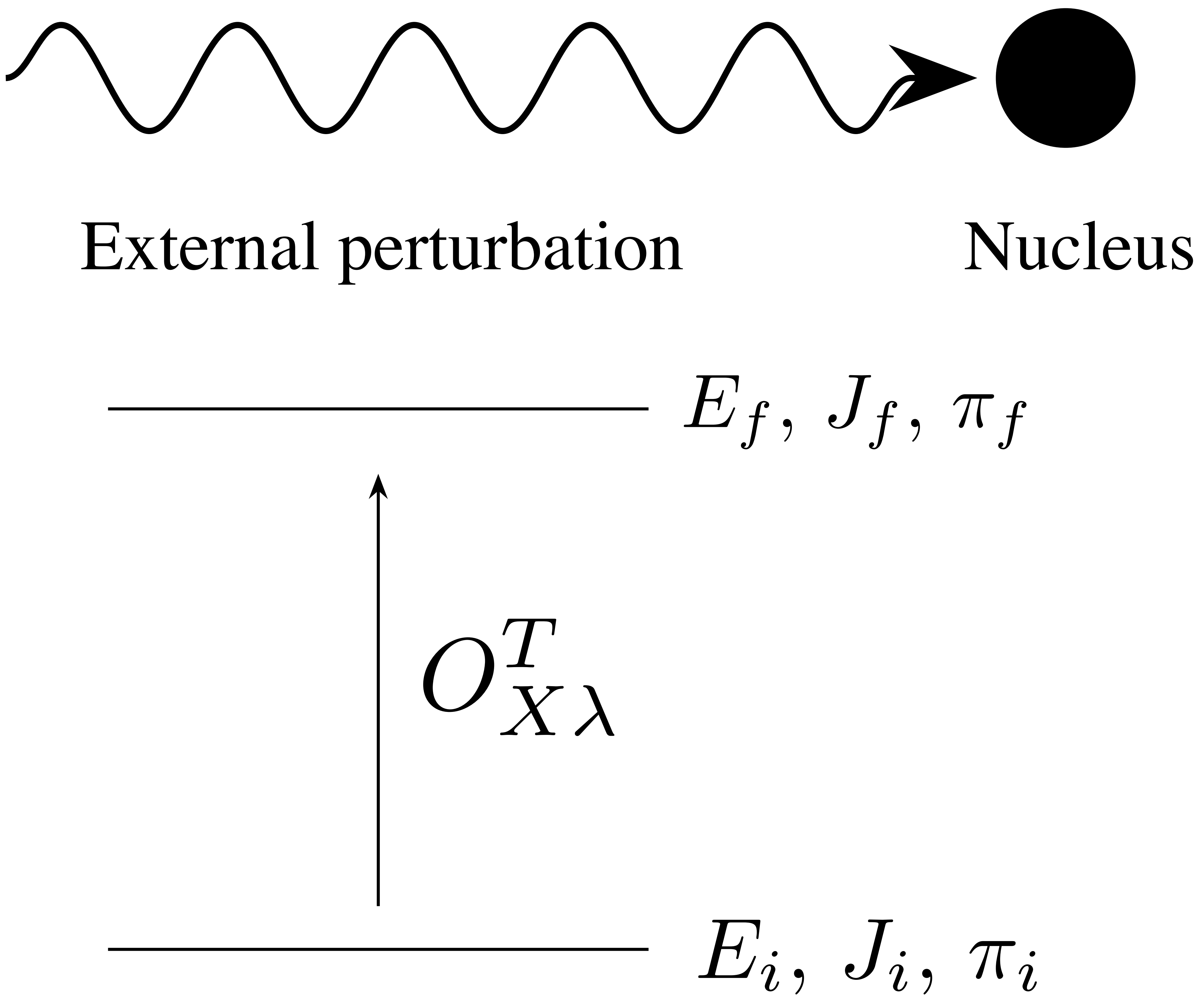
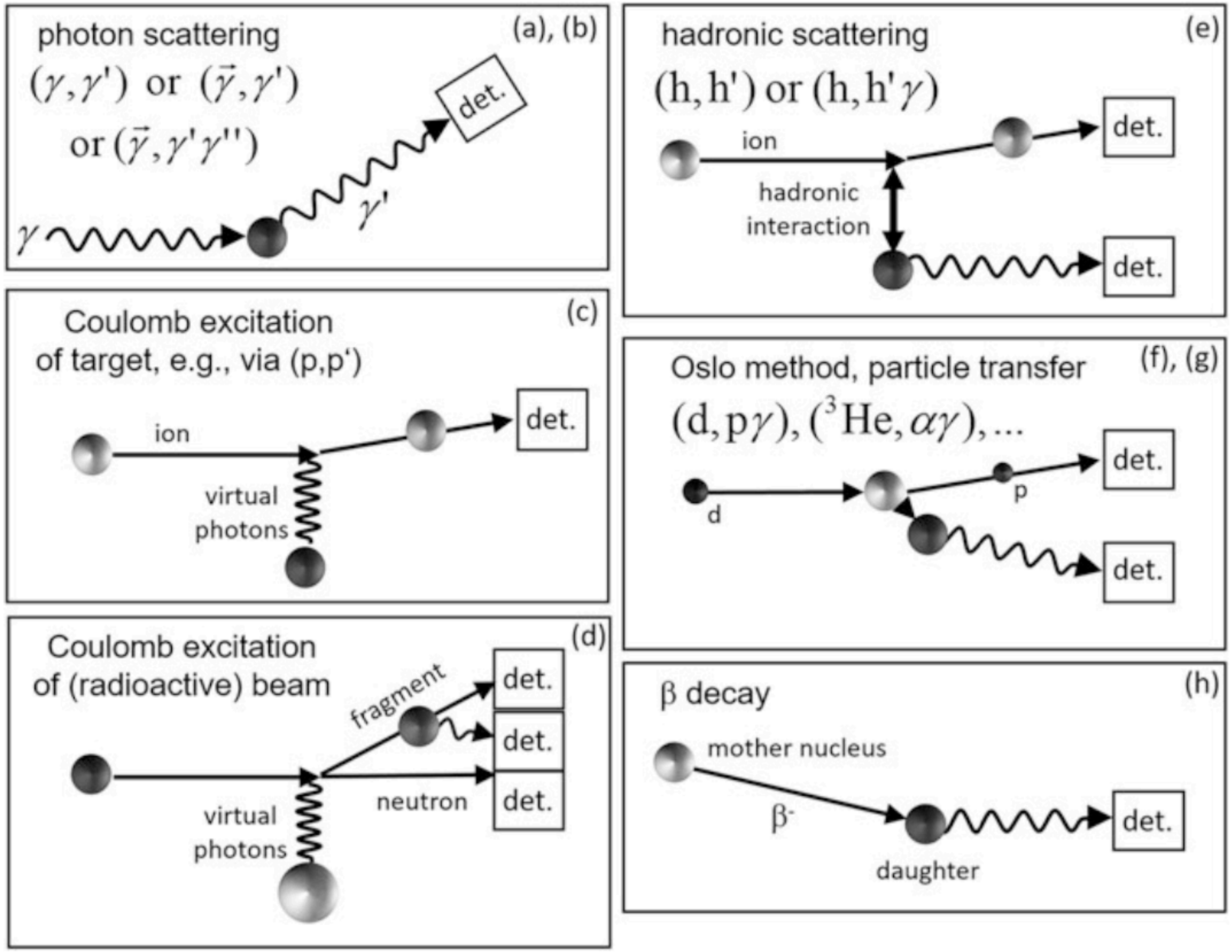
A Configuration-Interaction Shell Model approach to the Pygmy Dipole Resonance

Oscar Le Noan

In collaboration with
Kamila Sieja

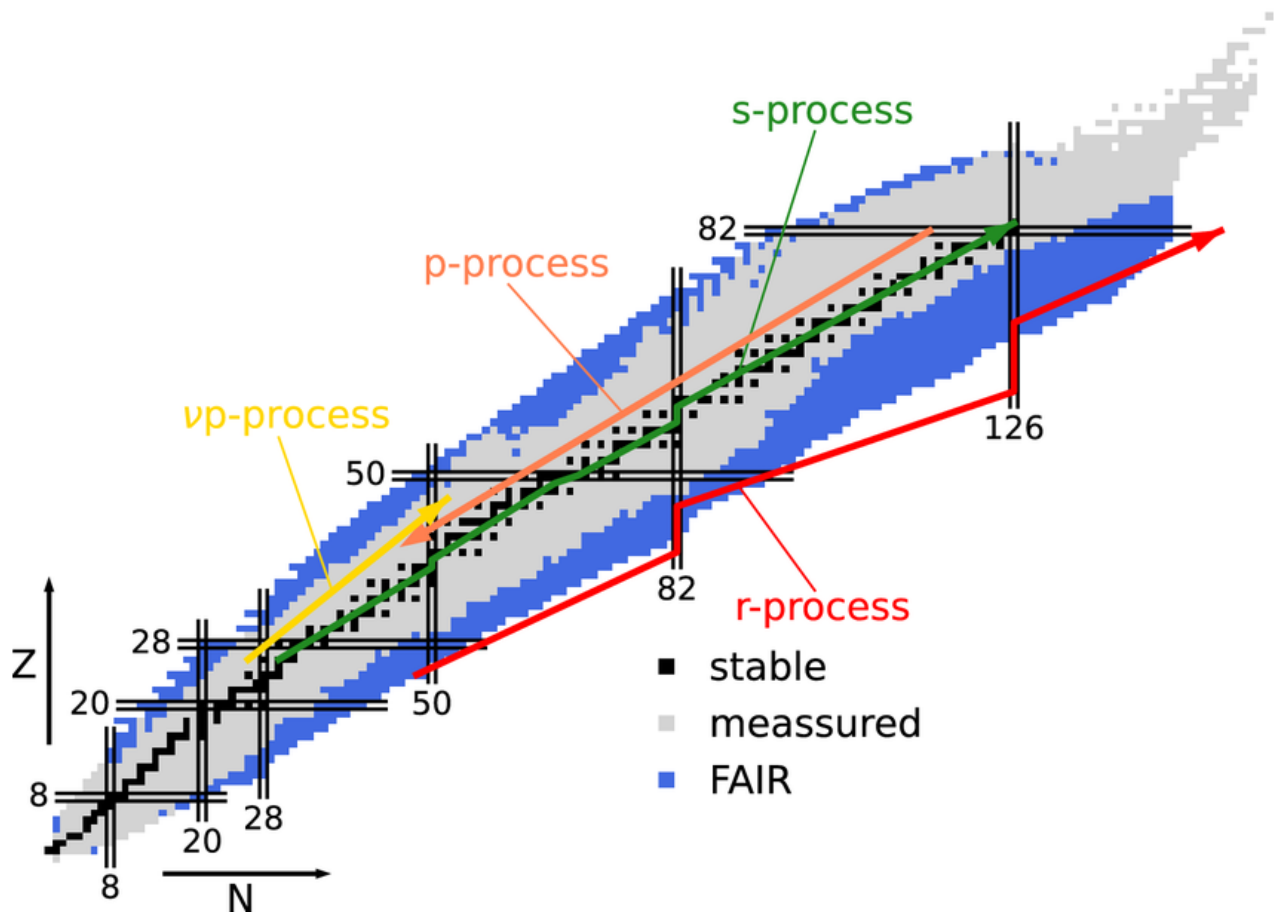
Nuclear electric dipole response (E1): definition





Nuclear dipole response: motivation

R-process nucleosynthesis



Neutron matter equation of state

$$\mathcal{E}(\rho,\alpha) = \mathcal{E}_{\mathrm{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$\rho = (\rho_n + \rho_p)$ $\alpha = (\rho_n - \rho_p)/\rho$

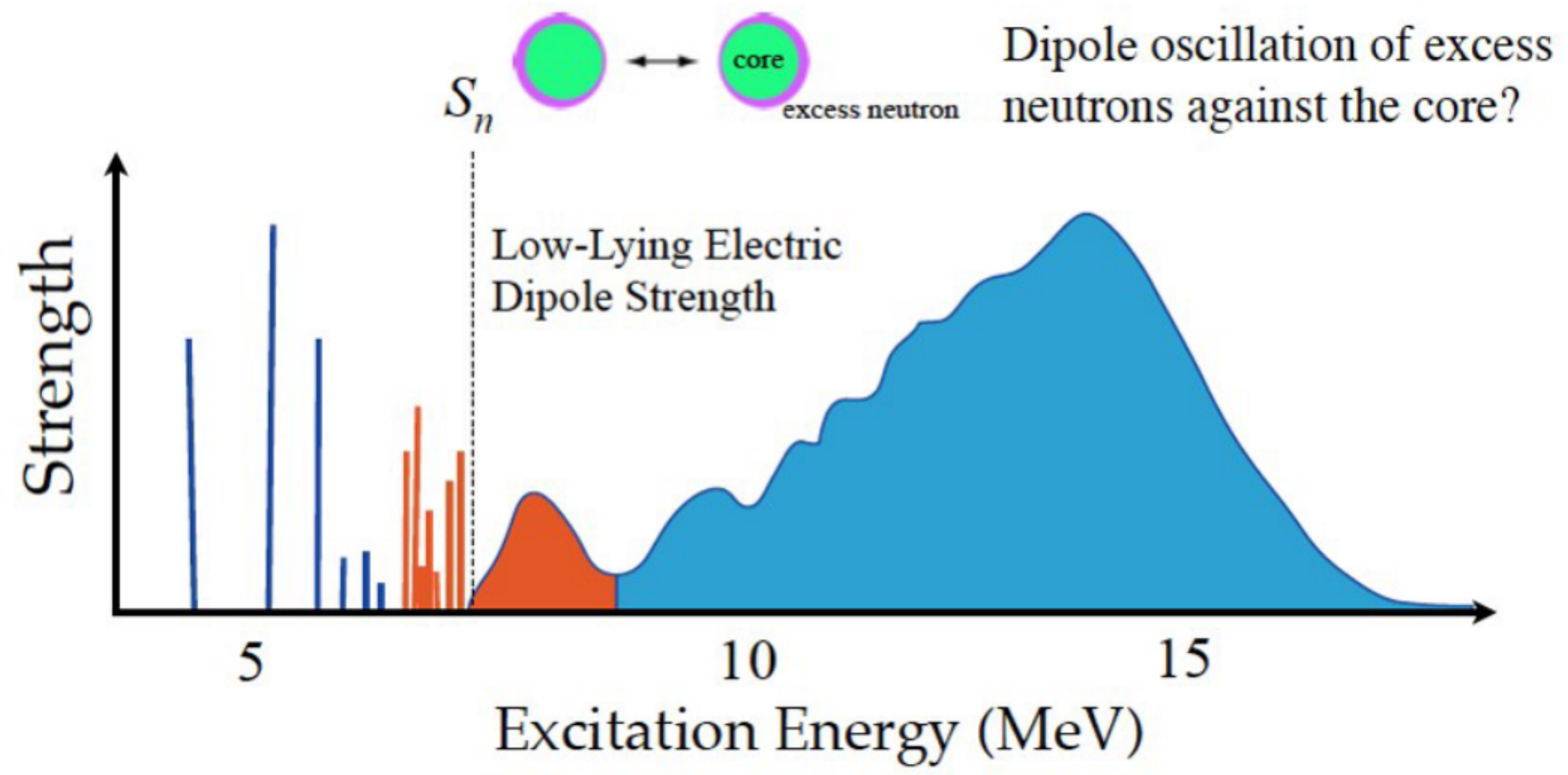
where the density-dependent symmetry energy is:

