Discrete flavour symmetries, neutrino mixing and leptonic CP violation

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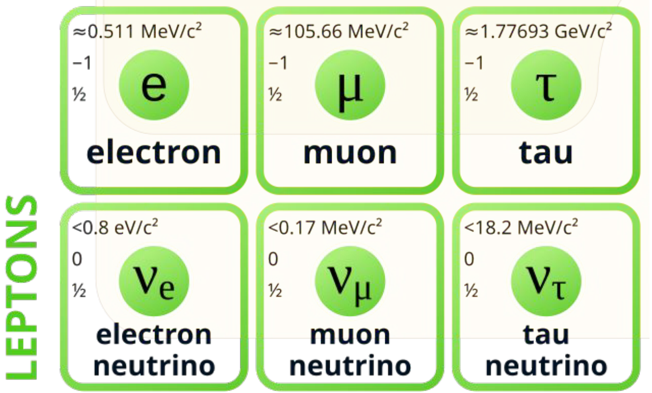
General overview

Are the values of the **neutrino masses** and **mixing angles** random or do they follow an underlying **symmetry** ?

The neutrino mixing pattern could be understood on the basis of a specific class of symmetry : **non-Abelian discrete flavour symmetry**.



New fundamental symmetry in the lepton sector ?

Theoretical Framework : Neutrino mixing

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \overline{l_L}(x)\, \gamma_\alpha \nu_{lL}(x)\, W^{\alpha\dagger}(x) + h.c.,$$

$$\nu_{lL}(x) = \sum_{j=1}^{n} U_{lj} \nu_{jL}(x),$$

Weak charged lepton current

$$U = VP, \quad P = \mathrm{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Matrice PMNS

Ordering of the **neutrino masses** :
(the squared differences are pretty well known experimentally)

(i) spectrum with normal ordering (NO): $m_1 < m_2 < m_3$, $\Delta m^2_{31(32)} > 0$, $\Delta m^2_{21} > 0$, $m_{2(3)} = (m_1^2 + \Delta m^2_{21(31)})^{\frac{1}{2}}$;

(ii) spectrum with inverted ordering (IO): $m_3 < m_1 < m_2$, $\Delta m^2_{32(31)} < 0$, $\Delta m^2_{21} > 0$, $m_2 = (m_3^2 + \Delta m^2_{23})^{\frac{1}{2}}$, $m_1 = (m_3^2 + \Delta m^2_{23} - \Delta m^2_{21})^{\frac{1}{2}}$.

Theoretical Framework : Neutrino mixing

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{l=e,\mu,\tau} \overline{l_L}(x)\, \gamma_\alpha \nu_{l\mathrm{L}}(x)\, W^{\alpha\dagger}(x) + h.c.,$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\, \nu_{j\mathrm{L}}(x),$$

Weak charged lepton current

$$U = VP, \quad P = \mathrm{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Matrice PMNS

$$\Delta P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha) \qquad \Delta P_{\alpha\beta} = \pm 16 J \sin\left(\frac{\Delta m^2_{12} L}{4E}\right) \sin\left(\frac{\Delta m^2_{13} L}{4E}\right) \sin\left(\frac{\Delta m^2_{23} L}{4E}\right)$$

$$J = \Im[U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}] = \frac{1}{8}\cos(\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\sin(\delta_{CP})$$

Asymmetry of the oscillation probability

Theoretical Framework : Discrete symmetry

Extending the SM with a discrete non-Abelian group, **unified at High Energy**, **broken at Low Energy** so that the particles get different masses.

$$\nu_{\tilde{l}L}(x) \xrightarrow{G_f} (\rho_r(g_f))_{\tilde{l}\tilde{l}'}\, \nu_{\tilde{l}'L}(x), \qquad g_f \in G_f,$$
$$\tilde{l}_L(x) \xrightarrow{G_f} (\rho_r(g_f))_{\tilde{l}\tilde{l}'}\tilde{l}'_L(x), \quad \tilde{l} = \tilde{e}, \tilde{\mu}, \tilde{\tau}.$$

Residual symmetries : $G_\ell, G_\nu \subset G_f$

$$\rho_r(g_\nu)^\dagger M_\nu^\dagger\, M_\nu\, \rho_r(g_\nu) = M_\nu^\dagger\, M_\nu.$$
$$\rho_r(g_e)^\dagger M_e M_e^\dagger \rho_r(g_e) = M_e M_e^\dagger,$$

$$U_{\rm PMNS} = U_e^\dagger\, U_\nu.$$

The PMNS matrix can be related to these subgroups

$$U_e^\dagger\, \rho_r(g_e)\, U_e = \rho_r^{\rm diag}(g_e)$$
$$(U_\nu^\circ)^\dagger\, \rho_r(g_\nu)\, U_\nu^\circ = \rho_r^{\rm diag}(g_\nu)$$
$$U_\nu = U_\nu^\circ\, P^\circ, \qquad P^\circ = {\rm diag}(1, e^{i\frac{\xi_{21}}{2}}, e^{i\frac{\xi_{31}}{2}})$$

Thus the PMNS matrix is either **completely determined or at least constrained** by the choice of $G_\ell, G_\nu, G_f, \rho_r(g_f)$

Example of discrete groups

S4 : (permutation of 4 elements).

