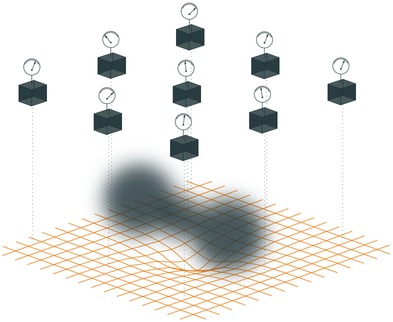
The general structure of quantum-classical dynamics



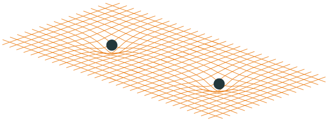
for gravity and more...

Antoine Tilloy
July 9th, 2025
Bridge QG First Annual Conference
Paris

MINES PARIS | PSL
ENS | PSL
Inria
erc

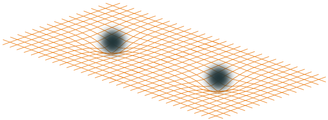
Prolegomena

Classical gravity



- **Matter** is classical
- Spacetime is classical

Semiclassical gravity

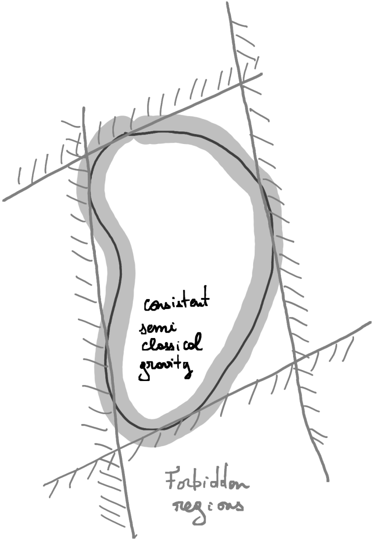


- **Matter** is quantum
- Spacetime is classical

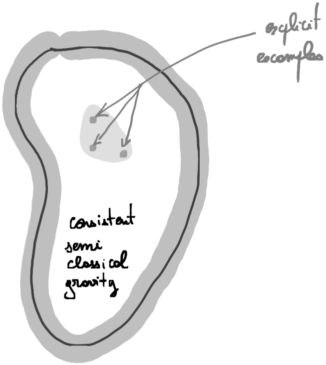
Fully quantum gravity

- **Matter** is quantum
- Spacetime is quantum

Two strategies to study the quantum nature of gravity



Bose et al., Marletto & Vedral

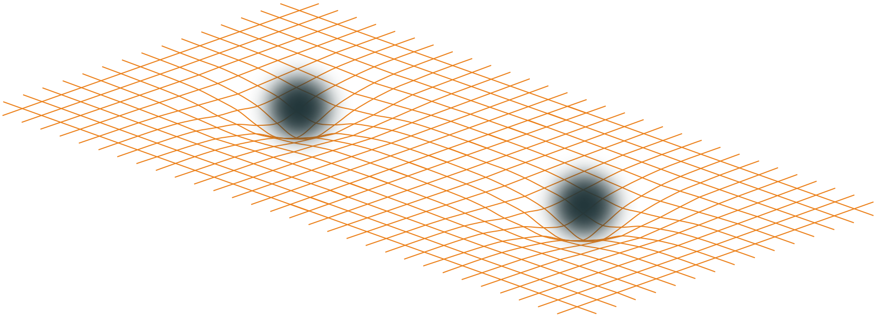


Kafri-Taylor-Milburn, Diósi, Oppenheim

Is the chimera possible?

I bet 99 to one that the outcome [of some proposed experiments] will be consistent with gravity having quantum properties. – Carlo Rovelli

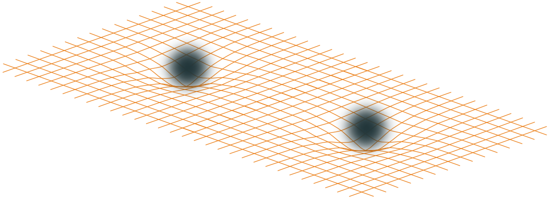
Standard semiclassical gravity



“Standard” semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

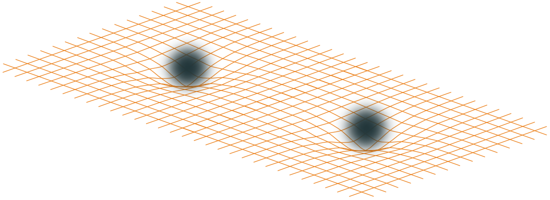
1. Quantum matter moves in a curved classical space-time
2. The classical space time is curved by quantum matter



“Standard” semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

1. Quantum matter moves in a curved classical space-time
2. The classical space time is curved by quantum matter



1 is known (QFTCST), **2** is not

The crucial question of semi-classical gravity is to know how quantum matter should source curvature.

