Flavour physics and CP violation
Lecture 1

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- fermion masses and Yukawa couplings
- flavour mixing and CKM matrix
- flavour changing neutral currents (kaon mixing)
- CP violation in the neutral kaon sector

Summer School on Neutrino Physics beyond the Standard Model
Université de Strasbourg, France, 29 June - 11 July 2025

Fermions in the electroweak Standard Model

<u>Gauge group</u>: $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{QED}$

(couplings) (g) (g') $(e = g \sin\theta_W)$ $\sin\theta_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$

Spontaneously broken by the vev $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $v = 246$ GeV

$\Rightarrow$ $M_W = \frac{1}{2} g v$, $M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\, v$, while A^μ is massless

<u>Fermions</u> come in three generations (family replication)

$$\begin{cases} \text{LH fermions} \rightarrow SU(2)_L \text{ doublets } (T^3 = \pm 1/2) \rightarrow \text{couple to } W^\pm \text{ (and } Z) \\ \text{RH fermions} \rightarrow SU(2)_L \text{ singlets } (T^3 = 0) \rightarrow \text{only couple to } Z \end{cases}$$

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ $\begin{pmatrix} c \\ s \end{pmatrix}_L$ $\begin{pmatrix} t \\ b \end{pmatrix}_L$ $Y = \frac{1}{3}$ $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ $Y = -1$

u_R c_R t_R $Y = \frac{4}{3}$

d_R s_R b_R $Y = -\frac{2}{3}$ e_R μ_R τ_R $Y = -2$

$\underbrace{\qquad\qquad\qquad}_{\text{quarks}}$ $\underbrace{\qquad\qquad\qquad}_{\text{leptons}}$

[no RH neutrinos in the SM]

Electric charge: $Q = T^3 + \dfrac{Y}{2}$

Fermion electroweak interactions

$$\mathcal{L}_{em} = e A_\mu \sum_i \left(\frac{2}{3} \bar{u}_i \gamma^\mu u_i - \frac{1}{3} \bar{d}_i \gamma^\mu d_i - \bar{e}_i \gamma^\mu e_i \right)$$

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \sum_i \left(\bar{u}_i \gamma^\mu P_L d_i + \bar{\nu}_i \gamma^\mu P_L e_i \right) + h.c.$$

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W} Z_\mu \sum_i \sum_{f=u,d,\nu,e} \left[g_L^f \bar{f}_i \gamma^\mu P_L f_i + g_R^f \bar{f}_i \gamma^\mu P_R f_i \right]$$

$$g_L^f = T_f^3 - Q_f \sin^2\theta_W \qquad g_R^f = -Q_f \sin^2\theta_W$$

$$\begin{cases} g_L^u = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \\ g_R^u = -\frac{2}{3}\sin^2\theta_W \end{cases} \quad \begin{cases} g_L^d = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \\ g_R^d = \frac{1}{3}\sin^2\theta_W \end{cases} \quad \begin{cases} g_L^e = -\frac{1}{2} + \sin^2\theta_W \\ g_R^e = \sin^2\theta_W \end{cases} \quad g_L^\nu = \frac{1}{2}$$

⚠ here u_i, d_i, ν_i, e_i $(i=1,2,3)$ refer to weak (or interaction) eigenstates, for which the charged weak current is diagonal (i.e., the W only couples fermions from the same generation)

The three generations of fermions have the same quantum numbers, hence interactions $\rightarrow$ <u>family replication</u> (also true for QCD)

Fermion masses and Yukawa couplings

$-m\bar{\Psi}\Psi = -m\bar{\Psi}_L\Psi_R + h.c.$ not invariant under $SU(2)_L \times U(1)_Y$

However, can write $SU(2)_L \times U(1)_Y$ invariant Yukawa couplings between fermions and the Higgs doublet, since both Ψ_L and H are $SU(2)_L$ doublets

$$\mathcal{L}_{\text{Yukawa}} = -y_u \bar{Q}\tilde{H}u_R - y_d \bar{Q} H d_R - y_e \bar{L} H e_R + h.c.$$

$$Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in (\underline{2}, \tfrac{1}{3}), \quad L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \in (\underline{2}, -1), \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \in (\underline{2}, +1)$$

$$\tilde{H} \equiv i\sigma^2 H^* = \begin{pmatrix} H^{0*} \\ -H^{+*} \end{pmatrix} \in (\underline{2}, -1)$$

