



Tracking with Hashing



27th November 2024



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Known high-level features

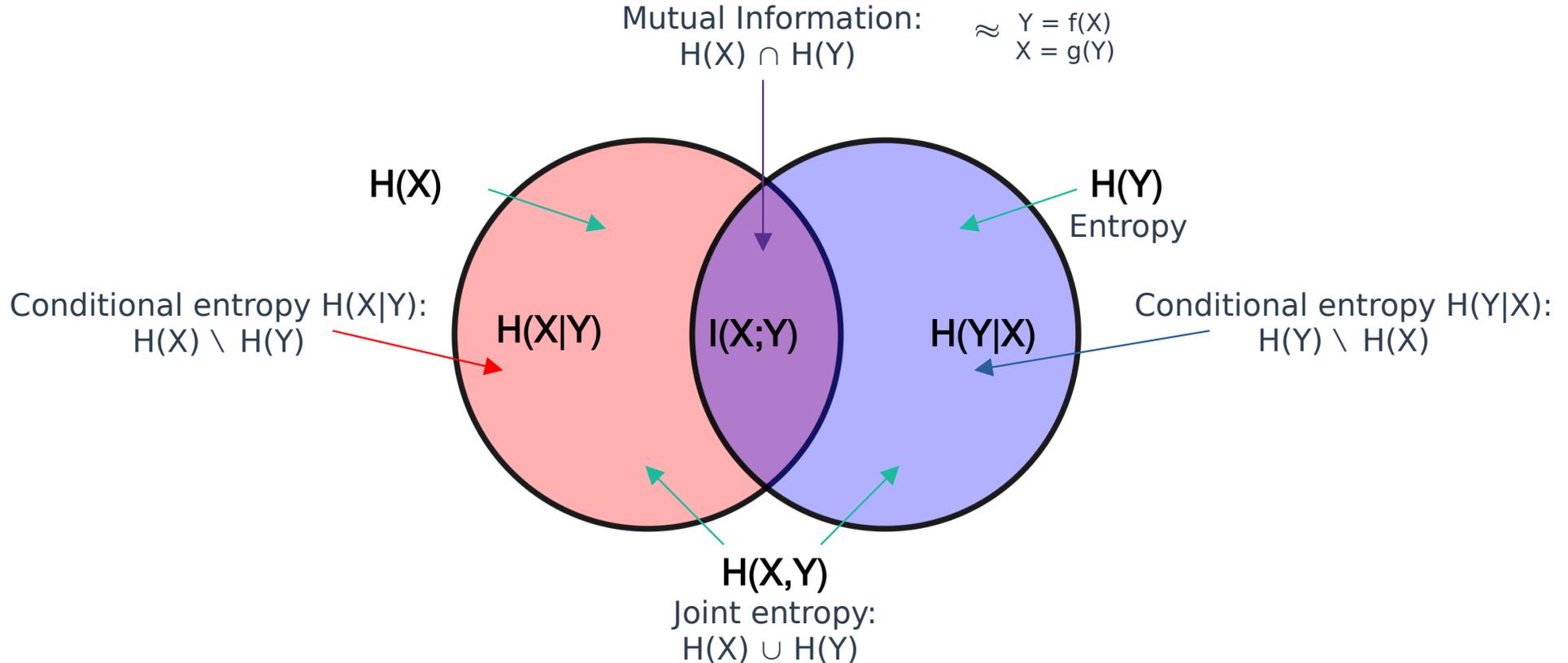
- **Assumption:**

- the model is using high-level features in the output latent space (12 neurons)

- **Approach:**

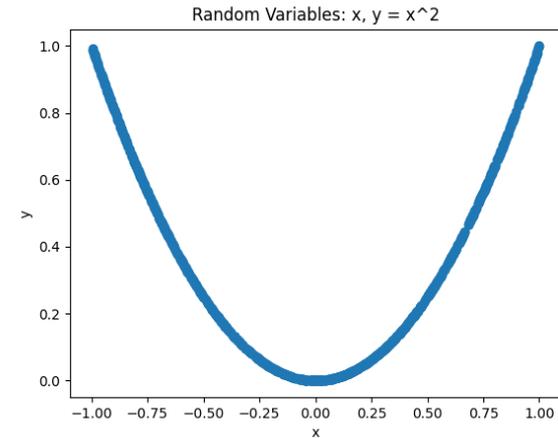
- Information theory: conditional entropy of high-level features conditioned on the output latent space → gives how much of the high-level feature can be predicted from the latent space alone

Entropy



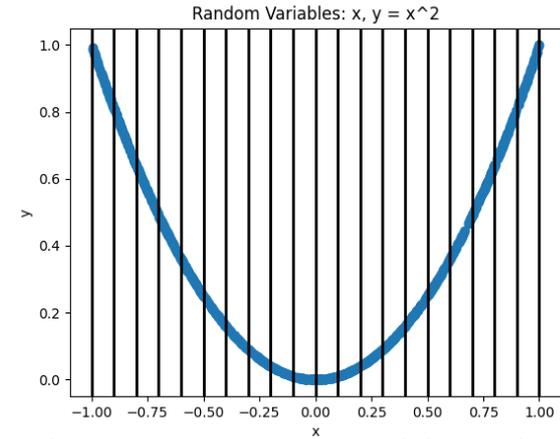
Conditional entropy

$$H(Y|X) = - \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} p(x, y) \log \frac{p(x, y)}{p(x)}$$