Tri-bimaximal mixing : $G_e = Z_3^T = \{1, T, T^2\}, \quad G_\nu = Z_2^S \times Z_2^U = \{1, S, U, SU\}, \quad U_\nu^\circ = V_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

ruled out by data because it implies $\theta_{13} = 0$.

Other possibility : $G_e = Z_3^T = \{1, T, T^2\}, \quad G_\nu = Z_2^{SU} = \{1, SU\},$

Which lead to the following equation for δ :

$$\cos\delta = \frac{\frac{1}{6} - c_{23}^2 + \frac{2}{3c_{13}^2}\,(c_{23}^2 - s_{23}^2\, s_{13}^2)}{2c_{23}\, s_{23}\, s_{13}\, c_{12} s_{12}} = \frac{(-1 + 5s_{13}^2)\,\cos 2\theta_{23}}{2\sqrt{2}\,\sin 2\theta_{23}\, s_{13}\,(1 - 3s_{13}^2)^{\frac{1}{2}}},$$

$$U_{\text{PMNS}} = U_\nu^\circ\, P^\circ = V_{\text{TBM}}\, U_{23}(\theta_{23}^\nu, \beta)\, P^\circ = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{c_{23}^\nu}{\sqrt{3}} & \frac{s_{23}^\nu}{\sqrt{3}}\, e^{i\beta} \\ -\frac{1}{\sqrt{6}} & \frac{c_{23}^\nu}{\sqrt{3}} + \frac{s_{23}^\nu}{\sqrt{2}} e^{-i\beta} & -\frac{c_{23}^\nu}{\sqrt{2}} + \frac{s_{23}^\nu}{\sqrt{3}}\, e^{i\beta} \\ -\frac{1}{\sqrt{6}} & \frac{c_{23}^\nu}{\sqrt{3}} - \frac{s_{23}^\nu}{\sqrt{2}} e^{-i\beta} & \frac{c_{23}^\nu}{\sqrt{2}} + \frac{s_{23}^\nu}{\sqrt{3}}\, e^{i\beta} \end{pmatrix} P^\circ, \tag{51}$$

where $c_{23}^\nu \equiv \cos\theta_{23}^\nu$ and $s_{23}^\nu \equiv \sin\theta_{23}^\nu$.

Example of discrete groups

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ruled out by data because it implies $\theta_{13} = 0$.

Other possibility : $G_e = Z_3^T = \{1, T, T^2\}, \quad G_\nu = Z_2^{SU} = \{1, SU\},$

$$U_{\mathrm{PMNS}} = U_\nu^\circ \, P^\circ = V_{\mathrm{TBM}} \, U_{23}(\theta_{23}^\nu, \beta) \, P^\circ$$
$$= \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{c_{23}^\nu}{\sqrt{3}} & \frac{s_{23}^\nu}{\sqrt{3}} e^{i\beta} \\ -\frac{1}{\sqrt{6}} & \frac{c_{23}^\nu}{\sqrt{3}} + \frac{s_{23}^\nu}{\sqrt{2}} e^{-i\beta} & -\frac{c_{23}^\nu}{\sqrt{2}} + \frac{s_{23}^\nu}{\sqrt{3}} e^{i\beta} \\ -\frac{1}{\sqrt{6}} & \frac{c_{23}^\nu}{\sqrt{3}} - \frac{s_{23}^\nu}{\sqrt{2}} e^{-i\beta} & \frac{c_{23}^\nu}{\sqrt{2}} + \frac{s_{23}^\nu}{\sqrt{3}} e^{i\beta} \end{pmatrix} P^\circ , \tag{51}$$

where $c_{23}^\nu \equiv \cos\theta_{23}^\nu$ and $s_{23}^\nu \equiv \sin\theta_{23}^\nu$.

Values of δ based on this group are **strongly disfavored by the current data** (but not completely ruled out yet).

Example of discrete groups

A4 : (even permutation of 4 elements).

