

Excursions with Emmanuel into Foldings, Colorings, Loops and Paths

(Philippe DI FRANCESCO)

- Prehistory 1987
Students @ IPHT / Brasov Summer School
- On the peaks of sharing an office 1992-2001
Membrane Folding / Quantum Gravity / Meanders / Hamiltonian-cycles
- On the peaks of sharing a student 2002-2007
Maps and Trees
- Many happy returns 2018-2023
Limit shapes of 2D Path models / 2D Vertex

PREHISTORY

INTERNATIONAL SUMMER SCHOOL

CONFORMAL INVARIANCE AND STRING THEORY

September 1 - 12, 1987

in

Poiana Brasov, Romania

ON THE PERKS OF SHARING AN OFFICE

- More windows , larger blackboard
- More visitors

ON THE PERKS OF SHARING AN OFFICE

- More windows , larger blackboard
- More visitors
- Emmanuel

Membrane folding meets Integrable Lattice Models

EUROPHYSICS LETTERS

20 May 1994

Europhys. Lett., **26** (6), pp. 455-460 (1994)

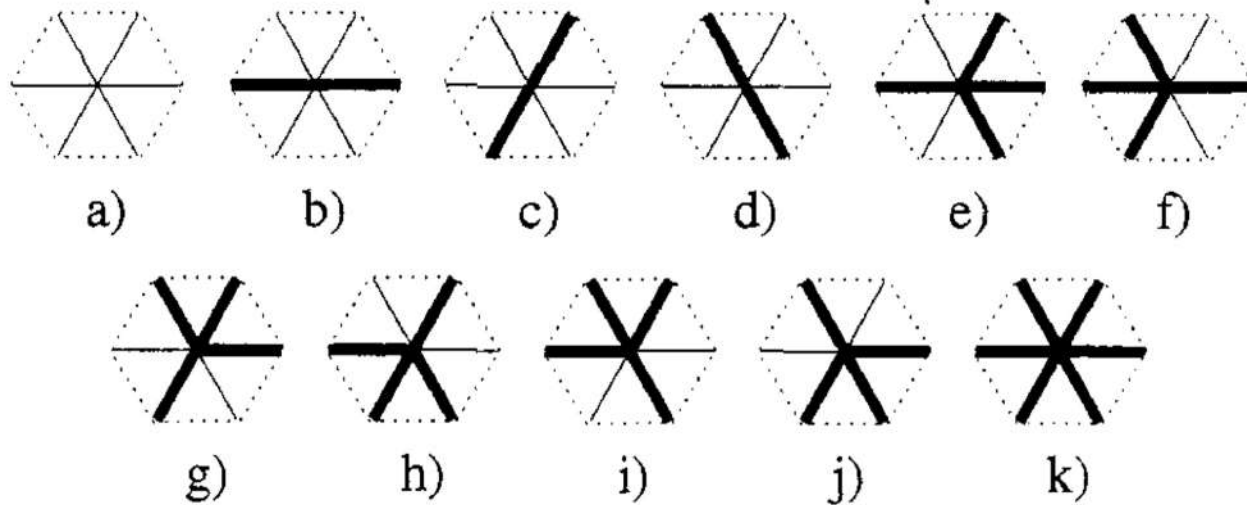
Entropy of Folding of the Triangular Lattice.

P. DI FRANCESCO and E. GUITTER

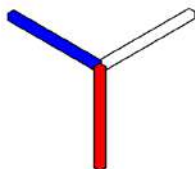
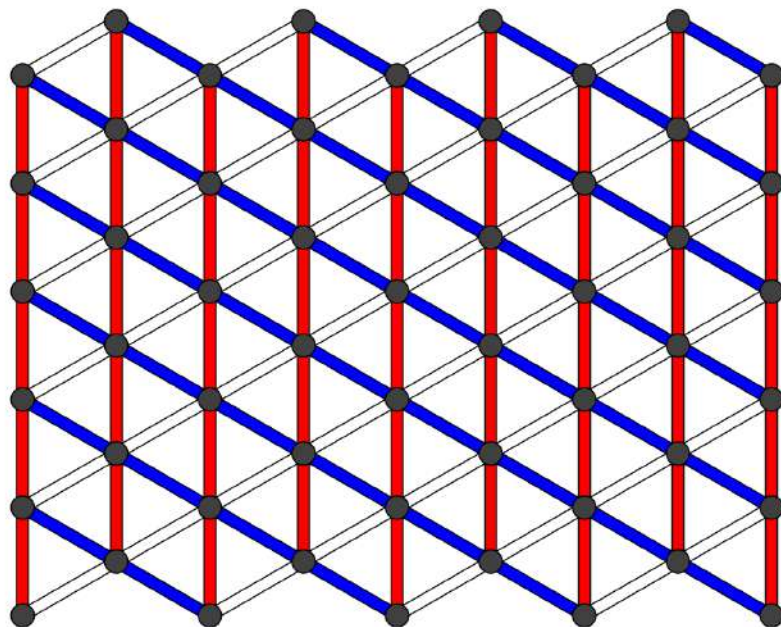
Service de Physique Théorique de Saclay() - F-91191 Gif-sur-Yvette Cedex, France*

(received 16 February 1994; accepted in final form 22 March 1994)

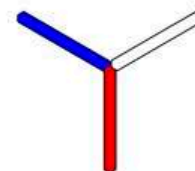
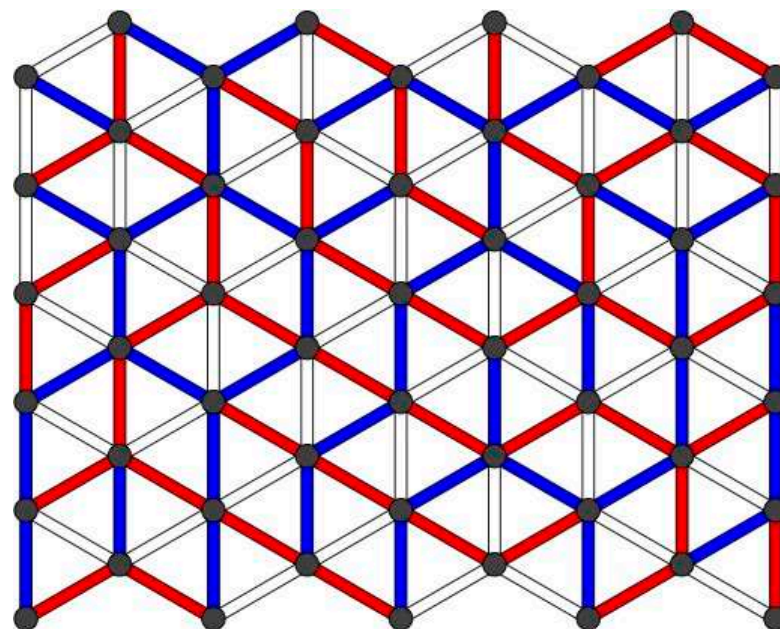
From 11-Vertex model



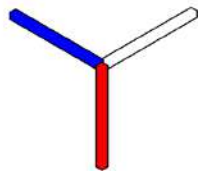
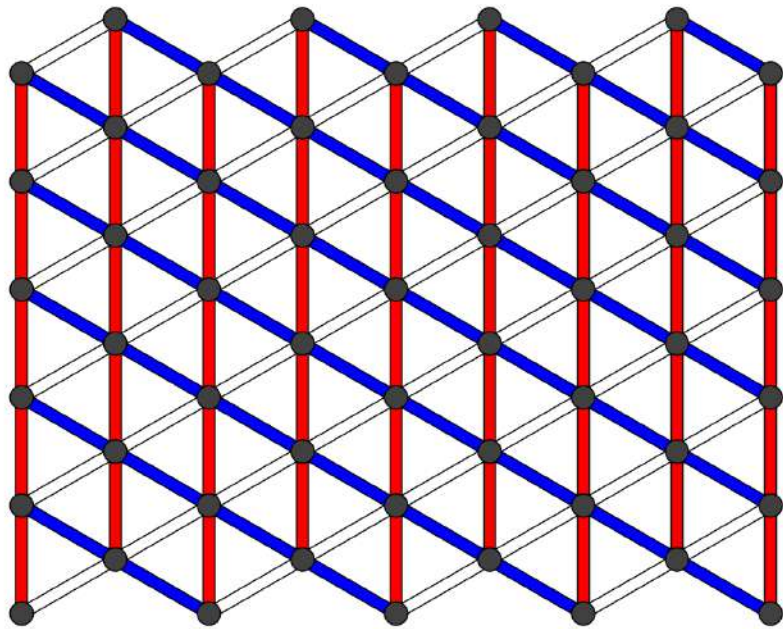
.....
To the bicolor model



