

Comprendre l'Infiniment Grand - Introduction to Cosmology

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June 29, 2026

Cosmology - Part I

1. Introduction

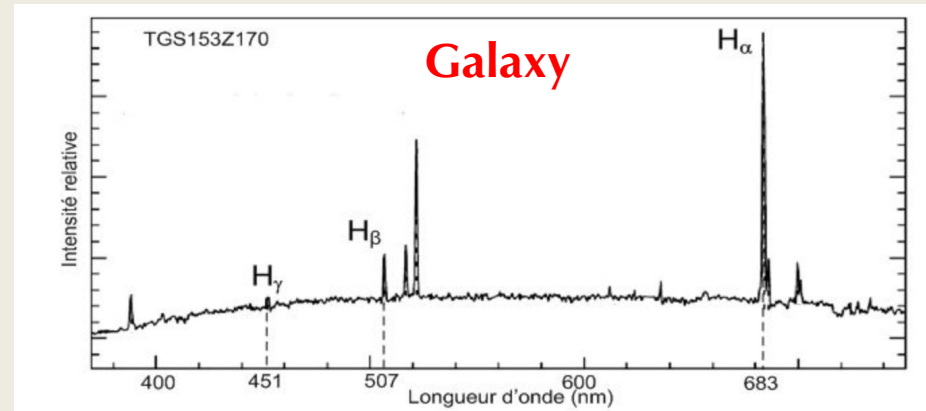
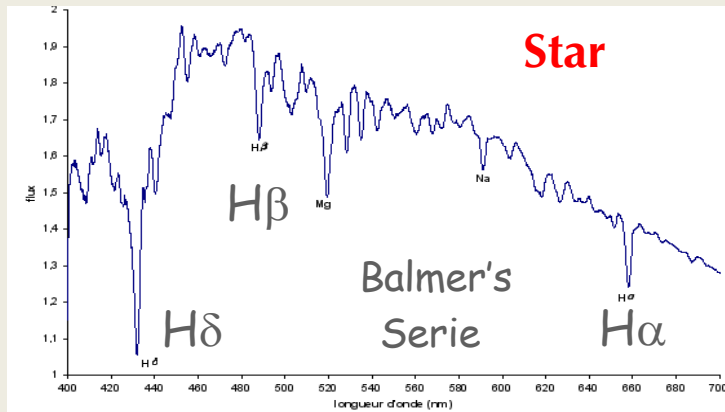
- Hubble law
- Content of the Universe

2. Gravitation and General Relativity

- Equivalence principle
- Tests of GR

1) Introduction

How do we measure velocity?



Stars spectra

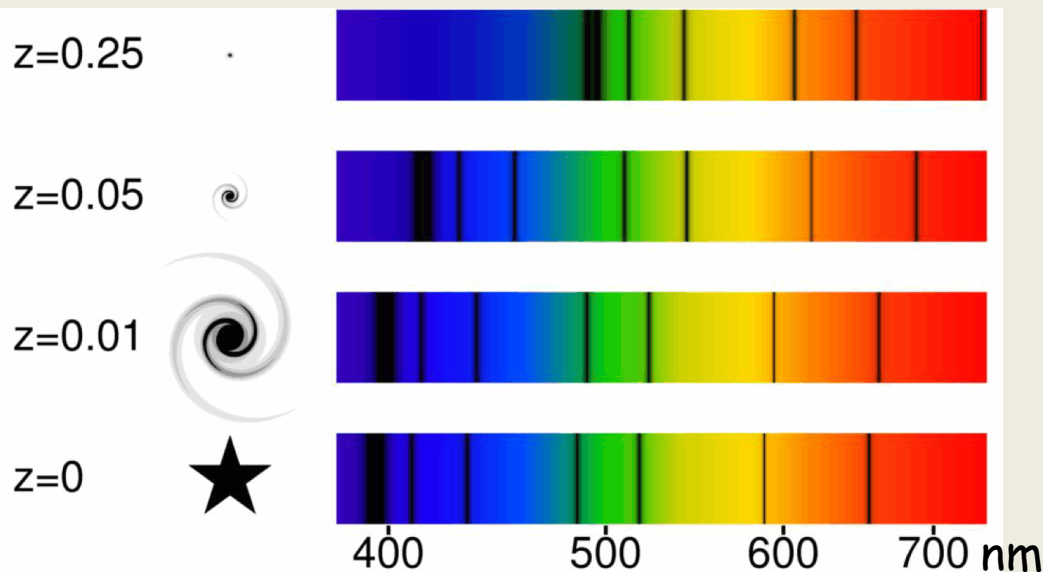
- Absorption lines

Galaxies

- Emission lines
- Balmer/Lyman breaks

Redshift

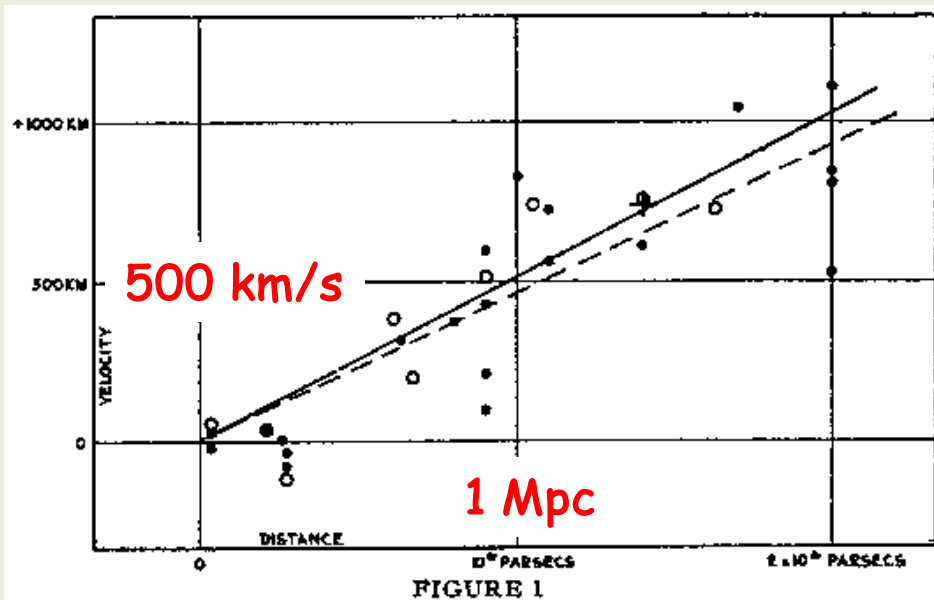
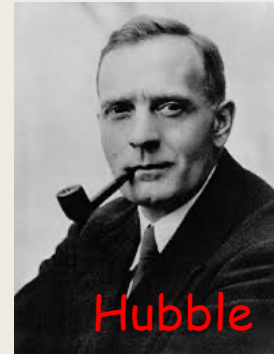
- Doppler effect
- $V/c = (\lambda - \lambda_0)/\lambda_0 = z$



