# Quantum Quenches from Quantum Fields

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Based on:

- C.K., & K.Zarembo, JHEP08 (2023) 18, JHEP02 (2025) 179
- A. Chalabi, C.K., C Su, ArXiv: 2503.22598, Phys.Lett.B 866 (2025) 139512
- Older works

L'esprit des cartes: une conférence en l'honneur d'Émmanuel Guitter
IPhT, Saclay, Paris
May 15th, 2025

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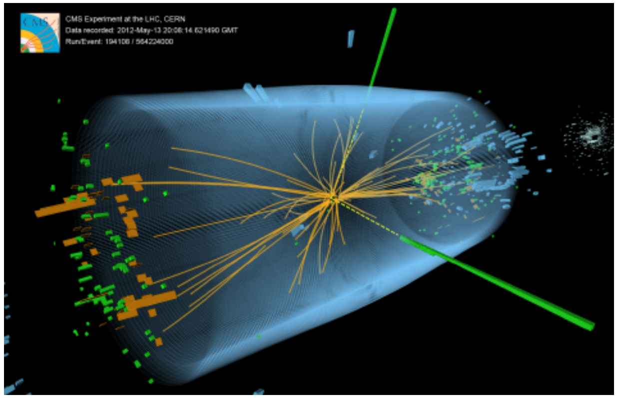
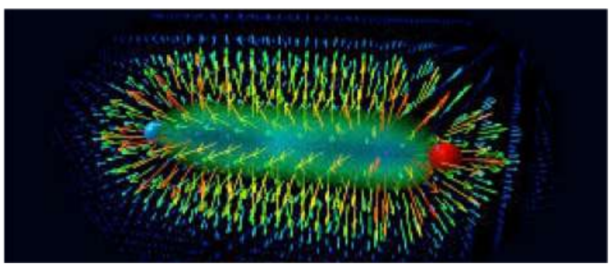
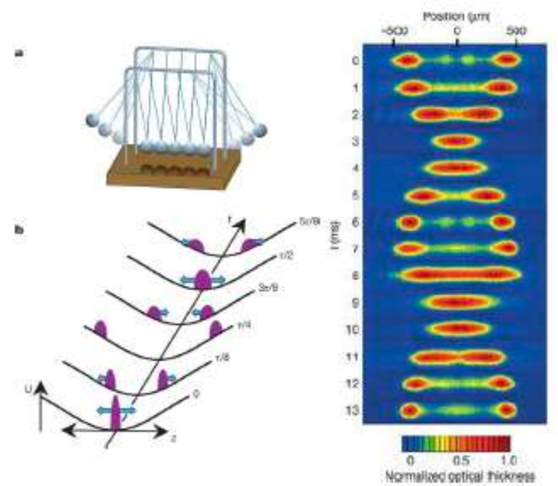
# Quantum Quenches from Quantum Fields




Higgs physics



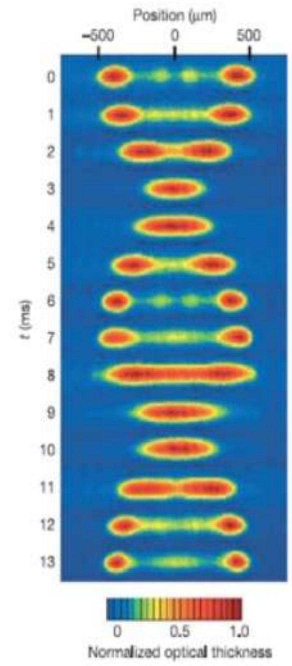
Quark confinement




Quantum Quench

What happens to a quantum many body system after a sudden disturbance ?

# Quantum Quenches and Overlaps




Set out quantum system in initial state $|\Psi_0\rangle$ which is not an eigenstate of its Hamiltonian $\mathcal{H}_0$

Study time development of local observable

$$\langle \mathcal{O}(t) \rangle = \langle \Psi_0 | e^{i\mathcal{H}_0 t} \mathcal{O} e^{-i\mathcal{H}_0 t} | \Psi_0 \rangle$$

$$= \sum_{\mathbf{u},\mathbf{v}} \langle \Psi_0 | \mathbf{u} \rangle \langle \mathbf{u} | \mathcal{O} | \mathbf{v} \rangle \langle \mathbf{v} | \Psi_0 \rangle e^{-i(E_{\mathbf{v}} - E_{\mathbf{u}})t},$$

$$\mathcal{H}_0 |\mathbf{u}\rangle = E_{\mathbf{u}} |\mathbf{u}\rangle$$

Assume $\mathcal{H}_0$ Hamiltonian of an integrable system

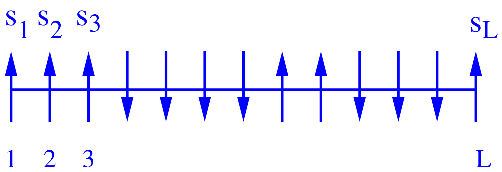
When and how can $\langle \Psi_0 | \mathbf{u} \rangle$ be calculated in closed form?