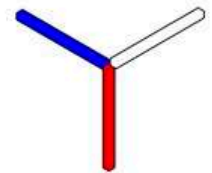
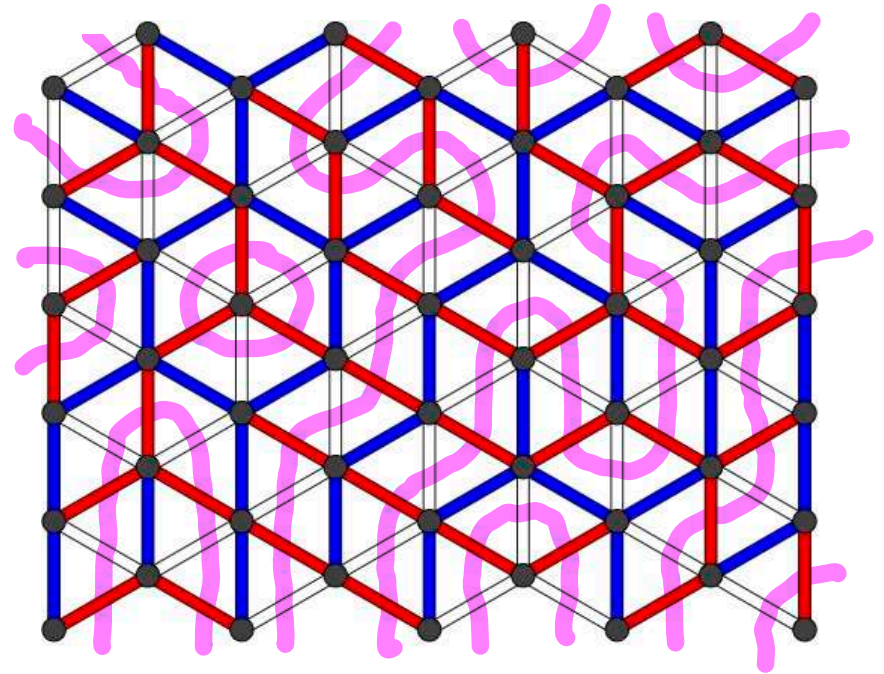
Flat sheet



Folded sheet



Flat sheet



Folded sheet

Fully-packed loop gas $O(n=2)$

[Baxter'70]

$$q = \prod_{n=1}^{\infty} \frac{(3n-1)}{\sqrt{3n(3n-2)}} = \frac{\sqrt{\Gamma(1/3)}}{\Gamma(2/3)} = \frac{\sqrt{3}}{2\pi} \Gamma(1/3)^{3/2}.$$



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Nuclear Physics B 450 [FS] (1995) 463–494

NUCLEAR
PHYSICS B [FS]

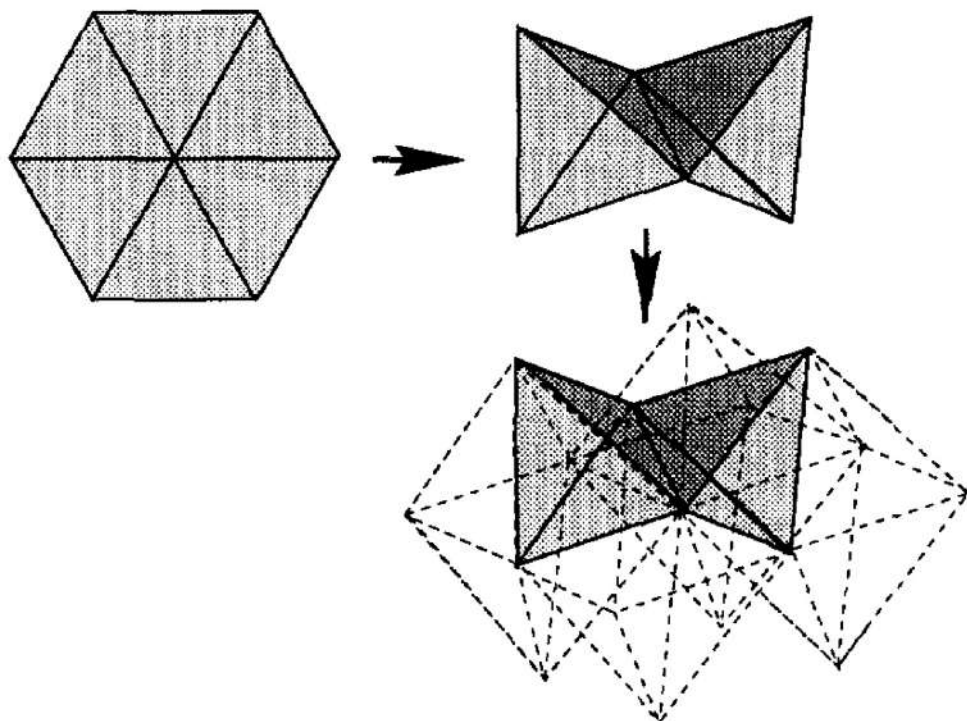
Three-dimensional folding of the triangular lattice

M. Bowick^a, P. Di Francesco^b, O. Golinelli^b, E. Guitter^b

^a *Physics Department, Syracuse University, Syracuse, NY 13244-1130, USA*

^b *CEA, Service de Physique Théorique de Saclay, F-91191 Gif sur Yvette Cedex, France*

Received 22 February 1995; revised 1 June 1995; accepted 2 June 1995



→ 96 vertex model!
No exact entropy but bounds

Folding transition of the triangular lattice

P. Di Francesco and E. Guitter

Service de Physique Théorique de Saclay, F-91191 Gif-sur-Yvette Cedex, France*

(Received 14 June 1994)

PHYSICAL REVIEW E

VOLUME 55, NUMBER 1

JANUARY 1997

J. Phys. A: Math. Gen. **30** (1997) L829–L838. Printed in the UK

LETTER TO THE EDITOR

Folding of the triangular lattice in the face-centred cubic lattice with quenched random spontaneous curvature

S Mori†§ and E Guitter‡||

† Department of Physics, School of Science, Kitasato University, Kitasato 1-15-1, Sagamihara, Kanagawa 228, Japan

‡ CEA, Service de Physique Théorique de Saclay, F-91191 Gif sur Yvette Cedex, France

Received 29 August 1997

Folding of the triangular lattice with quenched random bending rigidity

P. Di Francesco, E. Guitter, and S. Mori*

Service de Physique Théorique, C.E.A. Saclay, F-91191 Gif-sur-Yvette Cedex, France

(Received 19 July 1996)



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Nuclear Physics B 495 [FS] (1997) 583–607

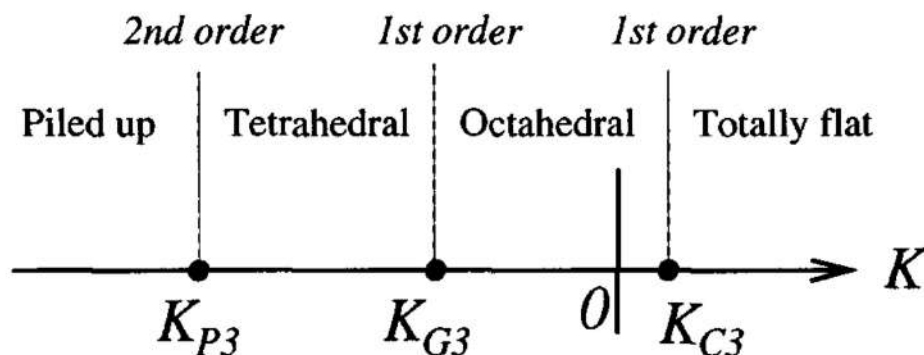
Geometrical folding transitions of the triangular lattice in the face-centred cubic lattice

M. Bowick^{a,1}, O. Golinelli^{b,2}, E. Guitter^{b,3}, S. Mori^{c,4}

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^c Department of Physics, Graduate School of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan

Received 20 November 1996; accepted 13 March 1997



2D, 3D, What about 1 D?



Nuclear Physics B 482 [FS] (1996) 497-535

NUCLEAR
PHYSICS B [FS]

Meanders: a direct enumeration approach

P. Di Francesco¹, O. Golinelli², E. Guitter³

Service de Physique Théorique, C.E.A. Saclay, F-91191 Gif sur Yvette, France

Received 1 July 1996; accepted 3 September 1996



Pergamon

Mathl. Comput. Modelling Vol. 26, No. 8-10, pp. 97-147, 1997

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PII: S0895-7177(97)00202-1

Meander, Folding, and Arch Statistics

P. DI FRANCESCO, O. GOLINELLI AND E. GUITTER

Service de Physique Théorique de Saclay

F-91191 Gif sur Yvette Cedex, France

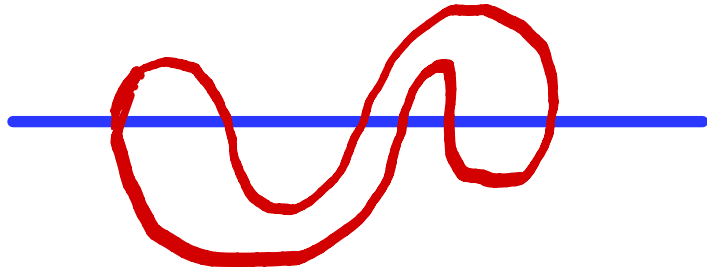
<philippe><golinelli><gutter>@amoco.saclay.cea.fr

"Le problème des timbres-poste"

[Edouard Lucas 1891, citing Emile Lemoine]

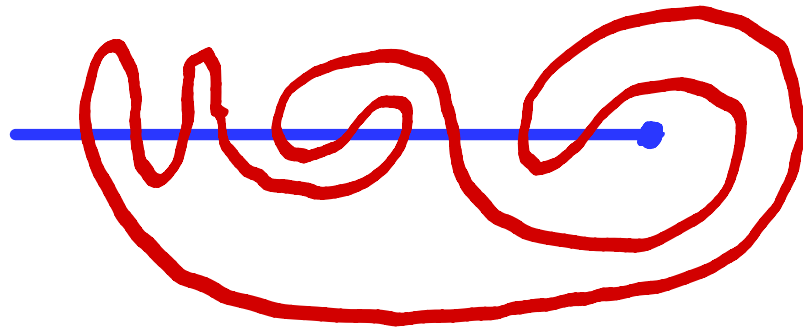
[Poincaré 1912] [Arnold 1991] [Lando-Zvonkin 1992-1993]

Self avoiding folding of a 1-D polymer.



closed meander

= meander M_n



open meander

= semi-meander $\tilde{M}_n$

Questions:

- count these for fixed n
- asymptotics? universality classes?