Electroweak symmetry breaking : $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, $\langle \tilde{H} \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$$\mathcal{L}_{\text{Yukawa}} \Rightarrow \mathcal{L}_{\text{mass}} = -m_u \bar{u}_L u_R - m_d \bar{d}_L d_R - m_e \bar{e}_L e_R + h.c.$$

$$m_u = y_u \frac{v}{\sqrt{2}}, \quad m_d = y_d \frac{v}{\sqrt{2}}, \quad m_e = y_e \frac{v}{\sqrt{2}}$$

[but $m_\nu = 0$, since no ν_R in the SM]

$\mathcal{L}_{\text{Yukawa}}$ also gives rise to Higgs-fermion interactions:

$$\mathcal{L}_{hff} = -\frac{m_u}{v} h \bar{u}_L u_R - \frac{m_d}{v} h \bar{d}_L d_R - \frac{m_e}{v} h \bar{e}_L e_R + h.c.$$

where in the unitary gauge : $H = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$ $\quad h =$ Higgs boson

Family replication and flavour mixing

Denote quarks and leptons of the 3 generations by u_i, d_i, ν_i, e_i $(i=1,2,3)$
$\Rightarrow$ the Yukawa couplings are now complex matrices in 3d flavour space:

$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_i Y^u_{ij} \tilde{H} u_{Rj} - \bar{Q}_i Y^d_{ij} H d_{Rj} - \bar{L}_i Y^e_{ij} H e_{Rj} + h.c.$$

⚠ Y_{ij} not diagonal a priori $\Rightarrow$ u_i, d_i, ν_i, e_i are not mass eigenstates, but <u>interaction eigenstates</u> (also called weak / flavour eigenstates)

<u>After EWSB</u>, and in vector notations:

$$\mathcal{L}_{\text{mass}} = -\bar{U}_L M^u U_R - \bar{D}_L M^d D_R - \bar{E}_L M_e E_R + h.c.$$

where $U_L = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L$, $U_R = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_R$, etc, are vectors in flavour space,
and $M^{u,d,e} = Y^{u,d,e} \frac{v}{\sqrt{2}}$ are 3×3 matrices

$\rightarrow$ diagonalized by a biunitary rotation:

$$M_u = R^u_L D_u R^{u+}_R \qquad D_u = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \qquad m_{u,c,t} \text{ real} > 0$$

such that $M_u M_u^+ = R^u_L D_u^2 R^{u+}_L$, $M_u^+ M_u = R^u_R D_u^2 R^{u+}_R$

$\rightarrow$ <u>mass eigenstates</u>: $U_L^{(m)} = R^{u+}_L U_L$, $U_R^{(m)} = R^{u+}_R U_R$

and similarly for M_d and M_e

Now the whole Lagrangian should be expressed in terms of fermion mass eigenstates (= physical fermions)

$\rightarrow$ does not affect kinetic terms $[\bar{\Psi} i \gamma^\mu \partial_\mu \Psi = \bar{\Psi}_L i \gamma^\mu \partial_\mu \Psi_L + \bar{\Psi}_R i \gamma^\mu \partial_\mu \Psi_R]$, nor strong / electromagnetic / neutral weak (Z) interactions, which involve currents $\bar{\Psi}_L \gamma^\mu \Psi_L$ and $\bar{\Psi}_R \gamma^\mu \Psi_R$

[also, Higgs-fermions interactions are diagonal in the mass eigenstate basis]

$\rightarrow$ <u>only charged weak interactions are affected:</u> (focus on quark sector from now on)

$$\mathcal{L}_{cc} \ni \frac{g}{\sqrt{2}} W_\mu^+ \bar{U}_L \gamma^\mu D_L + h.c.$$

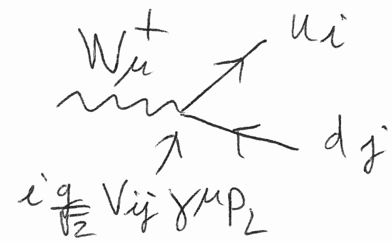
$$\bar{U}_L \gamma^\mu D_L = \bar{U}_L^{(m)} R_L^{u+} \gamma^\mu R_L^d D_L^{(m)} \equiv \bar{U}_L^{(m)} \gamma^\mu V_{CKM} D_L^{(m)}$$

where $\boxed{V_{CKM} \equiv R_L^{u+} R_L^d}$ is the CKM matrix (Cabibbo-Kobayashi-Maskawa)