Of relevance for

- Time development after quantum quench
  (post-quench steady state, post-quench entanglement dynamics)

- Correlation functions in AdS/dCFT

# Integrable Quenches




$s_{L+m} = s_m$ $\quad |\Psi\rangle = |s_1 s_2 s_3 \ldots s_L\rangle$

Eigenstates: $H_0|\mathbf{u}\rangle = E_0|\mathbf{u}\rangle$

Integrable Quench: $\langle\Psi_0|\mathbf{u}\rangle$ computable in closed form

Identified types of relevance for AdS/dCFT:

Matrix product states: $|\Psi_0\rangle = |\text{MPS}\rangle = \sum_{\{s_i\}} \text{Tr}(t_{s_1} \ldots t_{s_L})|s_1 \ldots s_L\rangle$

De Leeuw, C.K., Zarembo '15

Ex: Heisenberg spin chain $\quad |\text{MPS}\rangle = \text{Tr} \prod_{l=1}^{L} (|\uparrow\rangle_l \otimes t_1 + |\downarrow\rangle_l \otimes t_2)^L$

# Integrable Quenches

## Identified types of relevance for AdS/dCFT:

Valence Bond States: $|\Psi_0\rangle = |\mathrm{VBS}\rangle = |K\rangle^{\otimes \frac{L}{2}}, \quad K = \sum_{s_1, s_2} K_{s_1, s_2} |s_1 s_2\rangle$

