Challenges in nuclear structure theory in connection with neutrinoless $\beta\beta$ decay

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with the help of Javier Menendez, University of Barcelona

Summer School on Neutrino Physics Beyond the Standard Model
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Outline

Creation of matter in nuclei: $0\nu\beta\beta$ decay

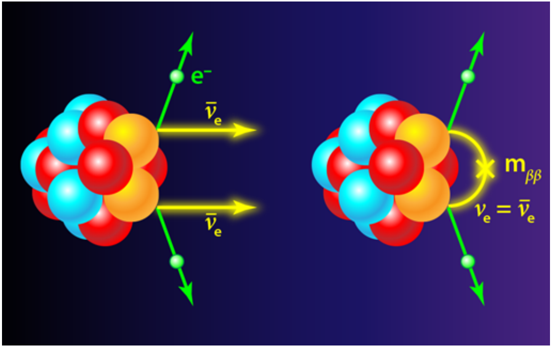
Lepton number is conserved in all processes observed:

single β decay,
$\beta\beta$ decay with ν emission...

Neutral massive particles (Majorana ν's) allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

Agostini, Benato, Detwiler, Menendez, Vissani, Rev. Mod. Phys. 95, 025002 (2023)



Creation of matter in nuclei: $0\nu\beta\beta$ decay

Lepton … (Majorana ν's)
in all pr … n:

single β …
$\beta\beta$ deca … (electrons)

SEPTEMBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Double Beta-Disintegration

M. GOEPPERT-MAYER, *The Johns Hopkins University*
(Received May 20, 1935)

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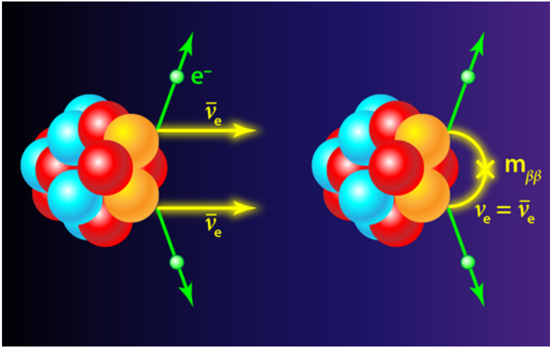

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TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

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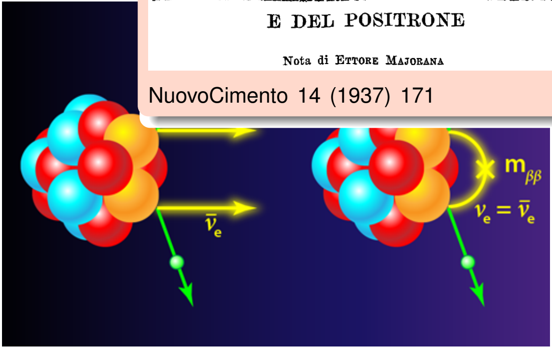

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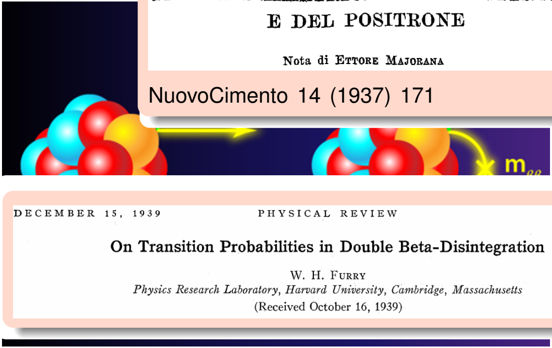


TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

NuovoCimento 14 (1937) 171

DECEMBER 15, 1939 PHYSICAL REVIEW VOLUME 56

On Transition Probabilities in Double Beta-Disintegration

W. H. Furry
Physics Research Laboratory, Harvard University, Cambridge, Massachusetts
(Received October 16, 1939)

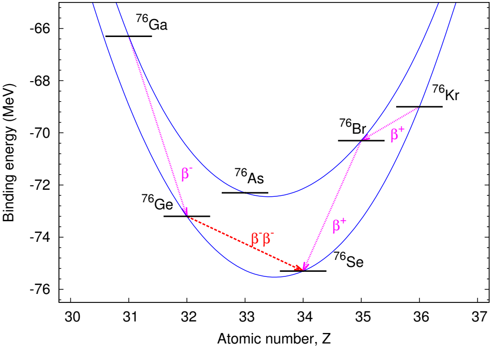
$\beta\beta$ decays

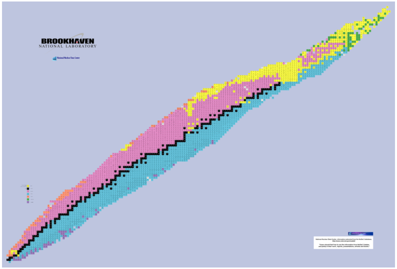
Second order process in the weak interaction

Only observable in nuclei where (much faster) β-decay is forbidden energetically due to nuclear pairing interaction

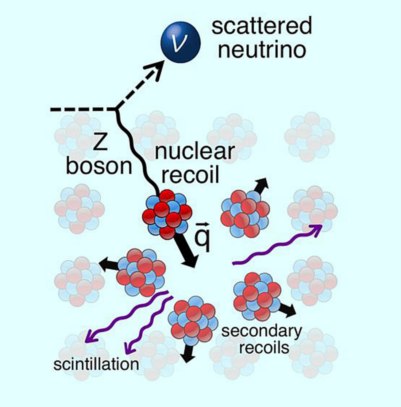
$$BE(A) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + \frac{(A-2Z)^2}{A} + \begin{cases} -\delta_{pairing} & \text{N,Z even} \\ 0 & \text{A odd} \\ \delta_{pairing} & \text{N,Z odd} \end{cases}$$

or where β-decay is very suppressed by ΔJ (total angular momentum) difference between mother and daughter nuclei





Coherent ν-nucleus scattering, dark matter detection



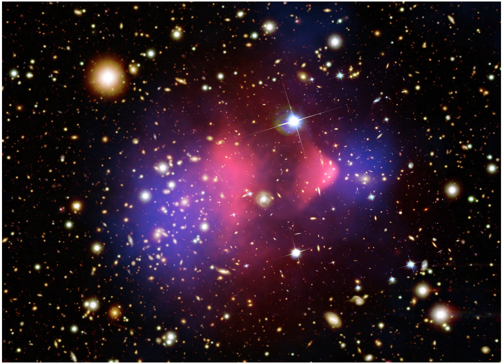
Dark matter scattering off nuclei

What is dark matter made of?

Coherent ν-nucleus scattering

Neutral current process, tiny cross-section

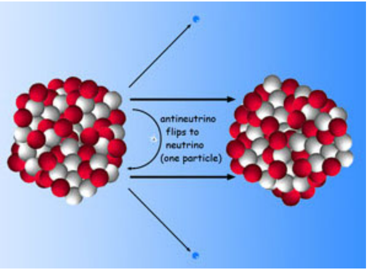
Neutrinos couple to neutrons, complements EM interactions



Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

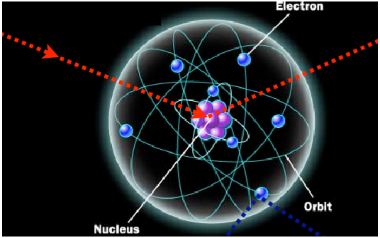
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments



$$0\nu\beta\beta\text{: } \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$



$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor

Different scales in new-physics searches using nuclei

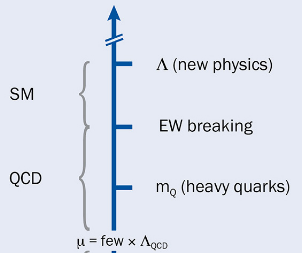
New physics scale: $\Lambda \gg 250$ GeV

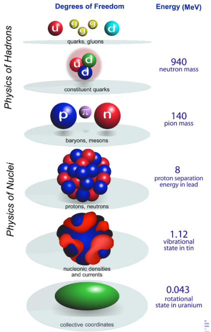
Electroweak scale:

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 250 \text{ GeV}$$

QCD (hadron) scale: $m_N \sim$ GeV

Nuclear scale: $k_F \sim m_\pi \sim 200$ MeV

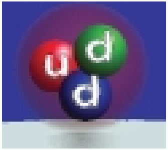


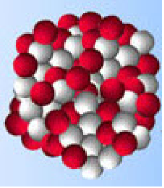


Particle, hadronic and nuclear physics

ν scattering off nuclei
interplay of particle, hadronic and nuclear physics:

ν's: interaction with quarks and gluons
Quarks and gluons: embedded in the nucleon
Nucleons: form complex, many-nucleon nuclei





General ν-nucleus scattering cross-section:

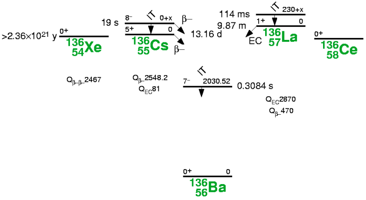
$$\frac{\mathrm{d}\sigma_{\nu\mathcal{N}}}{\mathrm{d}\boldsymbol{q}^2} \propto \left| \sum_i c_i\, \zeta_i\, \mathcal{F}_i \right|^2$$

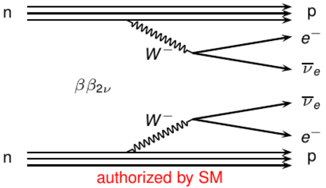
ζ: kinematics $(q^2, \cdots)$

c coefficients:
ν couplings to quark, gluons (Wilson coefficients), particle physics
convoluted with hadronic matrix elements, hadronic physics

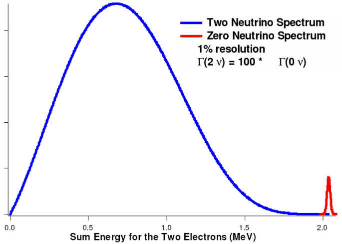
$\mathcal{F}$ functions: $\mathcal{F}^2 \sim$ structure factor, nuclear structure physics

$\beta\beta$ decay





Transition	$Q_{\beta\beta}$ (keV)	Abundance ($^{232}Th = 100$)
$^{110}Pd \rightarrow ^{110}Cd$	2013	12
$^{76}Ge \rightarrow ^{76}Se$	2040	8
$^{124}Sn \rightarrow ^{124}Te$	2288	6
$^{136}Xe \rightarrow ^{136}Ba$	2479	9
$^{130}Te \rightarrow ^{130}Xe$	2533	34
$^{116}Cd \rightarrow ^{116}Sn$	2802	7
$^{82}Se \rightarrow ^{82}Kr$	2995	9
$^{100}Mo \rightarrow ^{100}Ru$	3034	10
$^{96}Zr \rightarrow ^{96}Mo$	3350	3
$^{150}Nd \rightarrow ^{150}Sm$	3667	6
$^{48}Ca \rightarrow ^{48}Ti$	4271	0.2



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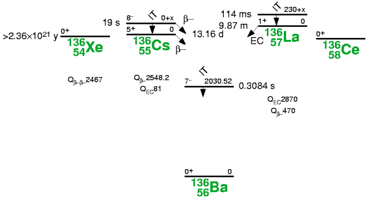

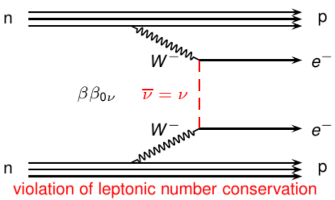

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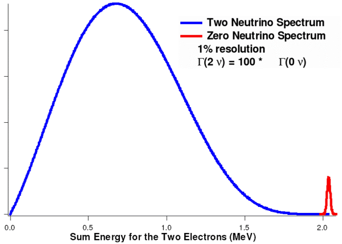


$(\beta\beta)_{0\nu}$ decay

Specificity of $(\beta\beta)_{0\nu}$:

NO EXPERIMENTAL DATA !!!

prediction for m_ν very difficult
easier for m_ν(A)/m_ν(A')

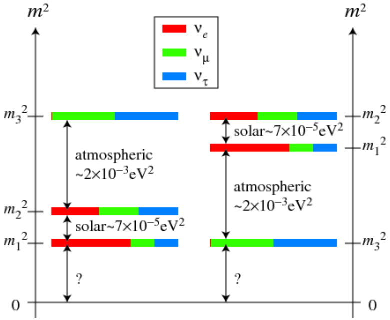
What is the best isotope to observe $(\beta\beta)_{0\nu}$ decay ?