Expanding Universe

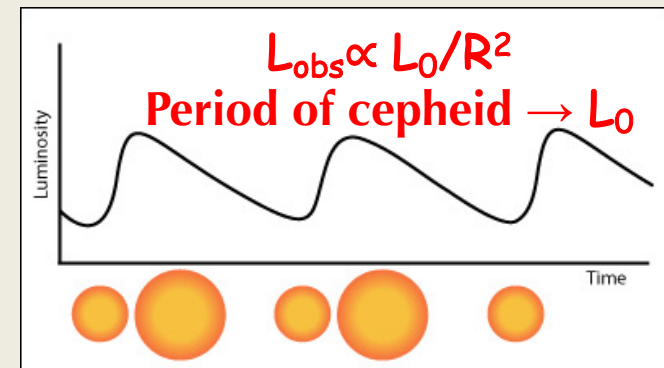
History of the discovery

- 1914, Slipher: farther the « nebula » (galaxy) is from us, the more it seems to be escaping away
- 1927, Lemaître: solutions of Einstein General Relativity for a non static universe $\Rightarrow$ velocity proportional to distance.

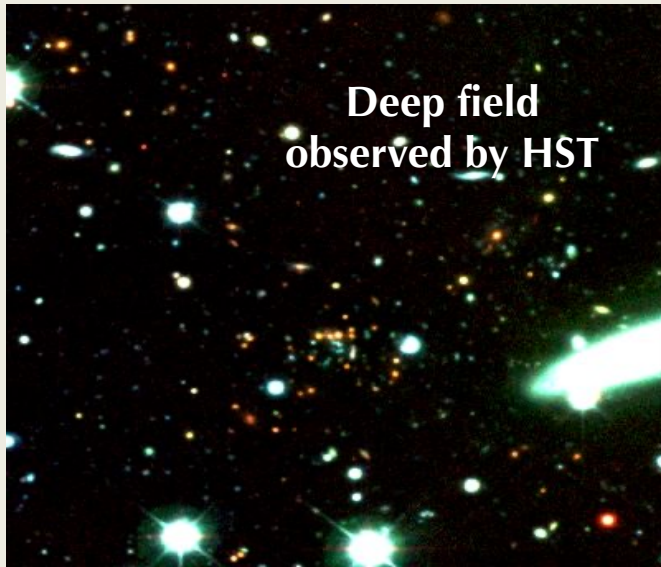


1 parsec = 3 light years

- 1929, Hubble: Relation distance – velocity thanks to cepheid in extragalactic “nebula”



Expanding Universe



Hubble's law

$$V = H_0 D$$

- Measurement of the velocity of galaxies with **their redshift (z)**
Doppler effect : $v/c = (\lambda - \lambda_0) / \lambda_0 = z$
- Increasing $z \Rightarrow$ Back in time

What value of H_0 ?

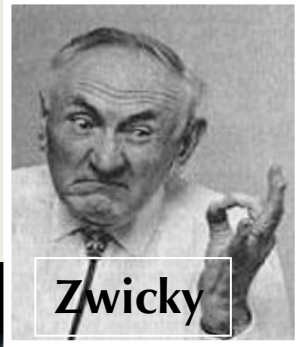
- Controversial and controverted measurement.

What about gravitation?

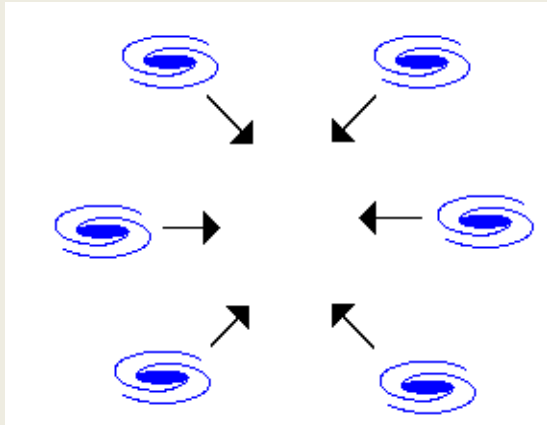
- It will slow the expansion of the universe for dark matter - Deceleration.
- It will accelerate the expansion of the universe for "repulsive" matter - Acceleration.



Discovery of Dark Matter

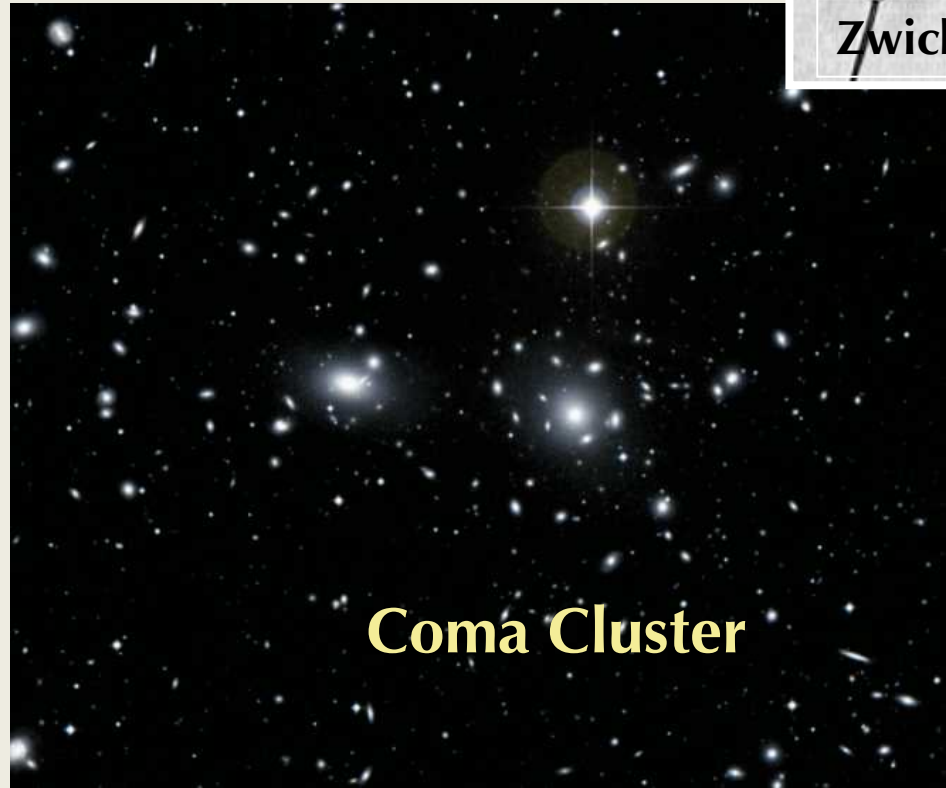


Zwicky, 1933



“Invisible” matter

- Galaxy cluster.
- Peculiar velocity of galaxies too high.
- Virial theorem.
- Visible galaxies are about 1-10% of the total mass.

