Flavour physics and CP violation

Lecture 2

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- flavour mixing and PMNS matrix
- flavour violation in the charged lepton sector
- neutrinos oscillations and CP violation
- interlude: Dirac versus Majorana neutrinos
- neutrinoless double beta decay

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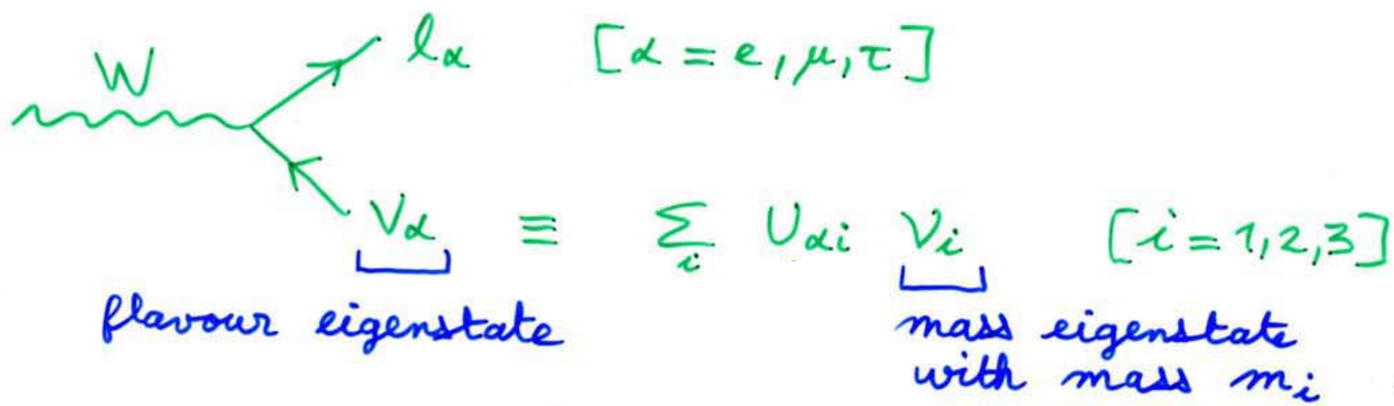
Flavour mixing – PMNS matrix

When neutrinos are massive, possibility of flavour mixing : the neutrino to which a given charged lepton (e, μ or τ) couples via the W is not a mass eigenstate, but a coherent superpositions of mass eigenstates



Like in the quark sector, the origin of flavour mixing is the mismatch between the basis of gauge (or flavour) eigenstates and of mass eigenstates. The relative rotation is the lepton mixing matrix, known as PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

Physical parameters in U_{PMNS}

U is a 3x3 unitary matrix $\Rightarrow$ 3 mixing angles and 6 phases (not all physical)

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} W_\mu^- \sum_\alpha \bar{e}_{\alpha L} \gamma^\mu \nu_{\alpha L} = \frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha, i} \bar{e}_{\alpha L} \gamma^\mu U_{\alpha i} \nu_{iL}$$

(i) if neutrinos are Dirac fermions : analogous to quarks and CKM

can rephase the lepton fields $e_{\alpha L} \to e^{i\phi_\alpha} e_{\alpha L}\,, \quad \nu_{iL} \to e^{i\phi_i} \nu_{iL}$ and absorb the phases in the PMNS matrix, so that CC interactions keep the same form

$$U_{\alpha i} \to e^{i(\phi_\alpha - \phi_i)} U_{\alpha i}$$

$\Rightarrow$ removes 2x3 - 1 = 5 relative phases $\Rightarrow$ a single physical phase $\delta_{\rm PMNS}$

(i) if neutrinos are Majorana fermions : cannot rephase the neutrino fields, since this would make neutrino masses complex

$$U_{\alpha i} \to e^{i\phi_\alpha} U_{\alpha i}$$

$\Rightarrow$ removes only 3 phases $\Rightarrow$ 3 physical phases : 1 "Dirac" phase $\delta_{\rm PMNS}$ and 2 "Majorana" phases

Standard parametrization of the PMNS matrix

Analogous to CKM: written as the product of three rotations with angles θ_{23}, θ_{13} and θ_{12}, the second (complex) rotation depending on the phase δ

$$U \equiv U_{23}U_{13}U_{12}P \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P$$

P is the unit matrix in the Dirac case, and a diagonal matrix of phases containing 2 independent phases ϕ_i in the Majorana case

$$c_{ij} = \cos\theta_{ij}\ ,\ \ s_{ij} = \sin\theta_{ij}$$

$$\theta_{ij} \in [0, \pi/2]\ , \quad \delta \in [0, 2\pi[\ , \quad \phi_i \in [0, \pi[$$

$\delta \neq 0, \pi \ \Rightarrow$ CP violation in oscillations: $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$

The Majorana phases play a role only in $\Delta L = 2$ processes like neutrinoless double beta decay

Flavour violation in the charged lepton sector (cLFV)

Lepton flavour mixing implies processes that violate lepton flavour (and CP) both in the neutrino and in the charged lepton sector

In the neutrino sector, this leads to flavour oscillations

In the charged lepton current, one would expect observable flavour violating processes such as the decays $\mu \to e\,\gamma$ or $\mu \to 3e$, as well as $\mu \to e$ conversion on nuclei

This is not the case due to a GIM mechanism: LFV is strongly suppressed (and in practice unobservable) in the Standard Model

Experimental status of charged lepton flavour violation

So far lepton flavour violation has been observed only in the neutrino sector (oscillations). Experimental upper bounds on LFV processes involving charged leptons:

[S. Davidson, talk at Planck 2022]

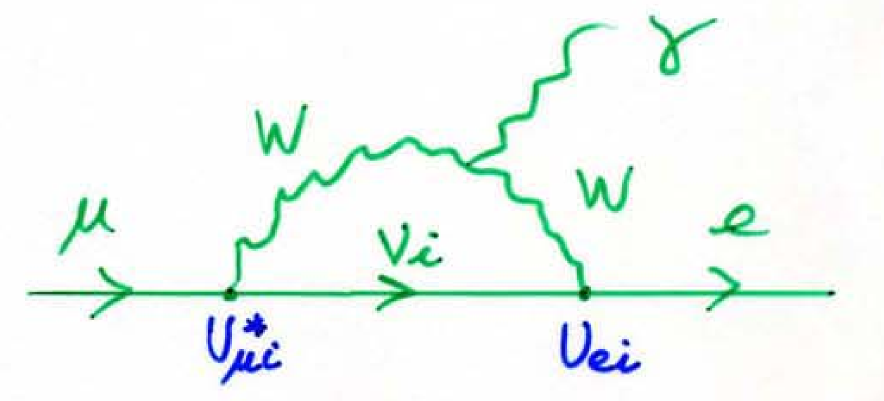
some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$ (SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(16 \to ?)}$ (Mu2e,COMET)
		$10^{-(18 \to ?)}$ (PRISM/PRIME/ENIGMA)
$K^+ \to \pi^+ \bar{\mu} e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
...		
$B^+ \to \bar{\mu}\nu$	$< 1.0 \times 10^{-6}$ (Belle)	$\sim 10^{-7}$ (BelleII)

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

some processes	current constraints on BR	future sensitivities
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to \ell\{\pi, \rho, \phi, K, ...\}$	$\lesssim \text{few} \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to ...$		
$h \to \tau^{\pm}\ell^{\mp}$	$< 1.5, 2.2 \times 10^{-3}$ (ATLAS/CMS)	
$h \to \mu^{\pm}e^{\mp}$	$< 6.1 \times 10^{-5}$ (ATLAS/CMS)	
$Z \to e^{\pm}\mu^{\mp}$	$< 7.5 \times 10^{-7}$ (ATLAS)	

[S. Davidson, talk at Planck 2022]

This is consistent with the Standard Model, in which LFV processes involving charged leptons are suppressed by the tiny neutrino masses (GIM mechanism much more powerful as in the quark sector)



e.g. μ → e γ :

$$\mathrm{BR}\,(\mu \to e\gamma) \;=\; \frac{3\alpha}{32\pi} \left| \sum_i U^*_{\mu i} U_{ei} \, \frac{m^2_{\nu_i}}{M^2_W} \right|^2$$

Using known oscillations parameters (U = PMNS lepton mixing matrix), this gives $\mathrm{BR}\,(\mu \to e\gamma) \;\lesssim\; 10^{-54}$: inaccessible to experiment!

