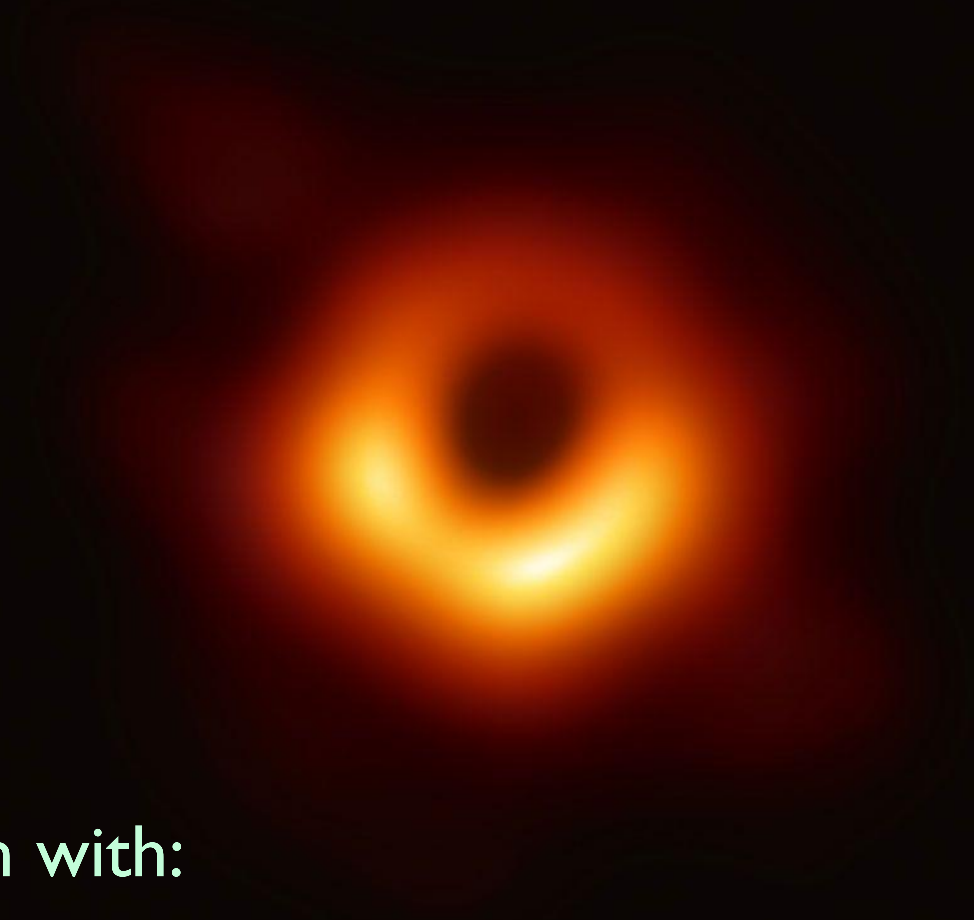
Nonperturbative Quantum Black Holes and Their Phenomenology



Saeed Rastgoo

UNIVERSITY OF ALBERTA

In collaboration with:

Ali Parvizi, Christian Pfeifer, Klaus Liegener

BridgeQG 2025

10/June/2025

Outline

- Spherically symmetric BHs in LQG
  - Interesting phenomena in the above BH
  - Effects of LQG on the above phenomena in BHs
- Rotating BH in LQG
  - BH Shadow and LQG constraints

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LQG BH Metrics

The 5 mainstream models can be written in the form

$$ds^2 = -f\left(r,\mathcal{K}\right)dt^2 + \frac{X\left(r,\mathcal{K}\right)}{f\left(r,\mathcal{K}\right)}dr^2 + h\left(r,\mathcal{K}\right)d\Omega^2$$

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Quantum parameters $\mathcal{K} = \mathcal{Q}$, $\mathcal{P}$, $\mathcal{R}$, $\mathcal{S}$ defined by us

To compare models with the same parameters (apples to apples)

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If $X = 1 \Rightarrow t - r$ symmetric