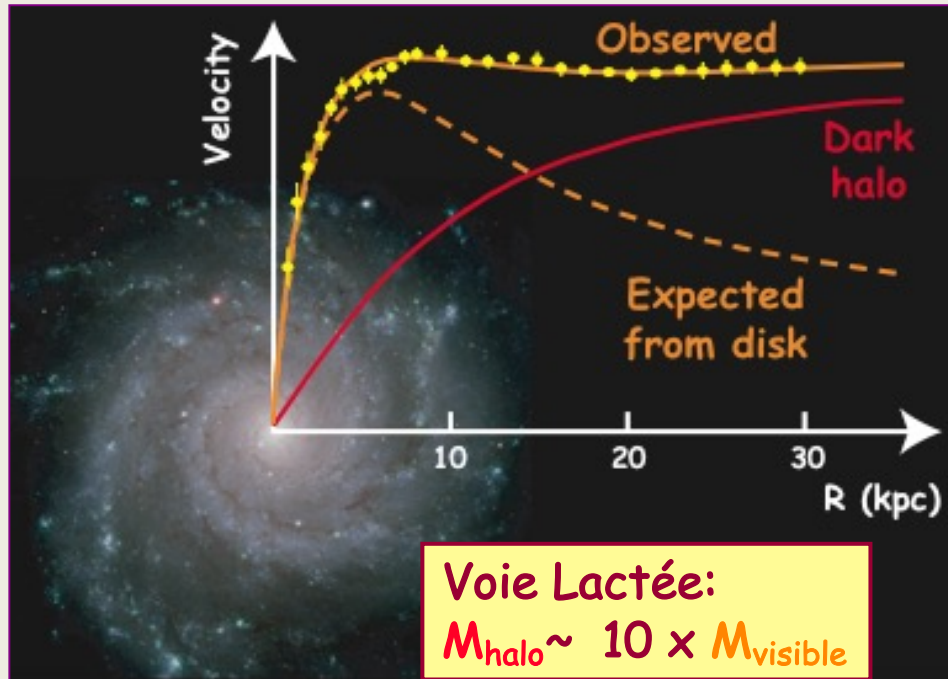


Coma Cluster

$$E_p + 2 E_c = 0$$
$$E_c = 1/2 M v^2 \text{ and } E_p = -1/2 GM^2/R$$

$$M = 2Rv^2/G$$

1970: how to weigh galaxies?



Galactic rotation curves

- Final proof by measuring the velocity of stars within galaxies
- Work of Vera Rubin and Kent Ford in the 70'

Newton Law

$$E_c + E_p = 0$$

$$V_{\text{rot}} = \sqrt{\frac{2GM}{R}}$$

Constant rotation curve



**Halo of
Dark Matter**

Dark energy

White dwarf - star

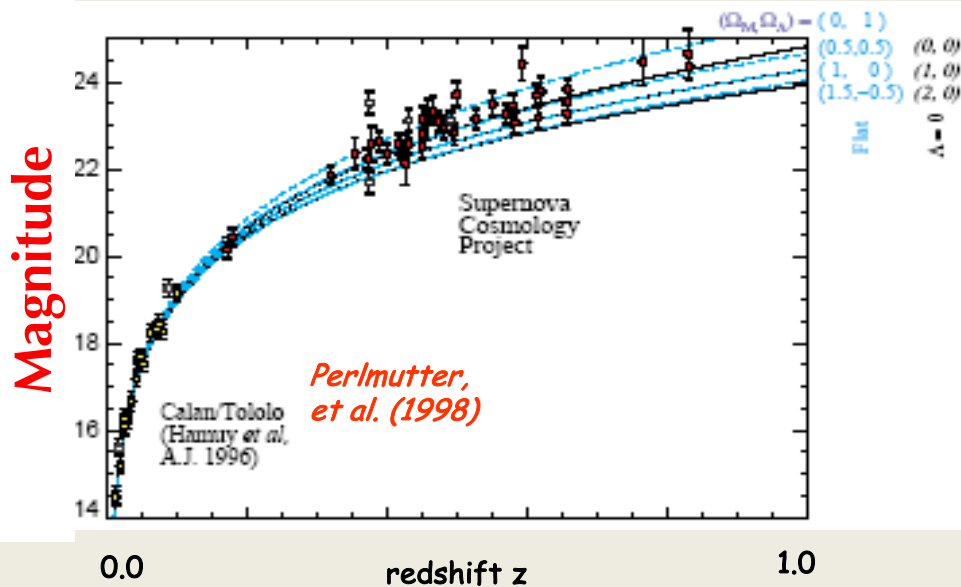


Discovery with supernovae

- In 1998, Hubble diagram (magnitude $\leftrightarrow z$) with standard candles (SN Ia)