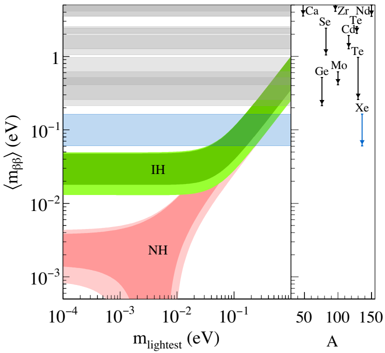
What is the influence of the structure of the nucleus on $(\beta\beta)_{0\nu}$ matrix elements ?

Next generation experiments: inverted hierarchy

Decay rate sensitive to neutrino masses, hierarchy
$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$

$$T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)^{-1} = G_{0\nu}\, g_A^4 \left|M_{\text{light}}^{0\nu}\right|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$





KamLAND-Zen, PRL117 082503(2016)

Matrix elements needed to make sure next generation ton-scale experiments full explore “inverted hierarchy”

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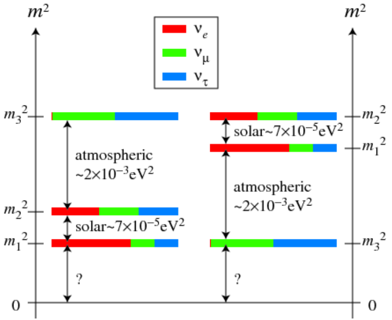
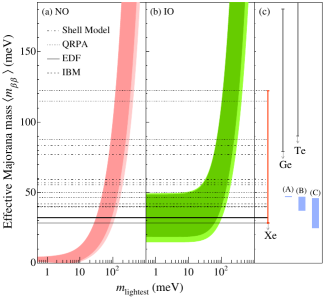




KamLAND-Zen (2024) arXiv:2406.11438

Matrix elements needed to make sure next generation ton-scale experiments full explore “inverted hierarchy”

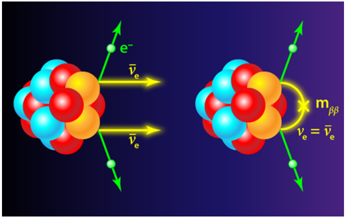
Calculating nuclear matrix elements

Nuclear matrix elements needed in low-energy new physics searches

$$\langle\, \text{Final}\, | \mathcal{L}_{\text{leptons}-\text{nucleons}} |\, \text{Initial}\,\rangle = \langle\, \text{Final}\,|\int dx\, j^{\mu}(x) J_{\mu}(x)\,|\,\text{Initial}\,\rangle$$

- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  GFMC, Coupled-cluster, IM-SRG...

- Lepton-nucleus interaction:
  Study hadronic current in nucleus:
  phenomenological approaches,
  effective theory of QCD



Double-beta decay emitters

Only decay candidates with $Q_{\beta\beta} > 2$ MeV
experimentally interesting due to extremely long lifetimes

ECEC, ECβ^+ and $\beta^+\beta^+$ also more suppressed

Transition	$T^{2\nu\beta\beta}$ (y)	$Q_{\beta\beta}$ (MeV)	Ab. (%)	
^{48}Ca $\rightarrow$ ^{48}Ti	$4.4 \cdot 10^{19}$	4.274	0.2	CANDLES
^{76}Ge $\rightarrow$ ^{76}Se	$1.7 \cdot 10^{21}$	2.039	8	GERDA, MAJORANA, LEGEND
^{82}Se $\rightarrow$ ^{82}Kr	$9.2 \cdot 10^{19}$	2.996	9	SuperNEMO
^{96}Zr $\rightarrow$ ^{96}Mo	$2.3 \cdot 10^{19}$	3.350	3	
^{100}Mo $\rightarrow$ ^{100}Ru	$7.1 \cdot 10^{18}$	3.034	10	AMoRE, CUPID
^{110}Pd $\rightarrow$ ^{110}Cd		2.013	12	
^{116}Cd $\rightarrow$ ^{116}Sn	$2.9 \cdot 10^{19}$	2.802	7	COBRA
^{124}Sn $\rightarrow$ ^{124}Te		2.288	6	
^{130}Te $\rightarrow$ ^{130}Xe	$6.9 \cdot 10^{20}$	2.530	34	CUORE, SNO+
^{136}Xe $\rightarrow$ ^{136}Ba	$2.2 \cdot 10^{21}$	2.462	9	nEXO, KamLAND-Zen, NEXT, DARWIN
^{150}Nd $\rightarrow$ ^{150}Sm	$8.2 \cdot 10^{18}$	3.667	6	

Worldwide running and planned experiments on different isotopes

Outline

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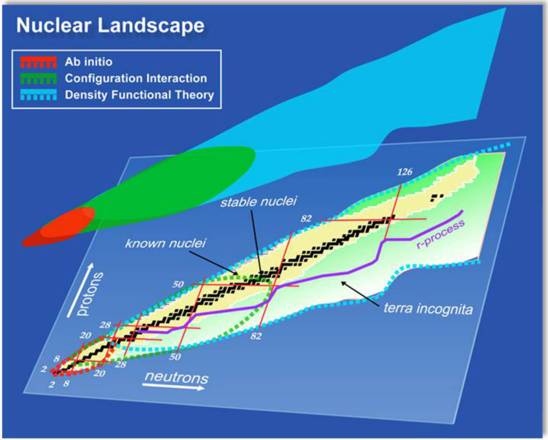
Nuclear many-body problem

The number of nucleons in nuclei is too large for an exact solution of A-body Schrödinger equation. Still, it is much too small for statistical methods



- Ab initio Methods
- Nuclear Shell Model (SM) / Configuration Interaction (CI)
- Density Functional Theory (DFT)

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \Longleftrightarrow \mathcal{E}_{EDF}[\rho]$$

$$\rho_{ij} = \langle \phi | a_j^\dagger a_i^\dagger | \phi \rangle \Longleftarrow | \phi \rangle = \prod a_i^\dagger | - \rangle$$

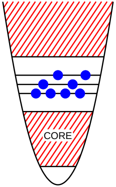
Many-body methods for $\beta\beta$ decay

Different many-body methods are used in $\beta\beta$ decay

- Nuclear shell Model
  Madrid-Strasbourg, Michigan, Bucharest, Tokyo
  Relatively small valence spaces (one shell), all correlations included
- Quasiparticle random-phase approximation (QRPA) method
  Tübingen, Bratislava, Jyvaskyla, Chapel Hill, Prague...
  Several shells, only simple correlations included
- Interacting Boson Model
  Yale-Concepción
  Small space, important proton-neutron pairing correlations missing
- Energy Density Functional theory
  Madrid, Beijing
  >10 shells, important proton-neutron pairing correlations missing

Ab initio many-body methods:
No Core Shell Model, Green's Function Monte Carlo, Coupled Cluster...

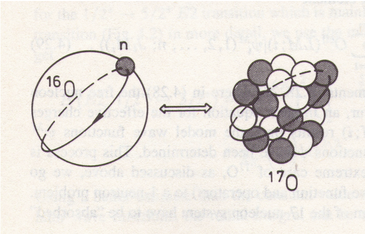
Shell Model problem



- Define a valence space
- Derive an effective interaction

$$\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

- Build and diagonalize the Hamiltonian matrix



- In general, effective operators also have to be introduced to account for the restrictions of the Hilbert space

$$\langle\Psi|\mathcal{O}|\Psi\rangle = \langle\Psi_{eff}|\mathcal{O}_{eff}|\Psi_{eff}\rangle$$

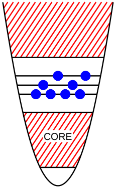
- In principle, all the spectroscopic properties are described simultaneously (Rotational band AND β decay half-life)

- A valence space can be adequate to describe some properties and completely wrong for others

$^{48}_{24}Cr_{24}$	$(f_{\frac{7}{2}})^8$	$(f_{\frac{7}{2}}p_{\frac{3}{2}})^8$	$(f_{\frac{7}{2}}f_{\frac{5}{2}})^8$	$(fp)^8$
$\langle n_{f7/2}\rangle$	8	7.21	7.60	6.55
$E(2^+)$	0.55	0.42	1.17	0.74
$Q(2^+)$	0.0	-26	-0.03	-29.5
$BE2(2^+ \rightarrow 0^+)$	77	150	82	215
B(GT)	0.80	0.96	4.54	4.25

- For the quadrupole properties $f_{\frac{7}{2}}p_{\frac{3}{2}}$ is a good space whereas for magnetic and Gamow-Teller processes the presence of the spin orbit partners is compulsory

Shell Model problem



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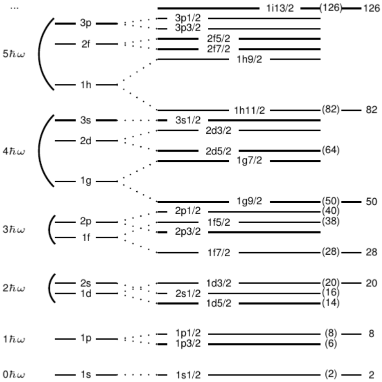

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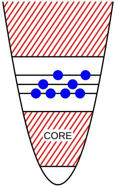

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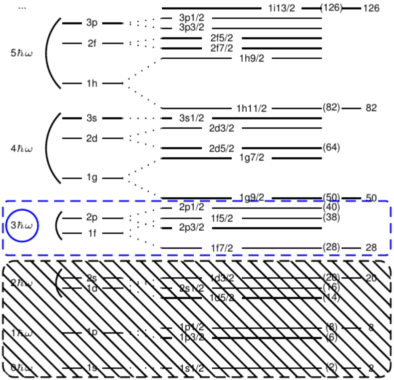

