Precision Tests of the MSSM at LHC

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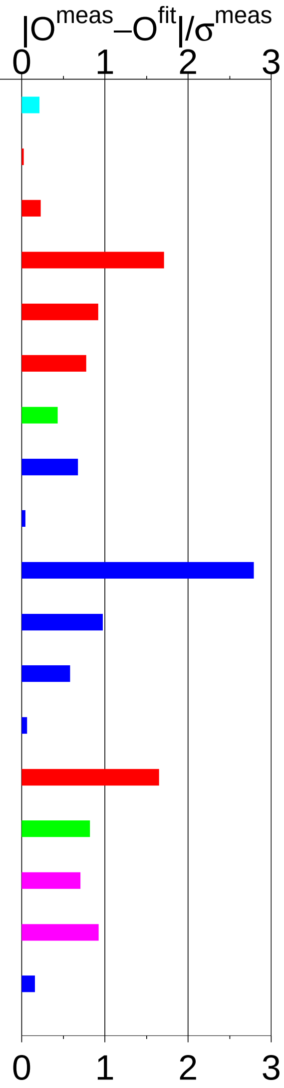
Outline:

- Higgs boson mass in the MSSM
- Coupling constant unification in the MSSM

Standard Model

The SM is amazingly successful: [LEP and TEVATRON data]

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^0 [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1479
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7



Motivation

The SM has deficiencies. Many open questions:

- . . .
- What is the origin of the mass spectrum?
- What is really the Higgs?
- Are the fundamental forces unified?
- What is the dark matter?
- Why is there matter-antimatter asymmetry?
- . . .

Possible answers in physics *Beyond the Standard Model*

- Supersymmetry, GUT, Extra Dimensions, String Theory, . . .

Supersymmetry

Unification of all forces of Nature $\rightsquigarrow$ include Gravity around $M_{Plank} \simeq 10^{19}$ GeV .

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  - SUSY generators:

$$Q\,|boson >= \,|fermion > \qquad Q\,|fermion >= \,|boson >$$

$\Rightarrow$ a partial unification of matter(fermions) with forces (bosons) naturally achieved.

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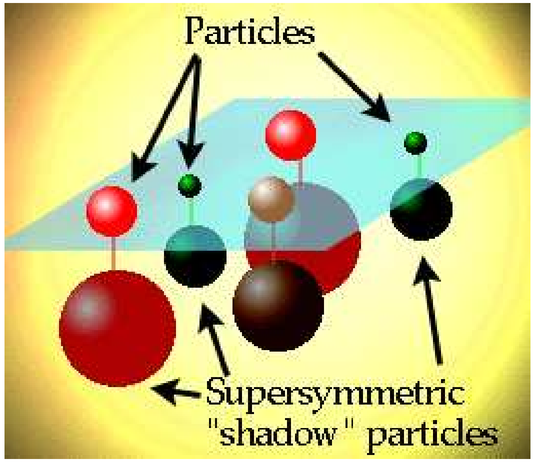

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- Minimal Supersymmetric extension of SM (MSSM)

	Bosons (spin=0)	Bosons(spin=1)	Fermions
Gauge		gluon weak photon	gluino wino, zino photino
Matter	sleptons squarks		leptons quarks
Higgs	Higgses		higgsinos

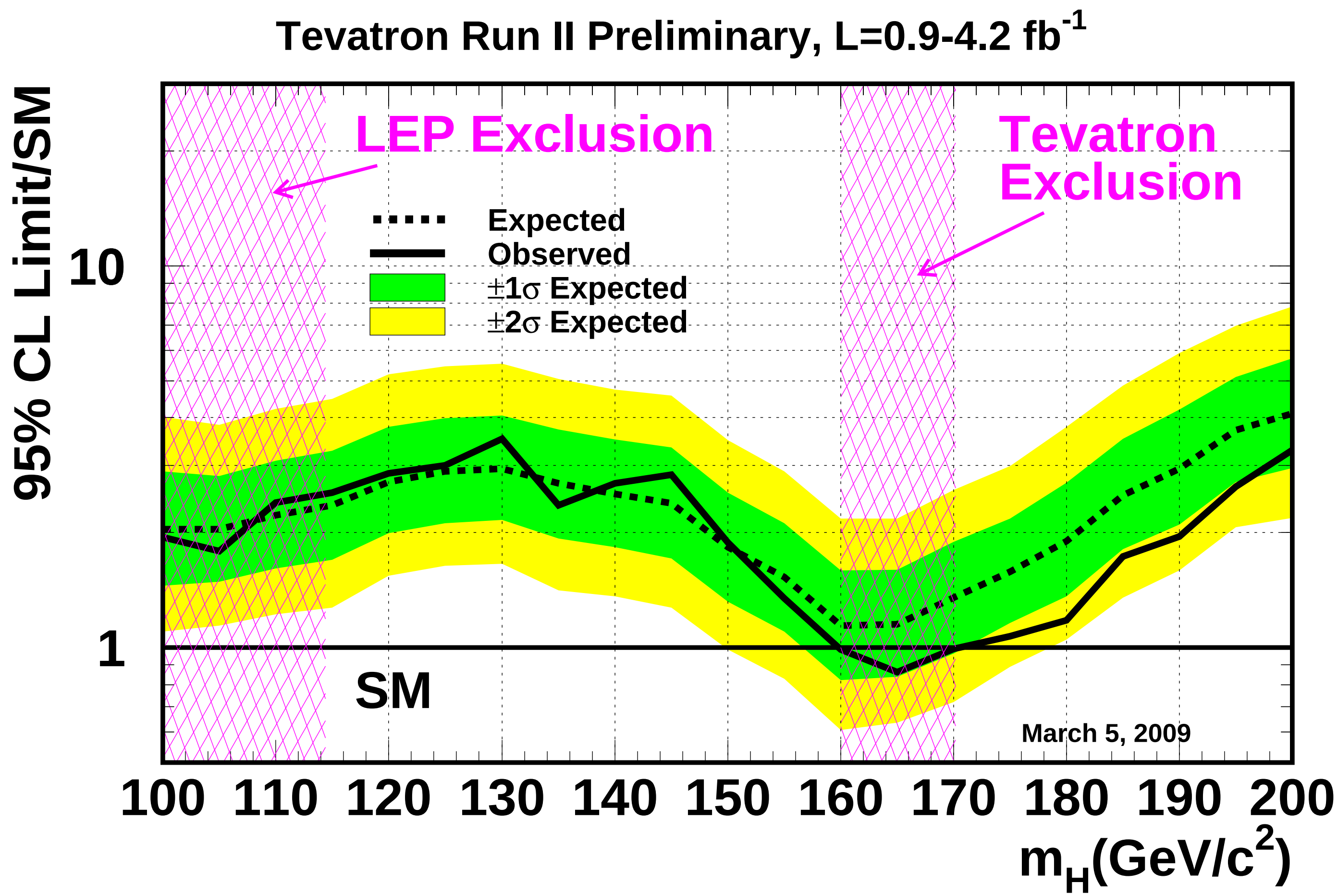
Supersymmetry

- SUSY can naturally address some of the basic questions:
  - the hierarchy problem ($M_W \ll M_{GUT}$) become natural
  - dark matter candidate
  - predicts a light Higgs boson !!
  - predicts gauge coupling unification !!
  - predicts SUSY particles at about 1 TeV
  - ...

Higgs boson mass in the MSSM

SM Higgs Search

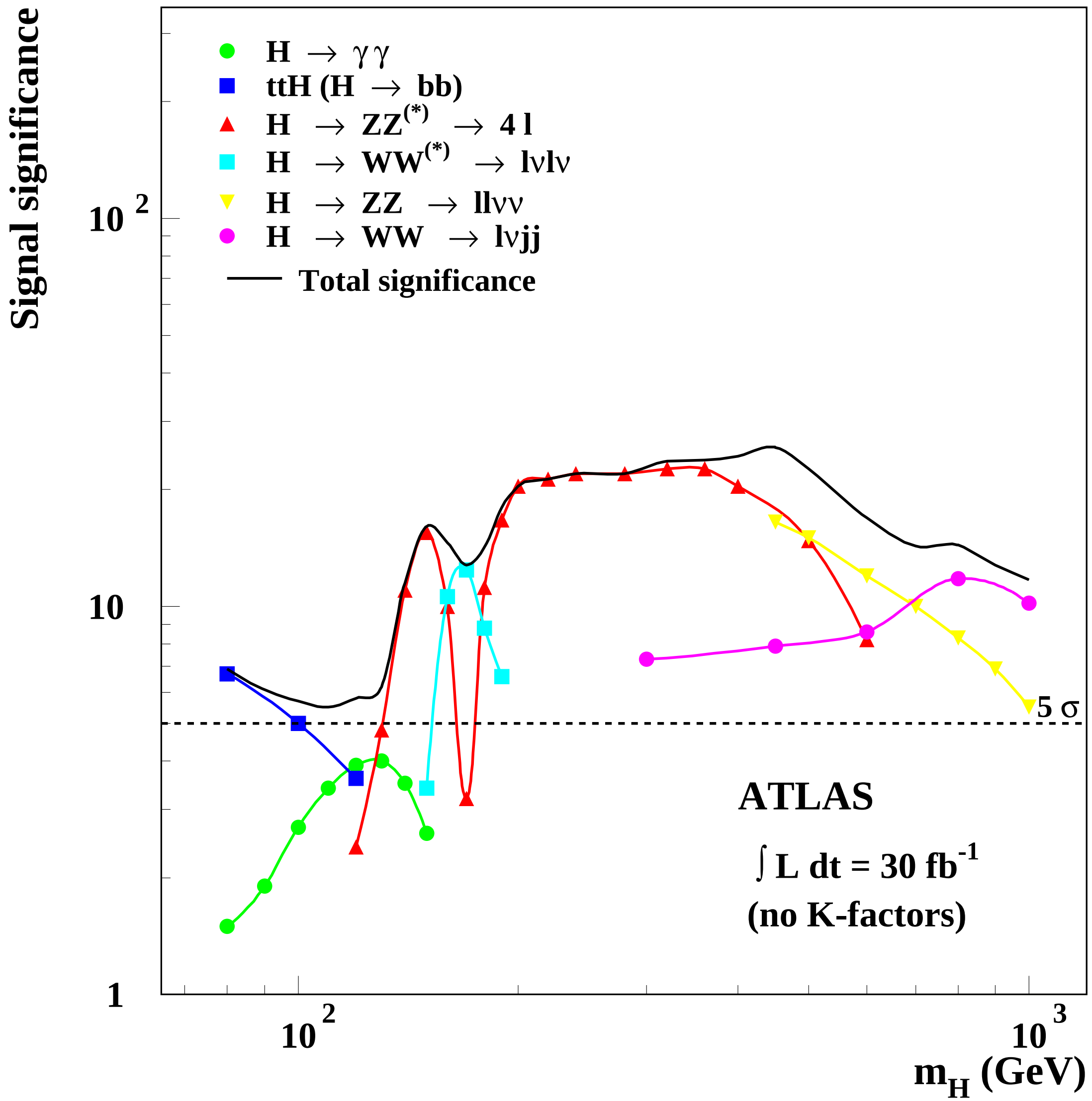
at present: direct searches



electroweak precision data: $m_H = 87^{+36}_{-27}$ GeV

SM Higgs Search

expected soon at the LHC



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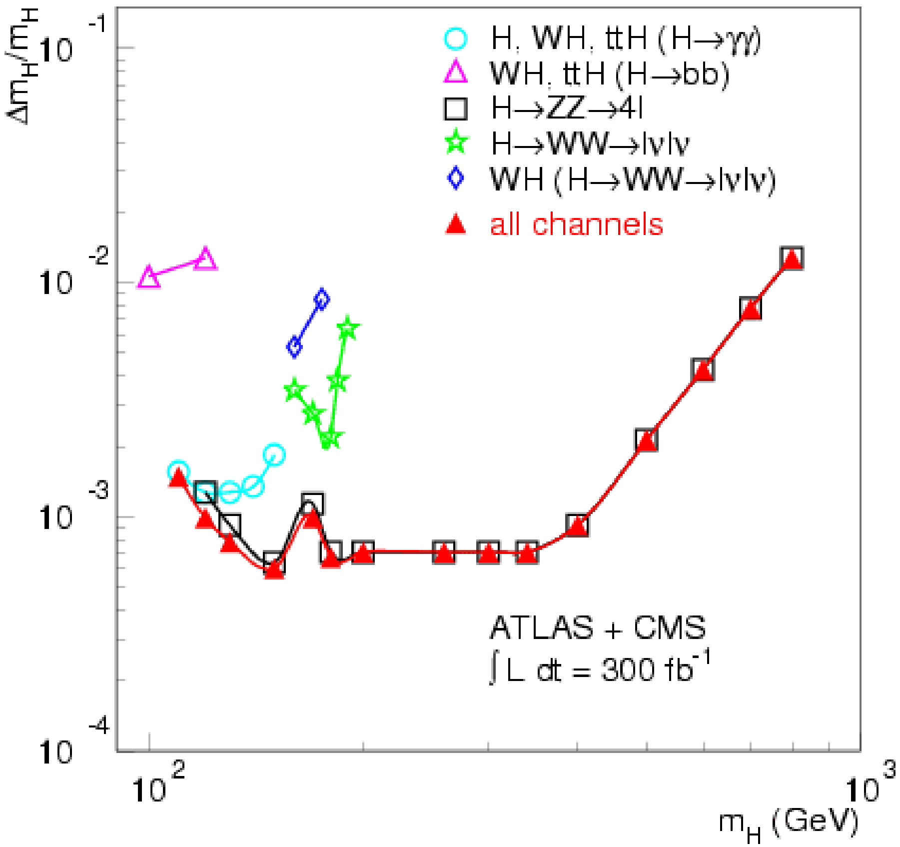

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- if discovered $\Rightarrow$ its mass is a precision measurement
- Higgs mass is very sensitive to top/stop sector through radiative corrections

  $\Rightarrow$ the mass of the light Higgs used as a consistency check of the MSSM
- MSSM electroweak precision data + heavy flavour + dark matter:

$$m_h = 110^{+10}_{-8} \pm 3(\mathrm{th})\ \text{GeV}$$

Theory vs. experiment

Experiment LHC: $\delta m_h^{\rm exp} = 100 - 200\,{\rm MeV}$ CERN-LHCC-2006-21

ILC: $\delta m_h^{\rm exp} = 50\,{\rm MeV}$ AguilarSaavedra et al '06

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- 2-loop $\mathcal{O}(\alpha_t\alpha_s, \alpha_t^2, \alpha_b\alpha_s, \alpha_b\alpha_t)$ in effective potential approximation ($p^2 = 0$)
  [Haber, Hempfling, Hoang '96], [Heinemeyer, Hollik and Weiglein '98], [Degrassi, Slavich, Zwirner '01], [Espinosa and Zang '00], [Brignole, Degrassi, Slavich, Zwirner '02] , [Carena et al '00], [Heinemeyer et al '05] , [S. Martin '03]
- Momentum-dependent corrections ($p^2 = m_h^2$): 2-loop SUSY-QCD [S.Martin '05]
- 3-loop LL and NLL $\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$ [S. Martin '07]

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  - full 2-loop corrections
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$$\mathcal{M}^2_{H,\text{tree}} = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_Z^2 \cot\beta + M_A^2 \tan\beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan\beta + M_A^2 \cot\beta \end{pmatrix}$$