LQG BH Metrics: KSW

The 5 mainstream models can be written in the form

KSW model: [Kelly, Santacruz, Wilson-Ewing, PRD 102, 106024 (2020)]

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Quantum parameter for phenomenology

$$\mathcal{Q} = \frac{\gamma^2 \Delta}{R_s^2}$$

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Schwarzschild radius

$$R_s = 2M$$

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Minimum area in LQG

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$$ds^2 = -f\left(r, \mathcal{Q}\right) dt^2 + \frac{X\left(r, \mathcal{Q}, \mathcal{P}\right)}{f\left(r, \mathcal{Q}\right)} dr^2 + h\left(r, \mathcal{Q}\right) d\Omega^2$$

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2nd Quantum parameter for phenomenology

$$\mathcal{P} = \frac{\delta_r}{\sqrt{\mathcal{Q}}R_s}$$

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spin networks lattice spacing

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2nd Quantum parameter for phenomenology

$$\mathcal{R} = \frac{\Delta}{8\pi \mathcal{Q} R_s^2}$$

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AOS model: [Ashtekar, Olmedo, Singh, PRD 98, 126003 (2018)]

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Seems asymptotic limit is not Schwarzschild

LQG BH Metrics: BMM

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BMM model: [Bodendorfer, Mele, Munch, Phys. Lett. B, 819:136390, 2021]

$$ds^2 = -f\left(r, \mathcal{Q}, \mathcal{S}\right) dt^2 + \frac{1}{f\left(r, \mathcal{Q}, \mathcal{S}\right)} dr^2 + h\left(r, \mathcal{Q}, \mathcal{S}\right) d\Omega^2$$

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2nd Quantum parameter for phenomenology

$$\mathcal{S} = \frac{\lambda_k}{\gamma^2 \Delta}$$

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Phenomenology – Photon Sphere

For null geodesics ($ds^2 = 0,\ \theta = \pi/2$), EoM becomes

$$f(r)g(r)\left(\frac{dr}{d\lambda}\right)^2 + \underbrace{f(r)\left(\frac{L^2}{h(r)}\right)}_{V_{\text{eff}}} = E^2$$

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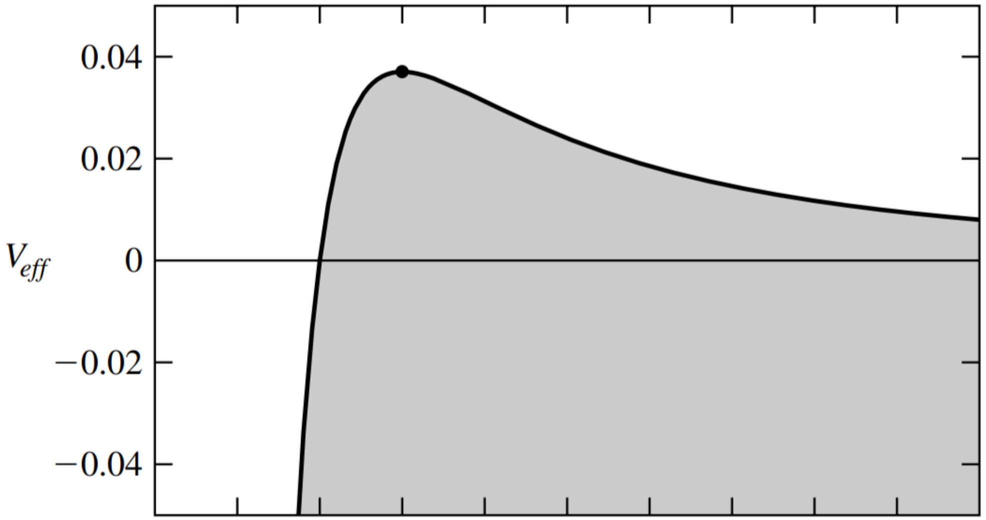

[Gravity, An Introduction to General Relativity, J. B. Hartle]