(relative rotation between LH up-type and LH down-type quarks)

<u>Renaming</u> $U_L^{(m)} \rightarrow U_L$, $D_L^{(m)} \rightarrow D_L$

$$\mathcal{L}_{cc} \ni \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}_L \; \bar{c}_L \; \bar{t}_L) \gamma^\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$



$$V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Parametrization of the CKM matrix

- Standard parametrization: in terms of 3 mixing angles and 1 phase
[unitary $\rightarrow$ 6 phases, but 5 can be removed by rephasing quark fields]

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{array}{l} c_{ij} \equiv \cos\theta_{ij} \\ s_{ij} \equiv \sin\theta_{ij} \end{array}$$

$\rightsquigarrow$ 2-generation limit (relevant for many processes in kaon physics):
the 2×2 CKM matrix is parametrized by a single parameter, the Cabibbo angle θ_c:
($\sin\theta_c \simeq 0.22$)

$$\mathcal{L}_{cc} \ni \frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{u}_L \; \bar{c}_L\right) \gamma^\mu \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}$$

- Wolfenstein parametrization: hierarchy $|V_{ub}| \ll |V_{cb}| \ll |V_{us}|$ ($s_{13} \ll s_{23} \ll s_{12}$) explicit

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \simeq 0.225 \;, \quad A \simeq 0.83 \;, \quad \rho \simeq 0.16 \;, \quad \eta \simeq 0.36 \qquad [\Rightarrow \delta \simeq 65°]$$

[corresponding to $|V_{us}| \simeq \lambda \simeq 0.225$, $|V_{cb}| \simeq A\lambda^3 \simeq 0.042$, $|V_{ub}| \simeq A\lambda^3\sqrt{\rho^2+\eta^2} \simeq 0.0037$]

$\rightsquigarrow$ can write an expression unitary to all orders in λ for V_{CKM} in terms of $A, \lambda, \bar{\rho}$ and $\bar{\eta}$
where $\bar{\rho} \equiv \rho\left(1-\frac{\lambda^2}{2}+\dots\right)$, $\bar{\eta} \equiv \eta\left(1-\frac{\lambda^2}{2}+\dots\right)$

Parameter counting (n families)

V_{CKM} $n \times n$ unitary $\Rightarrow$ n^2 parameters $\begin{cases} \frac{n(n-1)}{2} \text{ angles} & (\text{\# pairs of axes in } \mathbb{C}^n) \\ \frac{n(n+1)}{2} \text{ phases} \end{cases}$

$(VV^+ = \mathbb{1})$

Only phases that cannot be removed from the Lagrangian by quark field rephasing are physical

$u_i(x) \to e^{i\phi_i^u} u_i(x) \qquad d_i(x) \to e^{i\phi_i^d} d_i(x)$

leave quark masses invariant; only affect W-quark couplings:

$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu V_{ij} P_L d_j \longrightarrow \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu V_{ij} e^{i(\phi_j^d - \phi_i^u)} P_L d_j$$

ie $\boxed{V_{ij} \to V_{ij} e^{i(\phi_j^d - \phi_i^u)}}$ $\qquad$ $\underline{2n-1 \text{ relative phases}}$

$$\Rightarrow \frac{n(n+1)}{2} - (2n-1) = \boxed{\frac{(n-2)(n-1)}{2} \text{ physical (CPV) phases in } V_{CKM}}$$

$\to$ CP violation requires $\underline{n \geq 3 \text{ quark generations}}$

<u>Jarlskog invariant</u>: phase convention invariant measure of CPV

$\boxed{J \equiv \mathrm{Im}(V_{ud} V_{cs} V_{us}^* V_{cd}^*)}$ unaffected by $V_{ij} \to V_{ij} e^{i(\phi_j^d - \phi_i^u)}$