COUNTING : Golirelli's algorithm

```

PARAMETER (nmax = 14)
INTEGER A(-nmax+1:nmax)
INTEGER Sm(nmax)
INTEGER n
INTEGER j
DATA n, Sm/0, nmax*0/
A(0) = 1
A(1) = 0
2 n = n + 1
  Sm(n) = Sm(n) + 1
  j = -n + 1
1 IF ((n.EQ.nmax).OR.(j.EQ.n+1)) GOTO 3
  A(A(j)) = n+1
  A(n+1) = A(j)
  A(j) = -n
  A(-n) = j
  GOTO 2
3 A(A(-n+1)) = A(n)
  A(A(n)) = A(-n+1)
  j = A(n)+1
  n = n - 1
  IF (n .GT. 1) GOTO 1
PRINT '(i3, i15)', (n, Sm(n), n = 1, nmax)
END

```

! maximal order
! arch representation
! semi-meander counter
! current depth (or order)
! next branch to visit
! n and Sm initialized to 0
! single-arch semi-meander

! a new node is visited

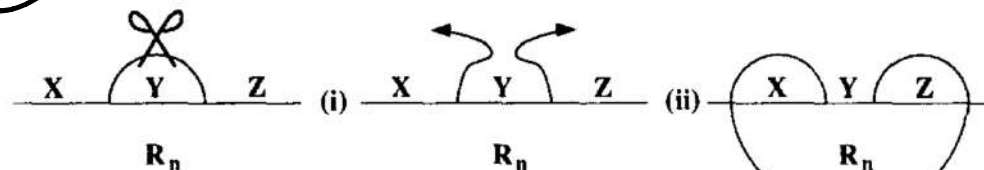
! leftmost (exterior) arch
! up or down?
! go down with process (I)

! going up

! next arch to break

2 Moves :

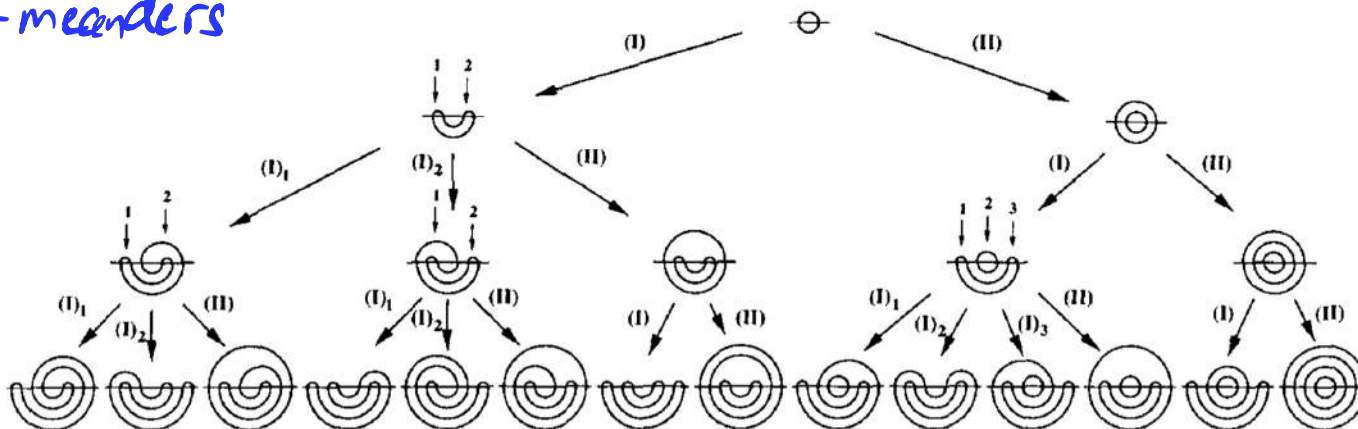
(I)



(II)



Tree of semi-meanders



ALTERNATIVE DESCRIPTIONS

- Matrix Integrals
- Temperley-Lieb Algebra

Commun. Math. Phys. 186, 1–59 (1997)

Communications in
Mathematical
Physics
© Springer-Verlag 1997

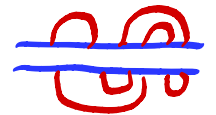
Meanders and the Temperley-Lieb Algebra

P. Di Francesco, O. Golinelli, E. Guitten*

Service de Physique Théorique, C.E.A. Saclay, F-91191 Gif sur Yvette Cedex, France

Received: 25 February 1996 / Accepted: 12 August 1996

Meander determinant



$$\left(\begin{array}{cc} \text{meander} & \text{meander} \\ \text{meander} & \text{meander} \end{array} \right) \rightarrow \begin{pmatrix} q^2 & q \\ q & q^2 \end{pmatrix} \rightarrow q^2(q^2 - 1)$$

$$D_n(q) = \det(\mathcal{G}_n(q)) = \prod_{i=1}^n U_i(q)^{a_{n,i}}$$

$$a_{n,i} = \binom{2n}{n-i} - 2\binom{2n}{n-i-1} + \binom{2n}{n-i-2}$$

(U_i = Chebyshev
of 2nd kind)

à suivre...

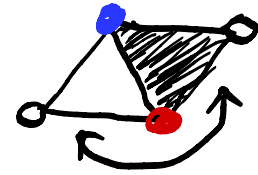
QUANTUM GRAVITY: NEW DIRECTIONS

(EPISODE I)

Vertex - Tricoloring

of Random Triangulations

⇒ compactly foldable →



Coloring random triangulations

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Received 12 November 1997; accepted 30 December 1997

$$Z = \sum_{\text{triangulations } \Delta} N^{2-2g} p^{V_R} q^{V_b} z^{V_w} t^N / \text{Aut}(\Delta)$$

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$
 genus(Δ) # RBW vertices # edges of Δ

• planar free energy (matrix model)

$$t \frac{\partial}{\partial t} \log Z = U_1 U_2 U_3 (1 - U_1 - U_2 - U_3)$$

$$U_1 (1 - U_2 - U_3) = p t$$

$$U_2 (1 - U_1 - U_3) = q t$$

$$U_3 (1 - U_1 - U_2) = r t$$

(uses rectangular
matrix
integrals)

p, q = potential

$r = N/M$

(EPISODE 2)



Nuclear Physics B 546 [FS] (1999) 731–750

NUCLEAR
PHYSICS B

Hamiltonian cycles on random Eulerian triangulations

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^a CEA-Saclay, Service de Physique Théorique, F-91191 Gif-sur-Yvette Cedex, France

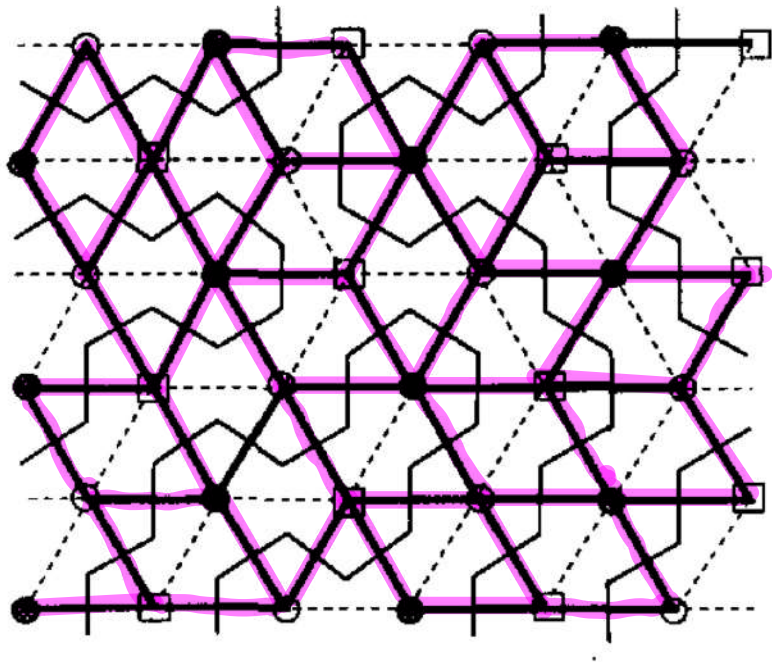
^b The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 23 November 1998; revised 15 January 1999; accepted 28 January 1999

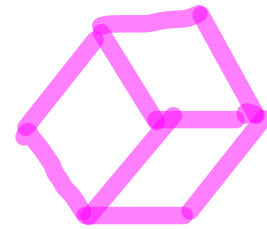
Eulerian triangulation =
face - bicolourable
= foldable!



- dream = solve $FPL(n=2)$ = folding of random E.T.
- reality = predictions on $FPL(n=0)$ = 1 Hamiltonian cycle visiting all triangles exactly once.
- First = understand the physics of the $FPL(n)$ model on a (flat) lattice



 Fully-Packed
Loops



Dual
Rhombus Tiling

$\Rightarrow$ extra height (1 more scalar field!)

Conformal theory description :

$$C_{FPL} = C_{\text{Dense}} + 1$$

[cf Blöte Nienhuis]

$\nearrow$
dense loop
model

$\uparrow$
height-
degree of freedom

• Next = switch on quantum gravity

KPZ formula = relates flat space scaling to random surface scaling.

(Based on the coupling of metric (Liouville) field to conformal (matter) field.)