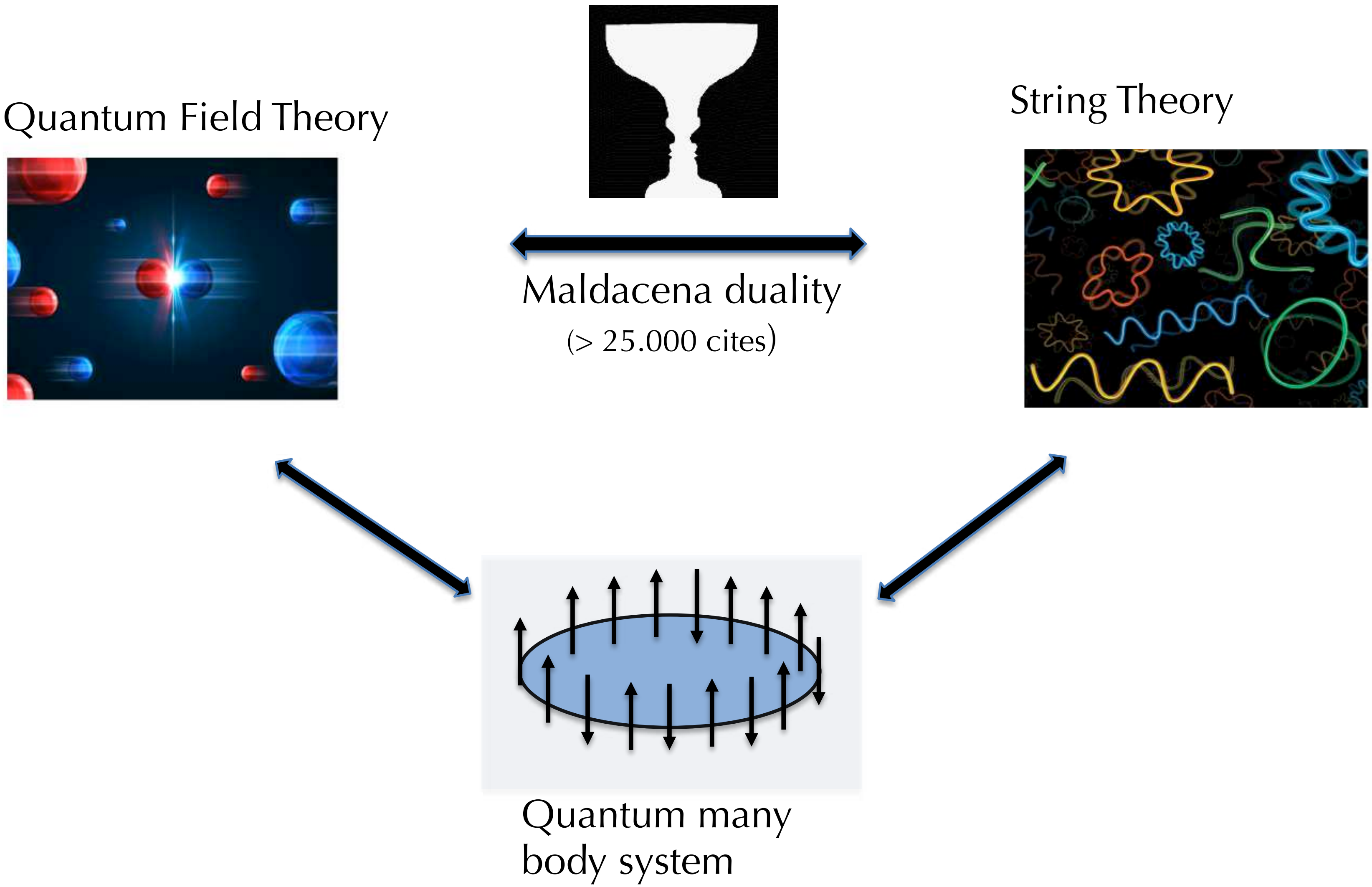
C.K., Müller, Zarembo '20

## Of possible relevance for AdS/CFT:

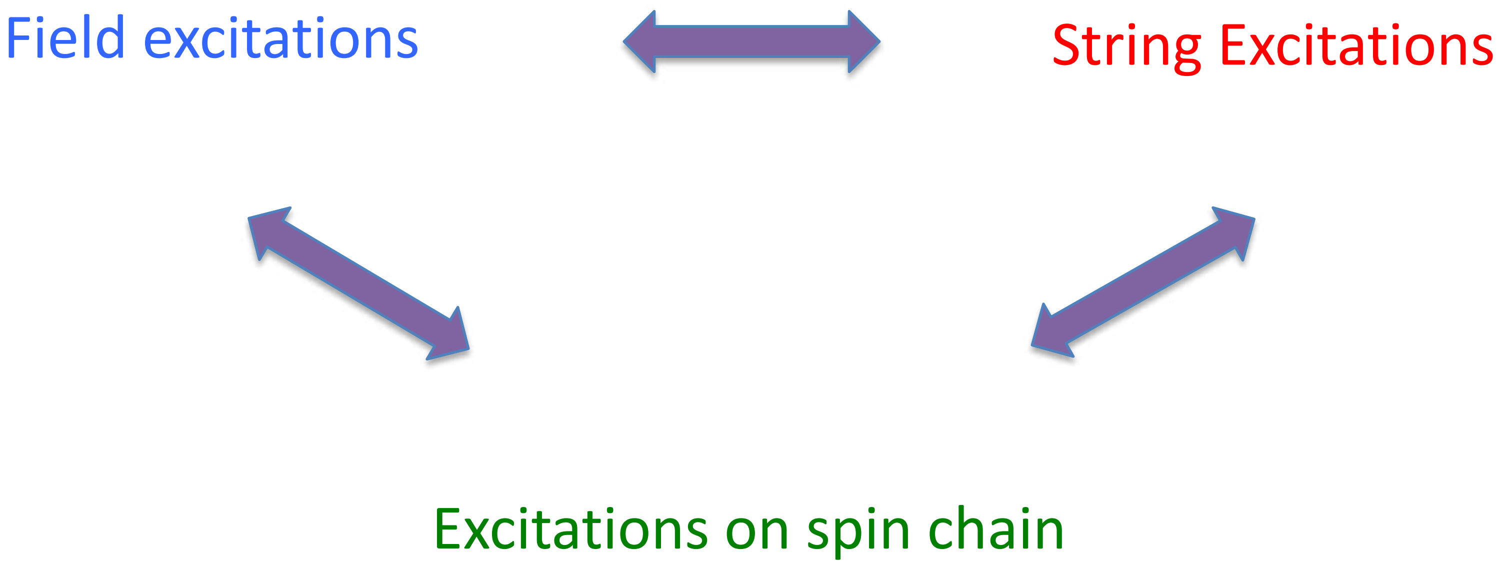
Cross cap states: $|C\rangle = |c\rangle\rangle^{\otimes L/2}$, where $|c\rangle\rangle = |\uparrow\rangle_j |\uparrow\rangle_{\frac{L}{2}+j} + |\downarrow\rangle_j |\downarrow\rangle_{\frac{L}{2}+j}$