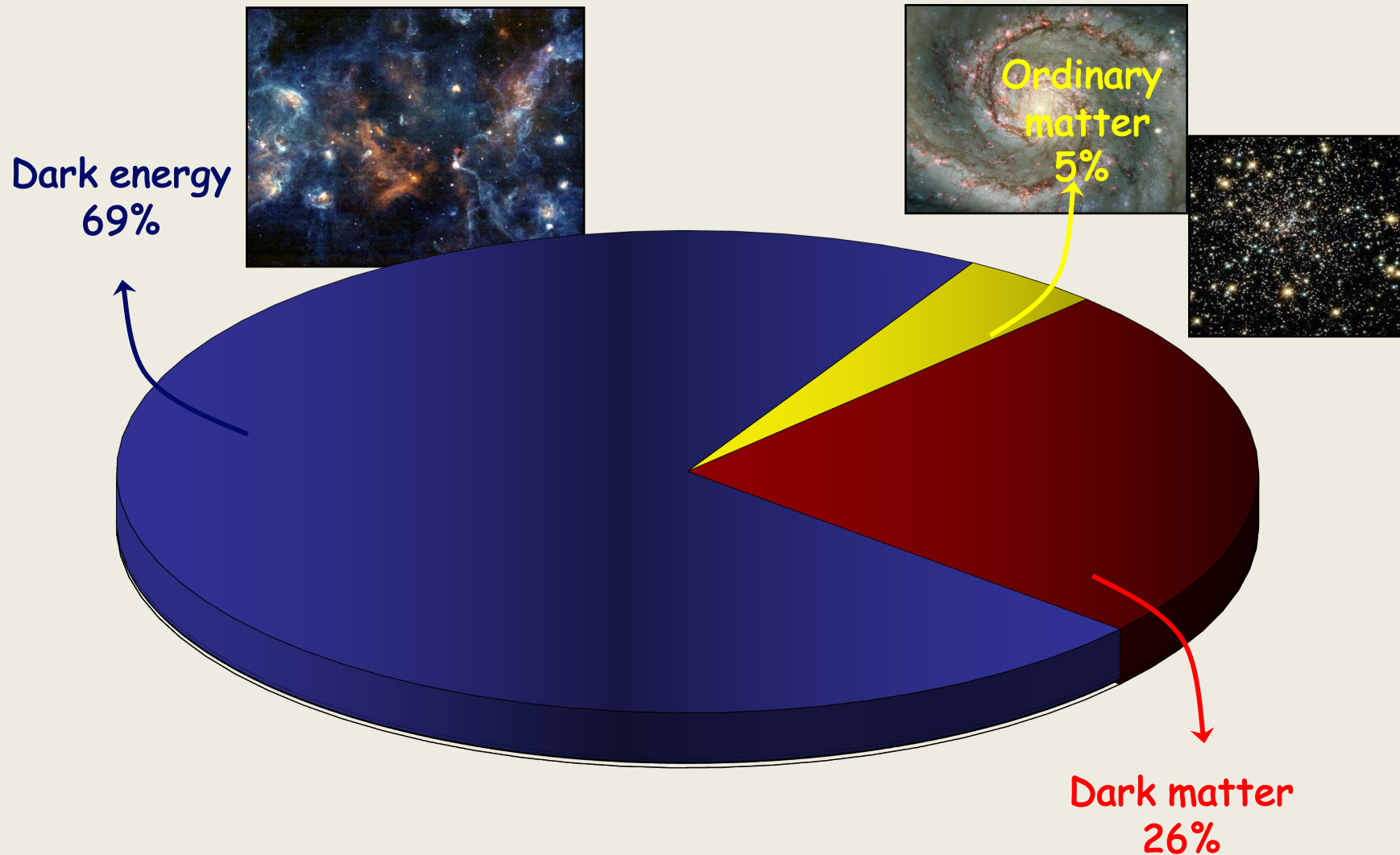
$$L_{\text{obs}} \propto L_0 / R^2$$

- Acceleration of expanding Universe



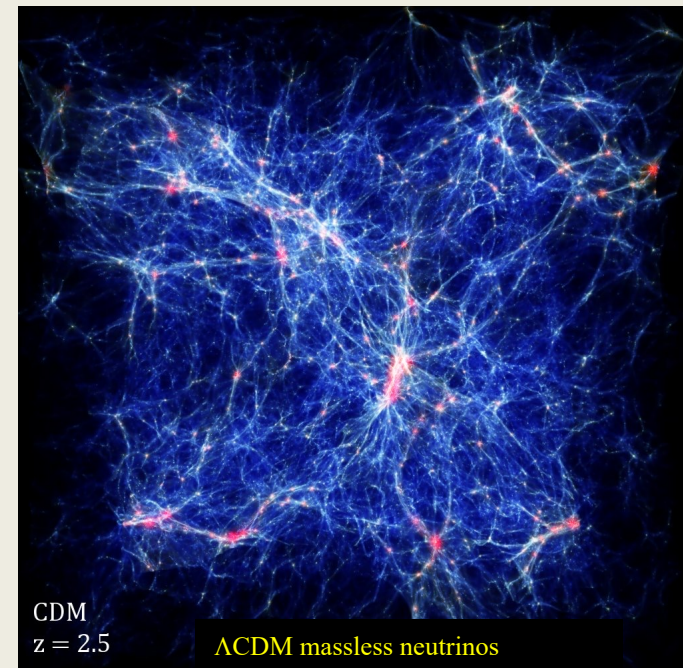
- ~2/3 of Dark Energy repulsive for gravitation
- ~1/3 “classical” matter

Content of the Universe



Content of the Universe

- Ordinary matter (baryonic) ~5% ~ 1 proton / 4 m³
 - Galaxies (stars, interstellar gas, dust)
 - Typical galaxy: $10^{12} M_{\odot}$
 - $\langle \rho_{\text{visible}} \rangle = 10^{-31} \text{ g / cm}^3$ 0.2%
 - Intergalactic gas
- Dark matter ~26%, many evidences:
 - Star rotation curves in galaxies
 - Galaxy velocity in clusters
 - Large Scale Structures, simulations
- Dark energy ~69%
 - Acceleration of the Universe expansion (SNIa)
 - Due to cosmological constant or dynamical fluid?



2) Gravitation and General Relativity

Gravitation and Relativity (Einstein)

- 1905 : Special Relativity (SR)

- Incompatible with Newton

$$F = \frac{G m_1 m_2}{|r_1(t) - r_2(t)|^2}$$

- Instantaneous force, $r_1(t)$ et $r_2(t)$ at the *same t*
- Newton = approximation of a more fundamental theory

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \text{ as Coulomb law approximation of Maxwell eq.}$$

- 1907 : Equivalence Principle

- A first step toward General Relativity
- Gravity bends light and affects the rate of clocks (1911)

- 1915 : General Relativity (GR)

- Not just a new theory of gravitation
- But a revolution in our conception of space and time
- Gravitation = curvature of spacetime → Pure geometry