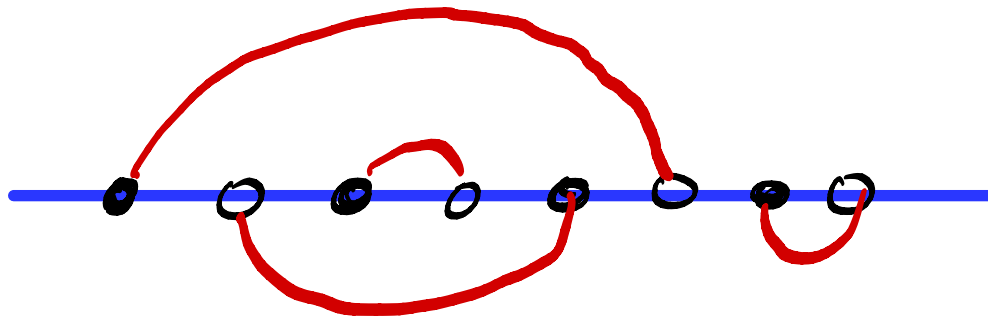
$$Z_{\substack{\text{random } E\Delta \\ \text{FPL}(n=0)}}^{N \text{ triangles}} \sim \frac{e^{\mu N}}{N^{\gamma-3}} ; \gamma = \frac{c-1 - \sqrt{(1-c)(25-c)}}{12}$$

$$\begin{aligned} c_{\text{Dense}} &= -2 \\ c_{\text{FPL}} &= -1 \end{aligned}$$

$\Rightarrow$

$$\gamma = \frac{-1 - \sqrt{13}}{6}$$

SOME FUN COMBINATORICS



← The Hamiltonian cycle
on the dual of Δ
 $= \phi^3$ graph.

(Note) = if we relax vertex bicolouration we kill
the extra degree of freedom $\Rightarrow C = C_{\text{Dense}} = -2$
 $\gamma = -1$

$$Z_{2n} = \sum_{m=0}^n \binom{2n}{2m} C_m C_{n-m} = C_n C_{n+1} \sim \frac{1}{\pi} \frac{4^{2n}}{n^3} \leftarrow 2-\gamma$$

Catalan numbers

MANY HAPPY RETURNS



Nuclear Physics B 549 [FS] (1999) 657–667

www.elsevier.nl/locate/npe



Fully packed $O(n = 1)$ model on random Eulerian triangulations

P. Di Francesco^{a,1}, E. Guitter^{a,2}, C. Kristjansen^{b,3}

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Received 24 February 1999; accepted 19 March 1999

$\Delta t = 24 \text{ years!}$



Available online at www.sciencedirect.com

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Nuclear Physics B 995 (2023) 116335

www.elsevier.com/locate/nuclphysb



Quantum Field Theory and Statistical Systems

Hamiltonian cycles on bicolored random planar maps

Bertrand Duplantier, Olivier Golinelli, Emmanuel Guitter^{*}

Université Paris-Saclay, CEA, CNRS, Institut de Physique Théorique, 91191 Gif-sur-Yvette, France

Received 12 May 2023; accepted 23 August 2023

Available online 29 August 2023

Editor: Hubert Saleur



Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 987 (2023) 116084

www.elsevier.com/locate/nuclphysb



Exponents for Hamiltonian paths on random bicubic maps and KPZ

Philippe Di Francesco^{a,b}, Bertrand Duplantier^a, Olivier Golinelli^a,
Emmanuel Guitter^{a,*}

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^b Department of Mathematics, University of Illinois, Urbana, IL 61821, USA

Received 26 October 2022; accepted 9 January 2023

Available online 13 January 2023

Editor: Hubert Saleur

new subtlety: KPZ

must be tweaked!

= different "stiffness" for the two fields. (by looking at operators) -

(EPISODE 3)

MEANDERS: THE END



Nuclear Physics B 570 [FS] (2000) 699–712



www.elsevier.nl/locate/npe

Meanders: exact asymptotics

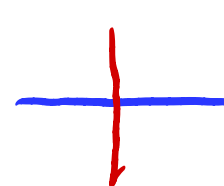
P. Di Francesco, O. Golinelli, E. Guitter

Service de Physique Théorique, C.E.A. Saclay, F-91191 Gif sur Yvette, France

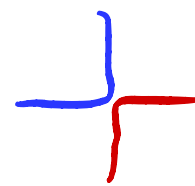
Received 5 November 1999; received in revised form 22 November 1999; accepted 30 November 1999

2-loop FPL ($n_1=n_2=0$) on
a random quadrangulation

2 types of
vertices



crossing



tangent

Dense

Dense

extra

$$c = (-2) + (-2) + 1 = -3$$

dual height $\in \mathbb{R}^2 = \langle A, B, C \rangle$

• Eulerian?
(vertex-bicolored)

if yes

• But not: $c = -4$ (no extra d.f.) but still 2 types of vertices

Field description:

$$\mathbb{R}^2 = \langle A, B \rangle$$

dual height $\in \mathbb{R}^2$ and tangent vertex irrelevant!

$$\begin{array}{c|c} h+A & h+A-B \\ \hline h & h-B \end{array}$$

$$\begin{array}{c|c} h+A & h \\ \hline h & \sqrt{h-B} \end{array}$$

Conclusion: $c = -4 \xrightarrow{\text{KPZ}} \gamma = \frac{5 - \sqrt{145}}{12}$.

$$M_n \sim e^{\mu n} / n^\alpha ; \quad \alpha = \frac{29 + \sqrt{145}}{12}$$

meander exponent



Nuclear Physics B 580 [FS] (2000) 757–795



Exact meander asymptotics: a numerical check

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^b LPTMS, bâtiment 100, Université Paris-Sud, F-91405 Orsay, France

Received 3 March 2000; accepted 1 May 2000

• numerical evidence

(third digit $\alpha = 3.420$ [7])

(also I. Jensen, T. Guttman)

• more exponents!

associated w/ multisources/sinks

LORENTZIAN GRAVITY



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Nuclear Physics B 567 [FS] (2000) 515–553



www.elsevier.nl/locate/npe

Integrable 2D Lorentzian gravity and random walks

P. Di Francesco^{a,1}, E. Guitter^a, C. Kristjansen^{b,2}

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^b The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 14 July 1999; accepted 11 October 1999

- time-ordered random $1+1$ D surfaces = trees!
- Integrability (commuting transfer matrices).



ELSEVIER

Nuclear Physics B 608 [FS] (2001) 485–526



www.elsevier.com/locate/npe

Generalized Lorentzian triangulations and the Calogero Hamiltonian

P. Di Francesco^a, E. Guitter^a, C. Kristjansen^{b,1}

^a CEA-Saclay, Service de Physique Théorique, F-91191 Gif sur Yvette cedex, France

^b The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received 31 October 2000; accepted 14 May 2001

INSTITUTE OF PHYSICS PUBLISHING

JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 897–927

PII: S0305-4470(02)24705-8

Critical and multicritical semi-random $(1 + d)$ -dimensional lattices and hard objects in d dimensions

P Di Francesco and E Guitter

Vicnot's heaps of pieces

TO SUMMARIZE:



Physics Reports
Volume 415, Issue 1, August 2005, Pages 1-88



Geometrically constrained statistical systems on regular and random lattices: From folding to meanders

P. Di Francesco  , E. Guitter 

ON THE PERKS OF SHARING A STUDENT

(especially if it's Jérémie!)

↳ see his talk...



- a new subject
- 11 papers
- as of last week, a brand new IHP director!

LIMIT SHAPES

Non-intersecting lattice paths (NILP) w/ fixed ends

Arctic curves for paths with arbitrary starting points: a tangent method approach

Philippe Di Francesco^{1,2} and Emmanuel Guitter¹

¹ Institut de physique théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France

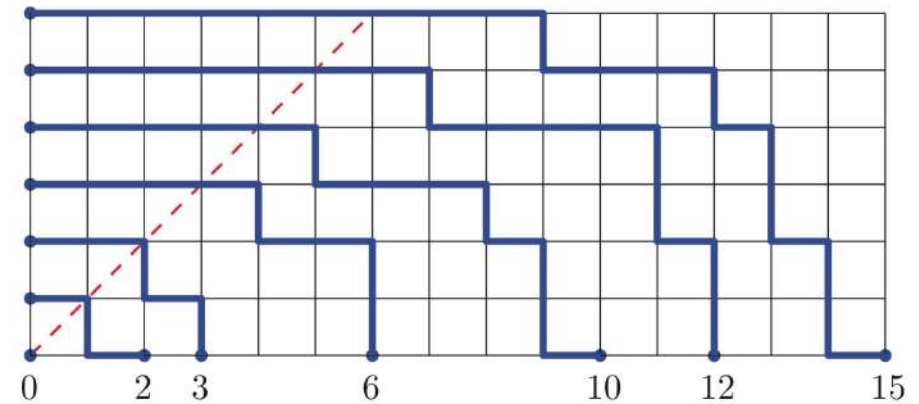
² Department of Mathematics, University of Illinois, Urbana, IL 61821, United States of America