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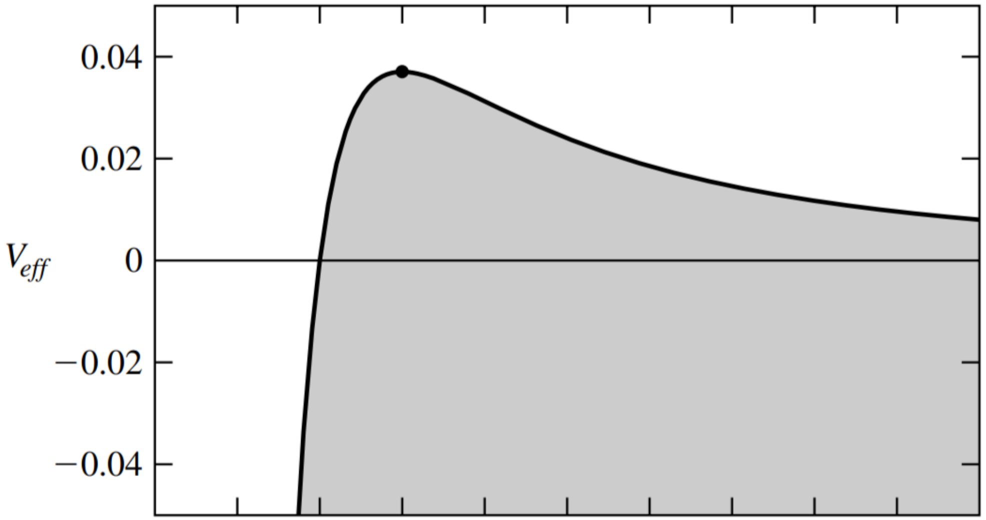
Max of V_{eff} : unstable closest circular orbit to BH = photon sphere



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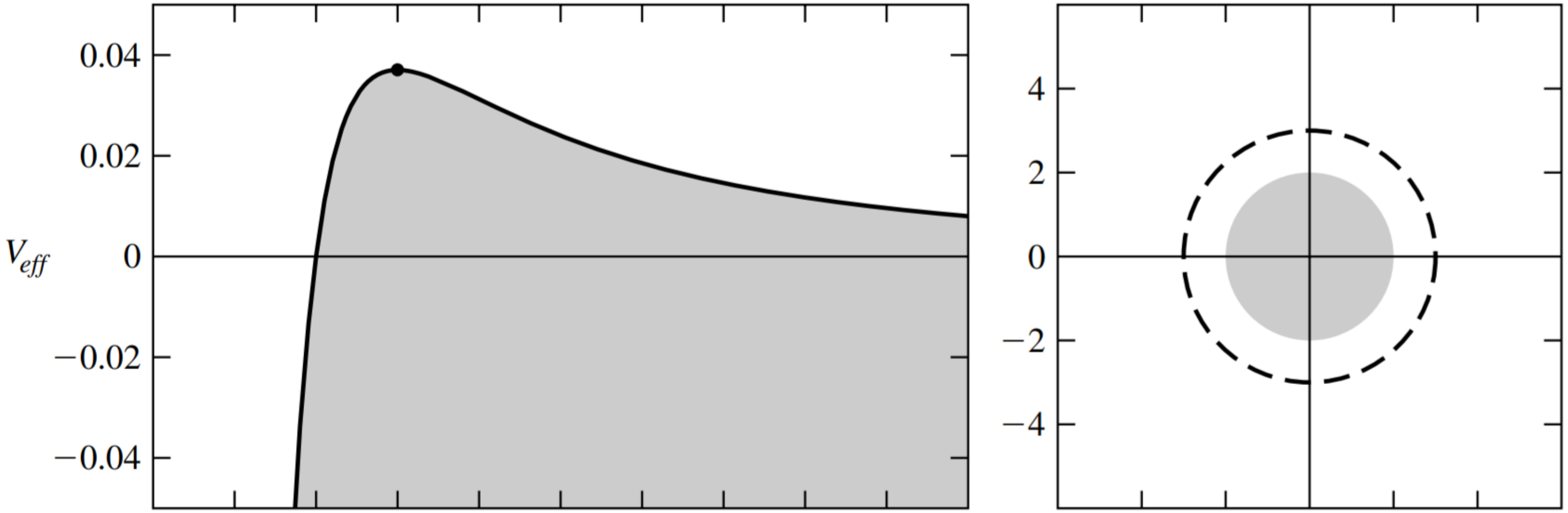