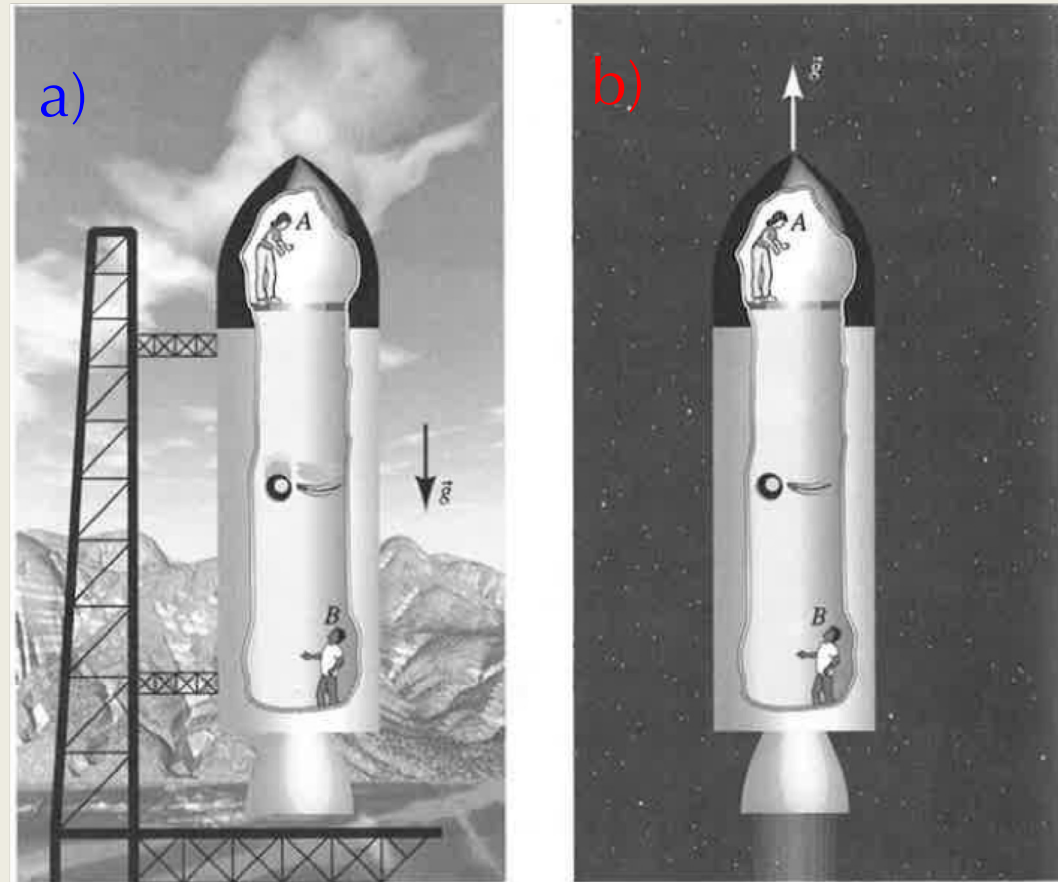
Equivalence principle

a) $m_i a = m_g g \Rightarrow$
the lead ball and the feather
experience the same
Acceleration

$\Rightarrow m_i = m_g$ and $a = g$

b) they have the same
constant speed but appear
with the same acceleration

- uniform gravitational field
= uniform acceleration



James B. Hartle

study effect of acceleration $\Rightarrow$ study gravitation

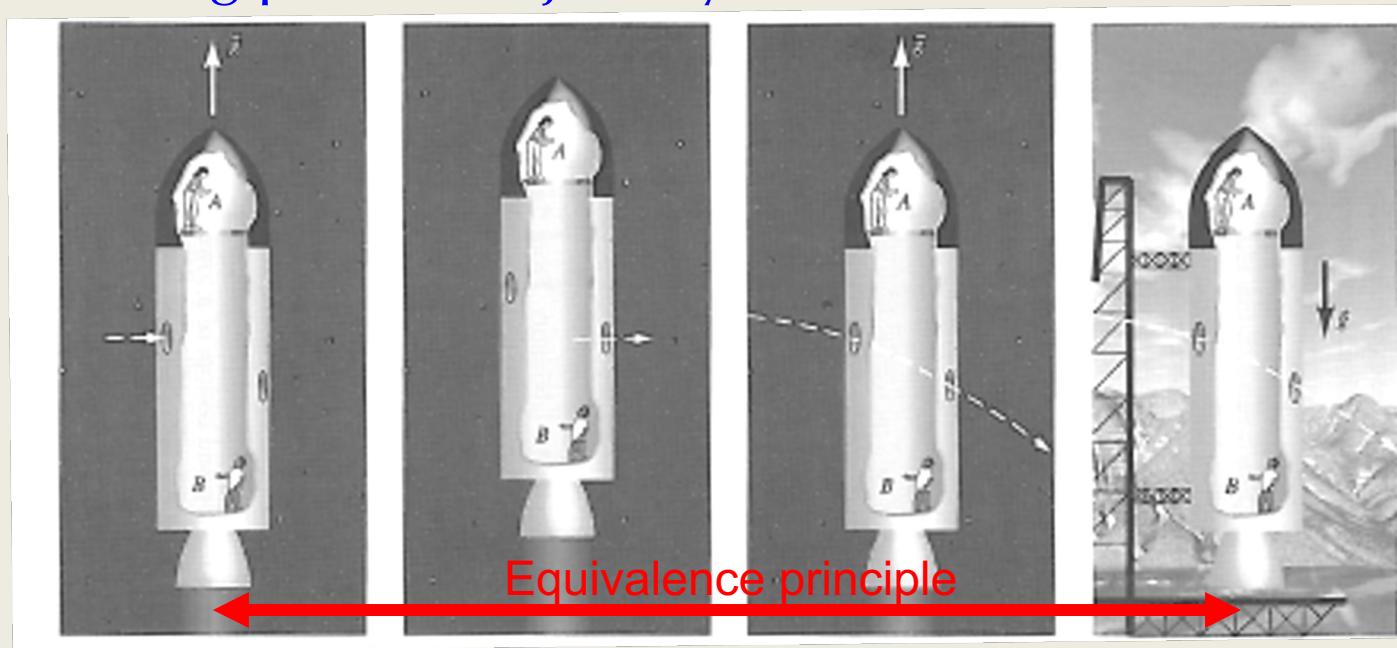
Equivalence principle

Equivalence Principle: An experiment in a freely falling laboratory, small enough and over a sufficiently small duration, is indistinguishable from the same experiment in an inertial frame away from all sources of gravitation

**Gravity can be removed by free fall
or conversely created by an acceleration**

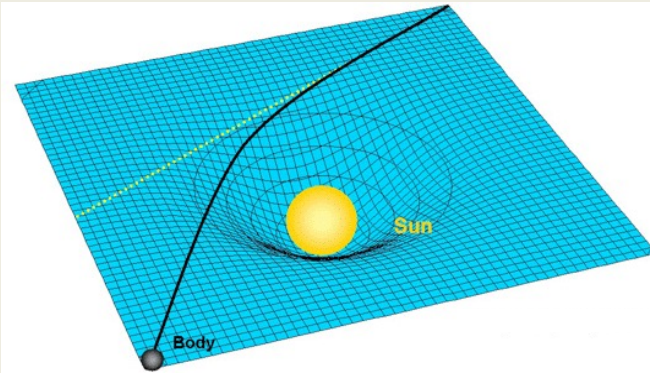
Light is falling !