E-mail: philippe@illinois.edu and emmanuel.guitter@ipht.fr

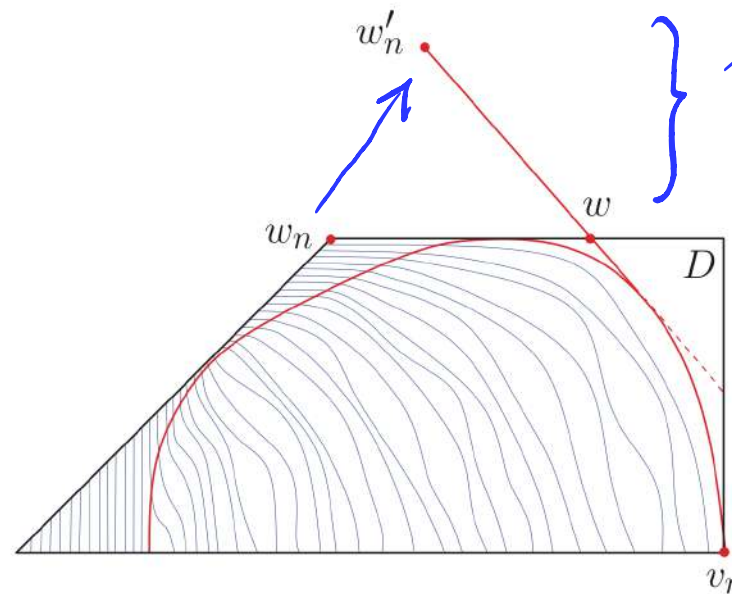
Received 5 April 2018, revised 15 June 2018

Accepted for publication 29 June 2018

Published 20 July 2018



fixed ends $\rightarrow$ distribution



1 single path
(geodesic)

exact

one-point
function w/ exit at w

The tangent method
(Colomo-Spottiello)

A tangent method derivation of the arctic curve for q -weighted paths with arbitrary starting points

Philippe Di Francesco^{1,2} and Emmanuel Guitter²

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E-mail: philippe@illinois.edu and emmanuel.guitter@ipht.fr

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Published 19 February 2019



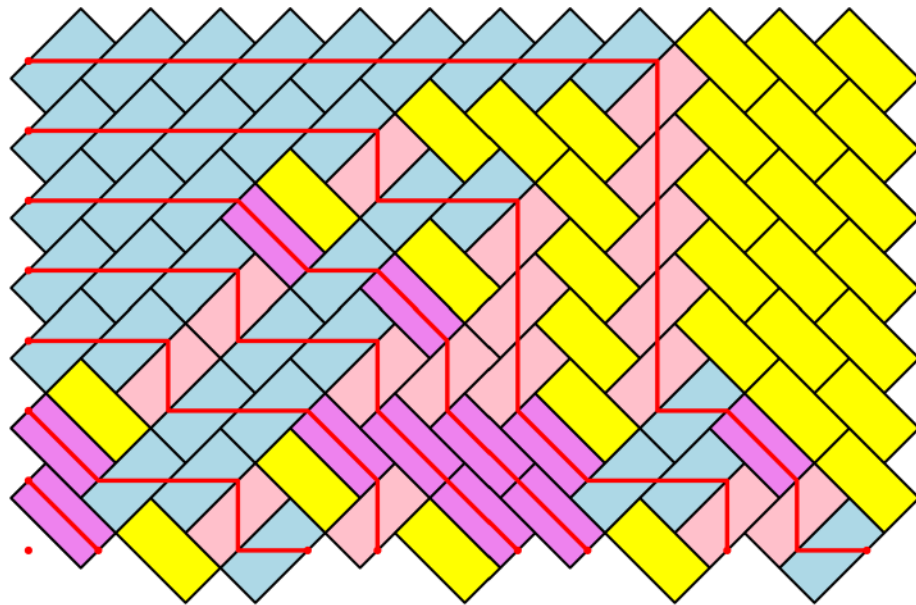
Journal of Statistical Physics (2019) 176:639–678
<https://doi.org/10.1007/s10955-019-02315-2>

The Arctic Curve for Aztec Rectangles with Defects via the Tangent Method

Philippe Di Francesco^{1,2} · Emmanuel Guitter²

Received: 5 March 2019 / Accepted: 15 May 2019 / Published online: 25 May 2019

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The tangent method applies but single path follows geodesics in curved background.

In all cases = arctic curve analytic for arbitrary end distributions (so-called free-fermion case)

Twenty Vertex Model

Twenty-Vertex model with Domain Wall Boundaries and Domino tilings

Philippe Di Francesco^{1,2 *} Emmanuel Guitter^{2 †}

1. Department of Mathematics
University of Illinois
Urbana, IL 61821, U.S.A.

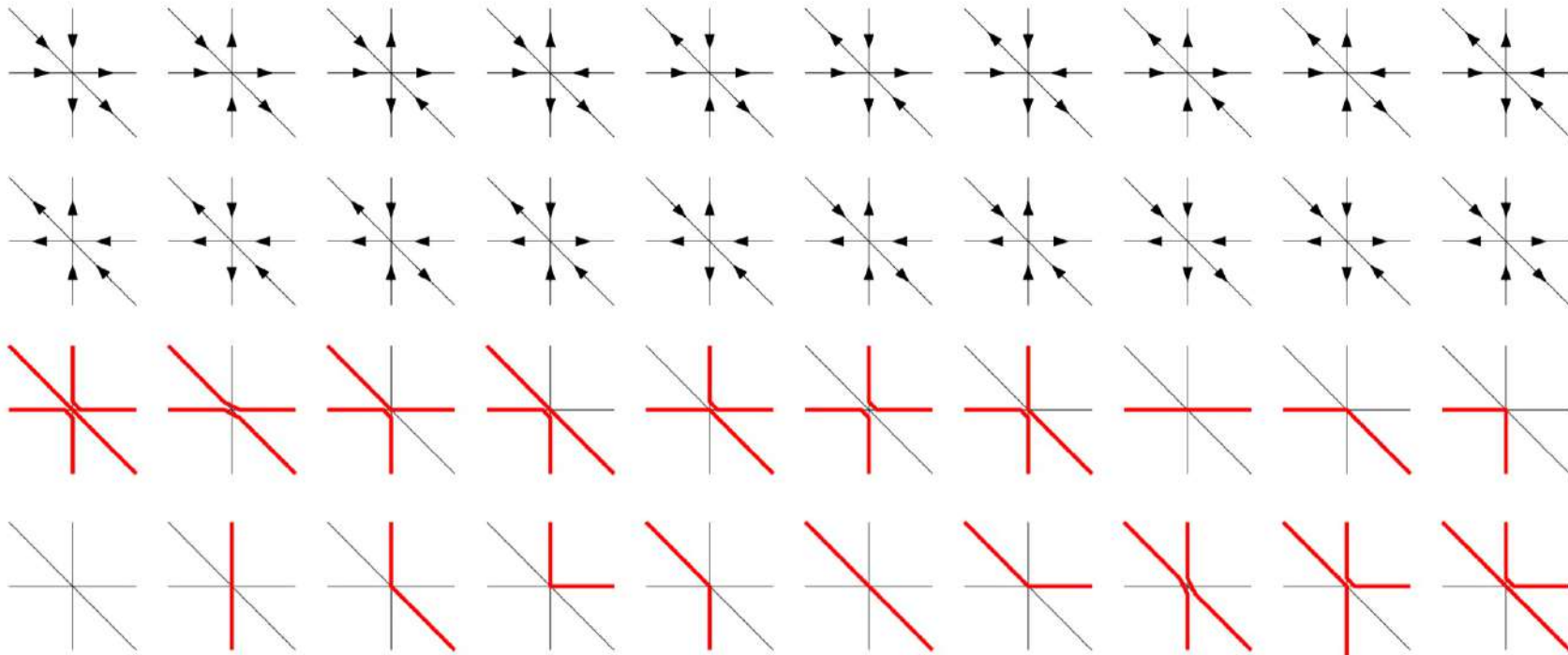
2. Université Paris Saclay, CNRS, CEA
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philippe@illinois.edu emmanuel.gutter@ipht.fr

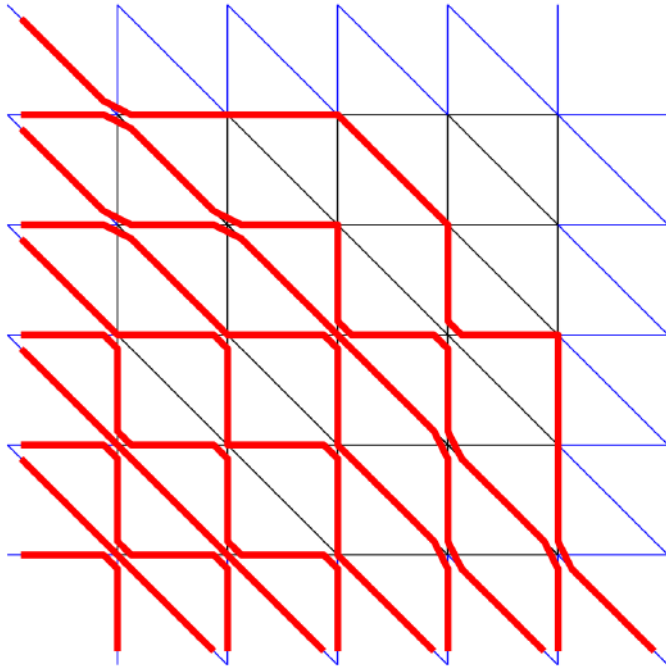
Submitted: Jun 20, 2019; Accepted: Mar 28, 2020; Published: May 1, 2020