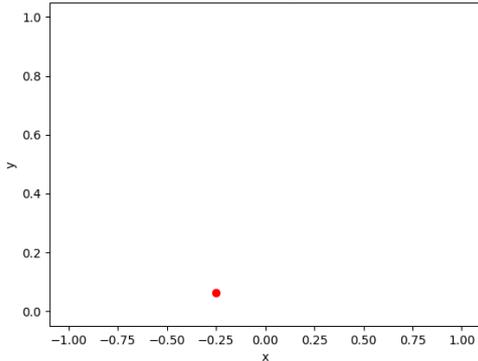
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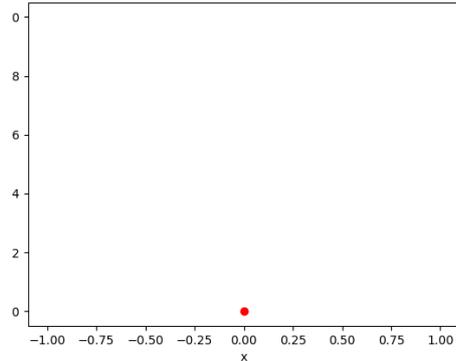
$$H(Y|x=-0.25) = 0$$

Conditional distribution: $y = x^2, x = -0.25$



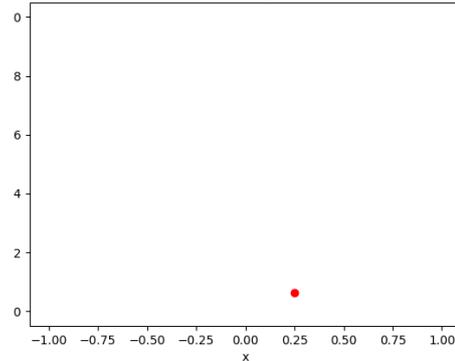
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Conditional distribution: $y = x^2, x = 0$



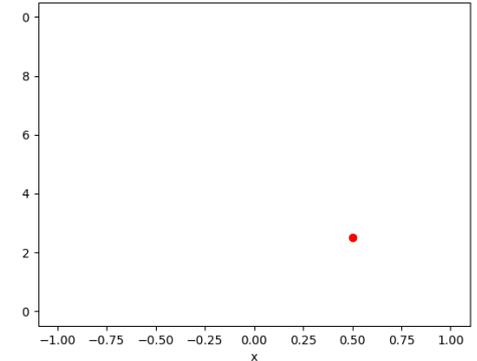
$$H(Y|x=0.25) = 0$$

Conditional distribution: $y = x^2, x = 0.25$



$$H(Y|x=0.5) = 0$$

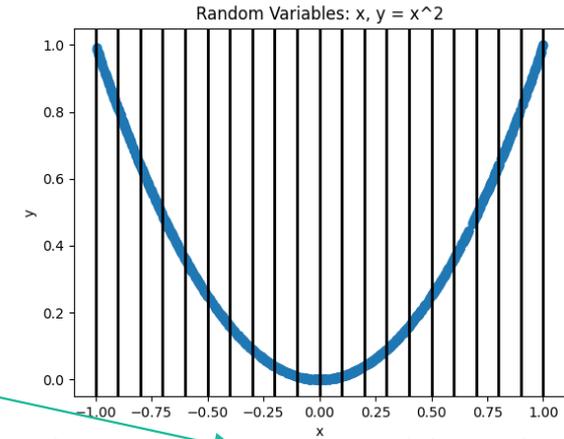
Conditional distribution: $y = x^2, x = 0.5$



Conditional entropy

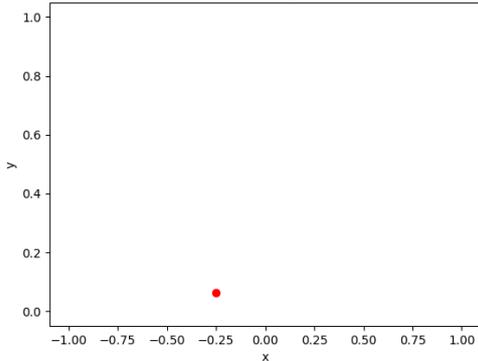
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$$H(Y|X) = 0$$



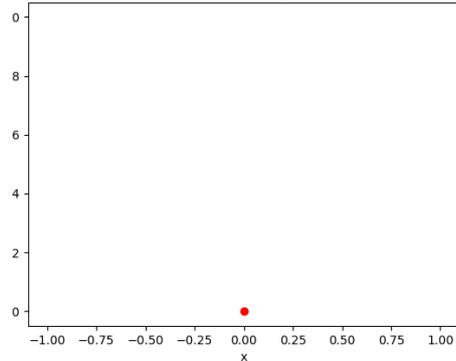
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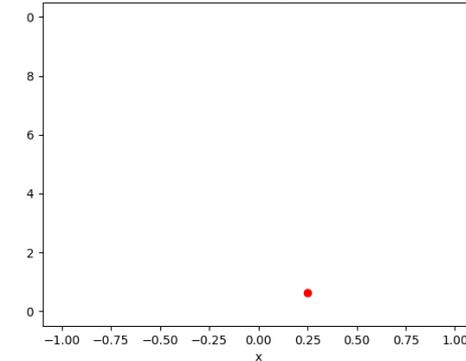
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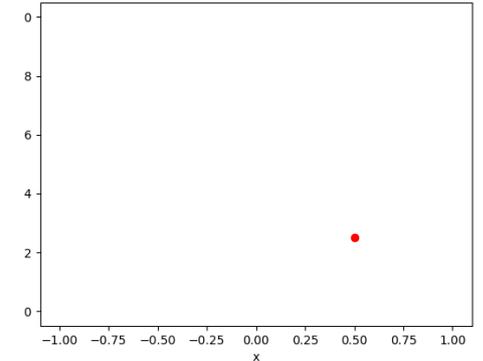
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Information coverage