$$\mathcal{S}(\rho) = J + L\frac{(\rho-\rho_0)}{3\rho_0} + \ldots.$$

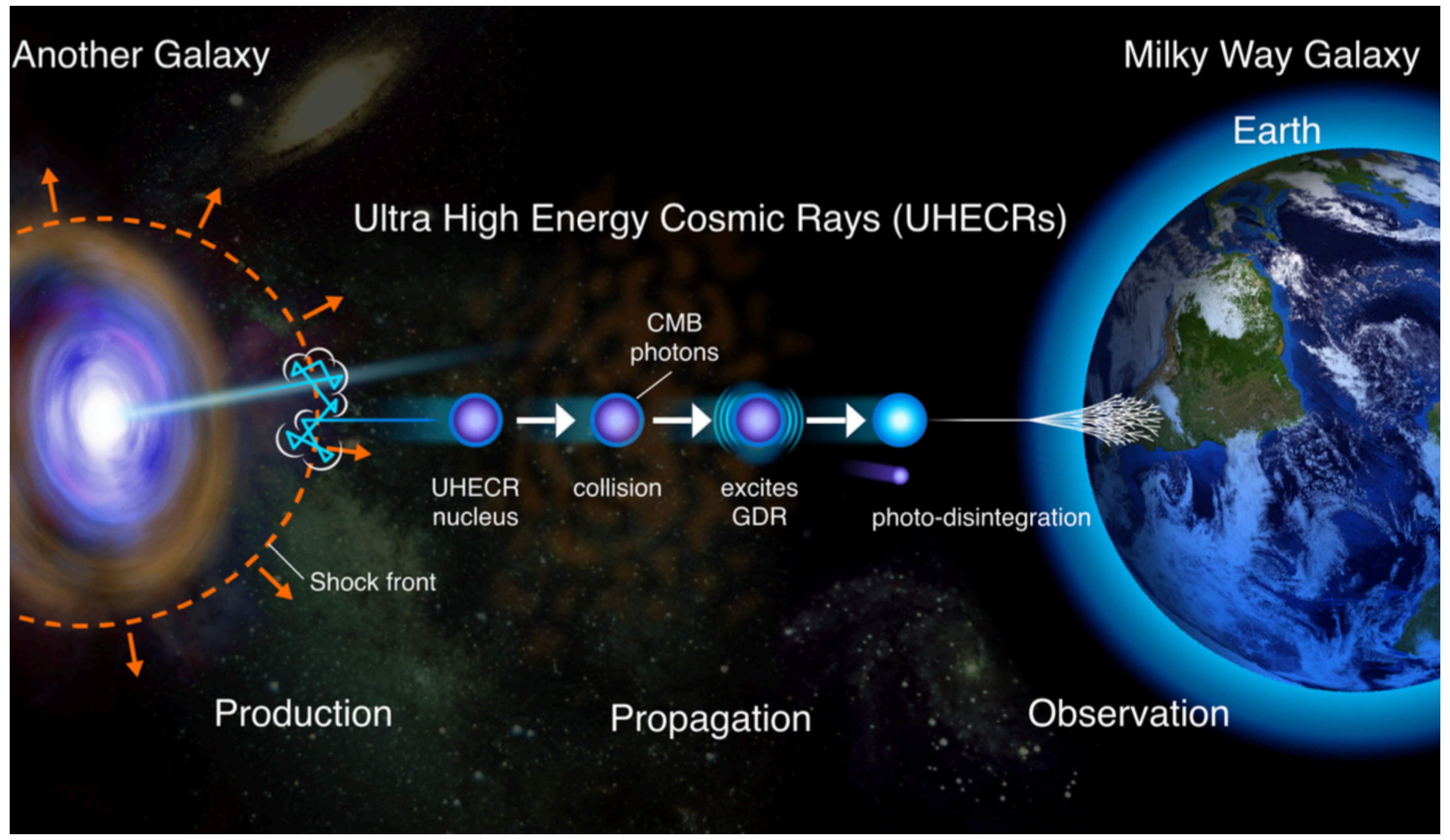
symmetry energy at saturation density

slope parameter, related to pressure of pure neutron matter at saturation density

Nuclear structure



Ultra High Energetic Cosmic Rays



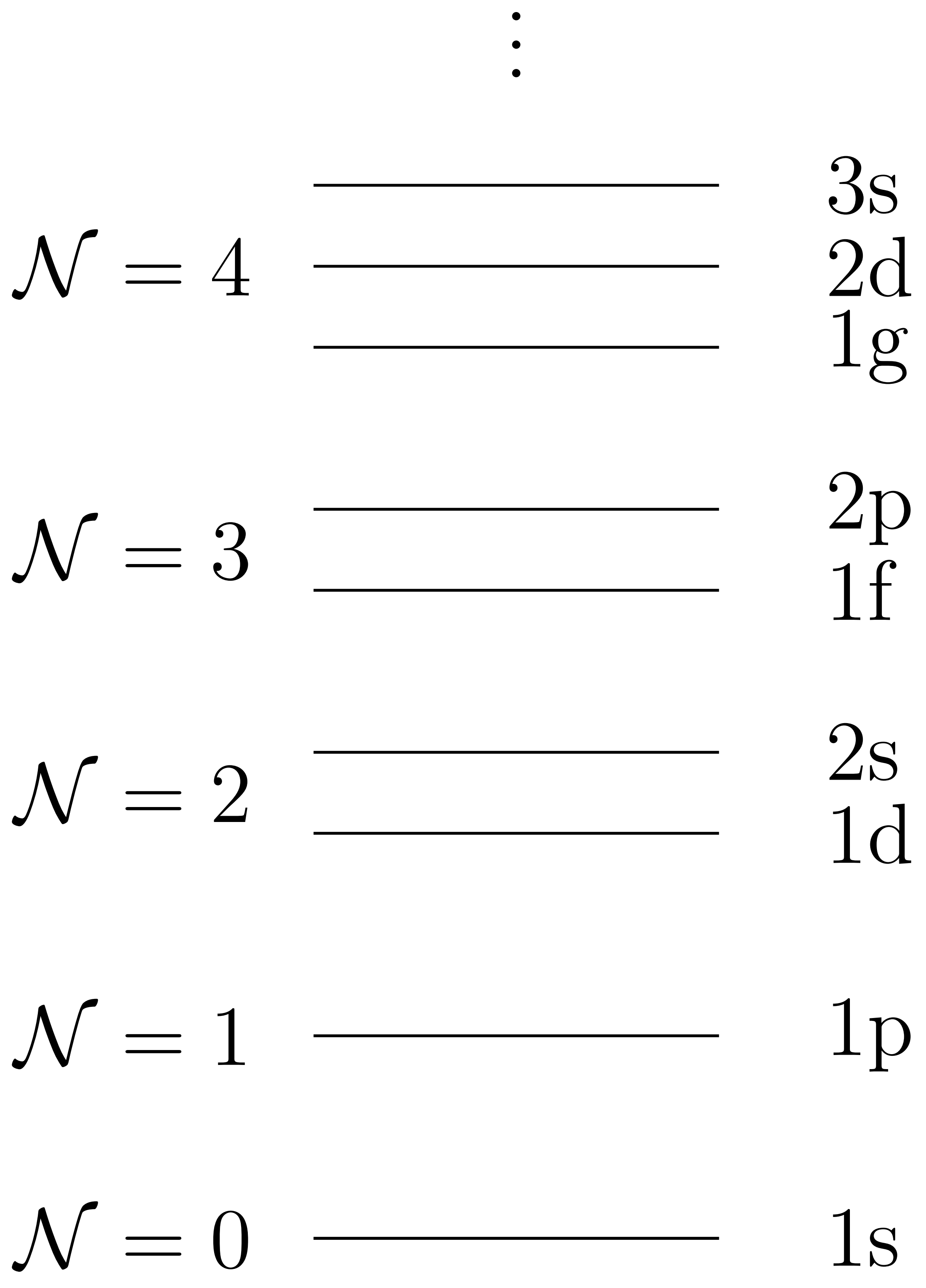
Theoretical framework: CI-SM

Quantity of interest: transition probability amplitudes

$$\langle \psi_f | |O_\lambda| |\mathrm{GS}\rangle$$

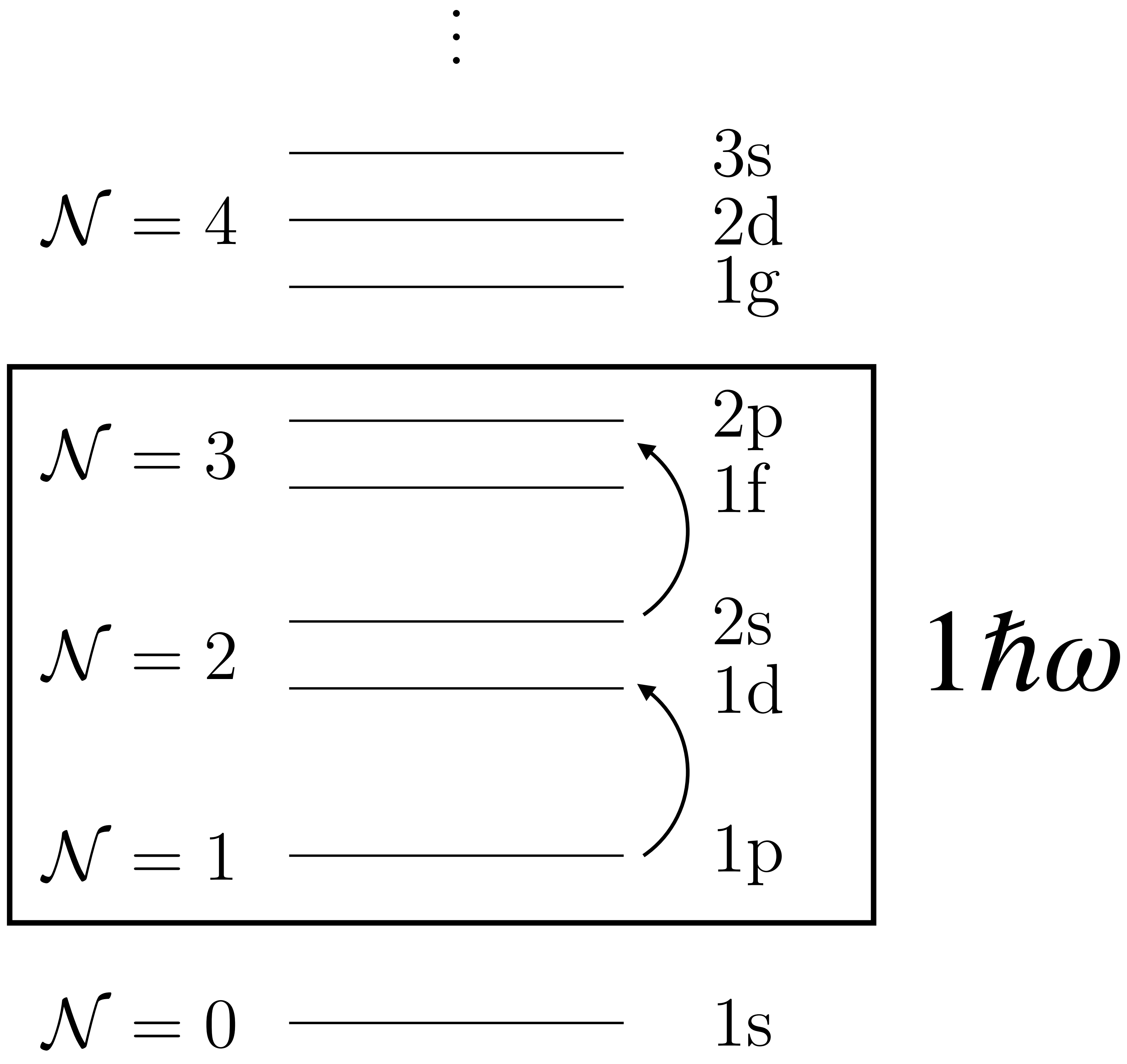
Discrete strength function

$$S(E) = \sum_f \frac{|\langle \psi_f | |O_\lambda| |\mathrm{GS}\rangle|^2}{2J_{\mathrm{GS}}+1}\delta(E_f - E)$$



Valence space mapping:
E1 sd-shell nuclei

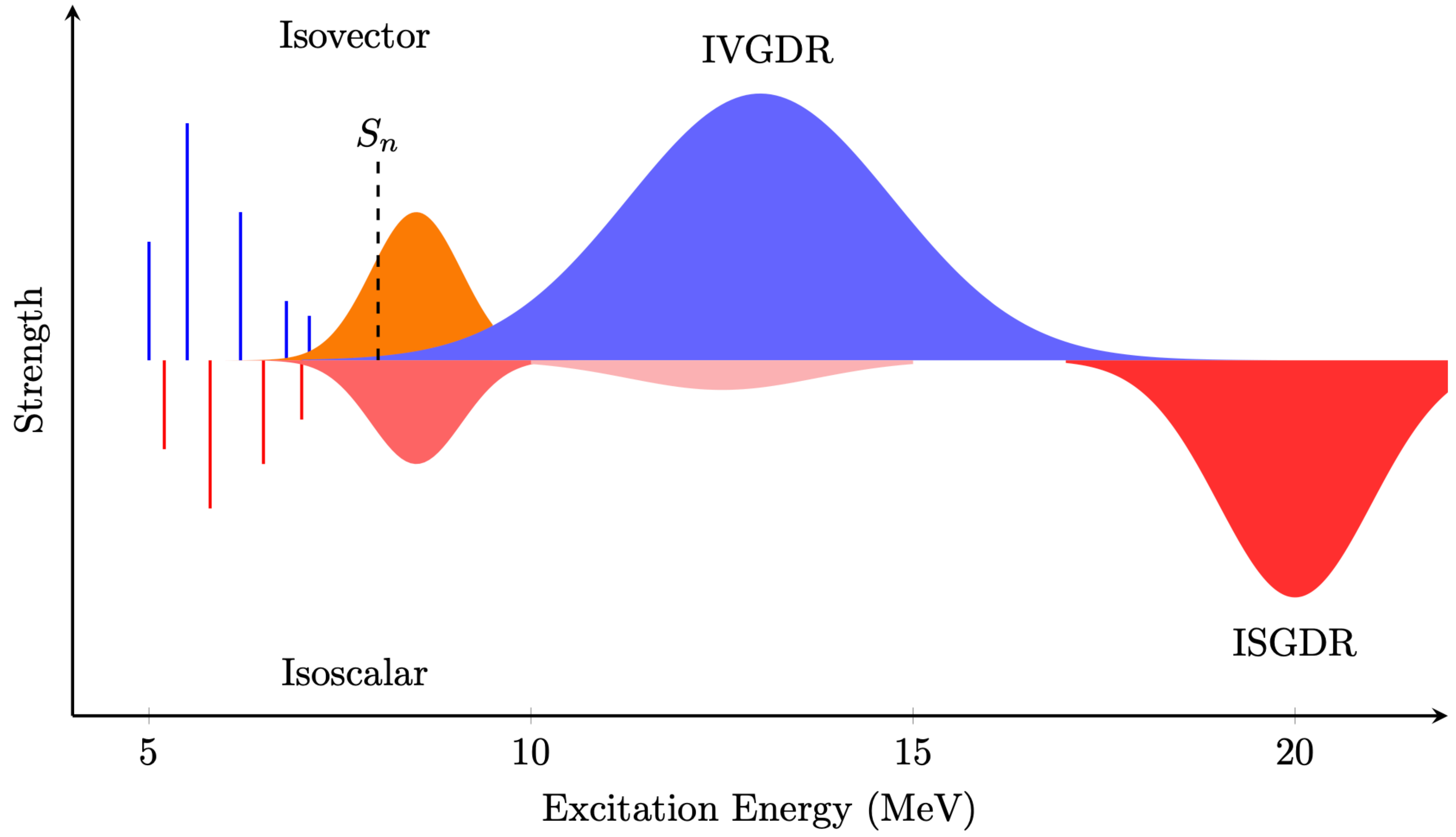
$$\mathcal{H} \longrightarrow \mathcal{H}_{\mathrm{valence}}$$
$$H \longmapsto H_{\mathrm{eff}}$$