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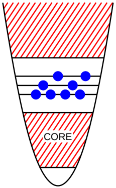

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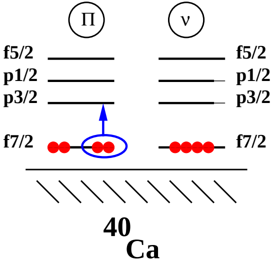

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Shell Model: Giant Computations

- exponential growth of basis dimensions:

$$D \sim \begin{pmatrix} d_\pi \\ p \end{pmatrix} \cdot \begin{pmatrix} d_\nu \\ n \end{pmatrix}$$

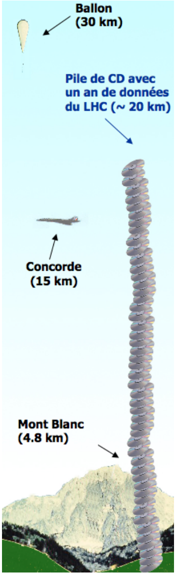
In *pf* shell :
^{48}Cr 1,963,461
^{56}Ni **1,087,455,228**
In *pf*-*sdg* space :
^{78}Ni **210,046,691,518**

- Actual limits in limits in giant diagonalizations: **0.2 10^{12}** for ^{114}Sn core excitations

- Some of the largest diagonalizations ever are performed in Strasbourg with relatively modest computationnal ressources:
Phys. Rev. C82 (2010) 054301, ibidem 064304

• m scheme ANTOINE code
• coupled scheme NATHAN code

E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427;
ANTOINE website



- Largest matrices up to now contain up to $\sim$ **10^{14}** non-zero matrix elements.

- This would require more than 1,000,000 CD-ROM's to store the information for a single matrix !

- They cannot be stored on hard disk and are computed on the fly.

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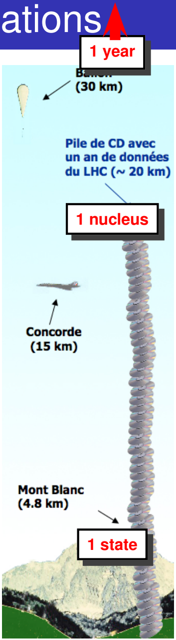

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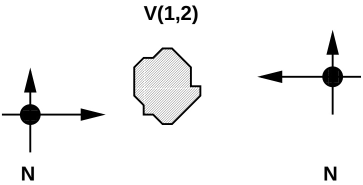
Historical Realistic Interactions

- Free nucleon-nucleon interaction:



- Spin state S=0,1
- Charge state pp,pn,nn (isospin T)
- Spatial state L=even,odd

$\Bigg\} \quad \delta_{(LS)J,T}$

- Deuteron properties

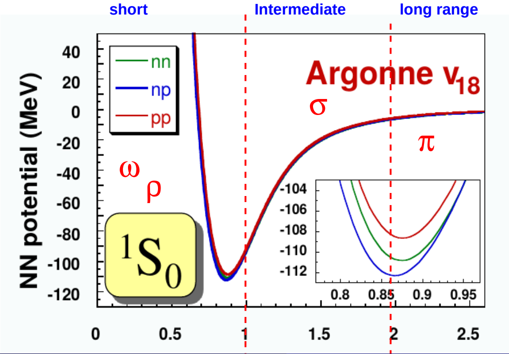
$\Longrightarrow$ Realistic potentials : Hamada-Johnston, Paris, CD-Bonn, Argonne, Idaho-A

$$v(NN) = v^{EM}(NN) + v^{\pi}(NN) + v^{SR}(NN)$$

Effective interactions for SM calculations

In the past in SM calculations schematic NN interactions were used, e.g. pairing plus quadrupole Hamiltonian or surface delta interactions. During last 40 years, effective interactions based on realistic NN potentials have been developed.

$$
\begin{aligned}
V(r) &= V_0(r) + V_\sigma \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_\tau \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + V_{\sigma\tau}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \quad \text{central} \\
&+ V_{LS} \boldsymbol{L} \cdot \boldsymbol{S} + V_{LS\tau}(\boldsymbol{L} \cdot \boldsymbol{S})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \quad \text{spin} - \text{orbit} \\
&+ V_T \boldsymbol{S}_{12} + V_{T\tau} \boldsymbol{S}_{12}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \quad \text{tensor} \\
&+ \ldots
\end{aligned}
$$



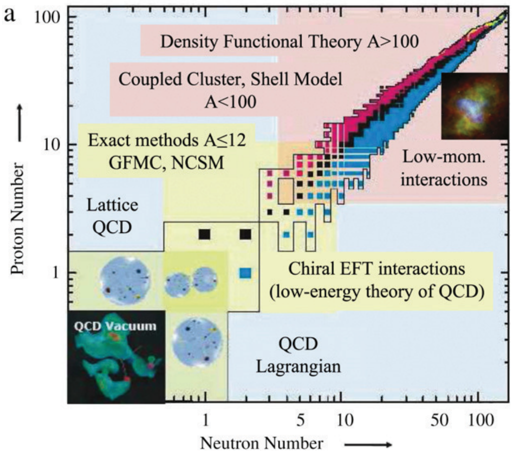
Proper calculations of OBEP are done within Quantum Field Theory. One has then to take the following path:

- Write down the Lagrangians for interactions of nucleons with mesons
- Using these Lagrangians, calculate Feynman diagrams that contribute to NN scattering

CD-Bonn, Argonne v18 *R. Machleidt, Phys. Rev. C63 (2001) 024001; R. Wiringa et al., Phys. Rev. C51 (1995) 38.*

Nuclear Structure from First Principles

All nuclear structure calculations are, to some extent, phenomenological



Relevant degrees of freedom: protons and neutrons
Many-body problem too hard in general, approximations are needed

Nuclear force at low (nuclear structure) energies: adjustments to reproduce finite nuclei needed

Can we connect nuclear structure calculations to quantum chromodynamics (QCD)?

Theory for nuclear forces

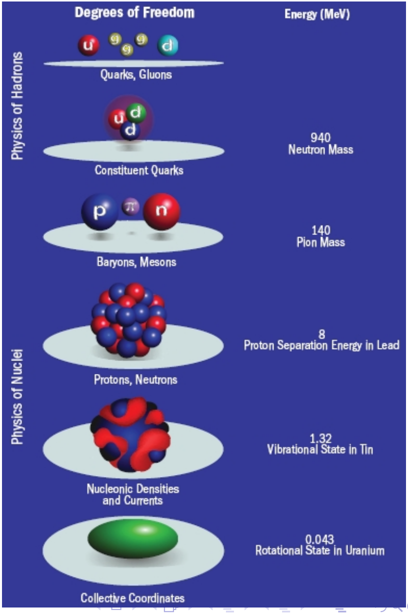
Difficult to find NN potential with consistent NNN forces and connected to QCD...

Use concept of separation of scales

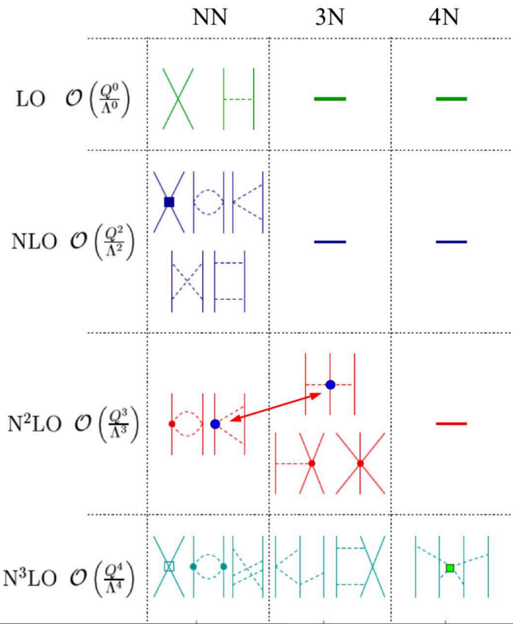
The energy scale relevant determines the degrees of freedom

For nuclear structure,
typical energies of interest
point to nucleons and pions
(pions are particularly light mesons!)

Effective theory with nucleons and pions as degrees of freedom,
with connection to QCD



Modern nuclear forces: N3LO



- We use pions and nucleons as degrees of freedom.
- The effective Lagrangian is classified using a systematic expansion based on a power counting in terms of $(Q/\Lambda_\chi)^\nu$, where ν is called chiral order and Λ_χ is the hard scale ($\sim$700MeV)
- ν=0 is called leading order, $\nu > 1$ are called next-to-$\nu - 1$ leading orders.
- Note hierarchy of nuclear forces.
- Coupling constants (LEC) adjusted to phase shifts and deuteron properties.

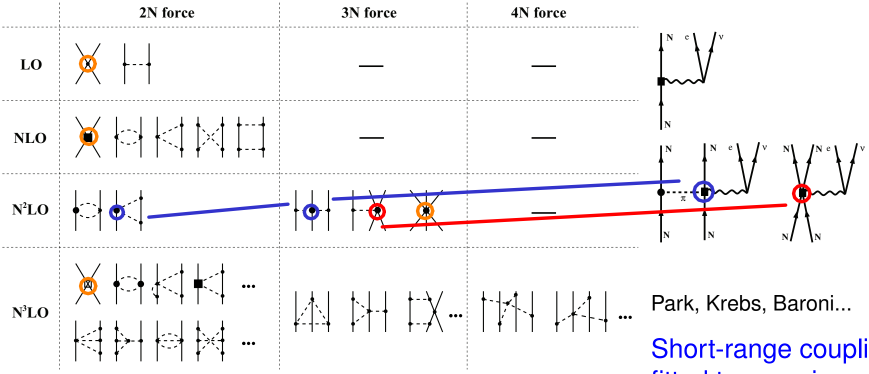
E. Epelbaum et al., Rev. Mod. Phys. 81 (2009) 1773.

Chiral Effective Field Theory (EFT)

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents



Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

Park, Krebs, Baroni...