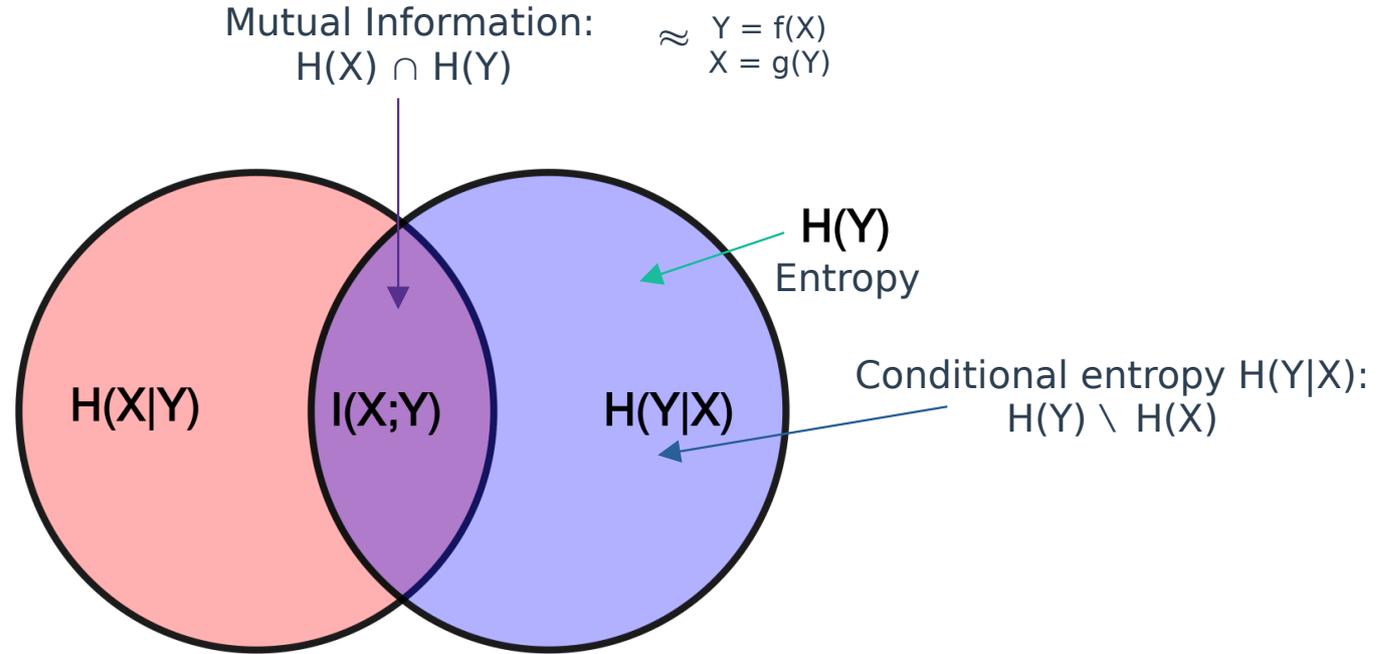
Information coverage
(*not an official name*):

$$\frac{H(Y) - H(Y|X)}{H(Y)}$$

$H(Y)$



= 1 when $H(Y|X) = 0$ (full dependency)
= 0 when $H(Y|X) = H(Y)$ (independency)



Kernel Density Estimation

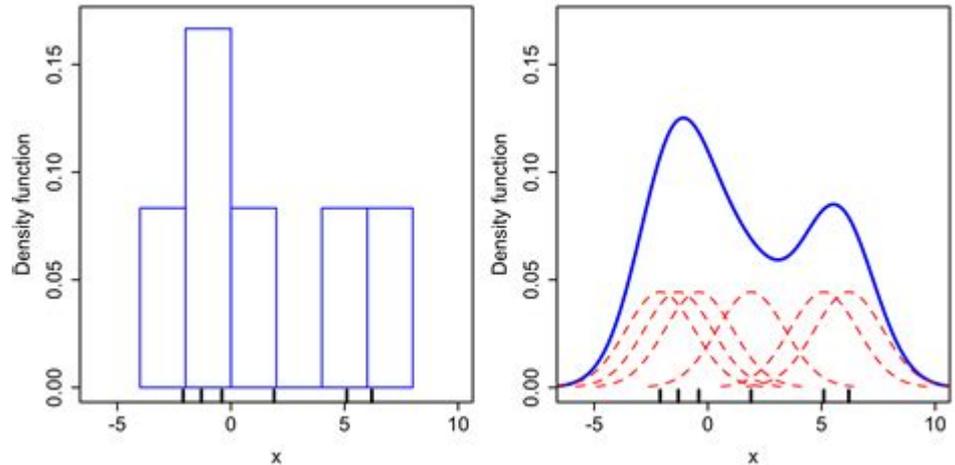
- **Need to have the probability distribution**