Study of the Pygmy dipole resonance

Definition

Low-lying E1 strength appearing in neutron-rich nuclei

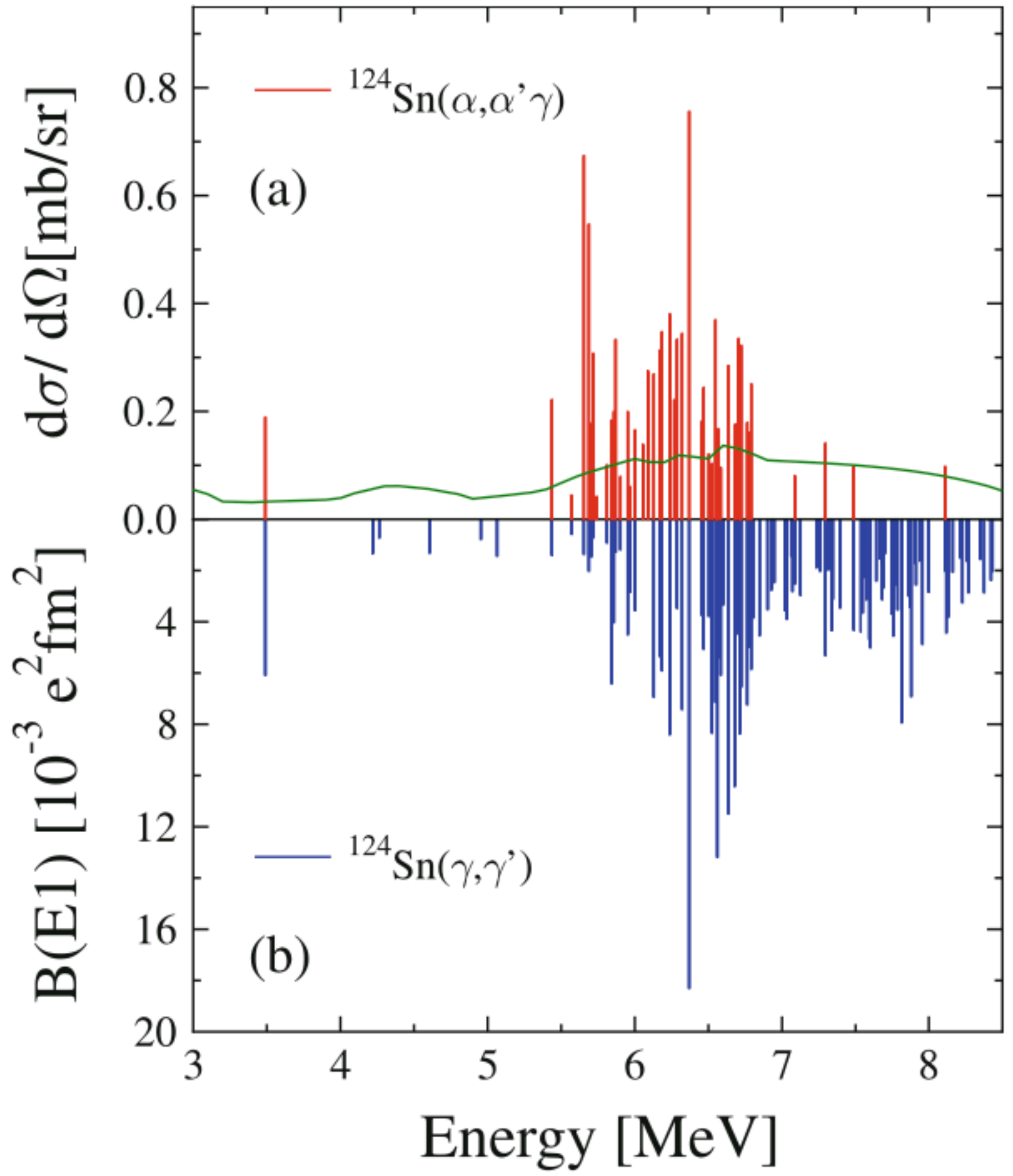


State of the art

- Observed only in neutron-rich nuclei, both stable and unstable.
- Found both below and above the particle emission threshold and exhibits strong isospin mixing.
- In medium-mass and heavy nuclei, there is an isospin splitting of the PDR.

$$O_{\mathrm{E1}} = O_{\mathrm{E1}}^{(0)} + O_{\mathrm{E1}}^{(1)}$$

Isoscalar $\sim r^3 Y_1$ Isovector $\sim r Y_1 \tau_z$

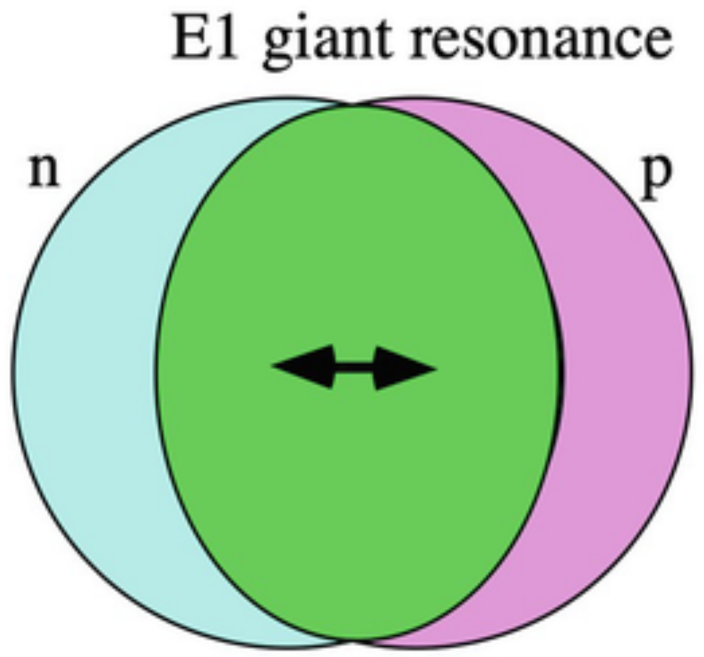


Study of the Pygmy dipole resonance

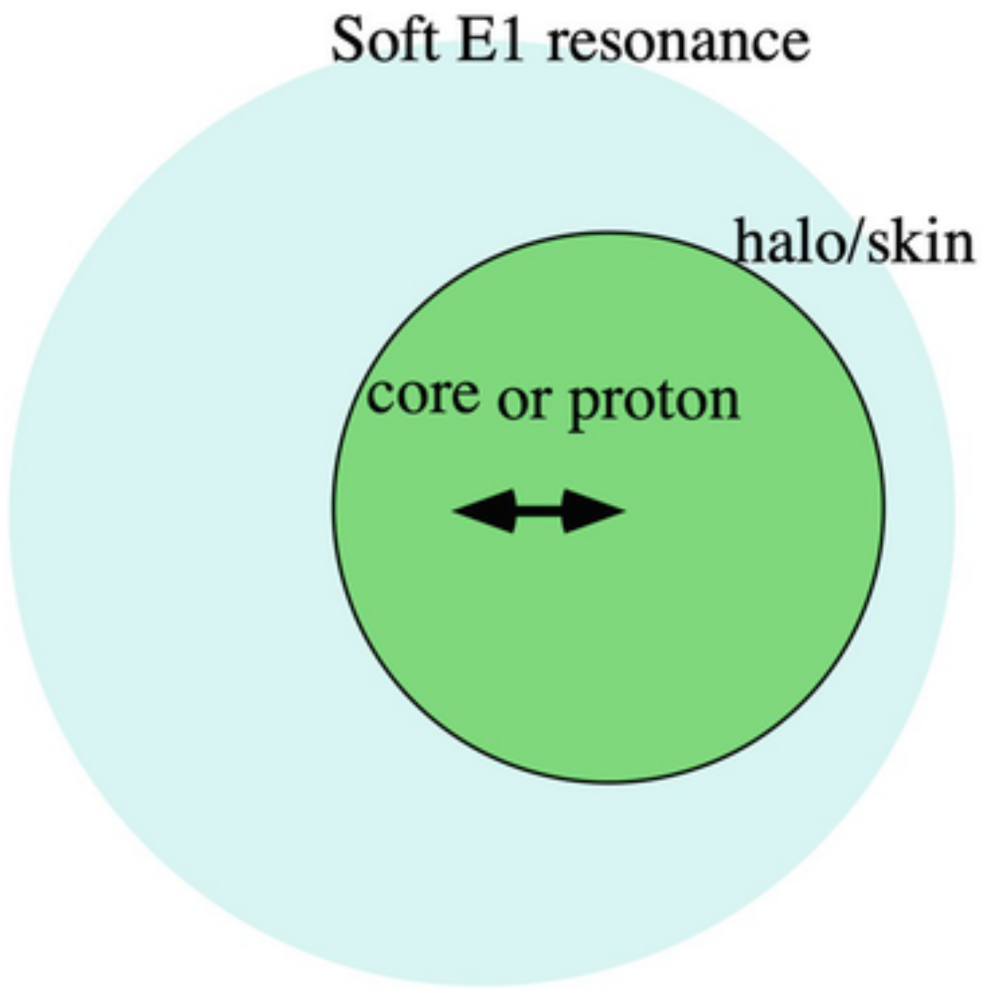
Open questions:

- Degree of collectivity ?
- Is the isospin splitting of the PDR an ubiquitous property ?
- Classical representation : neutron excess oscillation ?
- In the PDR energy region, are there states that cannot be interpreted as neutron-skin oscillation ?
- How is the transition between PDR and GDR states ?
- …

