Asymptotic symmetries and observables in 4d gravity

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Bridging high and low energies in search for quantum gravity - First Annual Conference, Paris 2025

The holographic principle

Black hole entropy in (3+1) dimensions $S_{BH} = \dfrac{A}{4G_N} \sim L^2 M_P^2$ scales like the entropy of a 3d QFT

$\Longrightarrow$ gravity has the same number of d.o.f. as a quantum theory in one lower number of dimensions

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['t Hooft' 93; Susskind '94]

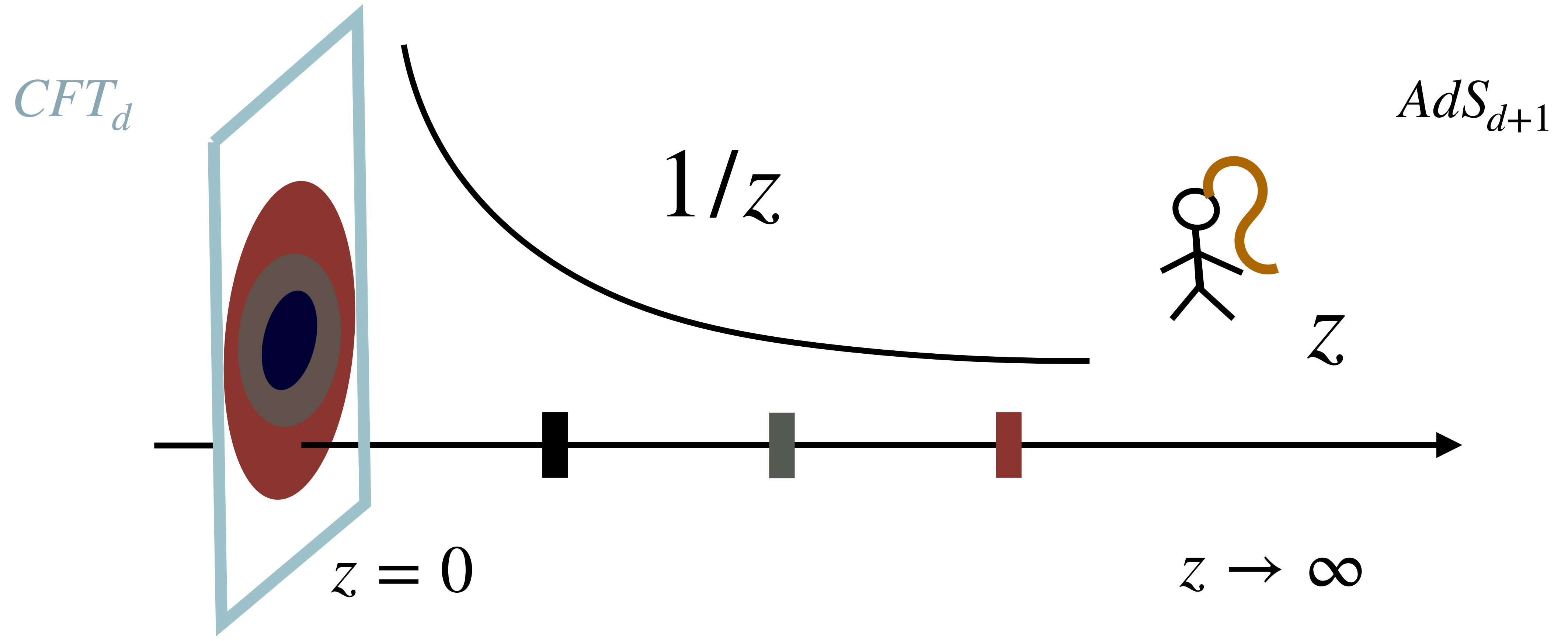
AdS/CFT correspondence

[Maldacena '98]

Conformal field theory (CFT)

- no dimensionful couplings

$(\text{scale})_{CFT} = z^2 \times \text{proper AdS scale}$



Anti-de-Sitter (AdS) gravity

$$ds^2 = R^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$

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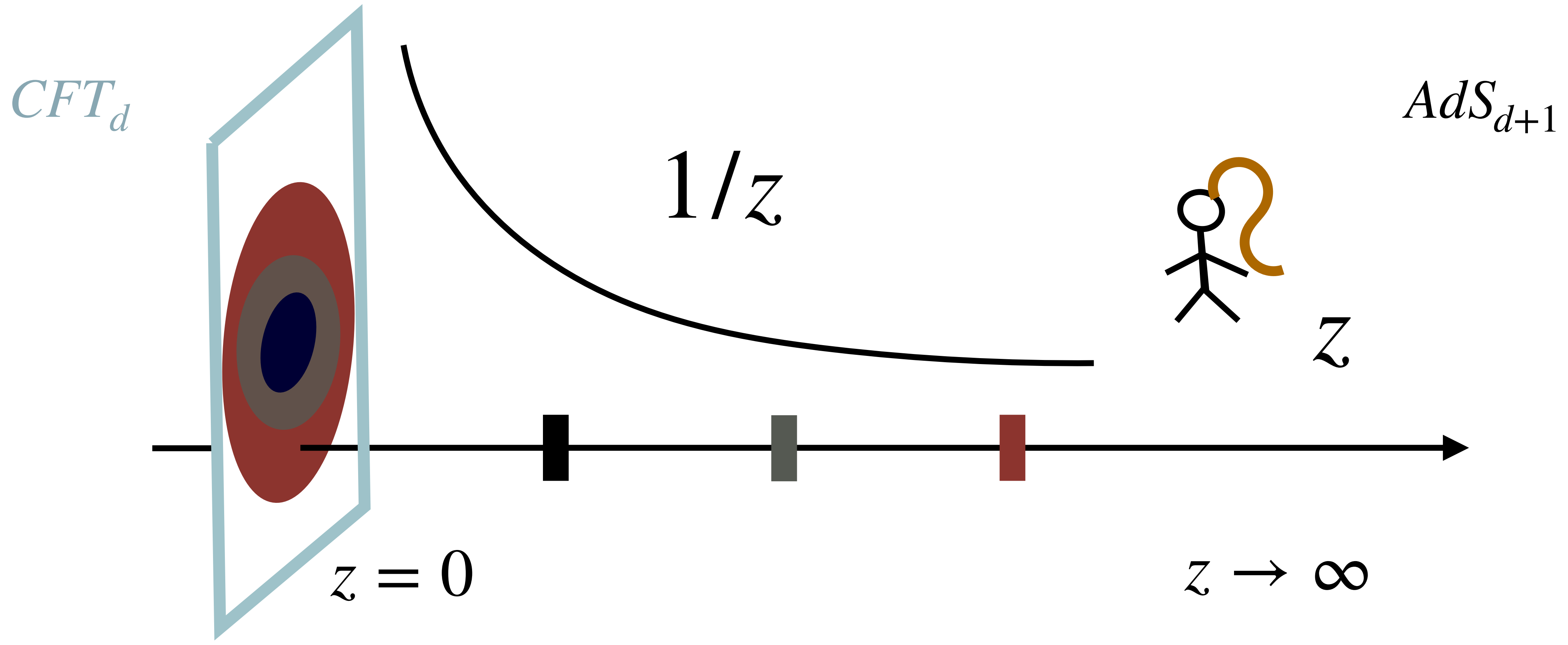

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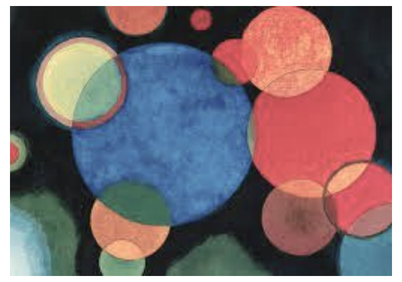
Short distances ⟷ Long distances