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Higher order corrections

$$\mathcal{M}_H^2 = \mathcal{M}_{H,\text{tree}}^2 - \begin{pmatrix} \hat{\Sigma}_{\phi_1} & \hat{\Sigma}_{\phi_1\phi_2} \\ \hat{\Sigma}_{\phi_1\phi_2} & \hat{\Sigma}_{\phi_2} \end{pmatrix}$$

$\hat{\Sigma}_{\phi_i}$ = renormalized self-energies

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$\Sigma_i(0)$ = bare self-energies

δV_i = Higgs potential counterterms

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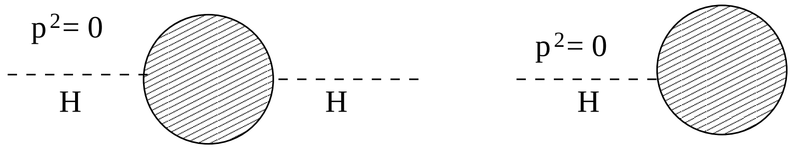

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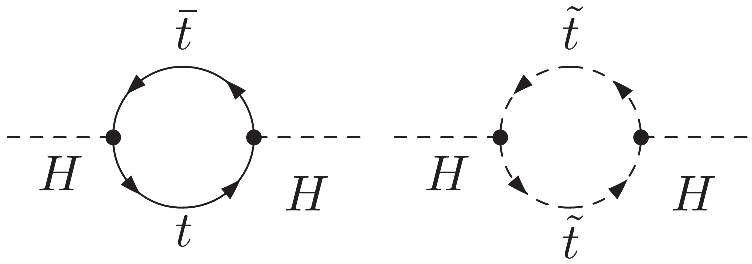

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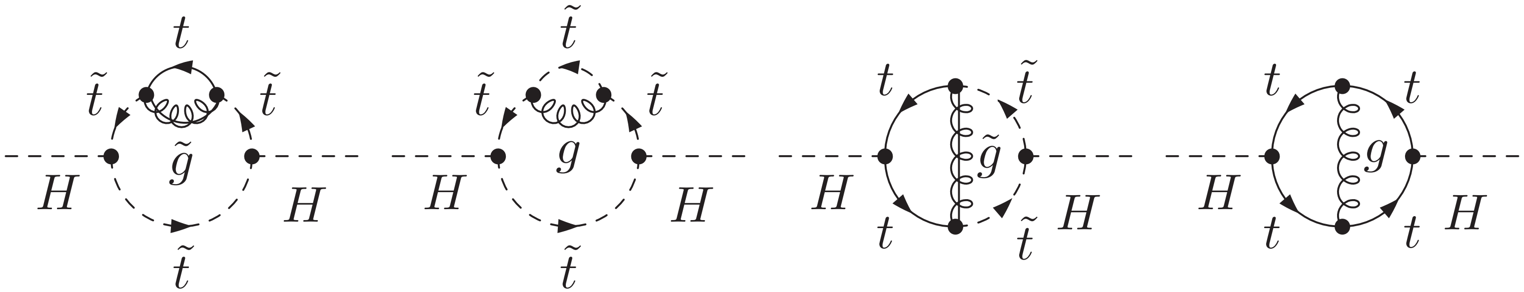

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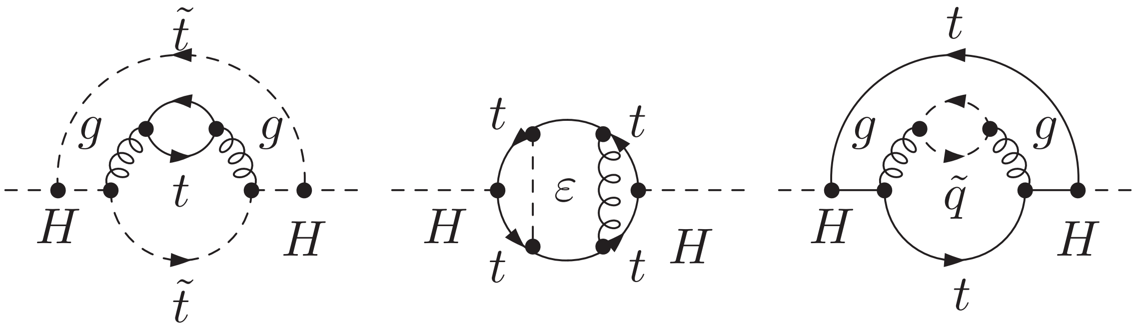

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- $\simeq$ 28.000 diagrams
- Computer programs: QGRAF, PERL, FORM, MINCER, MATAD, EXP, . . .

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- Asymptotic expansion $\rightsquigarrow$ 3-loop tadpole integrals $\rightsquigarrow$ MATAD

Numerical Results (no stop-mixing)

Input SM parameters: $\mu = M_t = 172.4$ GeV $\quad G_F = 1.16637 \times 10^{-5}$ GeV^{-2}

$M_Z = 91.1876$ GeV $\quad \alpha_s^{(5)}(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

MSSM parameters: $M_A = 1$ TeV $\quad \tan\beta = 40 \quad ; \quad A_t = 0 \quad M_{\tilde{q}} = 2$ TeV

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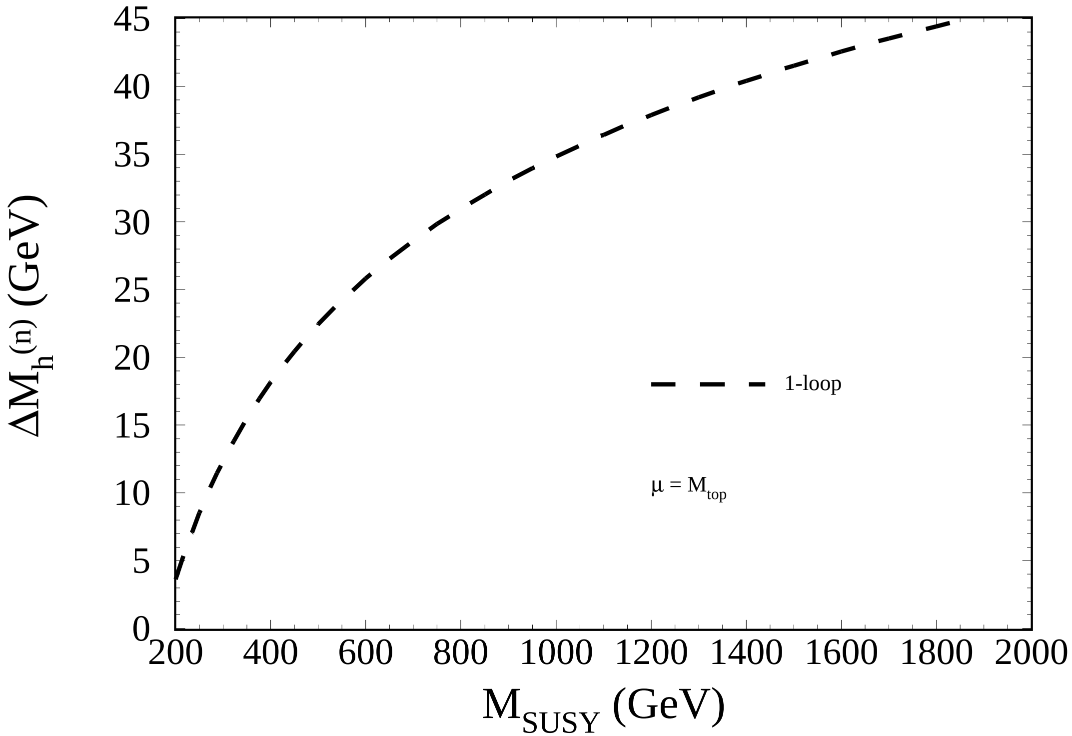

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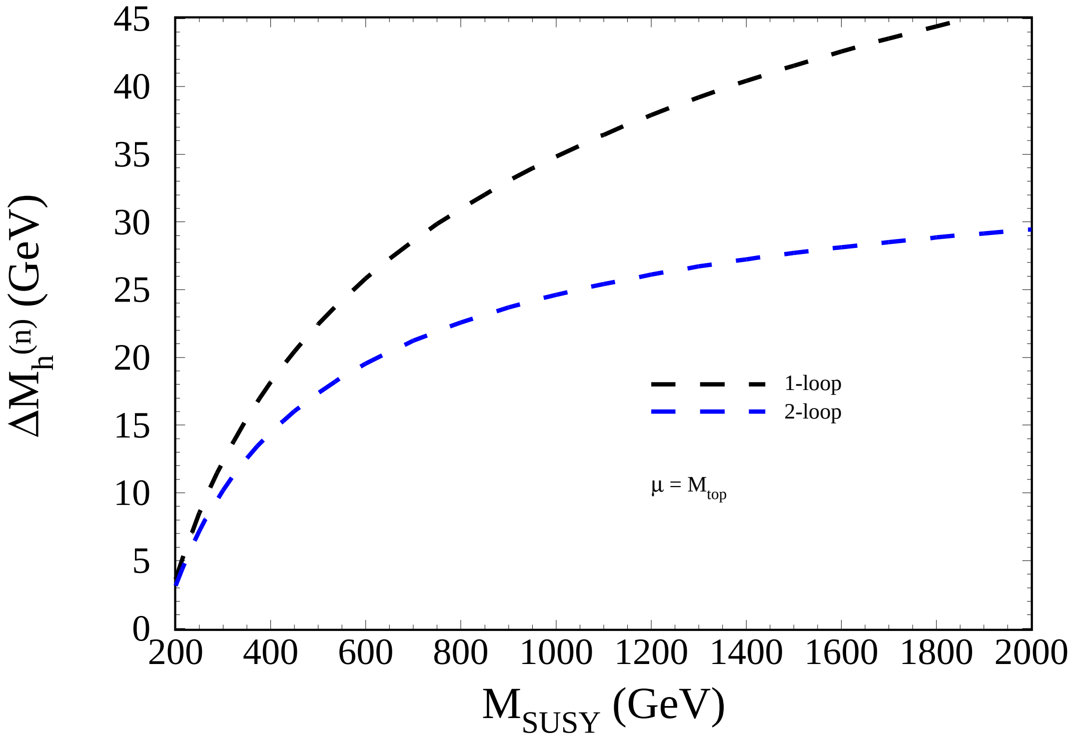

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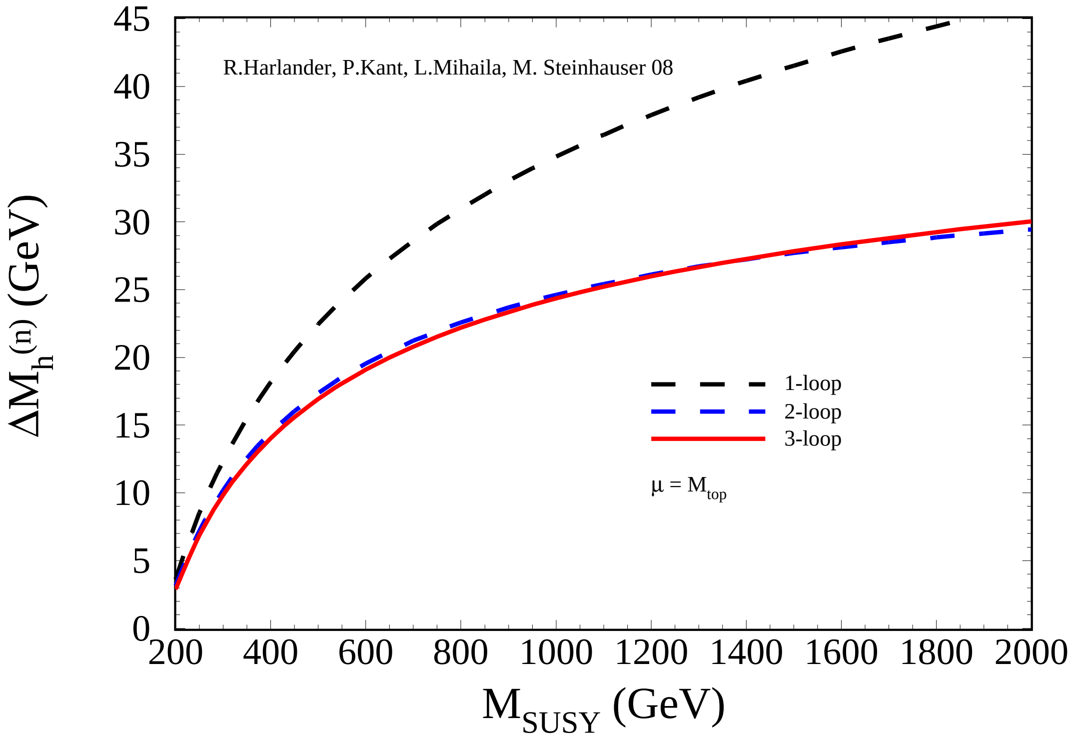

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OS-scheme $\quad \Delta M_h^{(n)} \equiv M_h^{(n-\text{loop})} - M_h^{\text{tree}}$

$M_{\text{SUSY}} = 0.3 - 1$ TeV : $\quad \Delta M_h^{(3)} \simeq 500$ MeV

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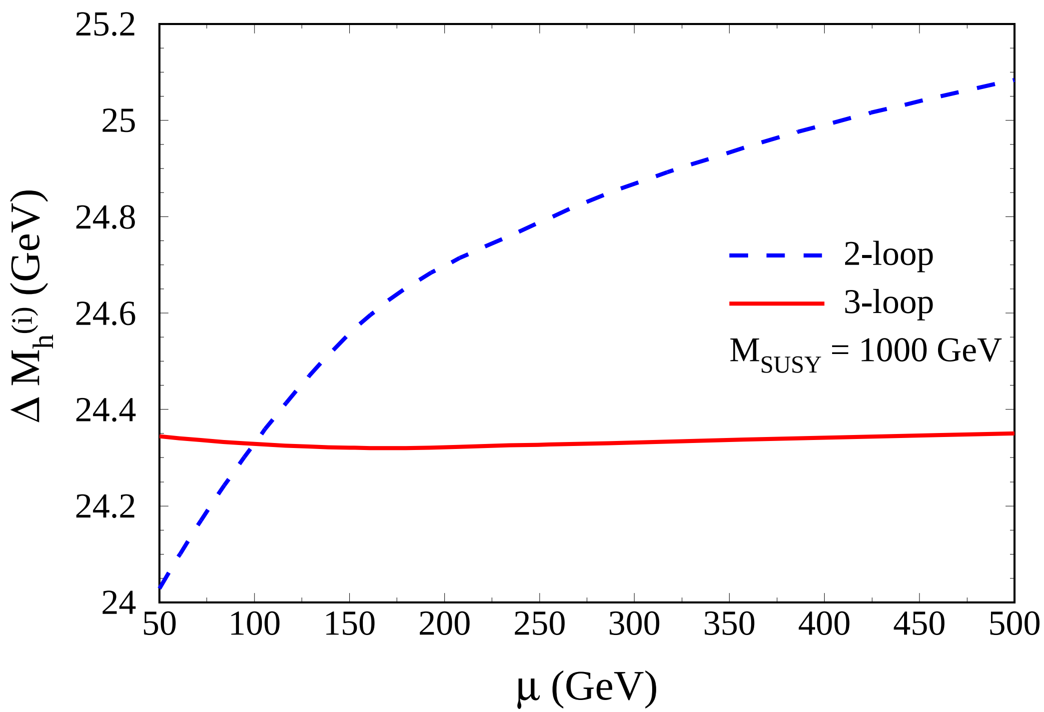

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Parametric uncertainties: dominated by M_t