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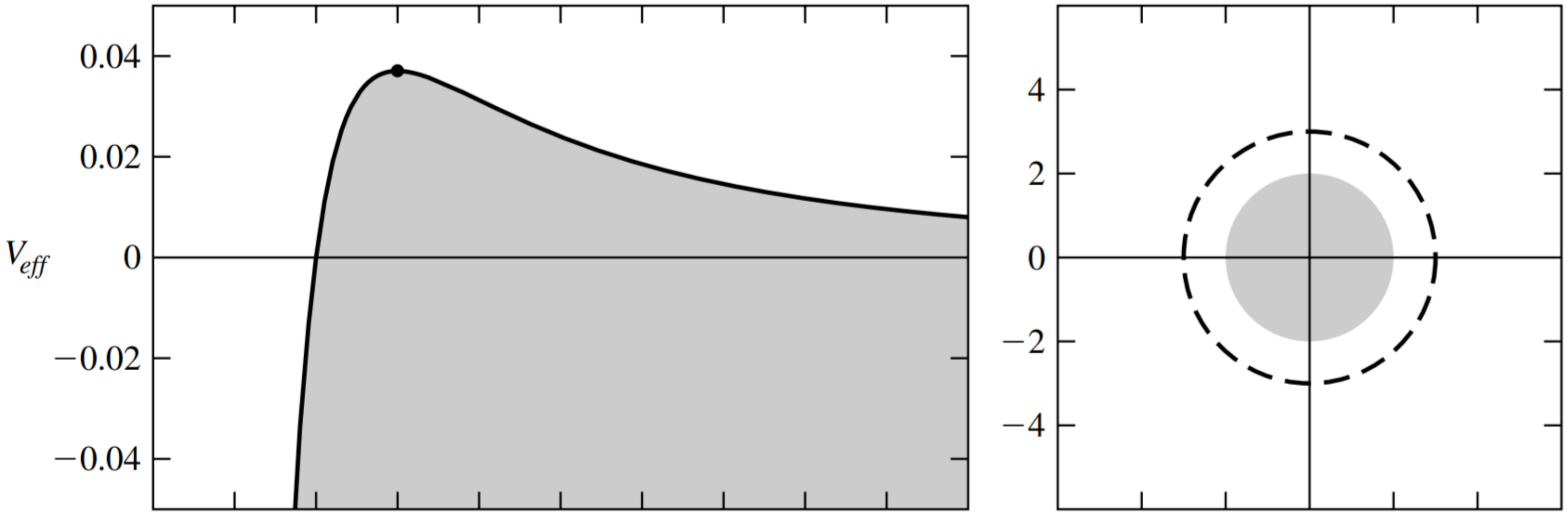
$$f'(r_{\text{ps}})h(r_{\text{ps}}) - f(r_{\text{ps}})h'(r_{\text{ps}}) = 0$$