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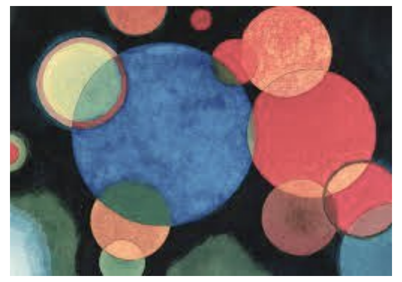
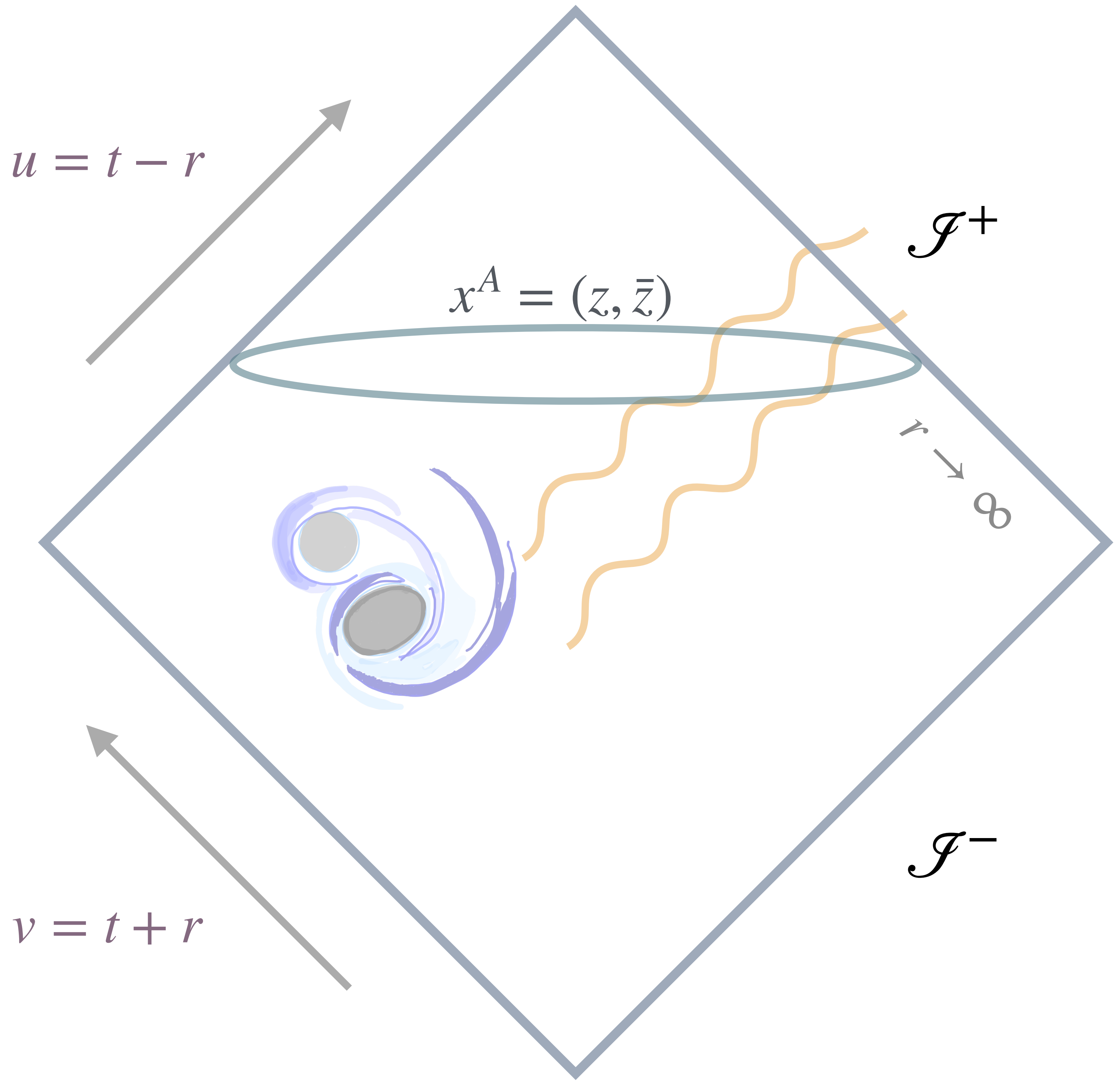
Holography in asymptotically flat spacetimes?

Outline
- — infinity of ``infrared'' symmetries in 3+1 dimensions ~ symmetries of 2d CFT
- — related to observables, such as gravitational memory effects
- — rich vacuum structure → observables beyond quantized linearized metric perturbation (graviton)

Short distances $\overset{?}{\longleftrightarrow}$ Long distances



Asymptotically flat spacetimes (4d)



- Gauge fix: $g_{rr} = g_{rA} = 0$ (radial propagation of GW)

  $\partial_r \det\left(r^{-2} g_{AB}\right) = 0$ (spherical wavefronts) [BBMS '62]

  $\implies \quad ds^2 = e^{2\beta} V du^2 - 2e^{2\beta} du dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$

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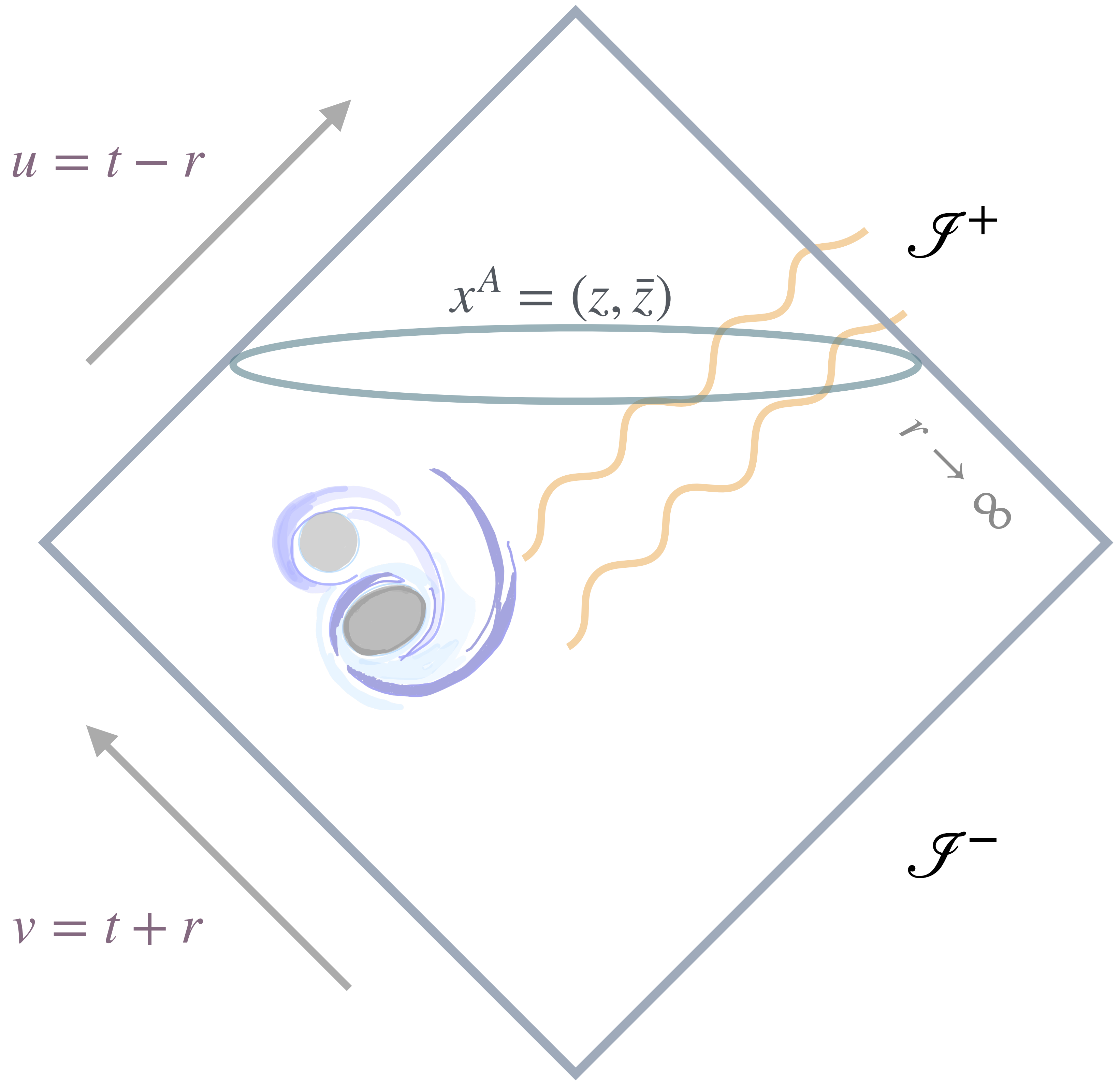