Short-range couplings fitted to experiment once

Outline

Weak interactions in nuclei

β and $\beta\beta$ decay processes are driven by the Weak interaction

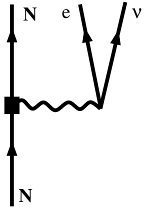
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$j_{L\mu}$ is the leptonic current (electron, neutrino): $j_{L\mu} = \overline{e}\gamma_\mu \left(1 - \gamma_5\right) \nu_{eL}$

The Lorentz structure is Vector − Axial-Vector ($V - A$) current,
as indicated by the Standard Model of Particle Physics

For neutrinos,
interaction eigenstates are not mass eigenstates:
$\nu_{eL} = \sum_i U_{ei} \nu_{iL}$,
with U the PMNS neutrino-mixing matrix

The treatment of electrons and neutrinos
is relatively easy because
they are elementary particles



Weak interactions: hadronic current

β and $\beta\beta$ decay processes are driven by the Weak interaction

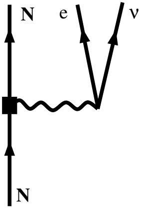
$$H_W = \frac{G_F}{\sqrt{2}} \left(j_{L\mu} J_L^{\mu\dagger} \right) + H.c.$$

$J_L^{\mu\dagger}$ is the hadronic current:
it is not so straightforward because the Standard Model predicts $J_L^{\mu\dagger}$ at the level of quarks and we need $J_L^{\mu\dagger}$ at the level of nucleons:

- Obtain $J_L^{\mu\dagger}$ phenomenologically
- Obtain $J_L^{\mu\dagger}$ using an effective theory: Chiral EFT

In nuclei (non-relativistic), β decay is simply

$$\langle F | \sum_i g_V \, \tau_i^- + g_A \, \boldsymbol{\sigma}_i \tau_i^- \, | I \rangle$$



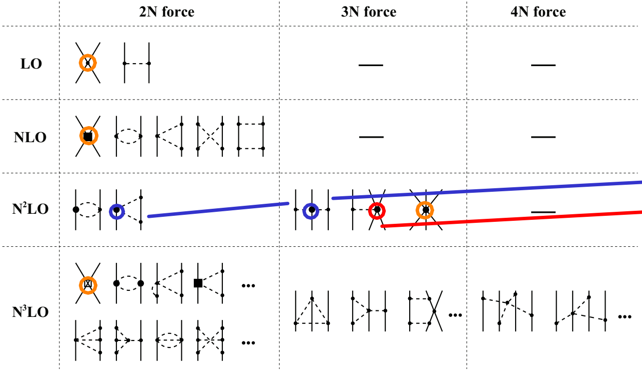
corresponding to Fermi and Gamow-Teller transitions,
corrections (forbidden transitions)
involve an expansion of the lepton wavefunctions

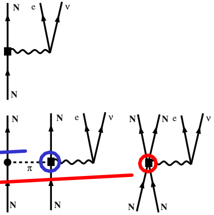
Chiral Effective Field Theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents





Weinberg, van Kolck, Kaplan, Savage, Wise, Meißner, Epelbaum...

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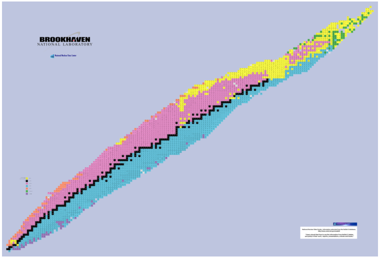
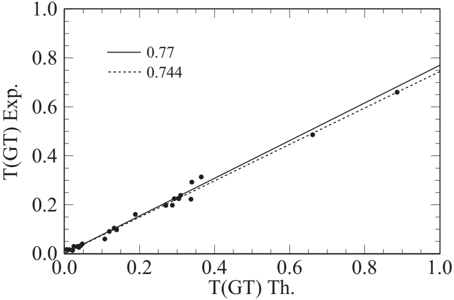
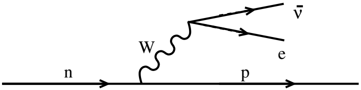
Short-range couplings fitted to experiment once

β decay: theory vs experiment

β decays (e^- capture) main decay model along nuclear chart
In general well described by nuclear structure theory: shell model...







Martinez-Pinedo et al. PRC53 2602(1996)

$$\langle F| \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]\, |I\rangle$$

β decay: “quenching”

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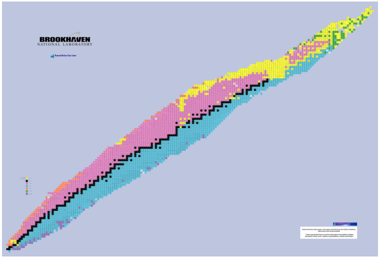

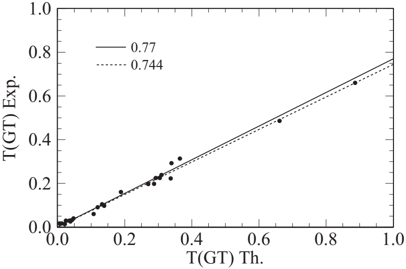

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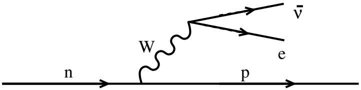


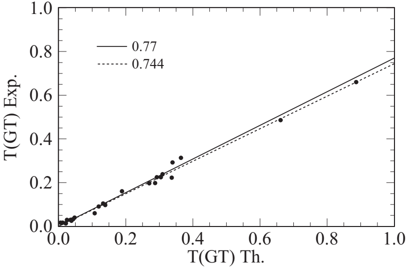
$$\langle F | \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} | I \rangle \,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Gamow-Teller transitions:
theory needs $\boldsymbol{\sigma}_i \tau$ “quenching”

Deficient many-body approach,
or transition operator?

Gamow-Teller β decay with IM-SRG

β decays (e^- capture) challenge for nuclear theory



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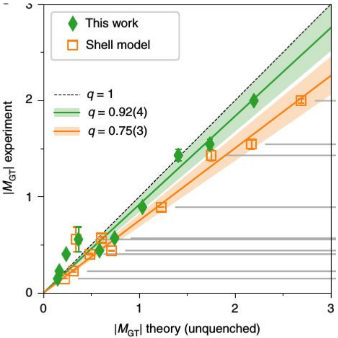
Phenomenological models
need $\boldsymbol{\sigma}_i \tau$ “quenching”



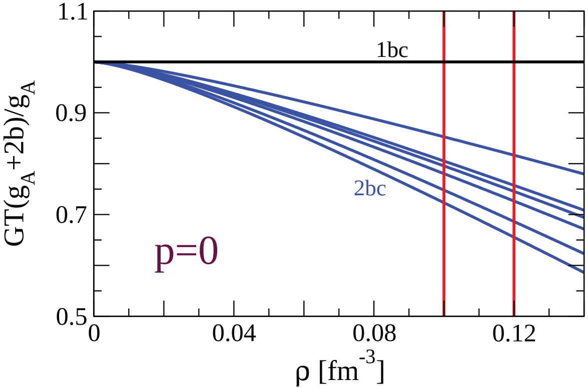
Gysbers et al. Nature Phys. 15 428 (2019)

Ab initio calculations including
meson-exchange currents
do not need any “quenching”

2b currents at zero momentum-transfer

2b currents at $p = 0$: relevant for Gamow-Teller decays, $2\nu\beta\beta$ decay

$$\mathbf{J}^{\text{eff}}_{n,2b} = -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$



Black horizontal line represents 1b current

Difference horizontal line and blue lines 2b effect

Nuclear density range $\rho = 0.10 \ldots 0.12$ fm^{-3}

Couplings c_3, c_4 taken from NN potentials

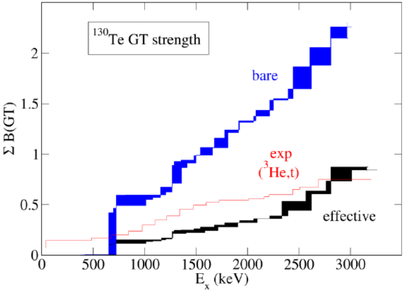
2b currents, in normal-ordered approximation predict quenching $q = 0.85 \ldots 0.66$

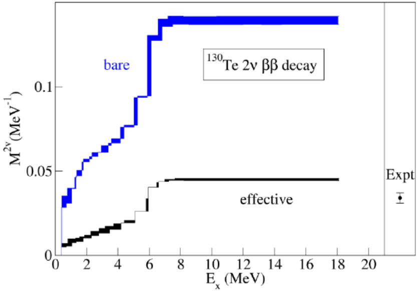
Renormalisation of the GT operator by MBPT

PHYSICAL REVIEW C **95**, 064324 (2017)

Calculation of Gamow-Teller and two-neutrino double-β decay properties for ^{130}Te and ^{136}Xe with a realistic nucleon-nucleon potential

L. Coraggio,[1,*] L. De Angelis,[1] T. Fukui,[1] A. Gargano,[1] and N. Itaco[1,2]





Renormalisation of the GT by Many-Body Perturbation Theory

$$\langle \Psi | \mathcal{O} | \Psi \rangle \; = \; \langle \Psi_{eff} | \mathcal{O}^{(1)}_{eff} + \mathcal{O}^{(1,2)}_{eff} | \Psi_{eff} \rangle$$

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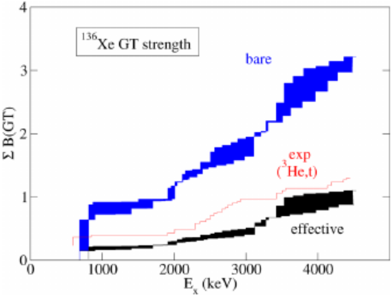

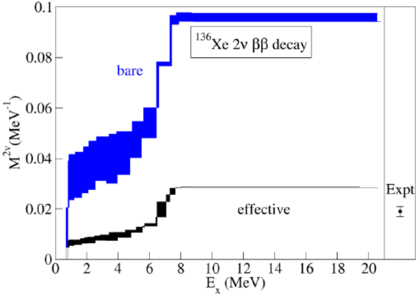

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Renormalisation of the GT operator by MBPT

Definitions

See also: "Value of the axial-vector coupling strength in β and $\beta\beta$ decays: A review" published in **Frontiers in Physics 5 (2017) 55**.