- Equivalence principle applies for all physical laws
including photon trajectory



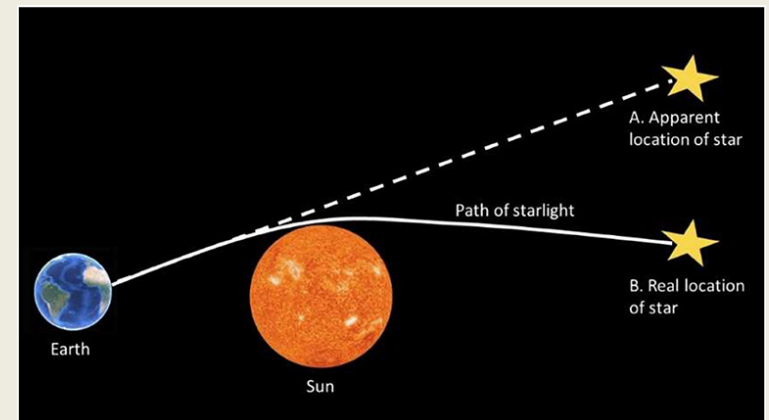
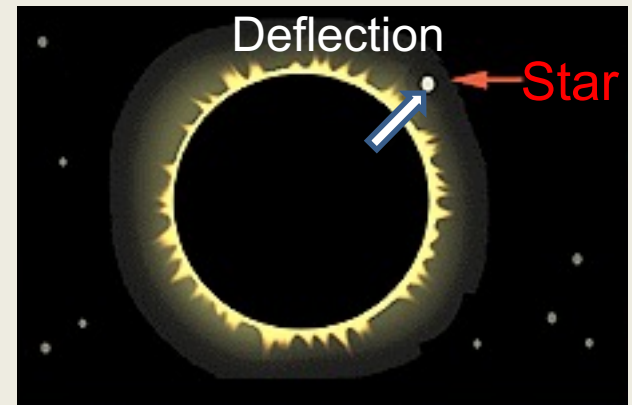
- $\Delta v = g \Delta t = g d/c \ll c \Rightarrow$ tiny effect on earth
 $\theta \sim \Delta v/c \sim gd / c^2 \quad d=10\text{m} \Rightarrow \theta \sim 9.81 \times 10 / (3 \times 10^8)^2 = 10^{-15} !$
 $g=GM/R^2 \Rightarrow \theta \sim 2GM / Rc^2 \sim 4 \mu\text{rad around sun!}$

Curved spacetime - Light rays are bent

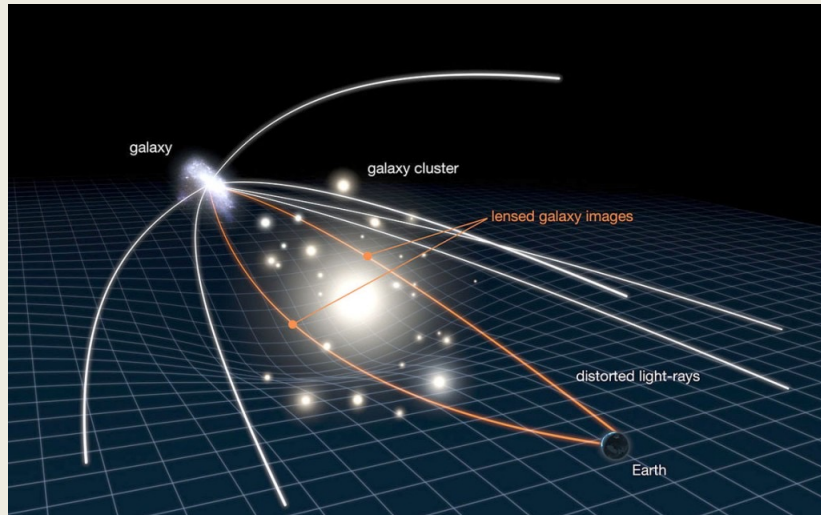


- 1915 : [Einstein](#), General Relativity mass curves spacetime and bends light

- 1919 : [Arthur Eddington](#) observes light deviation by the sun during a solar eclipse:
 - 1.75 arc second = $8.5 \mu\text{rad}$ as predicted by Einstein
 - Twice the deflection predicted by first computation (based on Eq. principle alone)



Curved spacetime - Gravitational lensing



Strong gravitational lensing
modern proof of RG

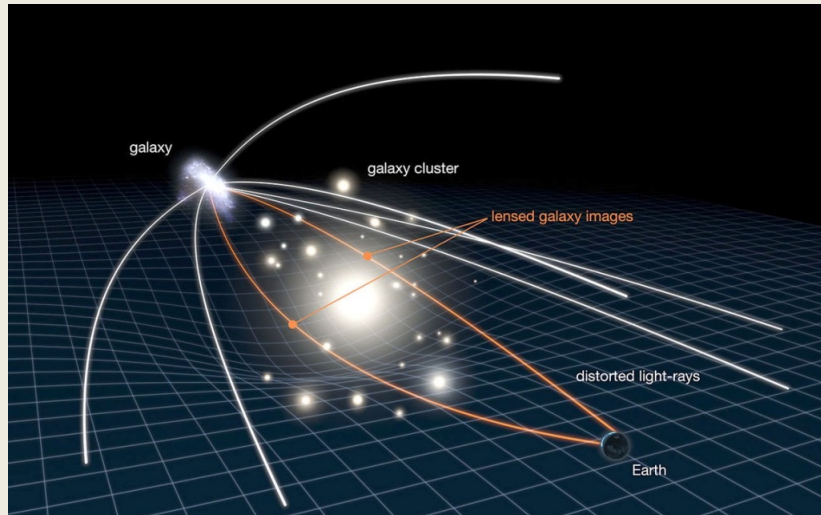


H0licow- Lensed quasars

- Image from HST
- Almost perfect align between the lens and the lensed galaxy
- One of the most complete Einstein rings ever seen observed by ESA satellite, Euclid in 2024