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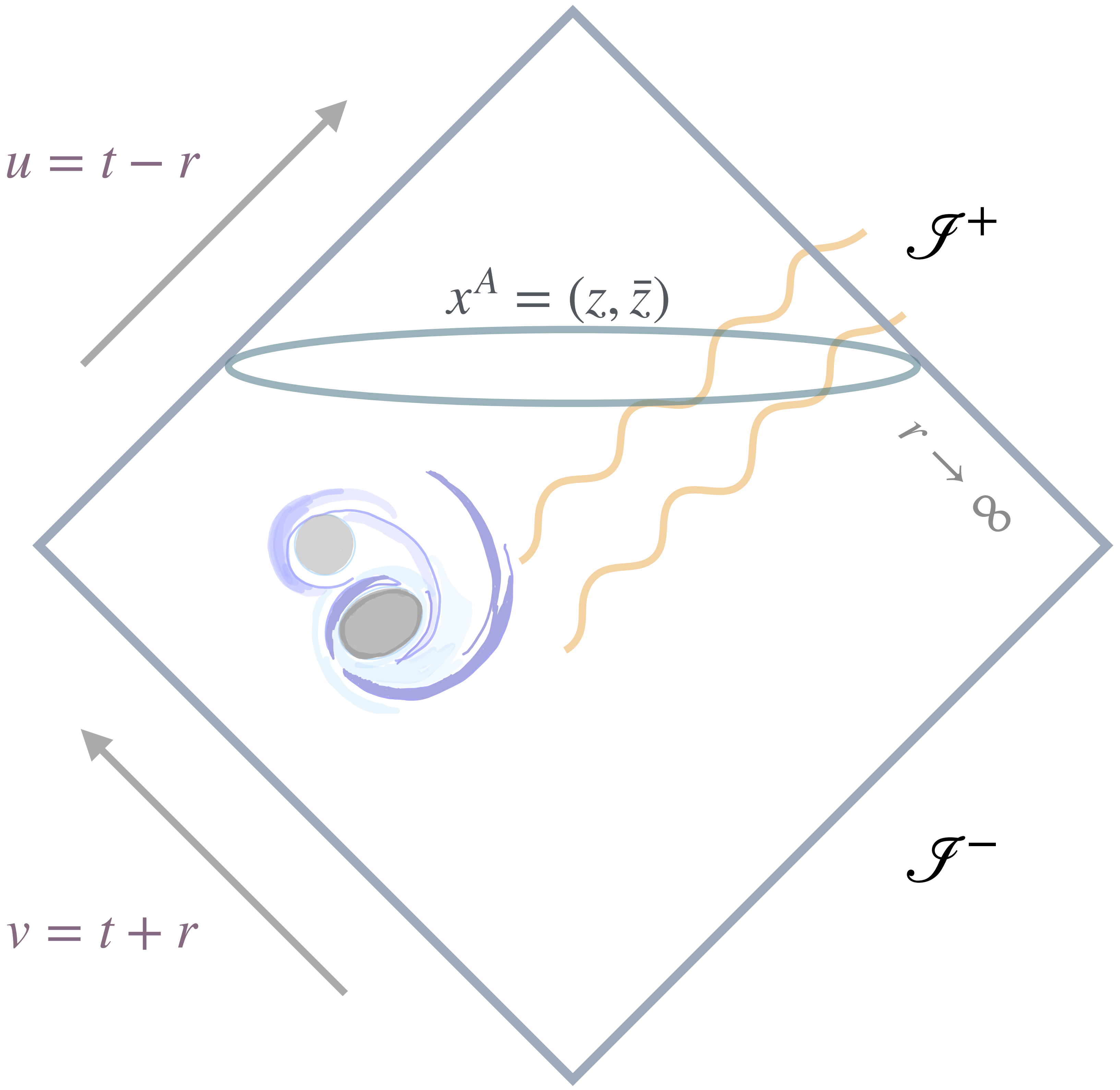

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- Solve the Einstein equations

- Impose boundary conditions:

  — allow for gravitational waves (not too fast fall-off in r^{-1})

  — finite asymptotic charges (energy, angular momentum, ...)

  $\Longrightarrow$ asymptotic expansion of V, β, U^A and g_{AB} in r^{-1}

Asymptotically flat spacetimes (4d)

Expand $G_{\mu\nu} \equiv R_{\mu} - \frac{1}{2}g_{\mu\nu}R = 0$ (no matter) in powers of r^{-1}:

$$ds^2 = e^{2\beta}Vdu^2 - 2e^{2\beta}dudr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$G_{r\mu} = 0 \quad @ \quad \mathcal{O}(r^{-2}) : \quad g_{AB} = r^2\gamma_{AB} + rC_{AB} + \cdots + \frac{T_{AB}}{r} + \mathcal{O}(r^{-2})$$

$$V = -\frac{\bar{R}[\gamma]}{2} + \frac{2M}{r} + \mathcal{O}(r^{-2})$$

$$U^A = -\frac{1}{2r^2}D_B C^{BA} - \frac{2}{3r^3}\left(N^A - \frac{1}{2}C^{AB}D^C C_{BC}\right) + \mathcal{O}(r^{-4})$$

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Bondi mass aspect (ADM mass - radiation)

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$G_{uu} = G_{uA} = G_{AB} = 0 \quad @ \quad \mathcal{O}(r^{-2}) :$ time evolution equations for $M, \;\; N^A \;\; \text{and} \;\; T_{AB},$ eg.

$$\partial_u M = \frac{1}{4} D_A D_B N^{AB} - \frac{1}{8} N_{AB} N^{AB}$$

[flux-balance law for Bondi mass aspect]

Towers of symmetries

- Analysis increasingly tedious at subleading orders in r^{-1}
- Simplification in the Newman-Penrose formalism → non-trivial components of the gravitational field encoded in the Weyl tensor components:

$$\Psi_0 = C_{\mu\nu\rho\sigma}\ell^\mu m^\nu \ell^\rho m^\sigma \equiv C_{\ell m \ell m}$$

$$\Psi_1 = C_{\ell n \ell m}, \quad \Psi_2 = -C_{\ell m \bar{m} n}, \quad \Psi_3, \quad \Psi_4$$

$\{\ell, n, m, \bar{m}\}$ null frame:

$$g_{ab} = -\ell_a n_b - n_a \ell_b + m_a \bar{m}_b + m_b \bar{m}_a$$

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- Leading components $\Psi_i^{(0)}$ of Ψ_i in a r^{-1} expansion are 4d Lorentz primaries at retarded time $u = 0$:

$$\mathscr{L}_Y \Psi_i^{(0)} = (Y^A \partial_A + h \partial_z Y^z + \bar{h} \partial_{\bar{z}} Y^{\bar{z}}) \Psi_i^{(0)}, \quad i = 0{,}1{,}2$$

[Freidel, Pranzetti '21]

For $Y^z \in \{1, z, z^2\}, \quad Y^{\bar{z}} \in \{1, \bar{z}, \bar{z}^2\}, \quad \xi_Y = Y^A \partial_A + \cdots$ generate 4d Lorentz transformations ∼ 2d global conformal transformations

Towers of symmetries

		$\Delta = h + \bar{h}$	$s = h - \bar{h}$
$M \to \mathscr{M} = M + \frac{1}{8}C_{AB}N^{AB}$	$\mathrm{Re}\Psi_2^{(0)} = \mathscr{M} \equiv \mathcal{Q}_0$	3	0
$N_A \to \mathscr{J}_A = \cdots$	$\Psi_1^{(0)} = m^A \mathscr{J}_A \equiv \mathcal{Q}_1$	3	1
$T_{AB} \to \mathscr{T}_{AB} = \cdots$	$\Psi_2^{(0)} = m^A m^B \mathscr{T}_{AB} \equiv \mathcal{Q}_2$	3	2

[Freidel, Pranzetti '21]

In terms of the Newman-Penrose variables, the flux balance laws become:

$$\boxed{\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2}\,, \quad s = 0,1,2}$$

$$\mathcal{Q}_{-1} = \frac{1}{2}DN, \quad \mathcal{Q}_{-2} \equiv \frac{1}{2}\partial_u N$$

$$N = \bar{m}^A \bar{m}^B \partial_u C_{AB}$$

Quantization

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2}\,, \quad s = 0,1,2$$

- Flux-balance laws impose a constraint on asymptotic states (phase space)

- C, N are canonically conjugate variables: $\{N(u,z,\bar{z}), C(u',z',\bar{z}')\} = 16\pi G\delta(u-u')\delta^{(2)}(z-\bar{z}')$ [Ashtekar ‘ 82; Iyer, Wald, Zoupas ’92]

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- Symplectic potential from boundary term in variation of action $\delta S = \int_M \mathrm{EOM}\cdot\delta g + \Theta[g,\delta g]$

$$\Theta^{\mathrm{AFS}} = \frac{1}{16\pi G}\int_{\mathscr{I}^+} dud^2z\left(\delta C_{zz}N_{\bar{z}\bar{z}} + \delta C_{\bar{z}\bar{z}}N_{zz} + \delta(\cdots)\right) \quad \Longrightarrow \quad \Omega = \delta\Theta^{\mathrm{AFS}} = \frac{1}{16\pi G}\int_{\mathscr{I}^+} dud^2z\left(\delta C\wedge\delta N + \delta\bar{C}\wedge\delta\bar{N}\right)$$

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Charge brackets

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2}\,, \quad s = 0,1,2$$

- Solve recursion relations to express $Q_s(u, z, \bar{z})$ in terms of $N,\ C$ $\implies \lim_{u\to-\infty} Q_s[N, C] \equiv q_s(z, \bar{z})$ + divergent terms for $s \geq 1$

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$Q_{ST} \equiv \int_{S^2} d^2z f(z, \bar{z}) q_0(z, \bar{z})$ and $Q_{SR} \equiv \int_{S^2} d^2z Y(z, \bar{z}) q_1(z, \bar{z})$ obey the 4d eBMS algebra $\{Q_{\xi_1}, Q_{\xi_2}\} = Q_{[\xi_1, \xi_2]}$

[Barnich, Troessaert 2011]

eBMS generators: $\xi = f(z, \bar{z})\partial_u + Y^A(z, \bar{z})\partial_A + \cdots$