Møller-Rosenfeld semi-classical gravity

The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi G \, \langle \hat{T}_{\mu\nu} \rangle$$

$\rightarrow$ source gravity via expectation values

Møller-Rosenfeld semi-classical gravity

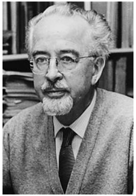
The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = 8\pi G \, \langle \hat{T}_{\mu\nu} \rangle$$

$\rightarrow$ source gravity via expectation values

There are:

- **technical relativistic** difficulties [renormalization of $\langle T_{\mu\nu} \rangle$]
- **conceptual non-relativistic** difficulties [Born rule, $\cdots$].


Christian Møller


Leon Rosenfeld

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x,t) = 4\pi G \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x, t) = 4\pi G \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

2. Non-relativistic limit of QFTCST (just external field)

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = -i \left(H_0 + \int \mathrm{d}x \, \Phi(x, t) \hat{M}(x) \right) |\psi_t\rangle,$$

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x,t) = 4\pi G \, \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

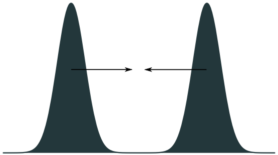
2. Non-relativistic limit of QFTCST (just external field)

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = -i \left(H_0 + \int \mathrm{d}x \, \Phi(x,t) \hat{M}(x) \right) |\psi_t\rangle,$$

Putting the two together: Schrödinger-Newton equation

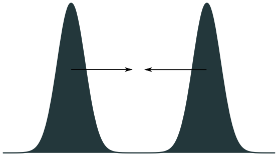
$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi_t\rangle = -i H_0 |\psi_t\rangle + i\, G \int \mathrm{d}x \, \mathrm{d}y \, \frac{\langle \psi_t | \hat{M}(x) | \psi_t \rangle \, \hat{M}(y)}{|x-y|} |\psi_t\rangle.$$

The problems with Schrödinger-Newton



Deterministic non-linearity is not allowed (Gisin, Diósi, Polchinski)

The problems with Schrödinger-Newton



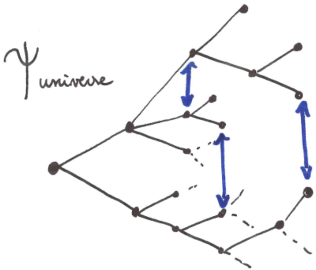
Deterministic non-linearity is not allowed (Gisin, Diósi, Polchinski)

Proof is **interpretation dependent**

- If there is **no fundamental collapse** [Many Worlds, Bohm,$\cdots$]
- If there is **fundamental collapse** [Copenhaguen, Collapse models]

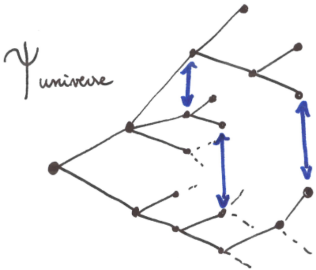
The problems with Schrödinger-Newton

Without collapse upon measurement (Bohm, Many Worlds, $\cdots$)

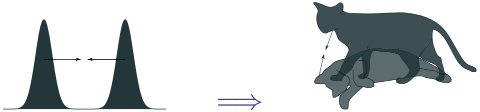


The problems with Schrödinger-Newton

Without collapse upon measurement (Bohm, Many Worlds,$\cdots$)



Decohered branches interact with each other $\rightarrow$ **empirically inadequate**