Caetano, Komatsu '21

# The AdS/CFT correspondence



# Spin chains connecting field theory and string theory

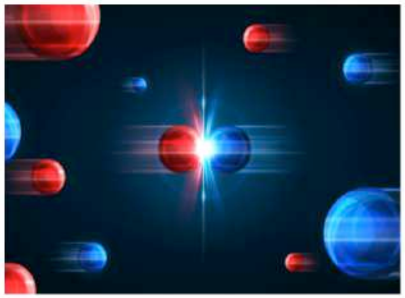


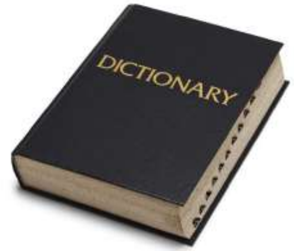

Interactions between excitations completely determined by symmetries

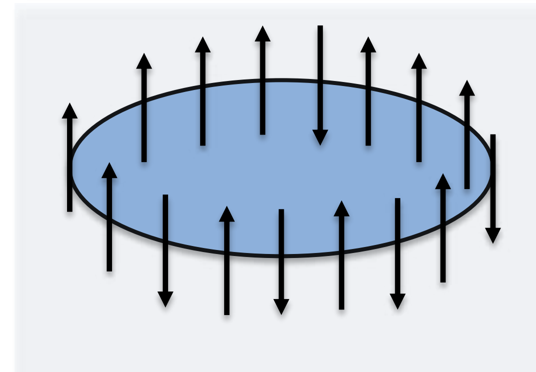
The spin chain turns out to be integrable, i.e. exactly solvable

Range of spin chain interaction: L+1, with L the loop order in the field theory

# Quantum Quenches from Quantum Fields

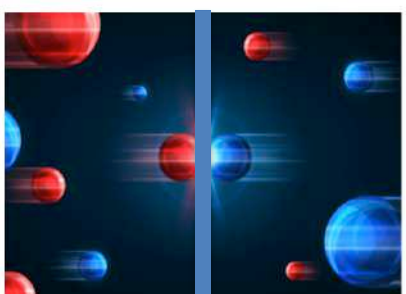


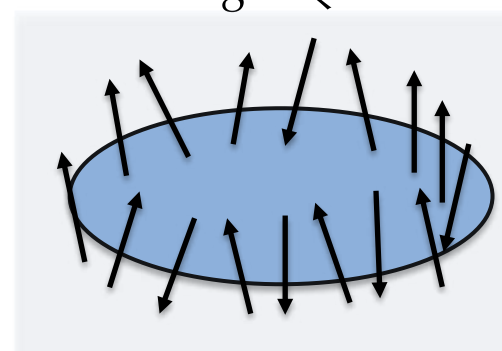


Introducing a Defect

Performing a Quantum Quench





De Leeuw, Kristjansen & Zarembo

Defect

Quench Initial State

## AdS/CFT

Conformal operators $\longleftrightarrow$ String states, (AdS/CFT)