Subgroups : $G_e = Z_3^T = \{1, T, T^2\}, \quad G_\nu = Z_2^S = \{1, S\},$

We have the following equation for δ :

$$\cos\delta = \frac{\cos 2\theta_{23}\ \cos 2\theta_{13}}{\sin 2\theta_{23}\ \sin\theta_{13}\ (2 - 3\sin^2\theta_{13})^{\frac{1}{2}}},$$

$$U_{\rm PMNS} = U_\nu^\circ\, P^\circ = V_{\rm TBM}\, U_{13}(\theta_{13}^\nu, \alpha)\, P^\circ$$
$$= \begin{pmatrix} \sqrt{\frac{2}{3}}c & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}s\, e^{i\alpha} \\ -\frac{c}{\sqrt{6}} + \frac{s}{\sqrt{2}}e^{-i\alpha} & \frac{1}{\sqrt{3}} & -\frac{c}{\sqrt{2}} - \frac{s}{\sqrt{6}}\, e^{i\alpha} \\ -\frac{c}{\sqrt{6}} - \frac{s}{\sqrt{2}}e^{-i\alpha} & \frac{1}{\sqrt{3}} & \frac{c}{\sqrt{2}} - \frac{s}{\sqrt{6}}\, e^{i\alpha} \end{pmatrix} P^\circ, \quad (34)$$

where $c \equiv \cos\theta_{13}^\nu$ and $s \equiv \sin\theta_{13}^\nu$.

It leads also to a correlation of $\sin(\theta_{13}), \sin(\theta_{23})$, which means that a good measure of those angles plus a decent measure of δ could **verify or rule out** model based on the A4 group.

Example of discrete groups

Comparison of different symmetry patterns with the expected data from **future experiments**

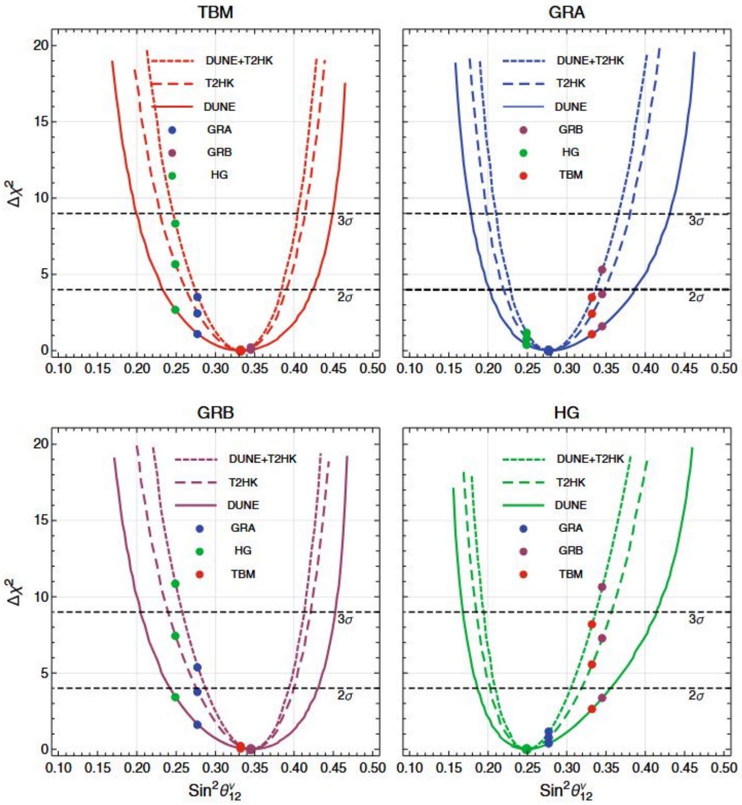

Fig. 3 Sensitivities of the experiments DUNE, T2HK and their combined (prospective) data to the symmetry form parameter $\sin^2\theta^{\nu}_{12}$ allowing to distinguish between the TBM, GRA, GRB, and HG symmetry forms under the assumption that one of them is realised in Nature. In the top left and right panels the assumed true symmetry forms are respectively TBM ($\sin^2\theta^{\nu}_{12} = 1/3$) and GRA ($\sin^2\theta^{\nu}_{12} = 0.276$), while in the bottom left and right panel these forms are GRB ($\sin^2\theta^{\nu}_{12} = 0.345$) and HG ($\sin^2\theta^{\nu}_{12} = 0.25$). See text for further details. (From Ref. [41].)

Generalised CP symmetry

With the previous models, the Majorana phases remain undetermined.

Their values are instead constrained by a **Generalised CP symmetry** (GCP) combining CP symmetry with flavour symmetry.

$$\tilde{l}_L(x) \xrightarrow{CP} i(X_L)_{\tilde{l}\tilde{l}'}\gamma_0 C\, \overline{\tilde{l}'_L(x')}^T,$$
$$\tilde{l}_R(x) \xrightarrow{CP} i(X_R)_{\tilde{l}\tilde{l}'}\gamma_0 C\, \overline{\tilde{l}'_R(x')}^T,$$
$$\nu_{\tilde{l}L}(x) \xrightarrow{CP} i(X_L)_{\tilde{l}\tilde{l}'}\gamma_0 C\, \overline{\nu_{\tilde{l}'L}(x')}^T,$$

Conclusion

Sufficiently precise measurement of the Dirac phase of the PMNS matrix as well as a refinement of the measures of the mixing angle will **provide informations as to which discrete group is correct** for the eventual flavour symmetry.