$\rightarrow$ even experimentally tested by Page and Geiker '81

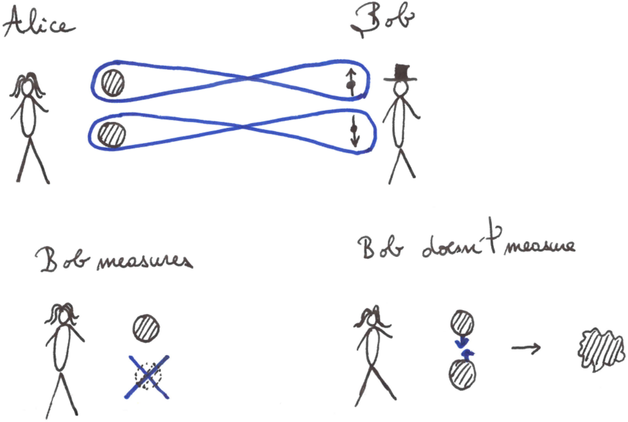
The problems with Schrödinger-Newton

With collapse upon measurement (either from pure Copenhaguen or collapse models).

Consider a mass entangled with a spin far away:

$$|\Psi\rangle \propto |\text{left}\rangle^{\text{Alice}} \otimes |\uparrow\rangle^{\text{Bob}} + |\text{right}\rangle^{\text{Alice}} \otimes |\downarrow\rangle^{\text{Bob}}.$$

Bob can decide to whether or not he measures his spin:



The big question

What mathematical object can one construct to source the gravitational field while keeping things consistent?

The big question, generalized

How can one consistently couple quantum and classical variables?

$$\rho_t \longleftrightarrow z_t$$

3 ways to do this

1. **Continuous measurement and feedback**
2. Spontaneous collapse models
3. **General continuous dynamics with a classical subspace** [used by Oppenheim]

Main result from `2403.19748` → `Scipost`: the 3 are **mathematically equivalent**

nt-ph] 5 May 2024

General quantum-classical dynamics as measurement based feedback

Antoine Tilloy*

Laboratoire de Physique de l'Ecole Normale Supérieure, Mines Paris - PSL, CNRS, Inria, PSL Research University, Paris, France

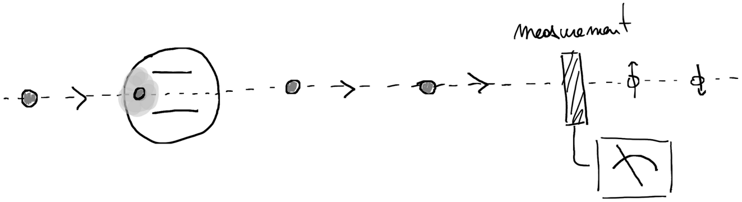
Abstract

This note derives the stochastic differential equations and partial differential equation of general hybrid quantum-classical dynamics from the theory of continuous measurement and general (non-Markovian) feedback. The advantage of this approach is an explicit parameterization, without additional positivity constraints. The construction also neatly separates the different effects: how the quantum influences the classical and how the classical influences the quantum. This modular presentation gives a better intuition of what to expect from hybrid dynamics, especially when used to construct possibly fundamental theories.

The First Way: Continuous measurement and feedback

Continuous quantum measurement – derivation

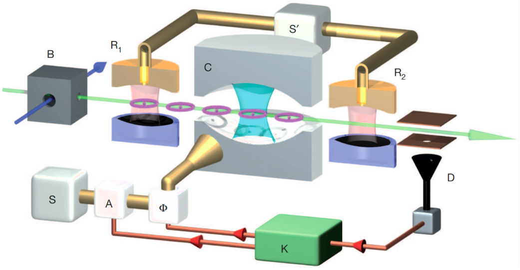
Continuous measurement – without Zeno effect



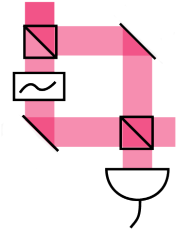
- time between ancillas $\Delta t \propto \varepsilon$
- interaction strength $\omega \propto \sqrt{\epsilon}$

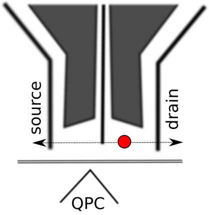
In practice

- Discrete situations “a la Haroche”, with **actual** repeated interactions



- Almost “true” continuous measurement settings (homodyne detection in quantum optics, quantum point contacts for quantum dots)





Continuous quantum measurement

Stochastic Master Equation (∼ 1987 – pre-theory by Gisin 1984)