- **Kernel density estimation:**

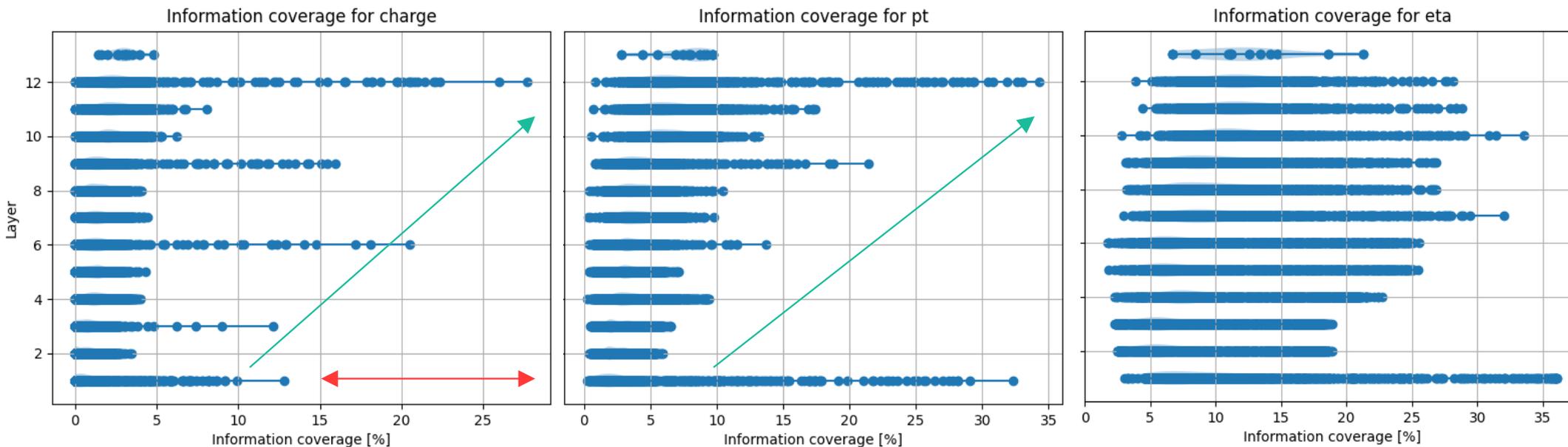
- Fit parameter: h
- Put a Kernel function $K(x,h)$ in each point and sum them to get the density estimation

- Gaussian kernel (`kernel = 'gaussian'`)

