Self-consistent simulations of the radiative transfer in gamma rays in neutron star magnetospheres

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Pulsar ARoMA: modeling of the system

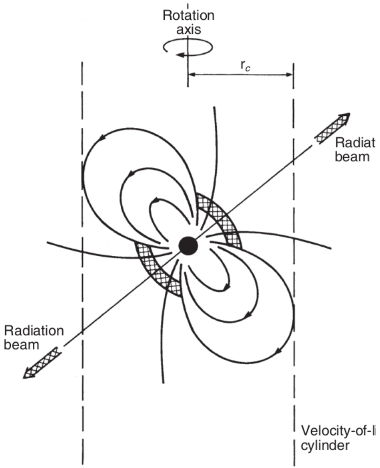


Figure 1: Pulsar magnetosphere. Credits: [Lyne, 2012].

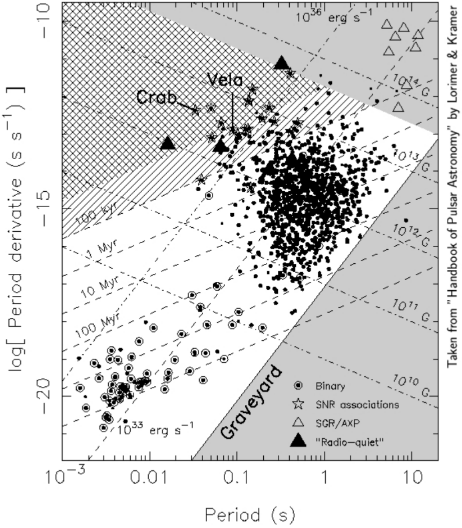
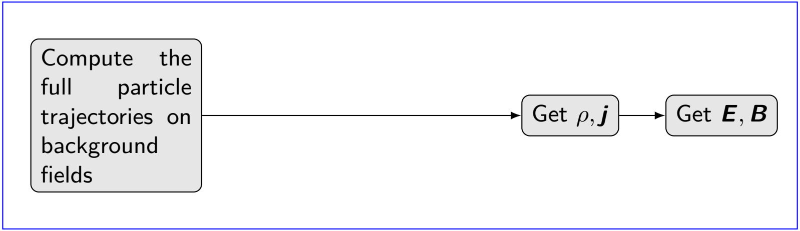


Figure 2: $P - \dot{P}$ diagram [Lorimer and Kramer, 2004].

Pulsar ARoMa: iteration scheme for stationary solutions

Pulsar Aroma [Mottez, 2024] computes self-consistent stationary solutions of electrospheres or magnetospheres.

repeat at each iteration until convergence



- f: distribution function of the particles
- ρ: charge density; $\boldsymbol{j}$: current density
- $\boldsymbol{E}$: electric field; $\boldsymbol{B}$: magnetic field

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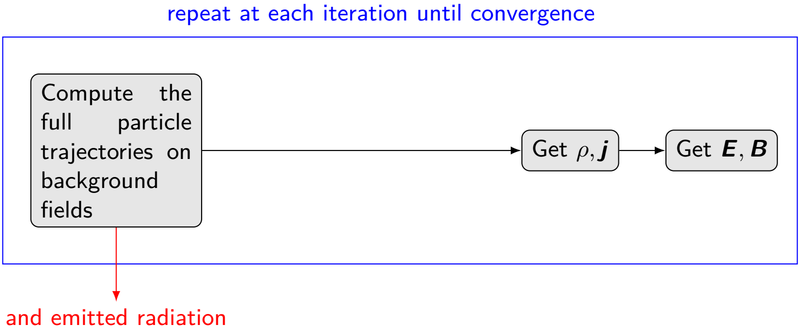

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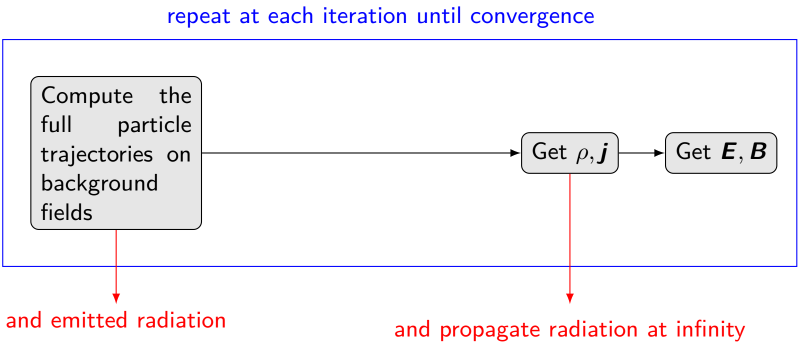