This makes LFV a unique probe of new physics: the observation of e.g. μ → e γ would be an unambiguous signal of new physics (no SM background)

→ very different from the hadronic sector

Conversely, the present upper bounds on LFV processes already put strong constraints on new physics

Neutrino oscillations in vacuum and CP violation

Oscillations are a quantum-mechanical process due to neutrino mass and mixing. An (ideal) oscillation experiment involves 3 steps:

1) production of a pure flavour state at t = 0 (e.g. a ν_μ from $\pi^+ \to \mu^+ \nu_\mu$)

This flavour state is a coherent superposition of mass eigenstates determined by the PMNS matrix, e.g. in the 2 flavour case

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

2) propagation

Each mass eigenstate, being an eigenstate of the Hamiltonian in vacuum, evolves with its own phase factor $e^{-iE_i t}$ $\Rightarrow$ modifies the coherent superposition, which is no longer a pure flavour eigenstate:

$$|\nu(t)\rangle = -\sin\theta\, e^{-iE_1 t}|\nu_1\rangle + \cos\theta\, e^{-iE_2 t}|\nu_2\rangle$$

3) detection via a CC interaction which identifies a specific flavour

probability amplitude : $\langle \nu_e|\nu(t)\rangle = -\cos\theta\sin\theta e^{-iE_1 t} + \cos\theta\sin\theta e^{-iE_2 t}$

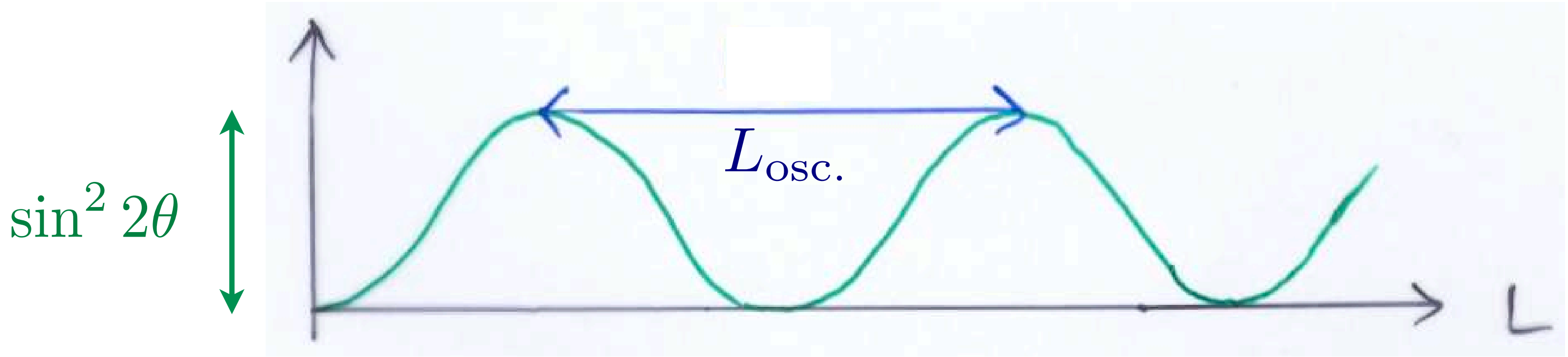
oscillation probability : $P(\nu_\mu \to \nu_e; t) = |\langle \nu_e|\nu(t)\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{E_2 - E_1}{2}\, t\right)$

2-flavour oscillations in vacuum

Assuming ultra-relativistic neutrinos $L \simeq ct$, $m_i^2 \ll p^2$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \quad \Rightarrow \quad \frac{E_2 - E_1}{2} \simeq \frac{m_2^2 - m_1^2}{4p}$$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



Amplitude of oscillations: $\sin^2 2\theta$

Oscillation length: $L_{\text{osc.}}(\text{km}) = 2.48\, E(\text{GeV})/\Delta m^2(\text{eV}^2)$

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

The above derivation gives the correct oscillation probability, but is a bit oversimplified

The propagating mass eigenstates ν_i where described as plane waves with well-defined (and equal) momenta ($p_i = p$)

Should instead be described by wave packets with mean momenta p_i

Under appropriate coherence conditions at production and detection, and neglecting decoherence due to separation of the wave packets, the above oscillation formula is recovered (without the ad hoc assumption $p_i = p$)

[See e.g. Akhmedov and Smirnov, arXiv: 0905.1903 for details]

N-flavour oscillations in vacuum

$$\nu_\alpha(x) = \sum_i U_{\alpha i}\, \nu_i(x) \quad \textbf{(fields)} \quad \Rightarrow \quad |\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\, |\nu_i\rangle \quad \textbf{(states)}$$

and for antineutrinos $$|\bar{\nu}_\alpha\rangle = \sum_i U_{\alpha i}\, |\bar{\nu}_i\rangle$$

1) production: $|\nu(t=0)\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$

2) propagation: $|\nu(t)\rangle = \sum_i U^*_{\alpha i}\, e^{-iE_i t} |\nu_i\rangle$

3) detection: $\langle \nu_\beta | \nu(t)\rangle = \sum_j U_{\beta j} \langle \nu_j | \nu(t)\rangle = \sum_i U_{\beta i} U^*_{\alpha i}\, e^{-iE_i t}$

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu(t)\rangle|^2 = \left|\sum_i U_{\beta i} U^*_{\alpha i}\, e^{-iE_i t}\right|^2$$

Assuming ultra-relativistic neutrinos, one obtains

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \mathrm{Re}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)$$

$$+ 2 \sum_{i<j} \mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right)$$

Oscillation probability = sum of oscillating terms with different « frequencies » $\Delta m^2_{ji} \equiv m^2_j - m^2_i$ and amplitudes (which depend on the θ_{ij} and Dirac-type CP-violating phases)

For $\alpha \neq \beta$, $P(\alpha \to \beta)$ is called appearance probability

$$P(\nu_\alpha \to \nu_\beta) \,=\, -\,4 \sum_{i<j} \mathrm{Re}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)$$

$$+\,2 \sum_{i<j} \mathrm{Im}\left[U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right)$$

For antineutrinos, U → U* (δ → - δ) and the last term changes sign if $\delta \neq 0, \pi \;\Rightarrow\; P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \,\neq\, P(\nu_\alpha \to \nu_\beta)$ ➞ CP violation

For $\alpha = \beta$ (disappearance or survival probability), the last term vanishes and the formula simplifies to

$$P(\nu_\alpha \to \nu_\alpha) \,=\, 1 - 4 \sum_{i<j} \left|U_{\alpha i} U_{\alpha j}\right|^2 \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)$$

No CP violation in disappearance experiments: $P(\nu_\alpha \to \nu_\alpha) \,=\, P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$

3-flavour oscillations

2 independent Δm^2: Δm^2_{32} (« atmospheric ») and Δm^2_{21} (« solar »)

U contains 3 mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ

$$U \equiv U_{23}U_{13}U_{12} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[omitting possible « Majorana » phases, which are relevant only for lepton number violating processes such as neutrinoless double beta decay, and have no effect on oscillations, since they cancel in in the combinations $U_{\alpha i}U^*_{\beta i}$]

In many experiments, oscillations are dominated by a single Δm^2 and can be described to a good approximation as 2-flavour oscillations:

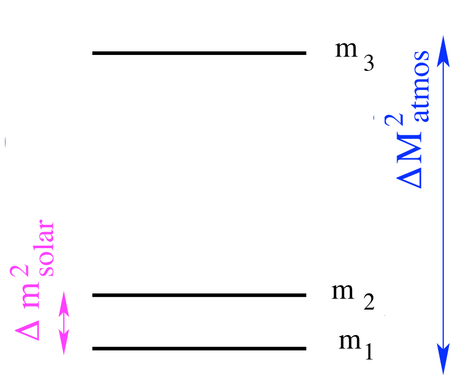
- solar neutrinos (*), LBL reactors $\Delta m^2_{21},\ \theta_{12}$ $\Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\mathrm{eV}^2$
  ($\nu_e/\bar{\nu}_e$ disappearance)
- atmospheric, LBL accelerators $\Delta m^2_{31},\ \theta_{23}$ $|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$
  ($\nu_\mu/\bar{\nu}_\mu$ disappearance)
- SBL reactor experiments $\Delta m^2_{31},\ \theta_{13}$ $\sin^2\theta_{13} \simeq 0.022$
  ($\bar{\nu}_e$ disappearance)

(*) matter effets dominate for high-energy solar neutrinos

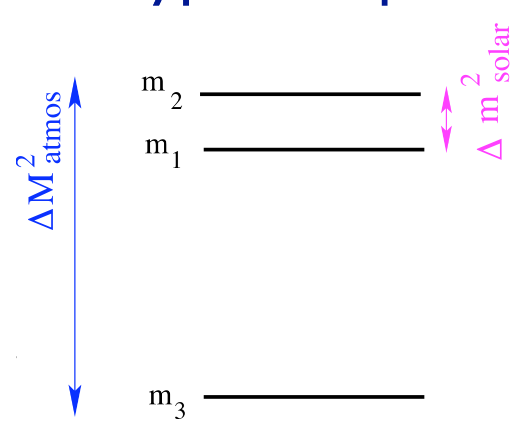
Notes : 1) θ_{13} is the only « small » leptonic angle $\theta_{13} < \theta_{12}, \theta_{23}$

