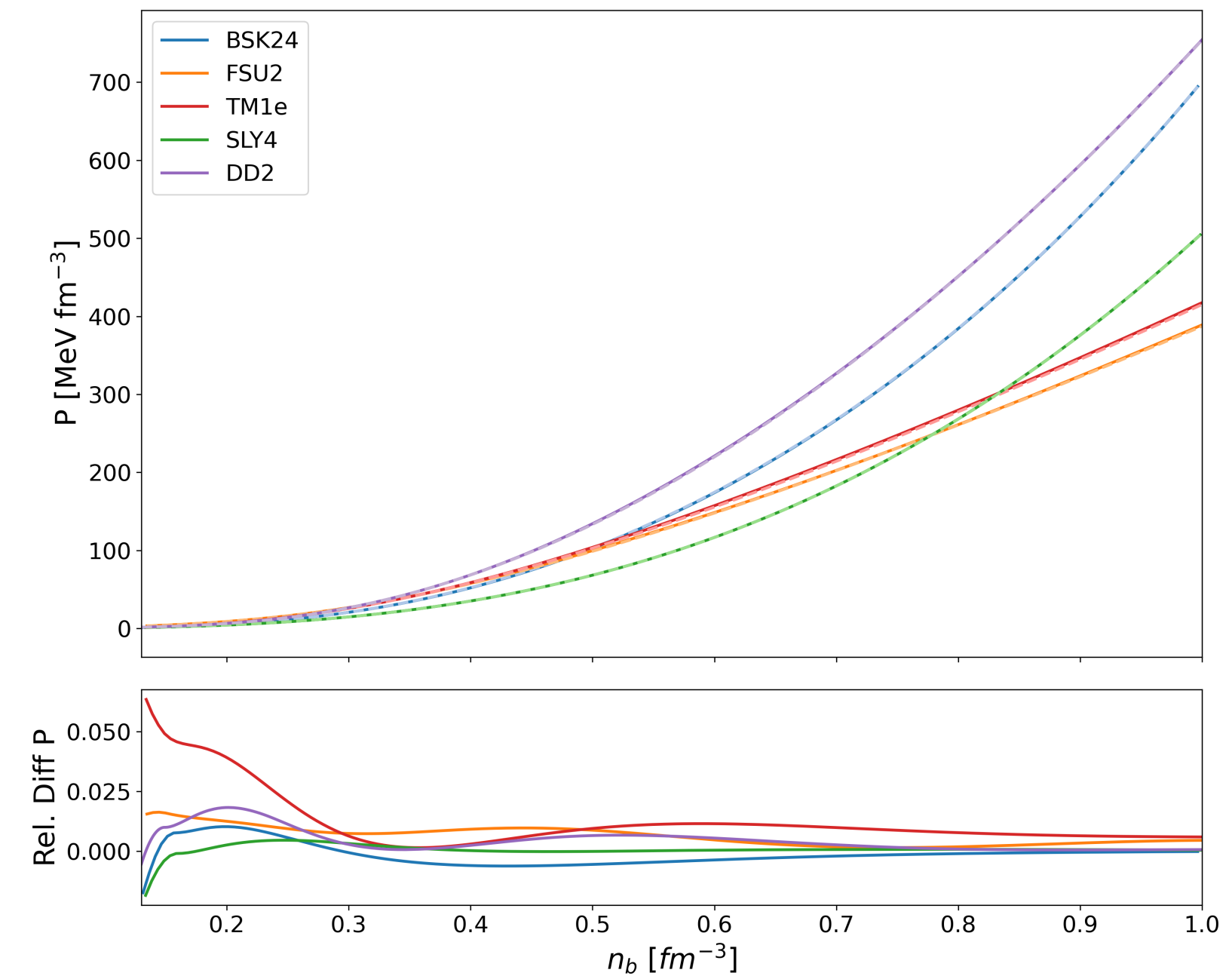
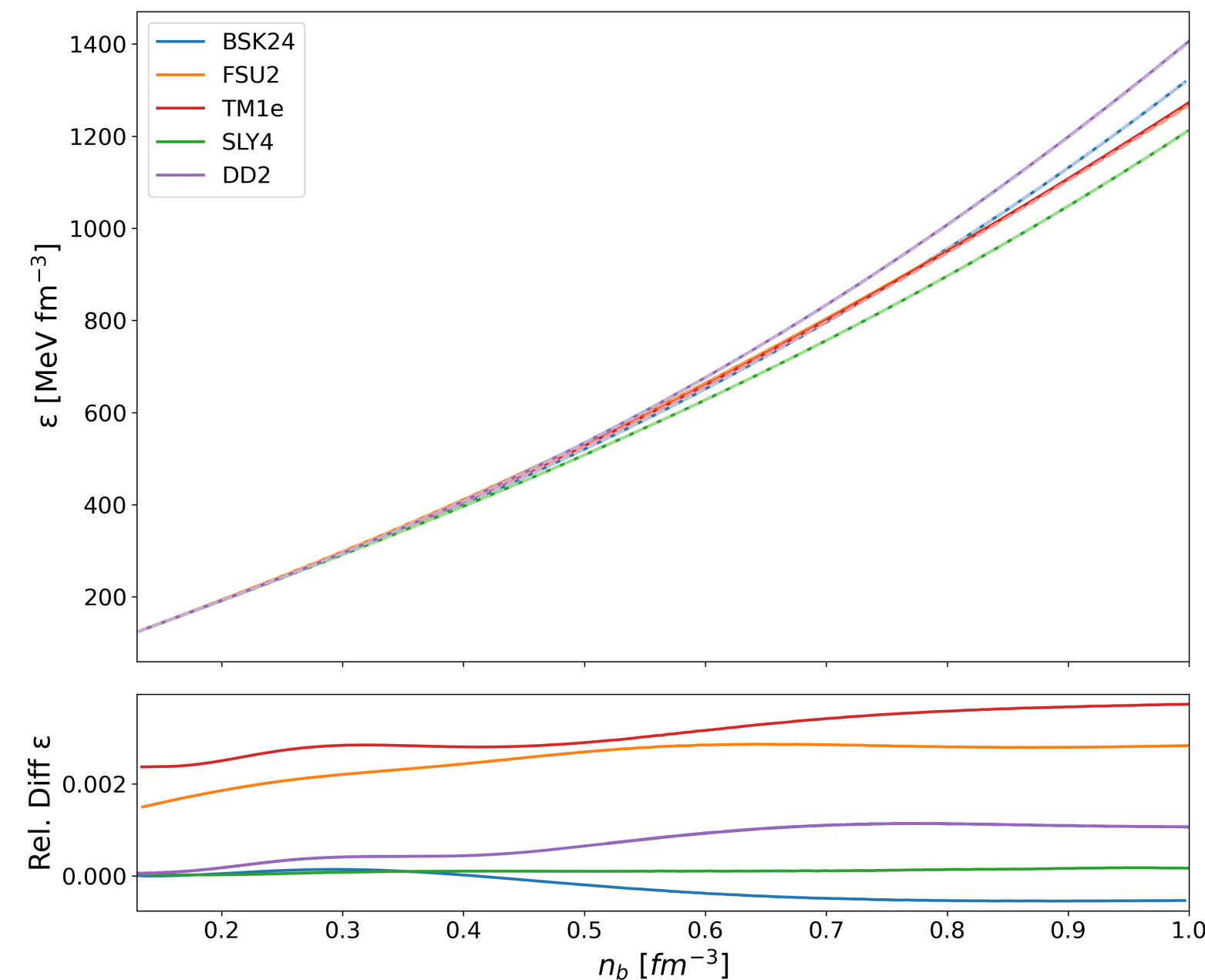
$$e_0(x) = V_0(x) + \frac{h_0 + h_1x + h_2x^2 + h_3x^3}{(1 + a_0x)(1 + b_0x)(1 + c_0x)}$$

# Almost causal meta-model: EoS reconstruction

Test the flexibility of the model to  
reproduce  $\beta$ -equilibrated EoS

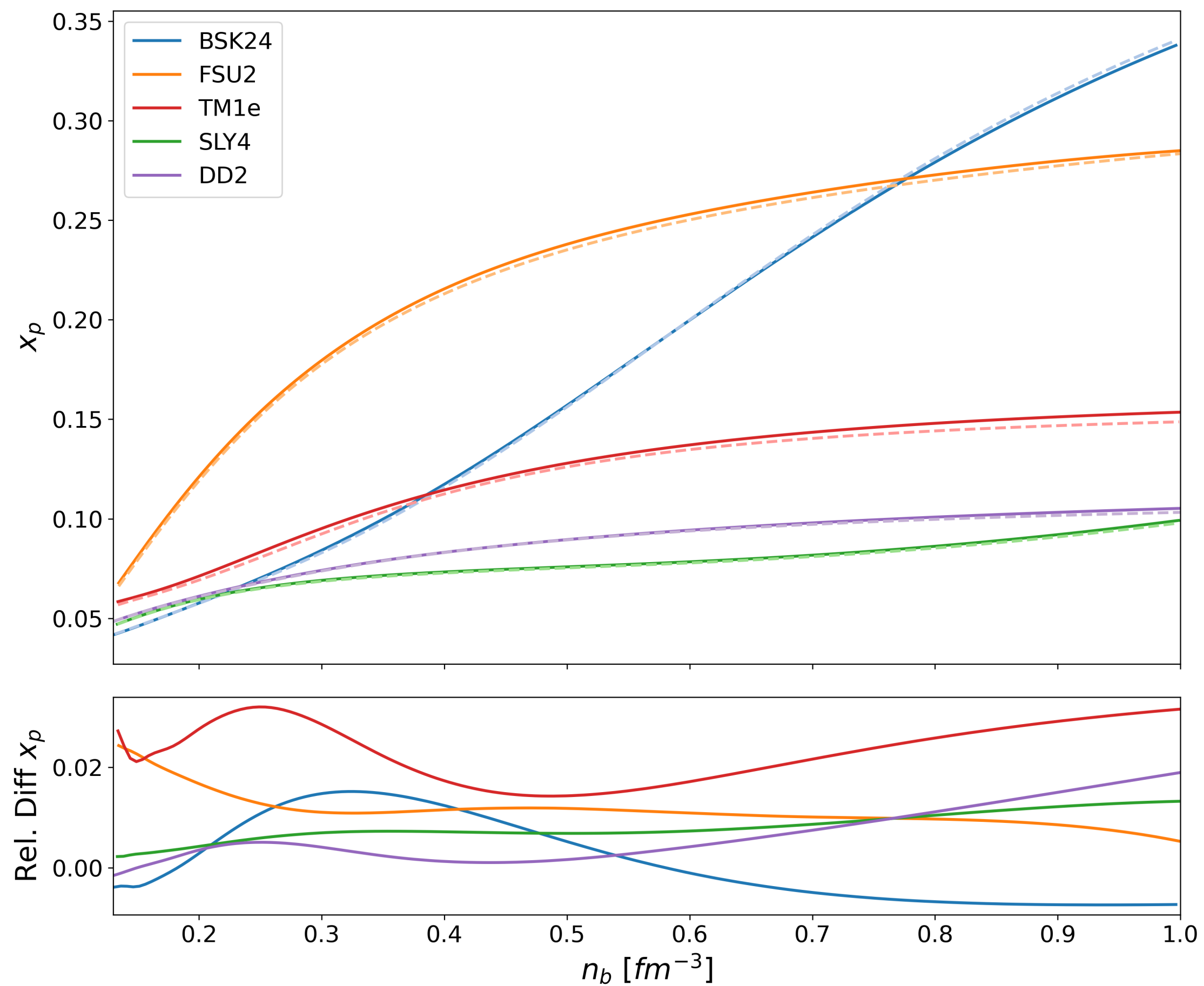
Constrain the space of the  
unphysical parameters

We have chosen: Sly4, BSK24,  
DD2, FSU2 and TM1e

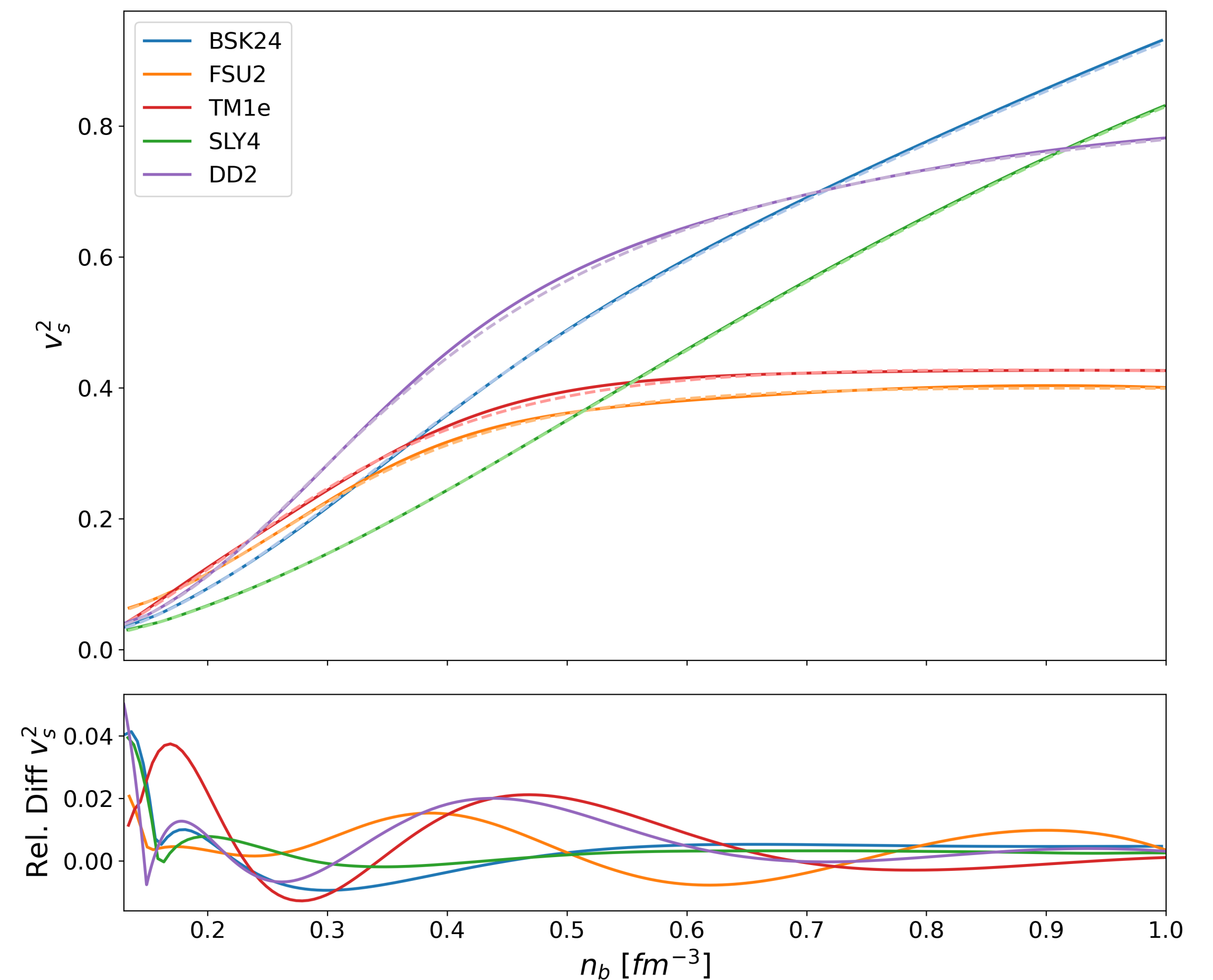


# Almost causal meta-model: EoS reconstruction

Proton fraction



Speed of sound



# Almost causal meta-model: Bayes Inference

$$\mathcal{M} : \mathbf{X} \rightarrow \{ \epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \dots \}$$

$$\mathcal{L}(\mathbf{X}) = \prod_j \mathcal{L}_j(\mathbf{X}) = \prod_j p \left( D_j | \mathcal{M}(\mathbf{X}) \right)$$