$\updownarrow$ $\updownarrow$

Eigenstates of integrable super spin chain: $|\mathbf{u}\rangle$

Main examples: 4D QFT ($\mathcal{N}=4$ SYM), 3D QFT (ABJM theory)
Planar limit

## AdS/dCFT

Co-dimension $d$ defect $\longleftrightarrow$ Probe brane

$\updownarrow$ $\updownarrow$

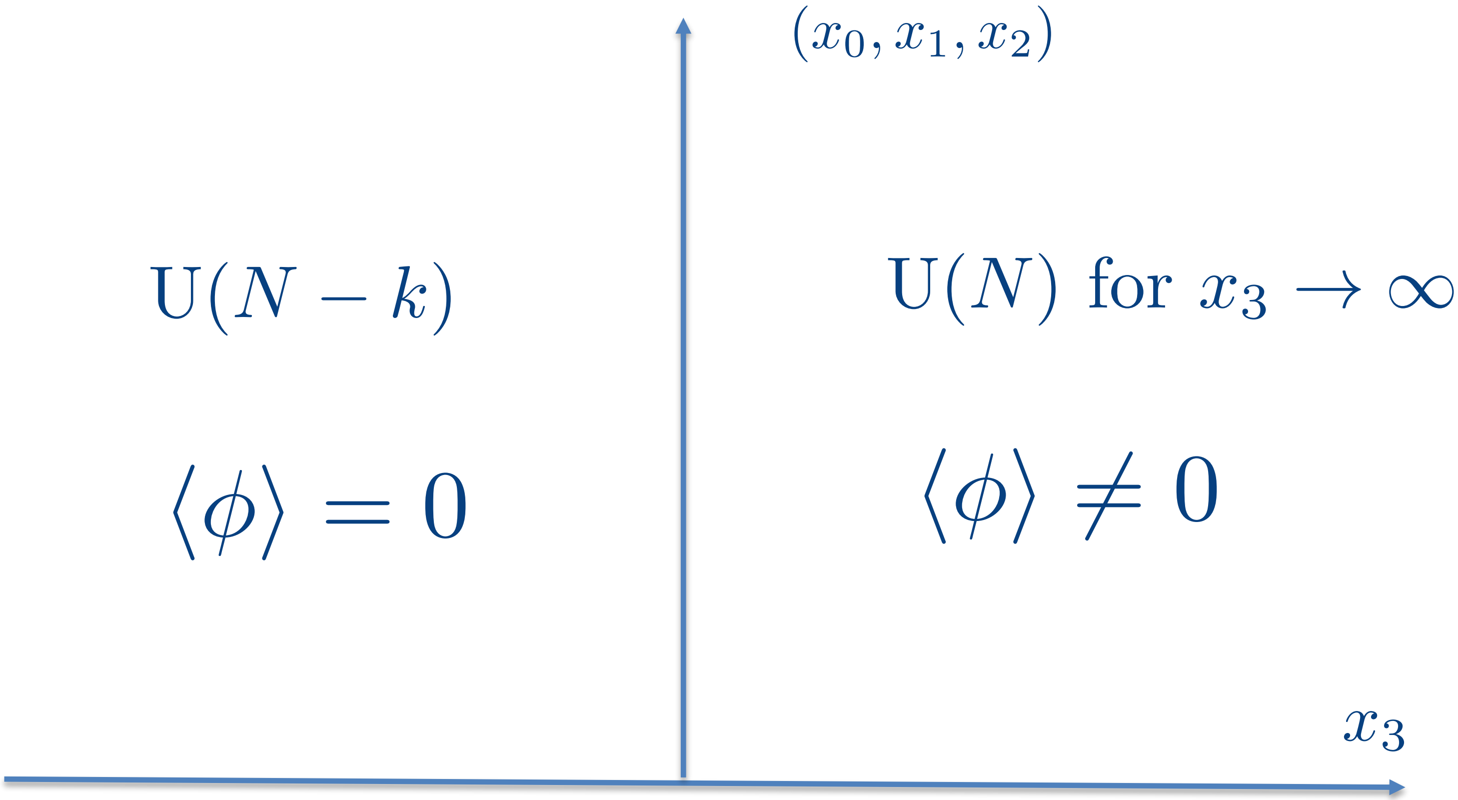
(Integrable) boundary state $|\Psi_0\rangle$ of spin chain

De Leeuw, C.K. Zarembo '15

$\langle\Psi_0|\mathbf{u}\rangle$ is a correlation function

# How |MPS⟩ enter the game

Ex: Domain wall set-up in 4D ($\mathcal{N} = 4$ SYM)




$k$ becomes the bond dimension of the |MPS⟩

# Higgs configuration

Field vacuum:

$$\langle\phi_i\rangle = \phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{pmatrix}, \; i=1,2,3, \quad x_3>0$$

$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

where $t_i$, $i=1,2,3$ constitute the generators of a $k$-dimensional irreducible repr. of $SU(2)$.

Origin:
Classical e.o.m.
($x_3$ distance from defect)

$$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = \left[\phi_j^{\rm cl}, \left[\phi_j^{\rm cl}, \phi_i^{\rm cl}\right]\right].$$

Assume only $x_3$ dependence and $x_3>0$,
$A_\mu^{\rm cl} = 0 \quad \Psi_\alpha^{\rm cl} = 0$

# One-point functions and $|\mathrm{MPS}\rangle$

General scalar conformal operator

$$\mathcal{O}_L(x) = \Psi^{i_1\ldots i_L}\,\mathrm{Tr}(\phi_{i_1}\ldots\phi_{i_L}) \qquad i_1,\ldots,i_L \in \{1,2,\ldots,6\}$$

$\equiv$ Eigenstate of integrable $SO(6)$ spin chain, $|\mathbf{u}\rangle$

$$\mathrm{Tr}(\phi_{i_1}\phi_{i_2}\ldots\phi_{i_L}) \sim |s_{i_1}s_{i_2}\ldots s_{i_L}\rangle$$

Minahan & Zarembo

Due to the vevs scalar operators can have 1-pt fcts already at tree level

$$\langle\mathcal{O}_L(x)\rangle = \frac{1}{x_3^L}\,\Psi^{i_1\ldots i_L}\,\mathrm{Tr}(t_{i_1}^{(k)}\ldots t_{i_L}^{(k)}) \equiv \frac{C_k}{x_3^L}$$

Matrix product state (of bond dimension $k>1$) associated with defect

$$|\mathrm{MPS_k}\rangle = \sum_{i_1,\ldots,i_L}\mathrm{tr}\,(t_{i_1}^{(k)}\ldots t_{i_L}^{(k)})|s_{i_1}\ldots s_{i_L}\rangle,$$