$\delta\,\Delta M_t^{\mathrm{LHC}} \simeq 1-2$ GeV $\Rightarrow$ $\delta\,\Delta M_h \simeq 1-2$ GeV

Numerical Results (no stop-mixing)

Input SM parameters: $\mu \quad = M_t = 172.4$ GeV $\quad G_F = 1.16637 \times 10^{-5}$ GeV^{-2}

$M_Z = 91.1876$ GeV $\quad \alpha_s^{(5)}(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

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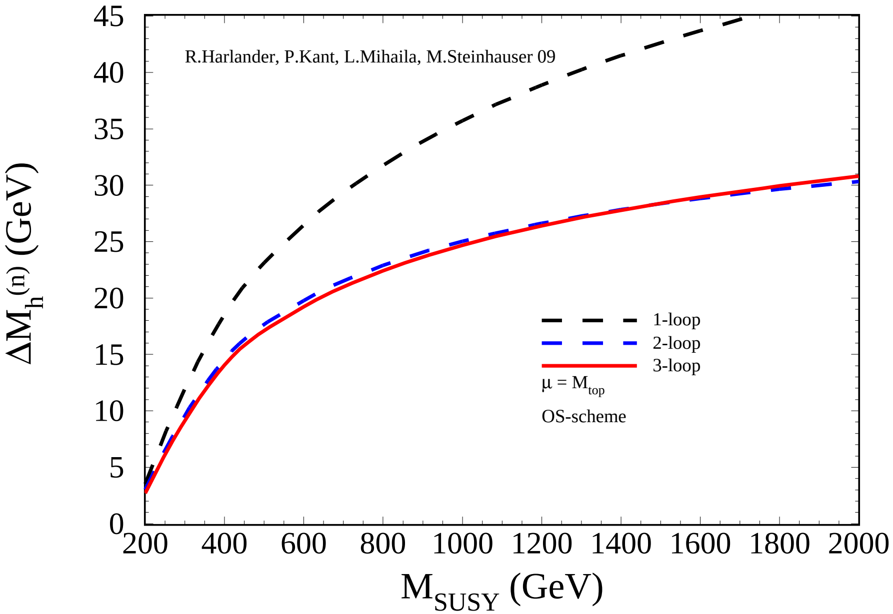
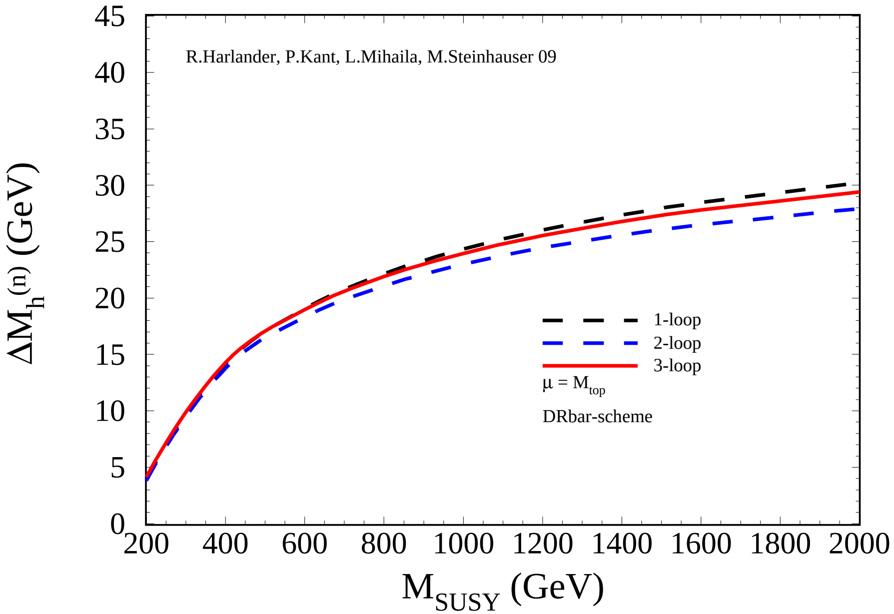
Renormalization scheme dependence:

OS



$\overline{\text{DR}}$



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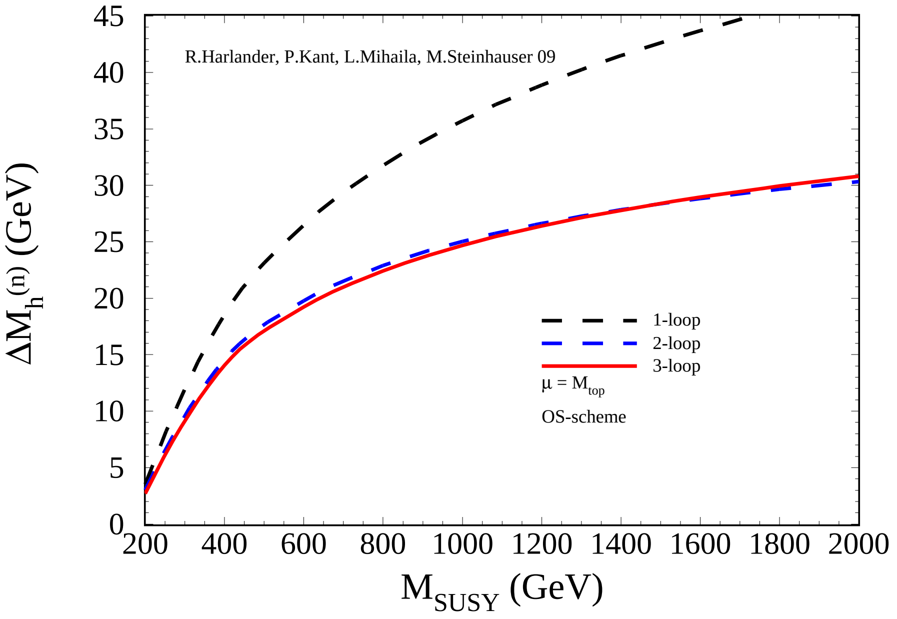
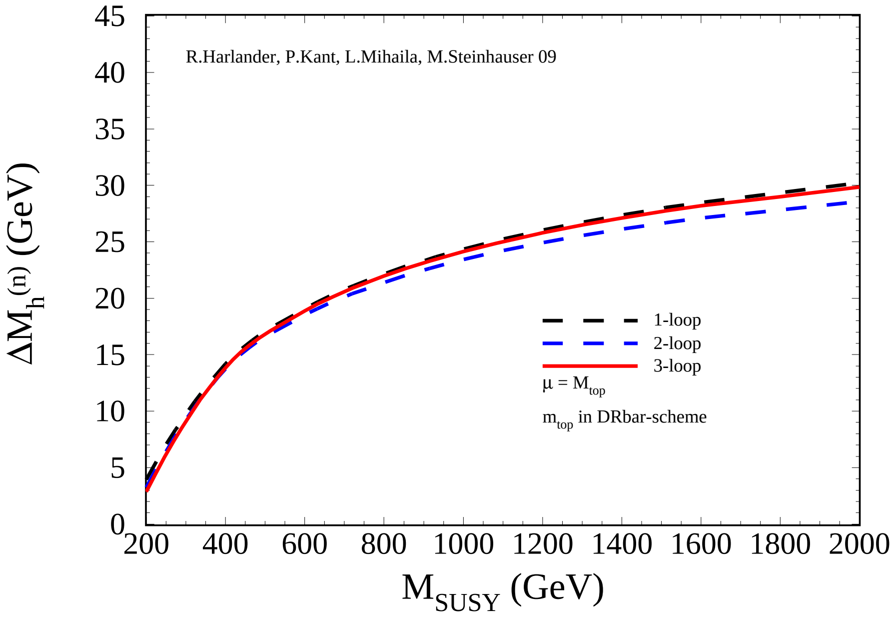


m_t in $\overline{\text{DR}}$-scheme: TSIL [S. Martin '05]



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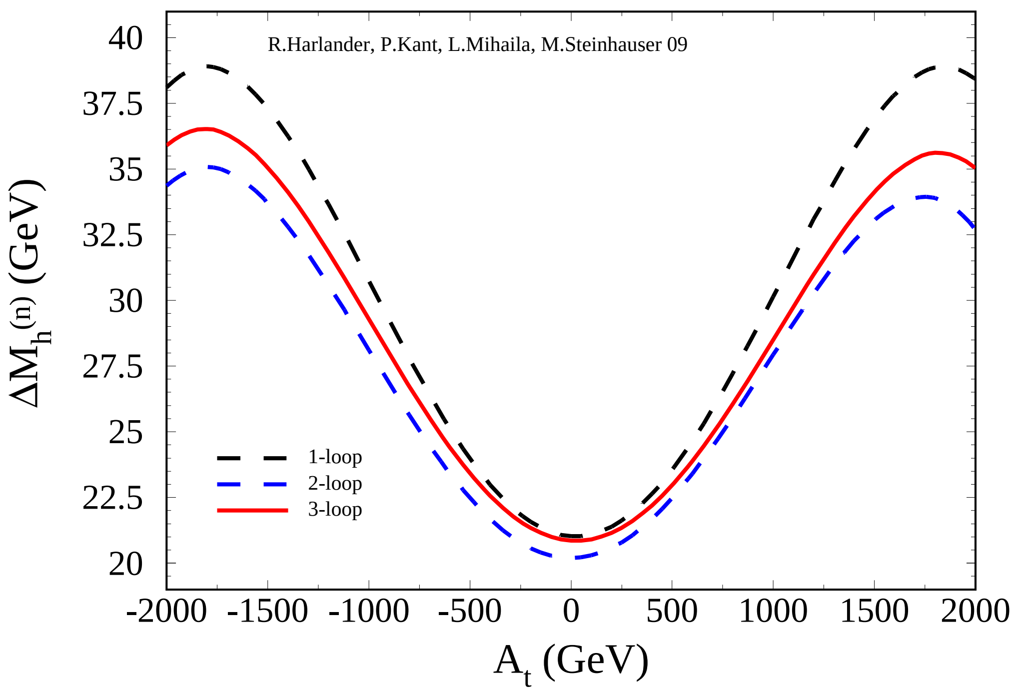
$\overline{\text{DR}}$ scheme $\quad A_t = 0\ :\ \pm 2$ TeV : $\Delta M_h^{(3)} = 0.5 - 1.5$ GeV



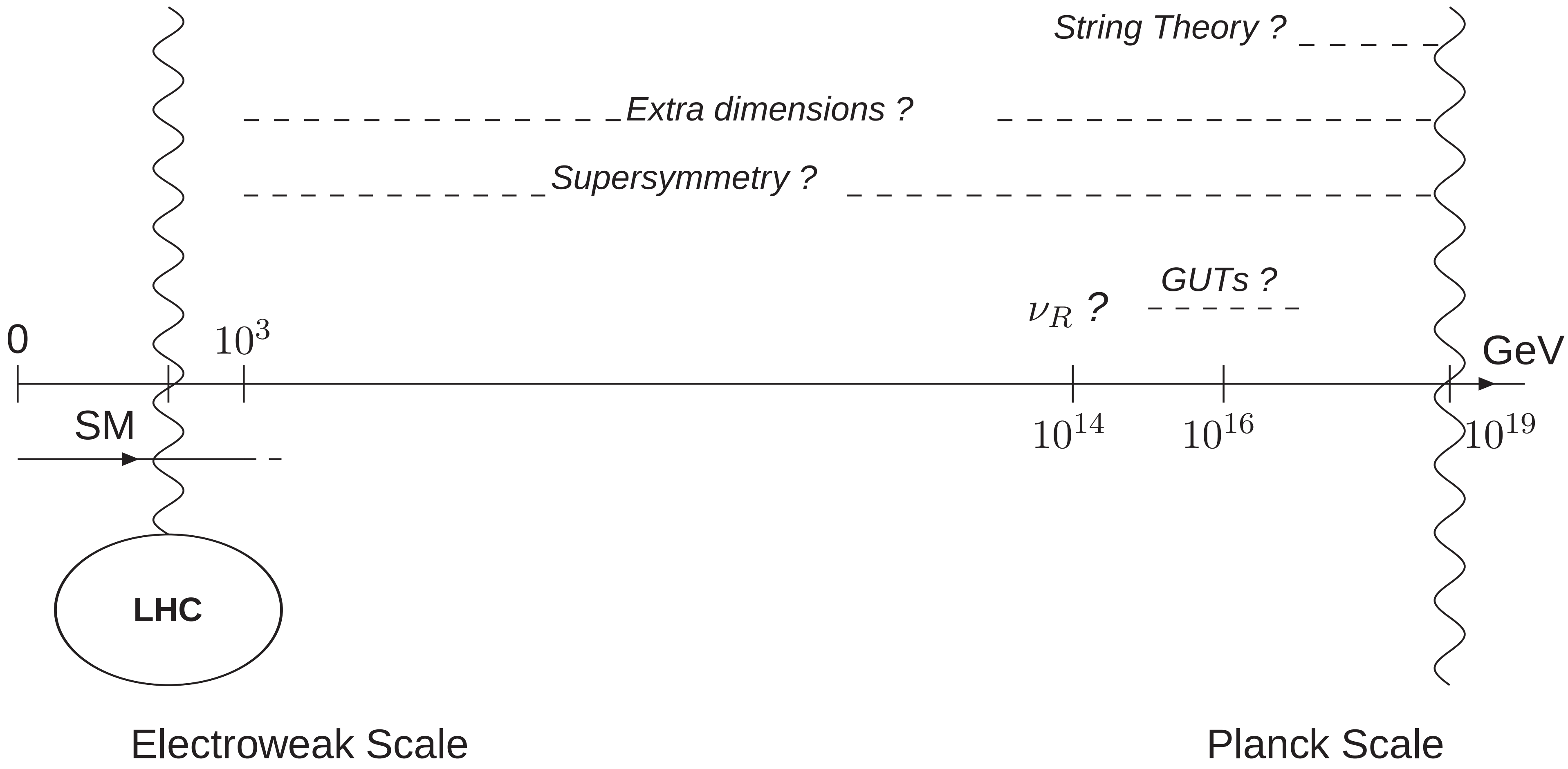
Conclusions

- m_h to 3-loop accuracy
  - 3-loop effects larger than experimental accuracy expected at LHC & ILC
  - 3-loop corrections stabilize the perturbative series
  - Stop-mixing & large $|A_t|$: $\Delta M_h^{(3)} \simeq 1.5$ GeV

- ToDo:
  - Computer code to compute m_h for realistic SUSY mass spectrum
  - 3-loop effects due to CP-violation in the MSSM

Coupling constant unification

Open Questions



Grand Unification

- Gauge symmetry increases with energy [Georgi, Quinn, Weinberg '74]

	Low energy		$\Rightarrow$	High energy
$SU_c(3)\otimes$	$SU_L(2)\otimes$	$U_Y(1)$	$\Rightarrow$	G_{GUT}
gluons	W, Z	photon	$\Rightarrow$	gauge bosons
quarks	leptons		$\Rightarrow$	fermions
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}