2) $\Delta m^2_{21} \ll |\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$ [by convention, $\Delta m^2_{21} > 0$]

sign of Δm^2_{31} unknown $\Rightarrow$ two types of spectra allowed



Normal hierarchy
(normal ordering)
$\Delta m^2_{31} > 0$



Inverted hierarchy
(inverted ordering)
$\Delta m^2_{31} < 0$

CP violation in oscillations

$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ at leading order in Δm^2_{21} $(\alpha \neq \beta)$:

$$\Delta P_{\alpha\beta} = \pm 8\, J \left(\frac{\Delta m^2_{21} L}{2E}\right) \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right), \qquad J \equiv \mathrm{Im}\left[U_{e1} U^*_{\mu 1} U^*_{e2} U_{\mu 2}\right]$$

Jarlskog invariant $\quad J = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$

→ condition for CP violation : $\quad \delta \neq 0, \pi$

→ for CP violation to be observable, sub-dominant oscillations governed by Δm^2_{21} must develop $\Rightarrow$ long baseline oscillation experiments (> 100 km), also sensitive to matter effects (which can mimic a CP asymmetry)

CP violation is only possible in <u>appearance experiments</u> $(\alpha \neq \beta)$
e.g. electron (anti-)neutrino appearance in a muon (anti-)neutrino beam
$(\nu_\mu \to \nu_e,\ \bar{\nu}_\mu \to \bar{\nu}_e)$

Disappearance experiments, e.g. at reactors, have no sensitivity to δ, implying $P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$

$\nu_\mu \to \nu_e$ appearance channel at long baseline experiments

Sensitivity to CP violation requires a long baseline (T2K, NOvA, DUNE, HK) such that subleading oscillations can develop

At leading order in Δm^2_{21} and θ_{13} :

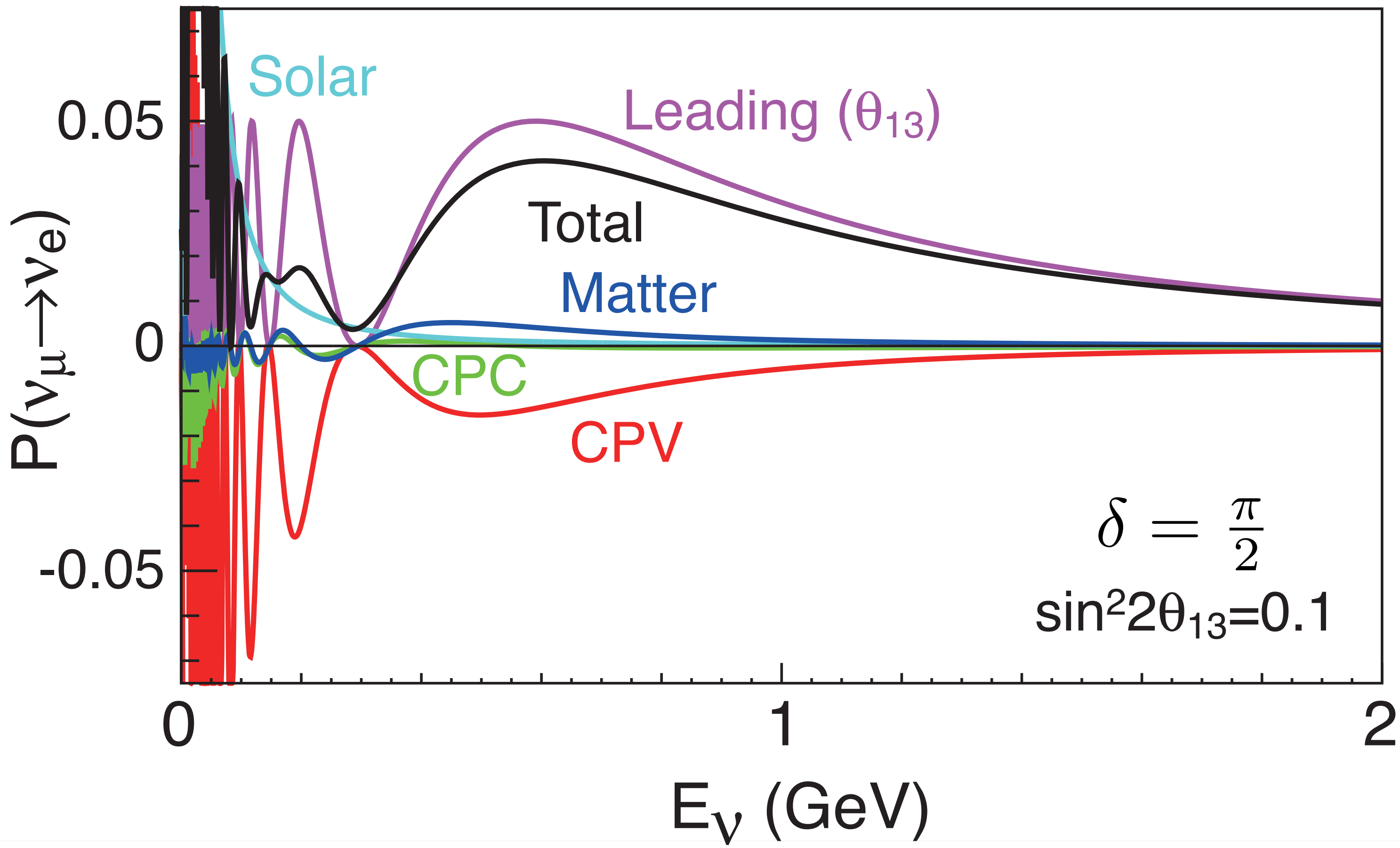
$$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cos^2\theta_{23}\sin^2 2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right)$$
$$+\frac{1}{2}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\cos\delta\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{2E}\right)$$
$$-\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

- first term: leading Δm^2_{31}-driven term, proportional to $\sin^2 2\theta_{13}$ and sensitive to the octant of θ_{23} (i.e. whether $\theta_{23} < \pi/4$ or $> \pi/4$)

- the third and fourth terms involve both Δm^2_{31} and Δm^2_{21} and are CP-even and CP-odd (changes sign for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations), respectively

- due to the long baseline, matter effects must be included (less important for T2K than for NOvA and DUNE)

Relative size of the different terms in the $\nu_\mu \rightarrow \nu_e$ oscillation probability for the long baseline accelerator experiment T2K (Japan, 295 km), in which matter effects are small

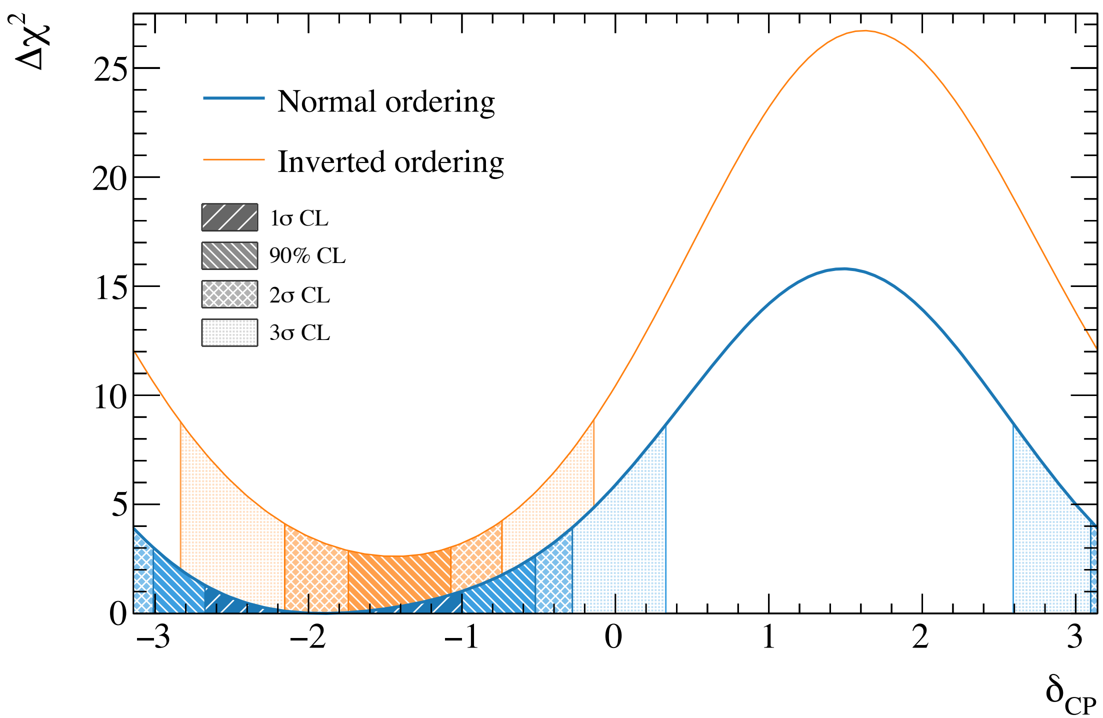


[arXiv:1412.4673]

First hints of CP violation at T2K

Long baseline accelerator experiment in Japan (295 km)

Observes more events in the neutrino mode $(\nu_\mu \to \nu_e)$ and less events in the antineutrino mode $(\bar{\nu}_\mu \to \bar{\nu}_e)$ than expected $\Rightarrow$ suggests CP violation (CP conservation excluded at more than 90% C.L.)