Density matrix:

$$\mathrm{d}\rho_t = \underbrace{-i[H,\rho_t]\,\mathrm{d}t}_{\text{standard quantum dynamics}} + \underbrace{\mathcal{D}[\hat{c}](\rho_t)\,\mathrm{d}t}_{\text{decoherence}} + \underbrace{\mathcal{H}[\hat{c}](\rho_t)\,\mathrm{d}W_t}_{\text{measurement backaction}}$$

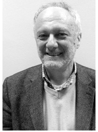
Signal:

$$\mathrm{d}r_t = \mathrm{tr}\left[(\hat{c}+\hat{c}^\dagger)\,\rho_t\right]\mathrm{d}t + \mathrm{d}W_t$$

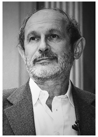
with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[(\mathcal{O}+\mathcal{O}^\dagger)\,\rho\right]\rho$
- $\frac{\mathrm{d}W_t}{\mathrm{d}t}$ "white noise"

V. Belavkin

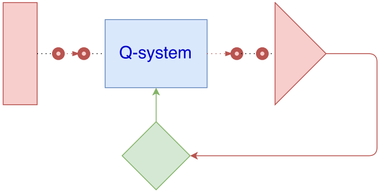


A. Barchielli



L. Diósi

Measurement based feedback



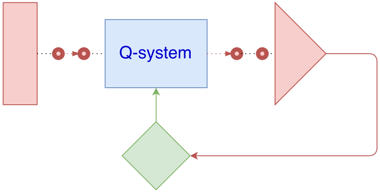
Step 1: Have the classical z_t depend on r_t

$$\mathrm{d}z_t = F(z_t)\,\mathrm{d}t + G(z_t)\,\mathrm{d}r_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Measurement based feedback



Step 1: Have the classical z_t depend on r_t

$$\mathrm{d}z_t = F(z_t)\,\mathrm{d}t + G(z_t)\,\mathrm{d}r_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Consistent by construction since derivable as *effective* from Copenhaguen QM

Most general stochastic equations

Quantum stochastic master equation

$$\mathrm{d}\rho = -i[H_0 + V(\mathbf{z}), \rho]\,\mathrm{d}t + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho)\,\mathrm{d}t + \sqrt{\eta_k}\,\mathcal{M}[\hat{c}_k](\rho)\,\mathrm{d}W_k,$$

Signal stochastic differential equation (the quantum classical glue)

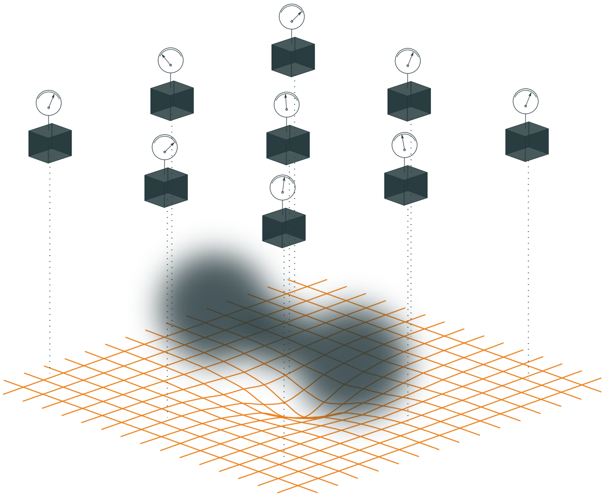
$$\mathrm{d}r_k = \frac{1}{2}\mathrm{tr}[(\hat{c}_k + \hat{c}_k^\dagger)\rho]\,\mathrm{d}t + \frac{1}{2\sqrt{\eta_k}}\mathrm{d}W_k.$$

Classical stochastic differential equation

$$\mathrm{d}z_a = F_a(\mathbf{z})\,\mathrm{d}t + G_{ak}(\mathbf{z})\,\sqrt{\eta_k}\,\mathrm{d}r_k$$

F, G are functions – $V(\mathbf{z})$, $\hat{c}_k$ are operators

“Intuition pump” picture for gravity



AS IF – “There are detectors in space-time measuring the mass density continuously and curving space-time accordingly.” → explains consistency

The Third Way: Embedding a classical sector in quantum dynamics

$$\rho(z, t)$$

Formulation of the problem

Quantum-classical state

A state diagonal in the classical variables z

$$\rho_{QC} = \int dz\, \rho_Q(z)\, |z\rangle\langle z|$$

- used early on (Diosi, Halliwell, Gisin)
- starting point of Oppenheim

PHYSICAL REVIEW X **13,** 041040 (2023)

Featured in Physics

A Postquantum Theory of Classical Gravity?