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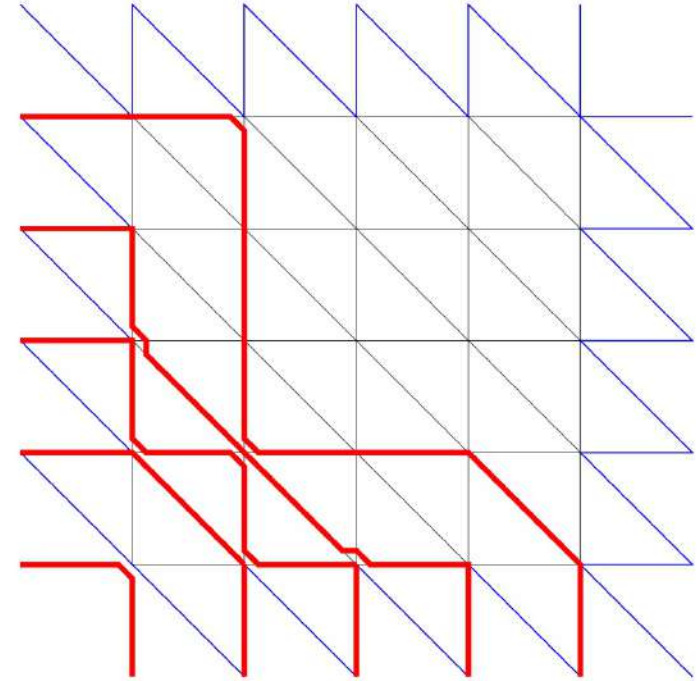
- ice model (like 6V) on triangular lattice
= osculating paths
- integrable weights from Kagome
(3 parameters)



DOMAIN WALL BOUNDARY CONDITIONS



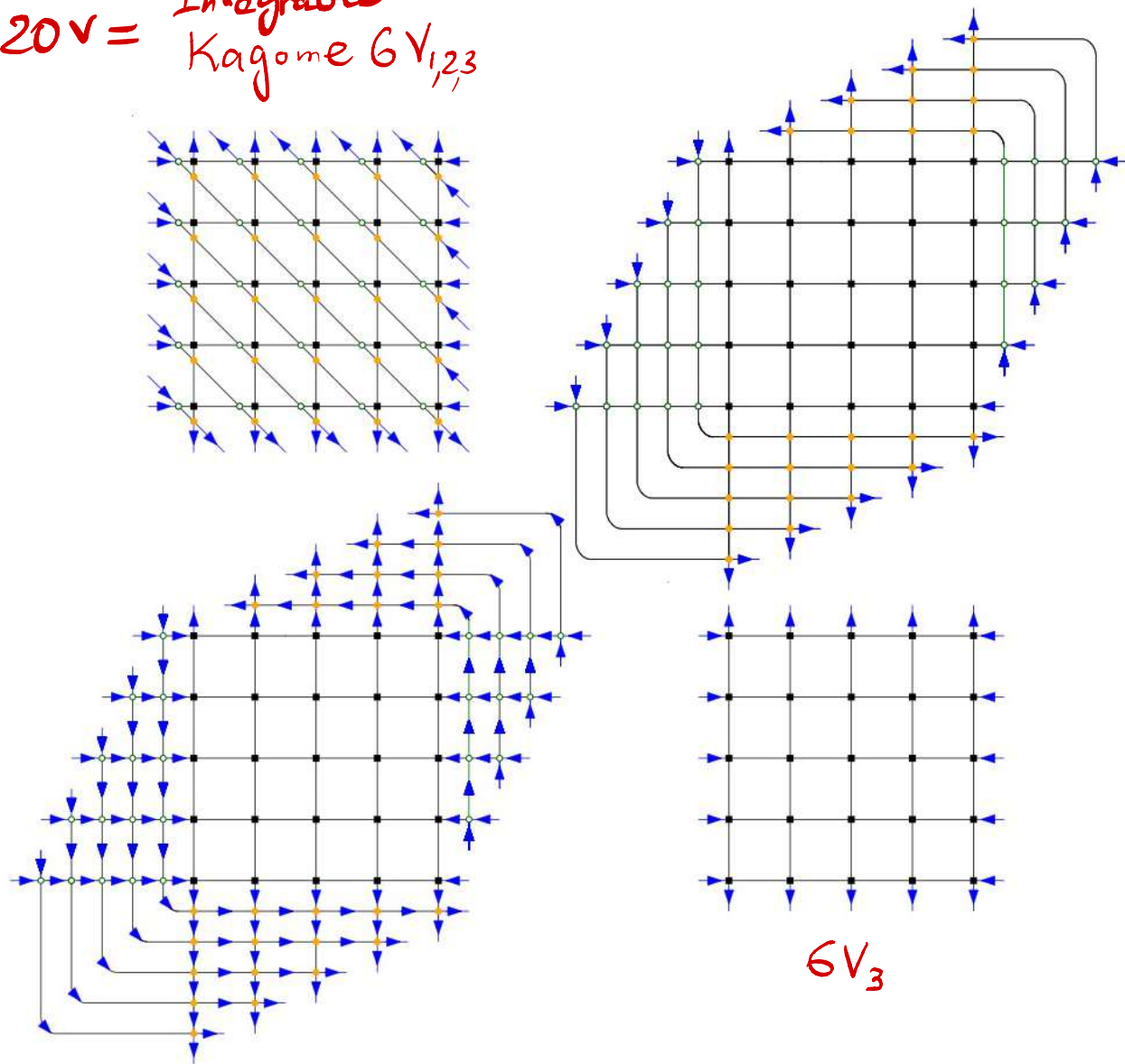
DWB1: 1, 3, 23, 433, 19705, ...



DWB3: 1, 3, 29, 901, 89893, ...

- enumeration of 2D DWBC1 configurations
by "mapping" to the 6V model

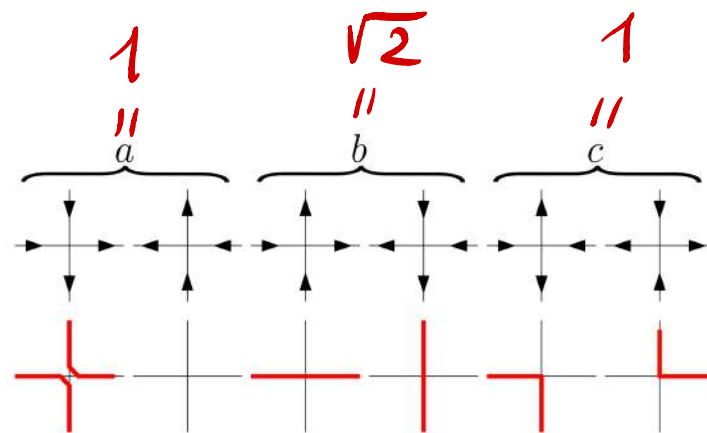
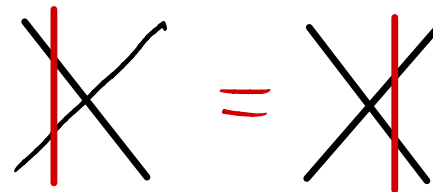
2DV = Integrable
Kagome $6V_{1,2,3}$



$6V_3$

uniform 2DV $\rightarrow$ $6V$ w/weights

Yang-Baxter:





A lattice of KAGOME (Daikokuya, Kitashirakawa)