Nucleon weak current in a nucleus:

$$j^{\mu}_{\mathrm{N}} = g_{\mathrm{V}}\gamma^{\mu} - g_{\mathrm{A}}\gamma^{\mu}\gamma^{5}$$

Quenching:

$$q = g_{\mathrm{A}}/g_{\mathrm{A}}^{\mathrm{free}}$$

Free value of g_{A} (Particle Data Group 2016) from the decay of free neutron:

$$g_{\mathrm{A}}^{\mathrm{free}} = 1.2723(23)$$

Effective value of g_{A}:

$$g_{\mathrm{A}}^{\mathrm{eff}} = q g_{\mathrm{A}}^{\mathrm{free}}$$

Jouni Suhonen (JYFL, Finland) MEDEX'19 7 / 31

From J. Suhonen, MEDEX 2019, Prague

Renormalisation of the GT by Many-Body Perturbation Theory

$$\langle\Psi|\mathcal{O}|\Psi\rangle = \langle\Psi_{eff}|\mathcal{O}^{(1)}_{eff} + \mathcal{O}^{(1,2)}_{eff}|\Psi_{eff}\rangle$$

Outline

Two-neutrino $\beta\beta$ decay matrix elements

Two-neutrino double-beta decay matrix element, second order process

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \tau_n^- + \sigma_n \tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \tau_m^- + \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | J_k^+ \rangle \langle J_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

$$= \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

- $\tau_n^- \tau_m^-$ transform two neutrons into two protons
- Only Gamow-Teller spin operator contributes:
  Fermi contribution vanishes due to isospin conservation:

$$\langle 0_f^+ | \sum_m \tau_m^- | J_k^+ \rangle = \langle 0_f^+ | T^- | J_k^+ \rangle \sim 0$$

- Neutrinos are emitted, do not appear in the transition operator

  $\Rightarrow$ Only intermediate nucleus $|1_k^+\rangle$ states contribute

Two-neutrino $\beta\beta$ decay calculations

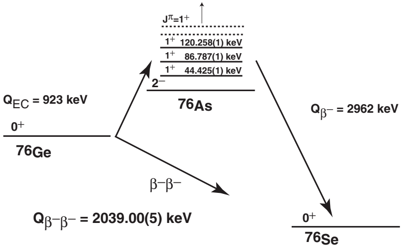
$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- \, | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- \, | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

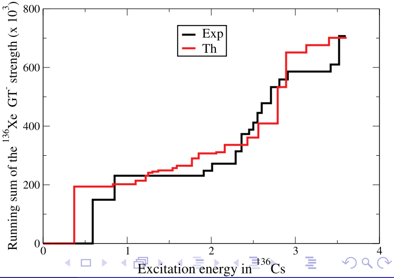
Shell Model $2\nu\beta\beta$ decay calculations
in good agreement to experiment

GT quenching is needed

Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.047	kb3
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.048	kb3g
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.065	gxpf1
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.116	gcn28:50
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.120	jun45
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.126	gcn28:50
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.124	jun45
$^{128}Te \rightarrow {}^{128}Xe$	0.049 ± 0.006	0.57	0.059	gcn50:82
$^{130}Te \rightarrow {}^{130}Xe$	0.034 ± 0.003	0.57	0.043	gcn50:82
$^{136}Xe \rightarrow {}^{136}Ba$	0.019 ± 0.002	0.45	0.025	gcn50:82





Gamow-Teller Strengths
(each leg of the $\beta\beta$ decay) are well reproduced

Two-neutrino $\beta\beta$ decay calculations

$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- | 1_k^+ \rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- | 0_i^+ \rangle}{E_k - (M_i + M_f)/2}$$

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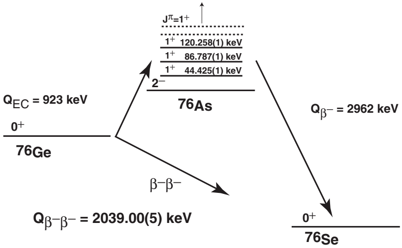
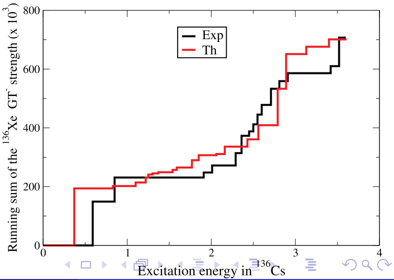
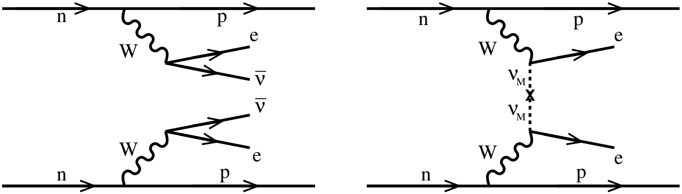




But so far no successfull
Ab initio calculations available ...

$0\nu\beta\beta$ decay vs $2\nu\beta\beta$ decays

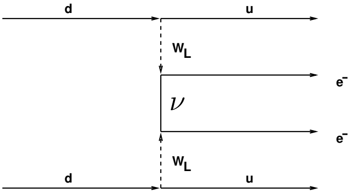
From the theoretical point of view, $0\nu\beta\beta$ and $2\nu\beta\beta$ decays are also different



- In $2\nu\beta\beta$ decay, the momentum transfer to the leptons is limited by $Q_{\beta\beta}$, while for $0\nu\beta\beta$ decay larger momentum transfers are permitted
- In $0\nu\beta\beta$ decay the Majorana neutrinos annihilate each other which is only possible if neutrinos have mass
- In $0\nu\beta\beta$ decay the Majorana neutrinos are part of the transition operator, via the so-called neutrino potential

Neutrinoless mode:

Exchange of a light neutrino, only left-handed currents



The theoretical expression of the half-life of the 0ν mode can be written as:

$$[T^{0\nu}_{1/2}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$$

Neutrinoless mode:

CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda,K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

Neutrinoless mode:

CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda,K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

two-body operator

Neutrinoless mode:

CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda,K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

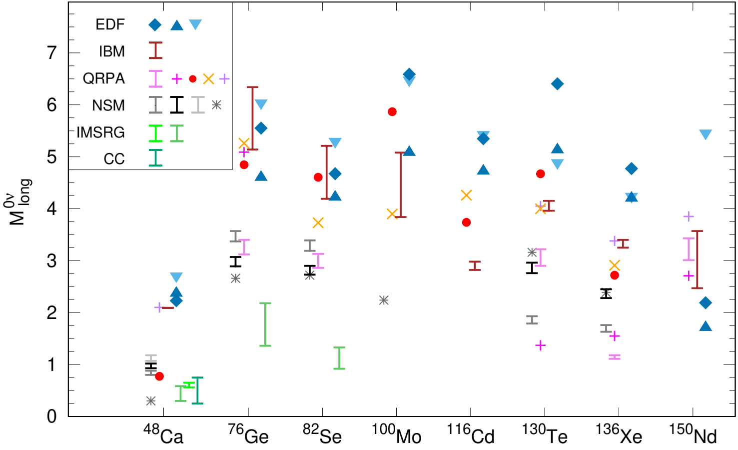
two-body operator

We are left with a “standard” nuclear structure problem

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - (\frac{g_V}{g_A})^2 M_F^{(0\nu)} - M_T^{(0\nu)}$$

$0\nu\beta\beta$ decay nuclear matrix elements

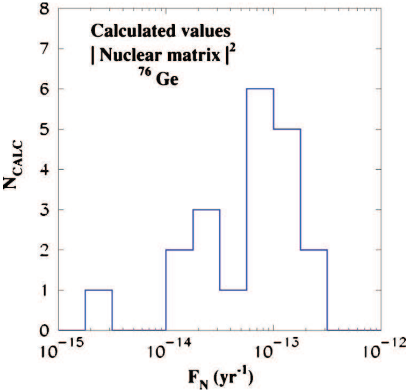
Large difference in nuclear matrix element calculations: factor ~ 3



Agostini, Benato, Detwiler, Menendez, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

$0\nu\beta\beta$ decay nuclear matrix elements

Spread about factor two – three in nuclear matrix element calculations



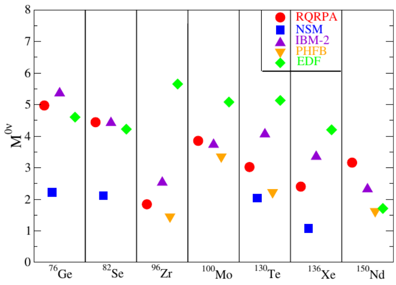
But this means a big improvement!

The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos. The essential problem is that the correct theory of nuclei

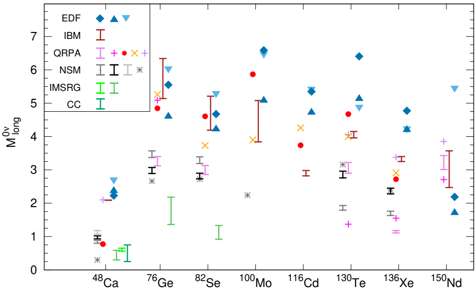
Bahcall, Murayama, Pena-Garay
PRD70 033012 (2004)

$0\nu\beta\beta$ matrix elements: last 15 years

Comparison of nuclear matrix elements calculations: 2012 vs 2023



P. Vogel, J. Phys. G39 124002 (2012)



Agostini, Rev. Mod. Phys.95, 025002 (2023)

What have we learned in the last 15 years ?

IM-GCM $0\nu\beta\beta$ NME for ^{48}Ca

Multi-reference calculation:
correlations systematically built on collective reference state
Generator coordinate method: deformation, isoscalar pairing

$$\langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_x H^x(r)\, \Omega^x \, | 0_i^+ \rangle$$

Best IM-GCM calculation reproduces EM transitions in ^{48}Ti

NME $\sim 0.4/30\%$ smaller than nuclear shell model

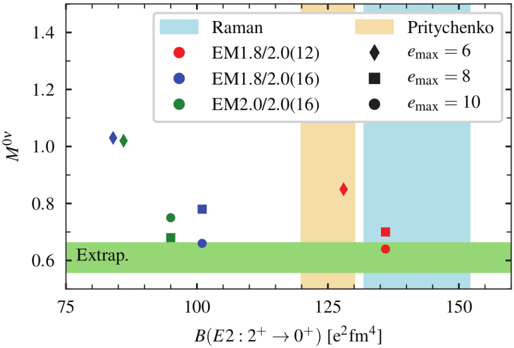
Yao et al.

PRL 124 232501 (2020)

Consistent with coupled cluster NME

Novario et al.

PRL 126 182502 (2021)



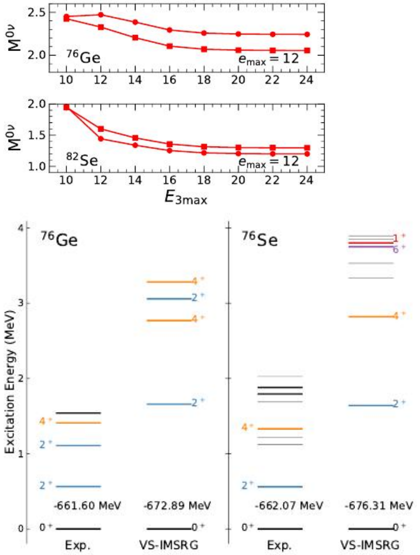
VS-IMSRG $0\nu\beta\beta$ NME for ^{76}Ge, ^{82}Se

VS-IMSRG reaches ^{76}Ge
one of the targets used in
most advanced experiments
(GERDA, MAJORANA)

VS-IMSRG NME converged
in 3N matrix elements included
Miyagi et al.
PRC105 014302 (2022)

Excitation spectra too spread
quadrupole correlations
not properly captured?