# Almost causal meta-model: Bayes Inference

$$\mathcal{M} : \mathbf{X} \rightarrow \{ \epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \dots \}$$

$$\mathcal{L}(\mathbf{X}) = \prod_j \mathcal{L}_j(\mathbf{X}) = \prod_j p(D_j | \mathcal{M}(\mathbf{X}))$$

Informed prior sampling the  $\chi_{EFT}$  band<sup>1</sup>  
of PNM energy with a metropolis MCMC

At this stage we have  $10^9$  models



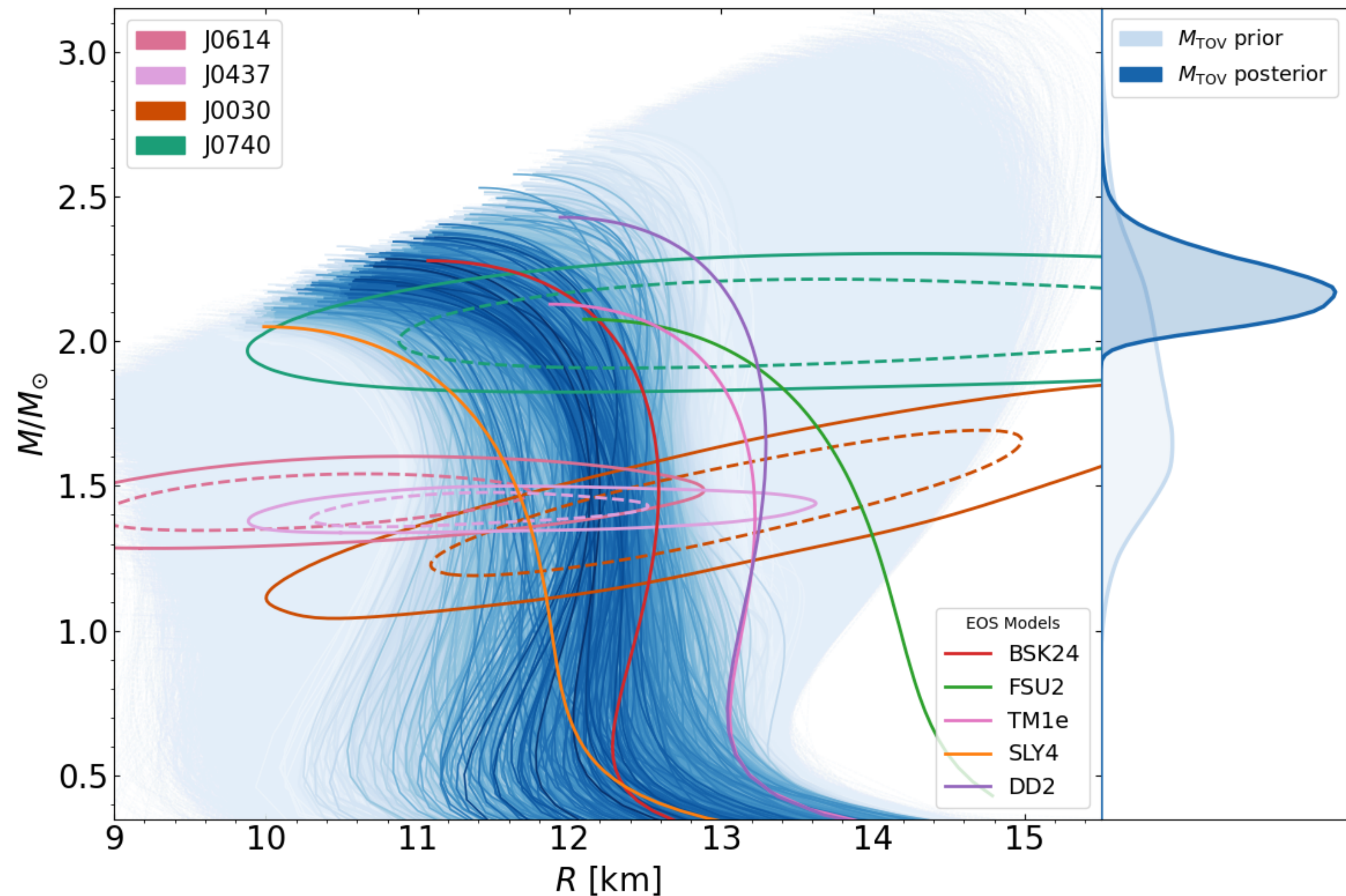
We extract  $5 \times 10^5$  models that  
pass through the remaining filter:

- AME2020 nuclear masses table
- Maximum observed NS mass from  
radio-timing of PSRJ0348 and  
PSRJ0740
- Tidal deformability from GW170817  
event detected by Ligo/Virgo  
collaboration
- NICER+XMN M-R measurements of  
PSRJ0030, PSRJ0347, PSRJ0614 and  
PSRJ0740

