# Neutrino mixing with **discrete symmetry**

Reference : S. King, C. Luhn (2016), Neutrino mass and mixing with discrete symmetry.

# Neutrino mixing

First approximations :

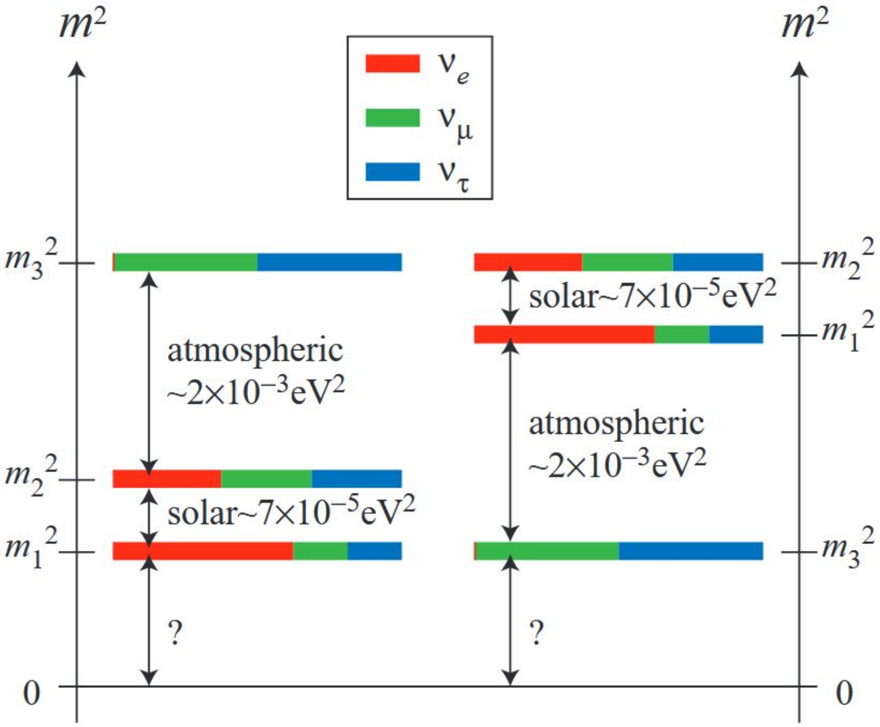
Tri-maximal mixing :

$$|U_{e2}| \approx |U_{\mu 2}| \approx |U_{\tau 2}| \approx \frac{1}{\sqrt{3}}$$

Bi-maximal mixing :

$$|U_{\mu 3}| \approx |U_{\tau 3}| \approx \frac{1}{\sqrt{2}}$$

$$|U_{e3}| \approx 0$$

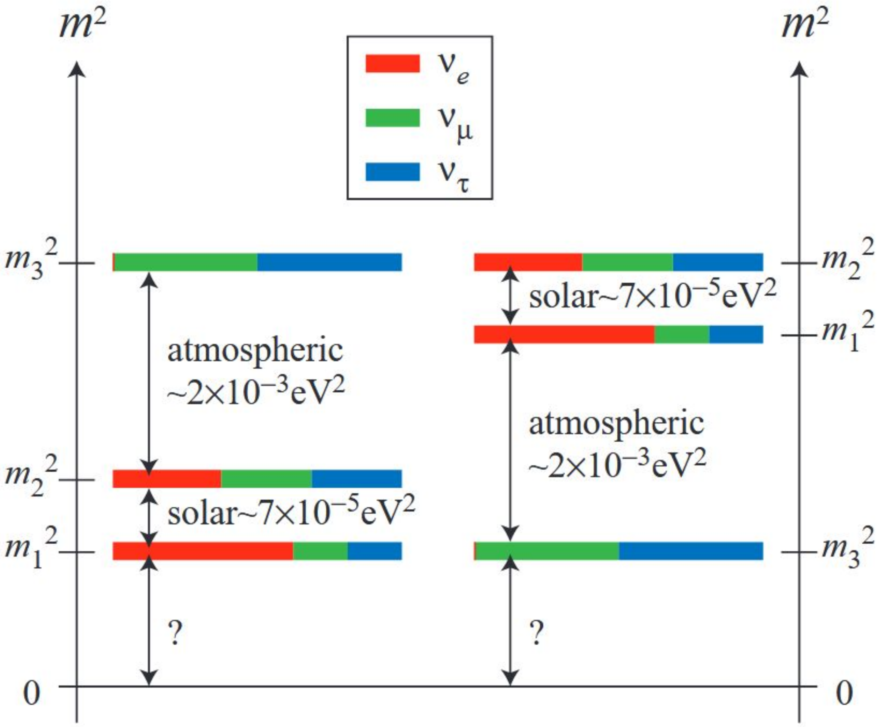



Probability that a particular mass state contains a particular flavour state. Normal ordering (left), inverted ordering (right)

# Neutrino mixing

PMNS matrix under the TBM approximation :

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$




Probability that a particular mass state contains a particular flavour state. Normal ordering (left), inverted ordering (right)

# Neutrino mixing

We now know that :