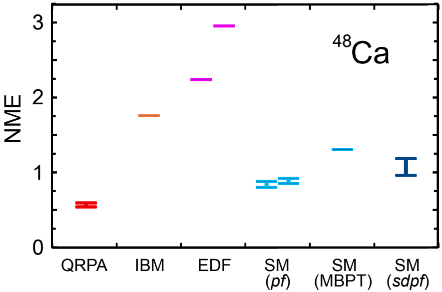
NME $\sim 20\%/50\%$ smaller
than nuclear shell model
Belley et al.
PRL126 042502 (2021)

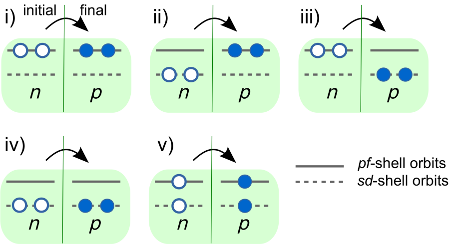


Shell model configuration space: two shells

^{48}Ca extended configuration space from *pf* to *sdpf*, 4 to 7 orbitals
dimension 10^5 to 10^9
^{48}Ca 0_2^+ state lowered by 1.3 MeV
nuclear matrix elements
enhanced only moderately 30%

Iwata et al. PRL116 112502 (2016)



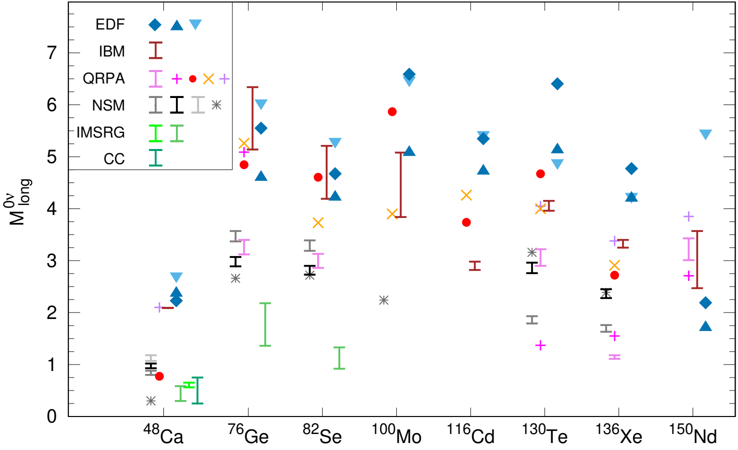


Terms dominated by pairing
2 particle – 2 hole excitations
enhance the $\beta\beta$ matrix element

Terms dominated by
1 particle – 1 hole excitations
suppress the $\beta\beta$ matrix element

$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor ~ 3

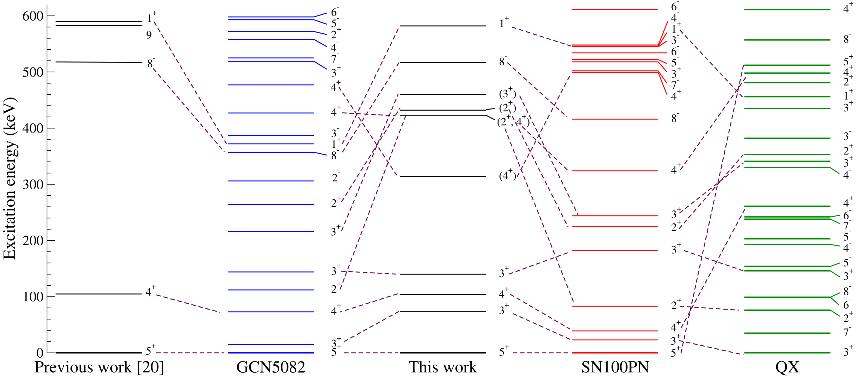


Agostini, Benato, Detwiler, Menendez, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

^{136}Cs experimental spectrum

While all these interactions are well, tested recent data on ^{136}Cs suggests GCN5082 results agree better with experiment than QX

Rebeiro, Triambak et al. arXiv:2301.11371

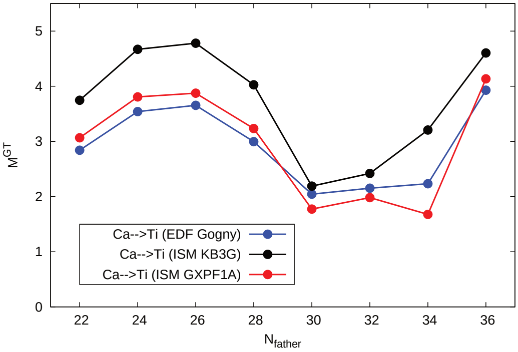


QX gives systematically smaller ^{136}Xe $0\nu\beta\beta$-decay nuclear matrix elements

$0\nu\beta\beta$ decay without correlations

Non-realistic spherical (uncorrelated) mother and daughter nuclei:

- Shell model (SM): zero seniority, neutron and proton $J = 0$ pairs
- Energy density functional (EDF): only spherical contributions



In contrast to full (correlated) calculation SM and EDF NMEs agree!

NME scale set by pairing interaction

Menendez, Rodríguez, Martínez-Pinedo, Poves PRC90 024311(2014)

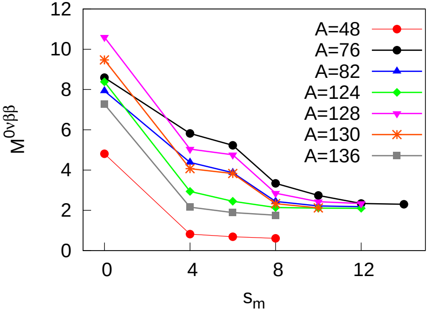
NME follows generalized seniority model:

$$M_{GT}^{0\nu\beta\beta} \simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{\Omega_\pi - N_\pi}\sqrt{N_\nu}\sqrt{\Omega_\nu - N_\nu + 1},$$ Barea, Iachello PRC79 044301(2009)

Pairing correlations and $0\nu\beta\beta$ decay

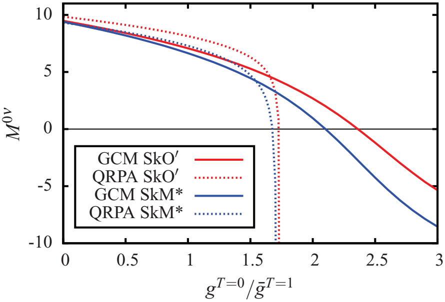
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



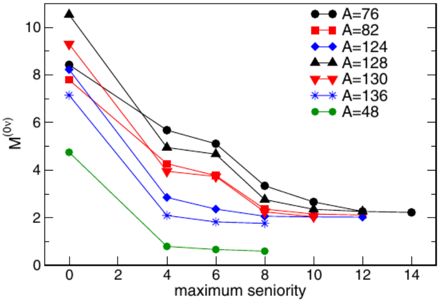
Hinohara, Engel PRC90 031301 (2014)

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\tau_i\tau_j$ operator

Pairing correlations and $0\nu\beta\beta$ decay

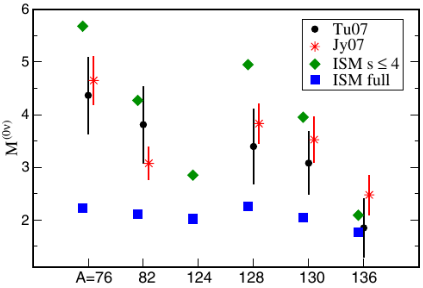
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



E. Caurier et al., PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



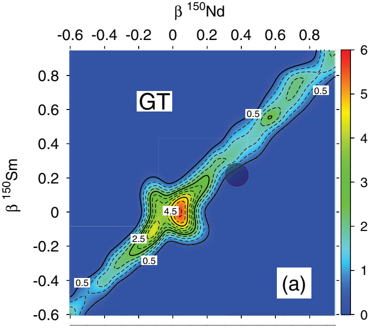
E. Caurier et al., PRL100 052503 (2008)

Related to approximate SU(4) symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator

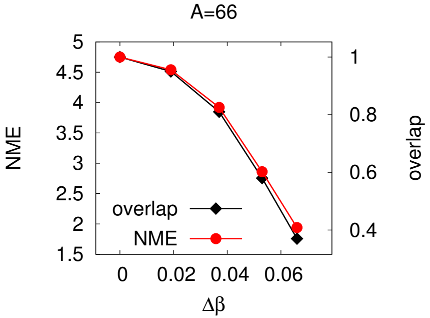
Deformation and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay is disfavoured by quadrupole correlations

$0\nu\beta\beta$ decay very suppressed when nuclei have different structure



Rodríguez, Martínez-Pinedo
PRL105 252503 (2010)



Menendez, Caurier, Nowacki, Poves
JPCS267 012058 (2011)

Suppression also observed with QRPA Fang et al. PRC83 034320 (2011)

Double Gamow-Teller strengths and $\beta\beta$ decay

Measurement of Double Gamow-Teller (DGT) resonance
in double charge-exchange reactions ^{48}Ca(pp,nn)^{48}Ti proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN (^{48}Ca), INFN Catania

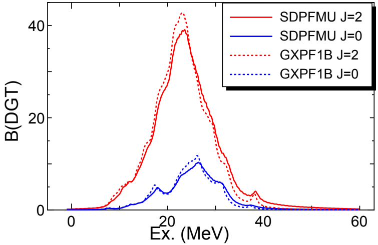
Takaki et al. JPS Conf. Proc. 6 020038 (2015)

Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Promising connection to $\beta\beta$ decay,
two-particle-exchange process,
especially the (tiny) transition
to ground state of final state

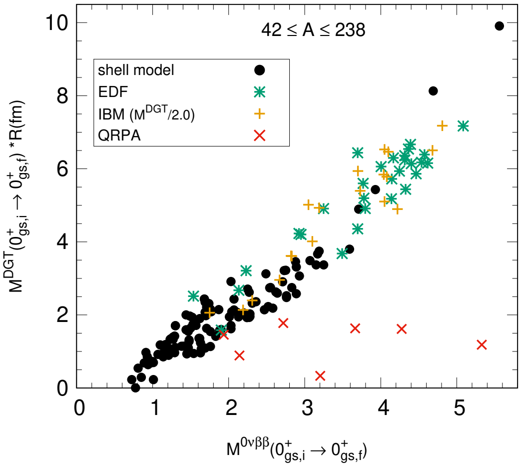
Shell model calculation

Shimizu, Menendez, Yako, PRL120 142502 (2018)



$$B(DGT^-;\lambda; i \to f) = \frac{1}{2J_i+1}\left|\left\langle {}^{48}\mathrm{Ti}\left|\left|\left[\sum_i \boldsymbol{\sigma}_i\tau_i^- \times \sum_j \boldsymbol{\sigma}_j\tau_j^-\right]^{(\lambda)}\right|\right|{}^{48}\mathrm{Ca}_{gs}\right\rangle\right|^2$$

Double GT transition to ground state
good linear correlation with $0\nu\beta\beta$ decay NMEs



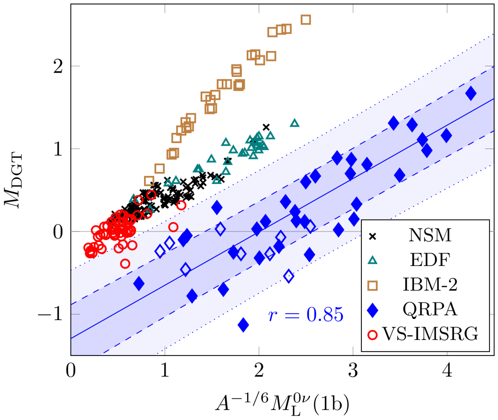
Double Gamow-Teller correlation with $0\nu\beta\beta$ decay holds across nuclear chart
Shimizu, Menendez, Yako
PRL120 142502 (2018)