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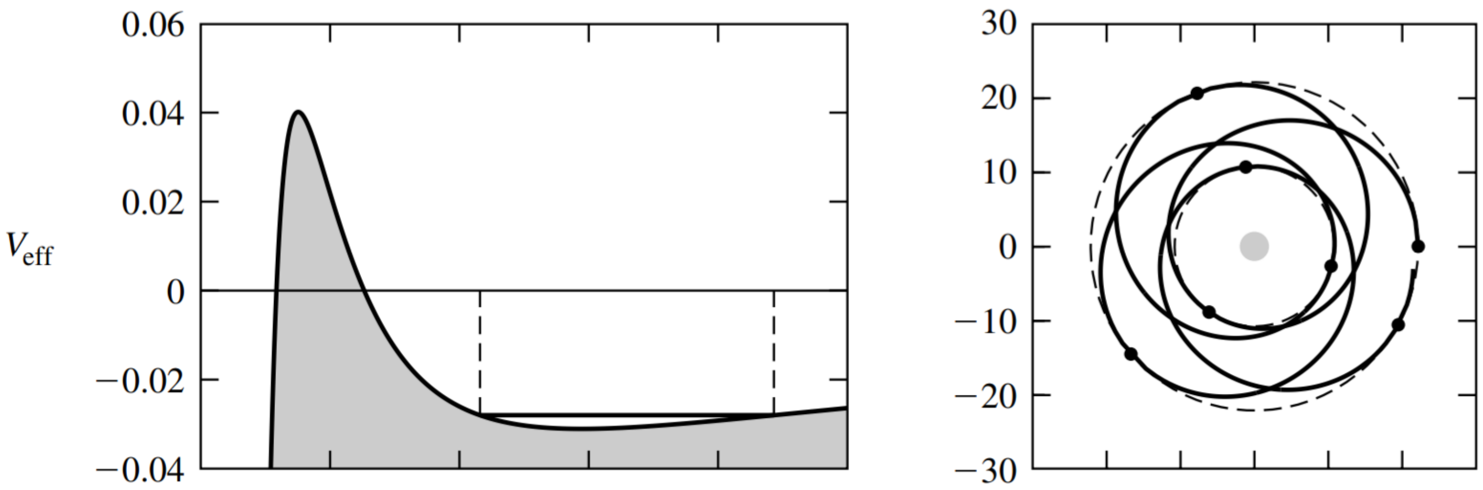
Phenomenology – Perihelion Shift

In GR, bound non-circular orbits do not close and oscillate between two radii.