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supertranslations

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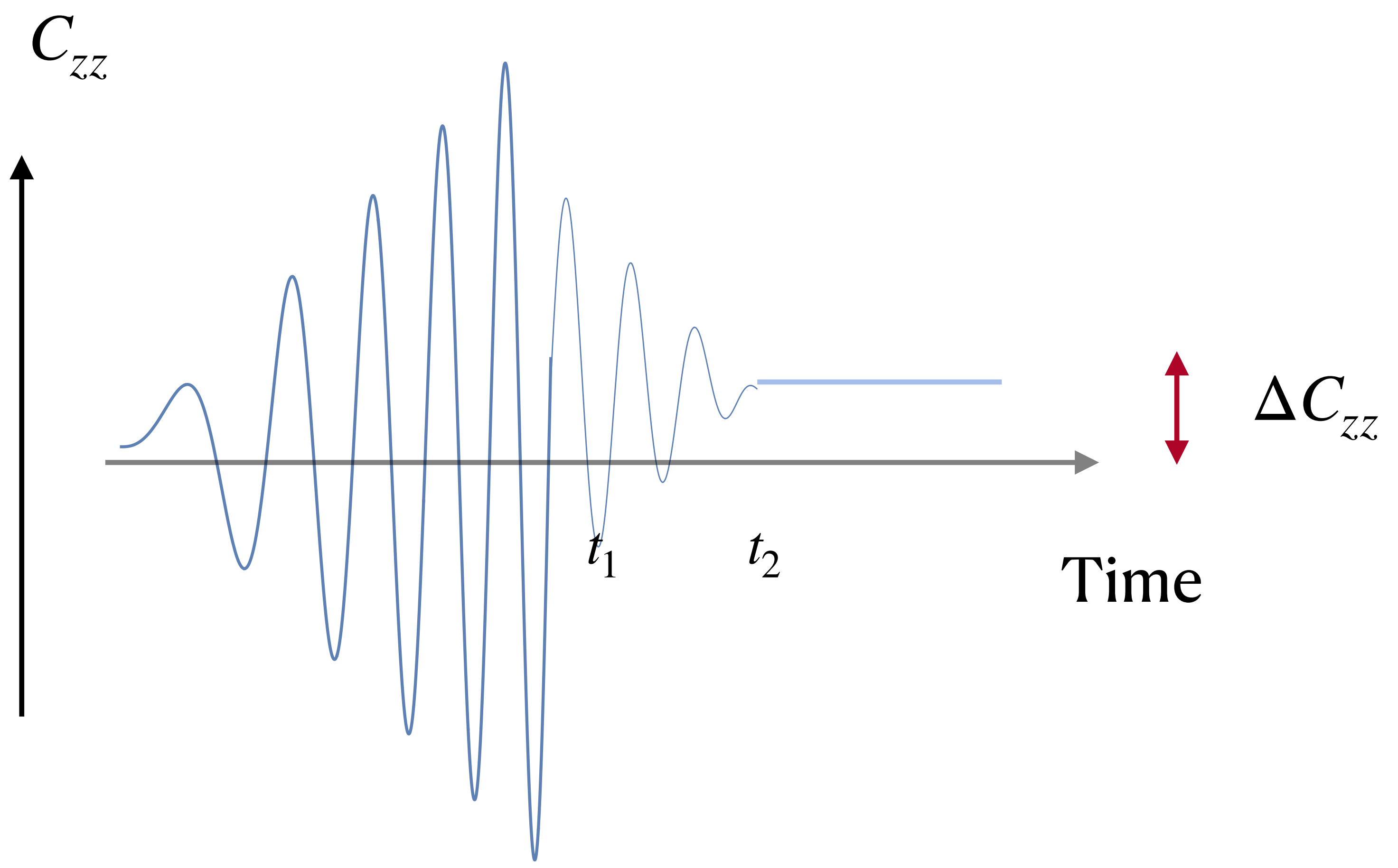
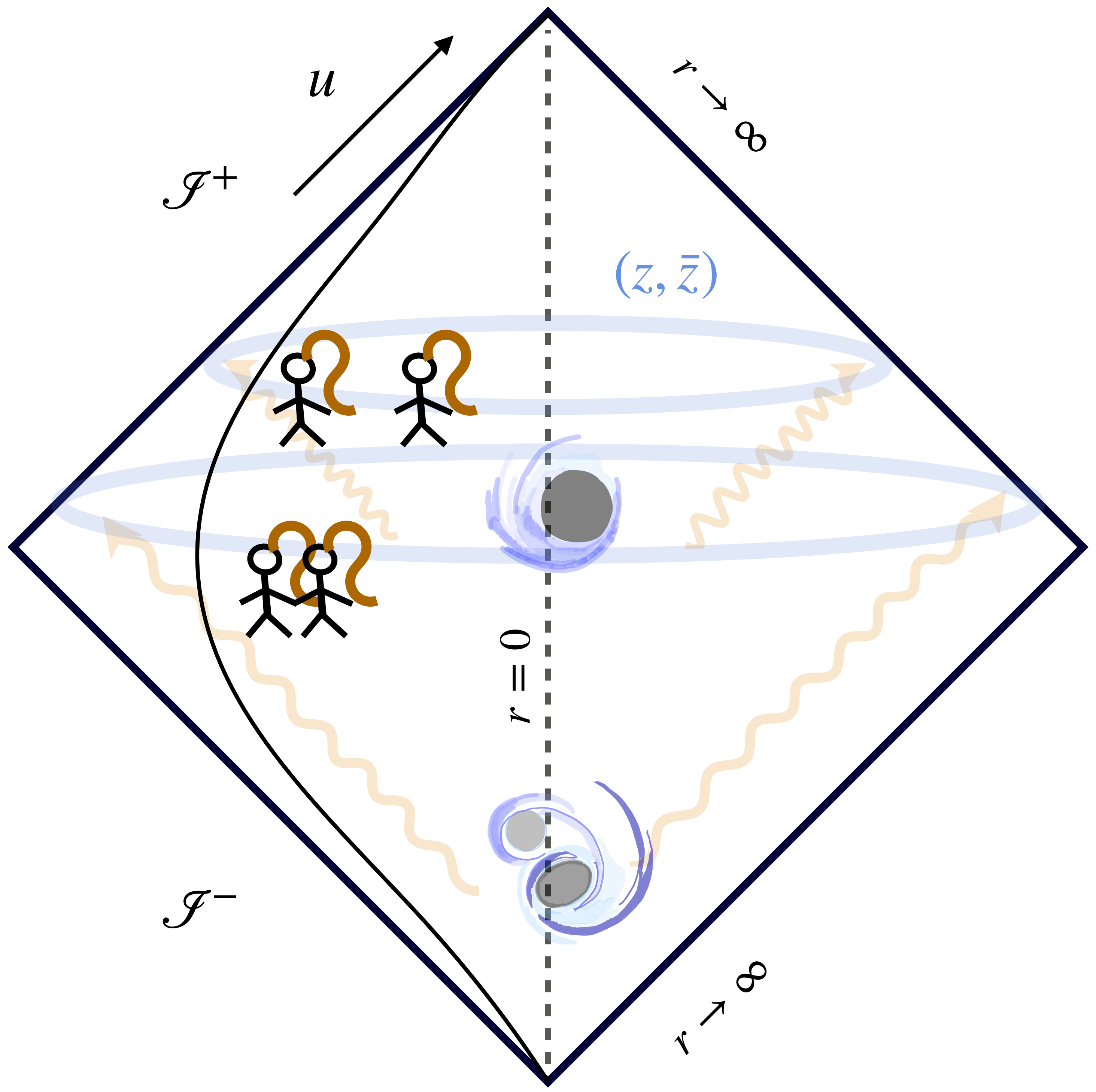
Superrotations