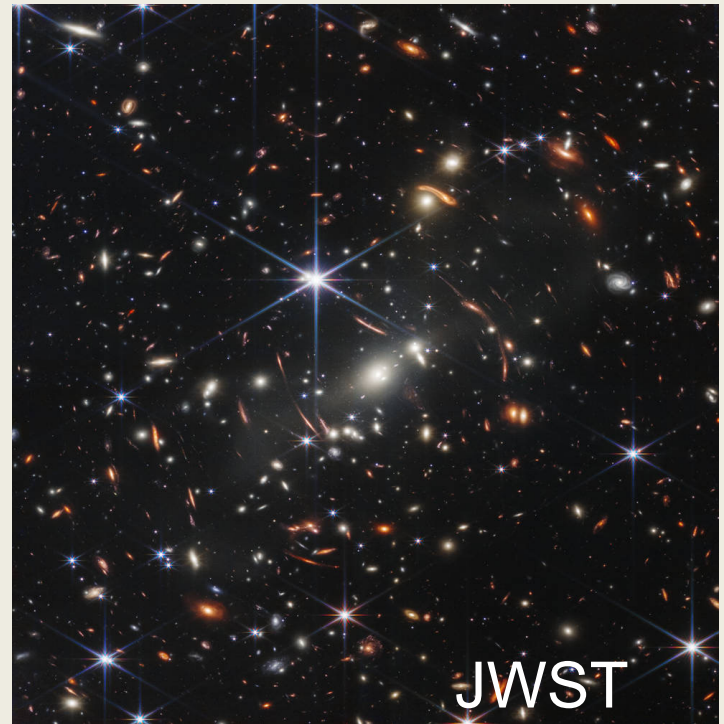


Curved spacetime - Gravitational lensing



- On July 11 2022 James Webb Space Telescope released this deep field
- Galaxies behind galaxy cluster SMACS 0723 ($z=0.39$, $R_{\text{vir}}=2.4\text{Mpc}$) are curved and warped

- Strong gravitational lensing: modern proof of RG



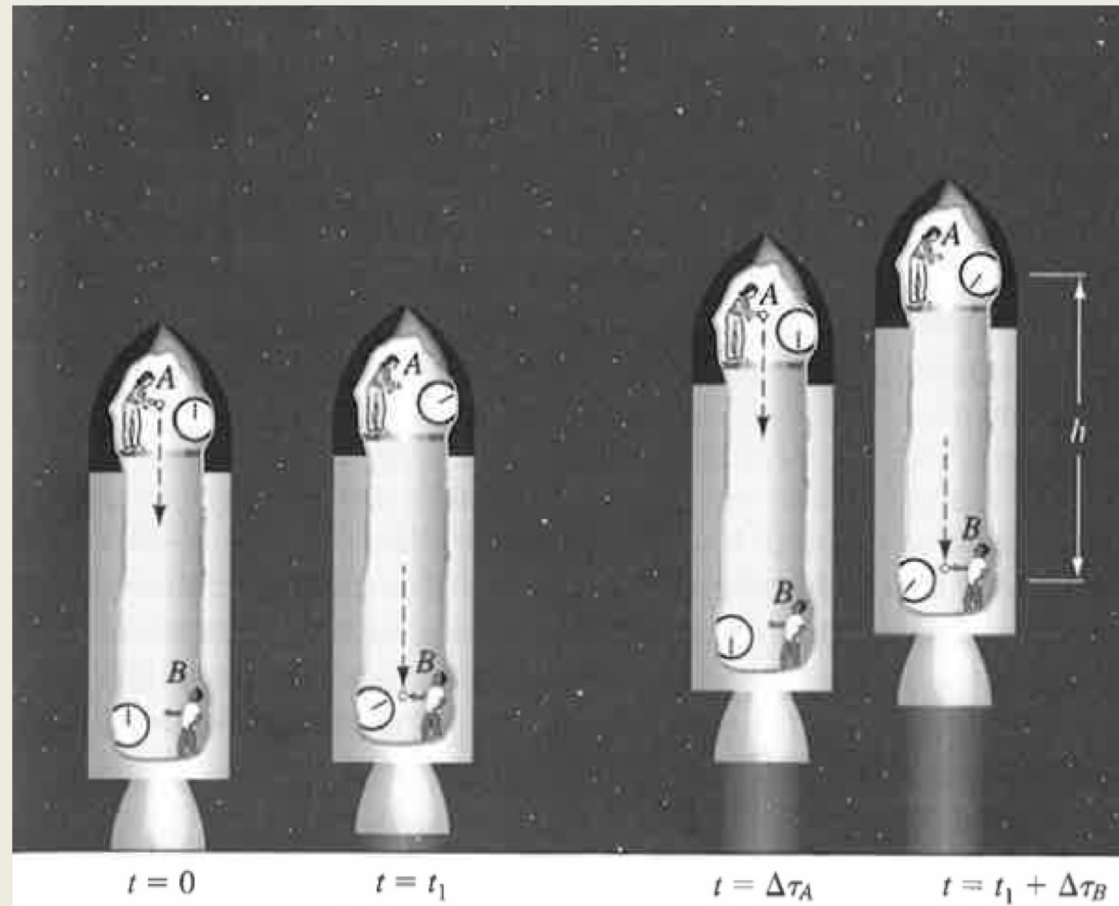
2.25 arcmin, 0.7Mpc at $z=0.39$

Clocks and gravitation

- a rocket in deep space with acceleration $+g$
 - A emits at $t=0$ and $\Delta\tau_A$
 - B receives at $t=t_1$ and $t_1 + \Delta\tau_B$
- Propagation time : $(t_1 - 0)$ acceleration $\Rightarrow$ faster

$$(t_1 + \Delta\tau_B) - \Delta\tau_A < t_1 - 0$$

$$\Rightarrow \Delta\tau_B < \Delta\tau_A$$
- Calculation gives (totally classic):



$$\Delta t_B = \left(1 - \frac{gh}{c^2}\right) \Delta t_A$$

Clocks and gravitation

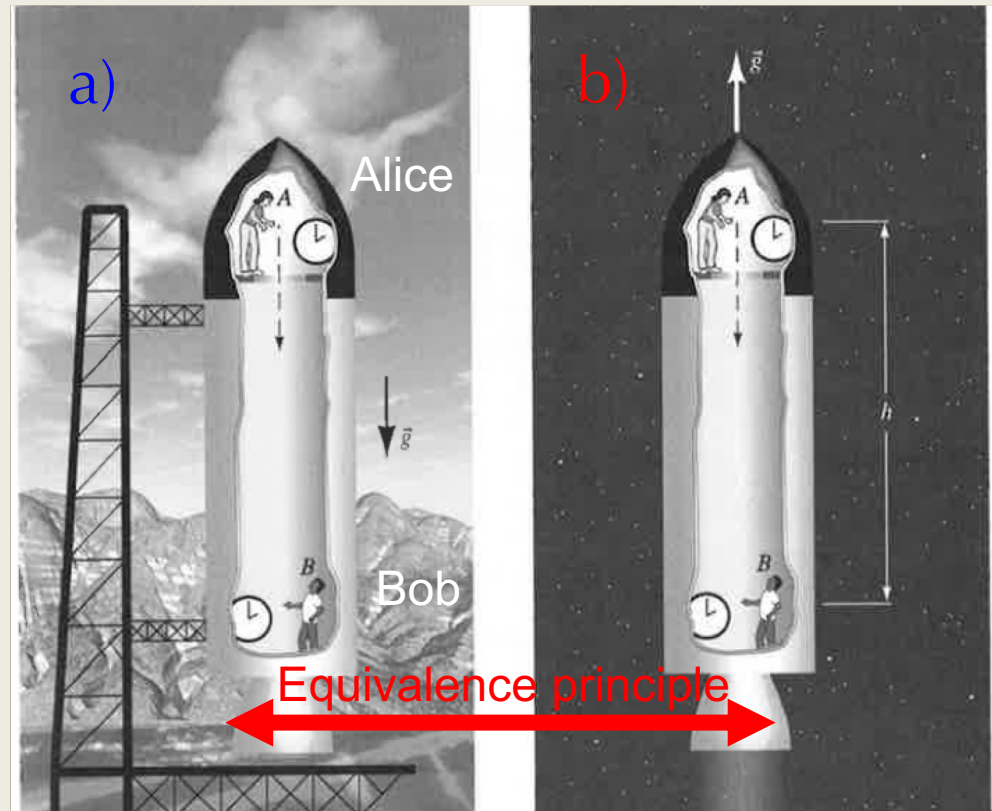
Equivalence Principle
(a) equivalent to (b)