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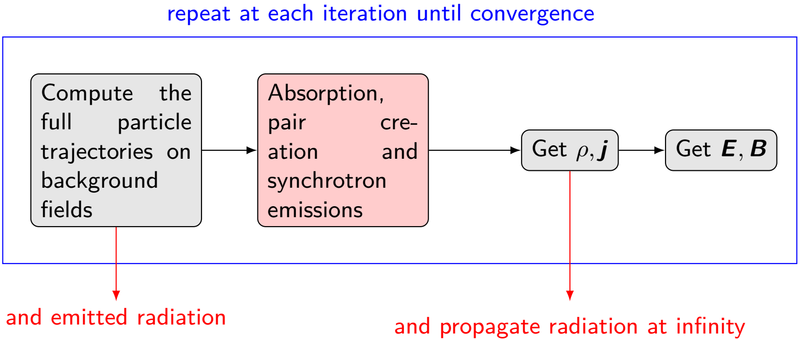

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We solve the radiative transfer equation for pure absorption,

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\alpha_\nu I_\nu \quad . \tag{1}$$

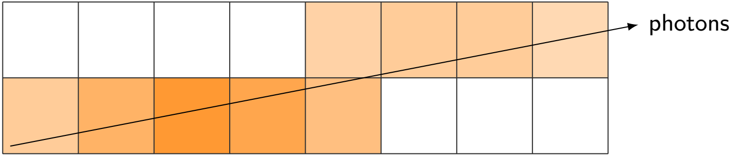

Figure 3: Trajectory of a photon packet and its 'peeled off' radiation.

Pair creation

- Photon-magnetic field pair creation:

$$\gamma + B \longrightarrow e^+ + e^- \quad .$$

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- From [Erber, 1966] and [Daugherty and Harding, 1983],

$$\alpha_{\nu 0} = \frac{\alpha B_0'}{\lambdabar} \begin{cases} 0.23 \exp -\frac{4(1+\nu_0'^{-2.7} B_0'^{-0.0038})}{3\chi_0} & , \quad \chi_0 \ll 1 \\ 0.30 \chi_0^{-\frac{1}{3}} \quad , \quad \chi_0 \gg 1 \quad , \end{cases} \tag{2}$$

with $\chi_0 = \nu_0' B_0'$, $\nu_0' = \frac{h\nu_0}{2m_e c^2}$ and $B_0' = \frac{B_0}{B_{\rm cr}}$.

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- The spectrum of the pairs in the frame of the observer $(\mathcal{R})$ is,

$$\frac{\mathrm{d}N}{\mathrm{d}\gamma} = \int \underbrace{\frac{1}{\alpha_\nu} \frac{\mathrm{d}\alpha_\nu}{\mathrm{d}\gamma}}_{\text{probability density}} \underbrace{\frac{E_{\nu \rm abs}}{h\nu}}_{\text{number of photons}} \mathrm{d}\nu \quad . \tag{3}$$

Population of pair created and synchrotron emission

- We treat the synchrotron radiation right after the creation because the cooling time is negligible ($\lesssim 10^3 \dfrac{m_e c}{qB} \sim 10^{-14}$ s).

$$e^+ + e^- \longrightarrow e^+ + e^- + 2\gamma_{\mathrm{sync}} \quad . \tag{4}$$

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$$E_{\nu \rm sync} = \int_0^{+\infty} \frac{2\sqrt{3}\pi q^2 \gamma}{c} \mathrm{F}\left(\frac{\nu}{\nu_c}\right) \frac{\partial N}{\partial \gamma} \mathrm{d}\gamma \quad . \tag{5}$$

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- We account for the quantum recoil so [Voisin et al., 2017],

$$\gamma = \frac{\gamma_{\mathrm{clas}}}{\sqrt{1 + \frac{h\nu}{\gamma_{\mathrm{clas}} m c^2}}} \quad . \tag{6}$$