E1

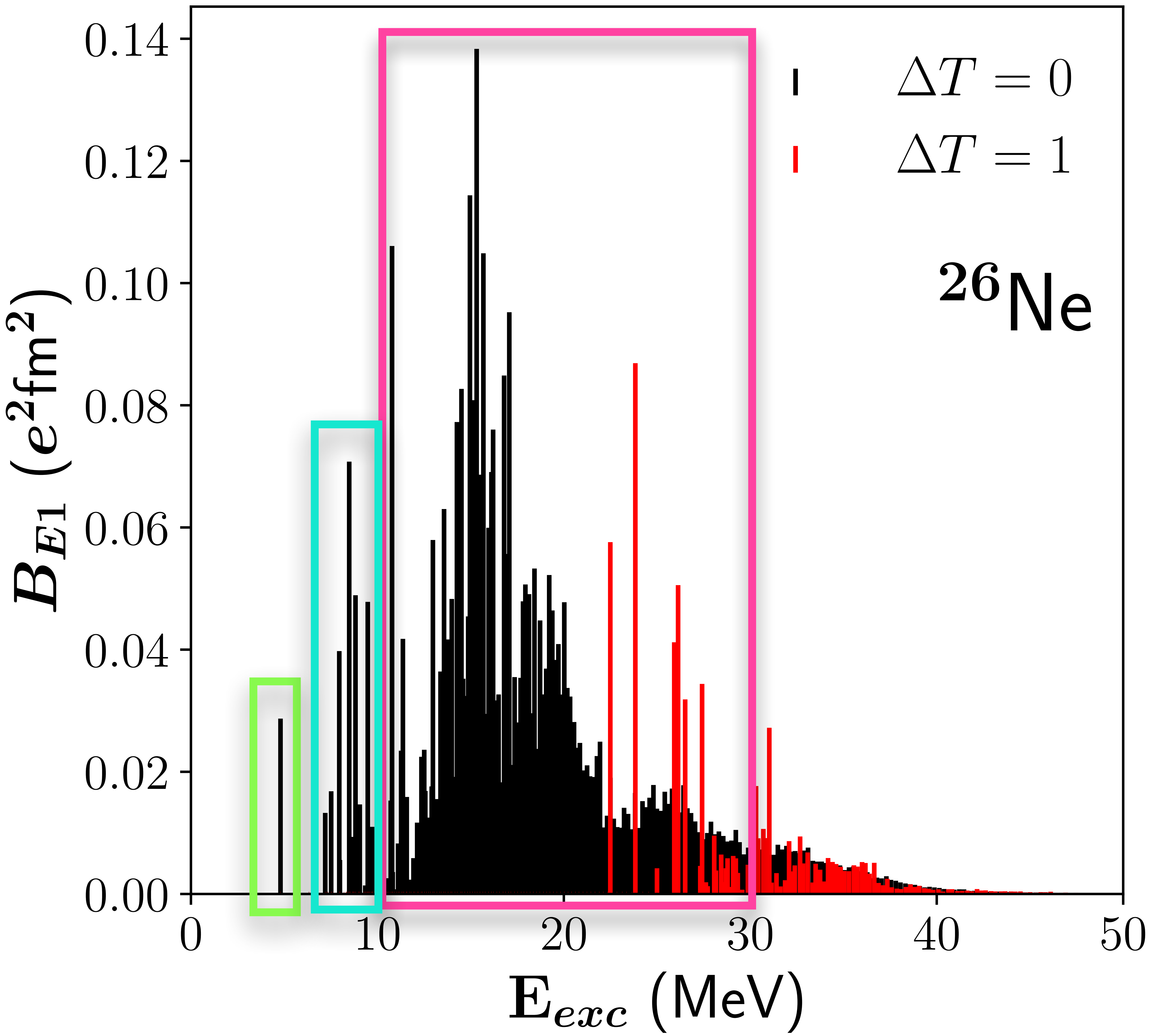


Soft E1



Study of the Pygmy dipole resonance

Comparison to experiment



CI-SM

$$\bar{S}_{\mathrm{PDR}} = 8.63 \text{ MeV}$$

$$S_0^{\mathrm{PDR}} = 0.37 \; e^2\mathrm{fm}^2$$

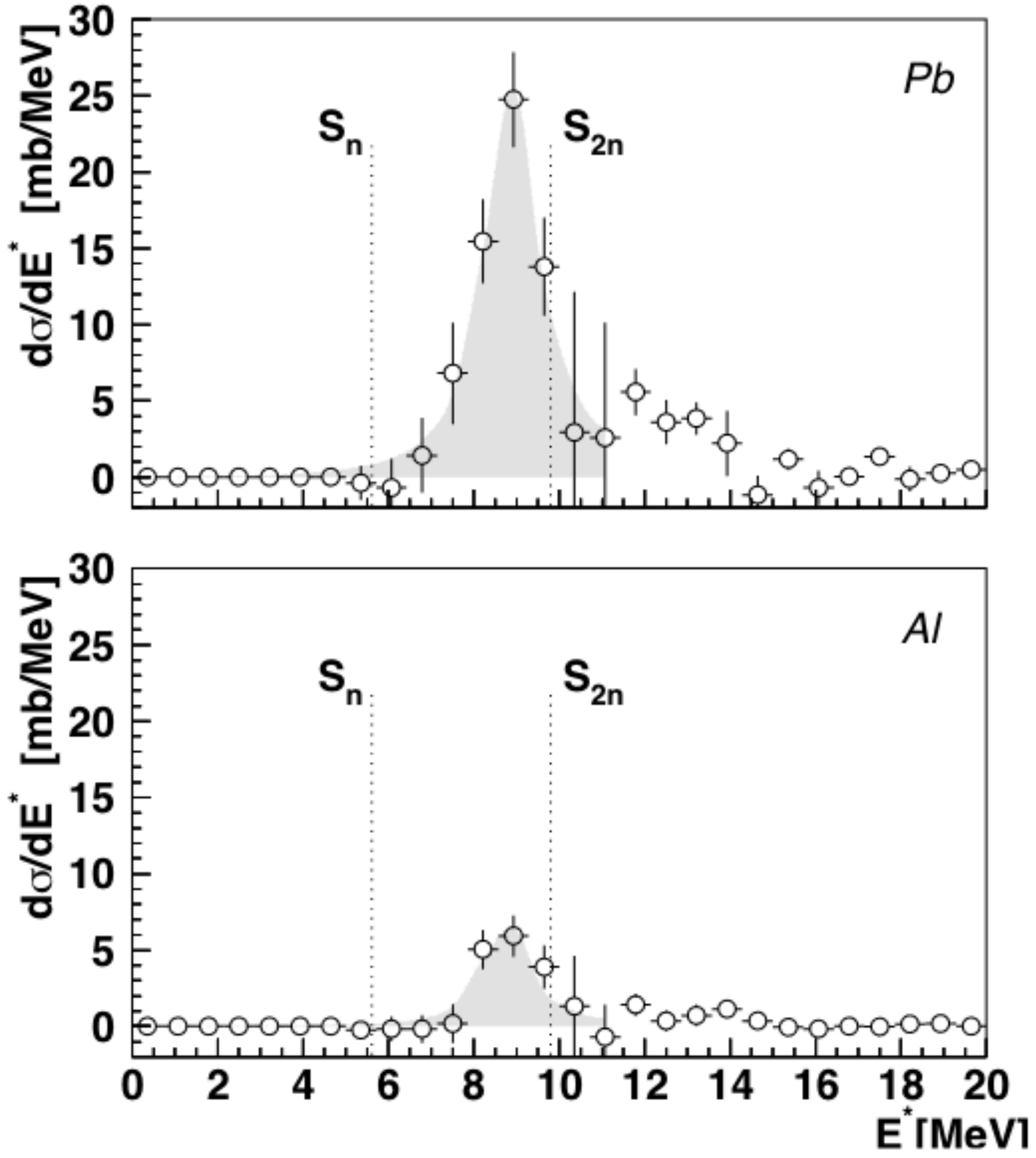
$\sim 4\%$ TRK

EXP

$$\bar{S}_{\mathrm{PDR}} \approx 9 \text{ MeV}$$

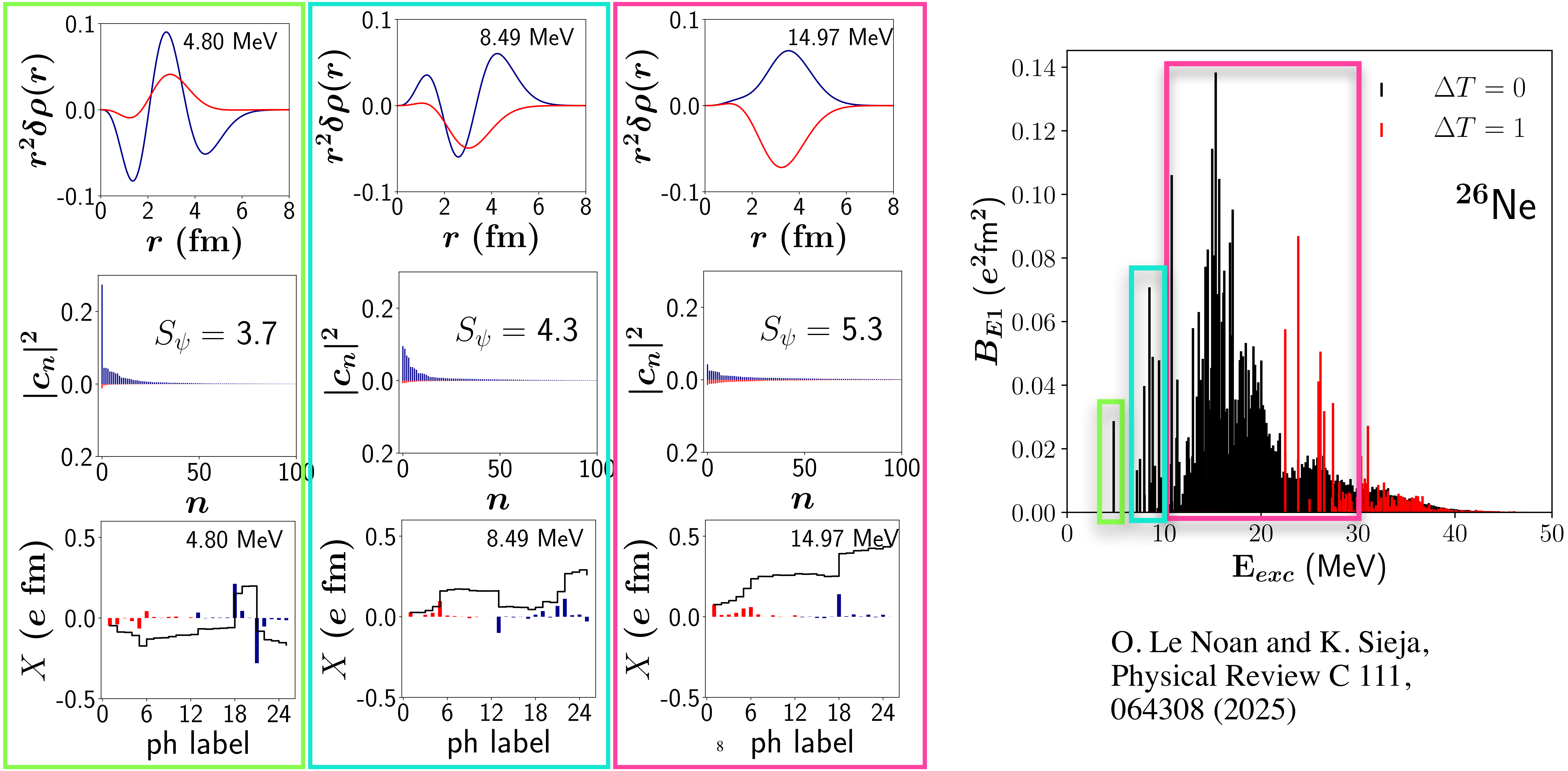
$$S_0^{\mathrm{PDR}} = 0.49 \pm 0.16 \; e^2\mathrm{fm}^2$$

$^{26}\mathrm{Ne} + \mathrm{Pb} \longrightarrow {}^{26}\mathrm{Ne}^*$



Gibelin, J. et al. Nuclear Physics A 2007, 788, 153–158.

Study of the Pygmy dipole resonance ^{26}Ne



O. Le Noan and K. Sieja, Physical Review C 111, 064308 (2025)

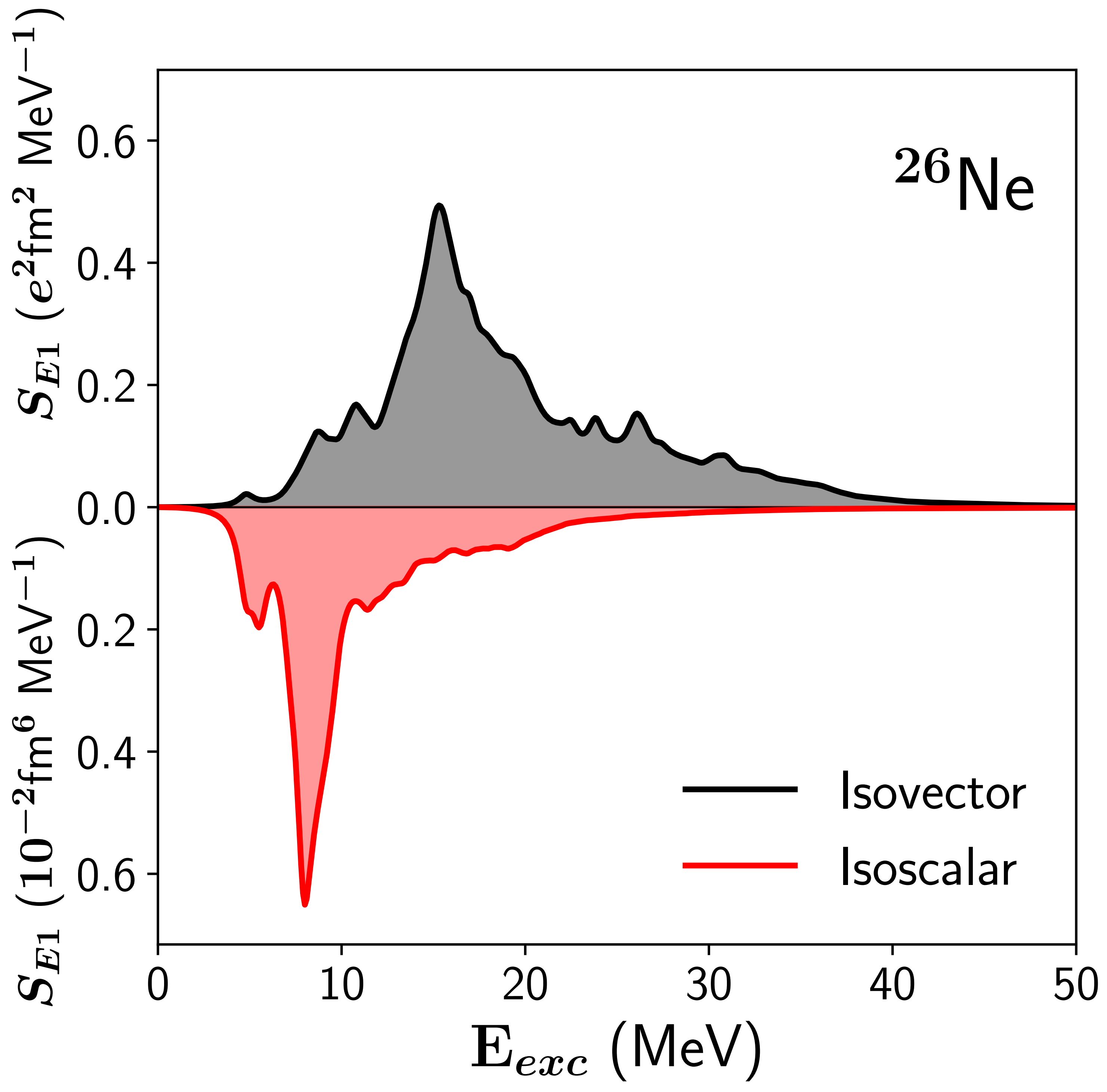
Study of the Pygmy dipole resonance ^{26}Ne

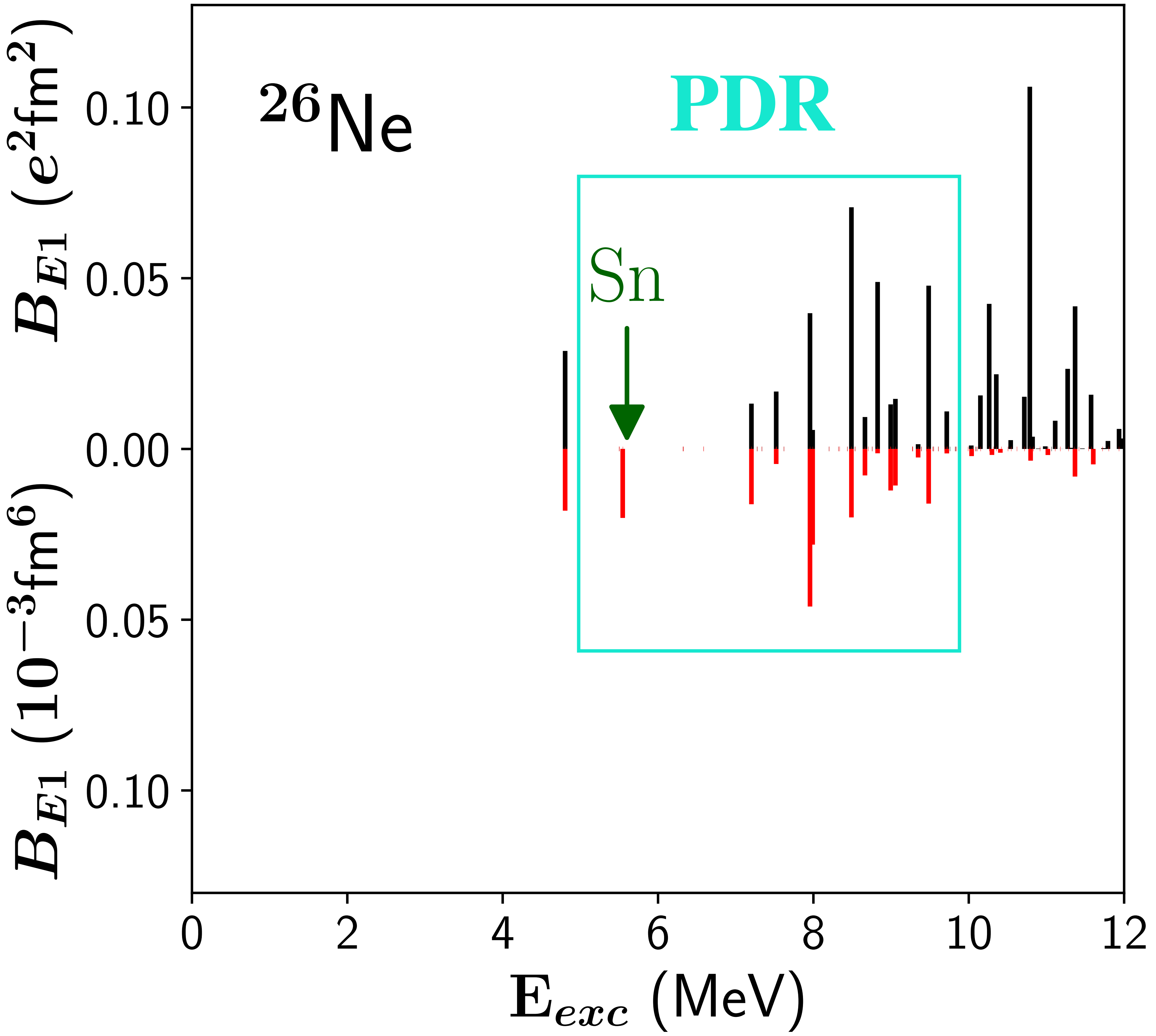
Isospin splitting of the PDR

$$O_{\mathrm{E1}} = O_{\mathrm{E1}}^{(0)} + O_{\mathrm{E1}}^{(1)}$$

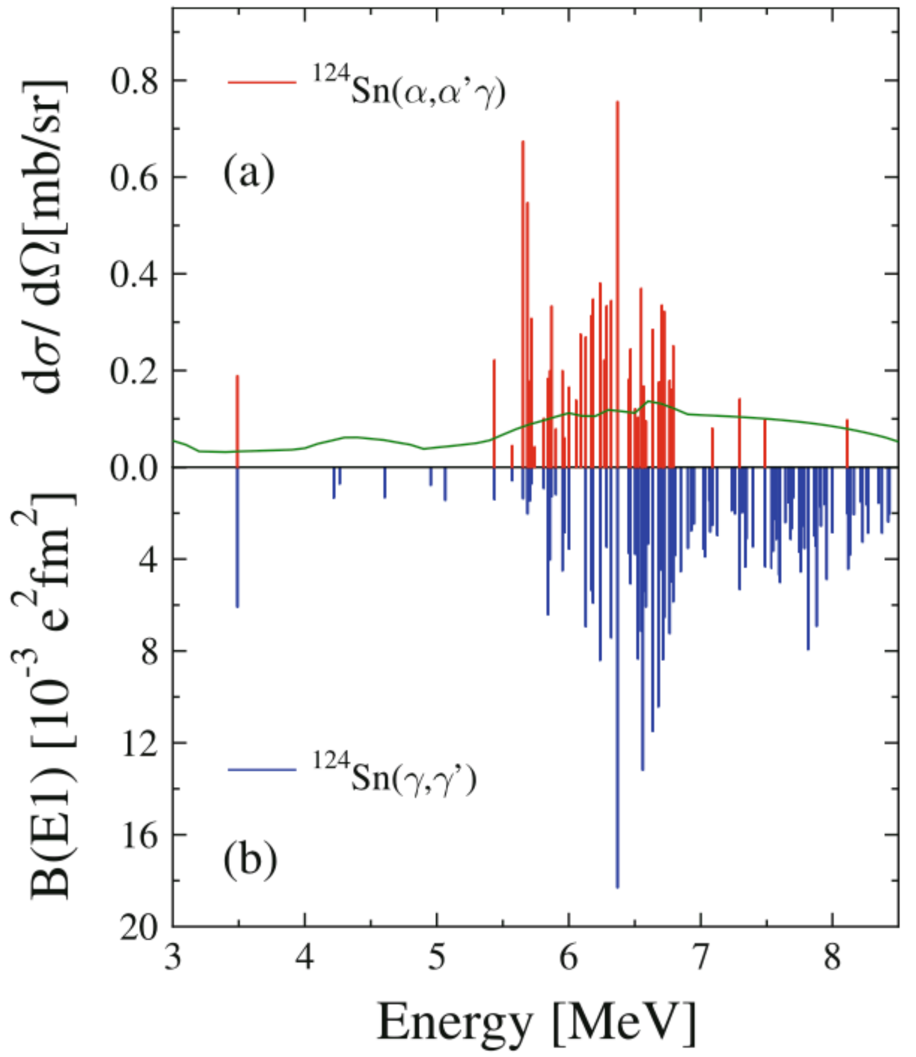
Isoscalar Isovector

$\sim r^3 Y_1$ $\sim r Y_1 \tau_z$





Experimentally:



Conclusion

Systematics & UHECR

- CI-SM provides good prediction of isovector E1 response for $A < 40$.
- UHECRs modelisation is essentially sensitive to the centroid position in nuclei close to stability.
- First CI-SM description of low-lying isoscalar E1 strength, good agreement with exp.
- Future work: CI-SM for pf-shell nuclei (A > 40).