[1] Huth et al, 2021, Phys. Rev. C, 103, 025803

# Almost causal meta-model: Mass - Radius

We cover a wide range of  
masses, radii and tidal



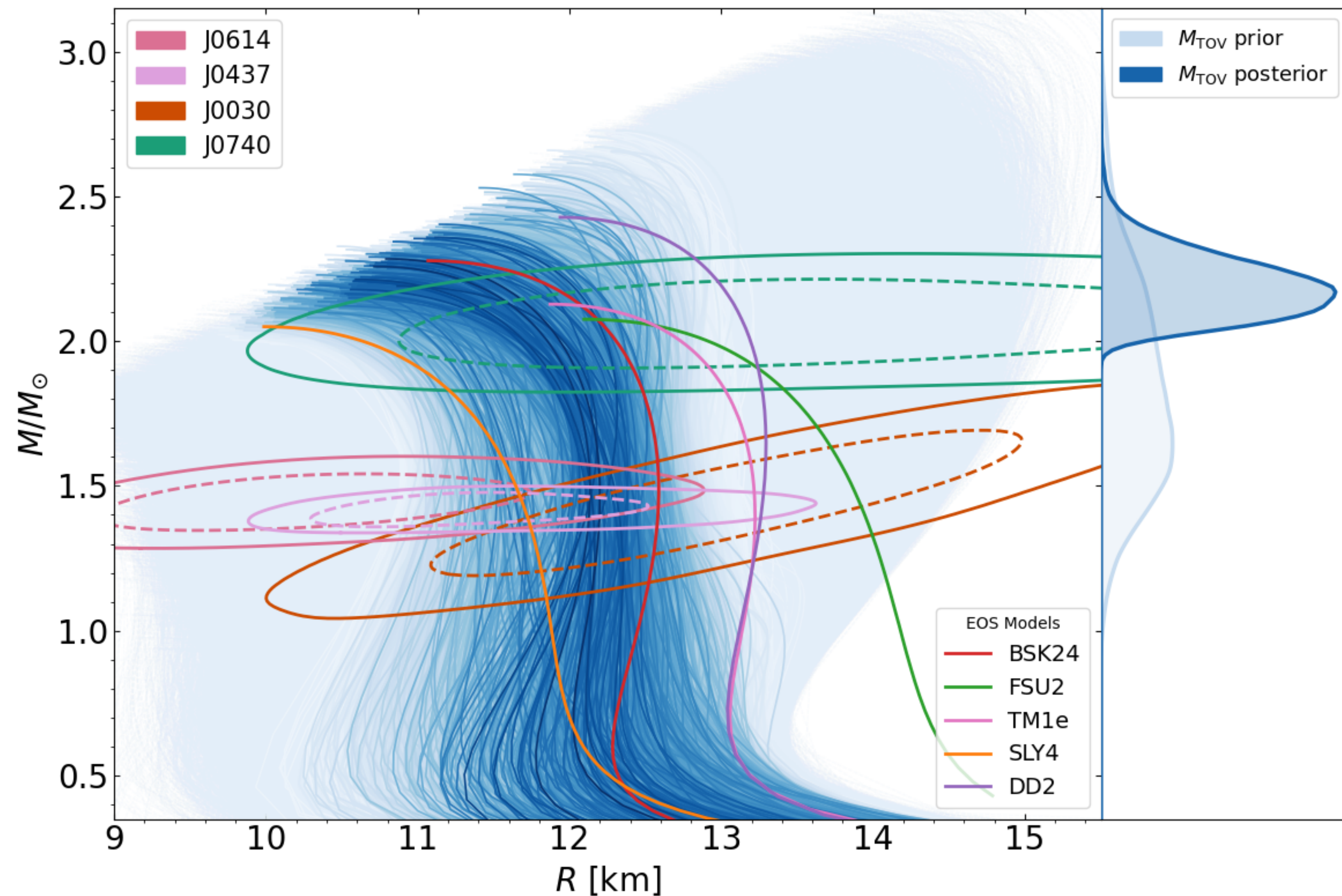


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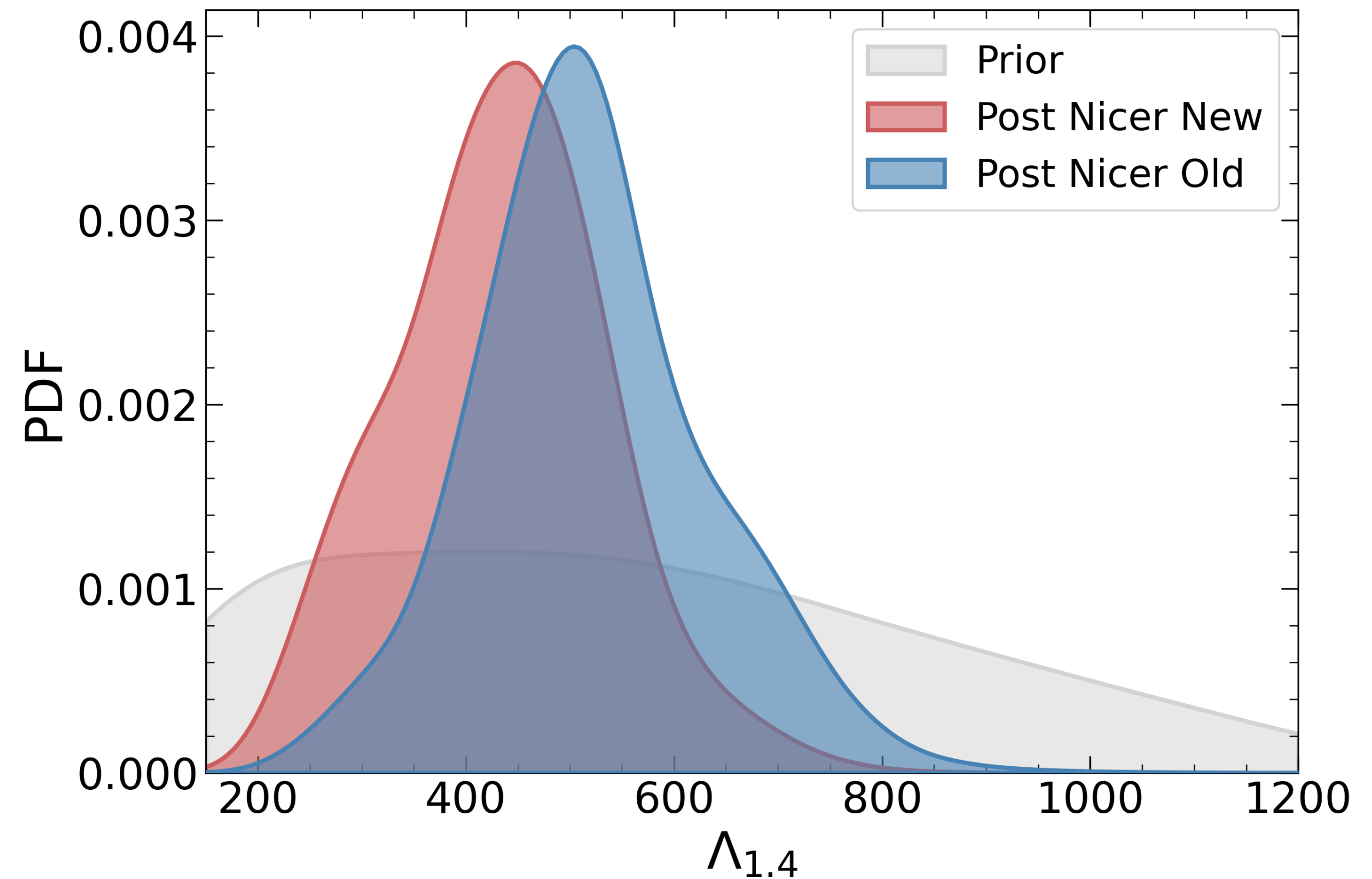
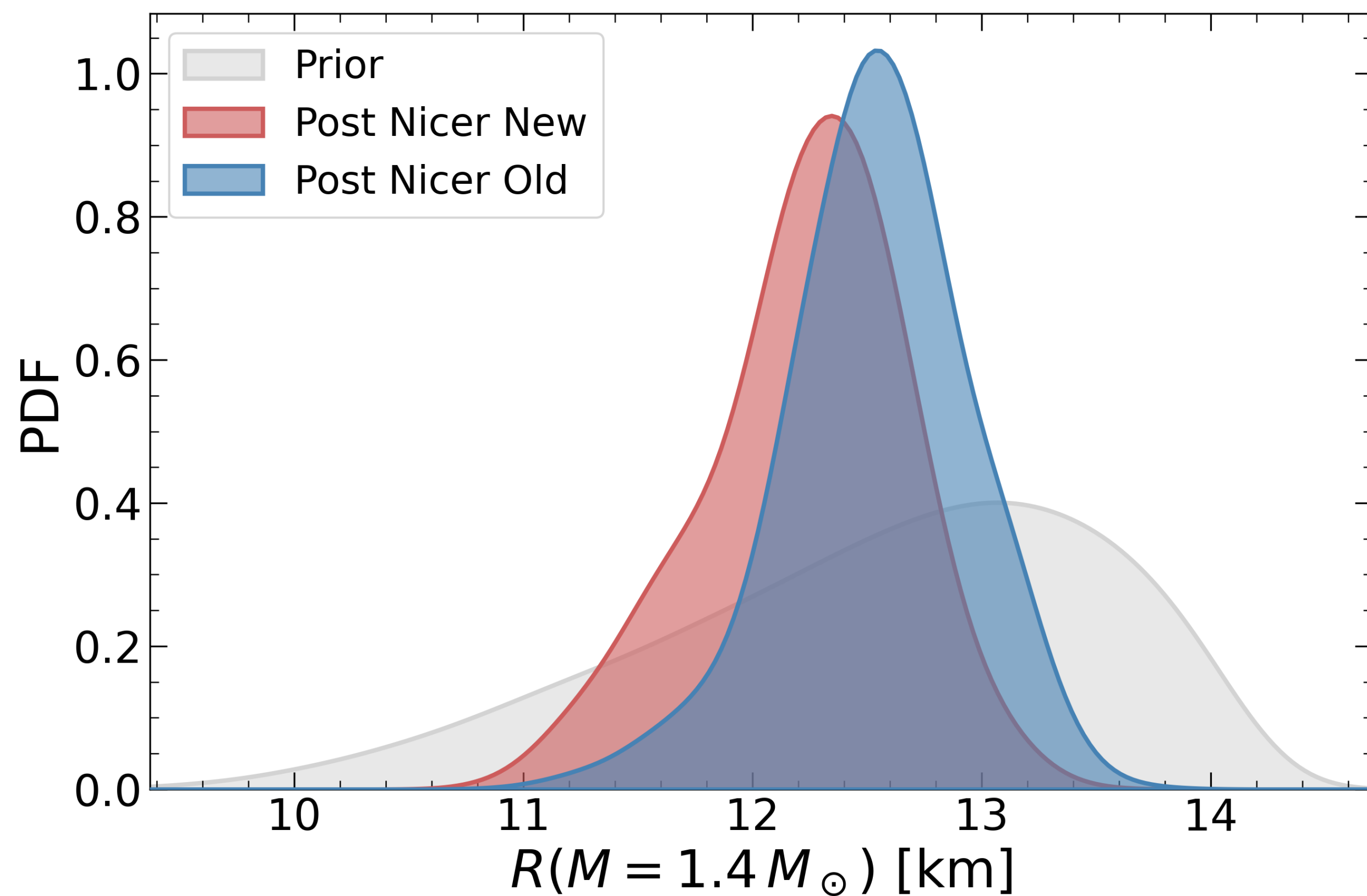
The two newest NICER data  
suggest a soft EoS

$M_{TOV} > 2.5M_{\odot}$  is disfavored





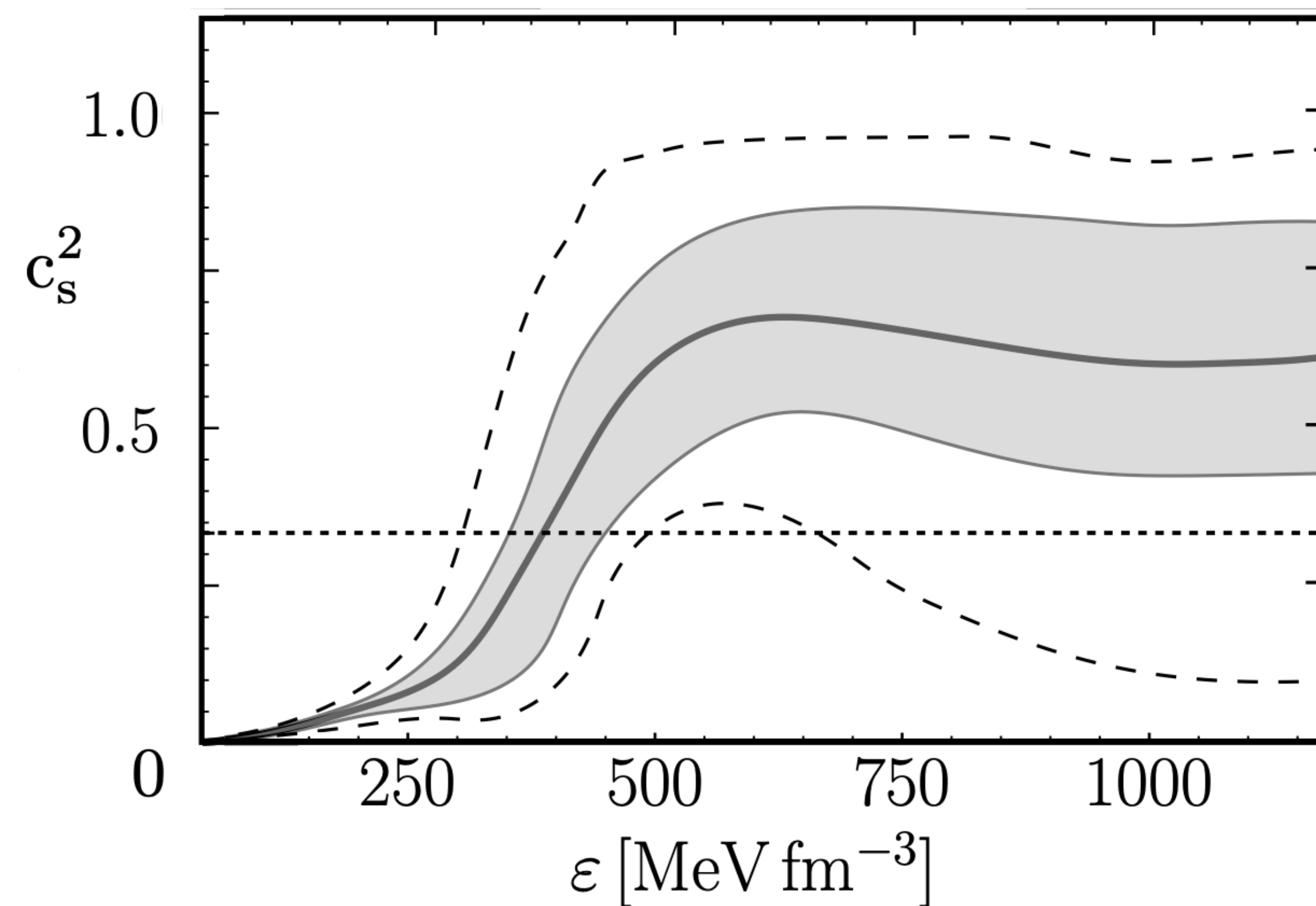
# Almost causal meta-model: NICER softening



# Almost causal meta-model: Speed of sound

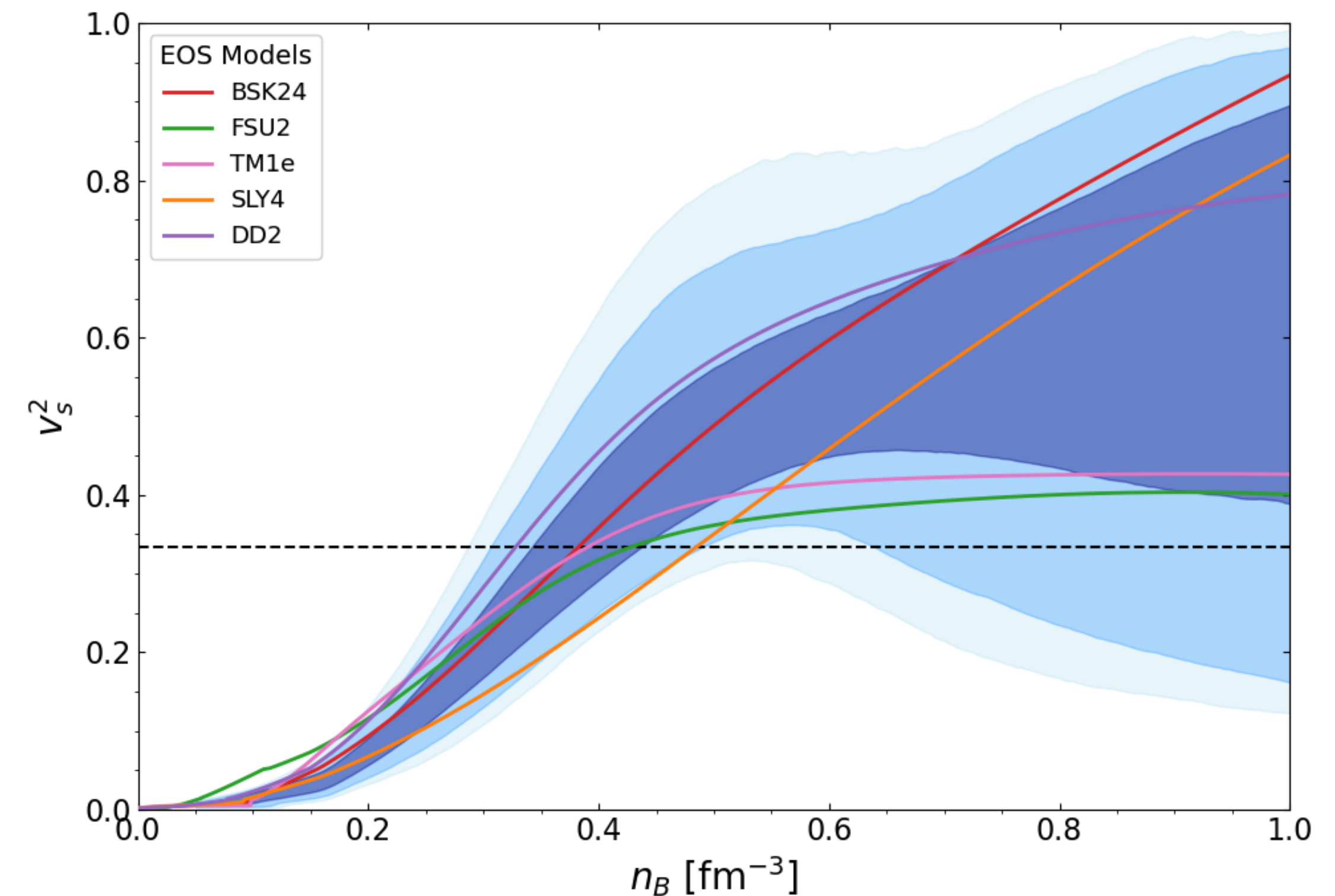
Brandes et Weise  
PhysRevD.111.034005 (2025)

## $c_s$ parametrization



Montefusco et al [in prep]