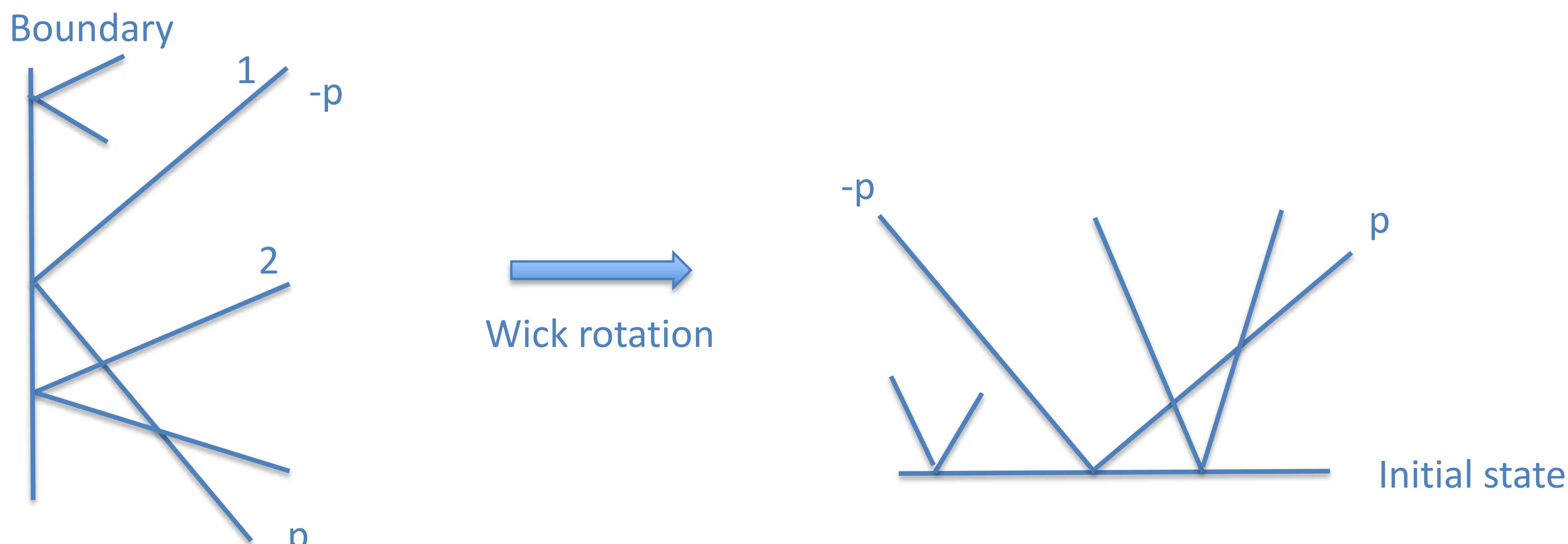
de Leeuw, C.K. & Zarembo '15

Object to calculate $\qquad C_k\,(\mathbf{u}) = \dfrac{\langle\mathrm{MPS}_k\,|\mathbf{u}\rangle}{\langle\mathbf{u}\,|\mathbf{u}\rangle^{\frac{1}{2}}}$

# Integrable boundaries in integrable QFTs

Ghoshal, Zamolodchikov '93

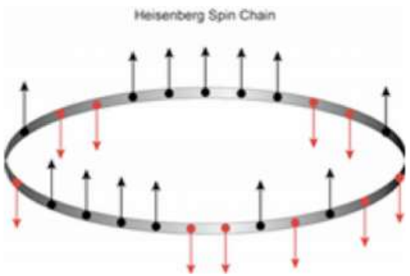
- No particle production or annihilation
- Pure reflection, possibly change of internal quantum numbers
- Yang-Baxter relations fulfilled (order of reflection does not matter)




Pure reflection
+BYBE for reflection matrix

Entangled (p,-p) pairs
+KYBE for initial state

# Integrability test




Ex: Heisenberg spin chain

Vacuum State: All spins down $|0\rangle$

Excited states with $M$ excitations: $|\{p_i\}_{i=1}^{M}\rangle$

$L$ conserved charges, $\hat{Q}_n$, with eigenvalues $Q_n$

$$Q_n\left(\{p_i\}\right) = (-1)^n\, Q_n\left(\{-p_i\}\right)$$

Integrable initial state: $\widehat{Q}_{2m+1}|\Psi_0\rangle = 0, \quad \forall\, m$