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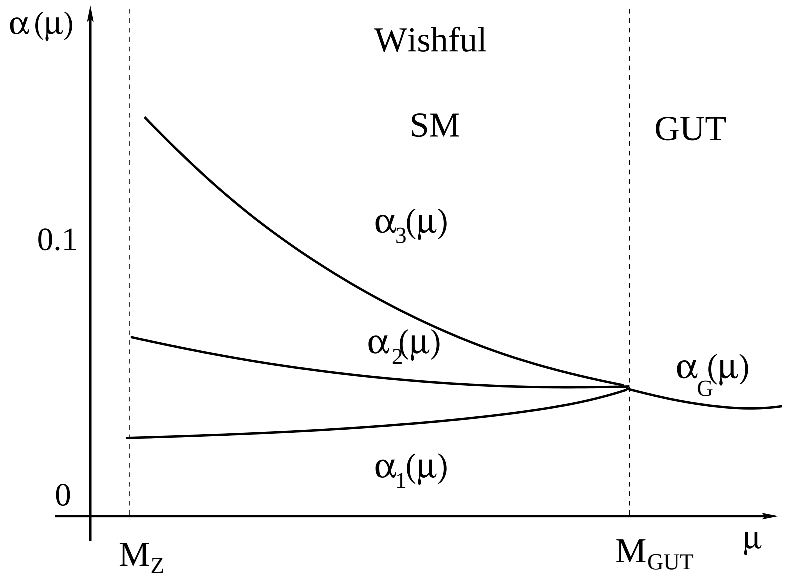
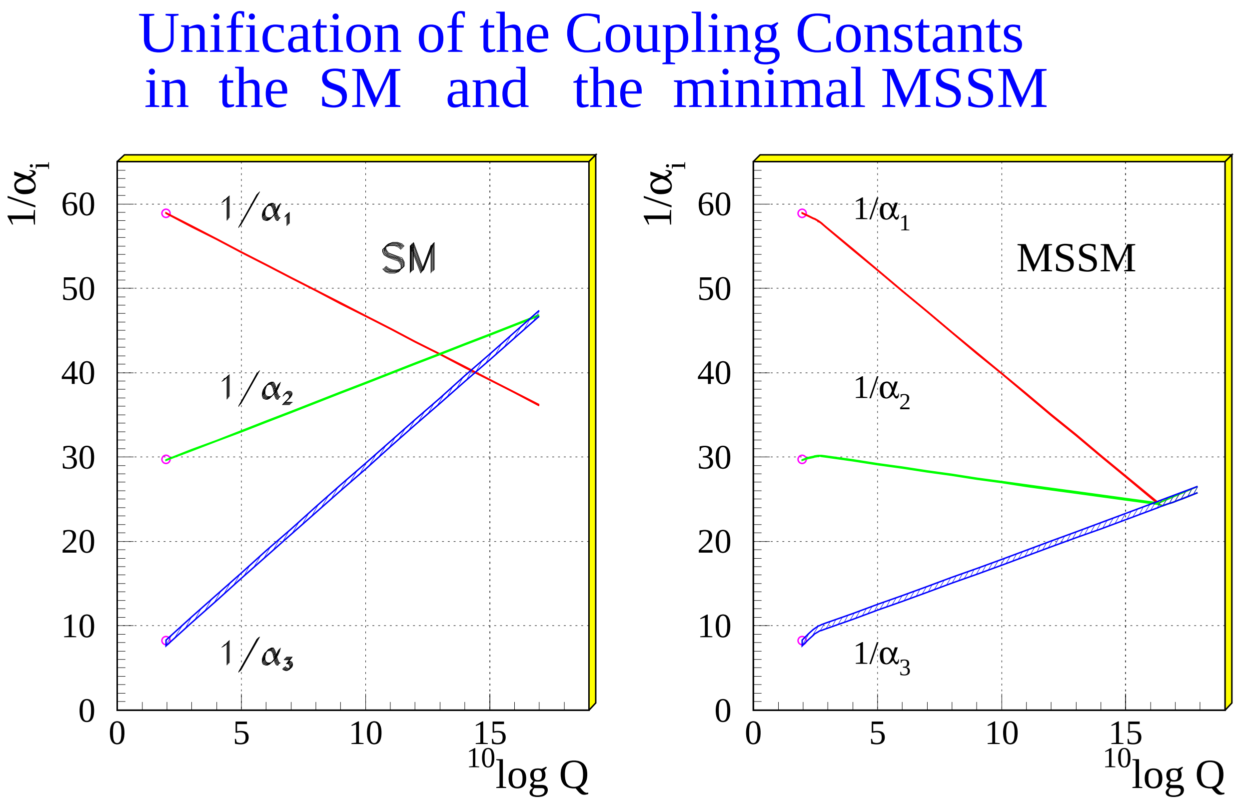
- Low energy interactions = branches of the unique interaction of a simple gauge group.

$$\alpha_i \equiv g_i^2/4\pi : \qquad \begin{aligned} \alpha_1(M_Z) &= 0.017\,, \\ \alpha_2(M_Z) &= 0.034\,, \\ \alpha_3(M_Z) &= 0.118\,, \\ M_Z &= 91.1876 \text{ GeV}\,. \end{aligned}$$



MSSM and LEP data



[Amaldi, Furstenau, de Boer]

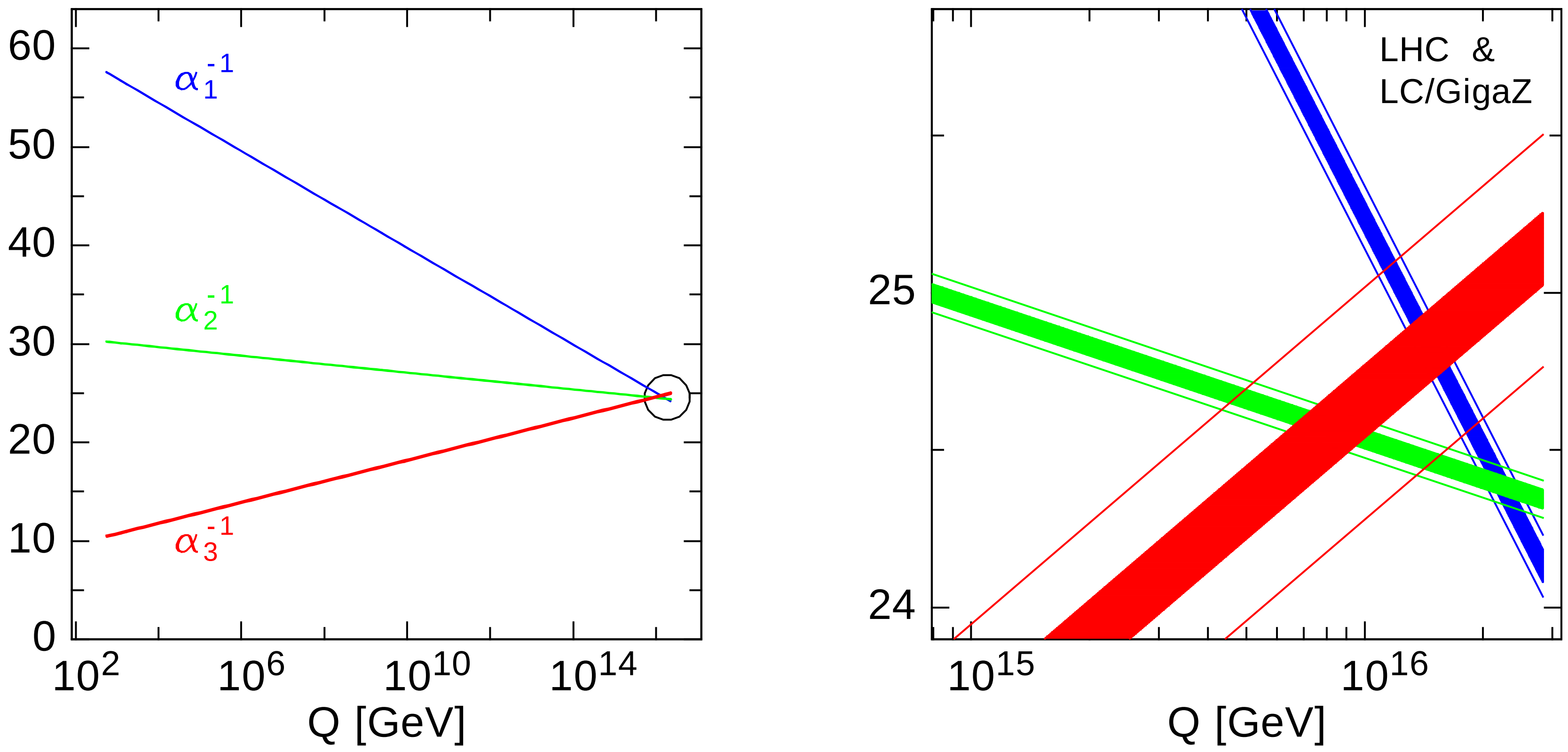
[Langacker, Luo]

[Ellis, Kelley, Nanopoulos]

- Gauge Coupling unification within SM excluded by about $12\,\sigma$.
- Gauge coupling Unification within SUSY GUTs works extremely well: it fits within 3σ the present low energy data.

High precision data

[Blair, Porod, Zerwas '04]



- Computation: common SUSY mass scale $\simeq$1 TeV
  2-loop Renormalization Group Running 1-loop threshold corrections at the weak scale (M_Z)
- Our aim: improve theoretical accuracy on $\alpha_s(M_{\rm GUT})$ calculated from $\alpha_s(M_{\rm Z})$

GUTs

- GUT threshold corrections (effects of heavy particles $M \geq M_{\rm GUT}$) $\Rightarrow$ gauge coupling no longer meet at a point $M_{\rm GUT}$

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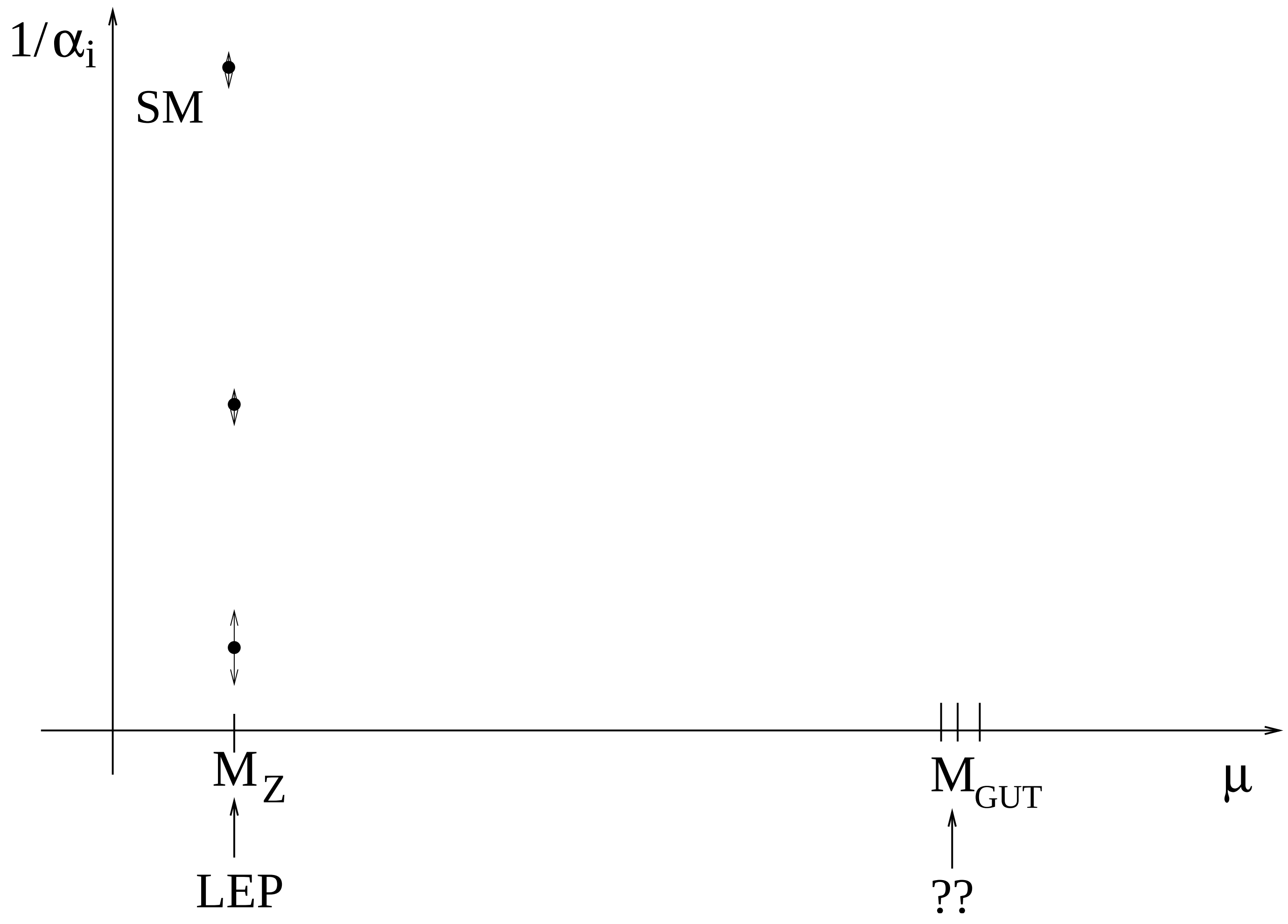

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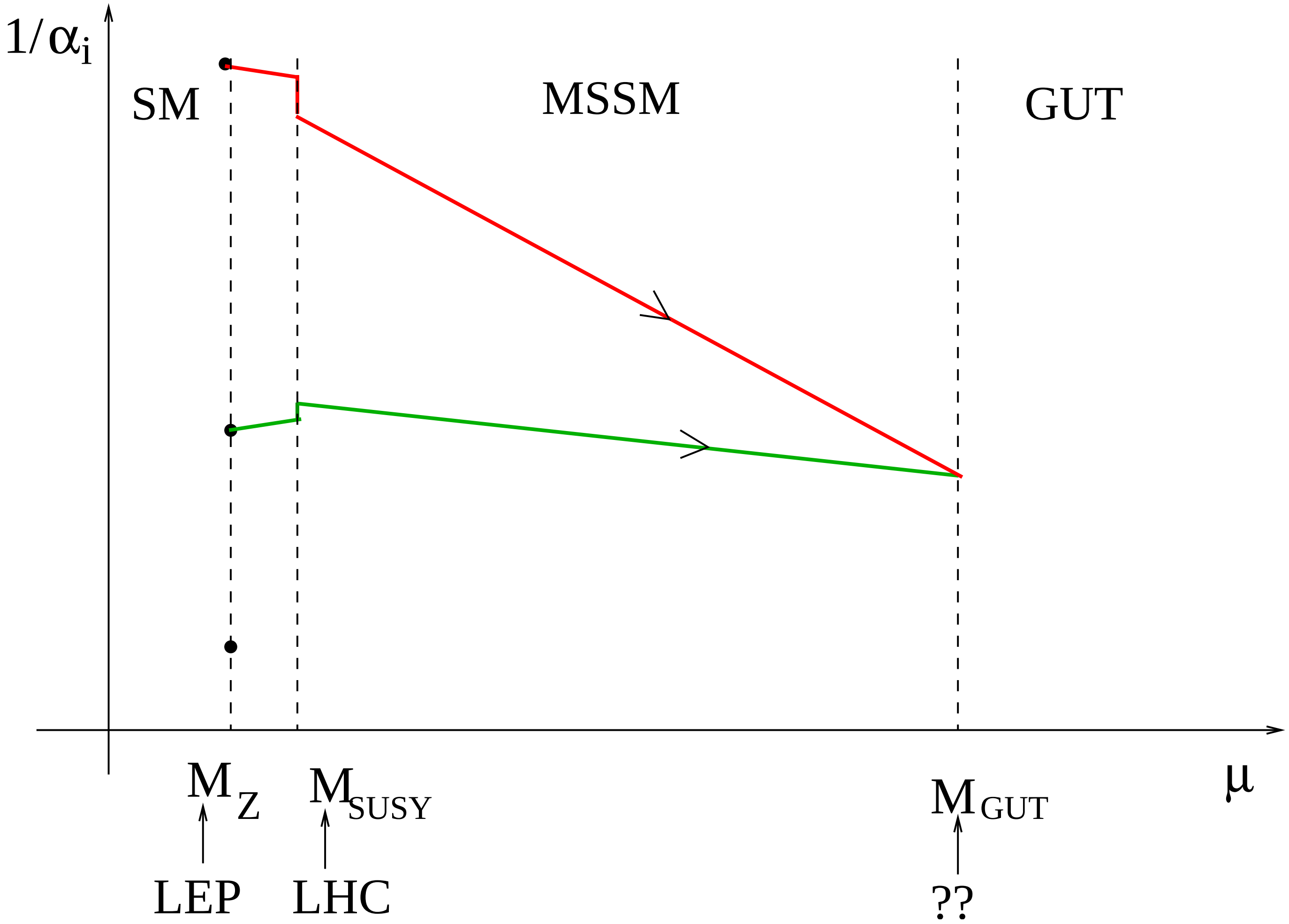