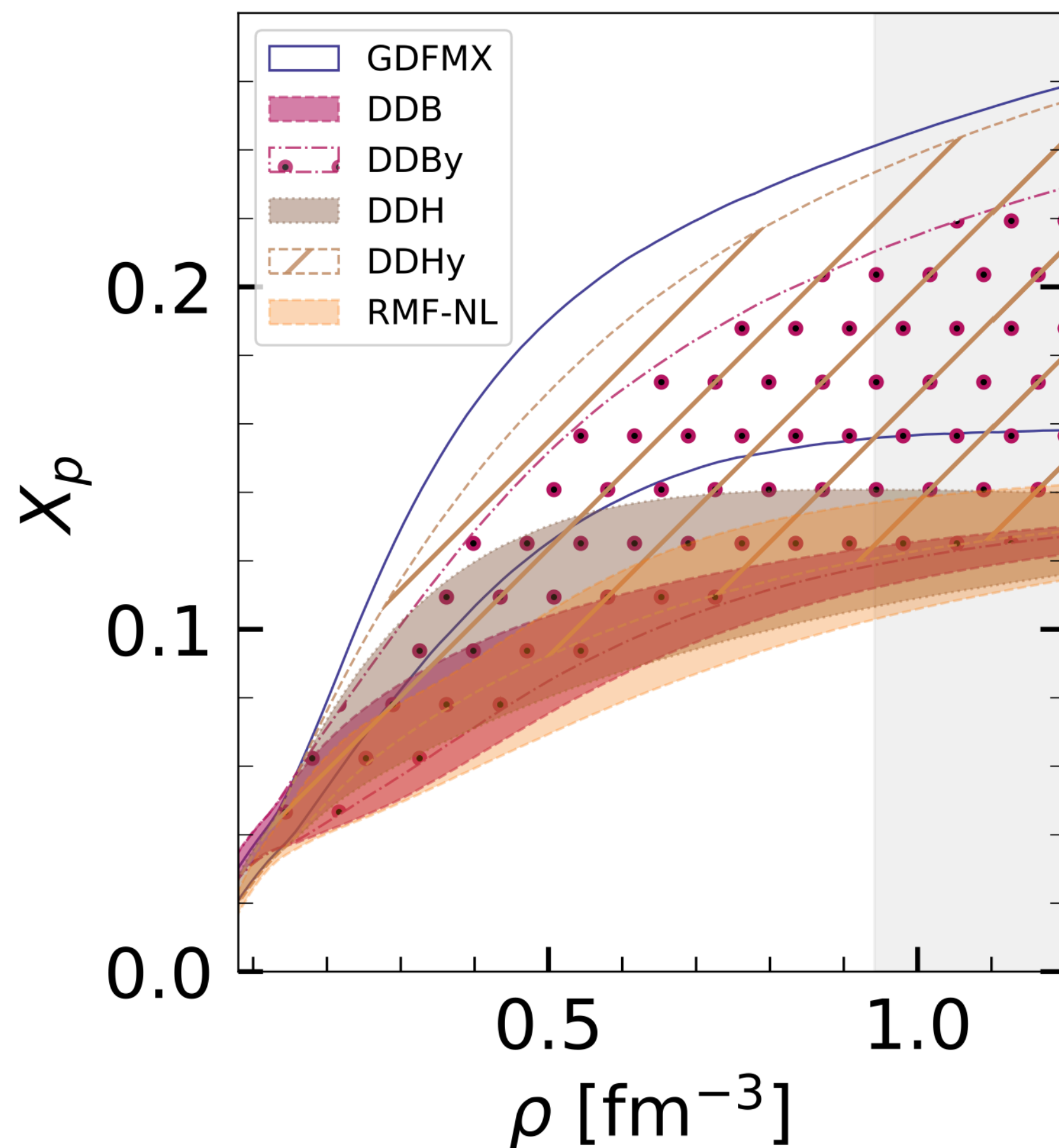
## Almost causal MM



# Almost causal meta-model: Composition

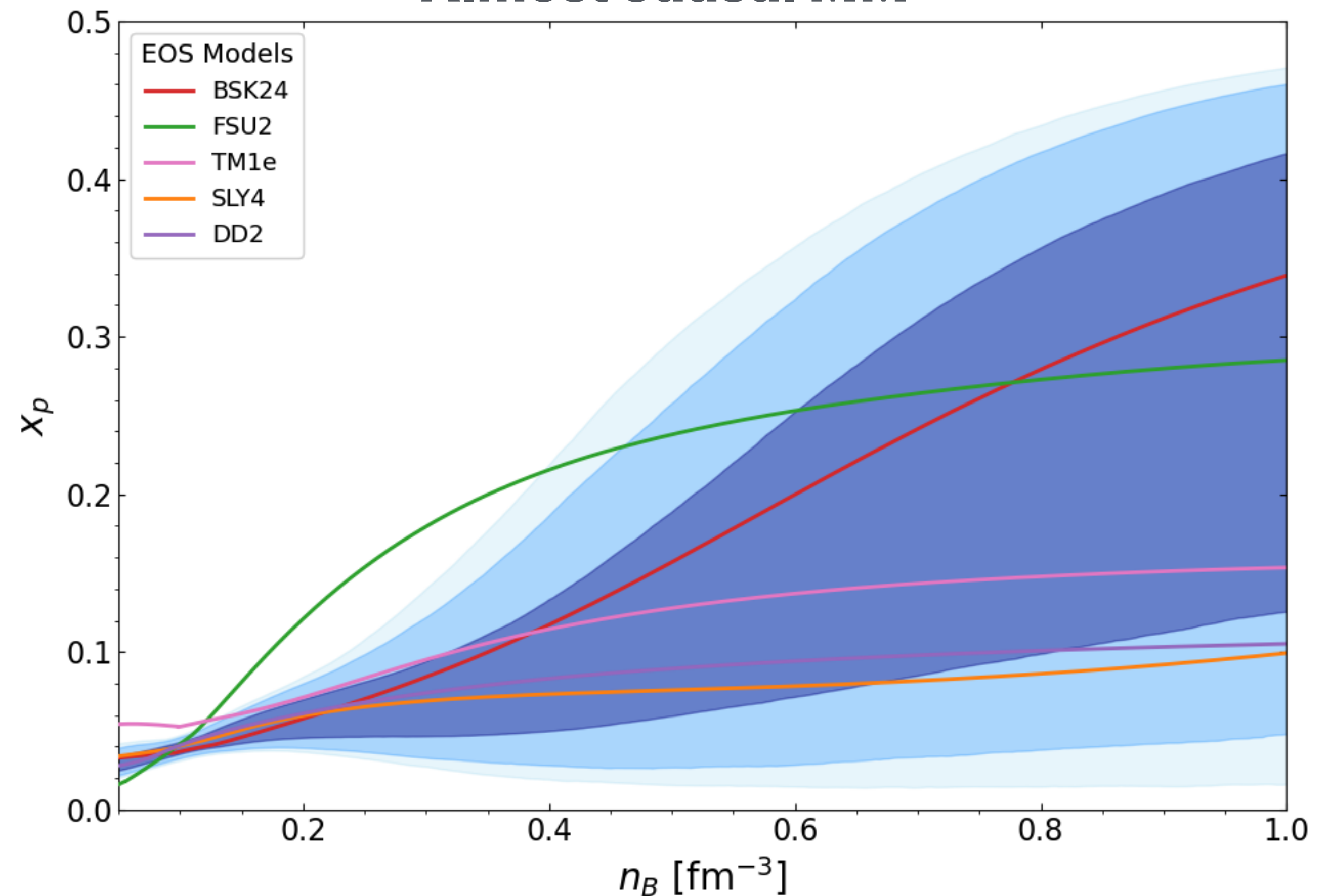
Cartaxo et al arXiv:2506.03112 (2025)

## Density functional approach



Montefusco et al [in prep]

## Almost causal MM





# Quasi-normal modes: Introduction

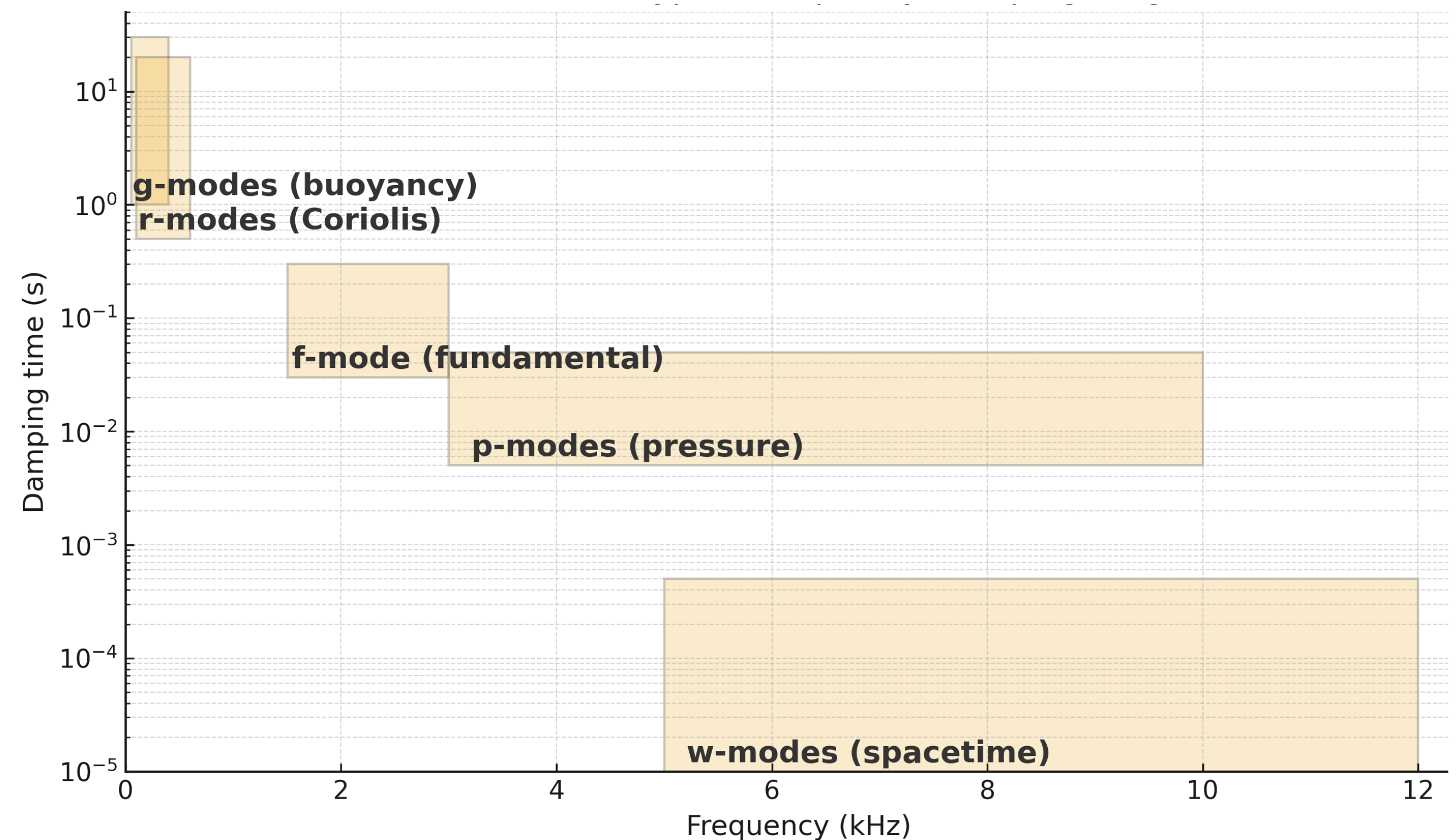
**Quasi-normal modes** are the **characteristic damped oscillations**, excited by perturbations of the star-spacetime system

**Classified by the restoring force**

**Discrete complex frequencies**

Real part  $\longrightarrow$  Oscillation

Imaginary part  $\longrightarrow$  Damping



# Quasi-normal modes: Introduction

**Quasi-normal modes** are the **characteristic damped oscillations**, excited by perturbations of the star–spacetime system