Jonathan Oppenheim

Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

(Received 25 June 2021; revised 20 March 2023; accepted 5 October 2023; published 4 December 2023)

The effort to discover a quantum theory of gravity is motivated by the need to reconcile the incompatibility between quantum theory and general relativity. Here, we present an alternative approach by constructing a consistent theory of classical gravity coupled to quantum field theory. The dynamics is linear in the density matrix, completely positive, and trace preserving, and reduces to Einstein's theory of general relativity in the classical limit. Consequently, the dynamics does not suffer from the pathologies of the semiclassical theory based on expectation values. The assumption that general relativity is classical necessarily modifies the dynamical laws of quantum mechanics; the theory must be fundamentally stochastic in both the metric degrees of freedom and in the quantum matter fields. This breakdown in predictability allows it to evade several no-go theorems purporting to forbid classical quantum interactions. The measurement postulate of quantum mechanics is not needed; the interaction of the quantum degrees of freedom with classical space-time necessarily causes decoherence in the quantum system. We first derive the general form of classical quantum dynamics and consider realizations which have as its limit deterministic classical Hamiltonian evolution. The formalism is then applied to quantum field theory interacting with the classical space-time metric. One can view the classical quantum theory as fundamental or as an effective theory useful for computing the backreaction of quantum fields on geometry. We discuss a number of open questions from the perspective of both viewpoints.

DOI: 10.1103/PhysRevX.13.041040 Subject Areas: Gravitation, Quantum Information

Q&A

The Physicist Who's Challenging the Quantum Orthodoxy

For decades, physicists have struggled to develop a quantum theory of gravity. But what if gravity — and space-time — are fundamentally classical?

Most general second order PDE

Constraints:

- Assuming z evolves continuously $\rightarrow$ at most second order derivatives
- $\rho(z)$ physical $\rightarrow$ positivity conditions

$$
\begin{aligned}
\frac{\partial \rho_t(\mathbf{z})}{\partial t} = & -i\,[H, \rho_t(\mathbf{z})] + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho_t(\mathbf{z})) \\
& - \frac{\partial}{\partial z_a}\left[F_a(\mathbf{z})\rho_t(\mathbf{z}) + \frac{\sqrt{\eta_k}\,G_{ak}(\mathbf{z})}{2}\left(\hat{c}_k\rho_t(\mathbf{z}) + \rho_t(\mathbf{z})\hat{c}_k^{\dagger}\right)\right] \\
& + \frac{1}{2}\frac{\partial^2}{\partial z_a \partial z_b}\left[\frac{G_{ak}(\mathbf{z})G_{bk}(\mathbf{z})}{4}\rho_t(\mathbf{z})\right]
\end{aligned}
$$

Equivalence via Ito's lemma

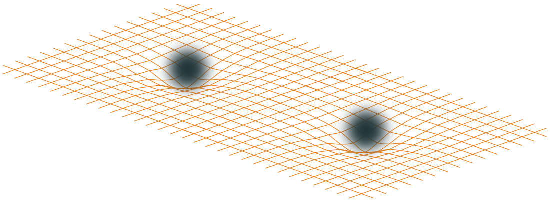
This PDE is the "**Fokker-Planck**" version of the "**Langevin**" dynamics of measurement $+$ feedback:

Equivalence

Using Itô's lemma, one has:

$$\forall f \quad \underbrace{\mathbb{E}[\rho_t\, f(\mathbf{z}_t)]}_{\text{measurement and feedback}} = \underbrace{\int f(\mathbf{z})\, \rho_t(\mathbf{z})\, \mathrm{d}\mathbf{z}}_{\text{hybrid PDE}}\,.$$

Back to gravity



History

Newtonian early work

Source gravity by measuring the mass density:

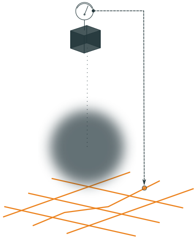
$$\nabla^2\Phi(x) = 4\pi G\mathscr{S}_{\hat{M}}(x)$$

toy model – [Kafri, Taylor, Milburn 2014]
full Newtonian potential – [Diósi & T 2015]