PDR

- We predict three kinds of low-lying dipole state: quadrupole-octuple, IS-LED and Pygmy.
- The PDR is collective in a shell-model picture (< GDR).
- The classical picture of a neutron skin oscillating against an N=Z core holds.
- We predict an isospin splitting of the PDR above particle emission threshold (! depends on the definition of the PDR…)
- Future work: toroidal nature of the PDR, E1 from PGCM

Back up

Theoretical landscape

Models commonly used to predict PSFs

Purely phenomenological approaches

- Simple Modified Lorentzian (SMLO)
- Artificial Neural Network (ANN)

Microscopic approaches

Linear response:

- Quasi-particle Random Phase approximation (QRPA)
- Quasi-particle Finite Amplitude Method (QFAM)

Beyond linear response:

- Second QRPA (SQRPA)
- QRPA + phonon coupling
- Projected Generator Coordinates Method (PGCM)
- Configuration Interaction Shell Model (CI-SM)
- …

Theoretical framework: CI-SM

Dressing of the E1 operator $O(\mathrm{E1}) = e\sum_{j=1}^{Z} rY_{10}$

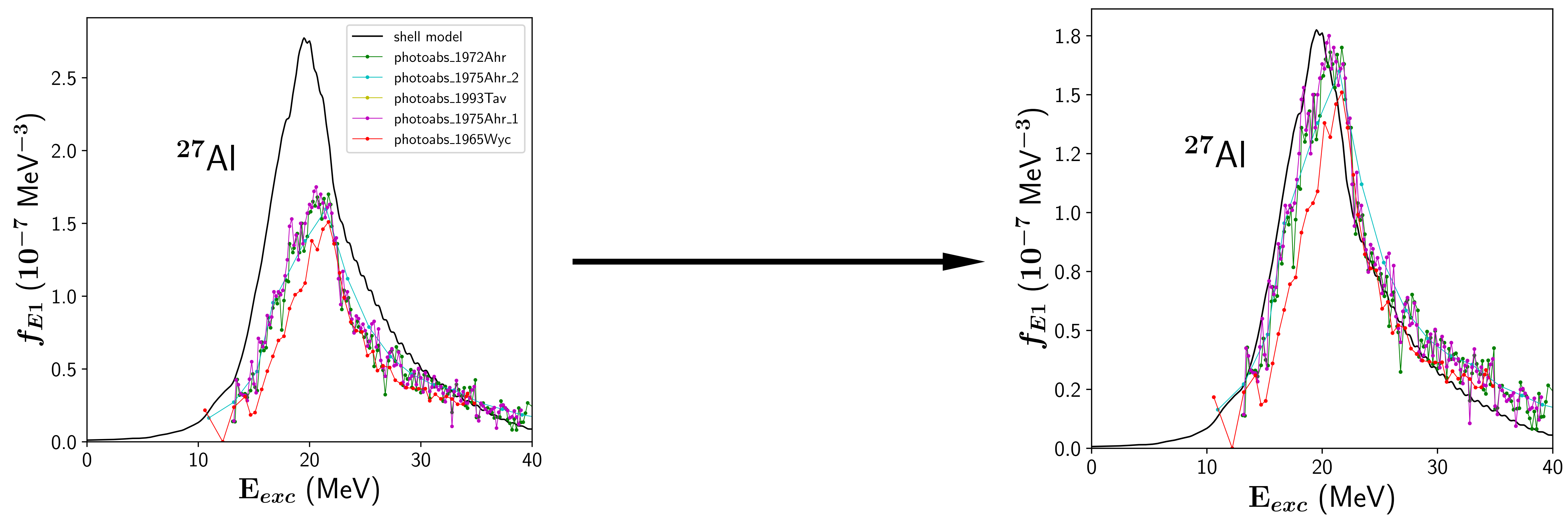
Valence space mapping: $\begin{array}{ccc} \mathcal{H} & \longrightarrow & \mathcal{H}_{\text{valence}} \\ H & \longmapsto & H_{\text{eff}} \end{array}$ Likewise for all observables $\begin{array}{ccc} \mathcal{H} & \longrightarrow & \mathcal{H}_{\text{valence}} \\ O(\mathrm{E1}) & \longmapsto & O_{\text{eff}}(\mathrm{E1}) \end{array}$

How to address this issue ?

Microscopic approaches (many body perturbation): Lee-Suzuki, IMSRG

Phenomenological approaches: scaling to data (effective charges),

scaling to TRK $S_1^{TRK} = \frac{9\hbar^2 e^2}{8\pi m}\frac{NZ}{A}$



Details of numerical calculations

300 Lanczos iterations

Lawson's method for COM

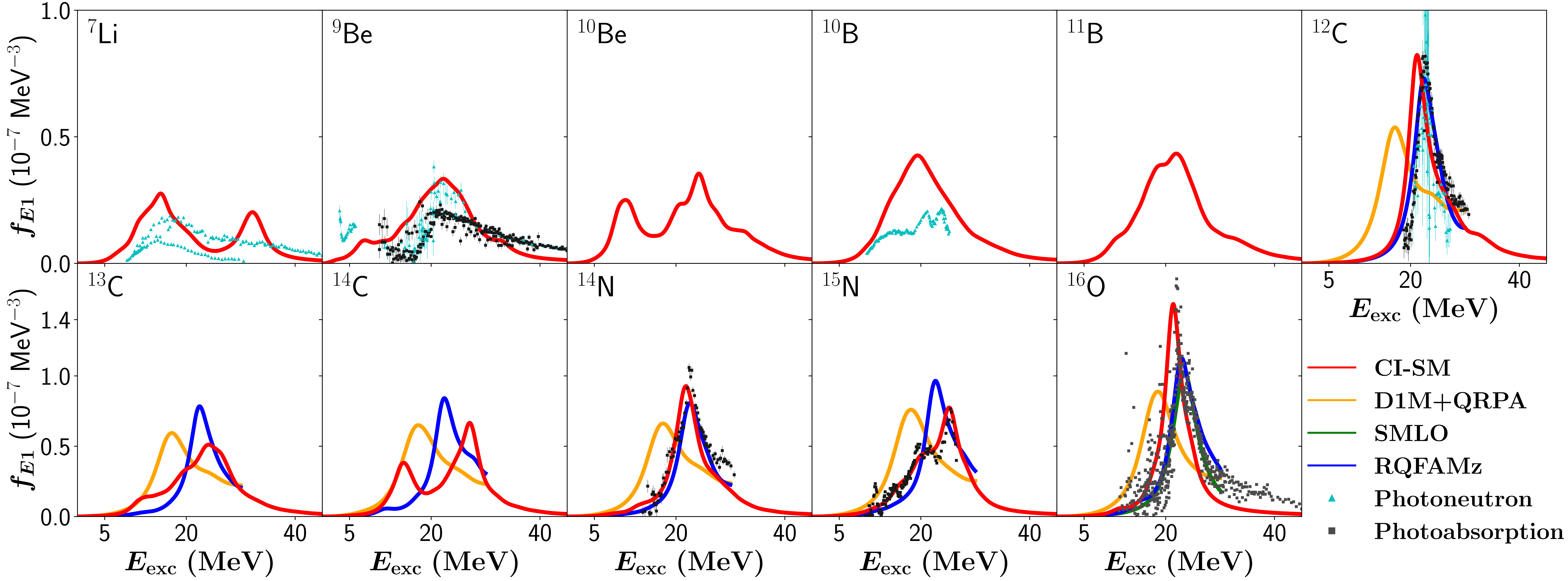
Effective interactions:

- E1 sd-shell: PSDPF
- E1 p-shell: WBP

E1 response p-shell nuclei

O. Le Noan and K. Sieja, Physical Review C, vol.111, Jun.2025

O. Le Noan S. Goriely, E. Khan, K. Sieja, To be published in PRC.



Theoretical framework: CI-SM

Lanczos Strength function

Step 1: compute the GS $[H_{\text{eff}}, J^z] = 0 \implies H_{\text{eff}}$ bloc diagonal in M

For a 0^+ GS we diagonalize H_{eff} in the $M = 0$ subspace using Lanczos algorithm $\implies |\text{GS}\rangle$

Step 2: compute the sum rule state $|\text{SR}\rangle = O\,|\text{GS}\rangle$

Step 3: Lanczos calculation using $|\text{SR}\rangle$

Unitary matrix $U_{ij} = \langle \mathcal{L}_i | \psi_j \rangle$ diagonalizing H_{eff} such that $U_{1j} = \langle \text{GS} | O \, |\psi_j\rangle$

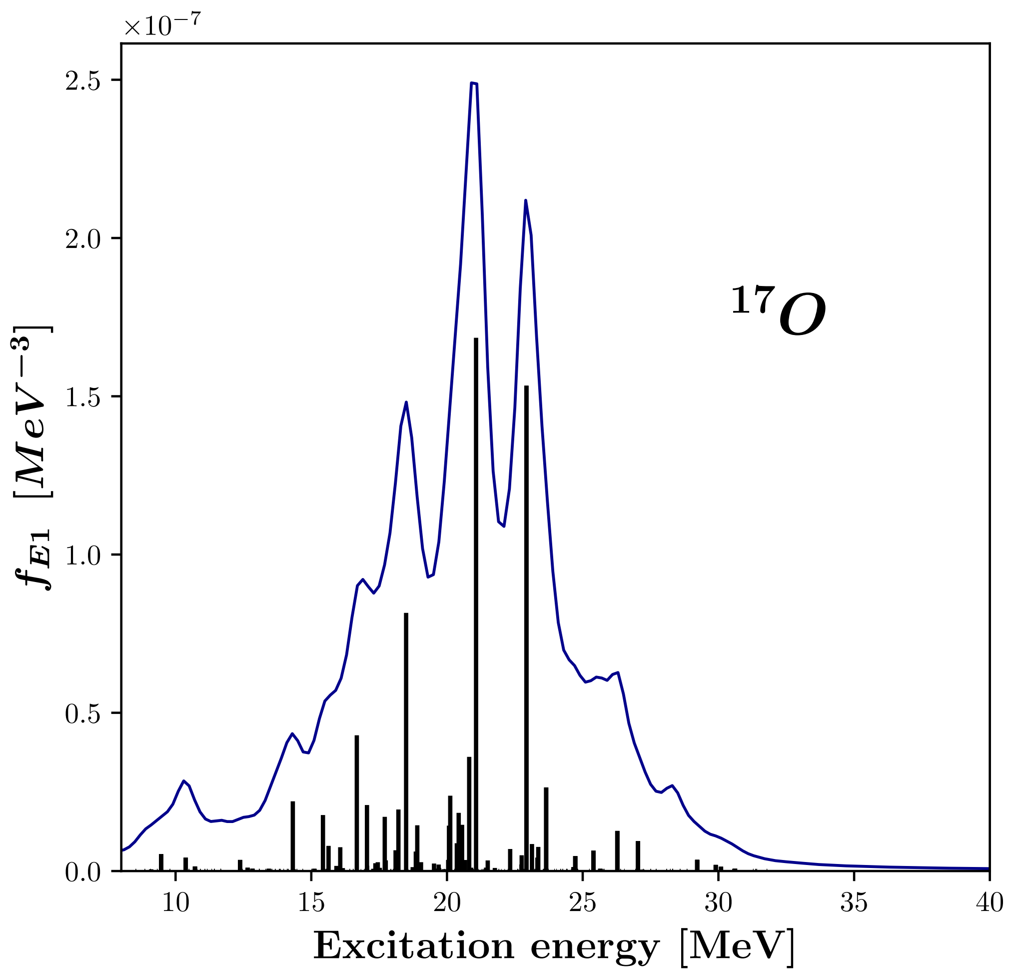
Details of numerical calculations

300 Lanczos iterations

Lawson's method for COM

Effective interactions:

- E1 sd-shell: PSDPF
- E1 p-shell: WBP



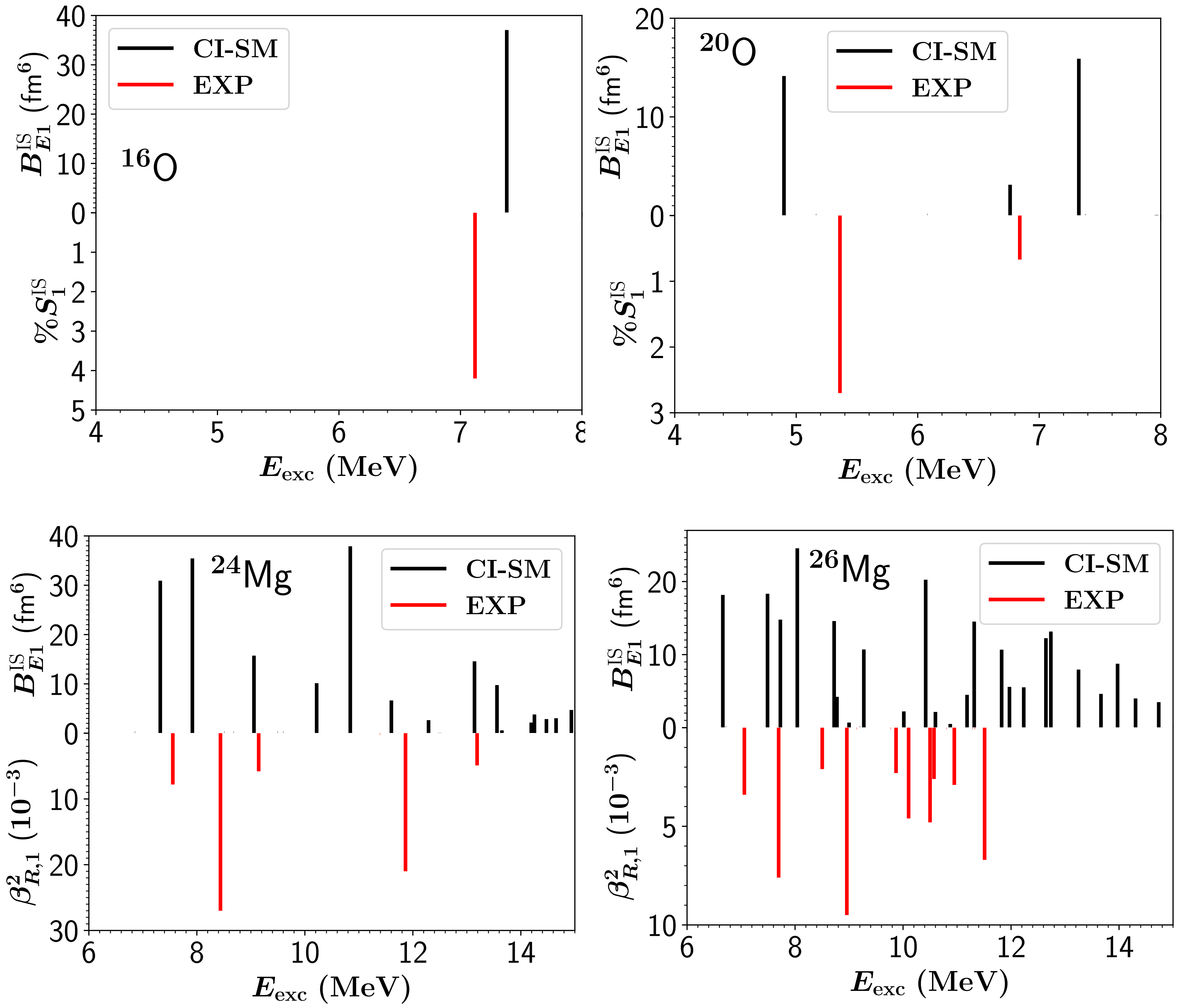
Theorem: for N_{it} Lanczos iterations the first $2N_{\text{it}}$ moments S_k of the distribution are exact

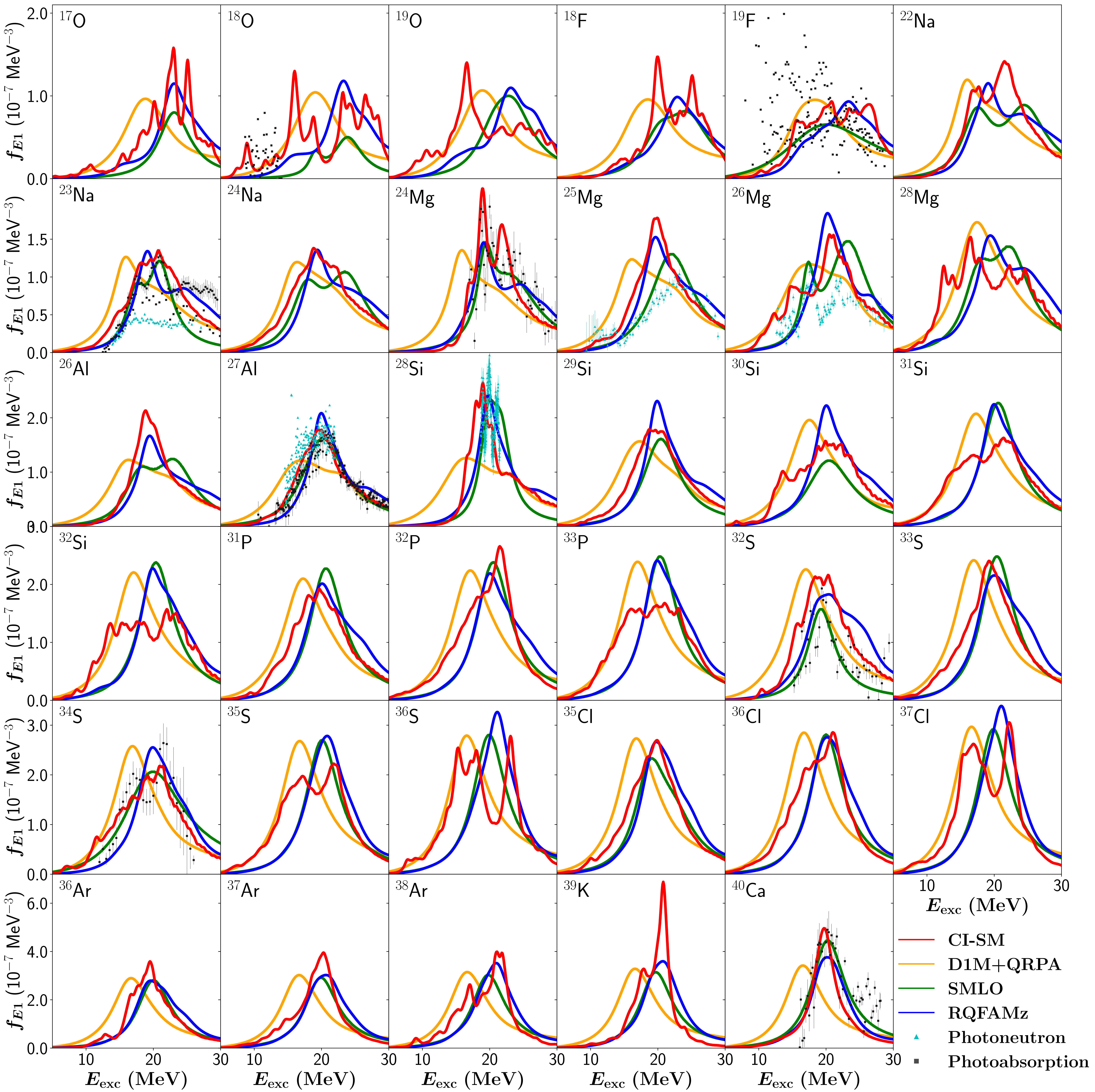
$$S_k = \sum_j E_j^k |\langle \psi_j | O \,|\text{GS}\rangle|^2 = \langle \text{SR}| H^k |\text{SR}\rangle$$