Zheng et al (2023)  
PhysRevD.107.103048

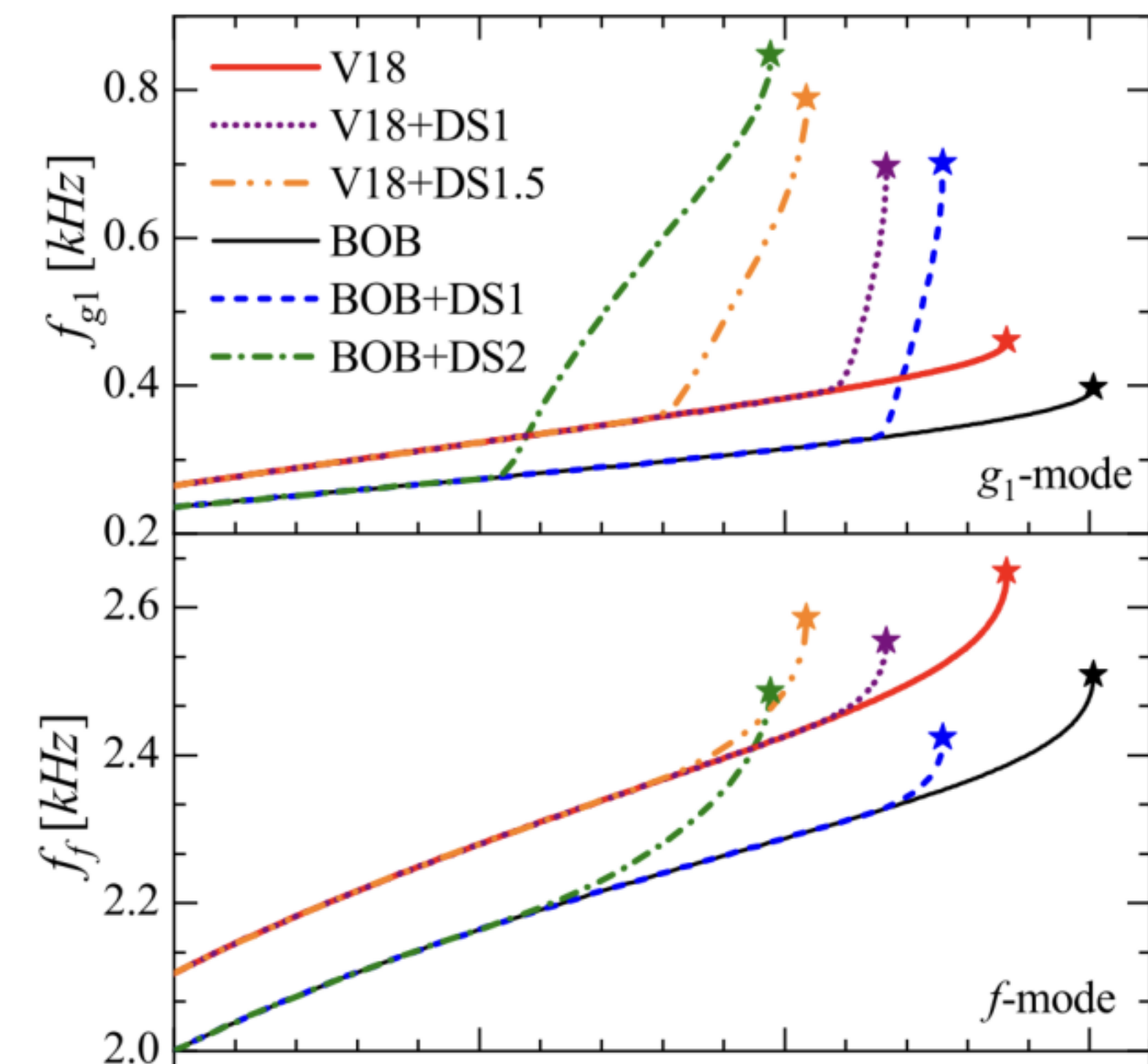
**Classified by the restoring force**

**Discrete complex frequencies**

Real part  $\longrightarrow$  Oscillation

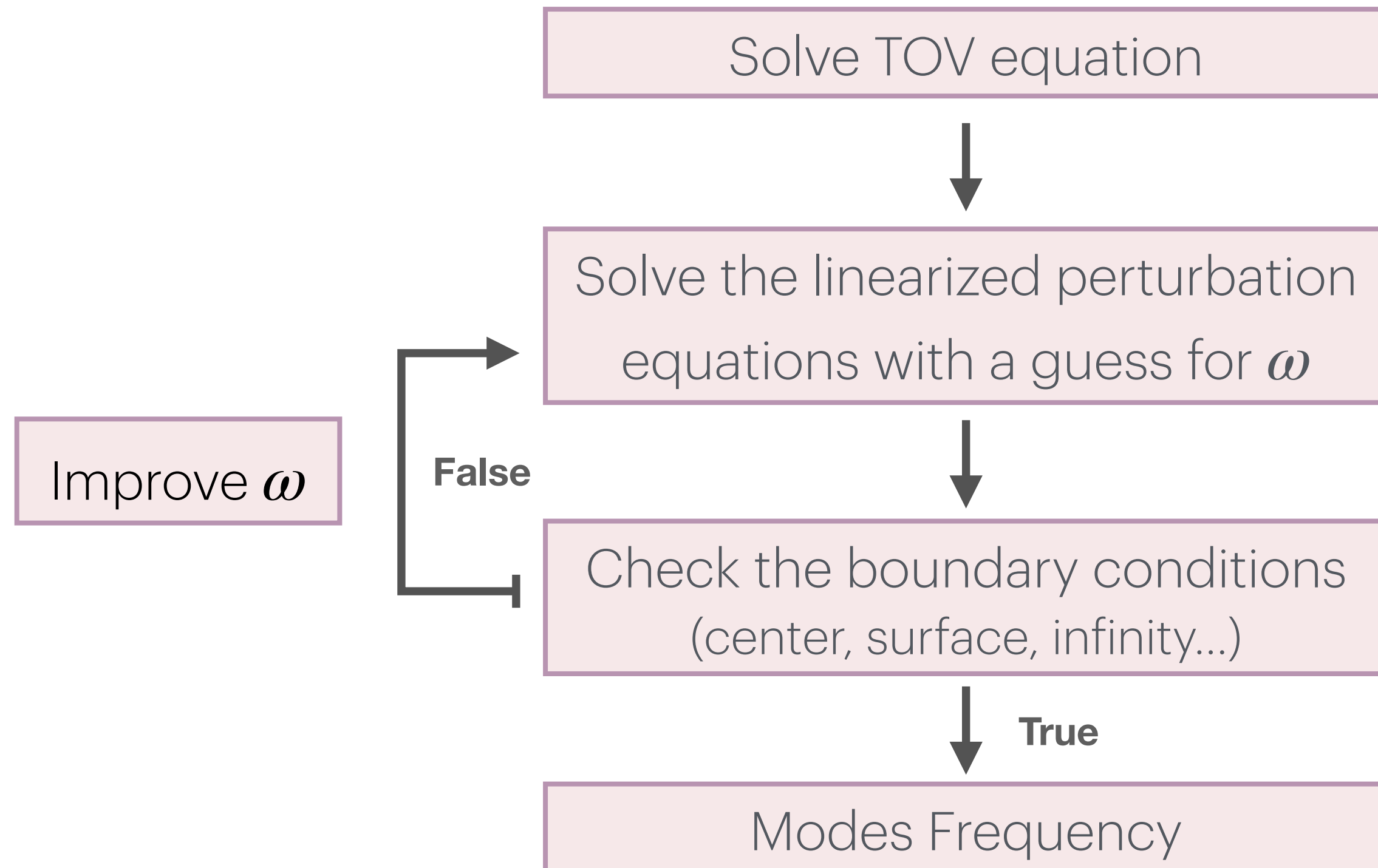
Imaginary part  $\longrightarrow$  Damping

**May be sensitive not only to the EoS  
but also to the internal composition**



# Quasi-normal modes: A difficult eigenvalue problem

## Eigenvalue problem



## NON-RADIAL PULSATION OF GENERAL-RELATIVISTIC STELLAR MODELS. I. ANALYTIC ANALYSIS FOR $l \geq 2$ \*

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Received February 24, 1967

$$\begin{aligned}
 H' + r^{-1}e^{\lambda}[l(l+1)/2 + 1 + 4\pi r^2(p - \rho)]H &= rK'' \\
 + e^{\lambda}(3 - 5m/r - 4\pi r^2\rho)K' - r^{-1}e^{\lambda}[l(l+1)/2 - 1 \\
 + 8\pi r^2(\rho + p)]K + 8\pi r^{-1}(\rho + p)e^{\lambda/2}W' + 8\pi r^{-1}\rho'e^{\lambda/2}W \\
 + 8\pi l(l+1)r^{-1}(\rho + p)e^{\lambda}V.
 \end{aligned} \tag{14a}$$

$$\begin{aligned}
 -e^{\nu-\lambda}K'' + 2r^{-1}e^{\nu}[-1 + m/r + 2\pi r^2(\rho - p)]K' - \omega^2K \\
 + r^{-2}e^{\nu}[l(l+1) - 2 + 8\pi r^2(\rho + p - \gamma p)]K \\
 + r^{-2}e^{\nu}[2e^{-\lambda} - 4\pi r^2(\rho + p + \gamma p)]H \\
 - 8\pi r^{-2}e^{\nu-\lambda/2}(\rho + p - \gamma p)W' - 8\pi r^{-2}e^{\nu-\lambda/2}(\rho' - p')W \\
 - 8\pi l(l+1)r^{-2}e^{\nu}(\rho + p - \gamma p)V = 0.
 \end{aligned} \tag{14b}$$

$$\begin{aligned}
 -\{\gamma p e^{\nu/2}[r^{-2}e^{-\lambda/2}W' + l(l+1)r^{-2}V - H/2 - K]\}' \\
 - \omega^2 r^{-2}e^{(\lambda-\nu)/2}(\rho + p)W + \frac{1}{2}(\rho + p)e^{\nu/2}(r^{-2}e^{-\lambda/2}\nu')'W \\
 - l(l+1)r^{-2}(\rho + p)(e^{\nu/2})'V - \frac{1}{2}(\rho + p)e^{-\nu}(He^{3\nu/2})' \\
 + (\rho + p)(Ke^{\nu/2})' = 0.
 \end{aligned} \tag{14c}$$

$$\begin{aligned}
 -\omega^2 e^{-\nu}(\rho + p)V + l(l+1)r^{-2}\gamma p V + r^{-2}\gamma p e^{-\lambda/2}W' + r^{-2}p'e^{-\lambda/2}W \\
 - \frac{1}{2}(\rho + p + \gamma p)H - \gamma p K = 0.
 \end{aligned} \tag{14d}$$



# Quasi-normal modes: Cowling Approximation

## **Cowling approximation**

Neglect perturbation of the  
metric



No dissipation due to  
GW emissions



Only real part of the  
frequency

# Quasi-normal modes: Cowling Approximation

## Cowling approximation

Neglect perturbation of the metric



No dissipation due to GW emissions



Only real part of the frequency

The equations become:

$$W' = \left( \frac{dP}{d\epsilon} \right)^{-1} [\omega^2 r^2 V + \Phi' W] - l(l+1)e^\Lambda V$$
$$V' = 2\Phi' V - e^\Lambda \frac{W}{r^2} - A \left[ \frac{\Phi'}{\omega^2 r^2} e^{2\Phi - \Lambda} W + V \right]$$

Where  $V$ ,  $W$  are the fluid displacement vectors and  $A$  is the Schwarzschild discriminant

By imposing the **boundary condition**  $\Delta P = 0$  at the stellar surface it is obtained

$$\omega^2 R^2 e^{\Lambda - 2\Phi} V + \Phi' W = 0$$

This let us obtain the frequency as the omega for which this BC is satisfied

# Quasi-normal modes: Frozen vs $\beta$ -equilibrated regime

We tested the impact on mode frequencies of  
assuming two opposite limits for the reaction rates

## Equilibrium regime

Each perturbed fluid element has time  
to re-equilibrate the composition

$$v_{\beta}^2 = \frac{dP}{d\epsilon} = \frac{dP(n, x_e(n), x_{\mu}(n))/dn}{d\epsilon(n, x_e(n), x_{\mu}(n))/dn}$$

Used in the studies that assumes agnostic  
barotropic or  $\beta$ -equilibrated EoS

## Frozen regime

Each perturbed fluid element remain at  
fixed composition

$$v_{FR}^2 = \frac{\partial P}{\partial \epsilon} \bigg|_{x_e, x_{\mu}} = \frac{\partial P(n, x_e(n), x_{\mu}(n))/\partial n}{\partial \epsilon(n, x_e(n), x_{\mu}(n))/\partial n}$$

This is the maximal speed of information in a  
reacting mixture<sup>1</sup>:

For stable and causal matter

$$0 < v_{\beta} < v_{FR} < 1$$

# Quasi-normal modes: Frozen vs $\beta$ -equilibrated regime

We tested the impact on mode frequencies of assuming two opposite limits for the reaction rates

## Equilibrium regime

Each perturbed fluid element has time to re-equilibrate the composition

$$v_{\beta}^2 = \frac{dP}{d\epsilon} = \frac{dP(n, x_e(n), x_{\mu}(n))/dn}{d\epsilon(n, x_e(n), x_{\mu}(n))/dn}$$

## Frozen regime

Each perturbed fluid element remain at fixed composition

$$v_{FR}^2 = \left. \frac{\partial P}{\partial \epsilon} \right|_{x_e, x_{\mu}} = \frac{\partial P(n, x_e(n), x_{\mu}(n))/\partial n}{\partial \epsilon(n, x_e(n), x_{\mu}(n))/\partial n}$$

$$W' = \left( \frac{dP}{d\epsilon} \right)^{-1} [\omega^2 r^2 V + \Phi' W] - l(l+1)e^{\Lambda} V$$

$$V' = 2\Phi' V - e^{\Lambda} \frac{W}{r^2} - A \left[ \frac{\Phi'}{\omega^2 r^2} e^{2\Phi - \Lambda} W + V \right]$$

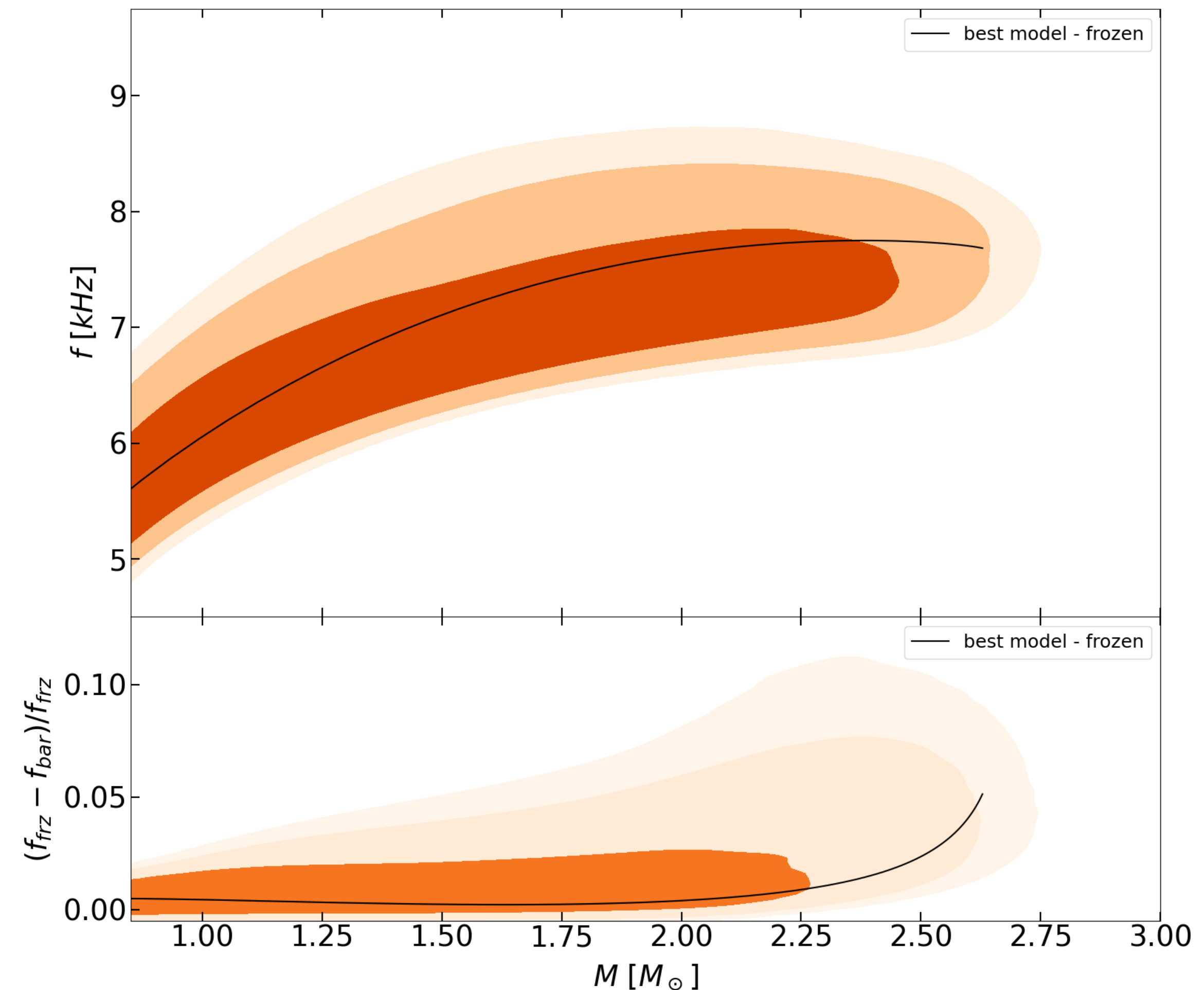


# Quasi-normal modes: Frozen vs $\beta$ -equilibrated regime

**$f$ -mode** differences  
smaller than 0.5%

**$p_1$ -mode**: difference < 5% except for  $M \rightarrow M_{max}$   
where they reach up to 10%

Since the differences are small, if  $v_{FR}$  is not available, it is possible to use the barotropic frequencies with very little errors.