Common to shell model energy-density functionals interacting boson model, disagreement to QRPA
Also correlation in VS-IMSRG (but weaker)
Yao et al. PRC106 014315(2022)

Experiments at RIKEN, INFN, RCNP? access DGT transitions

Correlation of $0\nu\beta\beta$ decay to DGT in QRPA

In QRPA, g_{pp} parameter
typically fitted to reproduce $2\nu\beta\beta$ half-life of measured transitions
but actually some tension between g_{pp} values to reproduce single-β decays
Faessler et al., J. Phys. G 35, 075104 (2008)



Jokiniemi, Menendez, PRC 107 044316 (2023)

Perform QRPA calculations with range of $g_{pp} = (0.6 - 0.9)$

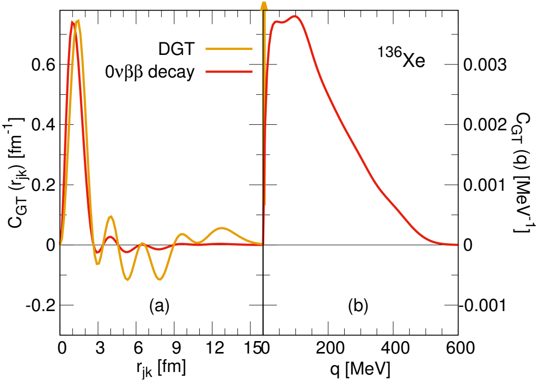
Correlation between DGT and $0\nu\beta\beta$ NMEs!
but different than for other many-body methods

Partially caused by relevance of $J > 1$ intermediate states in QRPA compared to eg shell model
Ejiri et al. Phys. Rept. 797 1 (2019)
Horoi et al, PRC 93, 044334 (2016)

Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and $0\nu\beta\beta$ decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

Long-range part dominant in QRPA DGT matrix elements

Shimizu, Menendez, Yako, PRL120 142502 (2018)

Short-range character of DGT, $0\nu\beta\beta$ decay

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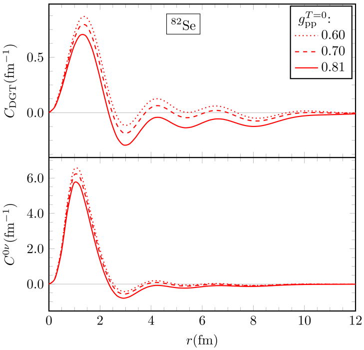
Bogner et al. PRC86 064304 (2012)



Jokiniemi, Menendez, PRC 107 044316 (2023)

$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

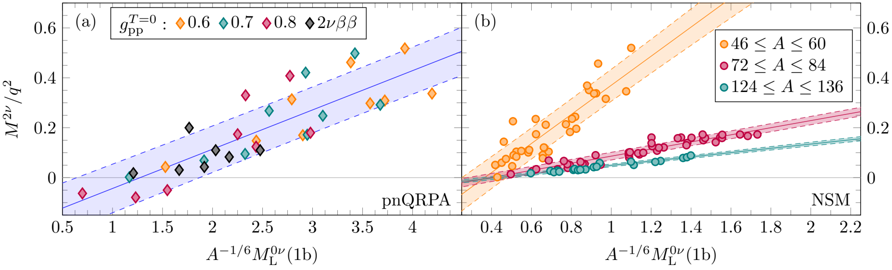
Long-range part dominant in QRPA DGT matrix elements

Shimizu, Menendez, Yako, PRL120 142502 (2018)

Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca, ^{136}Xe
Horoi et al. PRC 106, 054302 (2022), PRC 107, 045501 (2023)



Jokiniemi, Romeo, Soriano, Menendez, PRC 107 044305 (2023)

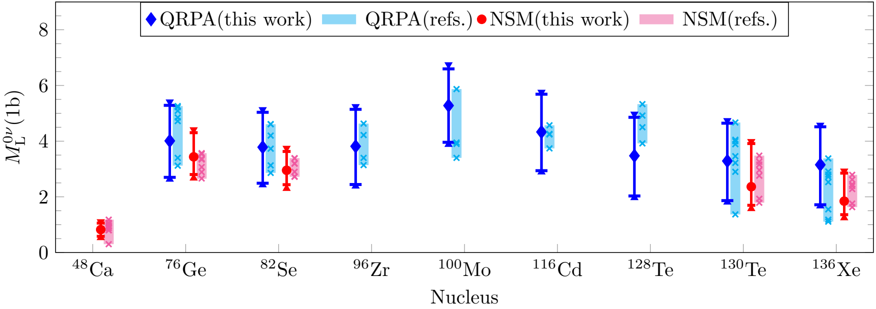
Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs!

$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

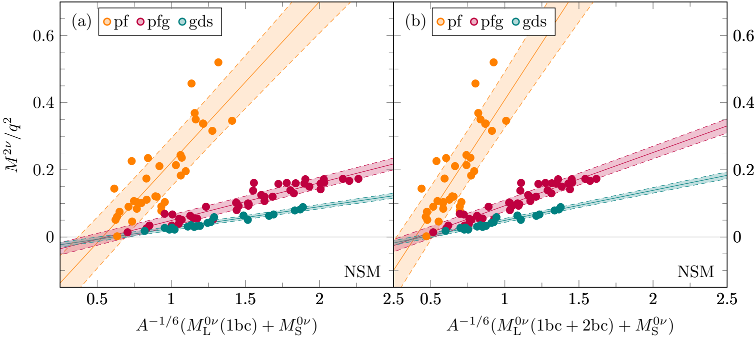
Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, Menendez, PRC 107 044305 (2023)

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: general case

A good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also appears when we include to the calculation of $0\nu\beta\beta$ NMEs
2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, Menendez, PRC 107 044305 (2023)

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents, short-range NME

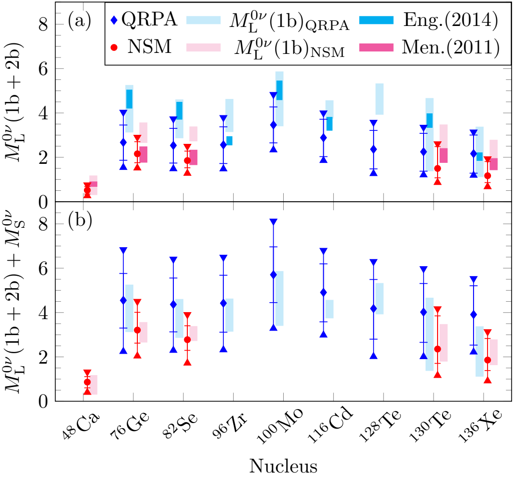
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation,
calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, Menendez, PRC 107 044305 (2023)



$0\nu\beta\beta$ decay light- and heavy-particle exchange

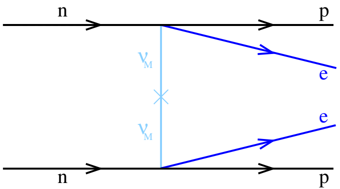
Neutrinoless $\beta\beta$ decay mediated by light or heavy particles

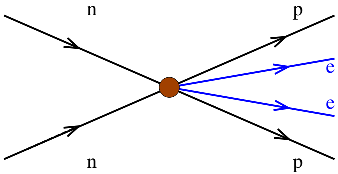
Barea, Horoi, Menendez, Šimkovic, Suhonen...

$$M^{0\nu\beta\beta} = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_X H^X(r)\,\Omega^X \,| 0_i^+ \rangle$$

$$H^X(r) = \frac{2}{\pi}\frac{R}{g_A^2}\int_0^\infty f^X(pr)\frac{h^X(p^2)}{\left(\sqrt{p^2+m_\nu^2}\right)\left(\sqrt{p^2+m_\nu^2}+\langle E^m\rangle - \frac{1}{2}(E_i-E_f)\right)}p^2\,dp$$

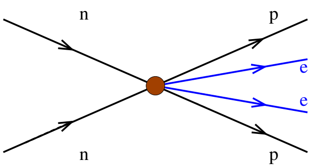
Same contributions in both channels
but in heavy-neutrino exchange the standard term becomes shorter range
$p \sim 100-200$ MeV, set by typical distance between decaying nucleons

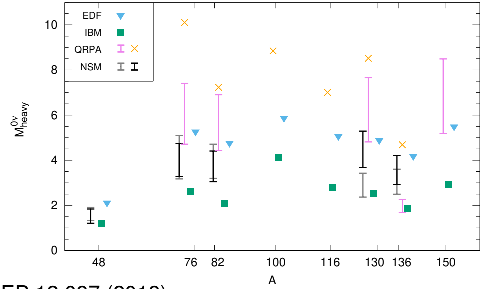




$0\nu\beta\beta$ mediated by BSM heavy particles

Extensions of the Stantard Model can also trigger $0\nu\beta\beta$ decay
typically mediated by exchange of heavy particle (heavy ν, M_R...)





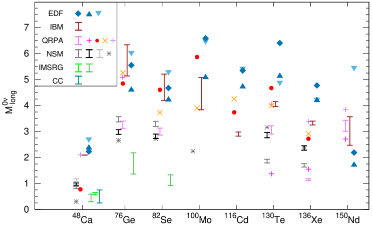
Effective field theory Cirigliano et al JHEP 12 097 (2018)
dimension-7 ($\sim 1/\Lambda^3$), dimension-9 ($\sim 1/\Lambda^5$) operators can lead to $0\nu\beta\beta$

$$T_{1/2}^{-1} = G_{01}\left(g_A^2\, M^{0\nu} + g_\nu^{NN}\, m_\pi^2\, M^{0\nu}_{cont}\right)^2 \frac{m_{\beta\beta}^2}{m_e^2} + \frac{m_N^2}{m_e^2}\tilde{G}\,\tilde{g}^4\,\tilde{M}^2\left(\frac{v}{\Lambda}\right)^6 + \frac{m_N^4}{m_e^2\,v^2}\tilde{G}'\,\tilde{g}'^4\,\tilde{M}'^2\left(\frac{v}{\Lambda'}\right)^{10} + \cdots,$$

Phase-space, hadronic/nuclear matrix elements, known or calculated
Present experiments constrain dim-7 / dim-9 operators $\Lambda \gtrsim 250$ / 5 TeV

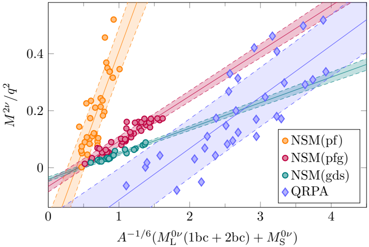
Summary

Calculations of $0\nu\beta\beta$ NMEs
challenge nuclear many-body methods,
searches demand reliable NMEs