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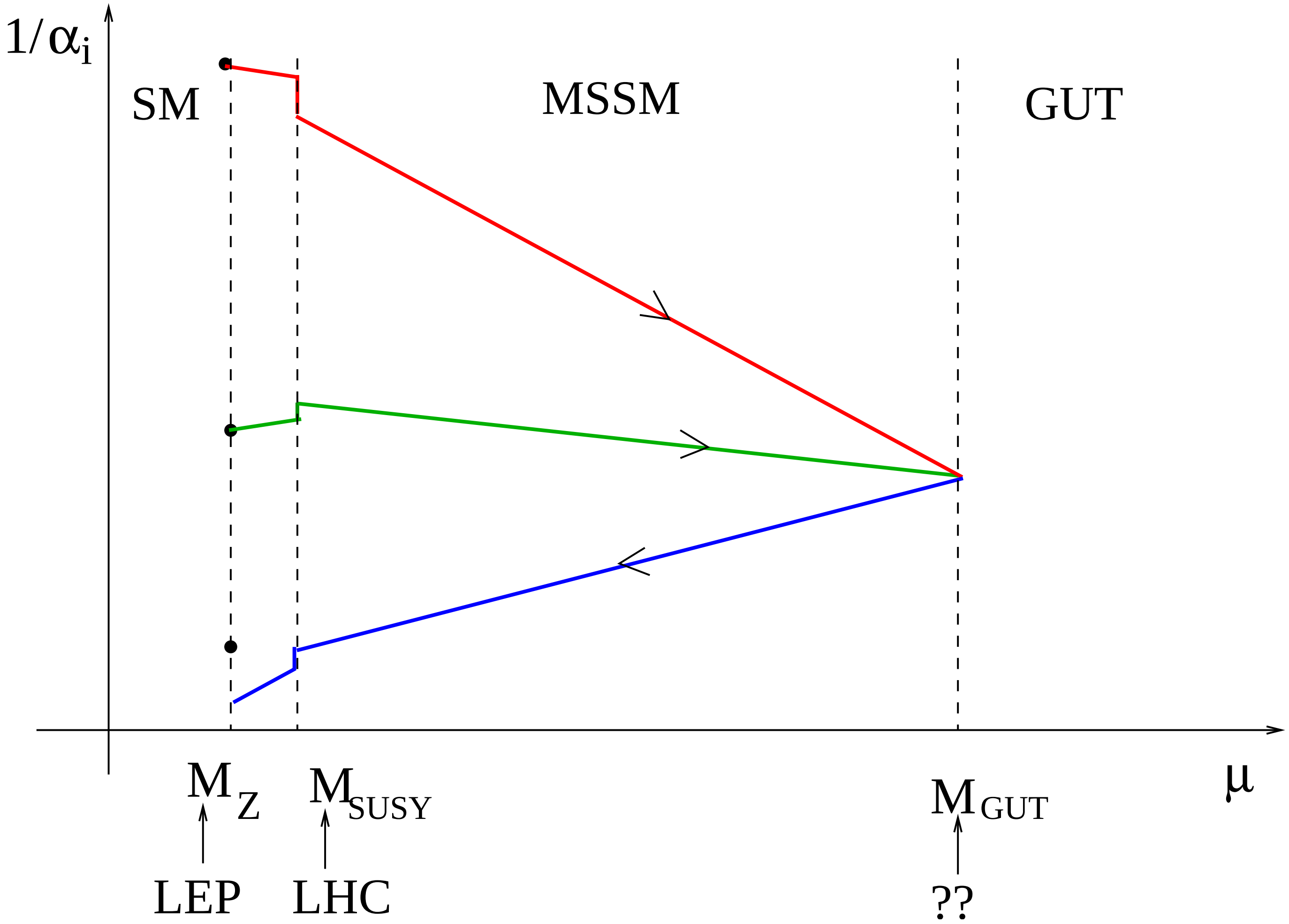

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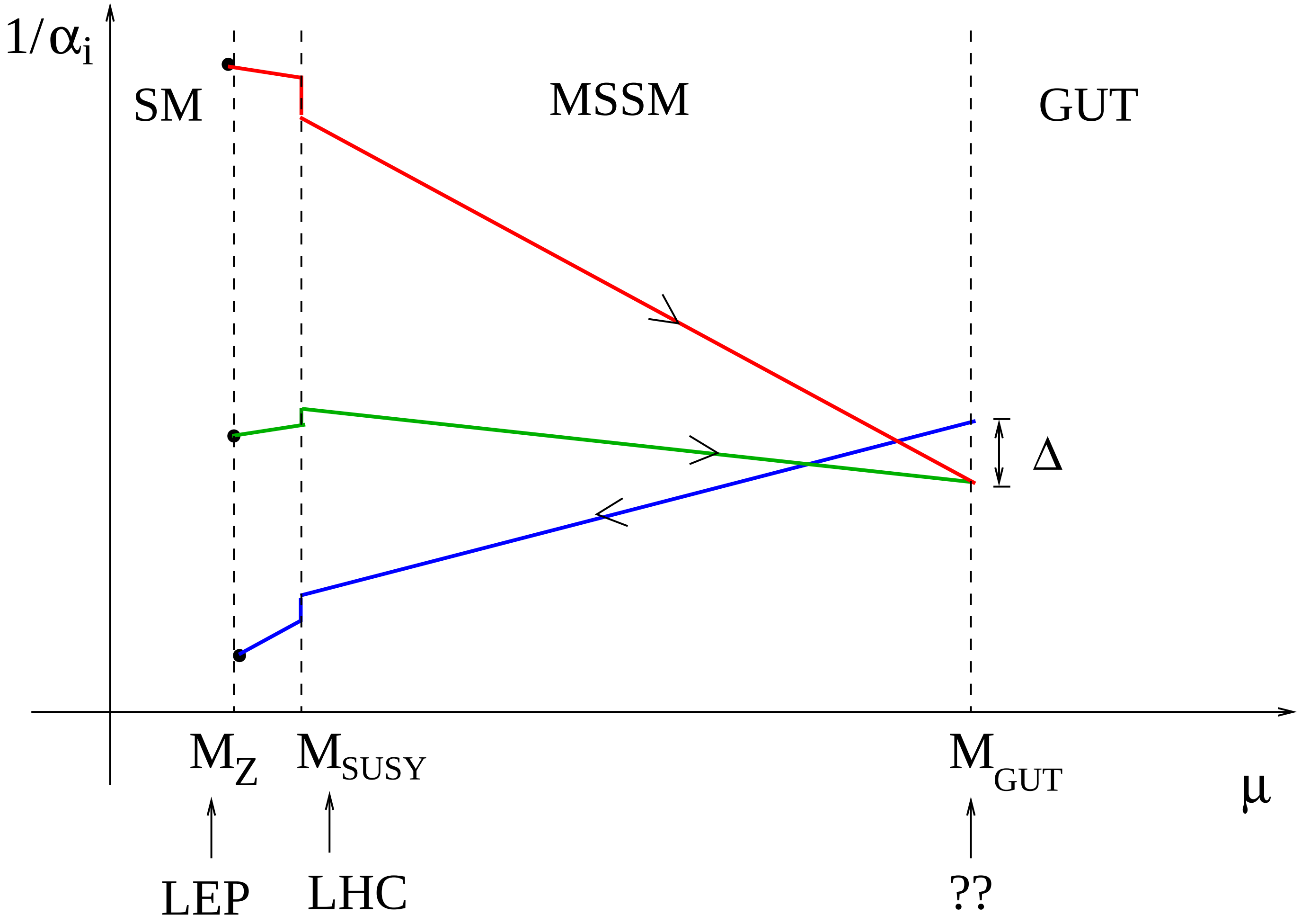

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- $M_{\rm GUT}$: $\simeq 2 \times 10^{16}$ GeV in SUSY GUTs
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  - 1-loop [K.Hagiwara, Y. Yamada '93],[J.Hisano, M. Murayama '94]:

$$\Delta \sim \alpha_G \ln\left(\frac{M_X}{M_{\text{GUT}}}\right) \qquad X = \text{gauge bosons, Higgs}\,, \ldots$$

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$\Rightarrow$ severe constraints on possible local gauge symmetries in GUTs

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Running = variation of coupling strength with the energy.

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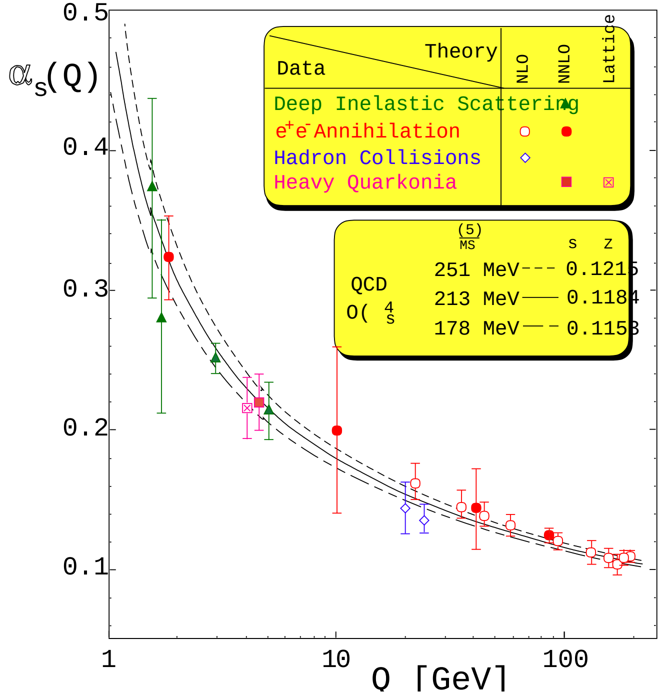
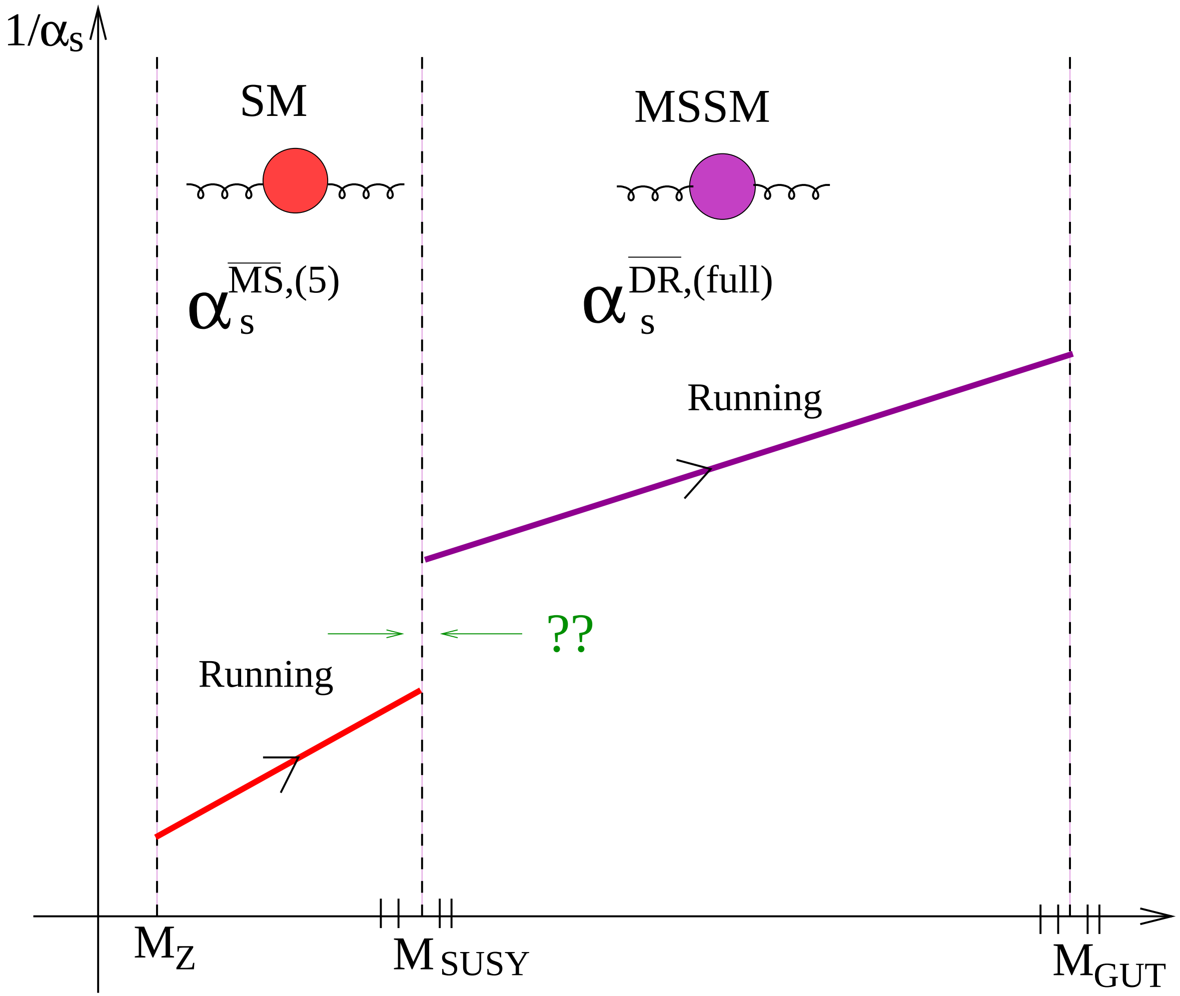
  - Vacuum can screen or anti-screen the gauge charges.
  - Anti-screening gives rise to the asymptotic freedom of strong interactions.