$$K(x; h) \propto \exp\left(-\frac{x^2}{2h^2}\right)$$

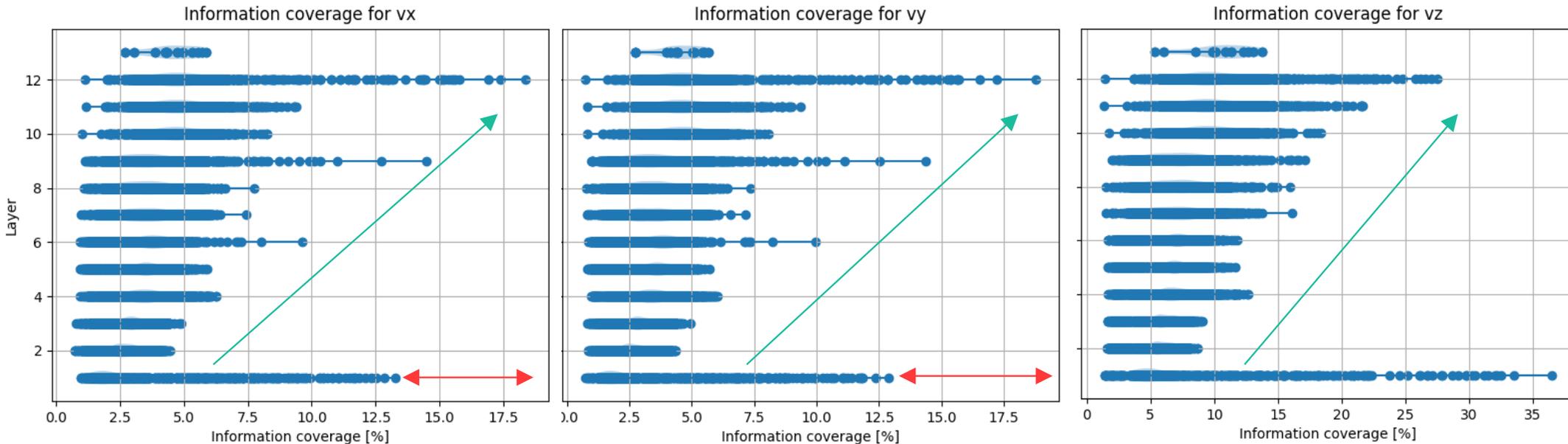


High-level variables single neuron



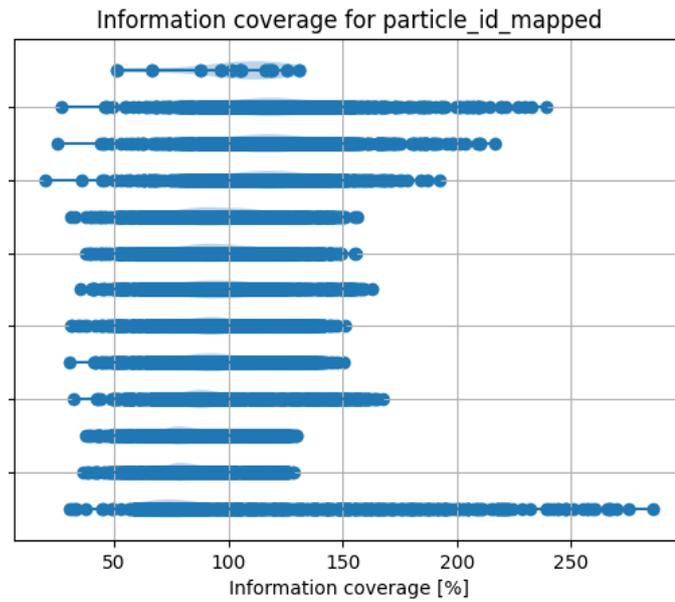
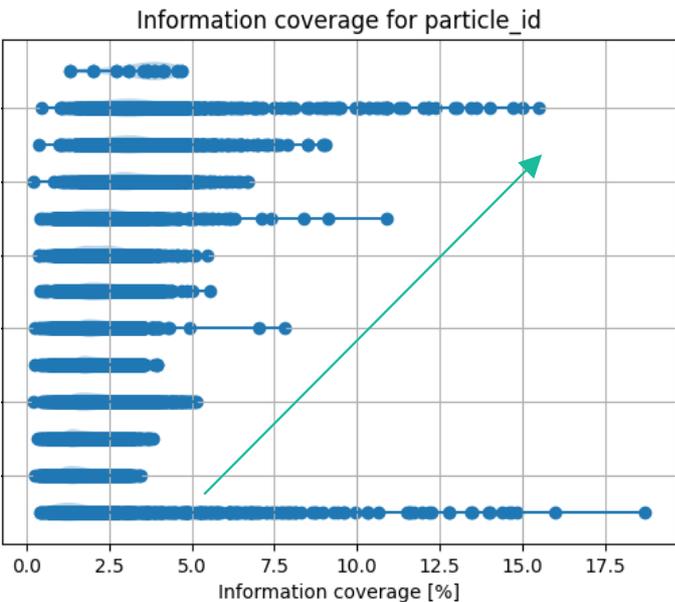
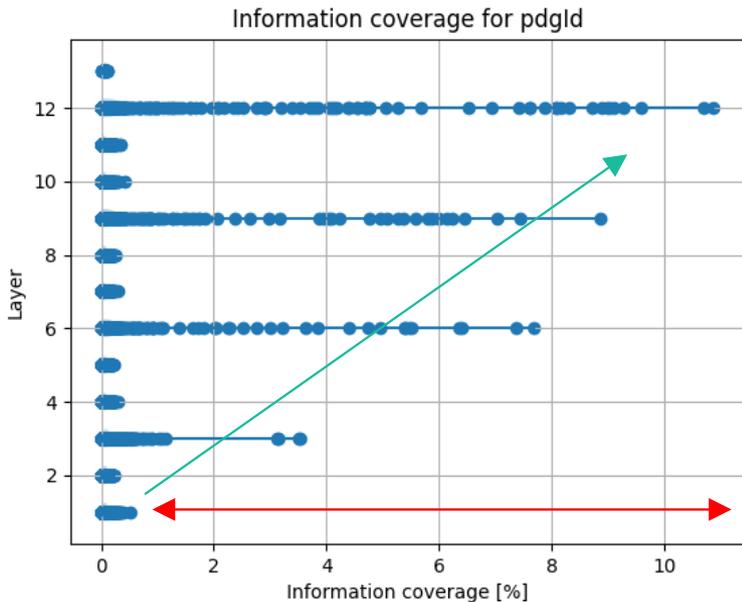
$h = 0.02$