General relativistic extensions

Construct a PDE for $\rho(z)$ for z the gravitational degrees of freedom in ADM general relativity

[Oppenheim, Weller-Davies, Layton, Soda, Russo, ... 2018 $\rightarrow$ today]



Markovian/Newtonian limit

$$\Delta\Phi(x) = 4\pi G M(x)$$

Technically easier

Newtonian limit = Markovian feedback limit

$$\texttt{classical variable at time t} \propto \texttt{measurement signal at time t}$$

$\implies$ technically infinitely easier $\implies$ one can say something

Markovian/Newtonian limit

$$\Delta\Phi(x) = 4\pi G M(x)$$

Technically easier

Newtonian limit $=$ Markovian feedback limit

$$\texttt{classical variable at time t} \propto \texttt{measurement signal at time t}$$

$\implies$ technically infinitely easier $\implies$ one can say something

Experimentally motivated

Hard to probe anything else in the near future **and unknown!**

Markovian/Newtonian limit

$$\Delta\Phi(x) = 4\pi G M(x)$$

Technically easier

Newtonian limit = Markovian feedback limit

$$\texttt{classical variable at time t} \propto \texttt{measurement signal at time t}$$

$\Longrightarrow$ technically infinitely easier $\Longrightarrow$ one can say something

Experimentally motivated

Hard to probe anything else in the near future **and unknown!**

Can be criticized

No "independent" gravitational degrees of freedom. Can it be extended?

Model

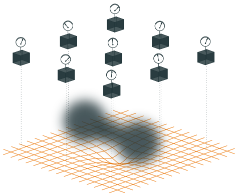
1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:

The equation for matter is now as before with

$\mathcal{O} \to \hat{M}(x), \ \forall x \in \mathbb{R}^3$

$\gamma \to \gamma(x, y)$ coding detector strength and correlation

and there is a "mass density signal" $\mathsf{S}(x)$ in every point.



Model

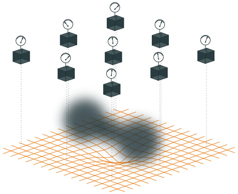
1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:

The equation for matter is now as before with

$$\mathcal{O} \to \hat{M}(x), \ \ \forall x \in \mathbb{R}^3$$

$\gamma \to \gamma(x, y)$ coding detector strength and correlation

and there is a “mass density signal” $\mathsf{S}(x)$ in every point.

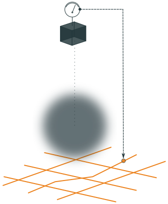


2. **Step 2: Feedback**

We take the mass density signal $\mathsf{S}(x)$ to source the gravitational field φ:

$$\nabla^2 \varphi(x) = 4\pi\, G\, \mathsf{S}(x)$$

which is **formally** equivalent to quantum feedback.



Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho = -i\left[H_0 + \frac{1}{2}\iint \mathrm{d}x\mathrm{d}y \mathscr{V}(x,y)\hat{M}(x)\hat{M}(y), \rho_t\right]$$
$$-\frac{1}{8}\iint \mathrm{d}x\mathrm{d}y\, \mathscr{D}(x,y)\left[\hat{M}(x), \left[\hat{M}(y), \rho_t\right]\right],$$

with the **gravitational pair-potential**

$$\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the **positional decoherence**

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V}\circ\gamma^{-1}\circ\mathscr{V}^{\top}\right](x,y)$$

Principle of least decoherence

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^{\top}\right](x,y)$$

There is still a (functional) degree of freedom $\gamma(x,y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

Principle of least decoherence

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^{\top}\right](x,y)$$

There is still a (functional) degree of freedom $\gamma(x,y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

There is an optimal (non-local) kernel that minimizes decoherence.

Principle of least decoherence

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^\top\right](x,y)$$

There is still a (functional) degree of freedom $\gamma(x,y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

There is an optimal (non-local) kernel that minimizes decoherence.