Evolution of the strong coupling

- Input parameter: $\alpha_s^{\overline{\text{MS}},(5)}(\text{M}_\text{Z})$ $\Rightarrow$ Output parameter: $\alpha_s^{\overline{\text{DR}},(\text{full})}(\text{M}_\text{GUT})$



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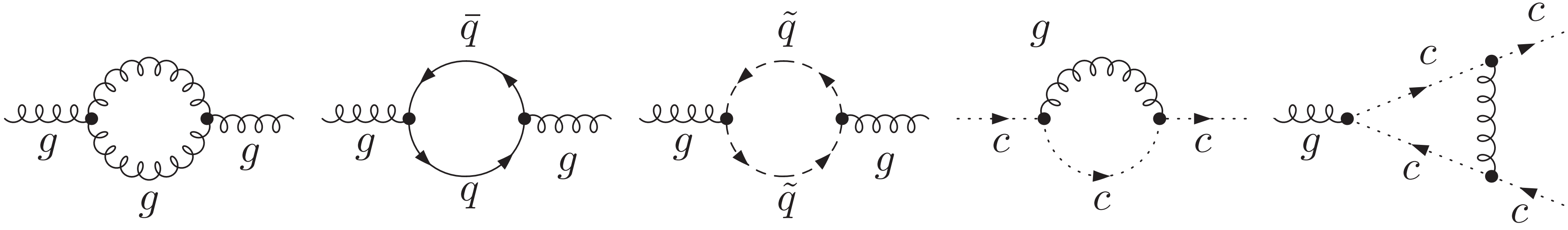

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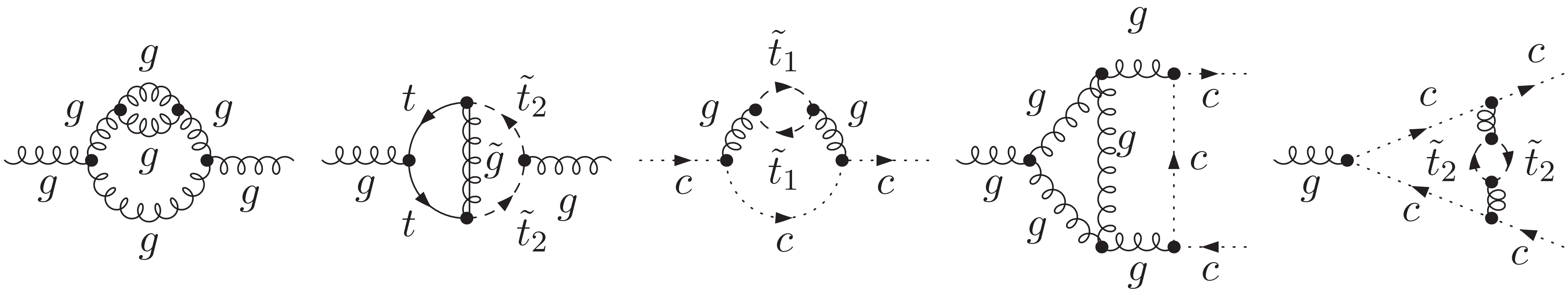

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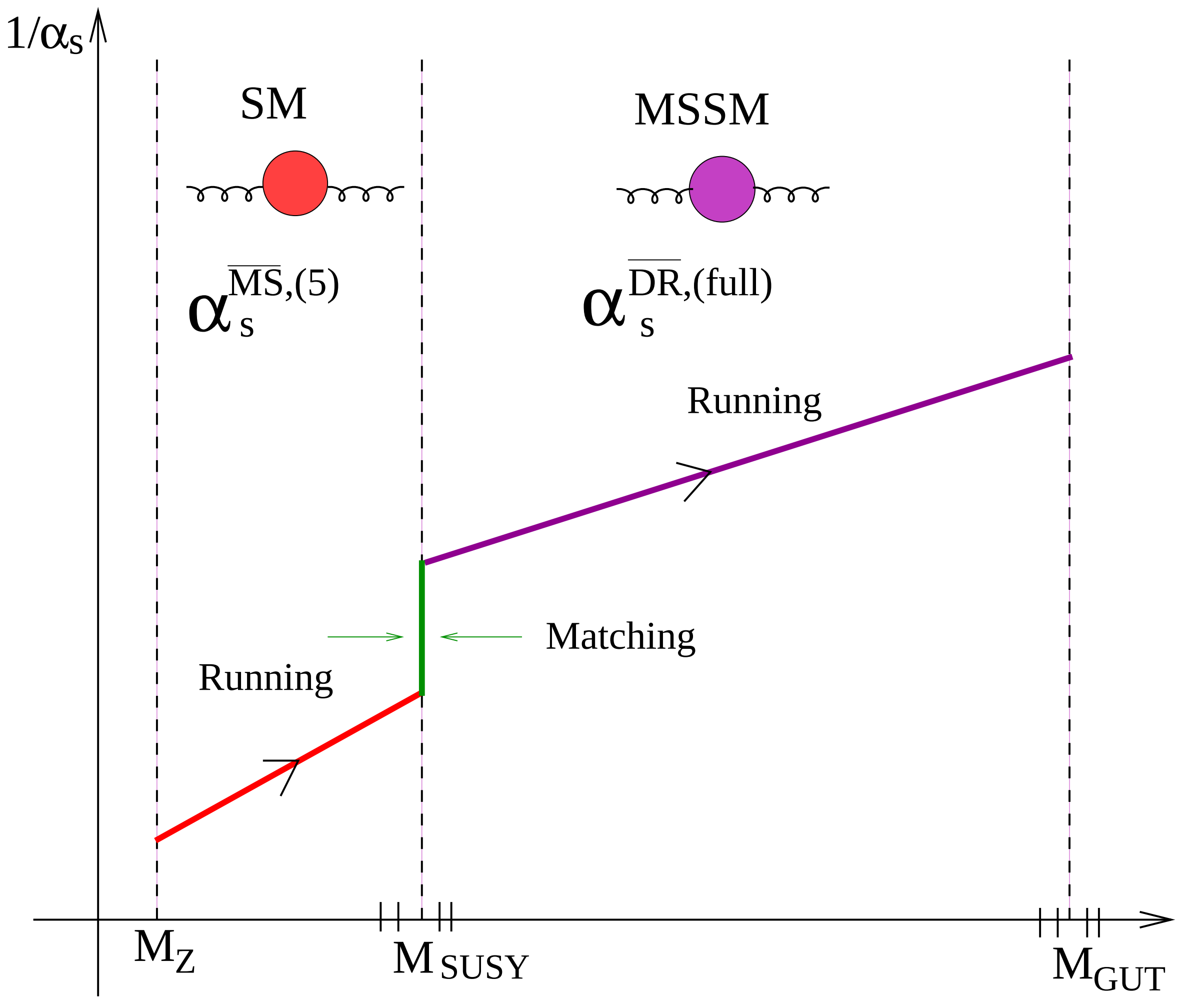
- 3-loop β_s in the MSSM
  - $\simeq$ 100.000 diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, . . .

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; . . .]

Matching

- Effective Field Theory:



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$$\mathcal{L}_{\text{MSSM}}(\alpha_s^{(\text{full})}, \ldots) \quad \rightarrow \quad \mathcal{L}(\alpha_s^{(5)}, \ldots) \quad \text{at energy } \mu$$

- "Matching" : low energy physics must be unchanged !!

$$\begin{aligned} \alpha_s^{(5)} &= \zeta_s\, \alpha_s^{(\text{full})} \\ &\vdots \\ \zeta_s &= \zeta_s(\alpha_s, M_{\text{SUSY}}, m_t, \mu) \\ \zeta_s &= \text{matching coefficient} \end{aligned}$$

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  - μ not predicted by theory
  - Physical quantities must be independent of μ
  - Quantum corrections improve stability

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Relate Green functions computed in the full and effective theory.

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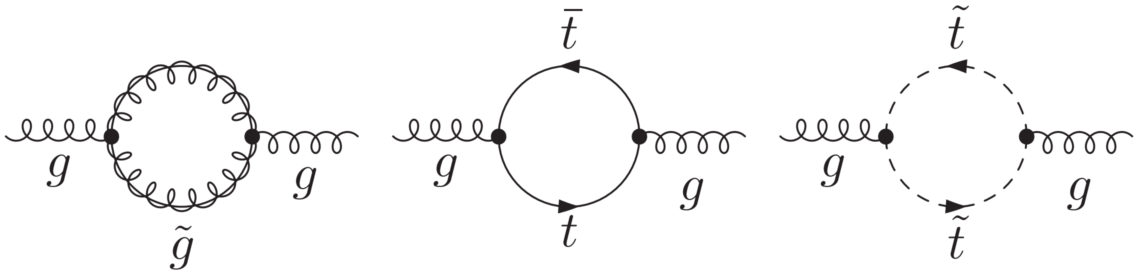

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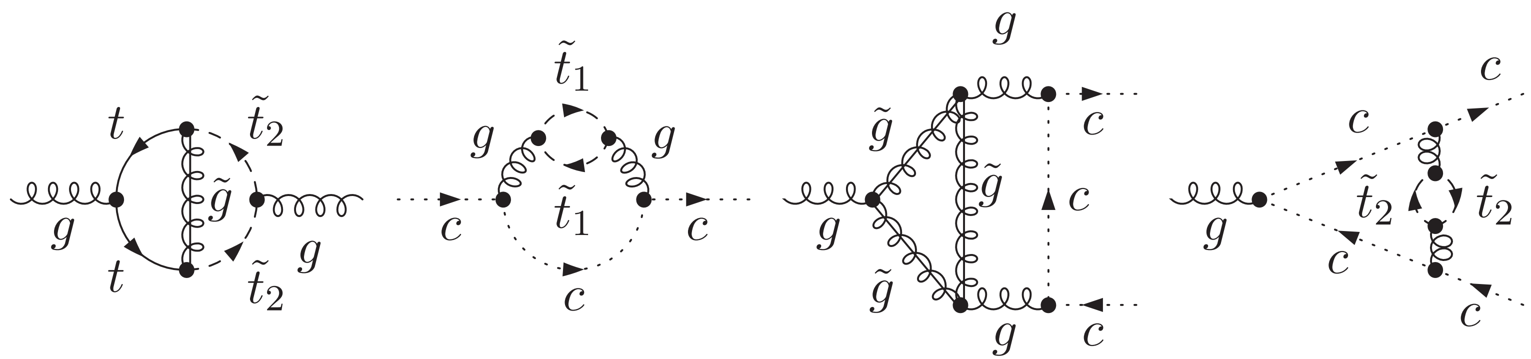

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$$\begin{aligned}\zeta_{s2}^{(5)} = & -\frac{215}{96} - \frac{19}{24}\ln\frac{\mu^2}{m_t^2} - \frac{5}{2}\ln\frac{\mu^2}{\tilde{M}^2} + \left[\frac{1}{6}\ln\frac{\mu^2}{m_t^2} + \ln\frac{\mu^2}{\tilde{M}^2}\right]^2 \\ & + \left(\frac{m_t}{\tilde{M}}\right)^2\left(\frac{5}{48} + \frac{3}{8}\ln\frac{m_t^2}{\tilde{M}^2}\right) - \frac{7\pi}{36}\left(\frac{m_t}{\tilde{M}}\right)^3 \\ & + \left(\frac{m_t}{\tilde{M}}\right)^4\left(\frac{881}{7200} - \frac{1}{80}\ln\frac{m_t^2}{\tilde{M}^2}\right) + \frac{7\pi}{288}\left(\frac{m_t}{\tilde{M}}\right)^5\end{aligned}$$

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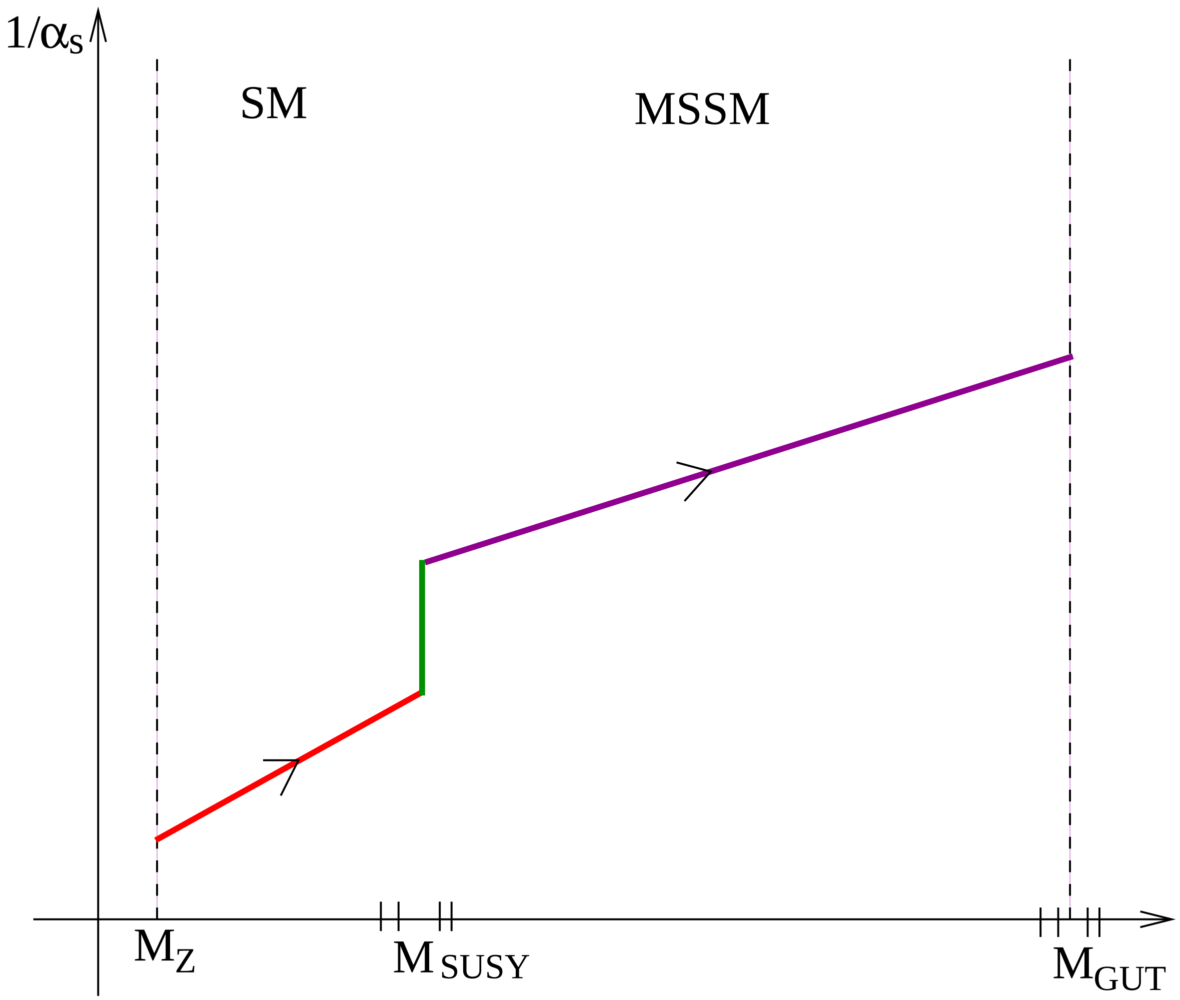