[T2K Coll., Eur. Phys. J. C 83 (2023)]

constraints from SBL reactor experiments imposed ($\Delta m^2_{31}, \theta_{13}$)

The long baseline accelerator experiment NOvA (USA, 810 km) does not confirm the hint for CPV in the NO case – more data / new experiments needed

Matter effect versus CP violation

In long baseline experiments, cannot neglect the impact of matter effects (forward scatterings of neutrinos off e-, p and n) on oscillations (especially if want to make precision measurements)

Matter effects are different for neutrinos and antineutrinos, leading to different oscillation probabilities even in the absence of CP violation
$\Rightarrow$ must disentangle the two effects to establish CPV in the lepton sector

Oscillations in matter with constant density (which is the case in the Earth crust) are governed by the same formula as in vacuum, with the replacements

$$\theta \ \rightarrow \ \theta_m \, , \qquad \frac{\Delta m^2}{4E} \ \rightarrow \ \frac{(E_m^2 - E_m^1)}{2}$$

$$\Rightarrow \ \ P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

Oscillation parameters in matter

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E}\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$\cos 2\theta_m = \frac{\left(1-\frac{n}{n_{\rm res}}\right)\cos 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$n_{\rm res} \equiv \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E}$$

for antineutrinos,

$$n \to -n$$

(n = n_e if only active neutrinos)

MSW resonance (Mikheev-Smirnov-Wolfenstein):

$$\sin 2\theta_m = 1 \quad \text{for } n = n_{\rm res}$$

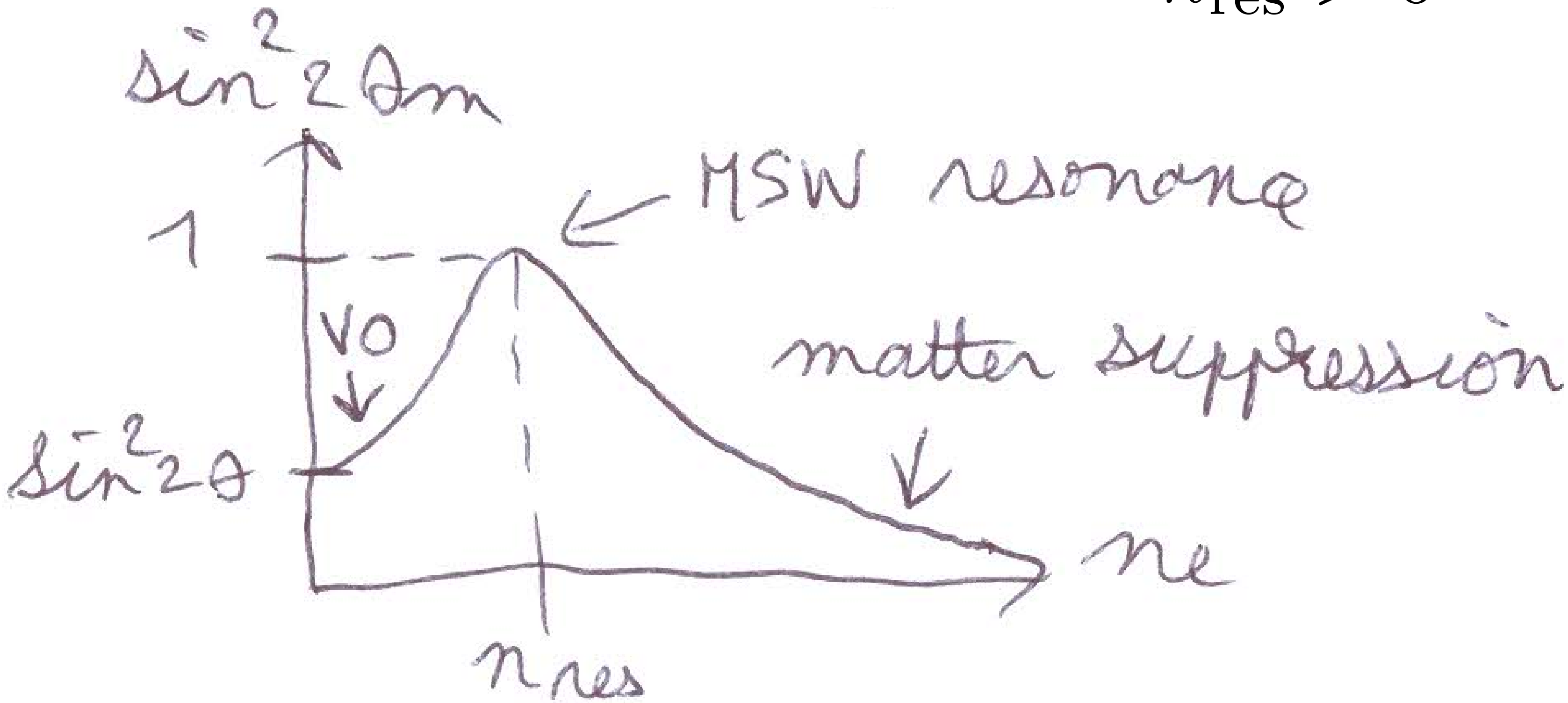
(irrespective of the value of the mixing angle in vacuum θ)

Resonance condition: $\begin{cases} \Delta m^2 \cos 2\theta > 0 & \text{for neutrinos} \\ \Delta m^2 \cos 2\theta < 0 & \text{for antineutrinos} \end{cases}$

When neutrino oscillations are enhanced, antineutrino oscillations are suppressed, and vice versa

Different regimes for oscillations in matter :

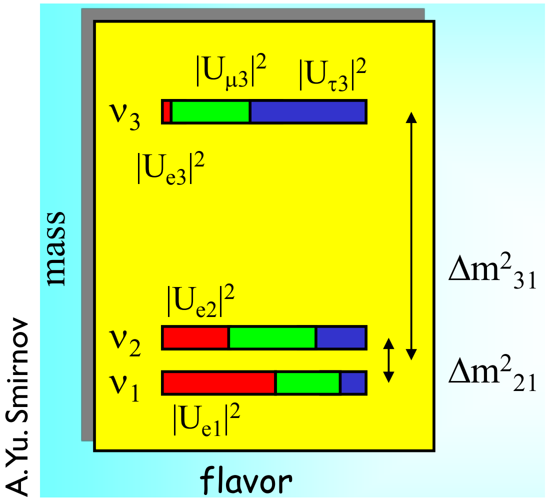
$$n_{\rm res} > 0$$



- low density ($n \ll n_{\rm res}$) : $\sin 2\theta_m \simeq \sin 2\theta$ $\Rightarrow$ vacuum oscillations
- resonance ($n = n_{\rm res}$) : $\sin 2\theta_m = 1$
- high density ($n \gg n_{\rm res}$) : $\sin 2\theta_m < (\ll) \sin 2\theta$ $\Rightarrow$ oscillations are suppressed by matter effects

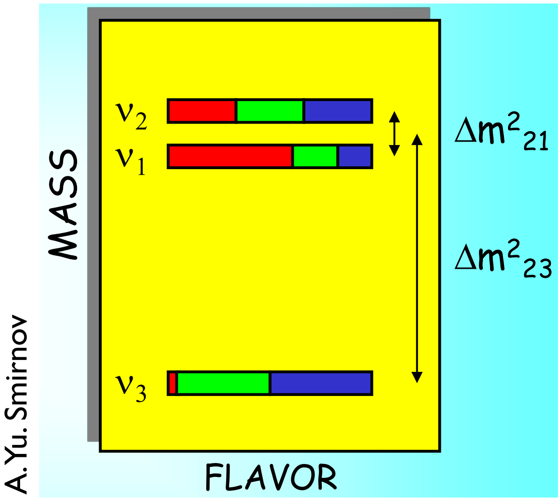
Application: determination of the mass hierarchy in long-baseline experiments

Two mass orderings allowed by experiments:



Normal hierarchy

$$\Delta m^2_{31} > 0$$



Inverted hierarchy

$$\Delta m^2_{31} < 0$$

In vacuum: $$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

For long baselines (> several 100 km), matter effects cannot be neglected

$$n_{\text{res}} = \frac{\Delta m^2_{31} \cos 2\theta_{13}}{2\sqrt{2}\, G_F E} \qquad \begin{cases} n_{\text{res}} > 0 & \text{for normal hierarchy} \\ n_{\text{res}} < 0 & \text{for inverted hierarchy} \end{cases}$$