$$\Delta t_B = \left(1 - \frac{gh}{c^2}\right) \Delta t_A$$

Times run slower in a
gravitational field !

$$h = z_A - z_B \Rightarrow gh \sim \Phi_A - \Phi_B$$

$$\Delta t_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right) \Delta t_A$$



Bob is younger than Alice....

Gravitational “redshift”

$$\Delta t_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right) \Delta t_A$$

- at the surface of a star: $\phi_A = -GM/R$
far away: $\phi_B = 0$

$$\Delta t_\infty = \left(1 + \frac{GM}{Rc^2}\right) \Delta t_*$$

$$\nu_\infty = \left(1 - \frac{GM}{Rc^2}\right) \nu_* < \nu_*$$

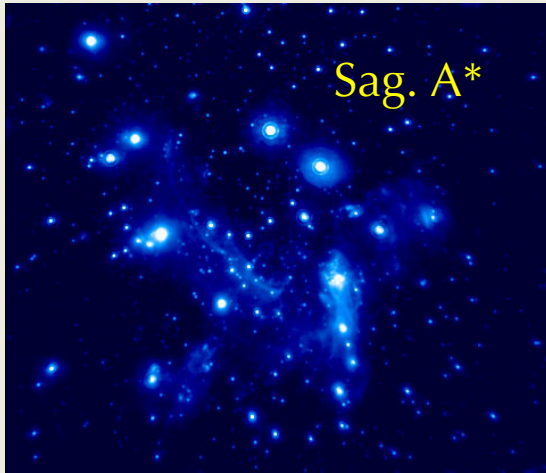
$\Rightarrow$ gravitational redshift

the photon loses energy going out of the potential well

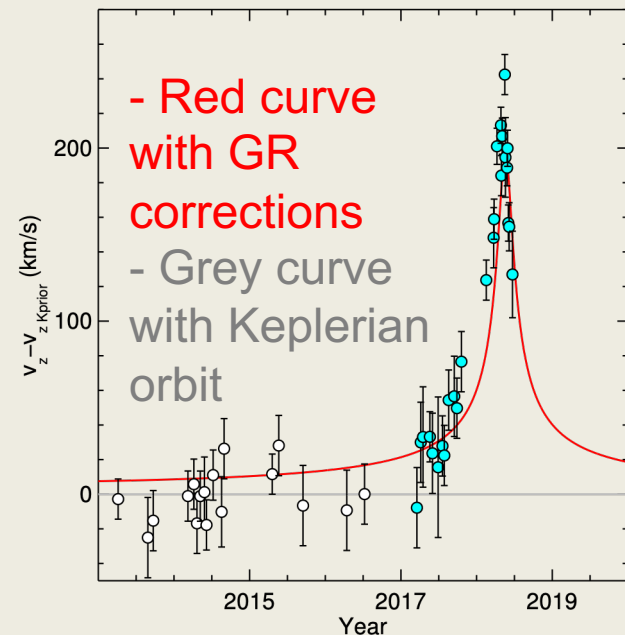
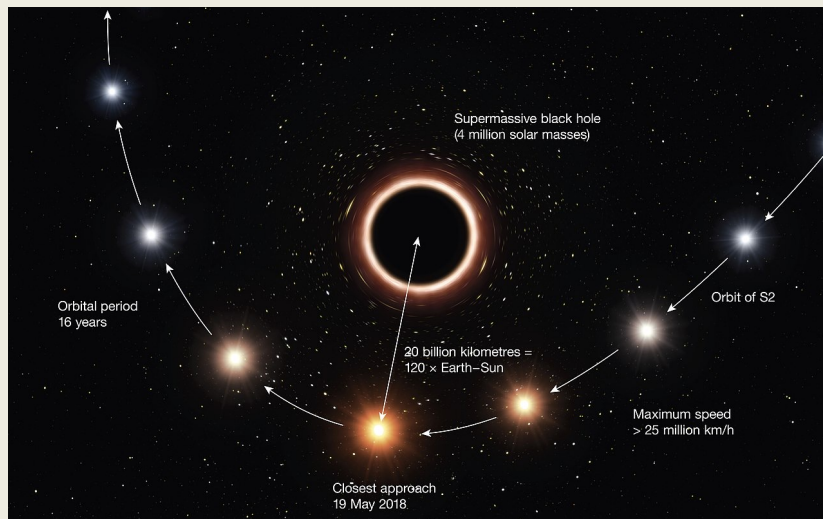
Positive shift in wavelength $\Delta\lambda/\lambda > 0$

- very important for GPS : $\Delta\nu/\nu \sim 4 \cdot 10^{-10}$
after 1h : $10^{-10} \times 3600 \text{ s}$ error $\Rightarrow \sim 400 \text{ m}$ error

S2 star close to MW black Hole



- Close to source Sagittarius A*, BH in the Milky way ($\sim 4 \cdot 10^6$ solar mass)
- S2 star very close to the BH on May 19 2018
- Verification of Einstein shift (plot below)
- Redshift ($c \cdot \Delta\lambda/\lambda \rightarrow$ speed km/s), note sign!



Science in movies



Planet of the Apes

- Twin paradox in Special Relativity (SR)
- Lorentz boost
$$ct = \gamma(ct' + \beta x)$$
$$\gamma = 1/(1 - \beta^2)^{1/2} > 1$$
- Time dilatation $T = \gamma T'$



Interstellar

- Strong gravitational field (GR)
- Proximity to a black hole (BH)
- $T = (1 + GM/(Rc^2)) \cdot T'$