$J \simeq A^2 \lambda^6 \eta$ in Wolfenstein parametrization

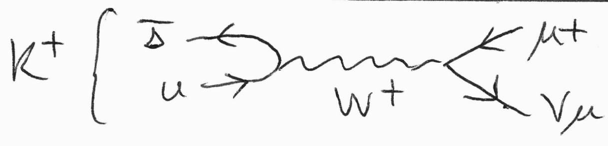
$J \simeq 3.1 \times 10^{-5}$

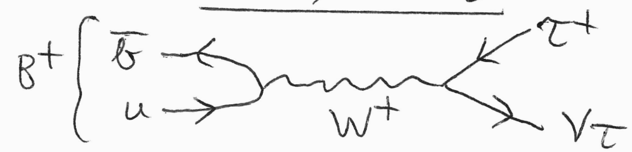
Flavour-violating processes

$$\mathcal{L}_{CC} \ni \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}_L\ \bar{c}_L\ \bar{t}_L)\, \gamma^\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

$V_{CKM} \neq \mathbb{1} \Rightarrow$ <u>weak interactions violate flavour</u> (charged current)
(induce transitions between quarks from different generations)

Processes such as <u>$K^+ \to \mu^+ \nu_\mu$</u> or <u>$B^+ \to \tau^+ \nu_\tau$</u>

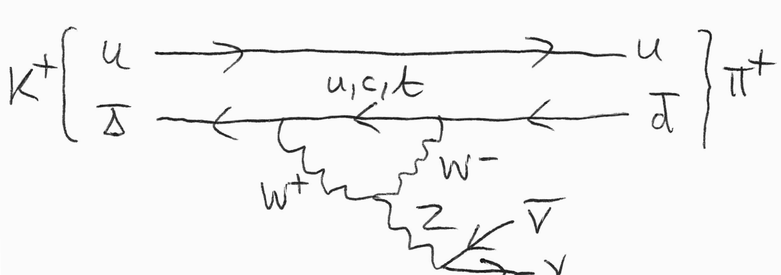


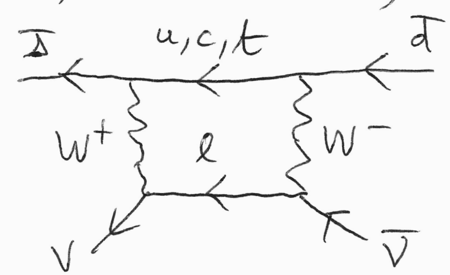


do not conserve the number of quarks of a given flavour (s or b)
[associated quantum number : strangeness (S), beauty (B)]

<u>Flavour-changing neutral currents (FCNCs)</u> (transitions between same-charge quarks : $s \to d$, $b \to s$, $b \to d$, $c \to u$)
are <u>forbidden at tree level</u>, since γ and Z couplings are flavour-diagonal in the mass eigenstate basis, as well as H couplings

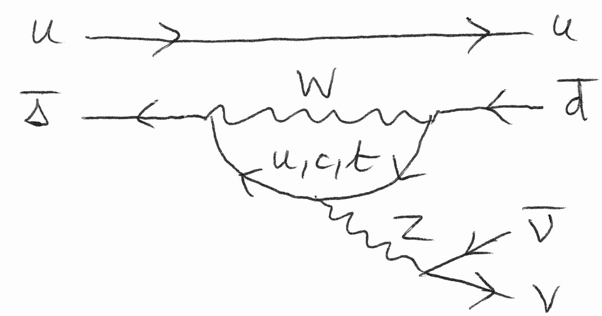
eg <u>$K^+ \to \pi^+ \nu\bar{\nu}$</u> [$K^+ = u\bar{s}$, $\pi^+ = u\bar{d}$ $\Rightarrow$ $s \to d$ transition]
arises only at 1 loop (loops involving a W, hence V_{CKM})





In addition to the loop factor, FCNCs are further suppressed by the GIM mechanism (Glashow, Iliopoulos, Maiani), which is a consequence of the unitarity of the CKM matrix

Naive estimate for $K^+ \to \pi^+ \nu\bar{\nu}$:



amplitude $\sim \underbrace{\frac{1}{M_W^2}}_{\text{4-fermion operator}} g^4 \sim \underline{G_F \alpha}$

$$G_F = \frac{g^2}{4\sqrt{2} M_W^2} = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{g^2 \sin^2\theta_W}{4\pi}$$

→ only suppression by α wrt the CC decay mode $K^+ \to \mu^+ \nu_\mu$, $\mathcal{O}(G_F)$