Physical implication I: gravitational memory



$\Longrightarrow$ Net relative displacement

between asymptotic observers



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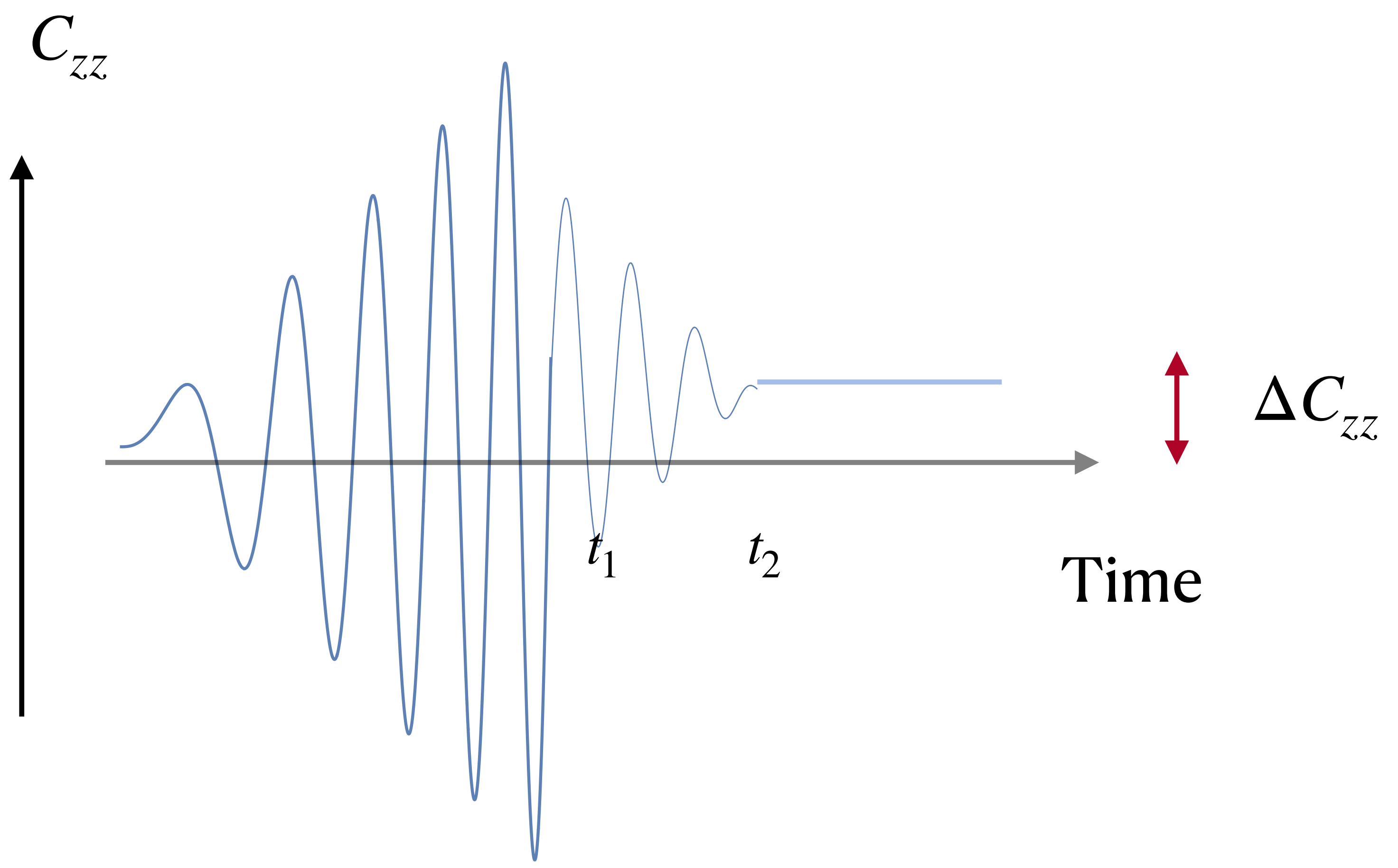

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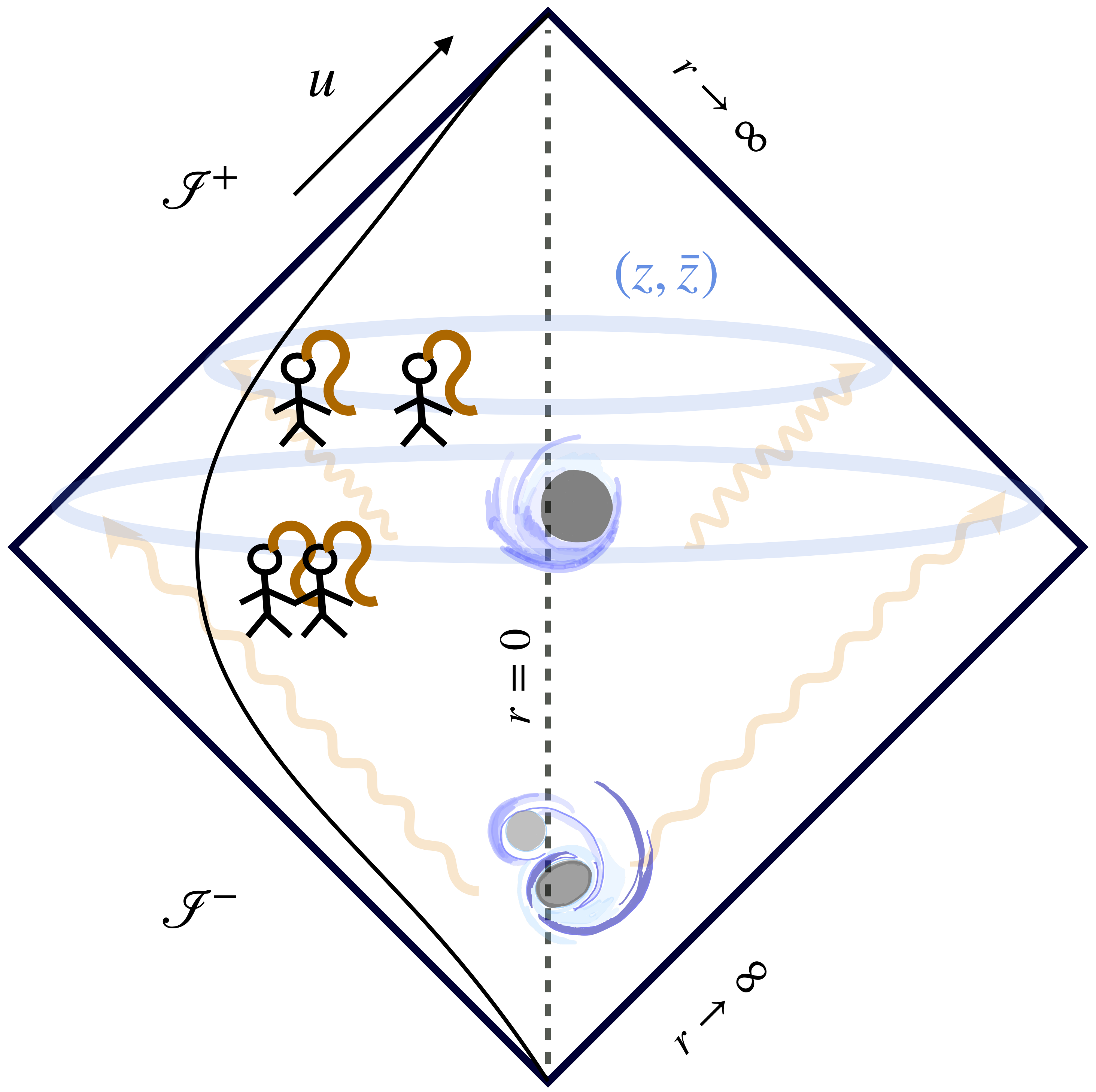

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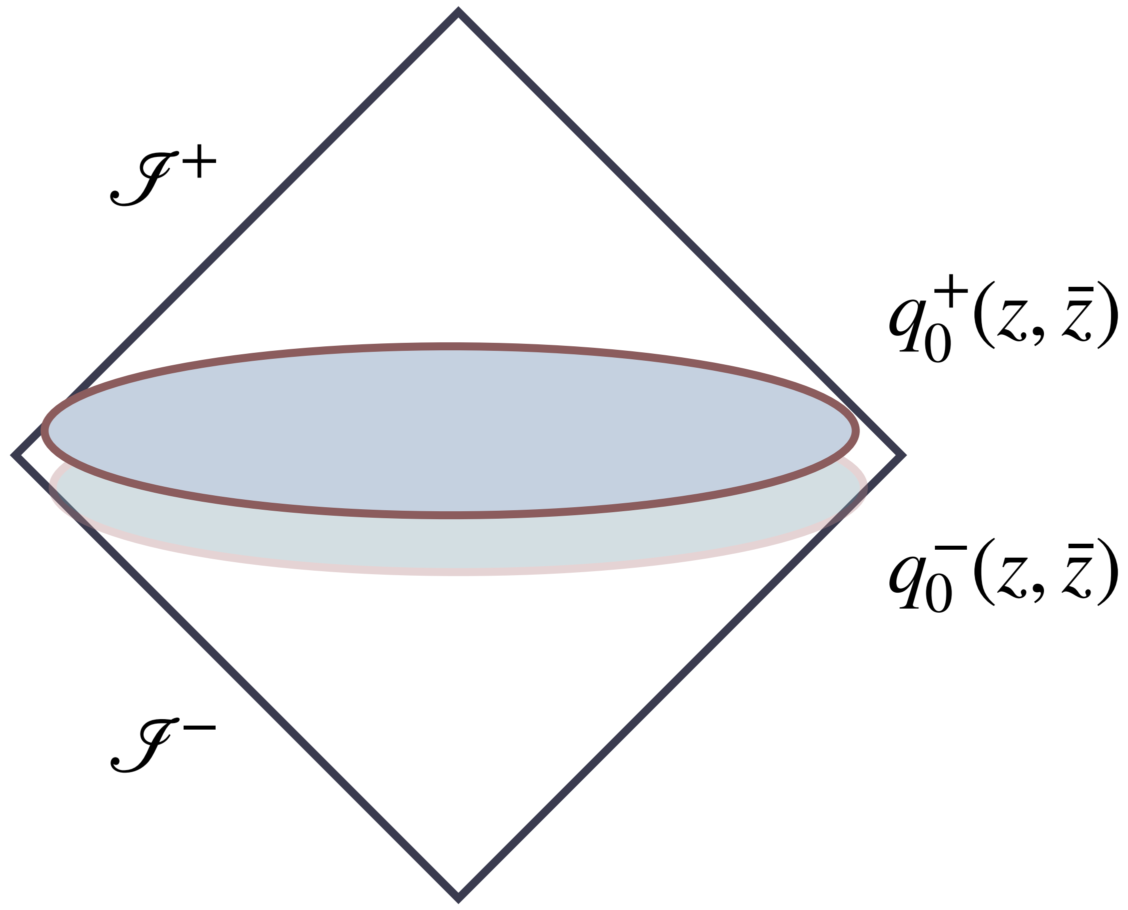
$Q_{ST}^{\text{soft}} = \int_{S^2} d^2z f(z,\bar{z}) \frac{1}{4}\left(D_z^2 \Delta C^{zz} + h.c.\right)$ generates the transformation:

$$C_{zz}(u,z,\bar{z}) \to C_{zz}(u,z,\bar{z}) + D_z^2 f(z,\bar{z})$$

Physical implication II: soft graviton theorem



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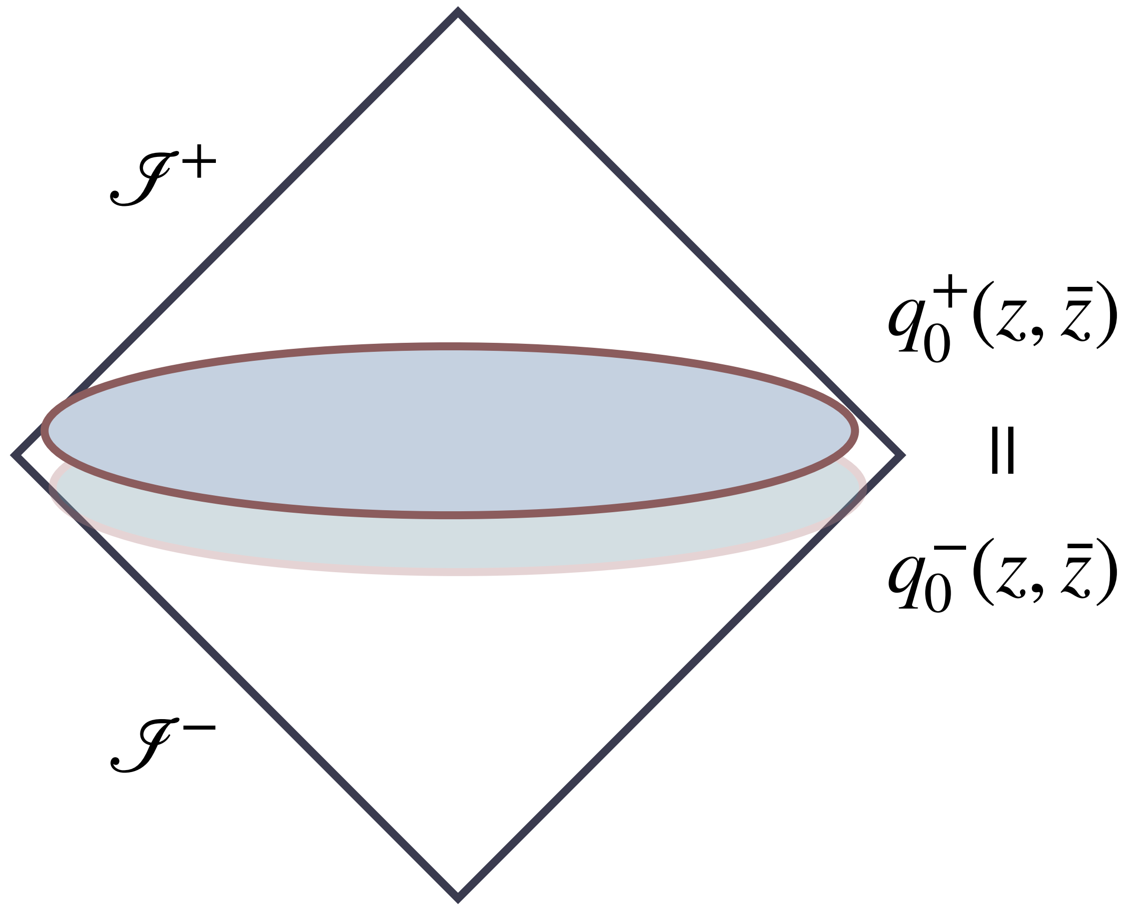
Physical implication II: soft graviton theorem



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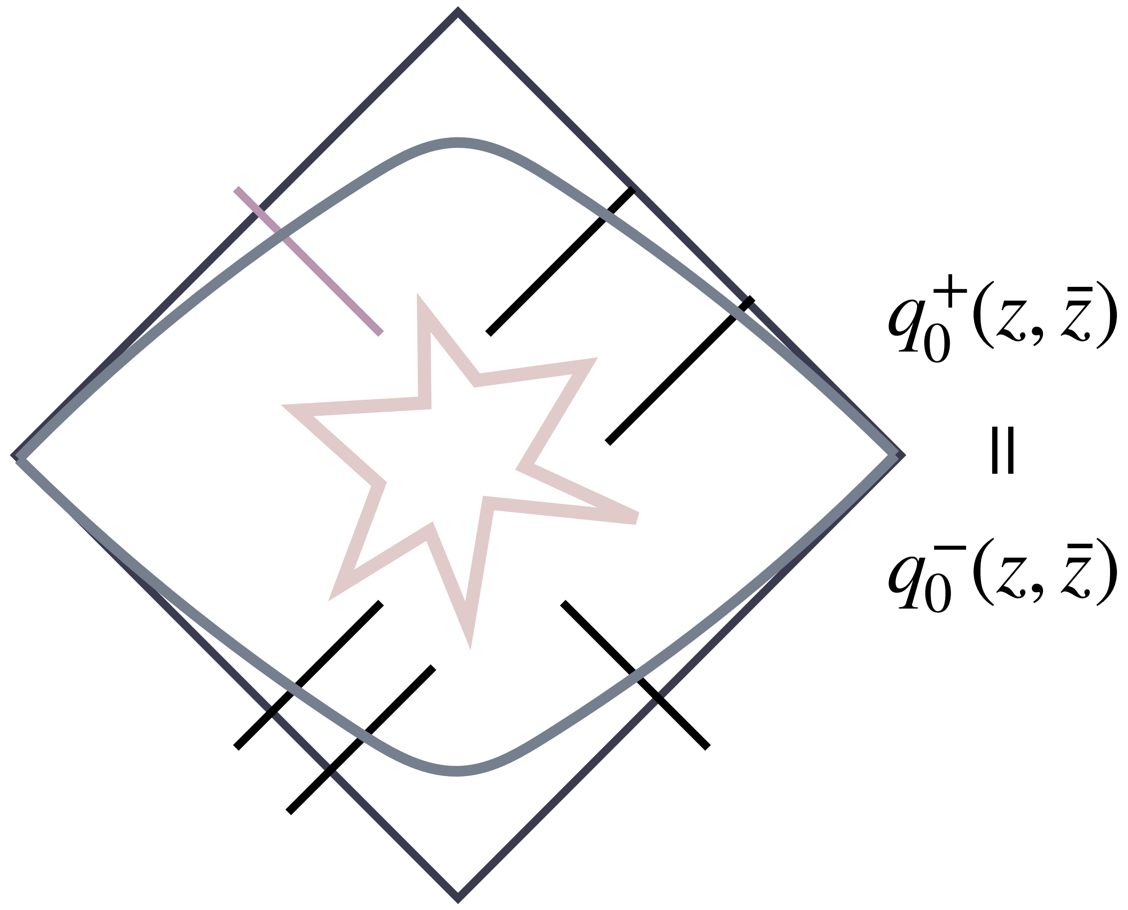

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use the constraints under the assumption that Q_0 vanish at $\mathscr{I}^\pm_\pm$

$\Longrightarrow$ charges are conserved in time, so they should commute with S-matrix