E1 response sd-shell nuclei

132 nuclei (all isotopic chains within the valence space)

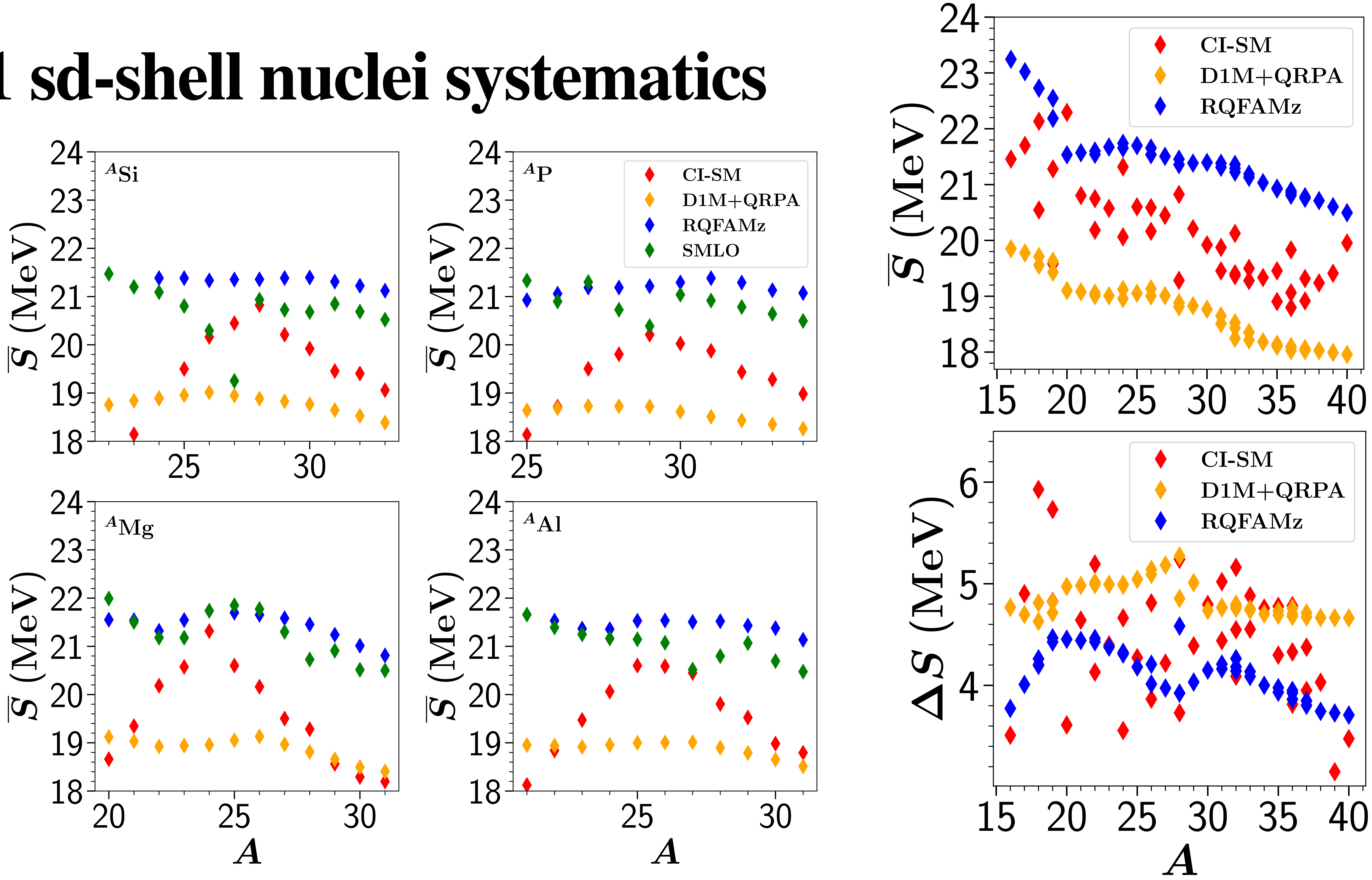
E1 PSF available in the Talys database, M1 to come.



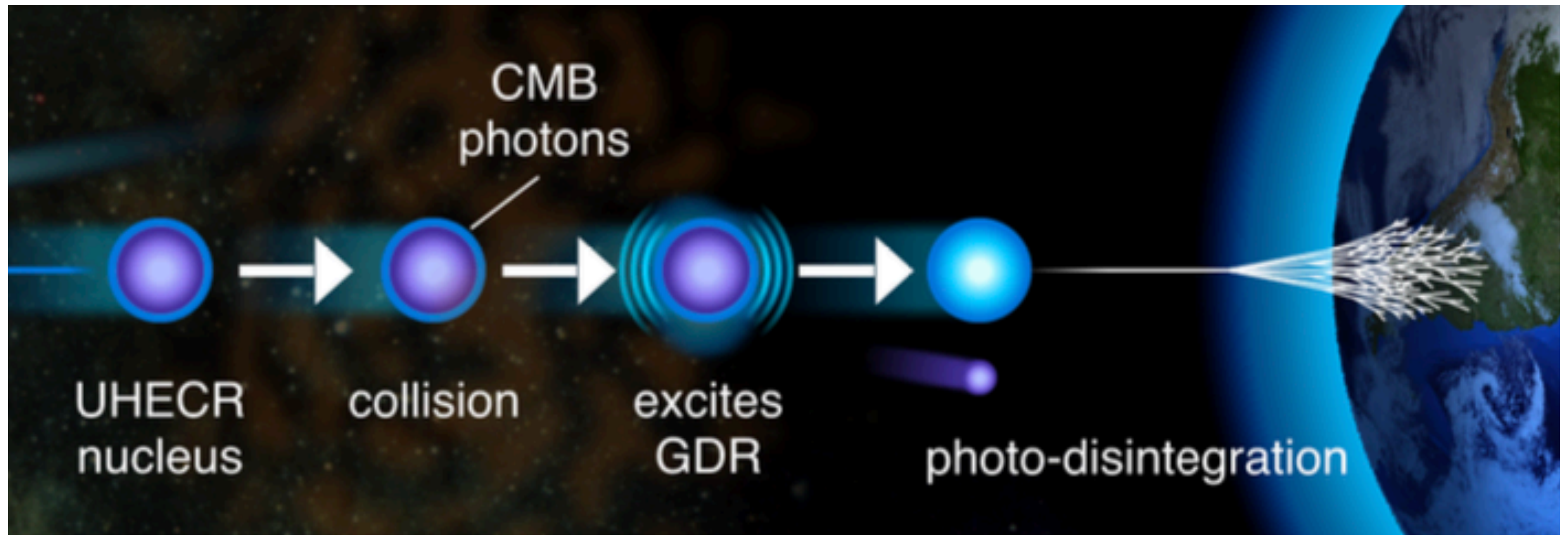


RMS			
	$\overline{S}$	0.84	MeV
	ΔS	0.56	MeV

E1 sd-shell nuclei systematics



Application to UHECR propagation



$$\frac{dN_{Z,A}}{dt} = N_{Z+1,A}\lambda_{\beta}^{Z+1,A} + N_{Z-1,A}\lambda_{\beta}^{Z-1,A}$$
$$+N_{Z,A+1}\lambda_{\gamma,n}^{Z,A+1} + N_{Z+1,A+1}\lambda_{\gamma,p}^{Z+1,A+1}$$
$$+N_{Z+2,A+4}\lambda_{\gamma,\alpha}^{Z+2,A+4} + N_{Z,A+2}\lambda_{\gamma,2n}^{Z,A+2}$$
$$+N_{Z+2,A+2}\lambda_{\gamma,2p}^{Z+2,A+2} + N_{Z+4,A+8}\lambda_{\gamma,2\alpha}^{Z+4,A+8}$$
$$+N_{Z+1,A+2}\lambda_{\gamma,np}^{Z+1,A+2} + N_{Z+2,A+5}\lambda_{\gamma,n\alpha}^{Z+2,A+5}$$
$$+N_{Z+3,A+5}\lambda_{\gamma,p\alpha}^{Z+3,A+5}$$
$$-N_{Z,A}\left[\lambda_{\beta}^{Z,A} + \sum_{x}\lambda_{\gamma,x}^{Z,A}\right] ,$$

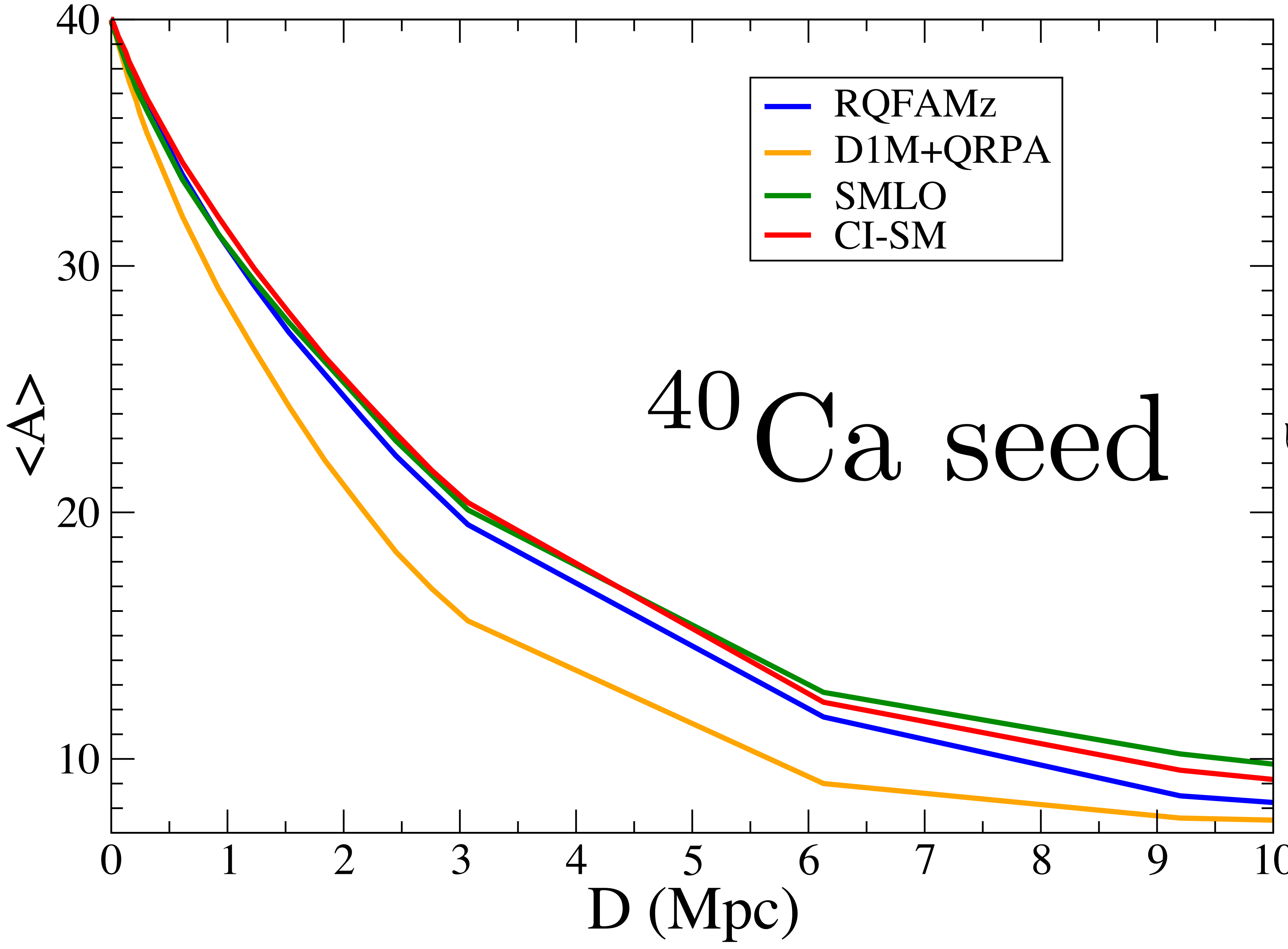
$$\lambda_{\gamma,x} = \int n(\epsilon)\ \sigma_{\gamma,x}(\epsilon)\ c\ d\epsilon$$