Piroli, Pozsgay Vernier '17

Bajnok, Gombor '20

$$\Pi\, T(u)\, \Pi\, |\Psi_0\rangle = T(u)|\Psi_0\rangle$$

Parity — Transfer matrix

$T$ explicitly known – Easy to carry out concrete checks

# Integrable Quenches in AdS/CFT

| String theory probe | Field theory defect | Matrix product state |
|---|---|---|
| D1-brane | Monopole | Diagonal matrices |
| D3-brane | Determinant operator | Diagonal matrices |
| D3-brane | (A subset of) Rigid Gukov Witten surface defects | SU(2) representations |
| D5-brane | Domain Wall | SU(2) representations |
| D7-brane | Domain Wall | SO(5) representations |

# Elements of the language of integrability

Eigenstates with $M$ excitations described in terms of $M$ momenta $p_1, \ldots, p_M$ or rapidities $u_i = \frac{1}{2}\coth(p_i/2)$

$$1 = \left(\frac{u_k - \frac{i}{2}}{u_k + \frac{i}{2}}\right)^L \prod_{j\neq k}^{K} \frac{u_k - u_j + i}{u_k - u_j - i} = e^{i\chi_k}, \qquad k = 1, \ldots M$$

Heisenberg spin chain

Can be encoded in Baxter polynomial $Q(u) = \prod_{i=1}^{M}(u - u_i)$

$$|\mathbf{u}\rangle = |\{u_i\}\rangle = \hat{B}(u_1)\ldots\hat{B}(u_M)|0\rangle$$

$$\langle\mathbf{u}|\mathbf{u}\rangle = \det G(\{u_i\}),$$

Gaudin determinant

$$G_{kj} = \frac{\partial\chi_k}{\partial u_j}$$

# Integrable overlaps and pairing

$$\hat{\mathbf{Q}}_{\mathbf{2n+1}}|\Psi_0\rangle = 0 \implies$$

$$\langle\Psi_0|\mathbf{u}\rangle \neq 0 \text{ iff roots are paired } \{u_i, -u_i\}_{i=1}^{K_u}$$

Gaudin matrix has block structure

$$\det G = \begin{vmatrix} A & B \\ B & A \end{vmatrix} = \begin{vmatrix} A+B & B \\ B+A & A \end{vmatrix} = \begin{vmatrix} A+B & B \\ 0 & A-B \end{vmatrix} = \det(A+B)\cdot\det(A-B)$$

$$= \det G_+ \cdot \det G_-$$

Quantity entering overlap formulas

$$\mathrm{SDet}\, G = \frac{\det G_+}{\det G_-} \equiv \mathbb{D}$$

Baxter polynomials

$$|\delta\rangle = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)^{\otimes L/2}, \qquad C^2 = \frac{\langle\delta|\mathbf{u}\rangle^2}{\langle\mathbf{u}|\mathbf{u}\rangle} = \frac{Q(0)}{Q(\frac{i}{2})}\,\mathrm{SDet}G$$

Pozsgay ’13
Brockman ‘14

# Integrable Super Spin Chains (of type $GL(M|N)$)

**Cartan matrix:** $M_{ab}$, **Dynkin labels** $q_a$, $a, b = 1, \ldots n$

Bethe equations

$$(-1)^{q_a} = \left( \frac{u_{a,j} - \frac{iq_a}{2}}{u_{a,j} + \frac{iq_a}{2}} \right)^L \prod_{b,k} \frac{u_{a,j} - u_{b,k} + \frac{iM_{ab}}{2}}{u_{a,j} - u_{b,k} - \frac{iM_{ab}}{2}} \equiv \mathrm{e}^{\,i\chi_{a,j}}\,.$$

$a = 1, \ldots n$ (# of nodes), $j = 1, \ldots, K_a$ (# of roots of type a)

Baxter polynomials: $Q_a(u) = \prod_{j=1}^{K_a} (u - u_{a,j})$, $\quad a = 1, \ldots n$

Gaudin matrix $\quad G_{aj,bk} = \dfrac{\partial \chi_{aj}}{\partial u_{bk}}$ $\quad$ of size $\sum_a K_a \times \sum_a K_a$

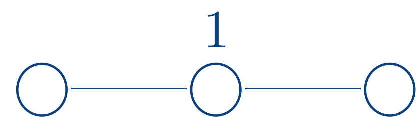
# Overlaps with $|\delta\rangle$-states from TBA

$SU(2): \quad |\delta\rangle = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)^{\otimes L/2}, \quad C^2 = \dfrac{Q(0)}{Q(\frac{i}{2})} S \det G$

Poszgay '18

$SO(6): \quad |\delta\rangle = (|XX\rangle + |YY\rangle + |ZZ\rangle + |\bar{X}\bar{X}\rangle + |\bar{Y}\bar{Y}\rangle + |\bar{Z}\bar{Z}\rangle)^{\otimes L/2}$ ,

$$C^2 = \frac{Q_1(0)Q_2(0)Q_3(0)}{Q_1\left(\frac{i}{2}\right)Q_2\left(\frac{i}{2}\right)Q_3\left(\frac{i}{2}\right)} S \det G$$