High-level variables single neuron



$h = 0.02$

High-level variables single neuron

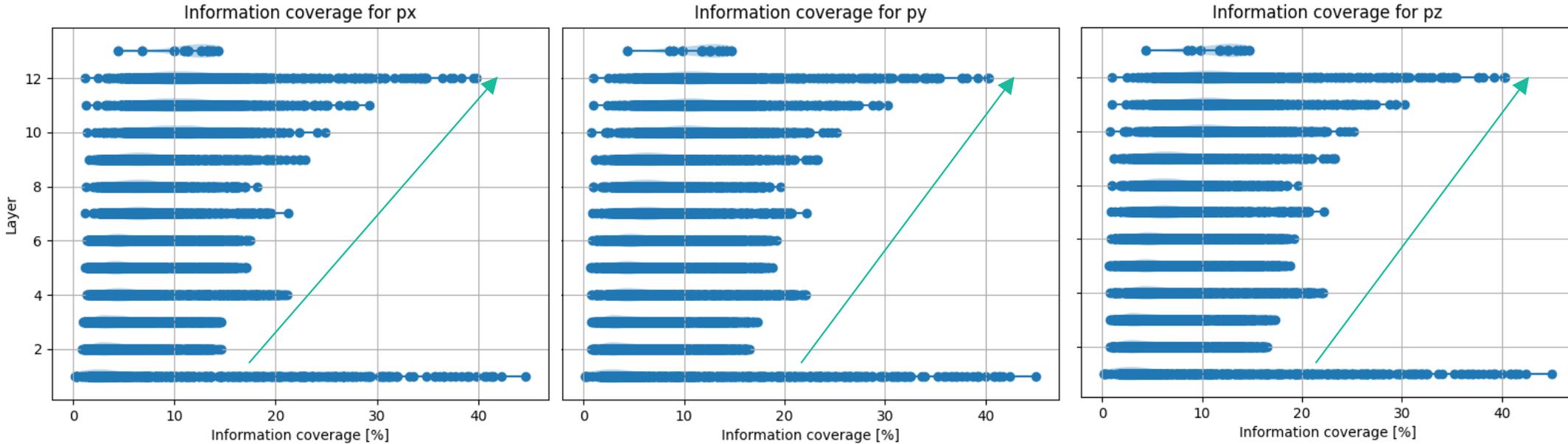


Learned to isolate some particles

Broken

$h = 0.02$

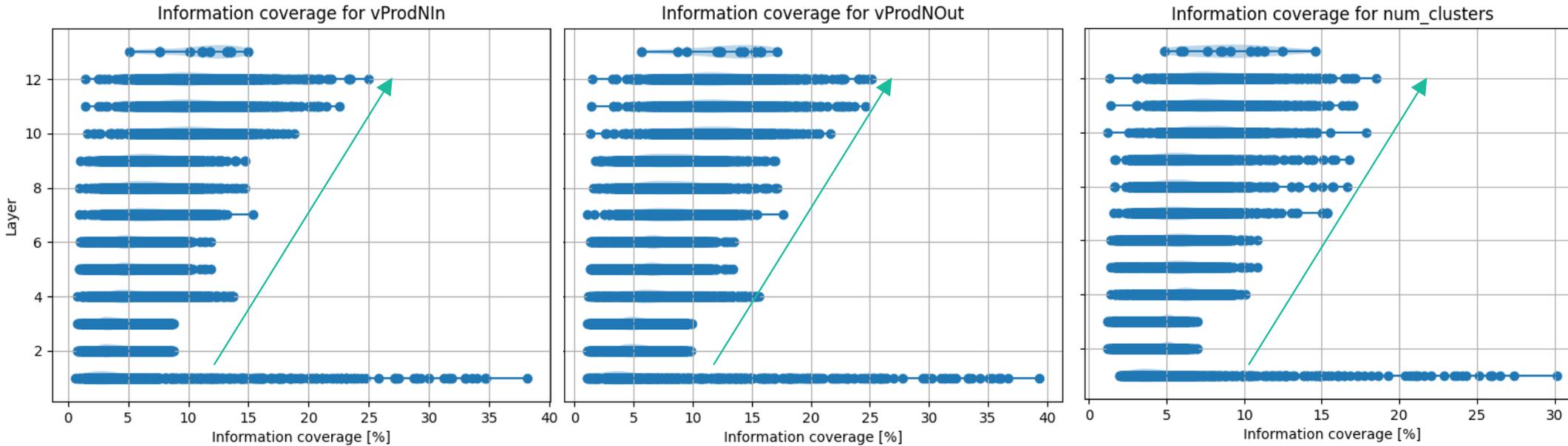
High-level variables single neuron



Same

$h = 0.02$

High-level variables single neuron



$h = 0.02$

Layer information coverage

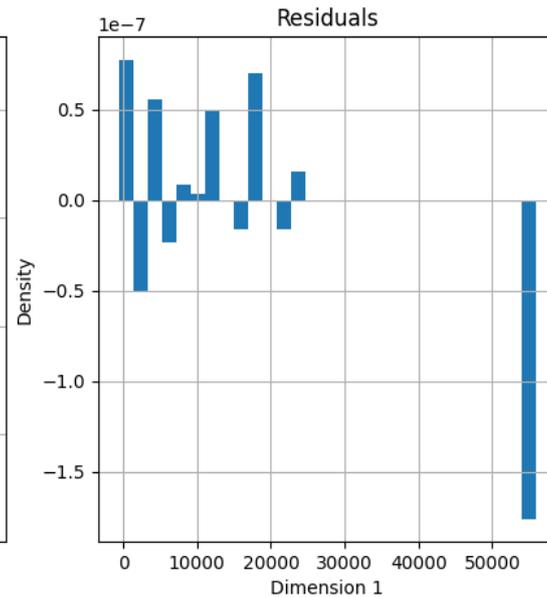
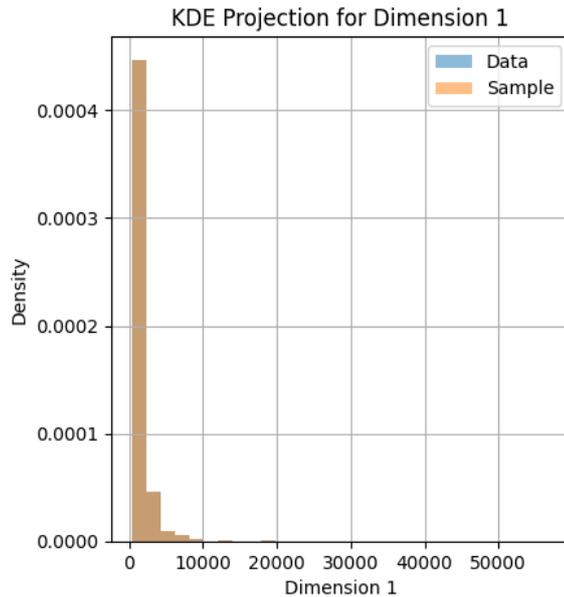
- Information coverage varies with h

$h=0.5$

l a y e r	p a r t i c l e _ i d	sub eve nt	bar cod e	px	py	pz	pt	eta	vx	vy	vz	radi us	s t a t u s	cha rge	pdg Id	vPr od NIn	vPr od NO ut
1 2	0. 5 7	0.3 8	0.5 5	0.5 4	0.5 4	0.5 4	0.5 1	0.5 6	0.3 8	0.3 8	0.3 8	0.3 5		0.0 9	0.1 7	0.3 1	0.3 4