A lattice of KAGOME (Daikokuya

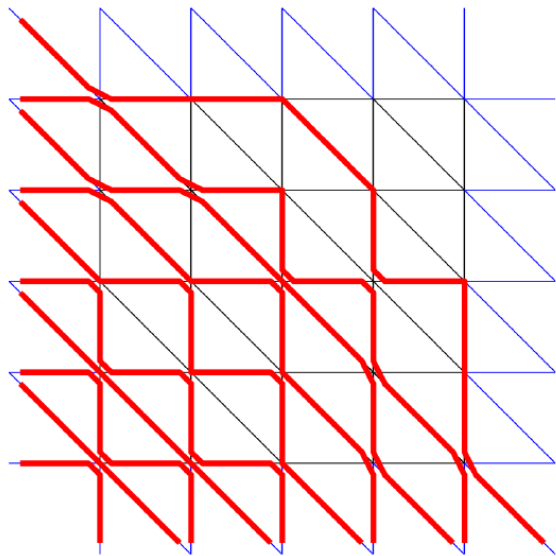




A lattice of KAGOME (Daikokuya

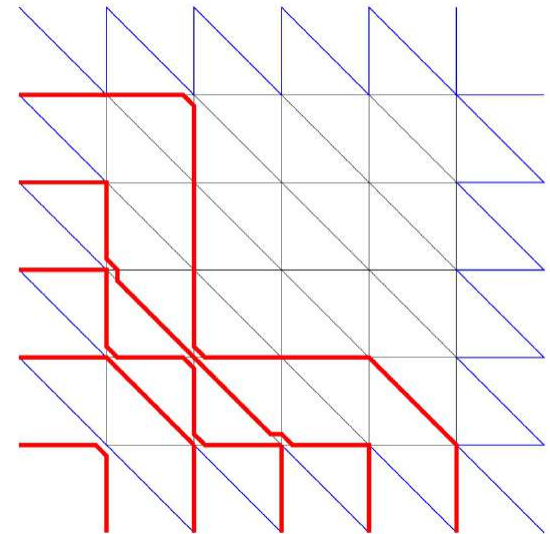


NON-BIJECTIONS

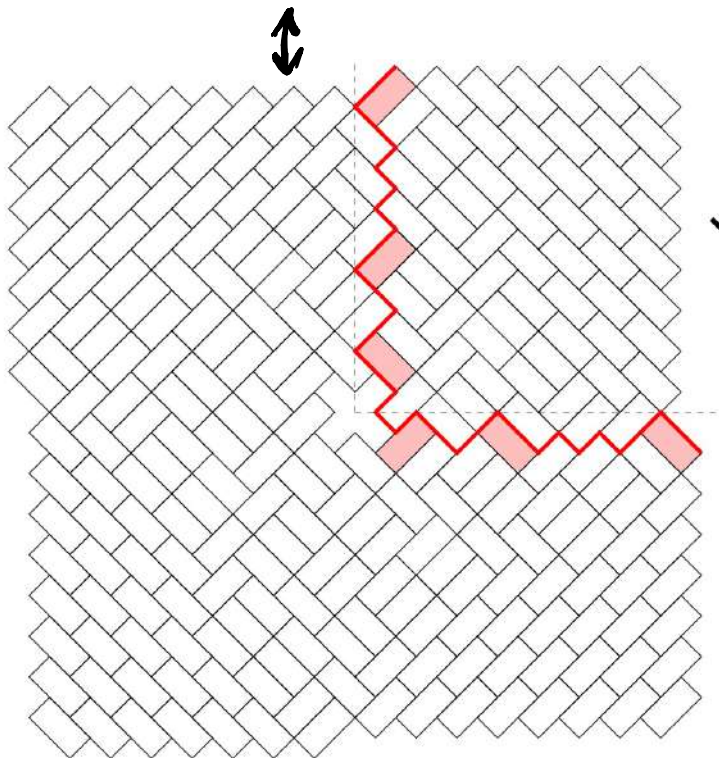


← 20V
DWBC 1

20V
DWBC 3 →

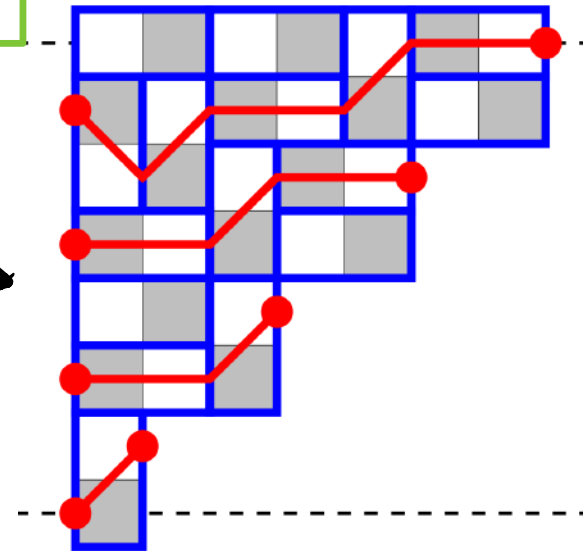


↕ conjecture!



← QTSHADT

Pachter triangle
demo tiling →



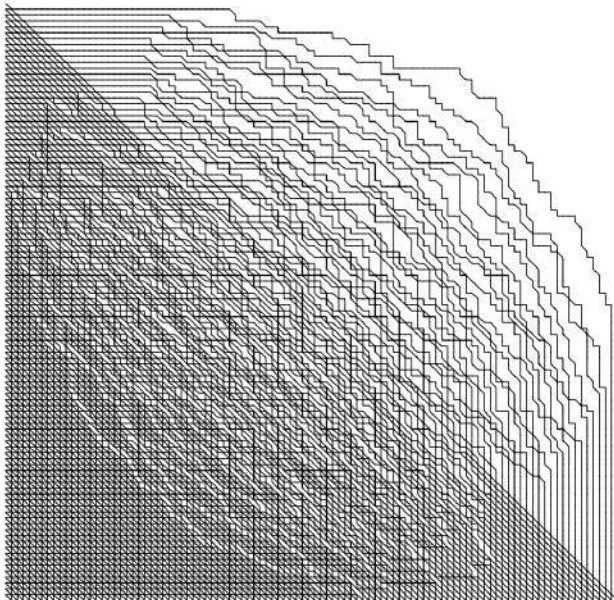
ARCTIC CURVES

Arctic Curves of the Twenty-Vertex Model with Domain Wall Boundaries

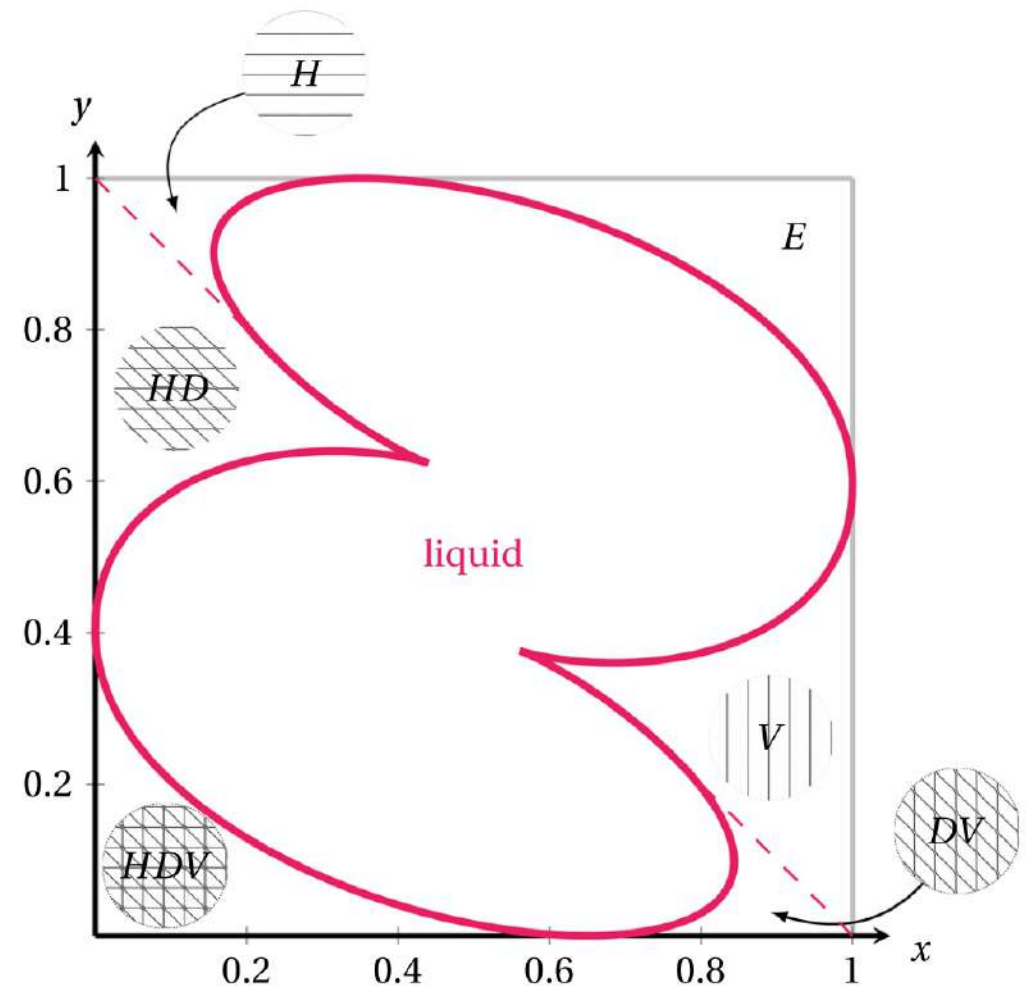
Bryan Debin¹ · Philippe Di Francesco^{2,3} · Emmanuel Guitter³ 

Received: 18 October 2019 / Accepted: 2 March 2020 / Published online: 16 March 2020
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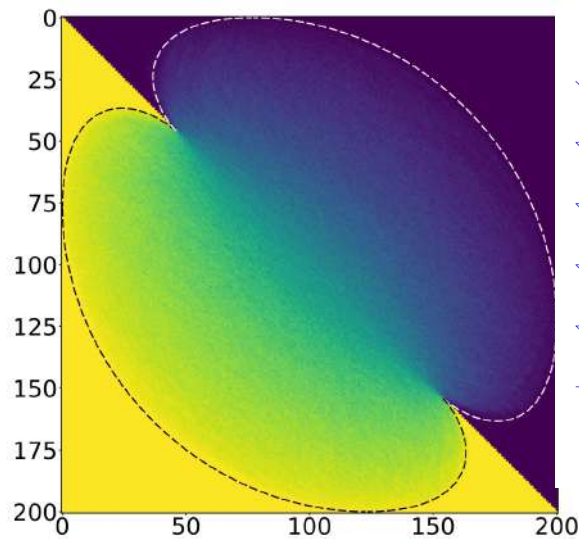
- use tangent method
+ mapping to the $6V$
+ refinements.



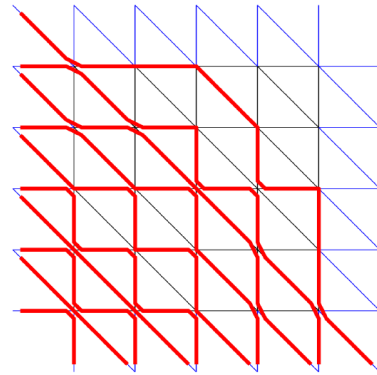
uniform weights



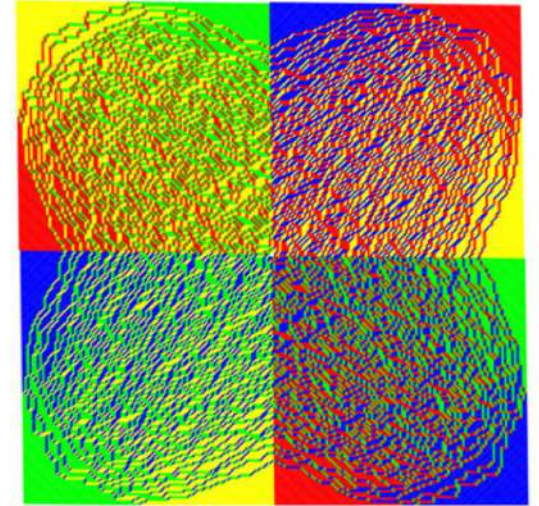
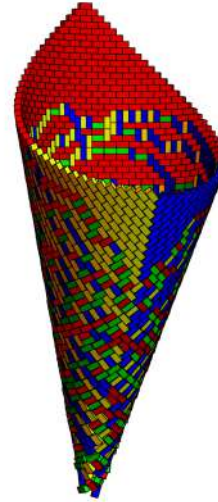
UNIFORM CASE : AN "ASYMPTOTIC" BIJECTION



