Leptogenesis in type-I seesaw models

Mathieu Ferey

Leptogenesis Meeting | 16/01/2025

Type-I Seesaw

$$\mathcal{L} = -\boxed{\sum_{\alpha,\beta} y^{e}_{\alpha\beta}\,\overline{\boldsymbol{L}^{\alpha}}\,\boldsymbol{\Phi}\, e_R^{\beta}} - \boxed{\sum_{\alpha,i} y^{\nu}_{\alpha i}\,\overline{\boldsymbol{L}^{\alpha}}\,\boldsymbol{\Phi}^{\mathrm{c}} N_R^{i}} - \boxed{\frac{1}{2}\sum_{ij} M_R^{ij} N_R^{i\top}\mathcal{C}^{\dagger} N_R^{j}} + \mathrm{h.c.}$$

- SM Yukawas $\xrightarrow{\text{SSB}}$ Dirac mass 3 × 3 matrix for charged leptons
- "Sterile" Yukawas $\xrightarrow{\text{SSB}}$ Dirac mass $3 \times N_s$ matrix for neutrinos $m_{\mathrm{D}} = \frac{v}{\sqrt{2}} y^{\nu}$
- RHS Majorana mass: $N_s \times N_s$ symmetric matrix

→ need to diagonalize to get definite mass states

Type-I Seesaw

Freely assume y^e and M_R diagonal. After SSB, regroup Dirac and Majorana masses:

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \boldsymbol{N}_L^{\mathrm{T}} \mathcal{C}^{\dagger} M \boldsymbol{N}_L + \text{h.c.}$$

with

$$\boldsymbol{N}_L = \begin{pmatrix} \nu_L \\ N_R^{\mathcal{C}} \end{pmatrix} \qquad M = \left[\begin{array}{c|c} \mathbb{0}_{3\times 3} & m_{\mathrm{D}} \\ \hline m_{\mathrm{D}}^{\top} & M_R \end{array}\right]$$

M is a complex and symmetric $(N_s + 3) \times (N_s + 3)$ matrix → bi-unitary transformation

$$M = \left[\begin{array}{c|c} U & A \\ \hline B & V \end{array}\right] \times \left[\begin{array}{c|c} m_\nu^{\text{diag}} & \mathbb{0} \\ \hline \mathbb{0} & M_N^{\text{diag}} \end{array}\right] \times \left[\begin{array}{c|c} U & A \\ \hline B & V \end{array}\right]^{\top}$$

$A, B = \mathcal{O}\left(m_{\mathrm{D}}/M_R\right)$, $M_N^{\text{diag}} = M_R + \mathcal{O}\left(m_{\mathrm{D}}/M_R\right)$, U is PMNS

`https://arxiv.org/pdf/0902.2469`

The Casas-Ibarra Parametrisation

It is straightforward to end-up to

$$m_\nu \equiv U m_\nu^{\mathrm{diag}} U^\top \simeq -m_{\mathrm{D}} M_R^{-1} m_{\mathrm{D}}^\top$$

flavour basis, mass basis

EFT result from tree-level matching when integrating out heavy neutrinos (happy Thomas sounds)! However, we also easily see that PMNS deviates from unitarity $UU^\dagger + AA^\dagger = 1$. One possible parametrisation for this result to hold is

$$\boxed{m_{\mathrm{D}}^\top = i M_R^{1/2} R \left(m_\nu^{\mathrm{diag}}\right)^{1/2} U^\dagger}$$

with R complex symmetrix $N_s \times 3$ matrix

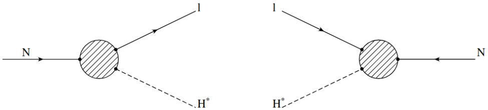
The Lepton Asymmetry

- How many RHN decay?
  - → In a strong hierarchical regime $M_{i>1} \gg M_1$, any asymmetry generated by the decay of the heaviest RHN $N_{i>1}$ can be washed out by the decay of the lightest RHN N_1.