# Quasi-normal modes: Quasi universal relation

$$\omega M = a_0 + a_1 \frac{M}{R} + a_2 \left( \frac{M}{R} \right)^2 + a_3 \left( \frac{M}{R} \right)^3$$

[Phys. Rev. C 103, 035810]

[Phys. Rev. D 103, 123015]

*f*-mode

accuracy > 95%

*p*<sub>1</sub>-mode

accuracy > 90%

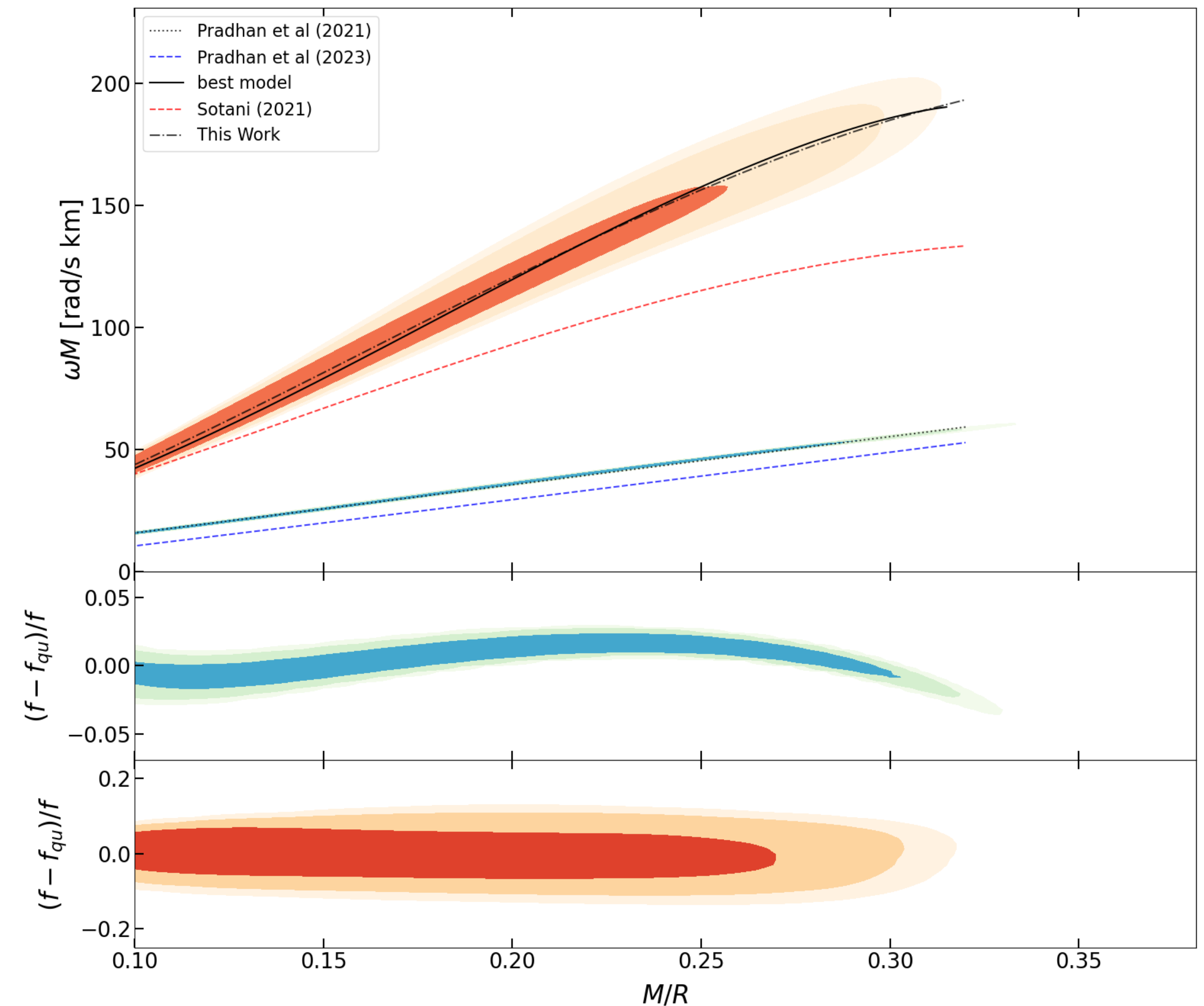
Every different set of EoS gave  
compatible fit



It is a **Quasi-universal** relation

It is possible to obtain the frequencies by  
evaluating only the M-R relation

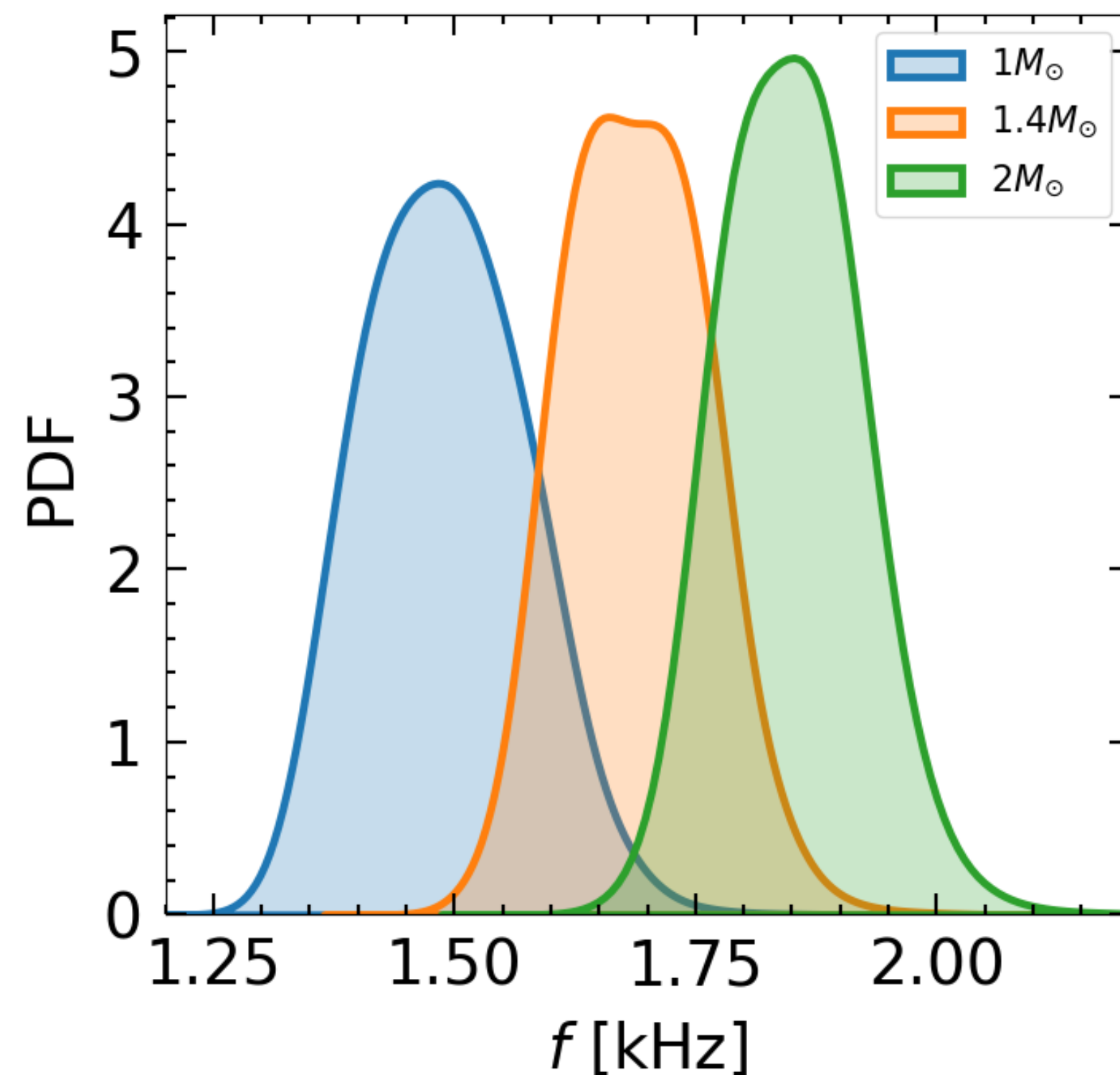
[ A&A, 694, A150 (2025) ]



# Quasi-normal modes: Synthetic full-GR frequency

[ A&A, 694, A150 (2025) ]

*f* – mode

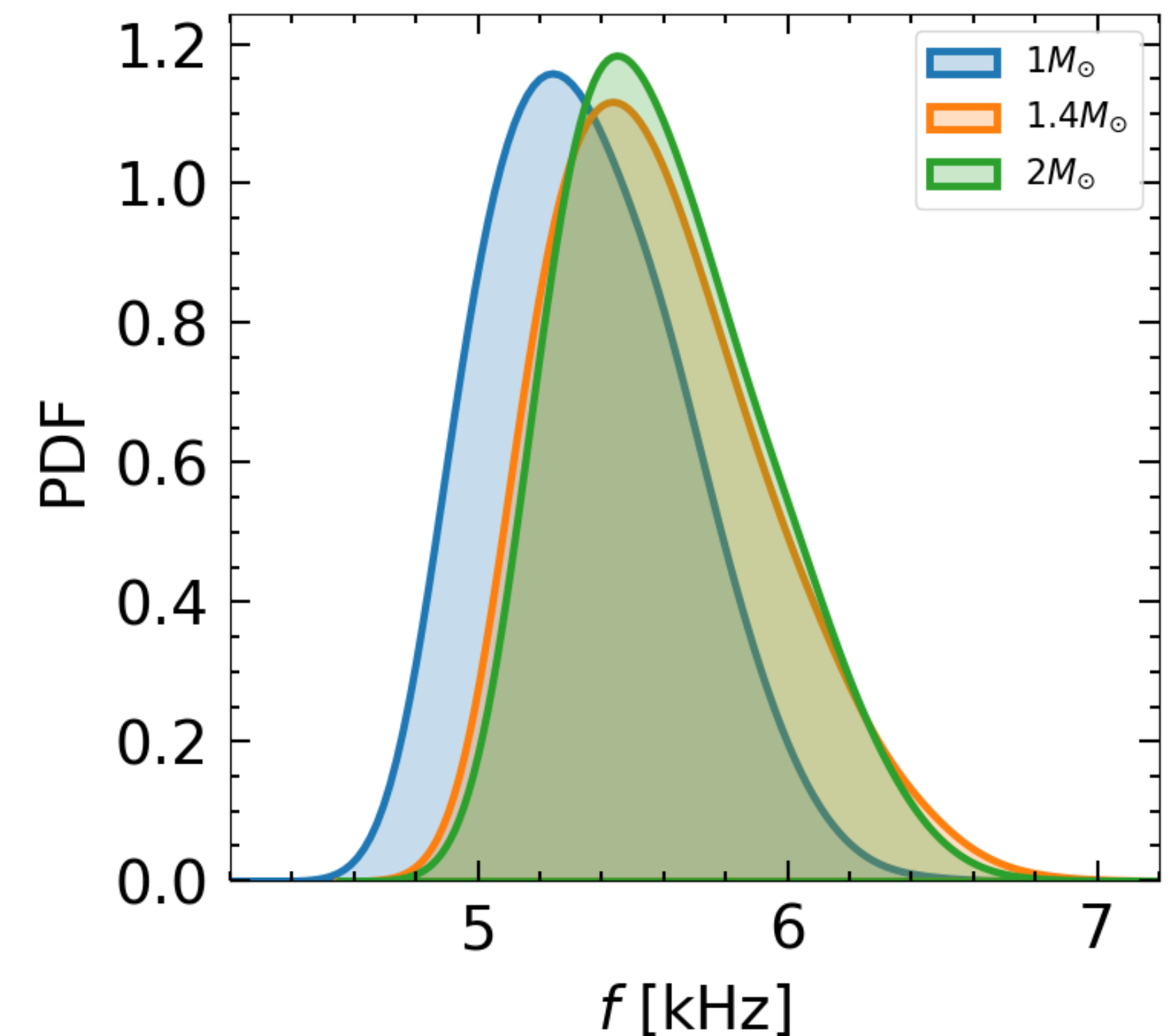


The split is larger for the *f*-mode  
than the *p*<sub>1</sub>-mode

It is possible to constrain the  
mass from an *f*-mode  
observation

This information is lost for  
the *p*<sub>1</sub>-mode

*p*<sub>1</sub> – mode





# Summary

— NS probe extreme densities and EoS can be univocally mapped to their static properties



This is the only way to complement experiment and pQCD

— No ab-initio model of nuclear matter for all densities



Bayesian techniques allow controlled extrapolations of low density constraints from nuclear theory and experiments

— The meta-modeling framework gives access to the composition without sacrificing the flexibility of agnostic model



It opens new channels to probe the EoS in a systematic way, e.g. modes, cooling, glitches

— The quasi-normal modes offers a deeper insight into the internal composition



With the next generation GW detectors may be possible to observe them and constrain the EoS