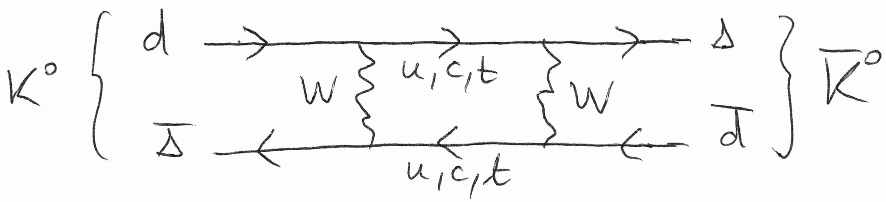
→ measured BR is actually much more suppressed:

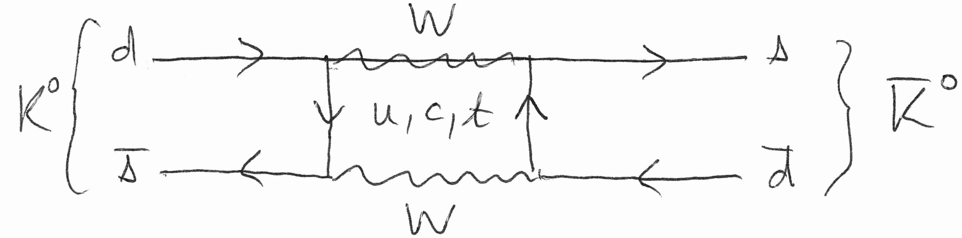
$$BR(K^+ \to \pi^+ \nu\bar{\nu}) = 1.14^{+0.40}_{-0.33} \times 10^{-10} \lll BR(K^+ \to \mu^+ \nu_\mu) = 0.6356 \pm 0.0011$$

→ GIM mechanism

$K^0-\bar{K}^0$ mixing (prototype of FCNCs)

$K^0 = \bar{s}d$, $\bar{K}^0 = s\bar{d}$ are not mass eigenstates due to mixing induced by W loops:





$\Rightarrow$ non-diagonal mass matrix: $\quad (K^0\ \bar{K}^0)\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$

The mass eigenstates, K_L and K_S, are very close in mass:

$\Delta m_K = m_{K_L} - m_{K_S} \ll m_K \qquad (\Delta m_K / m_K \simeq 7 \times 10^{-15})$

$K^0-\bar{K}^0$ mixing $\Rightarrow$ oscillations (with frequency Δm_K)

pure K^0 beam $\rightarrow$ $K^0-\bar{K}^0$ admixture $\rightarrow$ pure $\bar{K}^0$ beam $\rightarrow$ $K^0-\bar{K}^0$ admixture $\rightarrow$ pure K^0 beam, etc

(this phenomenon competes with decay)

Box diagrams $\Rightarrow$ $\dfrac{\Delta m_K}{m_K} = \dfrac{G_F}{\sqrt{2}} \dfrac{g^2}{24\pi^2} f_K^2 B_K |\mathcal{F}_0|$

$m_K = \dfrac{m_{K_L} + m_{K_S}}{2} = 497.6$ MeV $\left\{ \begin{array}{l} f_K \simeq 160 \text{ MeV} = \text{kaon decay constant} \\ B_K \simeq 0.75 \text{ (hadronic parameter} \end{array} \right.$

$$\mathcal{F}_0 \equiv \sum_{i,j} V_{u_i d} V_{u_i s}^* V_{u_j s}^* V_{u_j d} \, f\left(\frac{m_{u_i}^2}{M_W^2}, \frac{m_{u_j}^2}{M_W^2}\right)$$

$$\equiv \sum_{i,j} \lambda_i \lambda_j \, f(x_i, x_j)$$

$$\lambda_i \equiv V_{u_i d} V_{u_i s}^* \Rightarrow \sum_i \lambda_i = (V^+ V)_{sd} = 0 \quad ; \quad x_i \equiv \frac{m_{u_i}^2}{M_W^2}$$

Expand $f(x_i, x_j) = A + (x_i, x_j$ dependent terms$)$

$$\Rightarrow \mathcal{F}_0 = \sum_{i,j} \lambda_i \lambda_j \left[A + \mathcal{O}(x_i, x_j)\right] = A \underbrace{\left(\sum_i \lambda_i\right)\left(\sum_j \lambda_j\right)}_{=0} + \mathcal{O}\left(\frac{m_q^2}{M_W^2}\right)$$

$\rightarrow$ GIM mechanism

Explicitly, $\mathcal{F}_0 \simeq \lambda_c^2 x_c = (V_{cd} V_{cs}^*)^2 \dfrac{m_c^2}{M_W^2}$ (leading term)