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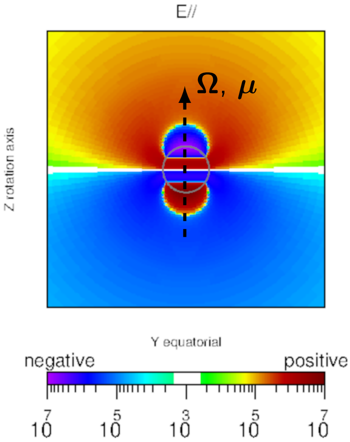
Cascade toy model in an aligned pulsar magnetosphere

Take a test particle at any position $\boldsymbol{r}$ in the magnetosphere.



$$E_{\parallel} = \frac{\boldsymbol{E} \cdot \boldsymbol{B}}{\|\boldsymbol{B}\|} \quad , \quad (\boldsymbol{E}, \boldsymbol{B}) \text{: vaccum fields.}$$

Cascade toy model in an aligned pulsar magnetosphere

Take a test particle at any position $\boldsymbol{r}$ in the magnetosphere.

↓

Compute the maximum possible Lorentz factor γ_m.

$$\gamma_m = \left(\frac{3}{2}\frac{E_{\parallel}}{qC^2}\right)^{\frac{1}{4}}$$

C is the local curvature of the field line.

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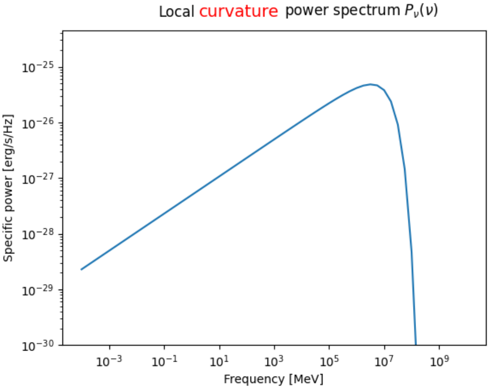
Deduce the local curvature power spectrum $\mathcal{P}_\nu$.



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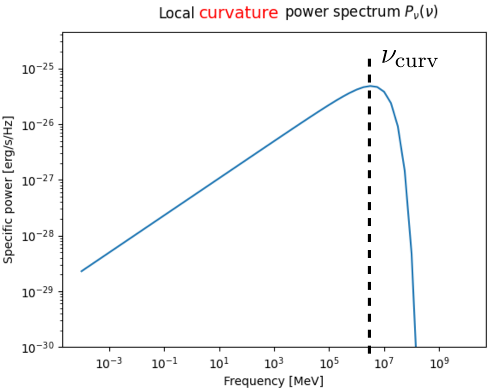
Propagate a new photon packet at the frequency $\nu_{\rm em} = \nu_{|\max(\mathcal{P}_\nu)}$.



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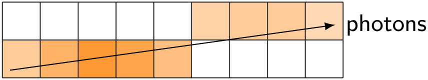
Derive the pair and synchrotron spectra.



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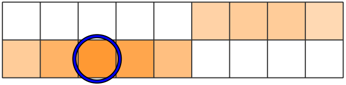

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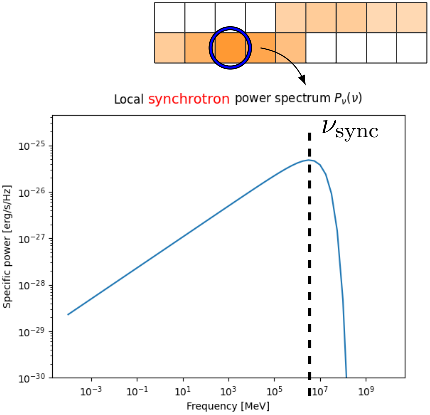


Loop over new photons created during the cascade until the photons escape the system.

Cascade toy model in an aligned pulsar magnetosphere

Take a test particle at any position $\boldsymbol{r}$ in the magnetosphere.

↓

Compute the maximum possible Lorentz factor γ_m.

↓

Deduce the local curvature power spectrum $\mathcal{P}_\nu$.

↓

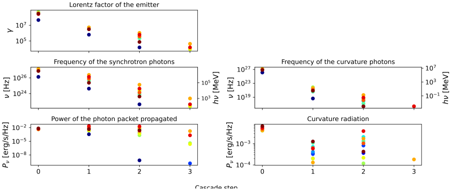
Propagate a new photon packet at the frequency $\nu_{\rm em} = \nu_{|\max(\mathcal{P}_\nu)}$.

↓

Derive the pair and synchrotron spectra.