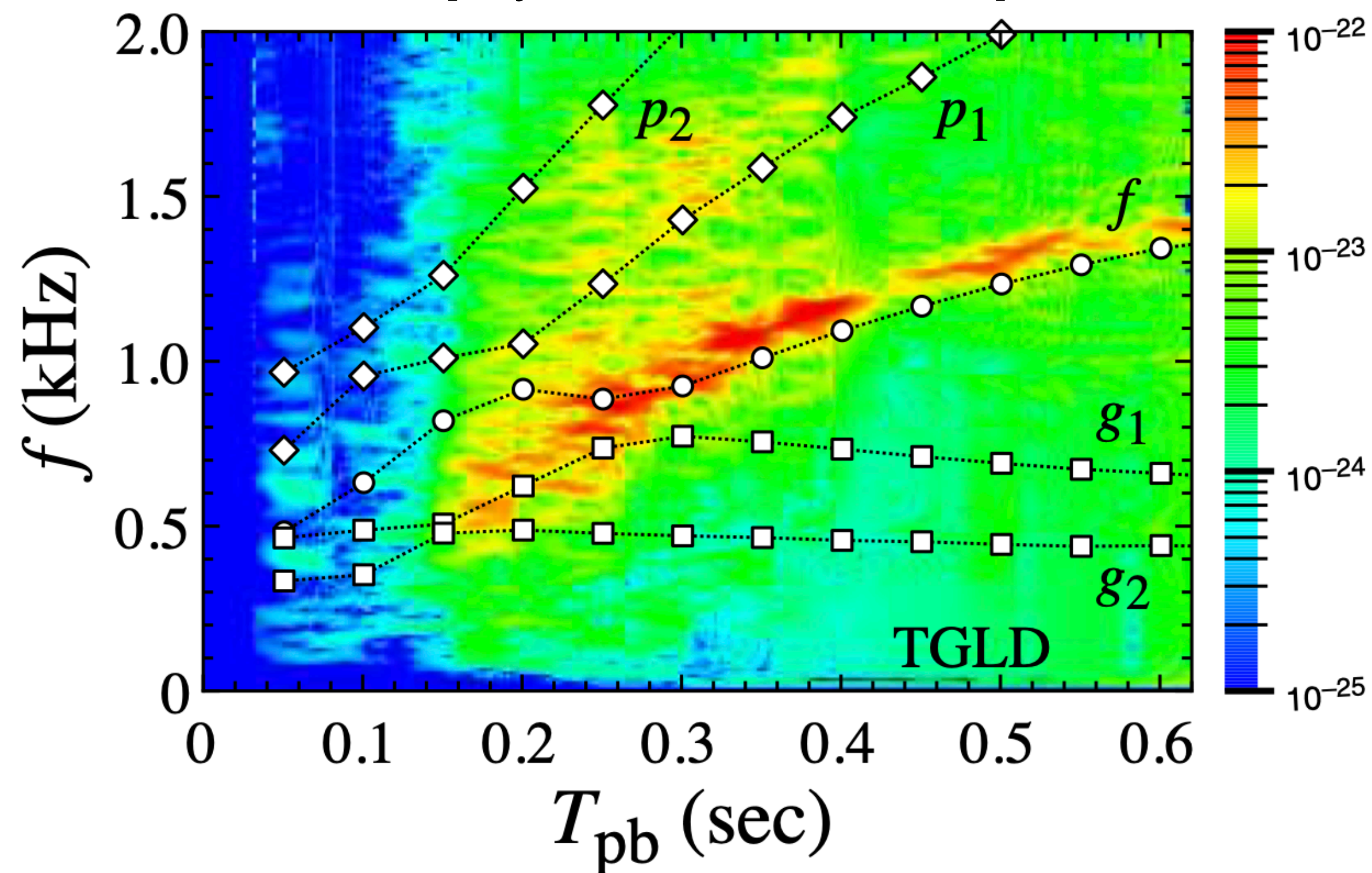
Next frontier: better understanding of finite temperature effects and magnetization for mergers and supernovae

BACKUP SLIDES

# Quasi-normal modes: Astrophysical scenario

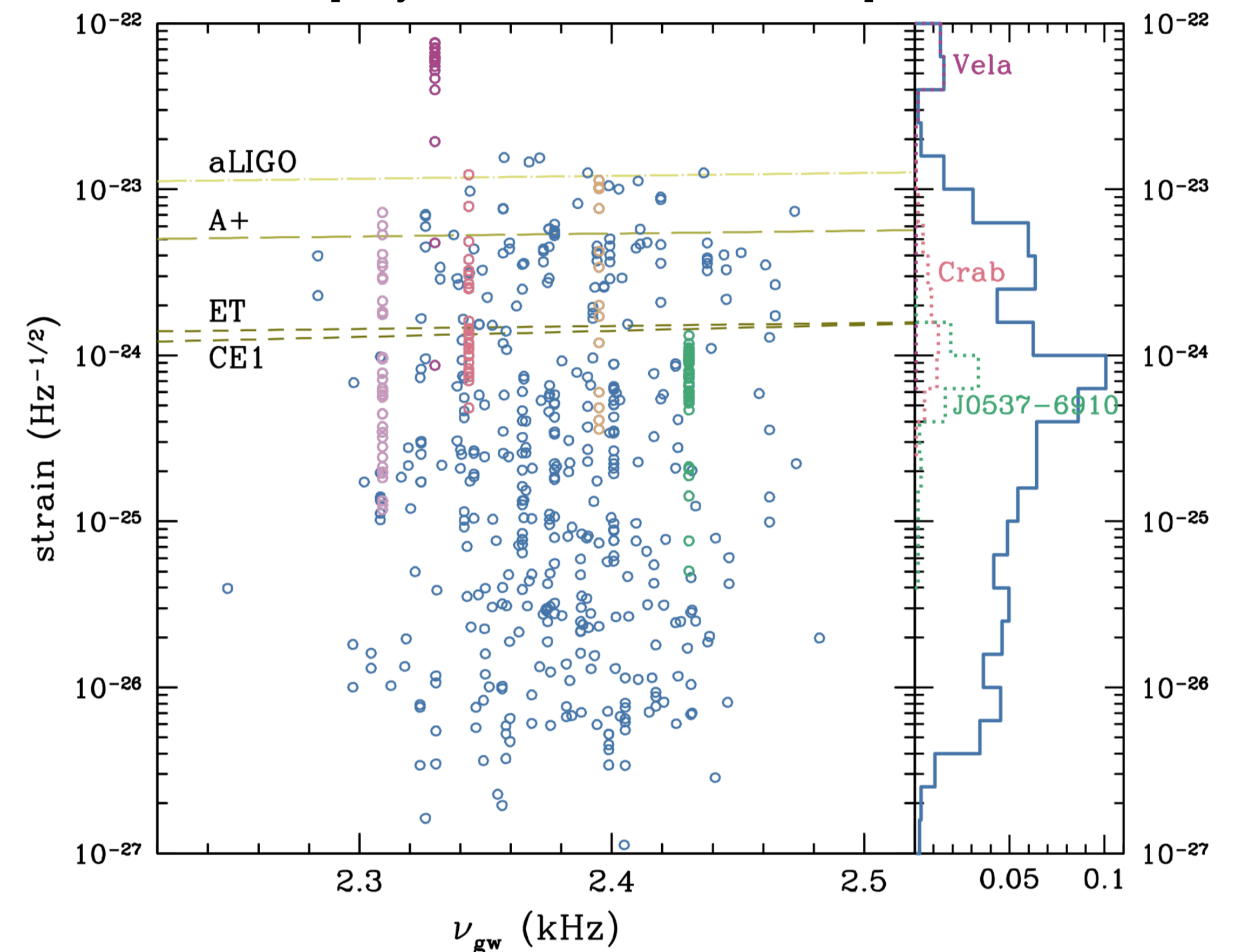
Comparison between **numerical simulations** and **eigenvalue obtained** frequencies for PNS

[Phys. Rev. D 104, 123009]



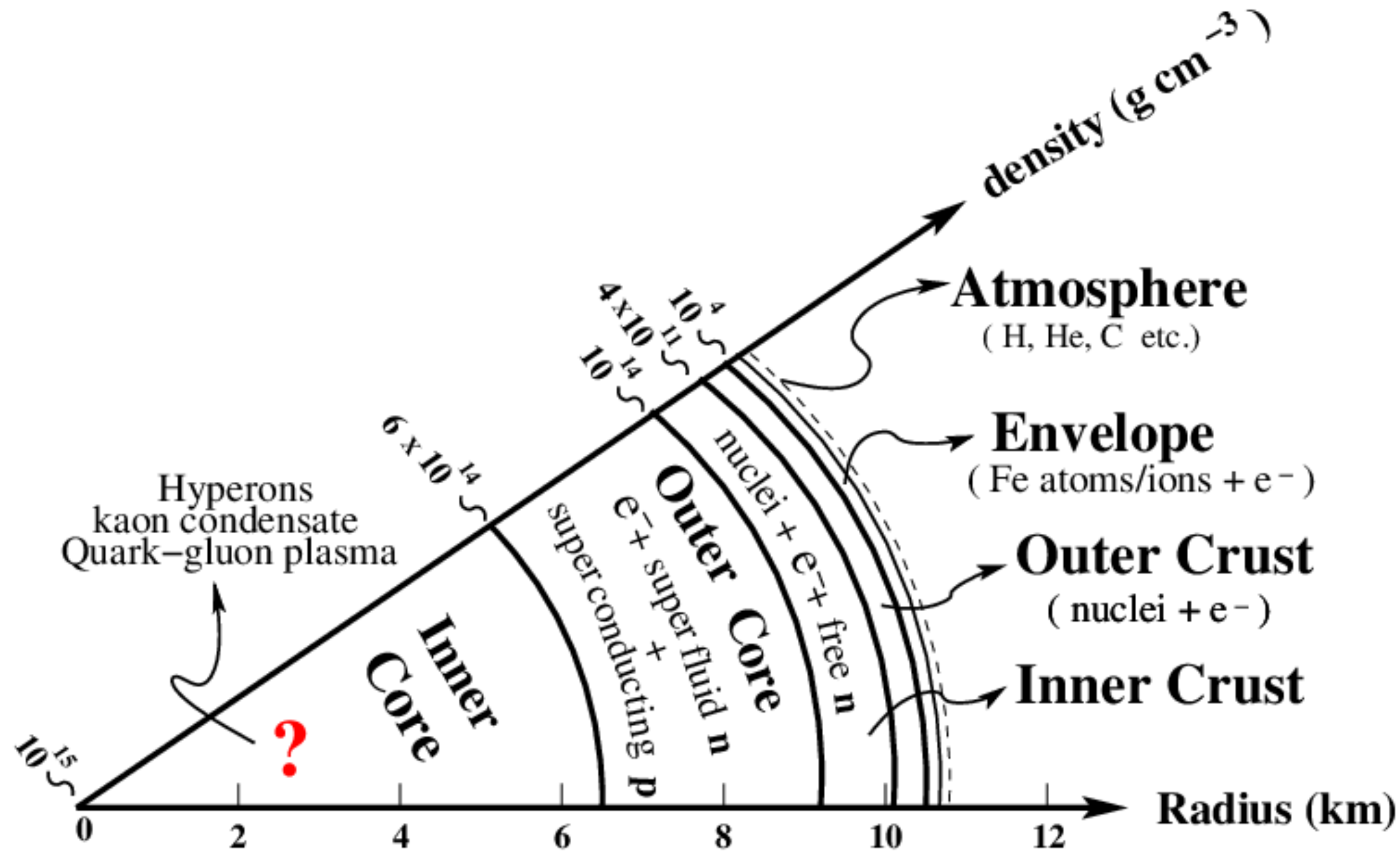
Strain prediction for the **f-mode excited**  
**by glitches**

[Phys. Rev. D 101, 103009]





# Radial profile of Neutron stars



# Metamodel representation of the nucleonic EoS

Which are the best uses for a metamodel?

EoS reconstruction

Bayesian inference

Numerical relativity simulation

Assessing uncertainty from  
astro observations

Can we decipher the  
composition?

# Bayes inference: filters and likelihood zoo

[Phys. Rev. X 15, 021014]

$$\mathcal{M} : \mathbf{X} \rightarrow \{\epsilon(n_B), P(n_B), \delta(n_B), v_\beta(n_B), v_{FR}(n_B), \dots\}$$

$$\mathcal{L}(\mathbf{X}) = \prod_j \mathcal{L}_j(\mathbf{X}) = \prod_j p(D_j | \mathcal{M}(\mathbf{X}))$$

Nuclear

pQCD

$\chi_{EFT}$

Nuclei information: NMP,  
AME masses, more?