If n_{res} is close to the Earth crust density, neutrino (antineutrino) oscillations are enhanced for NH (IH), while antineutrino (neutrino) oscillations are suppressed

[may have to disentangle CP violation from matter effect] an outdated but informative plot

$$R = \frac{\overline{\nu}_e \to \overline{\nu}_\mu}{\nu_e \to \nu_\mu}$$

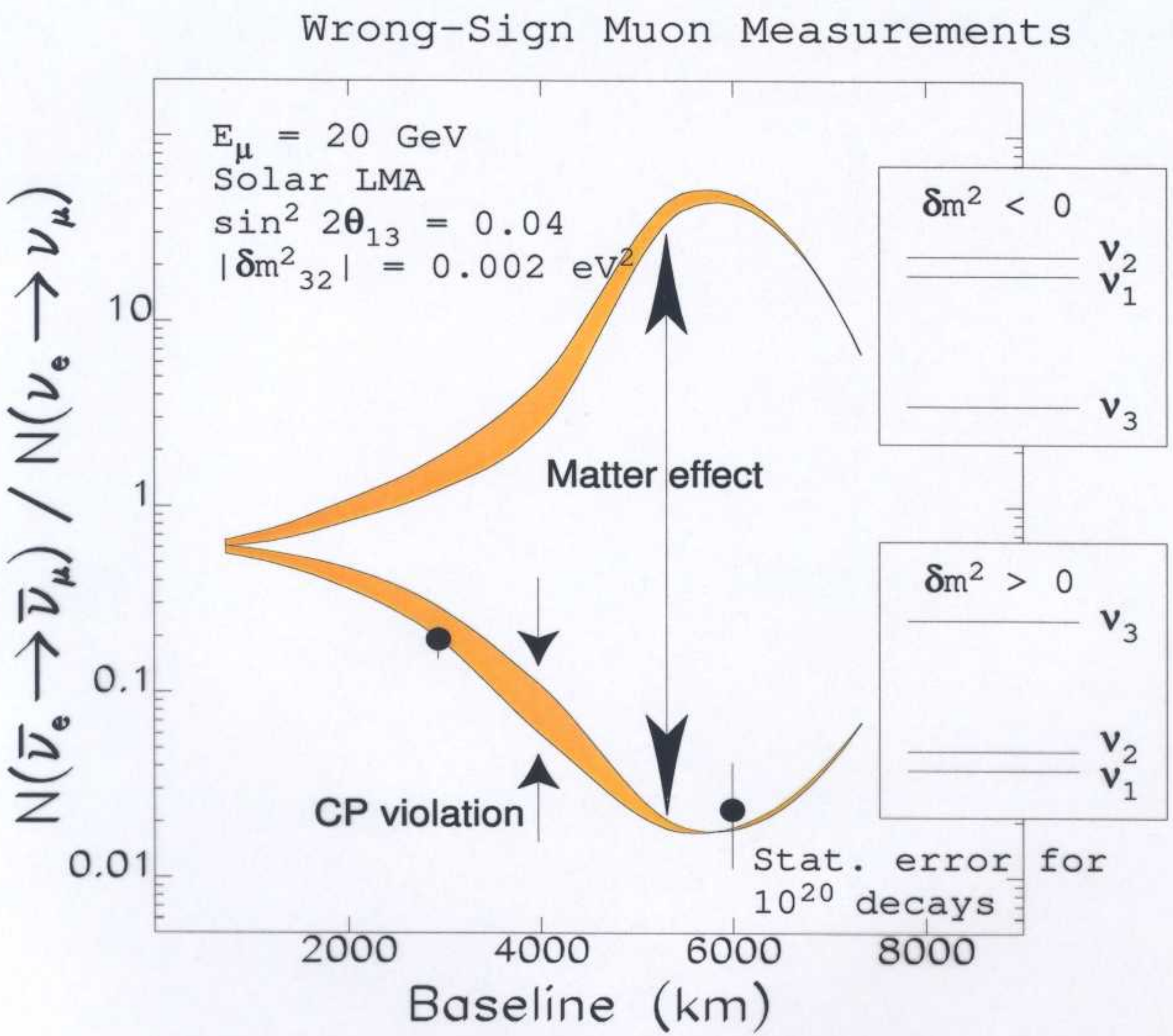


Figure 2: Predicted ratios of $\bar{\nu}_e \to \bar{\nu}_\mu$ to $\nu_e \to \nu_\mu$ rates at a 20 GeV neutrino factory. The statistical error shown corresponds to 10^{20} muon decays of each sign and a 50 kt detector.

[Barger, Geer, Raja, Whisnant]

Interlude: Dirac versus Majorana neutrinos

Neutrinos are the only SM fermions that do not carry electric charge
$\Longrightarrow$ can be their own antiparticles (Majorana fermions)

Experimentally, only ν_L (the "neutrino") and its CP conjugate ν_R^c (the "antineutrino") have been observed. We don't know if the neutrino also has a RH component ν_R (which would be an SM gauge singlet, hence unobservable)

The observed (LH) neutrino and (RH) antineutrino can be described equally well by a Dirac or Majorana neutrino. The only difference is that the RH component of a Dirac neutrino, ν_R , is independent of ν_L and is an SM gauge singlet, while the RH component of a Majorana neutrino coincides with ν_R^c , the CP conjugate of its LH component

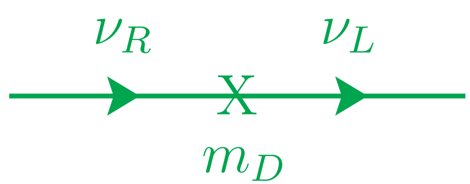
Dirac and Majorana mass terms

Dirac mass term

The simplest way to describe a massive neutrino is to add a ν_R to the SM and to write a Dirac mass term, as for the other fermions:

$$\mathcal{L}^{\mathrm{Dirac}}_{\mathrm{mass}} = -m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L\right) \equiv -m_D\, \bar{\nu}_D \nu_D \qquad \nu_D \equiv \nu_L + \nu_R$$

The massive neutrino ν_D is a Dirac fermion (2 independent chiralities)



$$\Delta L = 0 \qquad \Delta T^3 = \frac{1}{2}$$

[note: ν_R is an SM gauge singlet]

not invariant under $SU(2)_L \times U(1)_Y$ but can be generated from a Yukawa coupling to the SM Higgs doublet (which has weak isospin 1/2)

$$\mathcal{L}_{\mathrm{Yuk.}} = -y_D\, \bar{L}\, i\sigma^2 H^* \nu_R + \mathrm{h.c.} \quad \longrightarrow \quad m_D = y_D \frac{v}{\sqrt{2}}$$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad m_\nu \lesssim 1\,\mathrm{eV} \;\Rightarrow\; y_D \lesssim 10^{-11}$$

caveat : possible to write a Majorana mass term for ν_R $\Rightarrow$ end up with two Majorana neutrinos rather than one Dirac neutrino (see later)

Majorana mass term

Preliminary remark: can form a RH spinor from ν_L $\quad (C\gamma_\mu^T C^{-1} = -\gamma_\mu)$

$\nu_R^c \equiv C\,\bar{\nu}_L^T \;\sim\;$ CP conjugate of ν_L $\quad (\bar{\nu}_L \equiv \nu_L^\dagger \gamma^0)$

C = charge conjugation matrix; enters the charge conjugate of a Dirac spinor

$\psi(x) \;\to\; \psi^c(x) \equiv C\bar{\psi}^T(x)$ describes the corresponding antifermion

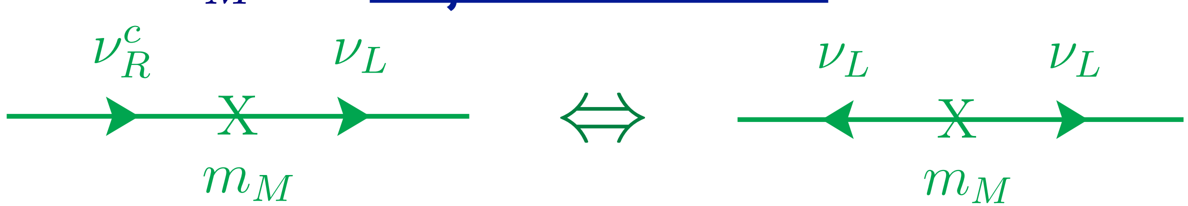
$\Rightarrow$ the existence of a LH neutrino (ν_L) implies the existence of a RH antineutrino ($C\bar{\nu}_L^T \equiv \nu_R^c \sim \bar{\nu}_R$)

Now, with ν_L and ν_R^c, can write a (Majorana) mass term :

$$\mathcal{L}_{\rm mass}^{\rm Maj.} = -\frac{1}{2}\, m_M\, (\bar{\nu}_L \nu_R^c + \bar{\nu}_R^c \nu_L) \equiv -\frac{1}{2}\, m_M\, \bar{\nu}_M \nu_M \qquad \nu_M \equiv \nu_L + \nu_R^c$$