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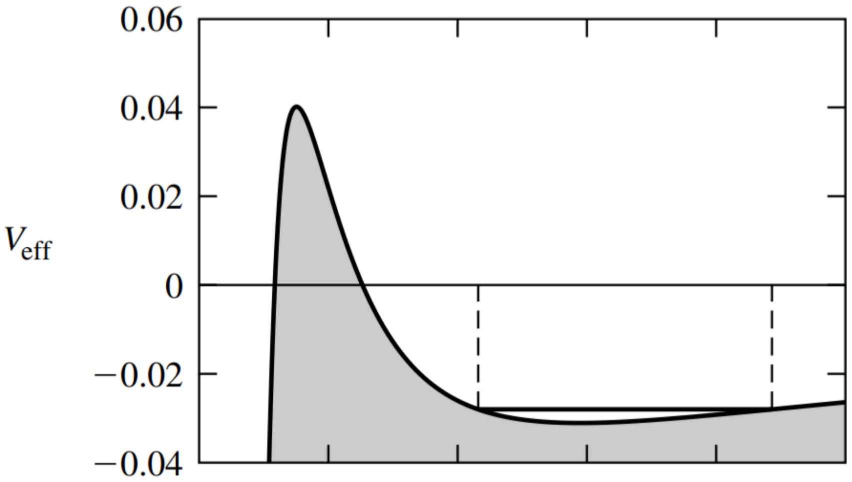
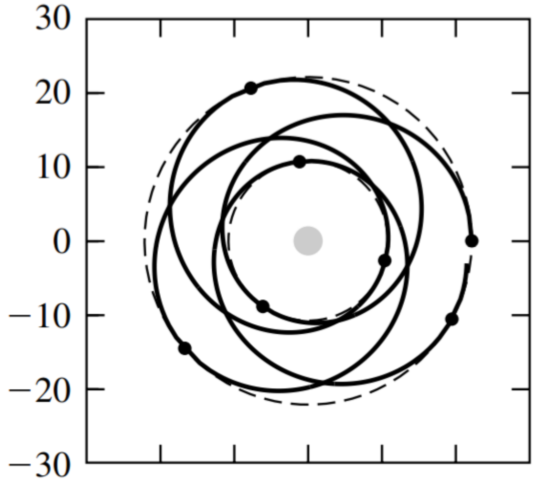
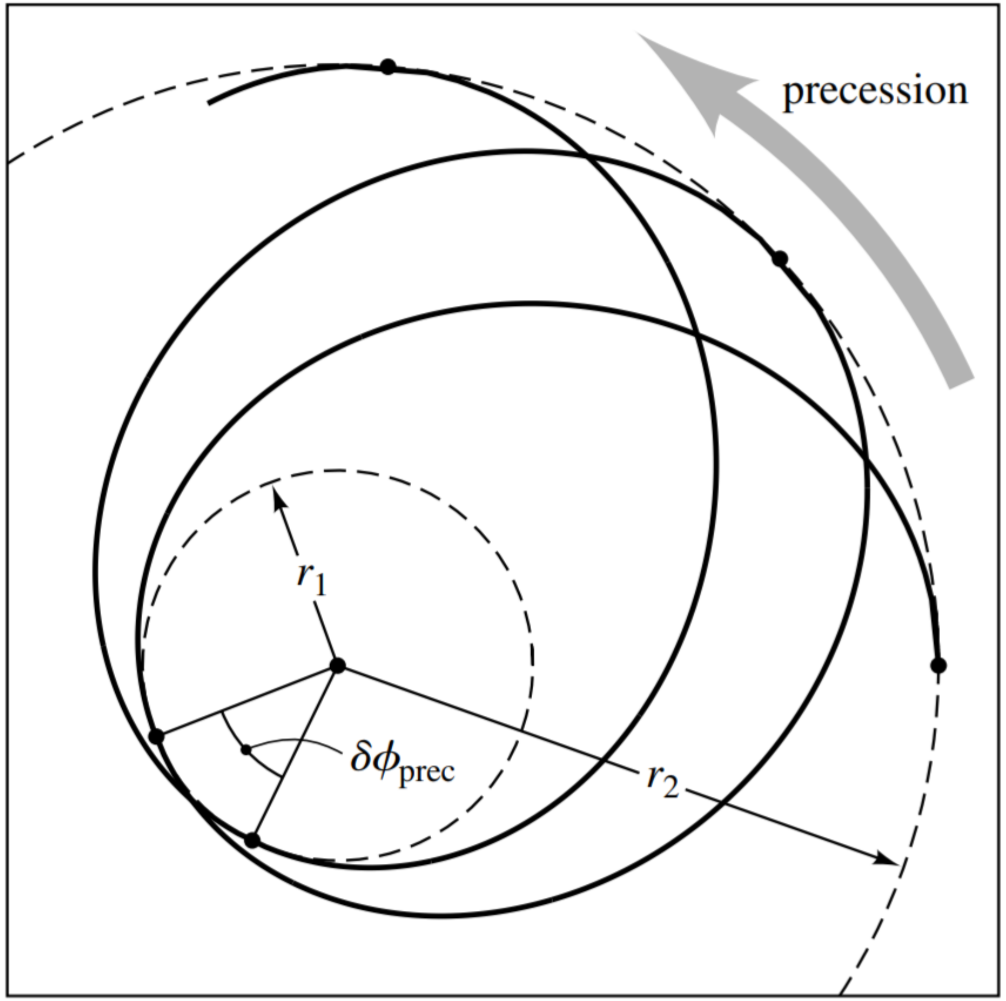
The angle between them is the precession of the perihelion per orbit.







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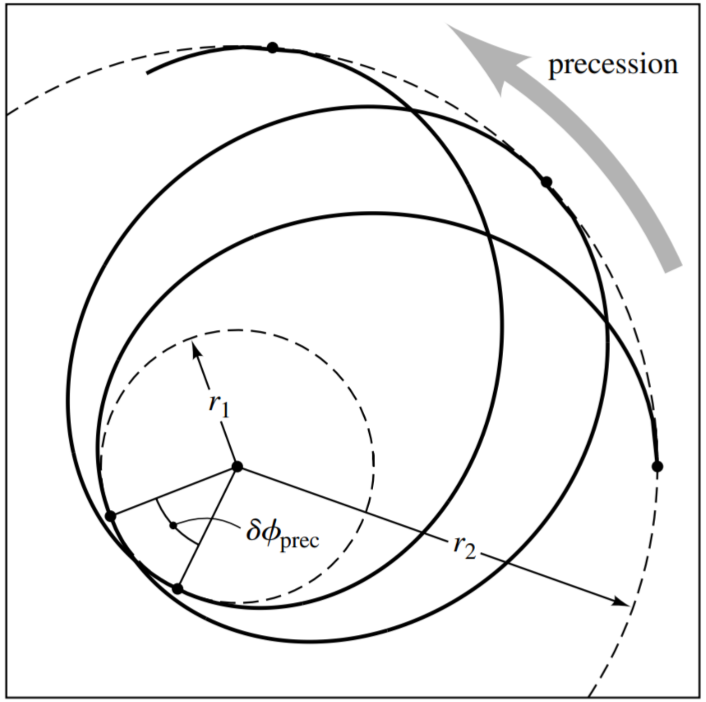
Phenomenology – Perihelion Shift