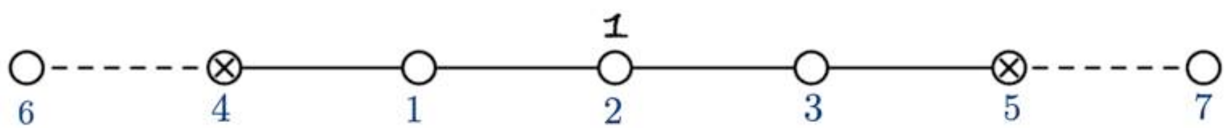



de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

Gombor '21

## AdS/CFT

$PSU(2,2|4)$ :




$$C^2 = \frac{Q_1(0)Q_3(0)Q_4(0)Q_5(0)Q_7(0)}{Q_2(0)Q_2(\frac{i}{2})Q_4(\frac{i}{2})Q_6(0)Q_6(\frac{i}{2})} S \det G$$

C.K., Zarembo '22

# From $|\delta\rangle$ to $|\mathrm{MPS}_k\rangle$ by dressing

Example for SU(2) chain

$$C_{|\mathrm{MPS}_k\rangle} = \sum_{a=-\frac{k-1}{2}}^{a=\frac{k-1}{2}} a^L \frac{Q\left(\frac{ik}{2}\right)Q\left(\frac{ik}{2}\right)}{Q\left((a-\frac{1}{2})i\right)Q\left((a-\frac{1}{2})\,i\right)} \sqrt{\frac{Q(0)Q\left(\frac{i}{2}\right)}{Q\left(\frac{ik}{2}\right)Q\left(\frac{ik}{2}\right)}\, S \det G}$$

Find a relation à la (systematic recursive strategy)

$$|\mathrm{MPS}_k\rangle = \hat{T}^{(k-1)}(\mathbf{u_{k-1}})|\delta\rangle + \alpha_1 \hat{T}^{(k-2)}(\mathbf{u_{k-2}})|\delta\rangle + \ldots$$

Transfer matrix

de Leeuw, Gombor, C.K., Linardopoulos, Pozsgay '19

Gombor, C.K., Qian '24

Take the inner product with eigenstate $|\mathbf{u}\rangle$

# More general integrability and pairing conditions

Gombor & Bajnok '20

## Generalized integrability condition

Factorizable $T$-matrix: $T(u) = T_+(u)T_-(u)$

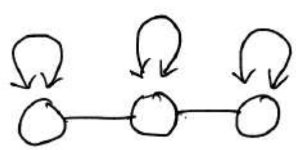
$$\langle\Psi_0|\Pi\, T_\pm(u)\,\Pi = \langle\Psi_0|T_\pm(u), \qquad \text{uncrossed}$$

$$\langle\Psi_0|\Pi\, T_\pm(u)\,\Pi = \langle\Psi_0|T_\mp(u), \qquad \text{crossed}$$

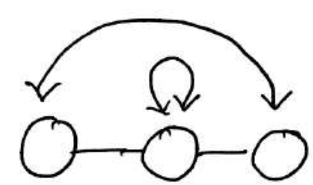
## Generalized pairing condition

$$\{u_{aj}\} = \{-u_{\sigma(a)j}\}, \qquad a = 1, \ldots, \#\text{ of nodes}$$

$\sigma(a) = \mathrm{Id}$, chiral overlap



$\sigma(a) \neq \mathrm{Id}$, achiral overlap



# Overlaps with $|\text{MPS}\rangle$ directly

Gombor '24

Solve KT-relation

$$\sum_{k,\gamma} K^{\alpha,\gamma}_{i,k}(u)\langle\Psi^0_{\gamma,\beta}|T_{kj}(u) = \sum_{k,\gamma}\langle\Psi^0_{\alpha,\gamma}|\widehat{T}_{ik}(-u)K^{\gamma\beta}_{kj}(u)$$

Overlap can be extracted from $K(u)$
by recursive procedure

Universal form

$$\frac{\langle \text{MPS}_{d_b}|\mathbf{u}\rangle}{\langle\mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}} = \left\{\sum_{k=1}^{d_b}\beta_k\prod_{j=1}^{n_+}F_j(u_j^+)\right\}\sqrt{\frac{\det G_+}{\det G_-}}$$

(Square roots of) Baxter polynomials

# What have we learnt ?

- Precise definition of an integrable quench —
  Concrete test method

- Overlap formulas in closed form
  contains info about post-quench behaviour
  e.g. time development of correlation functions, entanglement entropy...