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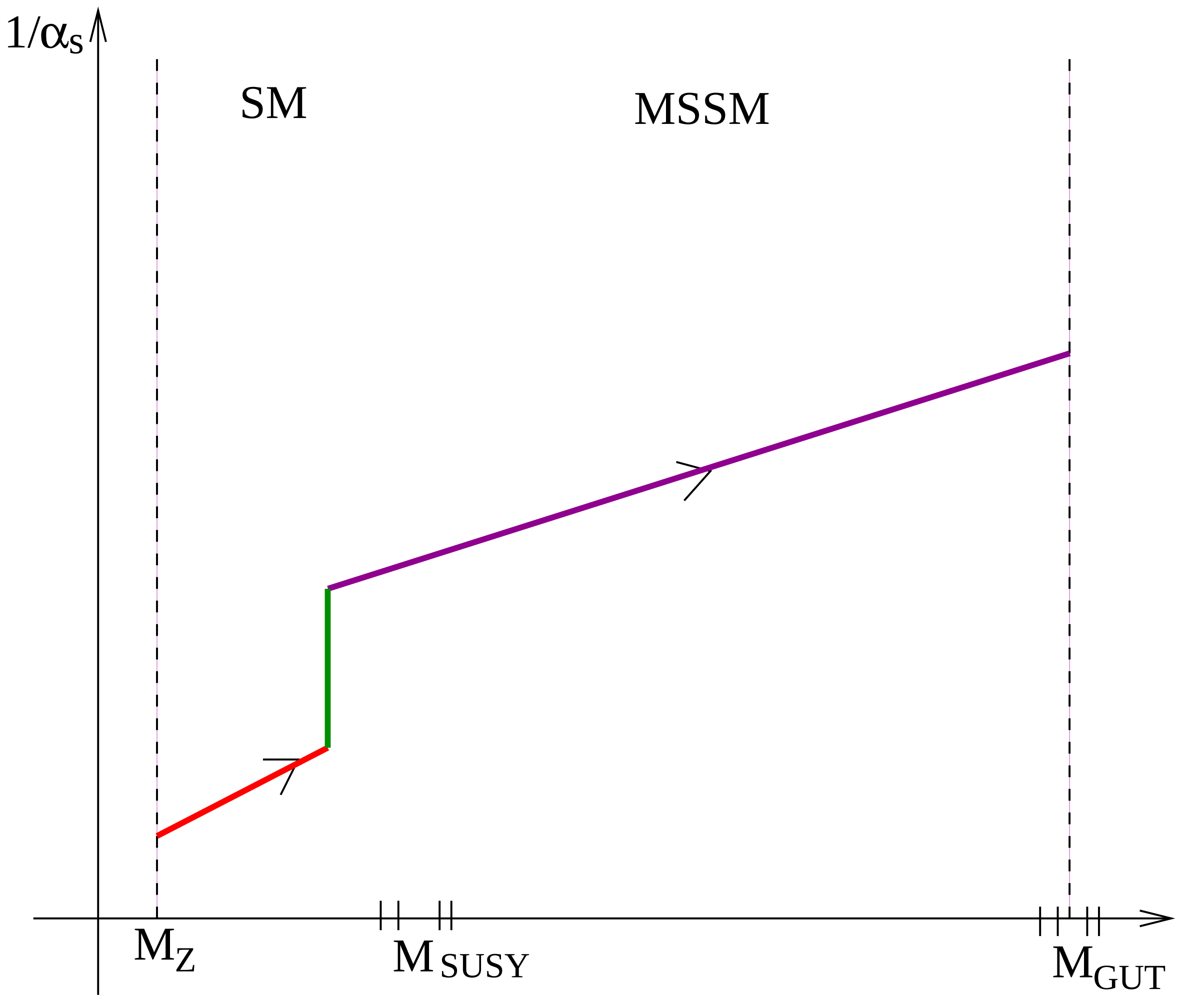

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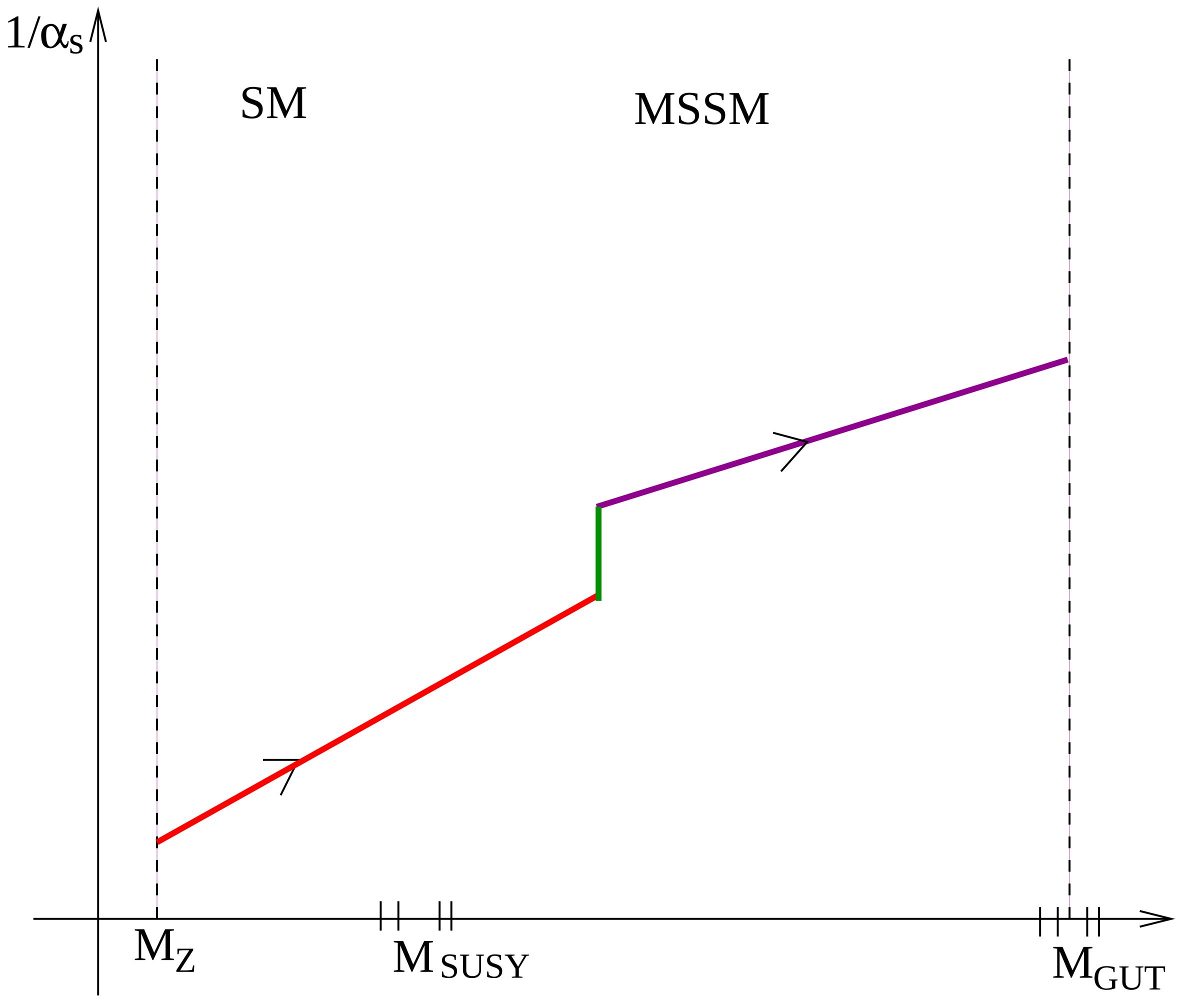

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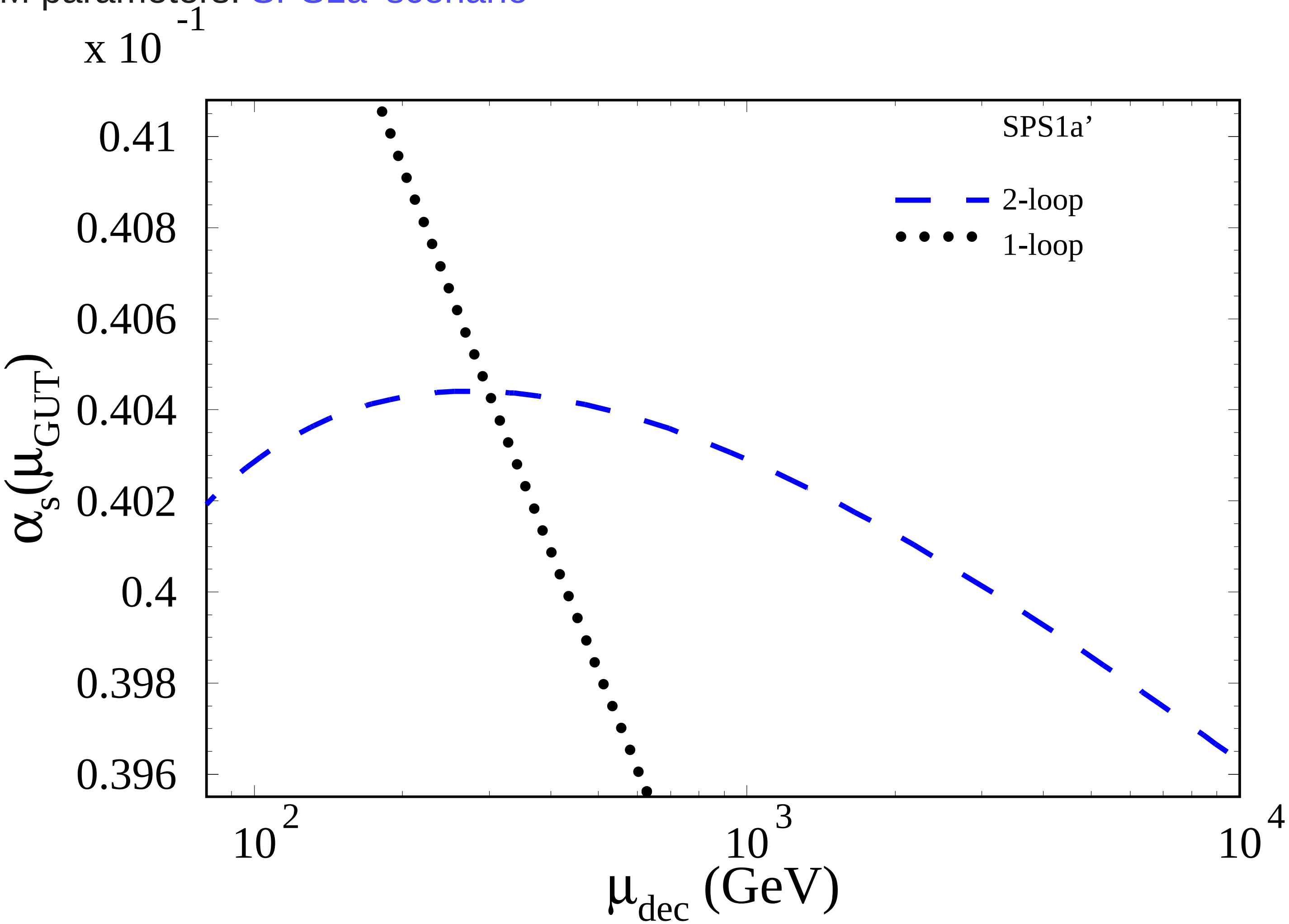

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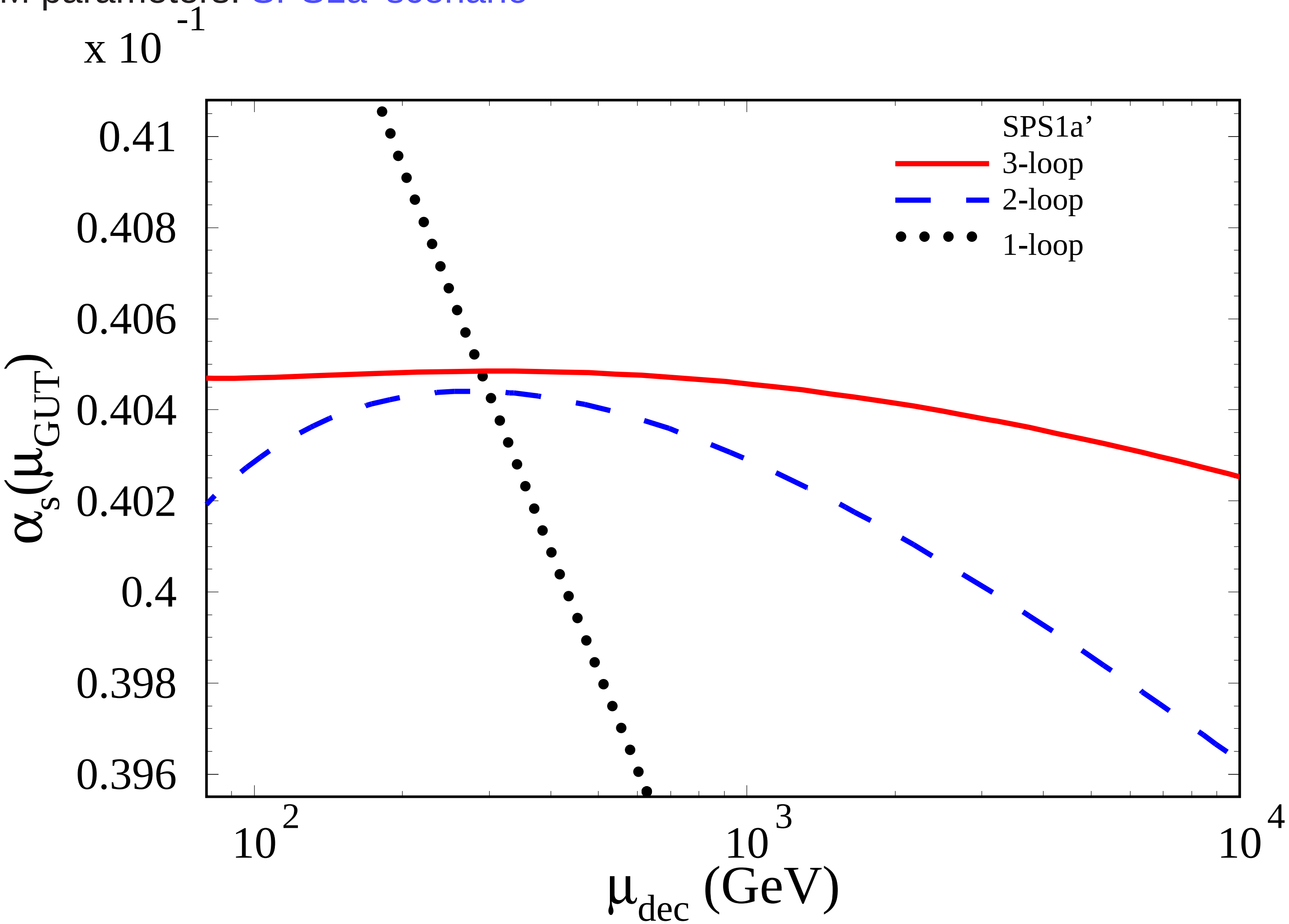