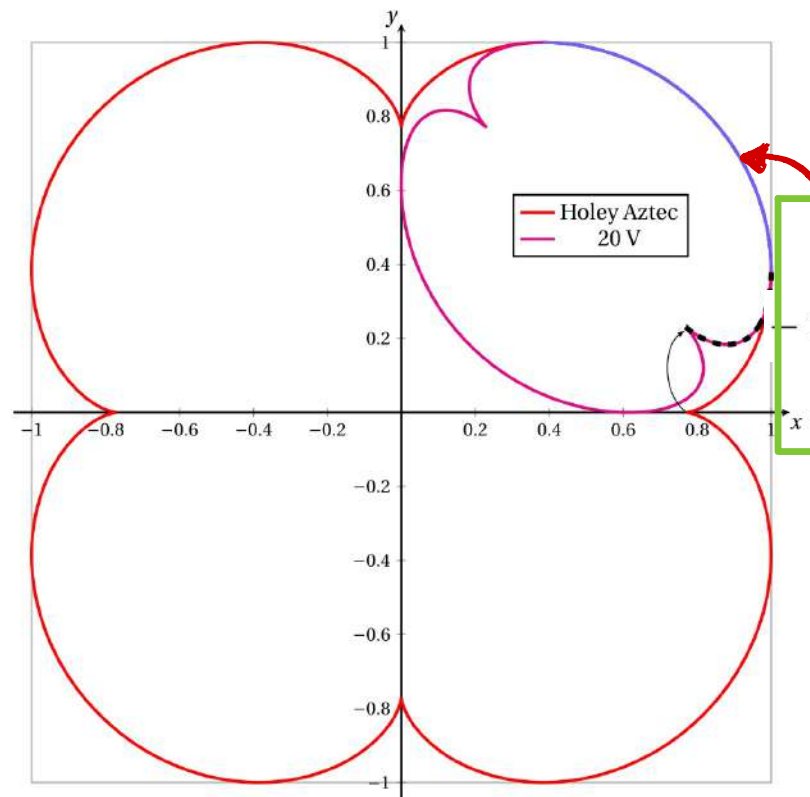
20V+DNBC 1



VS



QSHADT

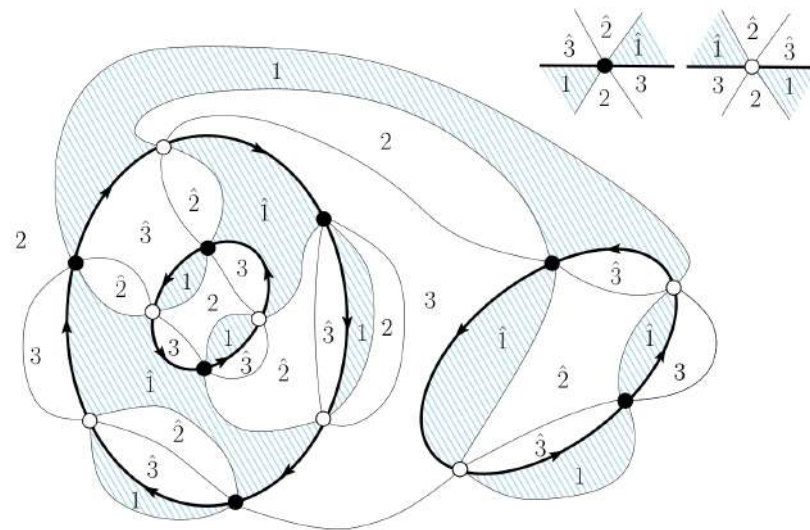
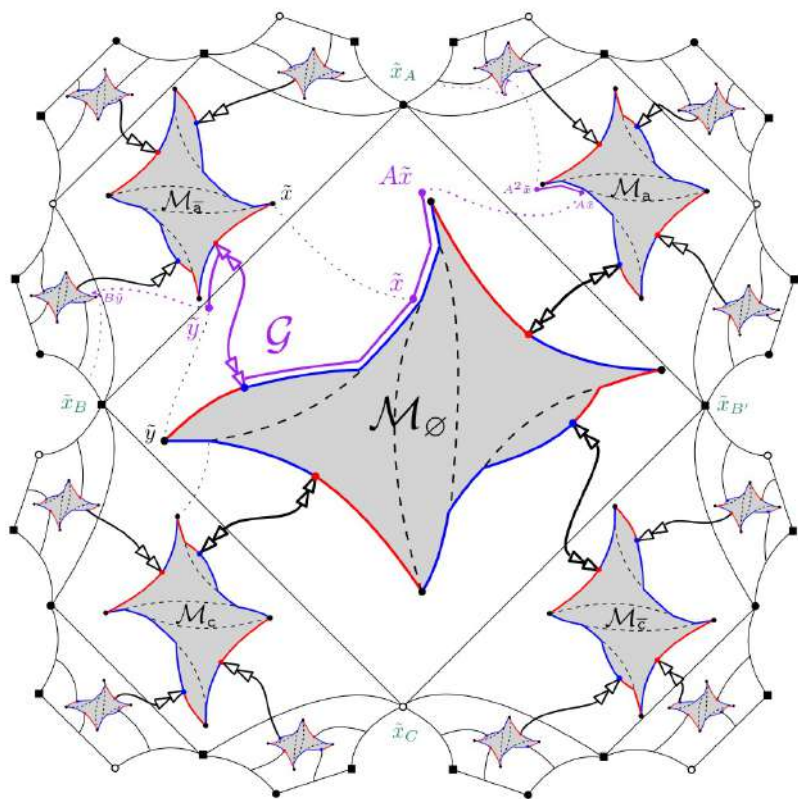
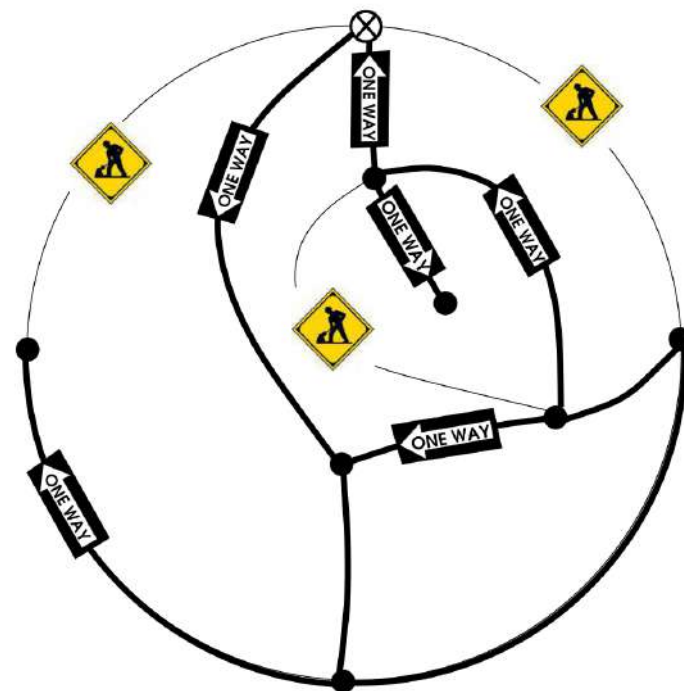
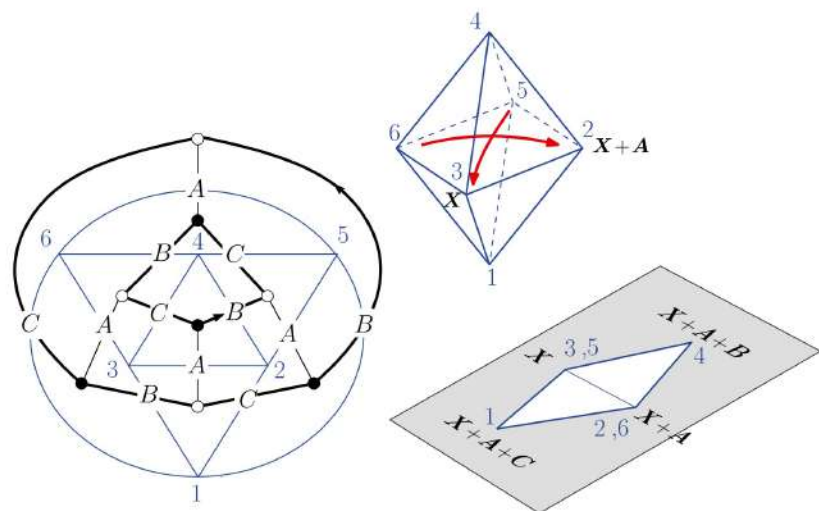


$$3^{11}(x^2 + y^2)^5 + 3^9 10(x^2 + y^2)^4 - 3^6 5(x^2 + y^2)^3 + 6^2 20(73(x^2 + y^2)^2 - 5^4 x^2 y^2) - 2^8 15(x^2 + y^2) - 2^{12} = 0$$

same portion of
arctic curve!

THE MANY TALENTS OF EMMANUEL

- A master of bijection / reformulation
- A fantastic intuition for field theory descriptions
- A true physicist / always asks the right questions
- A meticulous (and unstoppable) calculator
- An artist (see next)





32 years ago...

Bon Anniversaire
Emmanuel !



June 2024