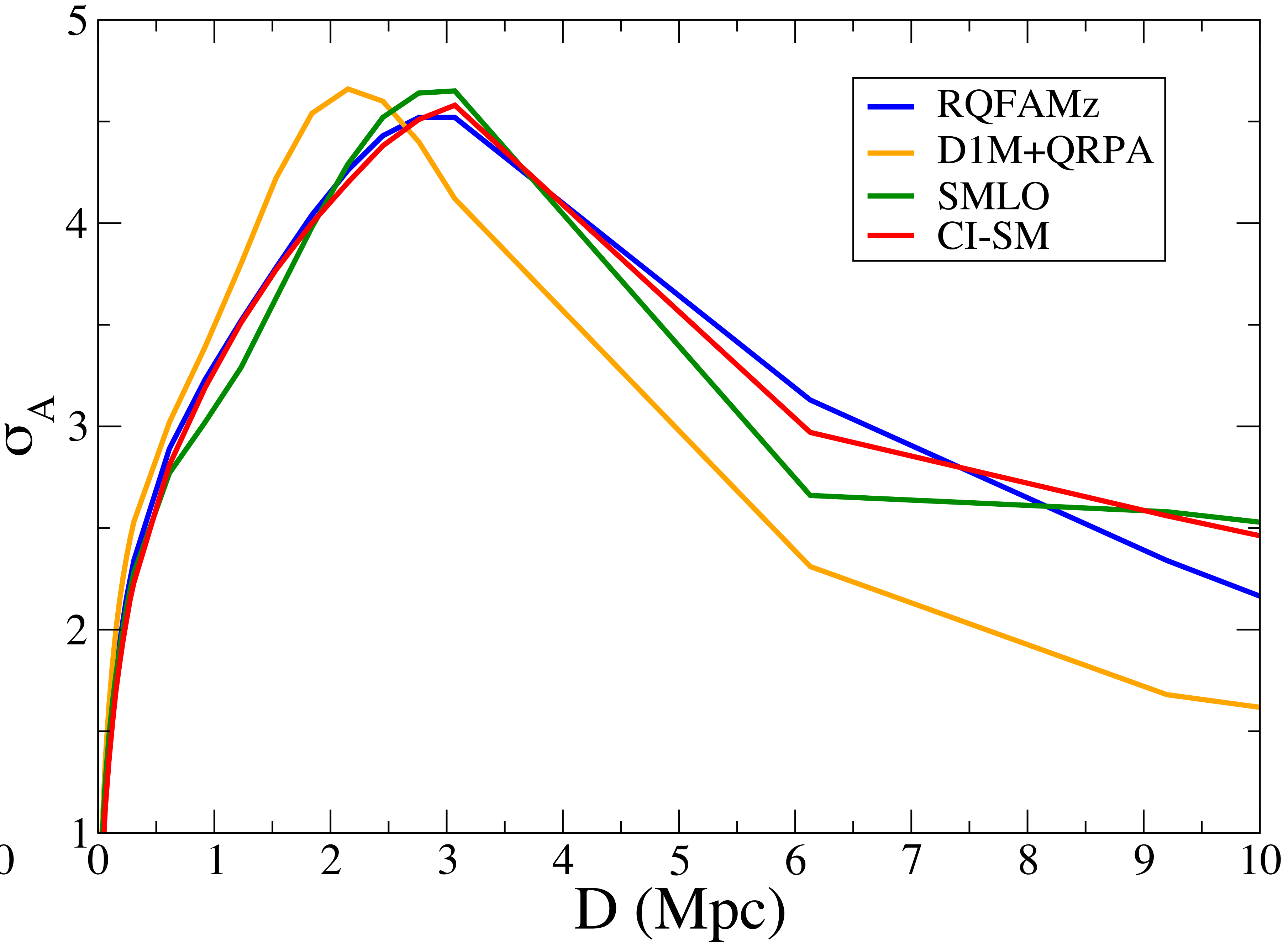
↑

TALYS

↑

CI-SM
PSFs



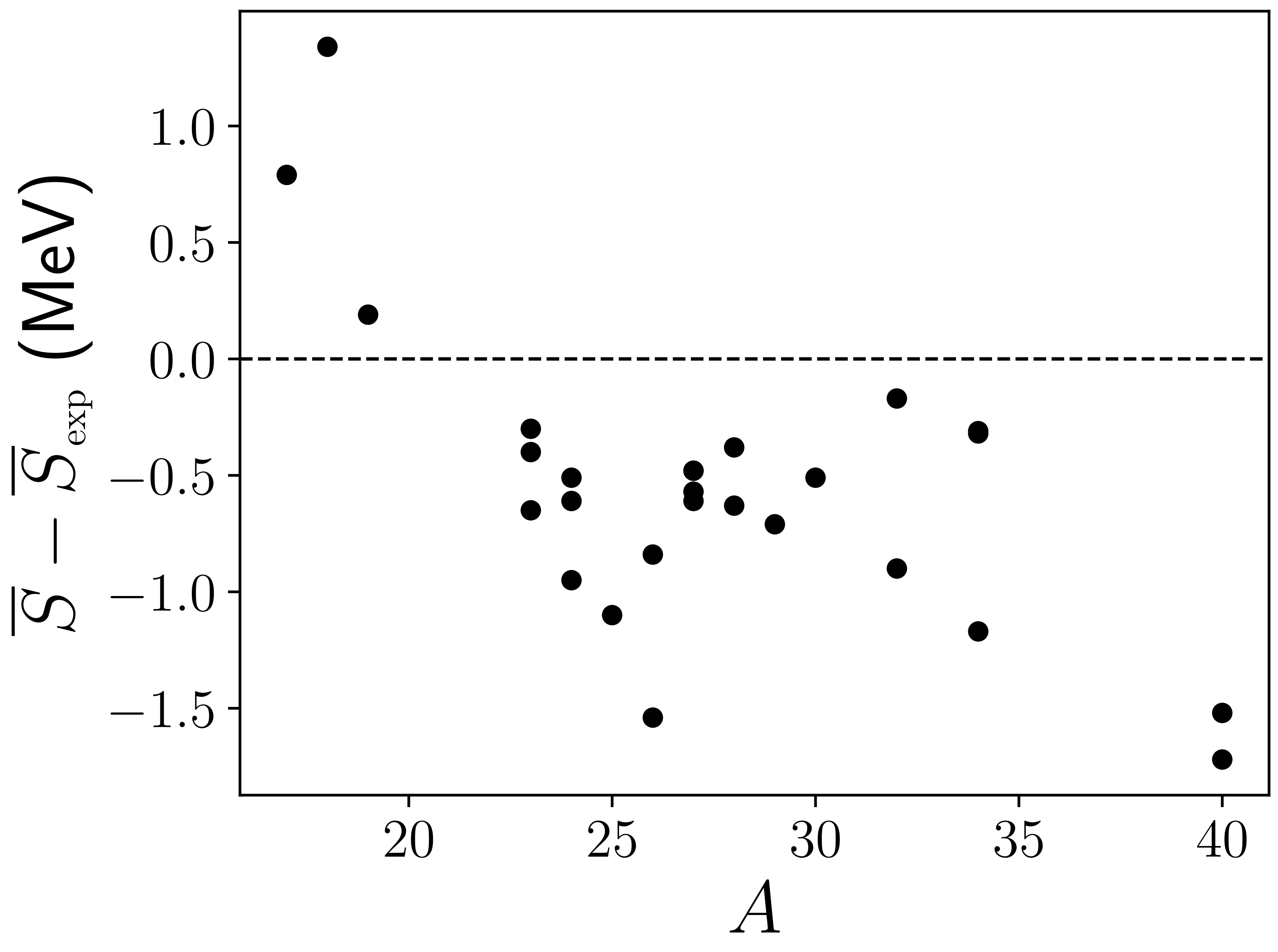


O. Le Noan, S. Goriely, E. Khan, and K. Sieja, Physical Review C 113,

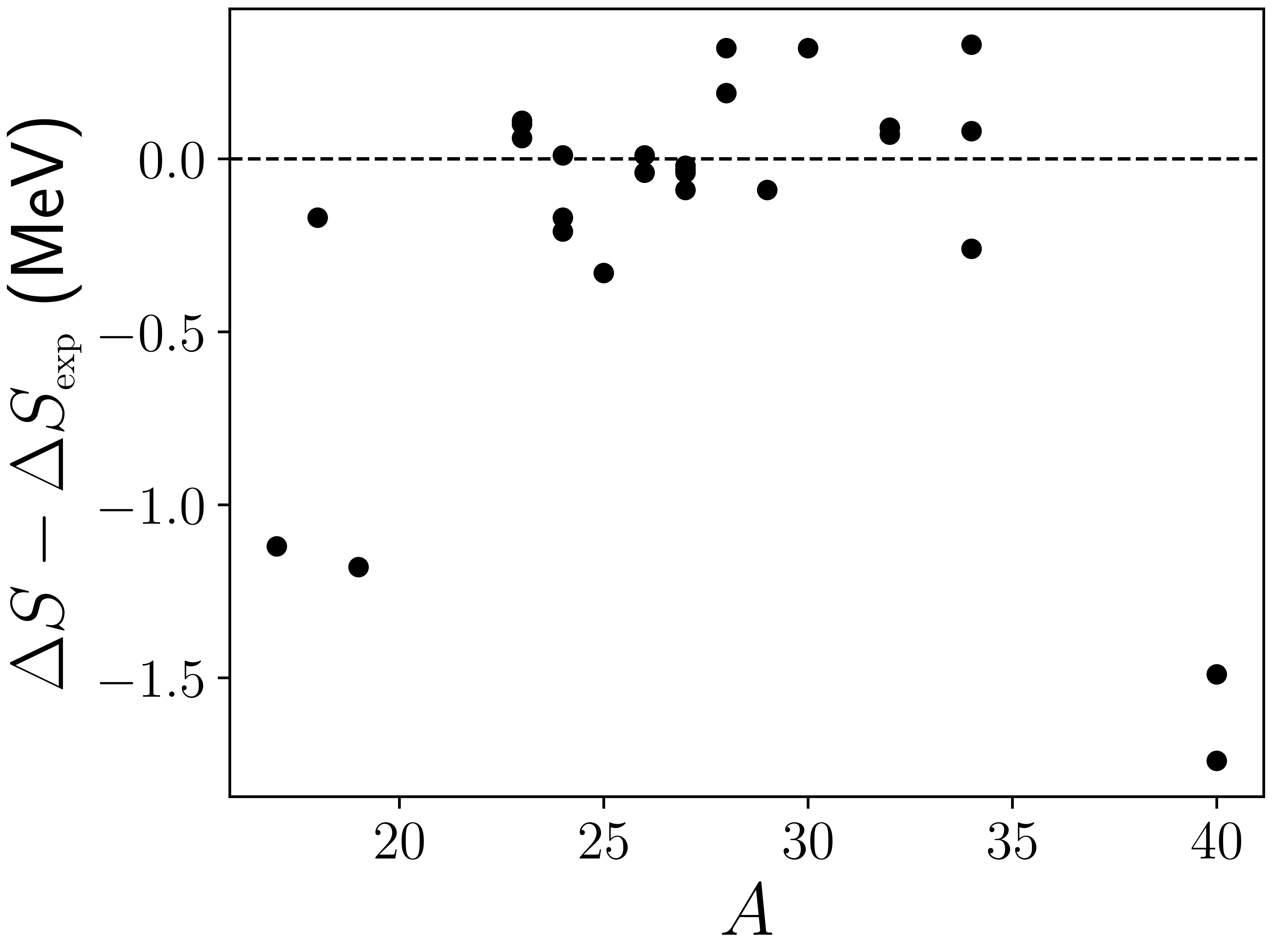
E1 sd-shell nuclei systematics

Models	ε_S	σ_S	$\varepsilon_{\Delta S}$	$\sigma_{\Delta S}$
CI-SM	−0.36	0.85	0.27	0.51
D1M + QRPA	−1.47	1.71	0.13	0.49
RQFAMz	0.54	0.89	−0.33	0.48

Centroids $\bar{S} = \frac{S_1}{S_0}$



Width $\Delta S = \sqrt{\frac{S_2}{S_0} - \bar{S}^2}$



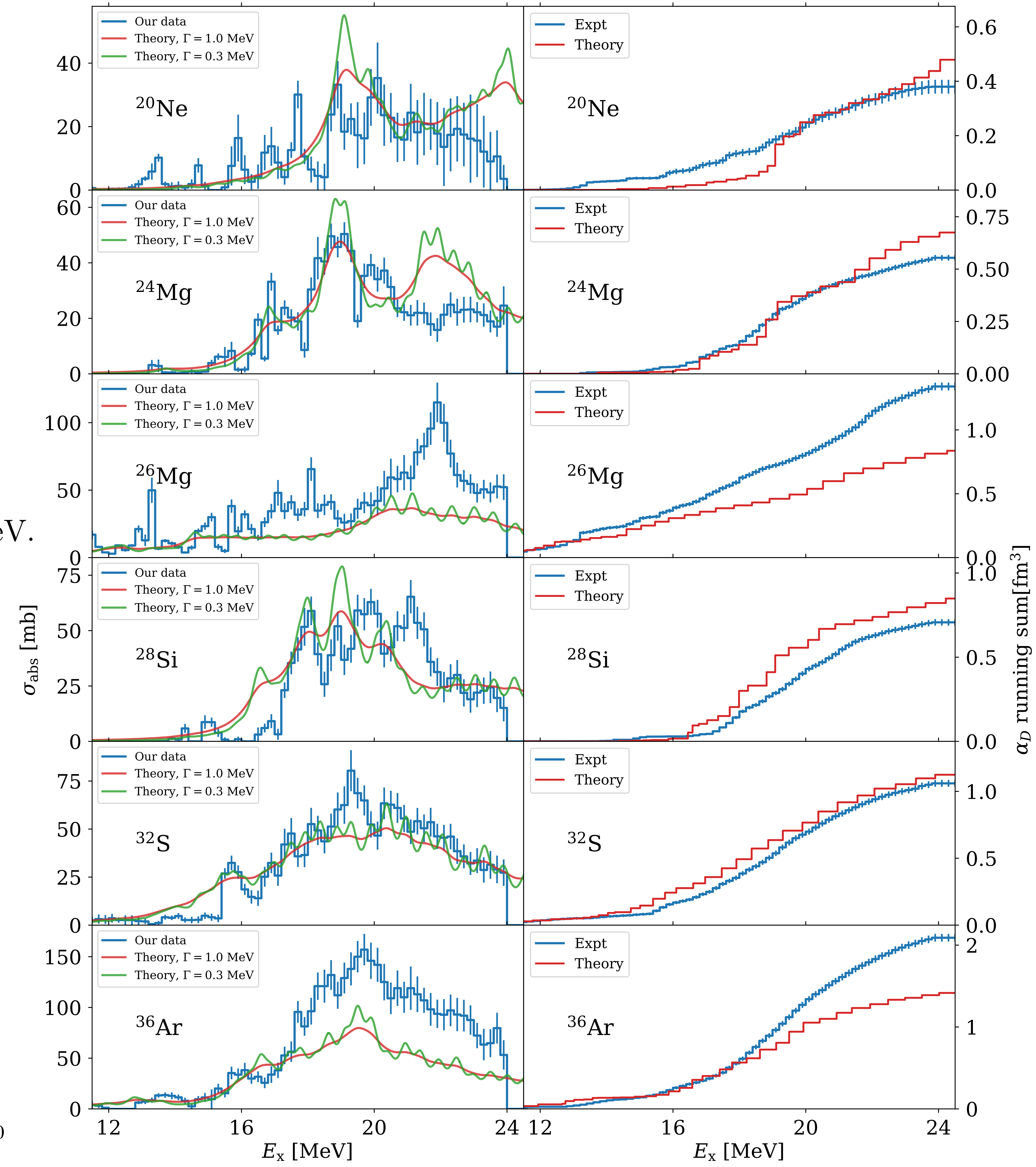
E1 response sd-shell nuclei

Electric dipole strength in sd-shell nuclei from small-angle proton scattering

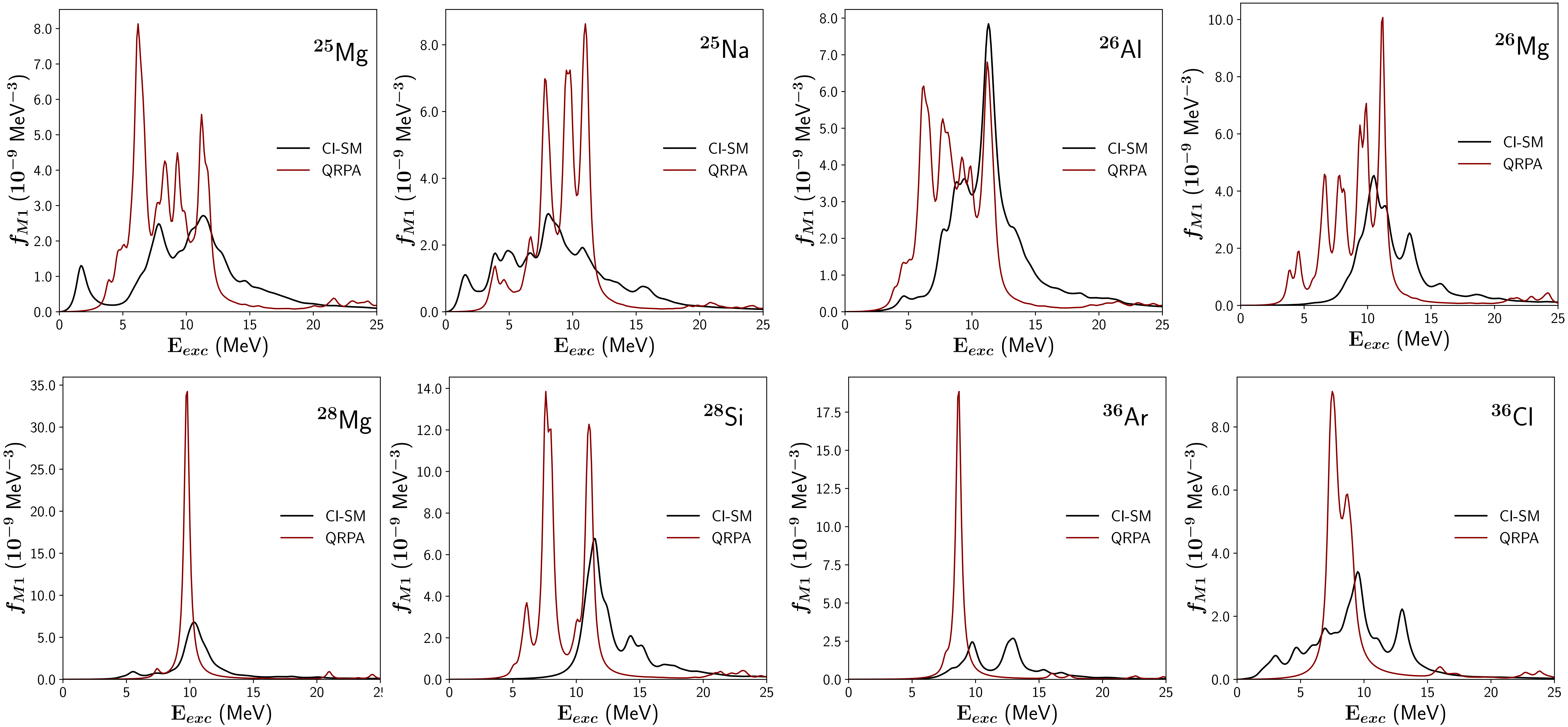
R. W. Fearick, O. Le Noan, H. Matsubara, P. von Neumann-Cosel, K. Sieja, and A. Tamii. To be published soon.