Goodness of fit

- **Sample from the fit KDE**

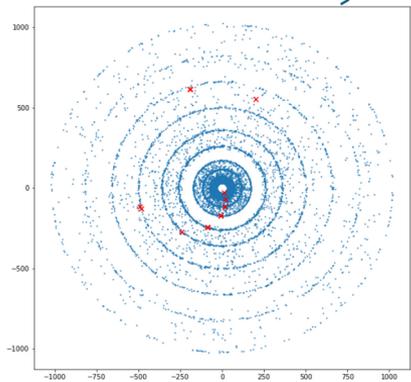


BACKUP

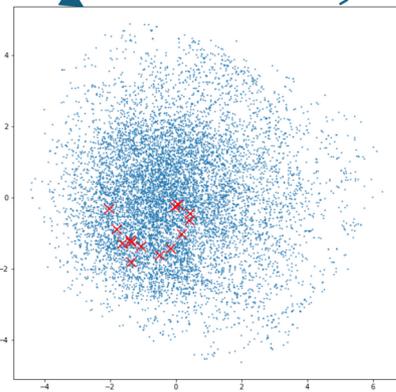
- **Plots en fonction de r - ϕ - z = pas la bonne approche**
- **Cherche à faciliter la reconstruction des traces → ce que les points d'une même trace ont en commun → doit regarder en fonction des paramètres des traces et comparer pour différentes traces**

GNN Metric Learning

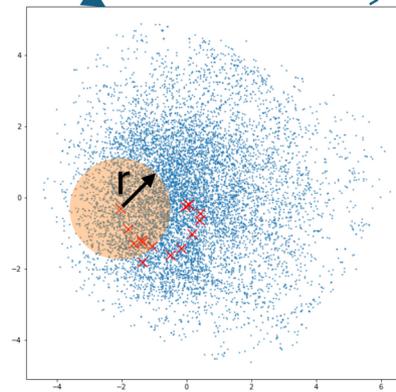
Embed into learned
latent space



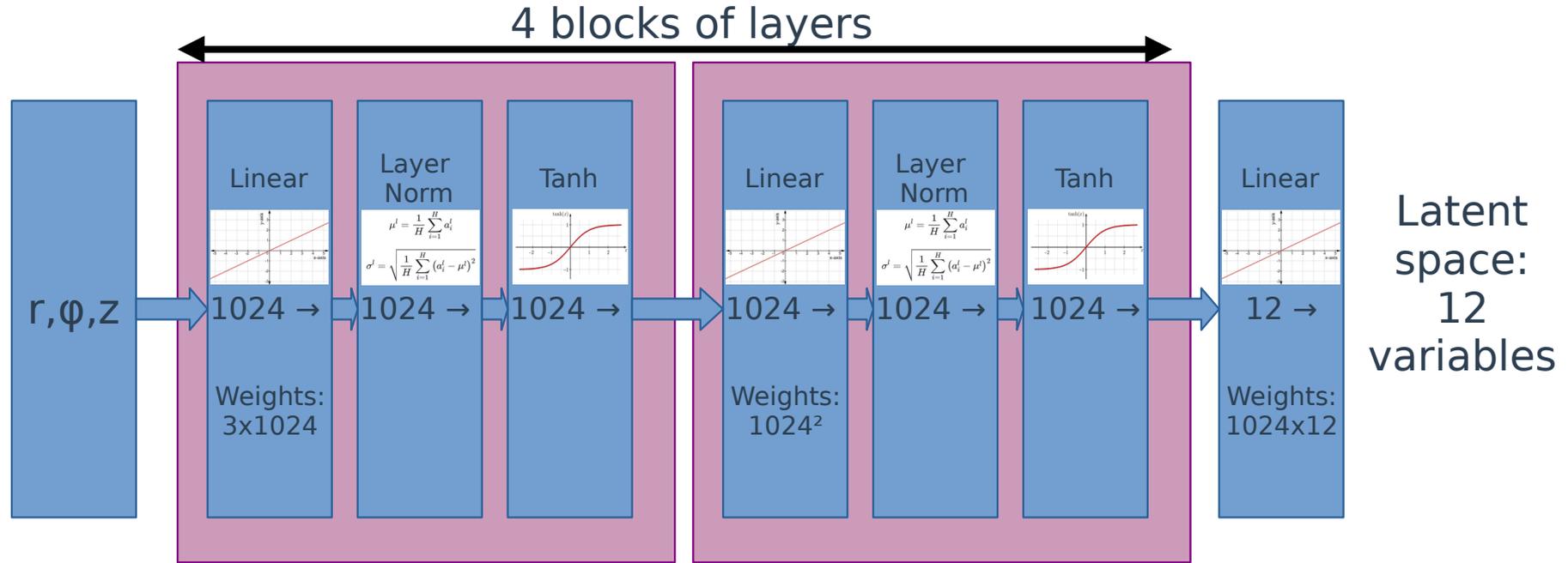
Connect all space points
within radius r