Diagonalizing in Fourier, global minimum for

$$\gamma = 2\sqrt{\mathscr{V} \circ \mathscr{V}^\top} = -2\mathscr{V}$$

Hence:

$$\mathscr{D}(x,y) = -\mathscr{V}(x,y) = \frac{G}{|x-y|}$$

aka the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.
Adding a regulator at a length scale σ has 2 effects:

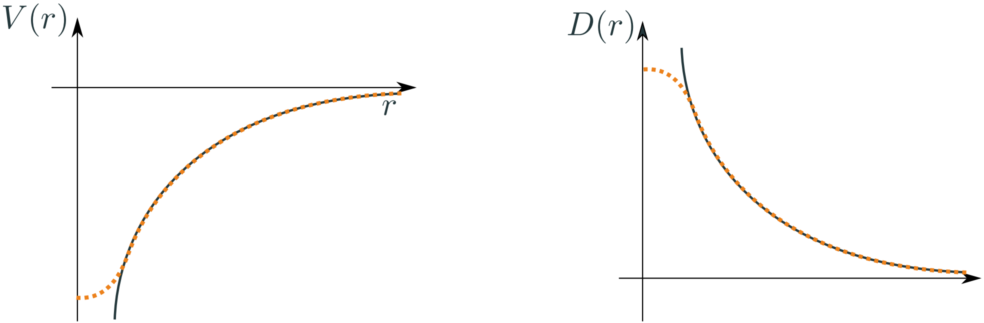
- It tames decoherence, making it finite
- It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.
Adding a regulator at a length scale σ has 2 effects:

- It tames decoherence, making it finite
- It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

$\Longrightarrow$ there is a **trade-off**.

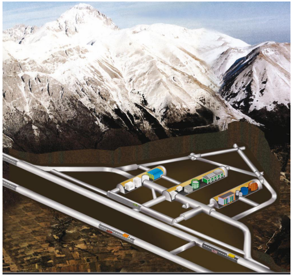


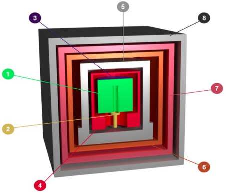
Tests of the cutoff length

Experimentally:

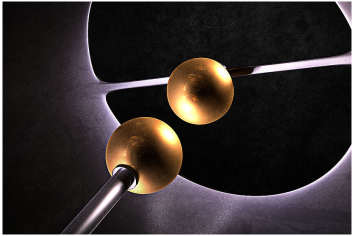
$$\underbrace{10^{-15} \to 10^{-10} m}_{\text{decoherence constraint}} \ll \sigma \leqslant \underbrace{10^{-4} m}_{\text{gravitational constraint}}$$

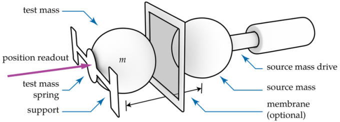
Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.





credit VIP collaboration – Catalina Curceanu





copyright Jonas Schmöle - Aspelmeyer group

Lack of measurement locality and entanglement

BMV idea: Ability to generate entanglement as a smoking of **quantum gravity**:

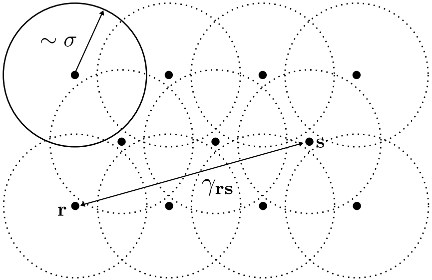
- Bose et al. / Marletto & Vedral 2017

Lack of measurement locality and entanglement

BMV idea: Ability to generate entanglement as a smoking of **quantum gravity**:

- Bose et al. / Marletto & Vedral 2017

Entanglement generated because measurement *not spatially local* (not strict LOCC)



[Trillo & Navascues 2024] [Feng, Marletto, Vedral 2025]

Very different entanglement generation, but not a YES/NO answer!

Summary

Conceptually: 3 equivalent ways to construct hybrid quantum-classical dynamics

1. **Measurement and feedback** which shows **consistency**
2. **Spontaneous collapse** which shows **empirical effects** + **measurement problem** solution
3. **Quantum Classical PDE** which shows **generality**

Quantum classical dynamics are *possible*, *well understood*, and *classified*

For gravity

- Newtonian limit: well defined models – minimizing decoherence gives Diosi-Penrose model
- General case: being explored by Oppenheim et al. – big progress, but not clear all constraints can be met
- Typically generates spatial entanglement if we ask for low decoherence