Heavy Ion collision

Radio and X-Ray

Heavy pulsar radio timing

Black widow pulsar

NICER M-R

Kilonovae and gamma  
ray bursts

Gravitational wave

GW170817

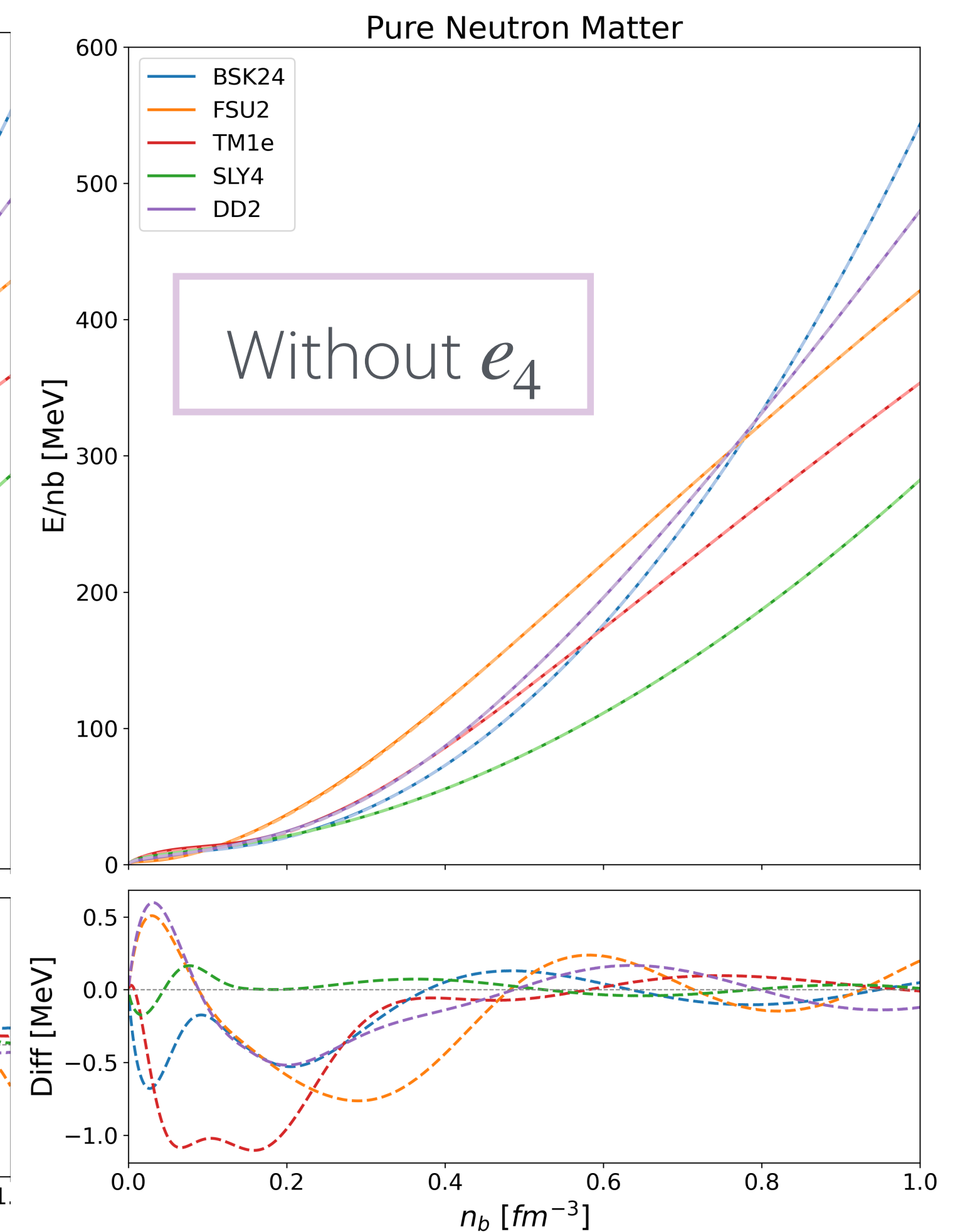
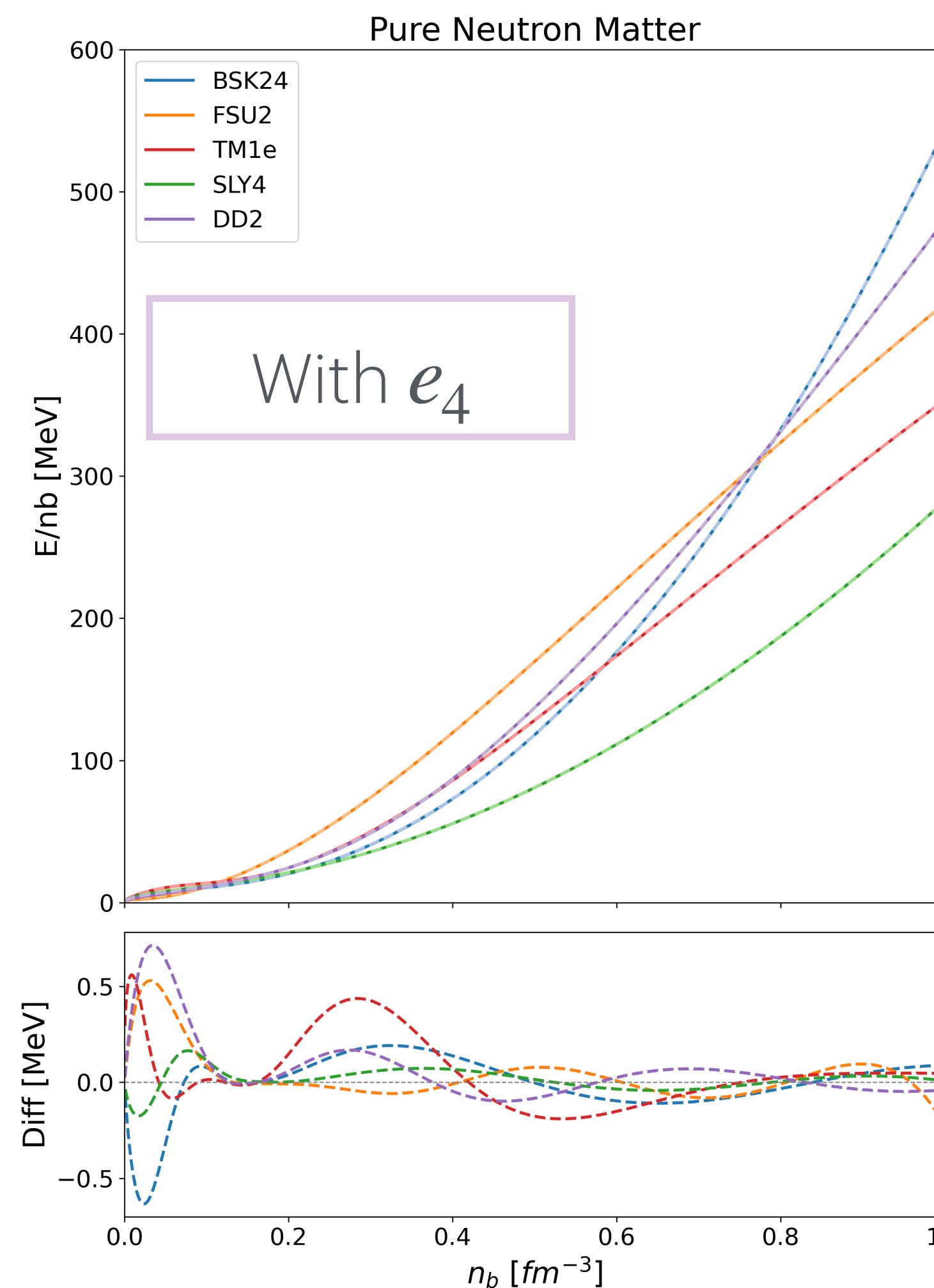
GW190425



# Fit without the quartic $\delta$ correction

Without the correction around  
saturation the PNM fit exhibit a  
 $\sim 0.5/1$  MeV of difference

The overall accuracy doesn't  
change



# FIT: results for SNM and PNM

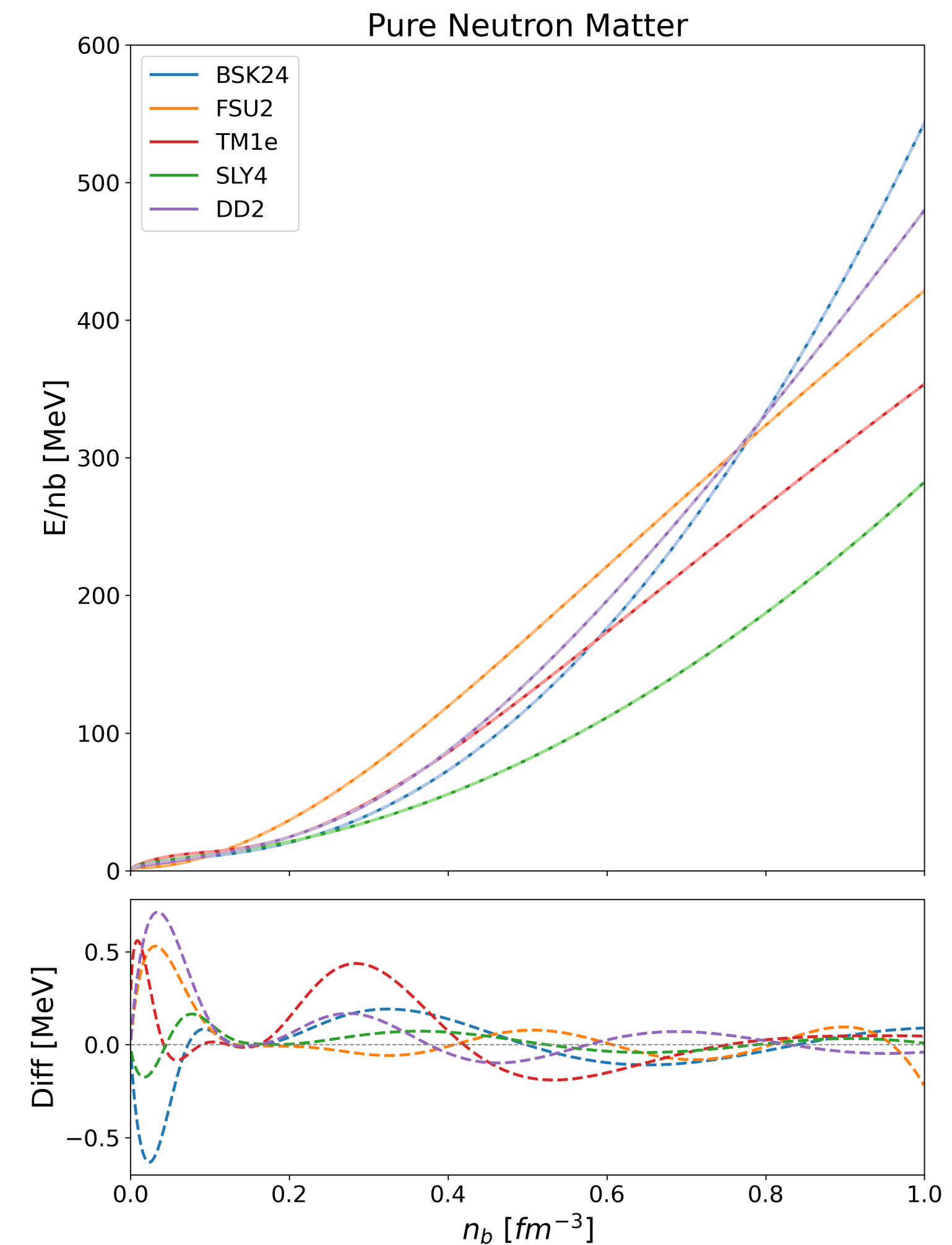
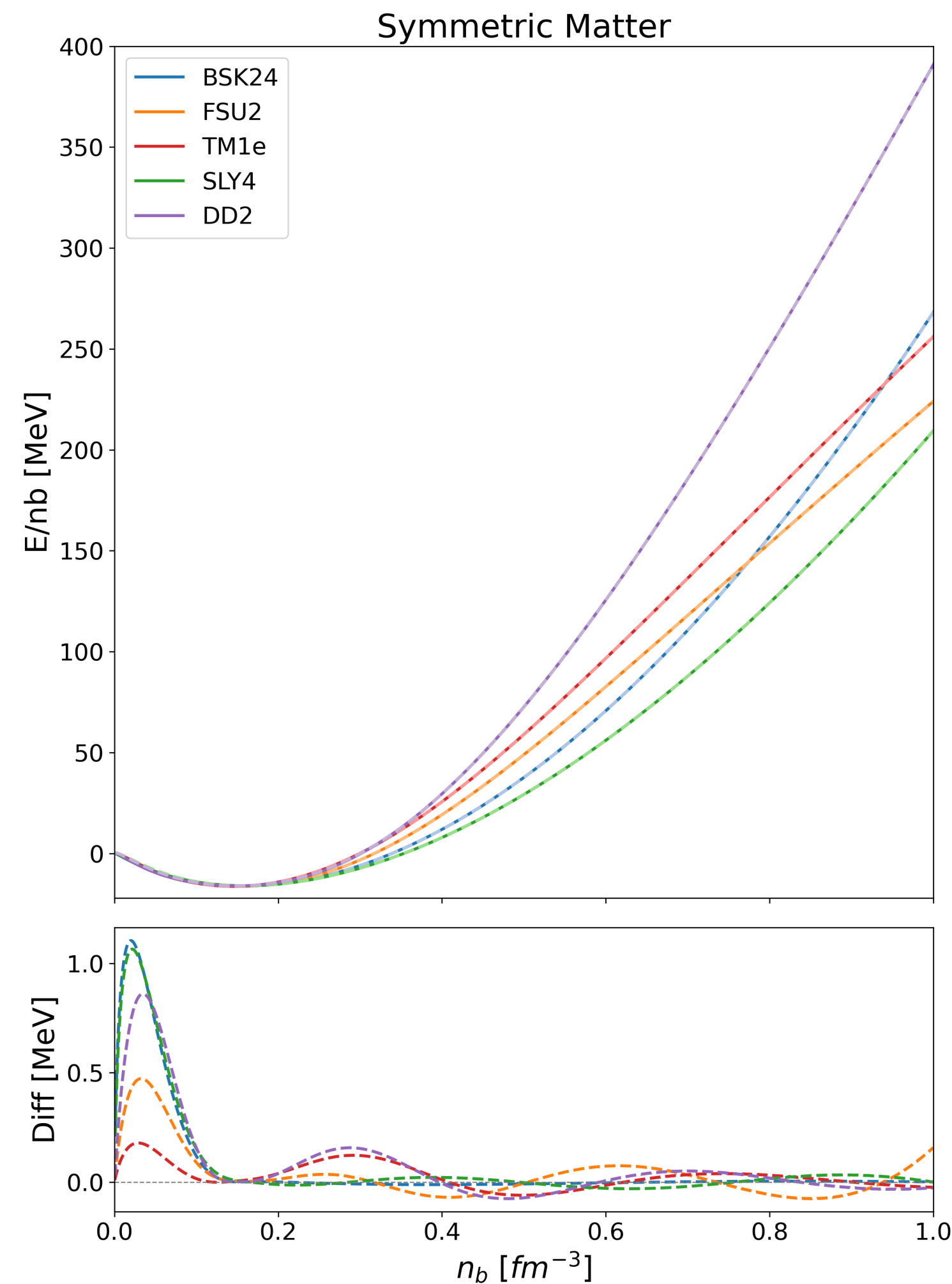
We found satisfying results with

$$a_{0,2} = b_{0,2} \text{ and } g_{0,2} = 0$$

Less Parameters!

Before saturation the accuracy is limited

Focus on  
astrophysics



# Nicer old vs nicer new:

$$M_{tov}$$

The two latest Nicer measures  
prefer soft EoS

We can see the effects on the  
PDF of  $M_{TOV}$  which is peaked  
on lower masses