All space point pairs
joined into graph

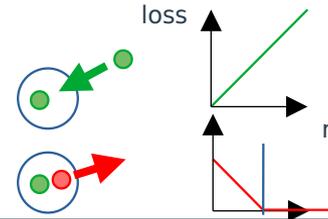


Architecture



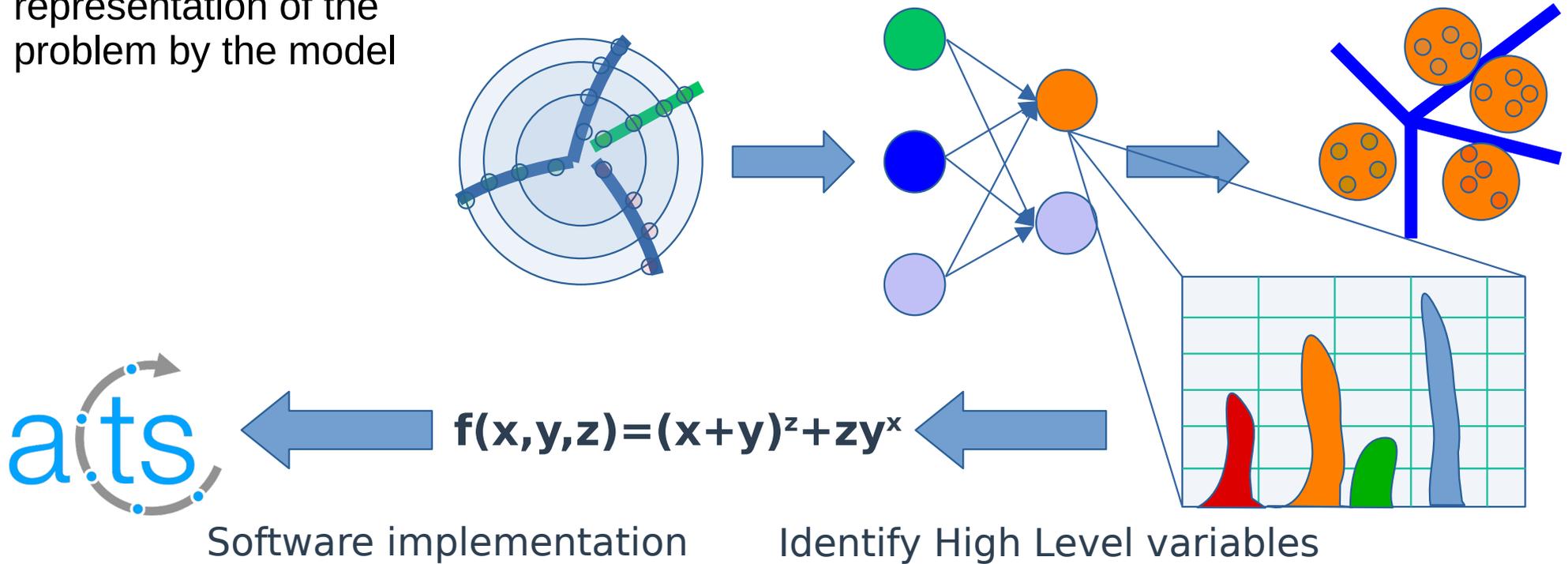
Hinge Loss

$$l_n = \begin{cases} x_n, & \text{if } y_n = 1, \\ \max\{0, \text{margin} - x_n\}, & \text{if } y_n = -1, \end{cases}$$



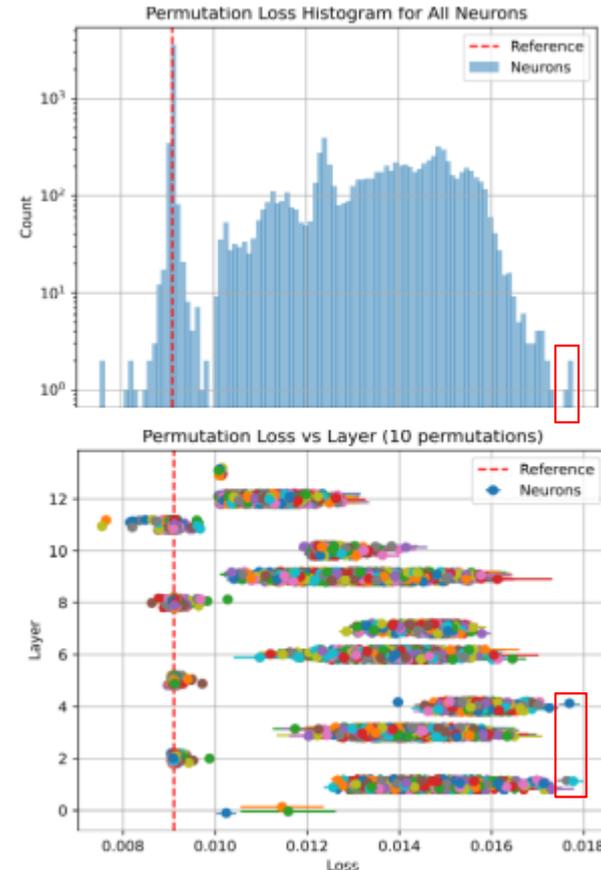
Interpretability

- study the internal representation of the problem by the model



Neuron identification: Permutation loss

- **3 promising neurons:**
 - 2 on layer 1 (*Linear* with input layer)
 - 1 on layer 4 (More complex)
- **Normalization Layers (3n-1) not perturbed by permutation → Information is shared among neurons**



Model

• Example 2

1. First, we build our input data from the raw Athena events:

```
acorn infer data_reader.yaml
```

2. We start the graph construction by training the Metric Learning stage:

```
acorn train metric_learning_train.yaml
```

3. Then, we build graphs using the Metric Learning in inference:

```
acorn infer metric_learning_infer.yaml
```

```
# Model inference parameters  
r_infer: 0.1  
knn_infer: 1000
```

```
hard_cuts:  
  pt: [1000, .inf]  
  
# Model parameters  
undirected: True  
node_features: [r, phi, z]  
node_scales: [1000, 3.14, 1000]  
emb_hidden: 1024  
nb_layer: 4  
emb_dim: 12  
activation: Tanh  
randomisation: 1  
points_per_batch: 50000  
r_train: 0.1  
knn: 50  
knn_val: 1000  
  
# Training parameters  
warmup: 5  
margin: 0.1  
lr: 0.01  
factor: 0.7  
patience: 10  
max_epochs: 100  
metric_to_monitor: f1  
metric_mode: max
```

Performance