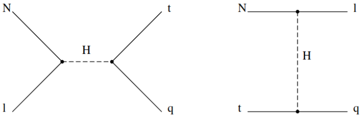
- Do we distinguish charged leptons? Equilibrium of charged lepton Yukawas:
  - → $T \gg 10^{12}\mathrm{GeV}$: 1 flavour approximation
  - → $10^{12}\mathrm{GeV} \gg T \gg 10^{9}\mathrm{GeV}$: 2 flavour approximation
  - → $T \ll 10^{9}\mathrm{GeV}$: full 3 flavour treatment

The simplest is the 1D1F approximation (One neutrino Decay and 1 Flavour)

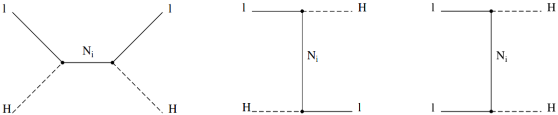
Neutrinos and leptons in the primordial plasma



(a) RHN decay and anti-decay



(b) $\Delta L = 1$ scattering



(c) $\Delta L = 2$ scattering

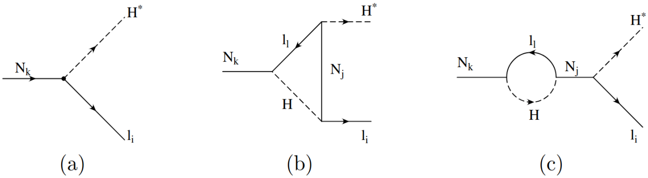
Boltzmann equations

Evolution of out-of-equilibrium particle distribution → Boltzmann equation

$$\frac{\mathrm{d}N_{N_1}}{\mathrm{d}z} = -(D + S)(N_{N_1} - N_{N_1}^{eq})$$
$$\frac{\mathrm{d}N_{B-L}}{\mathrm{d}z} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{eq}) - W N_{B-L}$$

- Decay
- $\Delta L = 1$ scattering
- Washout ($\Delta L = 1$ and $\Delta L = 2$ scattering, inverse decay)

CP-asymmetry



$$\varepsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \bar{\ell}_\alpha \overline{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \bar{\ell}_\alpha \overline{H})\right]} \simeq -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_\rho m_\rho^2 {R_{1\rho}}^2\right)}{\sum_\beta m_\beta \left|R_{1\beta}\right|^2}$$

Casas-Ibarra! Leptogenesis uncorrelated from low-energy parameters...

Saved by flavour

When distinguishing the three lepton flavours

$$\frac{\mathrm{d}N_{N_1}}{\mathrm{d}z} = -D_1\left(N_{N_1} - N_{N_1}^{\mathrm{eq}}\right)$$
$$\frac{\mathrm{d}N_{B-L}}{\mathrm{d}z} = \sum_{l=e,\mu,\tau}\left(\varepsilon_1^l D_1\left(N_{N_1} - N_{N_1}^{\mathrm{eq}}\right) - p_{1\alpha} W_1 N_l\right),$$

with flavour dependent CP-asymmetries

$$\varepsilon_1^l = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{l\beta}^* U_{l\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta \left|R_{1\beta}\right|^2}, \qquad l = e, \mu, \tau.$$

Low energy and High energy!

A quick summary

1. Type-I seesaw: 10 different regimes given the masses of the RHN, their hierarchy and how they compare to the equilibrium temperatures of charged lepton Yukawas
2. The out-of-equilibrium decay of RHN leads to lepton asymmetry, whose evolution is given by coupled Boltzmann equations
3. $\mathrm{SU}\,(2)$ sphalerons, $\mathrm{SU}\,(3)$ instantons, quark and charged lepton Yukawas in equilibrium, as well as the neutrality of plasma convert L asymmetry into B asymmetry

$$B = \frac{28}{79}\,(B-L)\,, \qquad L = \left(\frac{28}{79} - 1\right)(B-L)$$

$$\eta_B \equiv \frac{N_B}{N_\gamma^{\mathrm{rec}}} = \frac{28}{79}\frac{1}{27} N_{B-L}$$

A whole Odyssey

The ULYSSES (Universal LeptogeneSiS Equation Solver) Python library:

- → choose mass regime
- → give low and high energy parameters
- → solve Boltzmann equations and computes baryon to photon ratio