The massive neutrino $\nu_M = \nu_L + \nu_R^c$ satisfies the Majorana condition

$\nu_M = \nu_M^c \;\to\;$ Majorana fermion



$\Delta L = 2 \qquad \Delta T^3 = 1$

A Majorana mass term violates lepton number (signature of a Majorana neutrino). It cannot be generated from a coupling to the SM Higgs doublet, which has a T = 1/2 $\Rightarrow$ neutrino masses require an extension of the SM

Dirac versus Majorana neutrino

A Dirac neutrino is different from its antiparticle ($\nu \neq \nu^c$)

$\Rightarrow$ describes 4 degrees of freedom: $\nu\uparrow,\ \nu\downarrow,\ \bar{\nu}\uparrow,\ \bar{\nu}\downarrow$ [or $\nu_R,\ \nu_L,\ \bar{\nu}_R,\ \bar{\nu}_L$]

Described by a 4-component spinor $\nu_D = \begin{pmatrix} \nu_{D,L} \\ \nu_{D,R} \end{pmatrix}$, with independent LH and RH components $\nu_{D,L}$ and $\nu_{D,R}$

A Majorana neutrino satisfies the condition $\nu = \nu^c = C\bar{\nu}^T$

$\Rightarrow$ describes only 2 degrees of freedom: $\nu\downarrow,\ \bar{\nu}\uparrow$ [or $\nu_L,\ \bar{\nu}_R$]

Can be described by a 4-component spinor $\nu_M = \begin{pmatrix} \nu_{M,L} \\ \nu_{M,R} \end{pmatrix}$, but its LH and RH components are not independent, as $\nu_M = \nu_M^c \Rightarrow \nu_{M,R} = \nu_{M,R}^c \equiv C\bar{\nu}_{M,L}^T$

The Majorana condition is inconsistent with any conserved additive quantum number: if ψ possesses a conserved quantum number q,

$$\psi \to e^{i\theta q}\,\psi \quad \Rightarrow \quad \psi^c \to e^{-i\theta q}\,\psi^c$$

Thus only neutrinos (not quarks, charged leptons) can be Majorana fermions

For the same reason, one cannot rephase a Majorana neutrino

$\Rightarrow$ 2 additional physical phases in the PMNS matrix wrt the Dirac case

How to distinguish Majorana from Dirac neutrinos?

Dirac and Majorana neutrinos have the same gauge interactions, since weak interactions only involve ν_L and its CP conjugate $\bar{\nu}_R$, which can be described either by a Dirac or a Majorana neutrino (ν_R and $\bar{\nu}_L$, if they exist, are SM gauge singlets and do not interact at all)

One commonly calls ν_L "neutrino" and $\bar{\nu}_R$ "antineutrino", irrespective of whether they are degrees of freedom of a Dirac or Majorana neutrino. This terminology is motivated by the fact that, via the charged weak interaction, ν_L ($\bar{\nu}_R$) creates a negatively charged (positively charged) lepton

In other words, one can define a U(1) charge – the lepton number L – which is preserved by SM interactions. This is done by assigning $L(\nu_L) = L(\ell^-) = +1$ and $L(\bar{\nu}_R) = L(\ell^+) = -1$

Similarly, oscillations probabilities are the same for Dirac and Majorana neutrinos (production and detection are weak interaction processes: only ν_L and $\bar{\nu}_R$ can be produced and detected) (*)

(*) note: $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ corresponds to CP violation, not C violation, and is possible both for Dirac and Majorana neutrinos, precisely because $\bar{\nu}_\alpha \equiv \bar{\nu}_{R\alpha} =$ CP conjugate of $\nu_\alpha \equiv \nu_{L\alpha}$

The only practical difference between Dirac and Majorana neutrinos lies in their mass term, which violates lepton number by 2 units in the Majorana case

→ the Majorana nature of neutrinos can be established in $\Delta L = 2$ processes such as neutrinoless double beta decay

(the only $\Delta L = 2$ process that is accessible to experiment, in practice, in the absence of other sources of lepton number violation than the masses of the SM Majorana neutrinos)

The simplest mechanism of neutrino mass generation

Simplest possibility: add a RH neutrino N_R to the Standard Model

In addition to the Dirac mass term $-m_D\,\bar{\nu}_L N_R + \text{h.c.}$, must write a Majorana mass term for the RH neutrino, which is allowed by all (non-accidental) symmetries of the SM (or justify its absence):

$$-\frac{1}{2}\,M\,\bar{N}_L^c N_R + \text{h.c.} = -\frac{1}{2}\,M\,N_R^T C N_R + \text{h.c.} \qquad \Delta L = 2 \qquad \Delta T^3 = 0$$

[only lepton number, if imposed, can forbid this term]

Mass eigenstates : write the mass terms in a matrix form and diagonalize

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{N}_L^c\end{pmatrix}\begin{pmatrix}0 & m_D \\ m_D & M\end{pmatrix}\begin{pmatrix}\nu_R^c \\ N_R\end{pmatrix} + \text{h.c.}$$

$$= -\frac{1}{2}\begin{pmatrix}\bar{\nu}_{L1} & \bar{\nu}_{L2}\end{pmatrix}\begin{pmatrix}m_1 & 0 \\ 0 & m_2\end{pmatrix}\begin{pmatrix}\nu_{R1}^c \\ \nu_{R2}^c\end{pmatrix} + \text{h.c.}$$

where
$$\begin{cases}\nu_{L1} = \cos\theta\,\nu_L - \sin\theta\,N_L^c \\ \nu_{L2} = \sin\theta\,\nu_L + \cos\theta\,N_L^c\end{cases}$$

Defining $\nu_{Mi} \equiv \nu_{Li} + \nu^c_{Ri}$ (such that $\nu_{Mi} = \nu^c_{Mi}$), one can see that the mass eigenstates are 2 Majorana neutrinos with masses m1 and m2 :

$$\mathcal{L}_{\rm mass} = -\frac{1}{2} \sum_{i=1,2} m_i\, \bar{\nu}_{Li} \nu^c_{Ri} + {\rm h.c.} = -\frac{1}{2} \sum_{i=1,2} m_i\, \bar{\nu}_{Mi} \nu_{Mi}$$

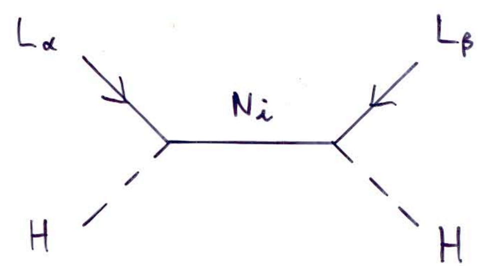
<u>"Seesaw" limit :</u> $M \gg M_W \gtrsim m_D$

Minkowski - Gell-Mann, Ramond, Slansky
Yanagida - Mohapatra, Senjanovic

(N_R = gauge singlet ⇒ M unconstrained by the electroweak symmetry)

$$m_1 \simeq -m_D^2/M \ll M_W \qquad m_2 \simeq M \gg M_W$$

$$\sin\theta \simeq \frac{m_D}{M} \ll 1 \quad \Rightarrow \quad \nu_{L1} \simeq \nu_L\,, \quad \nu^c_{R2} \simeq N_R$$



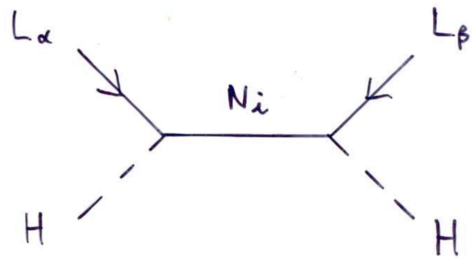
→ the light Majorana neutrino is essentially the SM neutrino

→ natural explanation of the smallness of neutrino masses

<u>New physics interpretation :</u> M = scale of the new physics responsible for lepton number violation – can a priori lie anywhere between $\sim 10^{15}\,{\rm GeV}$ (a larger M would give $m_\nu < \sqrt{|\Delta m^2_{31}|} \simeq 0.05\,{\rm eV}$, unless $y_D > 1$) and the weak scale (low-scale seesaw mechanism), or even below

3-generation (type I) seesaw mechanism ($i = 1, 2, 3;\ \alpha = e, \mu, \tau$)

$$\mathcal{L}^{I}_{\rm seesaw} = -Y_{i\alpha}\overline{N}_{Ri}L_\alpha H - \frac{1}{2}M_i\overline{N}_{Ri}N^c_{Ri} + \text{h.c.}$$



$$\Longrightarrow \qquad (M_\nu)_{\alpha\beta} = -\sum_i \frac{Y_{i\alpha}Y_{i\beta}}{M_i}v^2$$