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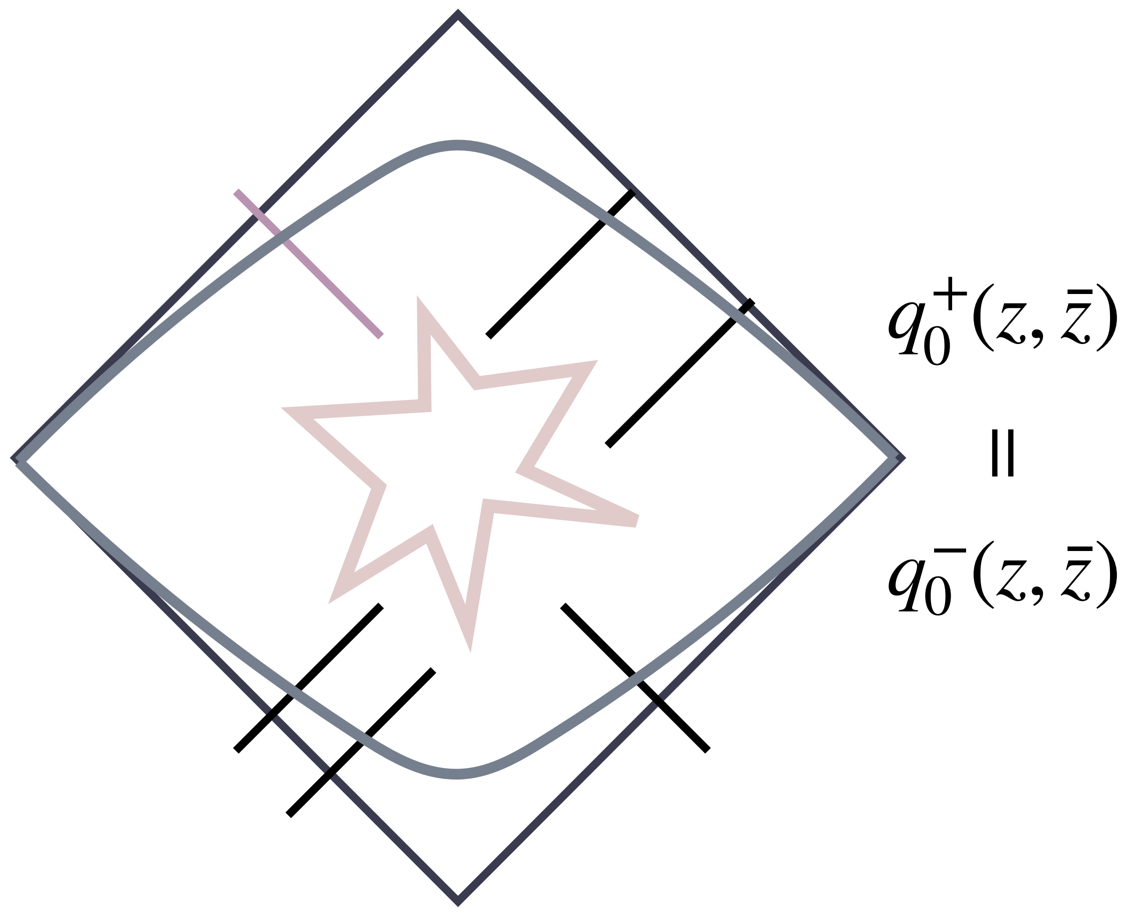

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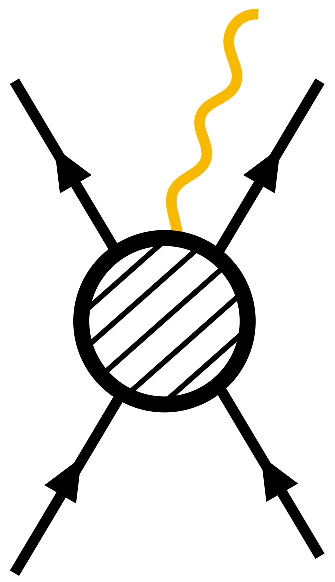
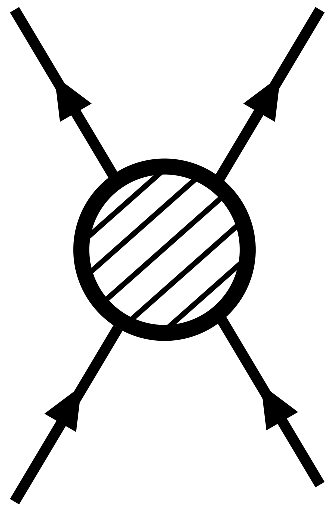
$$q_0 \propto \frac{1}{4}\left(D_z^2 \Delta C^{zz} + D_{\bar{z}}^2 \Delta C^{\bar{z}\bar{z}}\right) - \frac{1}{4}\int du N_{zz} N^{zz}$$

$$\boxed{\langle \mathrm{out} \,|\, q_0^+ \mathcal{S} - \mathcal{S} q_0^- \,|\, \mathrm{in} \rangle = 0}$$

$\lim\limits_{q\to 0}$



$$= \sum_{i=1}^{n} \frac{p_i^\mu p_i^\nu \epsilon_{\mu\nu}^\pm}{p_i \cdot q} \times$$



$+\mathcal{O}(q^0)$

[Weinberg '65; Strominger et. al '14]

Tower of soft theorems

$$\boxed{\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2}C\mathcal{Q}_{s-2}\,, \quad s = 0,1,2}$$

Imply new soft theorems:

- conservation of superrotation charge $q_1 \Longrightarrow$ subleading soft graviton theorem

[Cachazo, Strominger '14]

- conservation of $q_2 \Longrightarrow$ sub-subleading soft graviton theorem

[Freidel, Pranzetti, A.R. '21]

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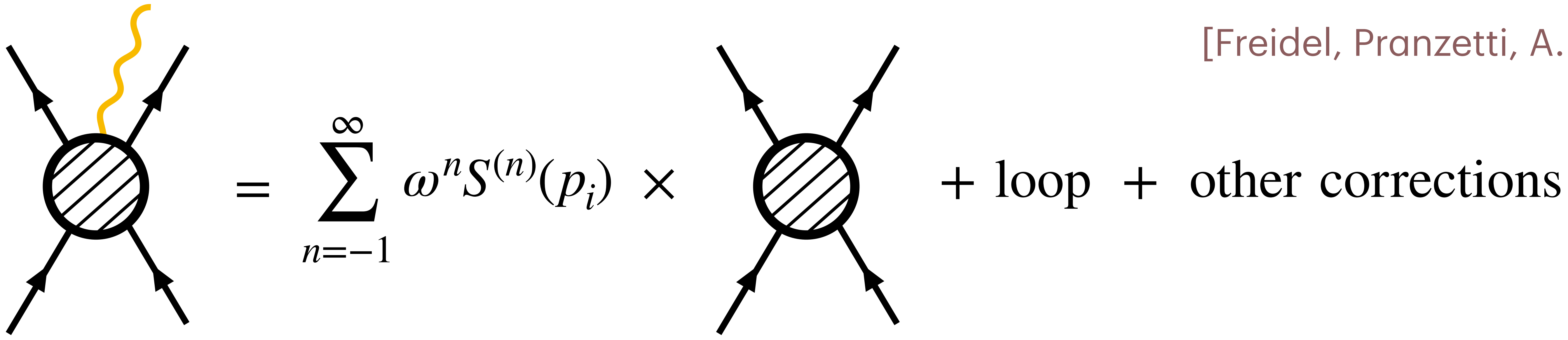
- conservation of $q_s,\ s \geq 3 \implies$ tower of (sub)^s- leading soft graviton theorems

[Freidel, Pranzetti, A.R. ’22]



$$= \sum_{n=-1}^{\infty} \omega^n S^{(n)}(p_i) \times \qquad + \text{ loop } + \text{ other corrections}$$

Tower of soft theorems

$$\frac{d\mathcal{Q}_s}{du} = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2} + \cdots \,, \qquad s \geq 3$$

- q_s generate a $w_{1+\infty}$ algebra on the gravitational phase space:

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Bracket of linear and

quadratic components

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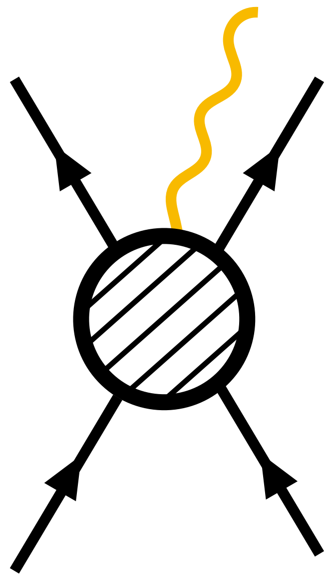
[Freidel, Pranzetti, A.R. '22]



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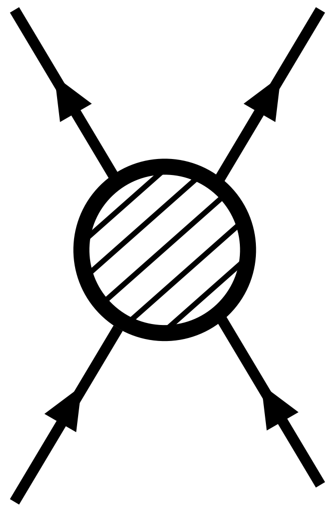

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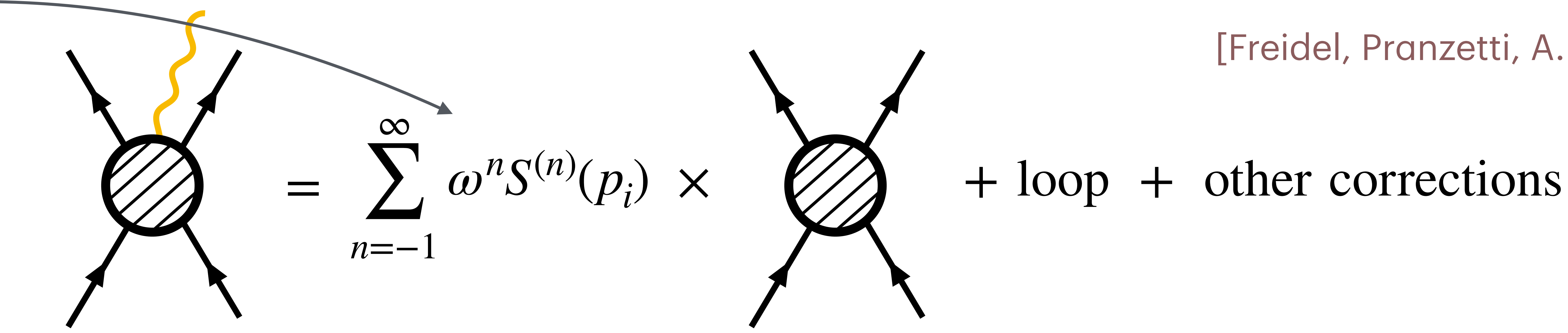
[Freidel, Pranzetti, A.R. '22]