$\Rightarrow \dfrac{\Delta m_K}{m_K} \propto G_F^2 m_c^2$ rather than $G_F \alpha$ due to GIM mechanism

$G_F m_c^2 \sim (10^{-5} \text{ GeV}^{-2})(1 \text{ GeV})^2 = 10^{-5}$ additional suppression

The same suppression typically happens for other FCNC processes

CP violation in the neutral kaon system

CP violation first discovered in $K^0 \to \pi\pi$ decays (1964), then in B meson (2001) and D meson (2019) decays

<u>K^0–$\bar{K}^0$ system</u>: $K^0 = \bar{s}d$ and $\bar{K}^0 = \bar{d}s$ are CP conjugates

$$CP|K^0\rangle = |\bar{K}^0\rangle \quad \Rightarrow \quad \underline{\text{CP eigenstates}}: \begin{cases} |K_1\rangle \equiv \dfrac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} & \text{(CP-even)} \\ |K_2\rangle \equiv \dfrac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}} & \text{(CP-odd)} \end{cases}$$

<u>mass eigenstates</u>: K_S, K_L

= eigenstates of the $(K^0, \bar{K}^0)$ mass matrix $H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$
(with $H_{11} = H_{22}$ due to CPT)

<u>if CP conserved</u> ($\delta_{CKM} = 0$), one has in addition $H_{21} = H_{12}$

$\Rightarrow$ the eigenstates of H are K_1, K_2 $\Rightarrow$ $\underline{K_S = K_1,\ K_L = K_2}$

$\Rightarrow$ the mass eigenstates are also CP eigenstates, and decay to CP-even or CP-odd final states:

$K_1 \to 2\pi$ ($\pi^+\pi^-$ or $\pi^0\pi^0$) CP even

$K_2 \to 3\pi$ ($\pi^0\pi^0\pi^0$ or $\pi^0\pi^+\pi^-$) CP-odd

However, CP is violated : $H_{21} \neq H_{12}$ due to $\delta_{CKM} \neq 0$

$\Rightarrow$ the mass eigenstates are

$$\begin{cases} |K_S\rangle \propto |K_1\rangle + \varepsilon |K_2\rangle & \text{mass } m_S \\ |K_L\rangle \propto |K_2\rangle + \varepsilon |K_1\rangle & \text{mass } m_L \end{cases}$$

where $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$ from the box diagrams

Observation of $K_L \to 2\pi$ (with a small BR) signals CP violation

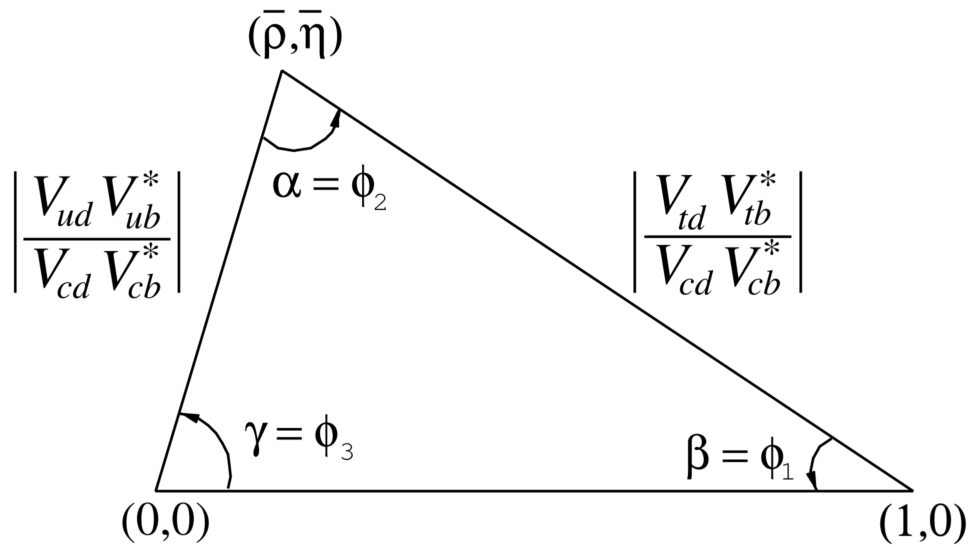
$$BR(K_L \to 3\pi) \simeq 0.32$$
$$BR(K_L \to 2\pi) \simeq 2.8 \times 10^{-3}$$