Table 1: Centroids and widths (in MeV), calculated between 12 and 20 MeV.

	$\overline{S}$	$\overline{S}_{\mathrm{exp}}$	$\overline{S} - \overline{S}_{\mathrm{exp}}$	ΔS	ΔS_{exp}	$\Delta S - \Delta S_{\mathrm{exp}}$
^{20}Ne	18.40	17.50	0.90	1.40	1.99	−0.59
^{24}Mg	18.06	18.15	−0.09	1.43	1.34	0.09
^{26}Mg	16.49	16.65	−0.16	2.22	2.15	0.06
^{28}Si	17.95	18.41	−0.45	1.38	1.22	0.23
^{32}S	17.23	17.82	−0.59	1.86	1.65	0.21
^{36}Ar	17.49	18.01	−0.52	1.87	1.64	0.23



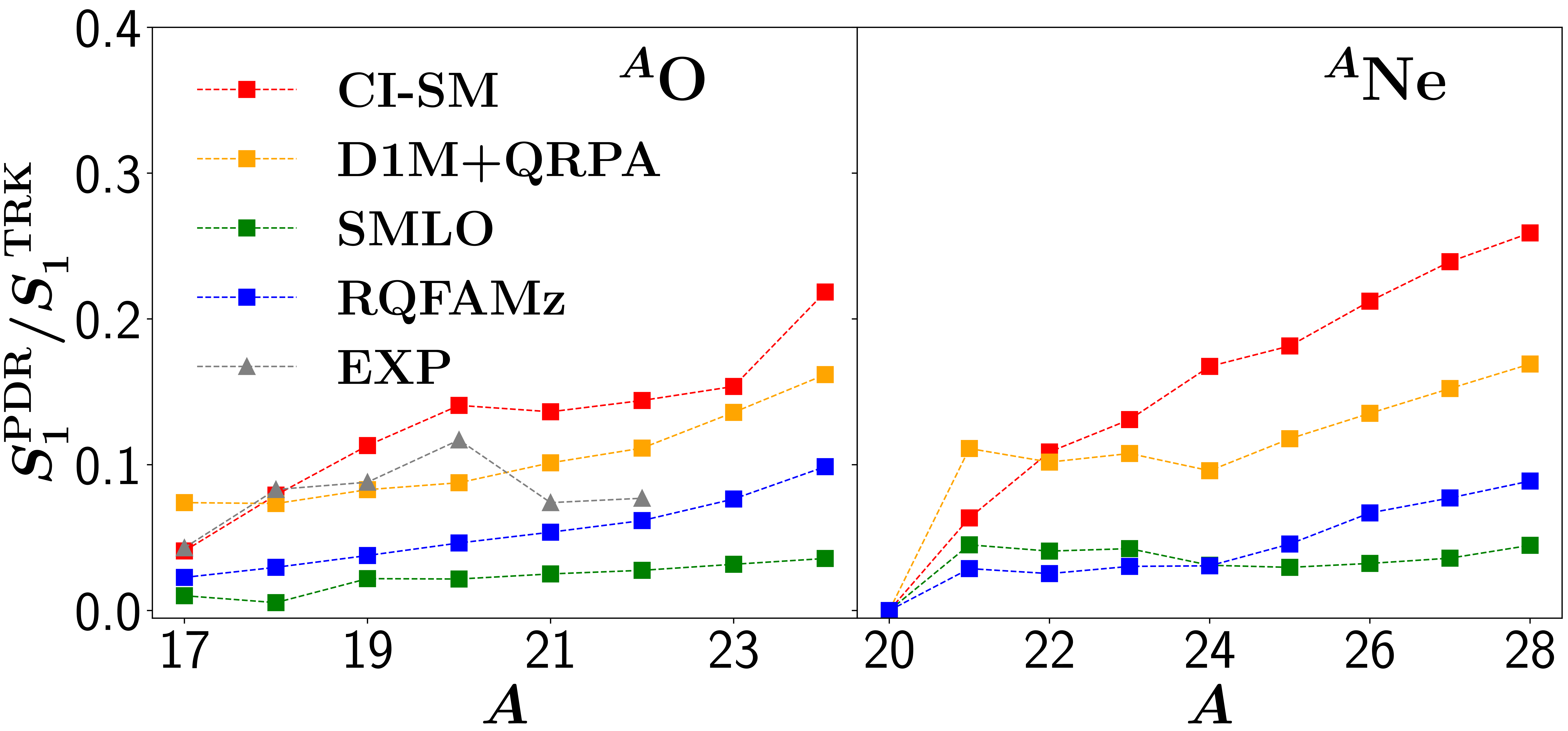
M1 p, and sd-shell nuclei



E1 response sd-shell nuclei

Neutron rich nuclei: model comparison

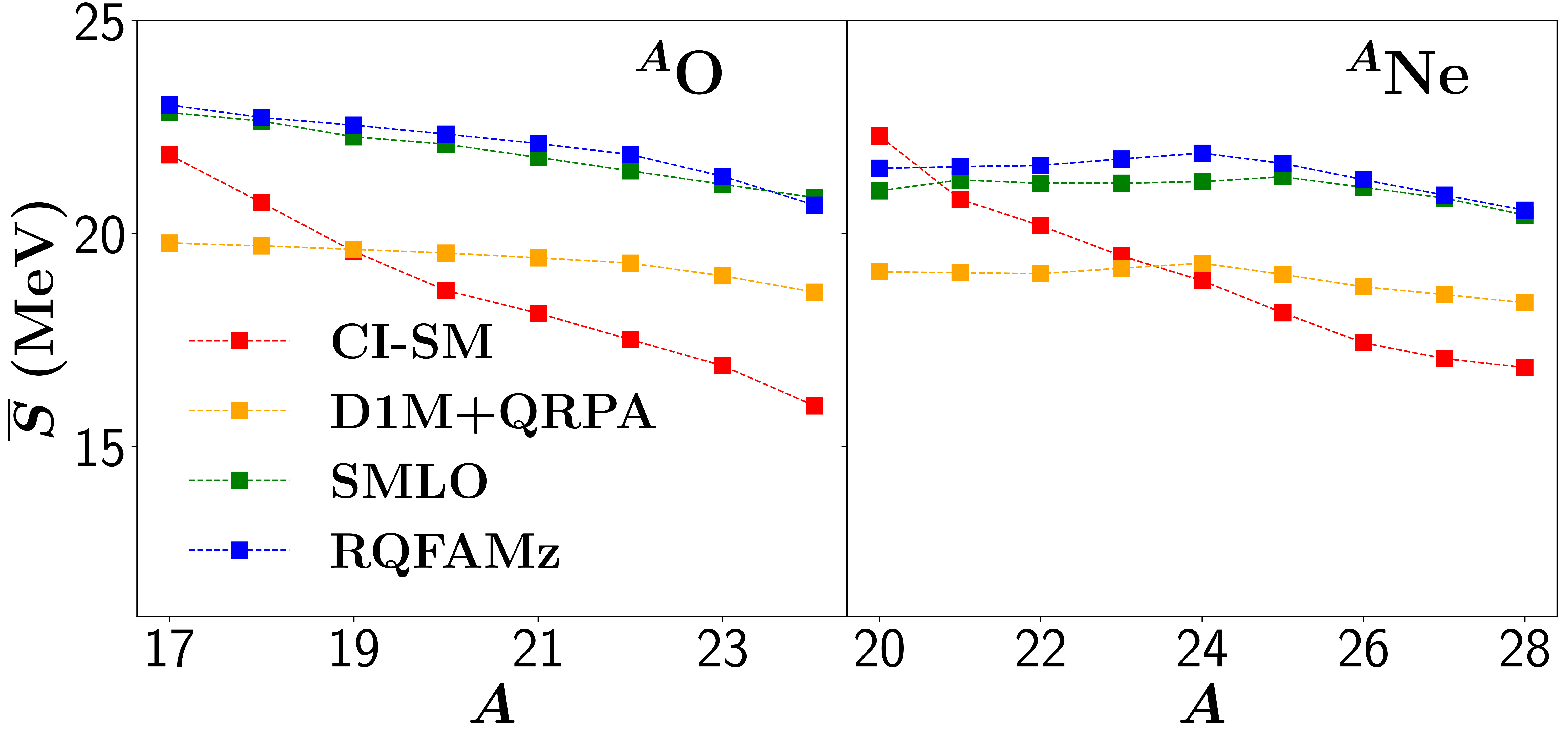
up to 30 MeV



from S_n to 15 MeV

Leistenschneider, A. et al. Physical Review Letters 2001, 86.

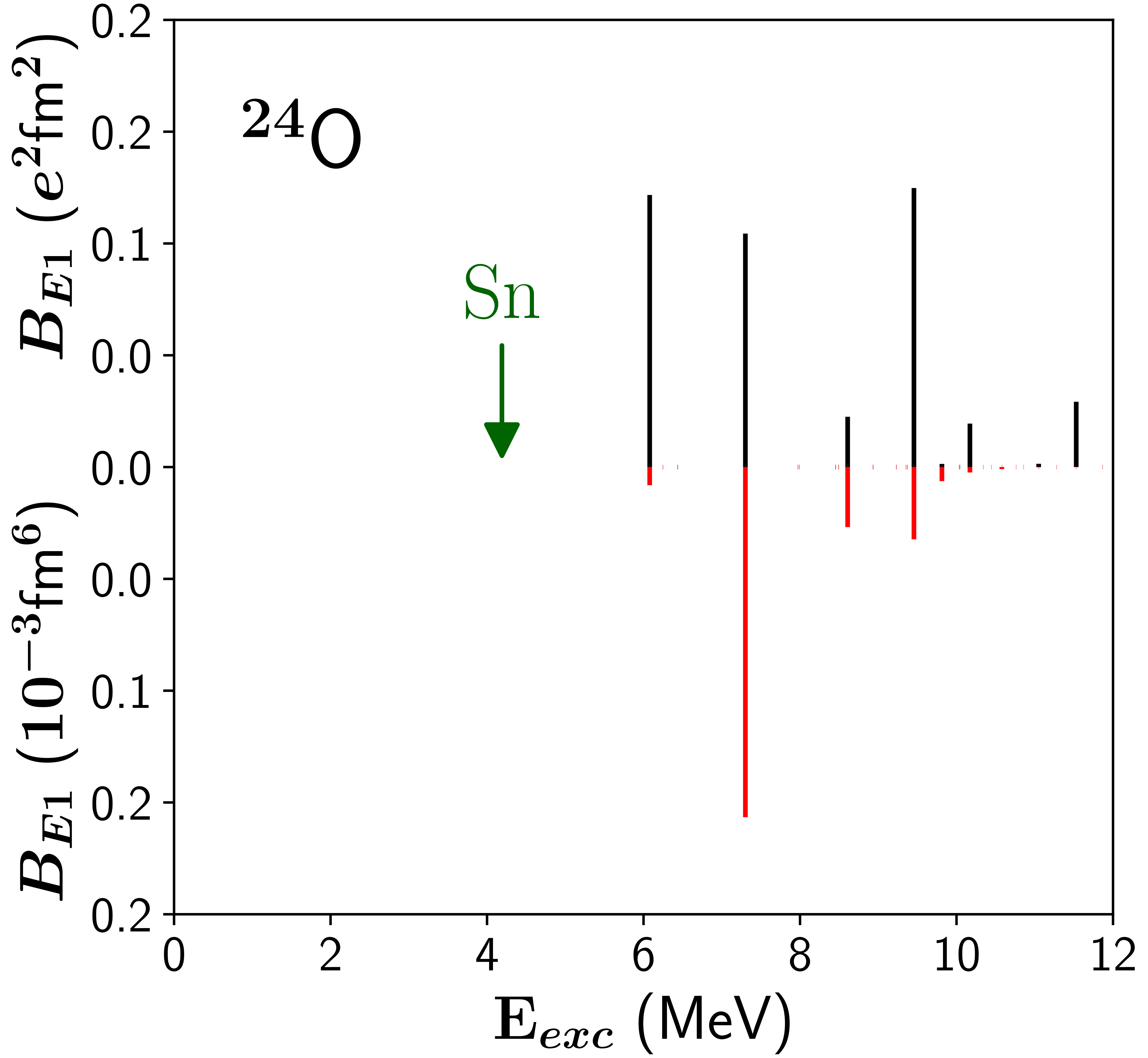
Study of the Pygmy dipole resonance

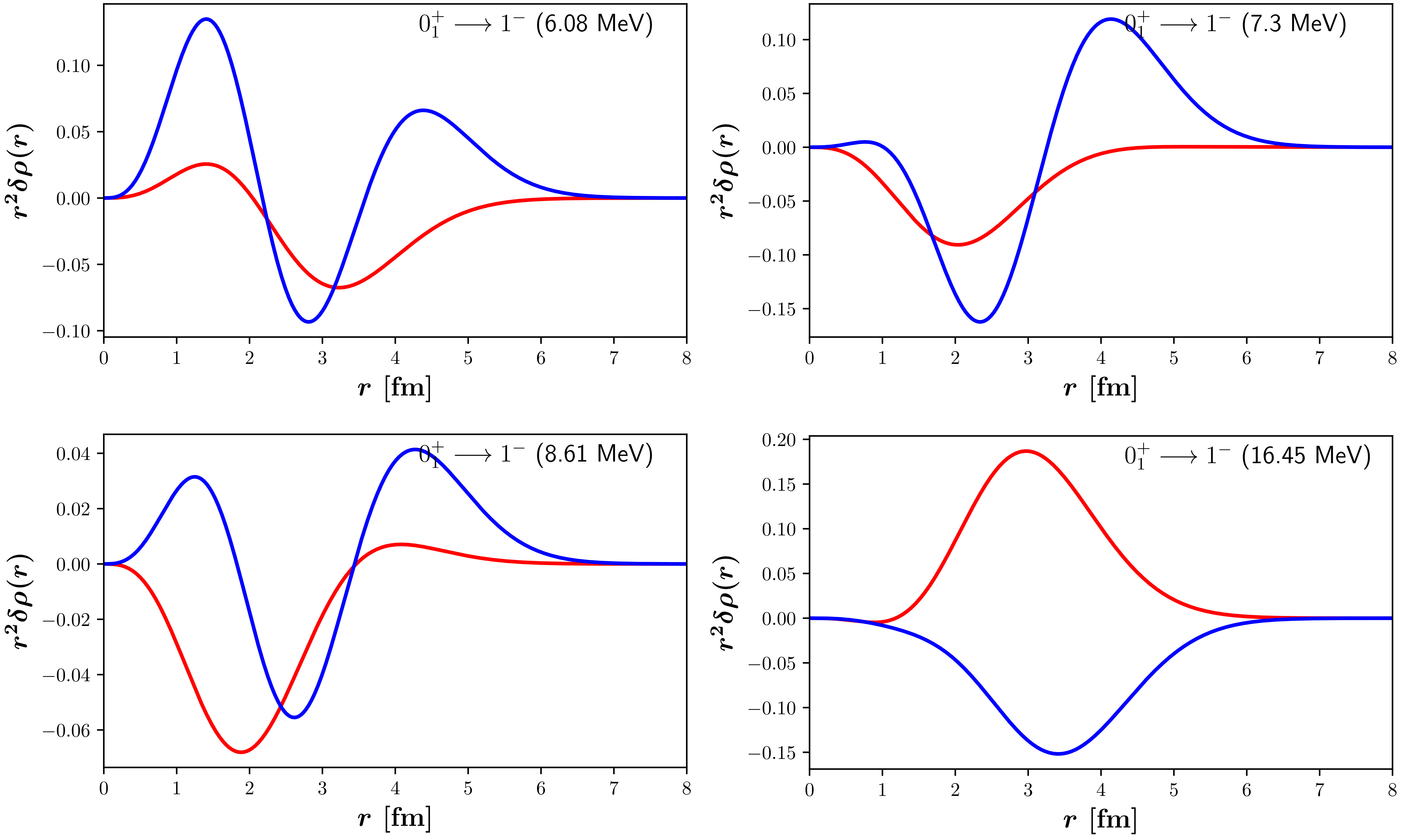


up to 30 MeV

Study of the Pygmy dipole resonance ^{24}O

Oxygen chain: model comparison





Study of the Pygmy dipole resonance

Isospin mixing in the neon chain