Light neutrino mass matrix: $M_\nu = -Y^T M^{-1} Y v^2 = U^* D_\nu U^\dagger$

U = lepton mixing (PMNS) matrix $\qquad \nu_\alpha = \sum_i U_{\alpha i}\,\nu_i$

$$D_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \qquad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i(\sigma+\delta)} \end{pmatrix}$$

Natural realization of the seesaw mechanism in Grand Unified Theories (GUTs) based on the SO(10) gauge group

- SM quarks and leptons fit in a single 16-dimensional representation of SO(10), which also contains a right-handed neutrino:

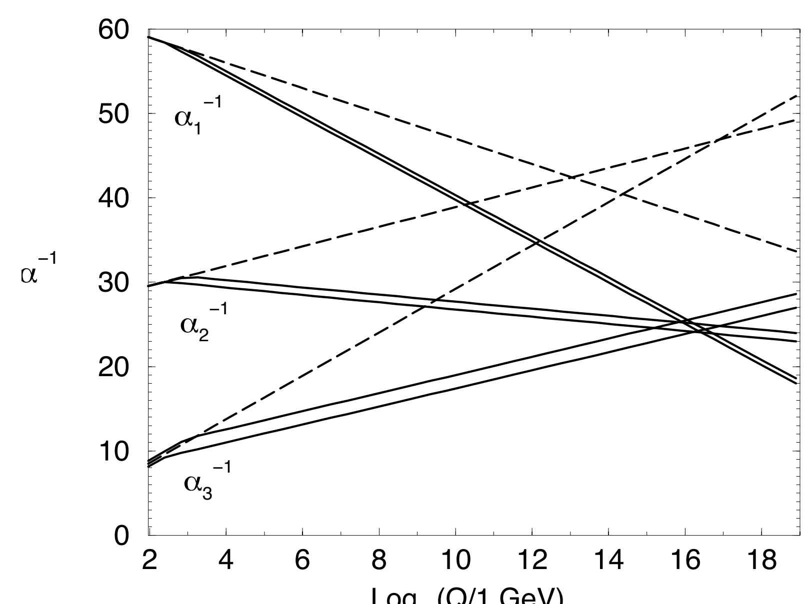
$$\mathbf{16}_i = (Q_i, u_i^c, d_i^c, L_i, e_i^c, N_i^c) \qquad (i = 1, 2, 3)$$

- the scale of RH neutrino masses is associated with the breaking of the B-L symmetry, which is a generator of SO(10), and is typically broken at or a few orders of magnitude below the GUT scale M_{GUT}

$$M_i \longleftrightarrow M_{B-L} \longleftrightarrow SO(10)\,\text{gauge symmetry breaking}$$

(≠ arbitrary scale, even if model dependent)

- natural values of the Dirac Yukawa coupling $y_D = \sqrt{2}\, m_D / v$ (i.e. $y_D \sim 1$) give $m_\nu = m_D^2 / M \sim 0.05\,\mathrm{eV}$ for $M \sim 10^{15}\,\mathrm{GeV}$, near the unification scale in supersymmetric extensions of the SM, $M_{\mathrm{GUT}} \approx 2 \times 10^{16}\,\mathrm{GeV}$



Right-handed neutrinos imply a deep (even if minimal) modification of the SM

- without RHNs, gauge invariance and renormalizability imply that B and L are global symmetries of the SM, only broken by quantum effects (anomalies)

- with RHNs, this is no longer true: a $\Delta L = 2$ Majorana mass term is allowed both by gauge invariance and renormalizability

Dirac neutrinos remain a viable possibility, but lepton number has to be imposed: no longer automatic

Theoretical prejudices against Dirac neutrinos:

- must impose lepton number

- need very small Yukawa couplings: $m_\nu = y_\nu \langle H \rangle \qquad \langle H \rangle = 174\,\mathrm{GeV}$

$$m_\nu \lesssim 1\,\mathrm{eV} \quad \Longrightarrow \quad y_\nu \lesssim 10^{-12} \qquad\qquad (y_e \lesssim 10^{-6})$$

[this makes the SM flavour puzzle, i.e. the unexplained hierarchy of fermion masses / Yukawa couplings even stronger, but it might be explained by a theory of flavour]

Theoretical prejudices for Majorana neutrinos:

- lepton number violated in many extensions of the SM

- any mechanism generating neutrino masses without RHNs gives Majorana neutrinos

- natural in SO(10) Grand Unified Theories (GUTs), left-right symmetric theories (based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or larger), supersymmetry without R-parity

- possible explanation of the small neutrino masses (seesaw mechanism...)

- open the possibility of generating the baryon asymmetry of the Universe via leptogenesis (B-L violation and CP violation are necessary ingredients of baryogenesis)

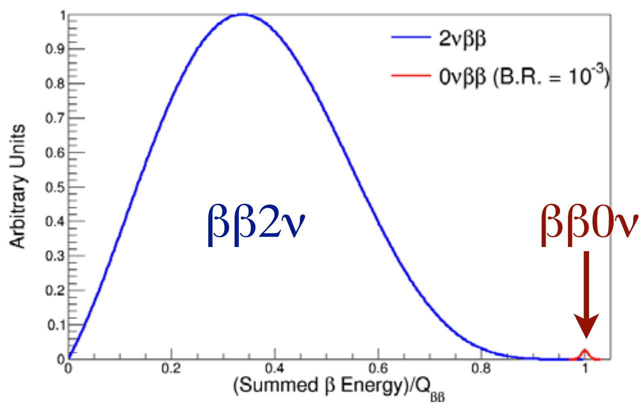
While Majorana neutrinos are theoretically compelling, only experiment (neutrinoless double beta decay, or possibly some other $\Delta L = 2$ leptonic process) will tell whether neutrinos are Dirac or Majorana particles

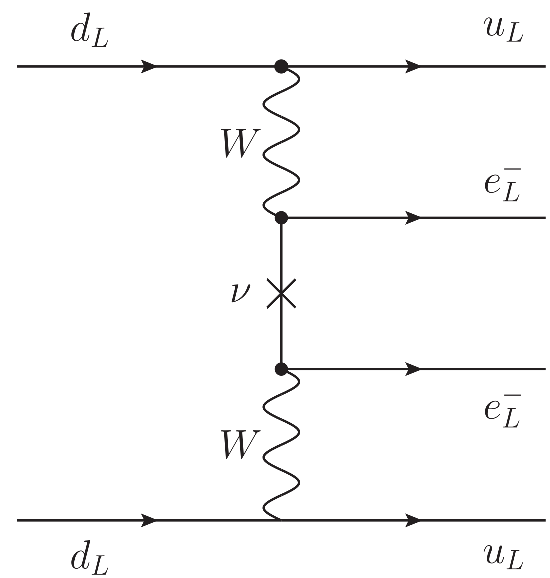
The neutrino nature: neutrinoless double beta decay

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

violates lepton number by 2 units

$\Rightarrow$ possible only for Majorana neutrinos





$$Q_{\beta\beta} \equiv M_i - M_f - 2m_e = T_{e_1} + T_{e_2}$$

Half-life: $$\left[T^{0\nu}_{1/2}\right]^{-1} = \Gamma_{0\nu} = G_{0\nu}(Q_{\beta\beta},Z)\ |M_{0\nu}|^2\ |m_{\beta\beta}|^2$$

integrated phase-space factor

nuclear matrix element (NME) (large theoretical uncertainty)

Sensitive to the effective mass parameter:

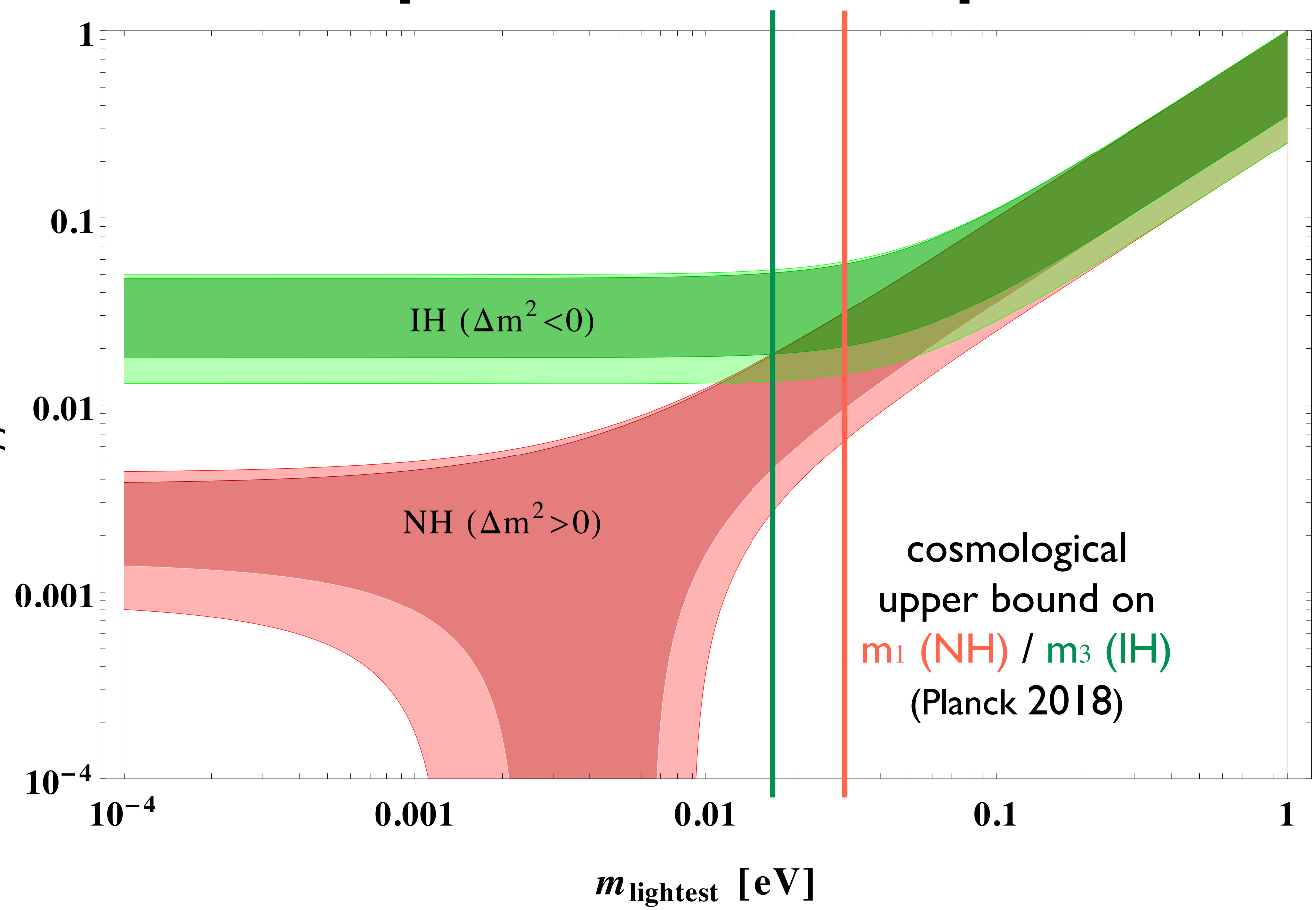
$$m_{\beta\beta} \equiv \sum_i m_i U^2_{ei} = m_1 c^2_{13} c^2_{12} e^{2i\alpha_1} + m_2 c^2_{13} s^2_{12} e^{2i\alpha_2} + m_3 s^2_{13}$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

[Dell'Oro et al., arXiv:1404.2616]

IH

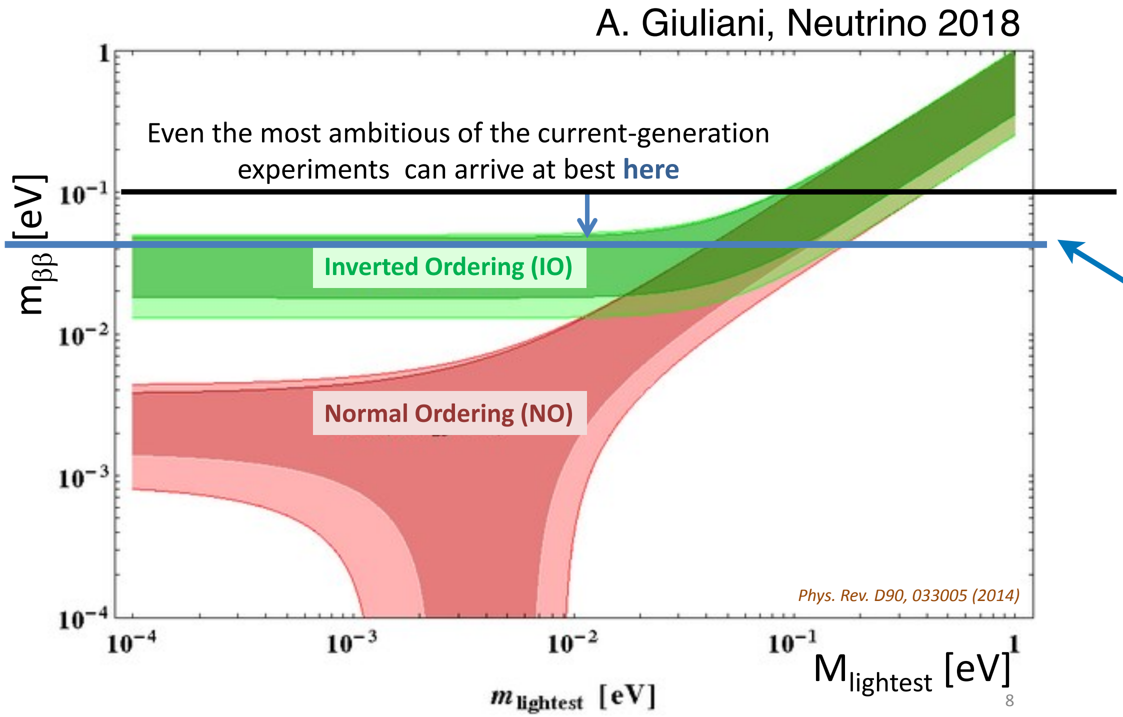
NH



dark shaded areas
= best fit values of
oscillation parameters
(only α_1, α_2 vary)

light shaded areas
= 3σ regions due
to uncertainties on
oscillation parameters
(+ dependence on α_i)

- need to reach 10 meV to exclude IH (lower bound on $m_{\beta\beta}$)
- need to reach few meV to test NH (if no mass degeneracy)
- if unlucky (m_1 ~ 1-10 meV), may not observe ββ0ν even if neutrinos are Majorana (cancellation in $m_{\beta\beta}$ due to α_1, α_2)



currently here, around 100 meV (experimental upper bounds depend on NME calculations ⇒ 2 - 4 uncertainty factor)

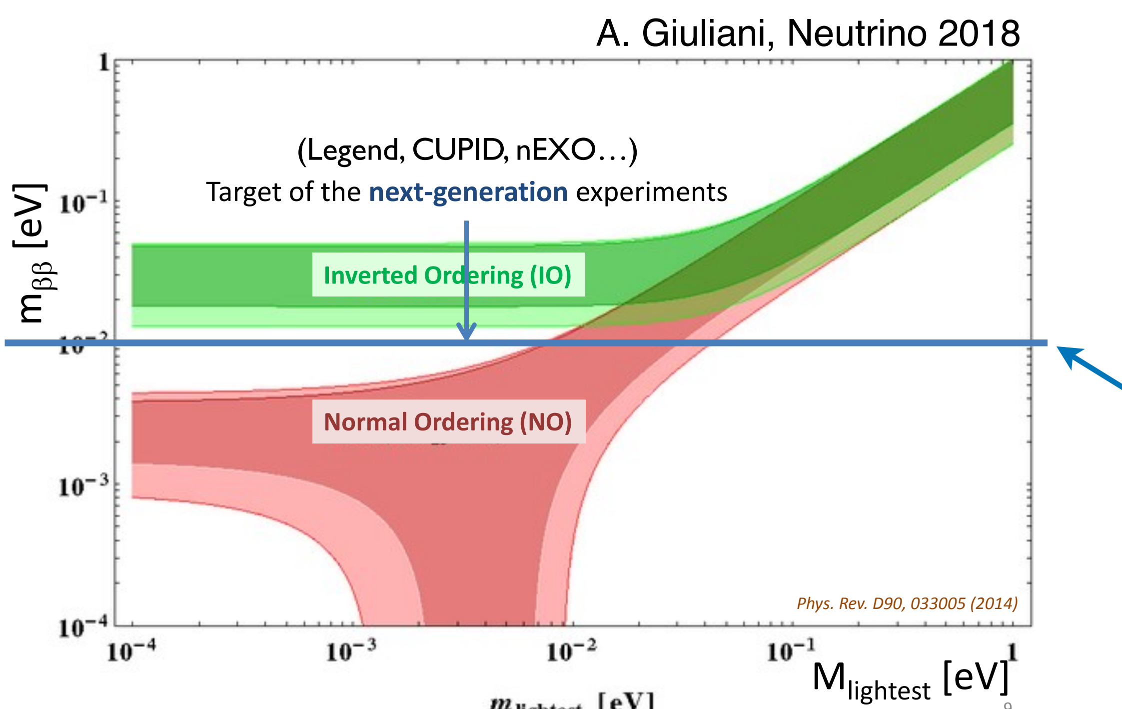
around 40 meV

Current best limit (90% C.L.) : KamLAND-Zen (2024) $^{136}\mathrm{Xe}$-loaded liquid scintillator

$$T^{0\nu}_{1/2} > 3.8 \times 10^{26}\,\mathrm{yr}$$

$$m_{\beta\beta} < (28-122)\ \mathrm{meV}$$

(uncertainty from NMEs)



around 10 meV