(loops back to "Propagate a new photon packet")

Cascade ends.



Cascade toy model in an aligned pulsar magnetosphere

Take a test particle at any position $\boldsymbol{r}$ in the magnetosphere.

Compute the maximum possible Lorentz factor γ_m.

Deduce the local curvature power spectrum $\mathcal{P}_\nu$.

Propagate a new photon packet at the frequency $\nu_{\mathrm{em}} = \nu_{|\max(\mathcal{P}_\nu)}$.

Derive the pair and synchrotron spectra.

Loop over all user specified positions in the magnetosphere to initiate new cascades.

Cascade ends.

Cascade efficiencies as a function of the initial position

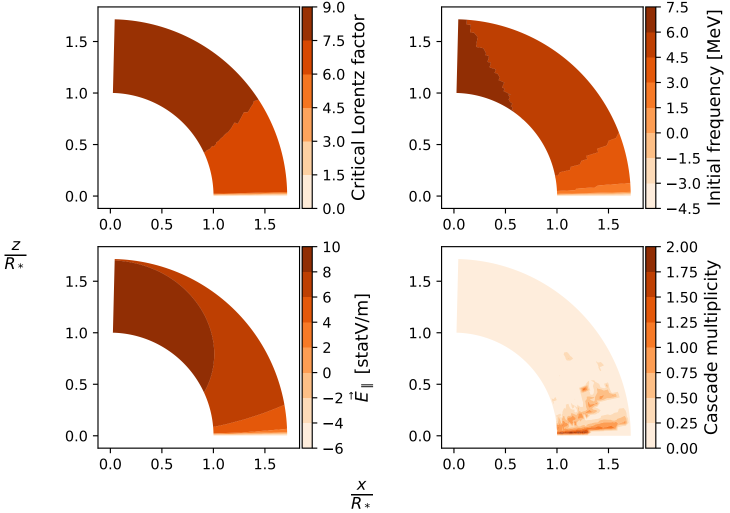


Figure 4: Upward cascades with different starting positions for particles, in a magnetosphere with $B = 10^{11}$ G and $P = 30$ ms.

Cascades in the polar cap and associated radiation

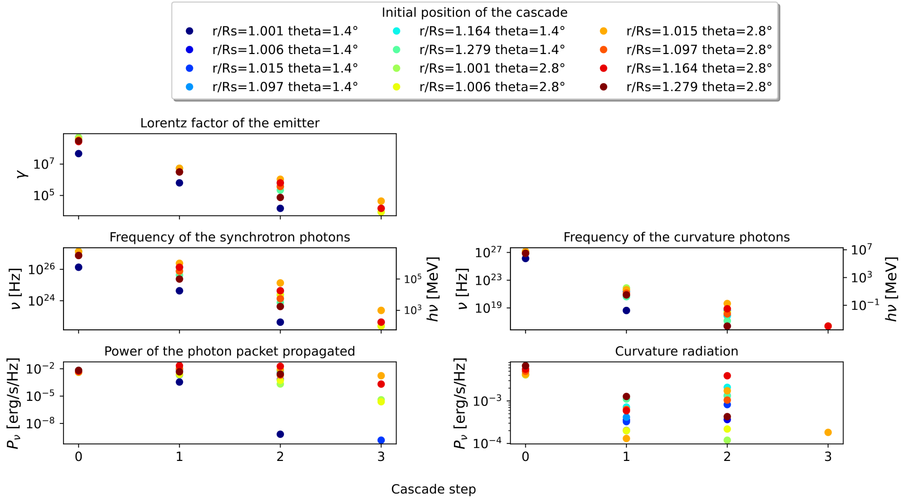


Figure 5: Upward cascades with different starting positions in the polar cap of a magnetosphere with $B = 10^{11}$ G and $P = 30$ ms.

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Conclusion

- Regions of high curvature are more efficiently filled with pairs.
- Synchrotron photons that escape the polar cap are mostly in the range $[1, 10]$ GeV.
- The cascades need to be more efficient to reproduce pulsars usual luminosities ($\gtrsim 10^{33}$ erg s^{-1}).
- With $B = 10^{12}$ G our computation of the cascades diverge in some regions of the magnetospheres.
- Taking into account quantum recoil in synchrotron radiation prevent some cascade divergence but not all of them.
- We are working on the implementation of the general case in Pulsar ARoMA.

Thank you for attention !

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