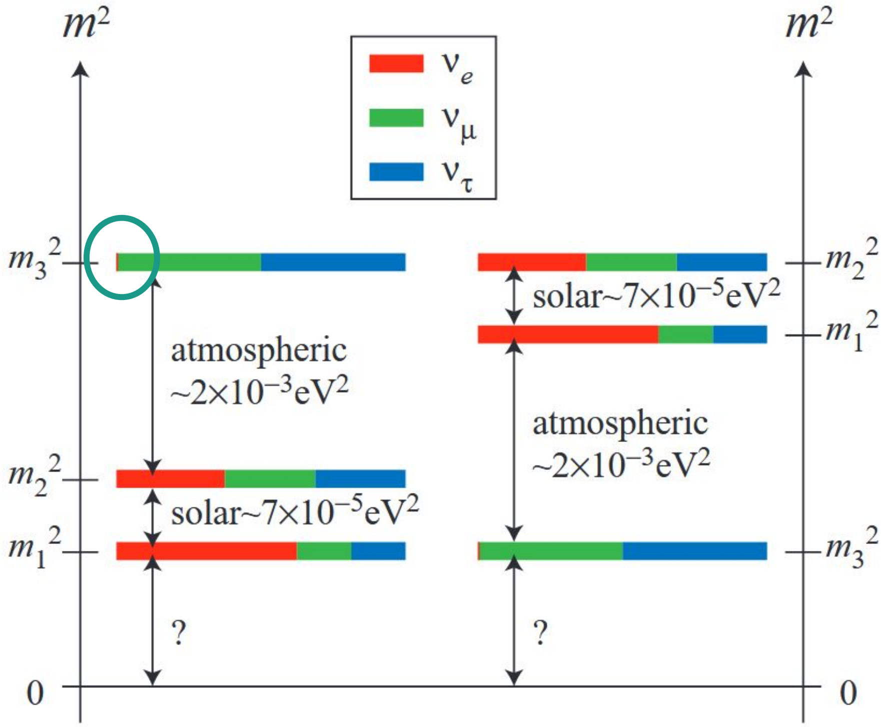
$$|U_{e3}| \approx 0.15 \neq 0$$

We can try to write the PMNS matrix with small deviations from the TBM matrix :

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a), \quad \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

With measured values we get :

$$s = -0.03 \pm 0.03, \quad a = -0.05 \pm 0.05, \quad r = 0.21 \pm 0.01$$




Probability that a particular mass state contains a particular flavour state. Normal ordering (left), inverted ordering (right)

# Explaining the mixing patterns with group symmetry

We need to find a symmetry group for neutrinos and charged leptons which is **broken at low energy**.

Group with irreducible triplet representations : we are limited to **U(3) and its subgroups**.

Why discrete subgroups ? Simpler and more natural.

Why non-abelian ? No fundamental reason, but abelian groups lead to quark-like hierarchical mixing.

# Explaining the mixing patterns with group symmetry

**Fundamental symmetries of the mass matrices :**

For the neutrino mass matrix :

$$\widetilde{K}_{p,q}^{T}\, m_{LL}^{\nu,\mathrm{diag}}\, \widetilde{K}_{p,q} = m_{LL}^{\nu,\mathrm{diag}}\,, \qquad \text{with} \quad \widetilde{K}_{p,q} = \begin{pmatrix} (-1)^p & 0 & 0 \\ 0 & (-1)^q & 0 \\ 0 & 0 & (-1)^{p+q} \end{pmatrix}$$

$$K_{p,q} = U_{\mathrm{PMNS}}^{*} \widetilde{K}_{p,q} U_{\mathrm{PMNS}}^{T}$$

These Ks form a group isomorphic to the Klein group Z_2 x Z_2

For the charged lepton mass matrix :

$$T^{\dagger}\left(M_e M_e^{\dagger}\right) T = M_e M_e^{\dagger}\,, \qquad \text{with} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{m}} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{m}} \end{pmatrix}$$

# Explaining the mixing patterns with group symmetry

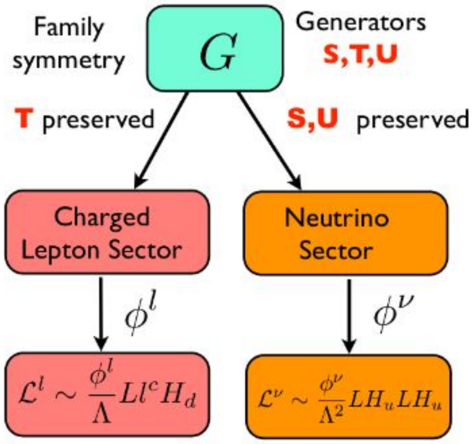
By combining the generators **S, U** of the Klein group, and the generator **T** of the charged lepton symmetry, we can obtain a first possible group : **S4**

It's the **direct** approach.

We can also use only the generator **S** of the Klein group and the generator **T**, we obtain a second possible group : **A4**

It's the **semi-direct** approach.

There is another possible approach : building the symmetry group from other principles (like type-I seesaw mechanism) : it's the **indirect approach**.




Direct approach