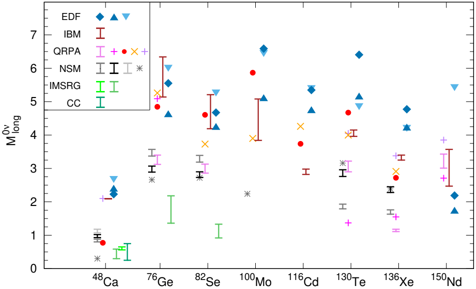
Ab initio calculations describe well β decay
They suggest reduced NMEs
due to nuclear correlations
and two-body currents

Likely enhancement by short-range NME
partially compensates reduction
due to correlations and currents



Good $0\nu\beta\beta - 2\nu\beta\beta$ correlation
in nuclear shell model, QRPA
exploit $2\nu\beta\beta$ data to obtain $0\nu\beta\beta$ NMEs
with theoretical uncertainties

$0\nu\beta\beta$ matrix elements: critical assessment



Agostini, Benato, Detwiler, Menendez, Vissani,

Rev. Mod. Phys. 95, 025002 (2023)

List of criteria for a critical assessment

- reproduce low-lying states spectroscopy in parent and daughter nuclei
- reproduce ElectroMagnetic properties
- reproduce single Gamow-Teller properties
- reproduce $(\beta\beta)_{2\nu}$ properties

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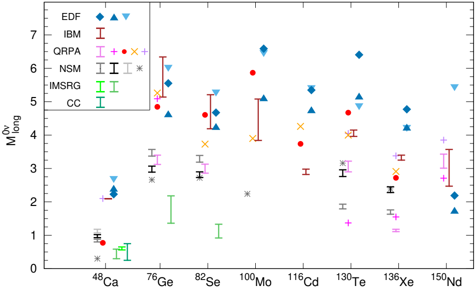

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Not much calculations left ...

Outline

$0\nu\beta\beta$ decay half-life

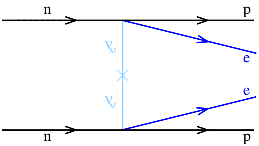
Half-life of $0\nu\beta\beta$ decay sensitive to
$m_{\beta\beta} \sim 1/\Lambda$ (dim-5 operator), new-physics scales $\widetilde{\Lambda}$ (dim-7) or $\widetilde{\Lambda}'$ (dim-9)

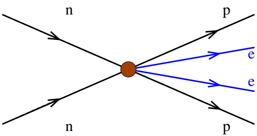
$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\text{light}}\right)^2 m_{\beta\beta}^2 + m_N^2\, \tilde{G}\, \tilde{g}^4\, \tilde{M}^2 \left(\frac{v}{\widetilde{\Lambda}}\right)^6 + \frac{m_N^4}{v^2} \widetilde{G}'\, \tilde{g}'^4\, \widetilde{M}'^2 \left(\frac{v}{\widetilde{\Lambda}'}\right)^{10}$$

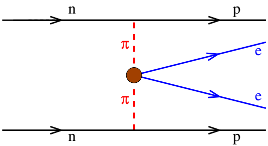
G_{01}, $\widetilde{G}$, $\widetilde{G}'$: phase-space factors (electrons), very well known

g_A, g_ν^{NN}, $\widetilde{g}$, $\widetilde{g}'$: coupling to hadron(s), experiment or calculate with QCD

$M^{0\nu}_{\text{long}}$, $M^{0\nu}_{\text{short}}$, $\widetilde{M}$, $\widetilde{M}'$: nuclear matrix elements, many-body challenge







$0\nu\beta\beta$ mediated by new-physics heavy particles

Stantard Model extensions trigger $0\nu\beta\beta$ decay (heavy ν, M_R...)

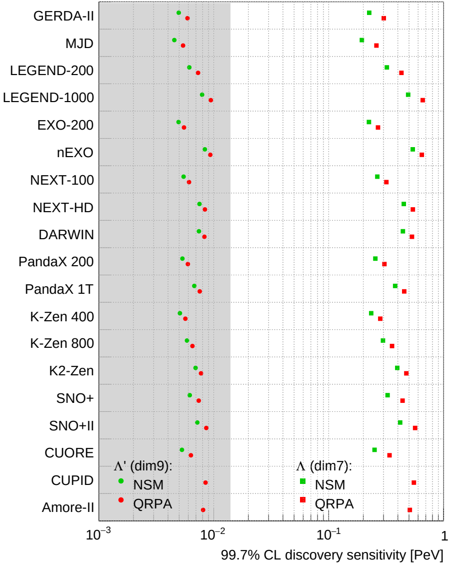
Phase-space, hadronic/nuclear matrix elements, known or calculated

Effective field theory

Cirigliano et al JHEP 12 097 (2018)

dimension-7 ($\sim 1/\Lambda^3$), dimension-9 ($\sim 1/\Lambda^5$) operators

constrained by current searches $\Lambda \gtrsim$ 250 TeV (dim-7)
$\Lambda \gtrsim$ 5 TeV (dim-9)

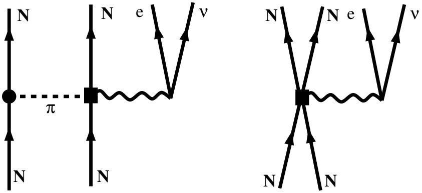


Two-body currents currents

At order Q^3 chiral EFT predicts contributions from two-body (2b) currents

Reflect interactions between nucleons

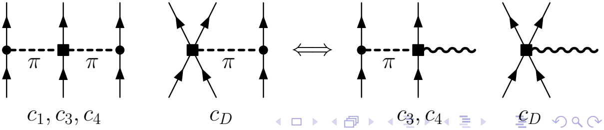
Long-range currents dominate



The expression for the leading Q^3 2b currents is

$$\mathbf{J}^3_{12} = -\frac{g_A}{4F_\pi^2}\frac{1}{m_\pi^2+k^2}\Big[2\Big(c_4+\frac{1}{4m}\Big)\mathbf{k}\times(\sigma_\times \times \mathbf{k})\tau^3_\times \\ +4c_3\mathbf{k}\cdot\big(\sigma_1\tau_1^3+\sigma_2\tau_2^3\big)\mathbf{k} - \frac{i}{m}\mathbf{k}\cdot(\sigma_1-\sigma_2)\mathbf{q}\tau^3_\times\Big]$$

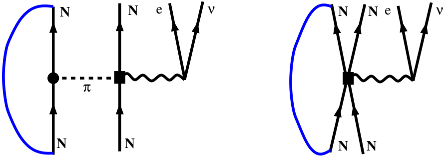
Long-range currents depend on c_3, c_4 couplings of nuclear forces



2b currents: normal-ordering

Approximate in medium-mass nuclei:
2b currents imply that the $\beta\beta$ decay operator is 4-body...
normal-ordered 1b part with respect to spin/isospin symmetric Fermi gas

Sum over one nucleon, direct and the exchange terms



$\Rightarrow \mathbf{J}^{\text{eff}}_{n,2b}$ normal-ordered 1b current

Corrections $\sim (n_{\text{valence}}/n_{\text{core}})$
in Fermi systems

The normal-ordered two-body currents modify GT operator

$$\mathbf{J}^{\text{eff}}_{n,2b} = \sum_{\boldsymbol{\sigma}_m}^{FG} \sum_{\tau_m}^{FG} \int \frac{p_m^2 dp_m}{(2\pi)^3} \, \mathbf{J}_{m,n,2b} \, (1 - P_{mn})$$

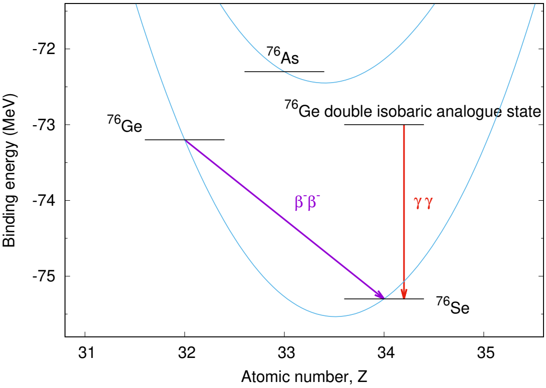
$$= -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[\frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} + I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$

long-range p dependent long-range p independent

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays,
focused on double-M1 transitions

$$M^{\gamma\gamma}_{M1\,M1} = \sum_k \frac{\langle 0_f^+ | \sum_n (g_n^l \boldsymbol{l}_n + g_n^s \boldsymbol{\sigma}_n)^{IV} | 1_k^+(\mathrm{IAS})\rangle \langle 1_k^+(\mathrm{IAS}) | \sum_m (g_m^l \boldsymbol{l}_m + g_m^s \boldsymbol{\sigma}_m)^{IV} | 0_i^+(\mathrm{DIAS})\rangle}{E_k - (E_i + E_f)/2}$$



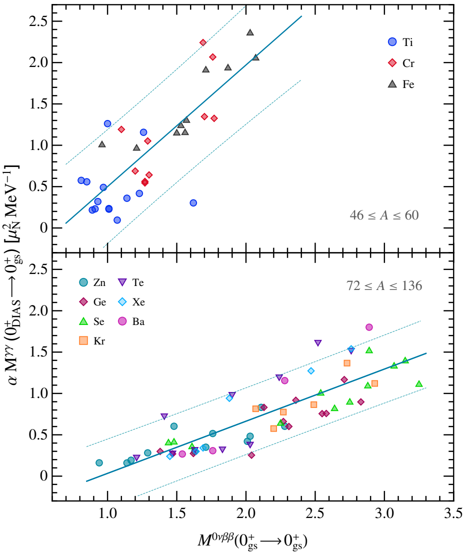
Similar initial and final states
but both in same nucleus
for electromagnetic transition

M1 and GT operators similar,
physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, Menendez, Pena-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



Good correlation between $M1M1$ same-energy photons and shell-model $0\nu\beta\beta$ NMEs

A dependence:
energy denominator dominant states at higher energy in heavier nuclei

Overall, study $\sim$ 50 transitions several nuclear interactions for each of them

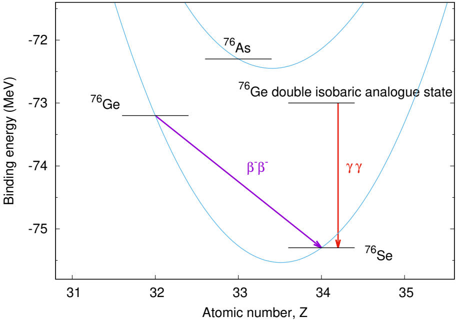
Romeo, Menendez, Pena-Garay
PLB 827 136965 (2022)

Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$ decays are very suppressed with respect to γ decays
just like $\beta\beta$ decays are much slower than β decays

$\gamma\gamma$ decays have been observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Outlook:

Study in detail leading decay channels for $M1M1$ decay in DIAS of $\beta\beta$ nuclei

Particle emission $M1$, $E1$ decay: BR $\sim 10^{-7} - 10^{-8}$

Experimental proposal for ^{48}Ti by Valiente-Dobon et al.

Valiente-Dobon, Romeo et al., in prep