All orders in ω ~

finite-energy graviton!



$$= \sum_{n=-1}^{\infty} \omega^n S^{(n)}(p_i) \times \quad + \text{loop} \ + \ \text{other corrections}$$

From IR to UV and back

$$G_{uu} = G_{uA} = G_{AB} = 0 \ @ \quad \mathcal{O}(r^{-2}) \implies \{q_s(z), q_{s'}(z')\}^{(1)} = (s+1)q^{(1)}_{s+s'-1}(z)D_{z'}\delta^{(2)}(z-z') - (s'+1)q^{(1)}_{s+s'-1}(z')D_z\delta^{(2)}(z-z'), \quad s+s'<3$$

- Bracket for $s = s' = 2 \implies q_3$! For algebra to close need tower of generators $q_s \forall s \in \mathbb{N}$
- $s \geq 2$ charges are related to Ψ_0 components to higher orders in a r^{-1} expansion;

  linear components = multipole moments of the gravitational field

[Compere, Oliveri, Seraj, '22]

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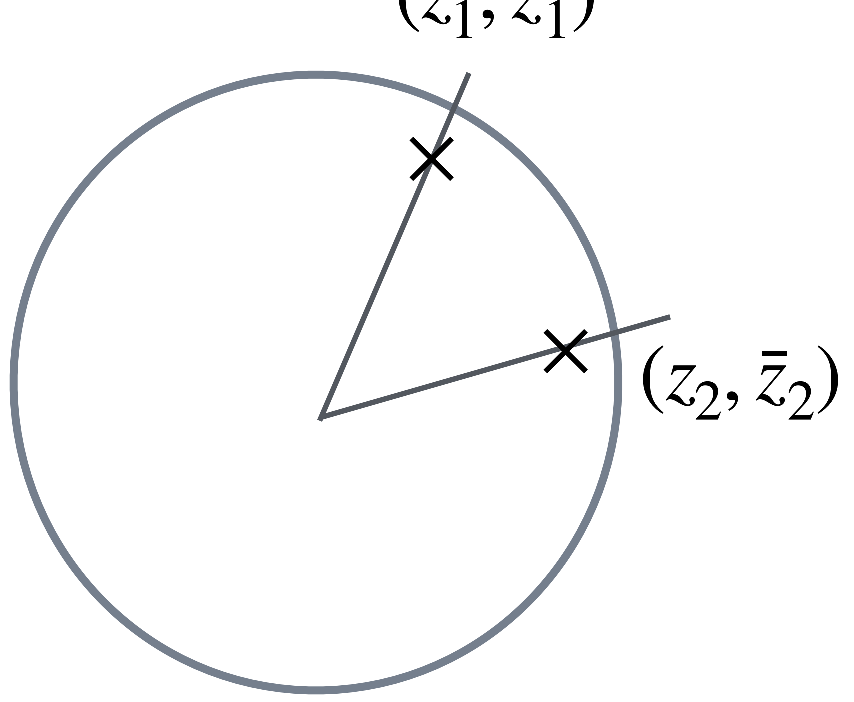
- q_s from increasingly subleading terms in the collinear expansion of two conformal primary gravitons:

$$G^{--}_{\Delta_1}(z_1)G^{\pm\pm}_{\Delta_2}(z_2) \sim -\frac{\kappa}{2}\frac{1}{\bar{z}_{12}}\sum_{n=0}^{\infty} B(2h_1+1+n, 2h_{2\pm}+1)\frac{z_{12}^{n+1}}{n!}\partial^n_{z_2}G^{\pm\pm}_{\Delta_1+\Delta_2}(z_2) + \mathcal{O}(\bar{z}^0_{12}),$$



$\Delta = 1 - s$: (sub)^s-leading soft mode, $\quad q_s(z,\bar{z}) \sim \mathrm{Res}_{\Delta=1-s}\partial_z^{s+2}G^{--}_{\Delta}(z,\bar{z})$

[Guevara, Himwich, Pate, Strominger ’21]

From IR to UV and back

subleading term in z_{12} ~ long-distance effect in CFT$_2$ ⟷ subleading term in r^{-1} ~ short-distance effect in 4d AFS

Seems to resonate with UV-IR relation in AdS/CFT?

4d gravity picture

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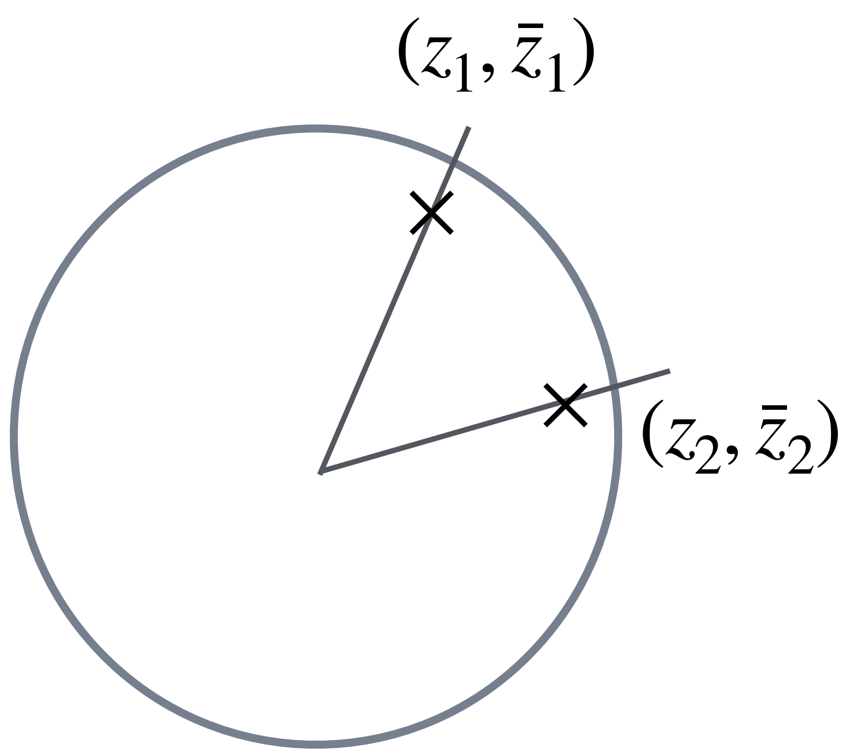
2d CFT picture

$$\begin{cases} G^{--}_{\Delta_1}(z_1)G^{\pm\pm}_{\Delta_2}(z_2) \sim -\frac{\kappa}{2}\frac{1}{\bar{z}_{12}}\sum_{n=0}^{\infty} B(2h_1+1+n, 2h_{2\pm}+1)\boxed{\frac{z_{12}^{n+1}}{n!}\partial^n_{z_2}G^{\pm\pm}_{\Delta_1+\Delta_2}(z_2)} + \mathcal{O}(\bar{z}^0_{12}), \\ \Delta = 1 - s : \text{(sub)}^s\text{-leading soft mode}, \qquad q_s(z,\bar{z}) \sim \mathrm{Res}_{\Delta=1-s}\partial_{\bar{z}}^{s+2}G^{--}_{\Delta}(z,\bar{z}) \end{cases}$$



[Guevara, Himwich, Pate, Strominger '21]

Quantum spacetime fluctuations?

- q_s and correlators thereof may capture quantum features of spacetime

- Leading soft sector: $C_{zz} = -\underbrace{2D_z^2 C(z,\bar{z})} + \underbrace{D_z^2 N(z,\bar{z})}\Theta(u - u_0)$

C (Goldstone mode) q_0^{soft} (memory mode) are canonically paired

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- Fixing C ~ fixing an asymptotic BMS frame; in quantum gravity ~ compute observables in C eigenstates (infinity of BMS vacua)

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Define the quantum operator $\hat{Q}_0 = \dfrac{1}{16\pi G_N}\displaystyle\int_{S^2} d^2z\,\hat{C}(z,\bar{z})\,\bar{\Box}^2\,\hat{N}(z,\bar{z})$ s.t. $\hat{Q}_0\,|C\rangle = q_0^{\text{soft}}\,|C\rangle$

bi-linear linear

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bi-linear $\nearrow$ $\nwarrow$ linear

Shares similarities with area operator whose fluctuations $\langle 0|\Delta\hat{Q}_0^2|0\rangle \propto \dfrac{A}{\ell_p^2}$ seem to be enhanced by IR scale... [He, A.R., Zurek '24]

Outlook

- Infrared sector of gravity in 3+1-dimensions is very rich:

  — infinite-dimensional asymptotic symmetry algebra [eBMS]

  — modes of graviton to all orders in a low energy expansion ~ higher-spin symmetry on phase space

  — phase-space symmetries related to chiral algebras in 2d CFT ⟶ holography, UV-IR connection?

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Thank you!