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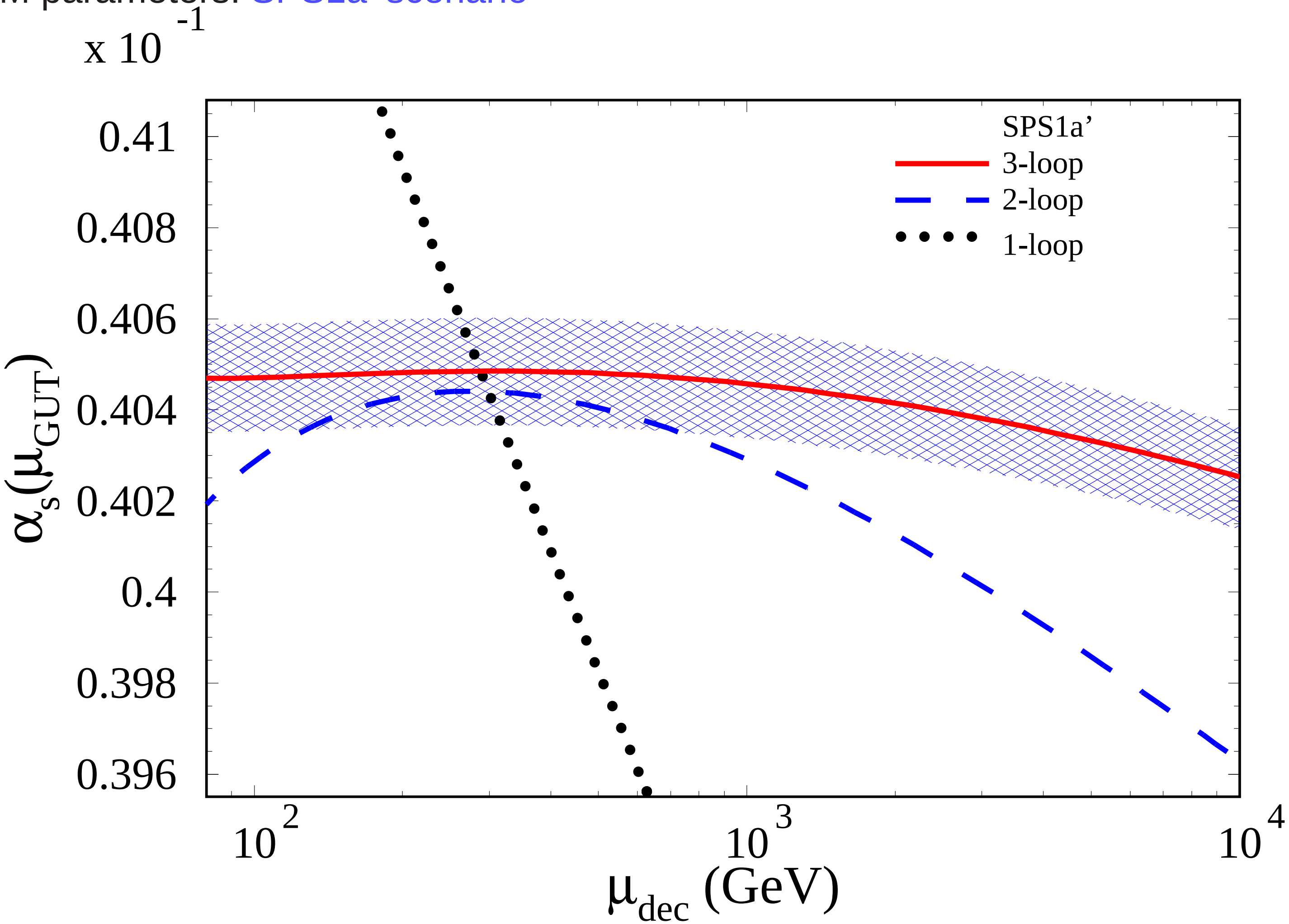

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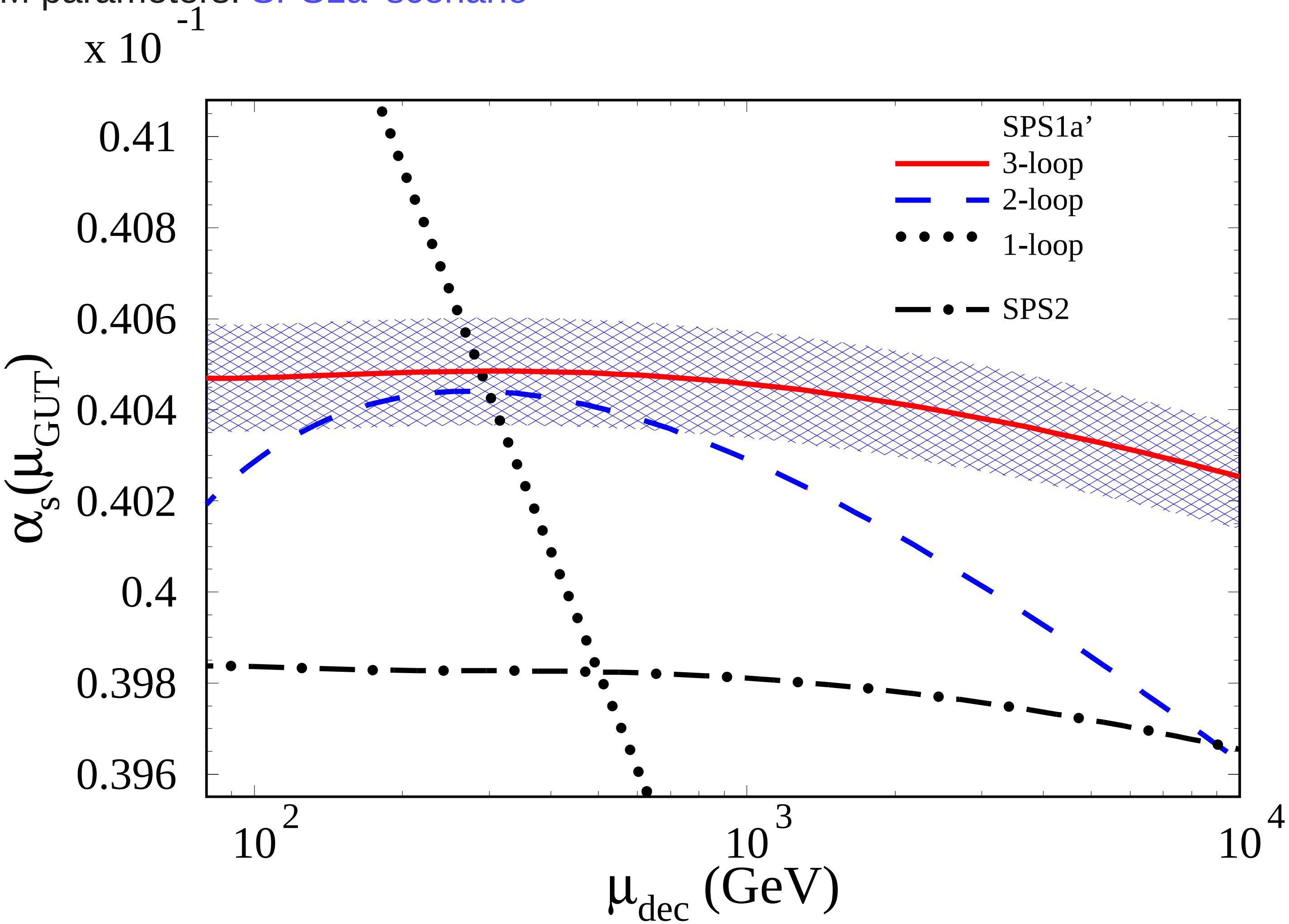
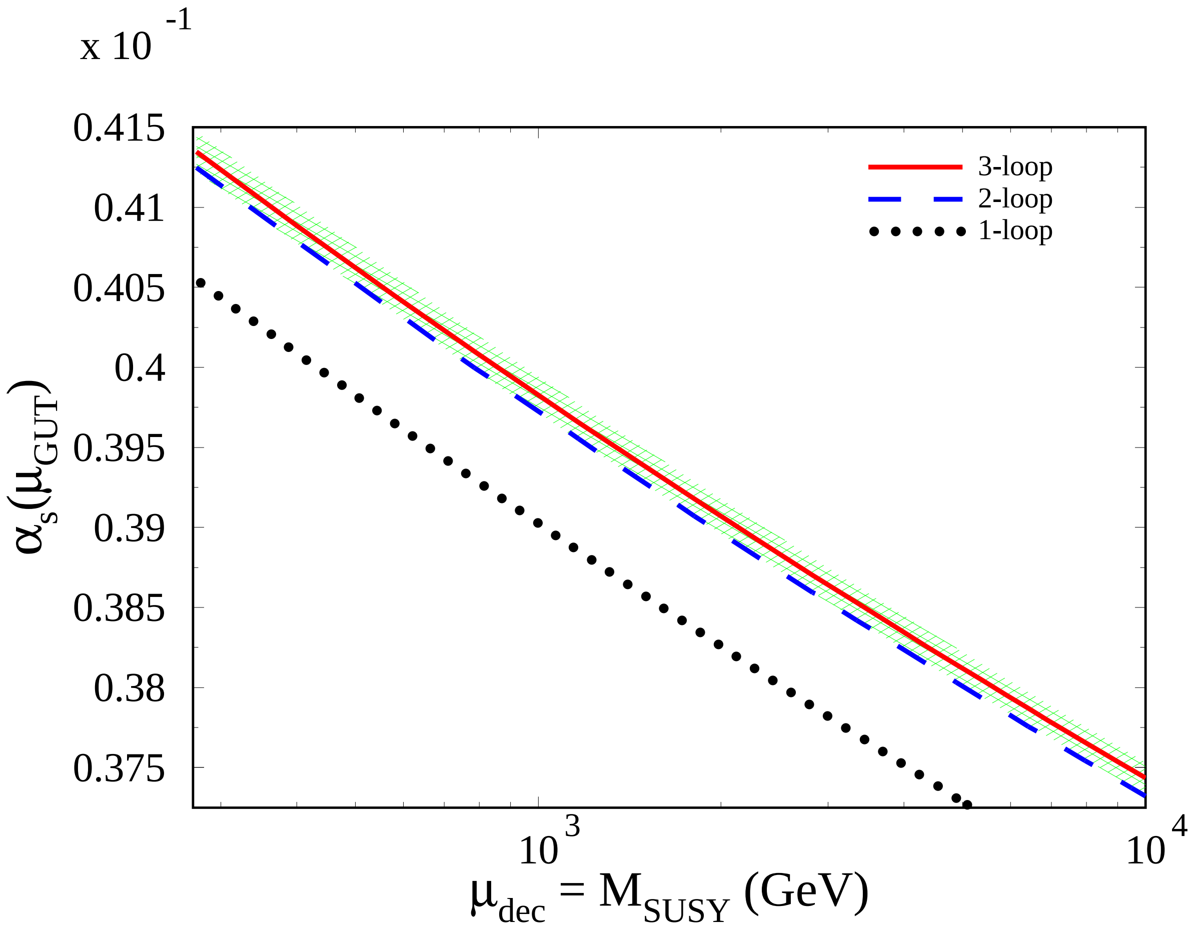


$M_{\rm SUSY}$

- Sensitivity of $\alpha_s(M_{\rm GUT})$ to SUSY-mass scale:



Bottom quark mass

- (SUSY)GUT models $\Rightarrow$ predictions for $m_{\rm t}, m_{\rm b}/m_\tau$
- Bottom quark mass in SM: known with 4-loop accuracy

  $\delta m_b^{\overline{\rm MS}}(m_b) = 25$ MeV [J. H. Kühn, M. Steinhauser, C. Sturm '07]
- Bottom quark in MSSM (models with large $\tan\beta$)
  - SUSY mass spectrum sensitive to bottom Yukawa coupling
  - $Y_b(\mu) \leftrightarrow m^{\overline{\rm DR}}(\mu)$ affected by large SUSY radiative corrections

$m_b(M_{\rm GUT})$

Input: $\alpha_s^{\overline{\rm MS},(5)}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06], $M_Z = 91.1876$ GeV ,

$m_b^{\overline{\rm MS}}(m_b) = 4.164 \pm 0.025$ GeV [Kühn, Steinhauser, Sturm '07]

simplified assumptions for the MSSM parameters

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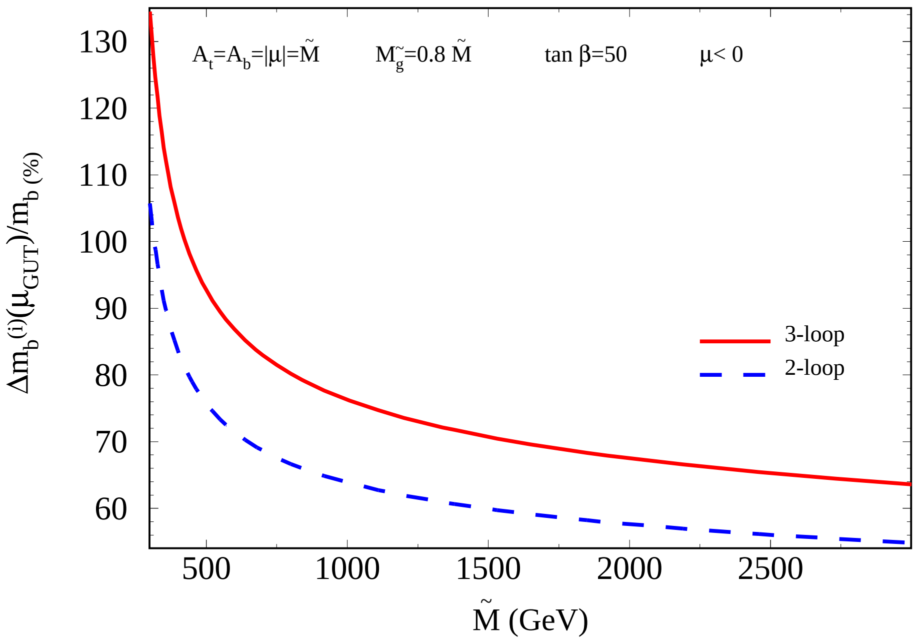
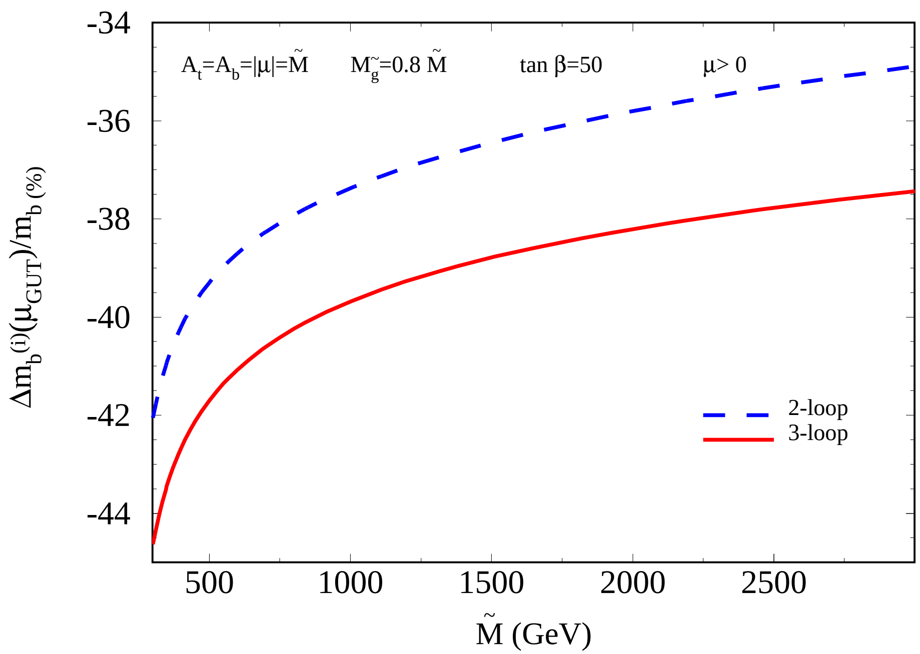
[A. Bauer, L. M., J. Salomon '08]





Conclusions

- $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$
  - The 3-loop effects comparable with the experimental accuracy for α_s
  - $\alpha_{\rm s}(M_{\rm GUT})$ very sensitive to SUSY-mass scale
- $m_b^{\overline{\rm DR}}(M_{\rm GUT})$
  - MSSM with large $\tan\beta$: 3-loop effects reach up to 30%
- ToDo:
  - combine: 3-loop running analysis for the strong sector

    known 2-loop running for the electroweak sector
  - extend analysis to SUSY-GUT models

“With the LHC, we will expand the frontiers of fundamental physics.

... We will learn whether existing indications for unification and supersymmetry have been Nature teaching us or Nature teasing us.”

[Franck Wilczek, SUSY’07]