Unitarity triangle

The unitarity of the CKM matrix leads to relations between its entries (hermitian products of two rows or columns) such as

$$\left(V^\dagger V\right)_{bd} = 0 \quad \Rightarrow \quad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

This unitarity relation can be represented as a triangle in the complex plane



where all sides have been divided by $V_{cd}V_{cb}^*$, such that the triangle is completely determined by the position of the $(\bar{\rho}, \bar{\eta})$ vertex

Different flavour physics measurements can be used to put constraints on the position of the $(\bar{\rho}, \bar{\eta})$ vertex and to test the consistency of the CKM model (i.e. , the ability of the CKM matrix to describe all observed flavour physics processes, such as decays of K, B and D mesons)

Particle Data Group, Review of Particle Physics 2024

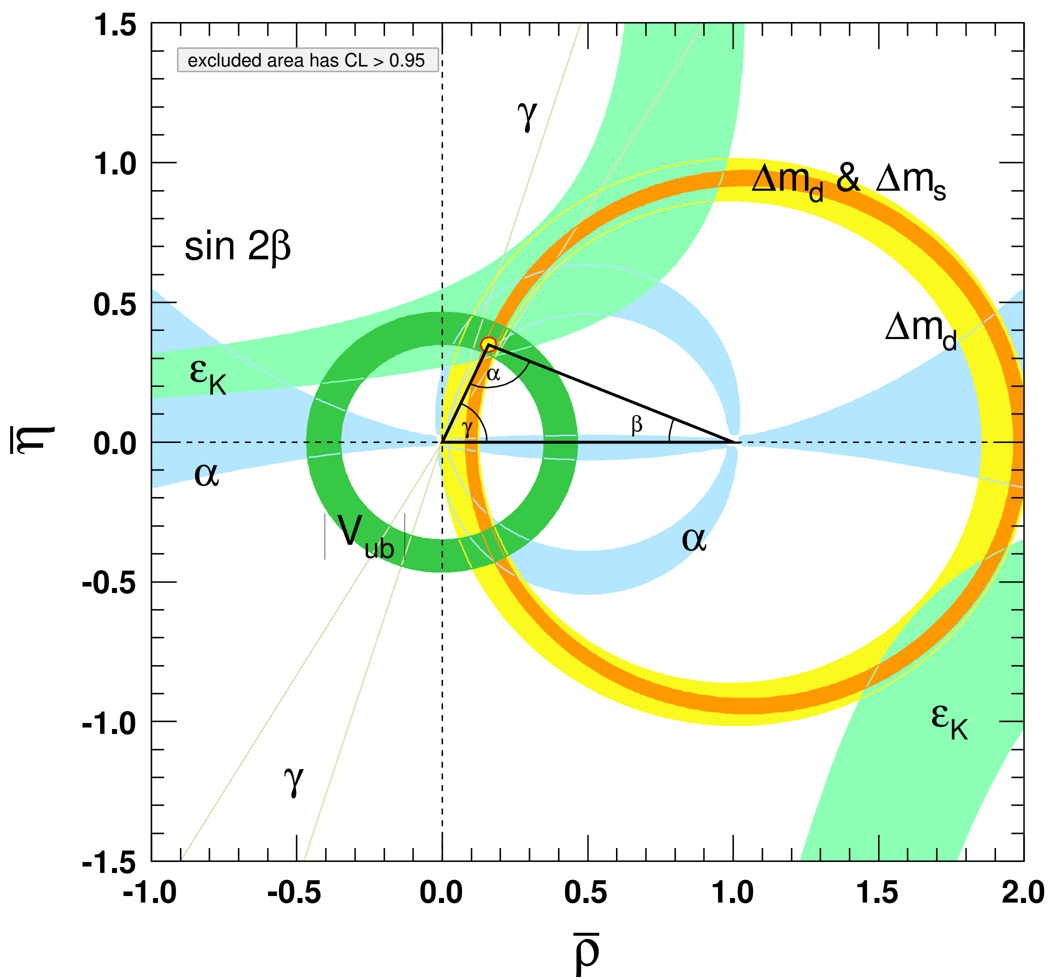


Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

The 95% C.L. shaded regions all overlap with the global CKM fit region (which assumes unitarity)