$$\delta\phi + 2\pi = \int_0^{2\pi+\delta\phi} d\phi = 2\left|\int_{R_-}^{R_+} \frac{d\phi}{dr} dr\right|$$

$$=2\left|\int_{R_-}^{R_+} dr \frac{\frac{1}{r^2}\frac{1}{\sqrt{f(r)g(r)}}\sqrt{f(R_-)-f(R_+)}R_- R_+}{\sqrt{f(R_-)f(R_+)\left(R_-^2-R_+^2\right)+\frac{f(r)}{r^2}\left[f(R_-)R_+^2\left(r^2-R_-^2\right)+f(R_+)R_-^2\left(-r^2+R_+^2\right)\right]}}\right|$$



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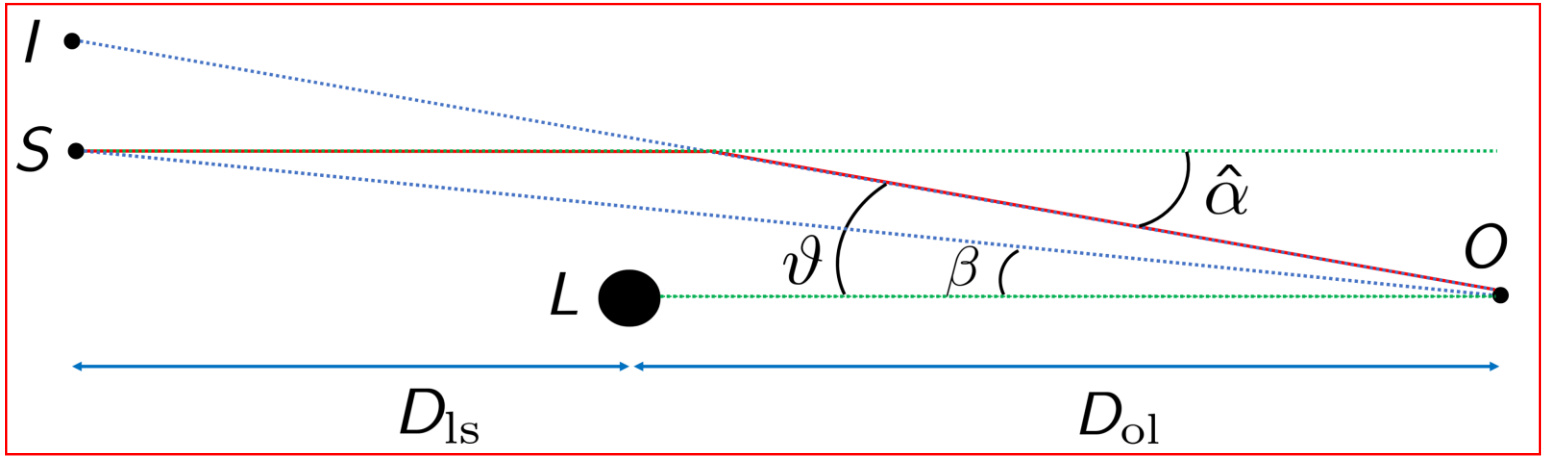
Phenomenology – Lensing

Observer O sees the source at I rather than S, due to the gravitational effects of the source S



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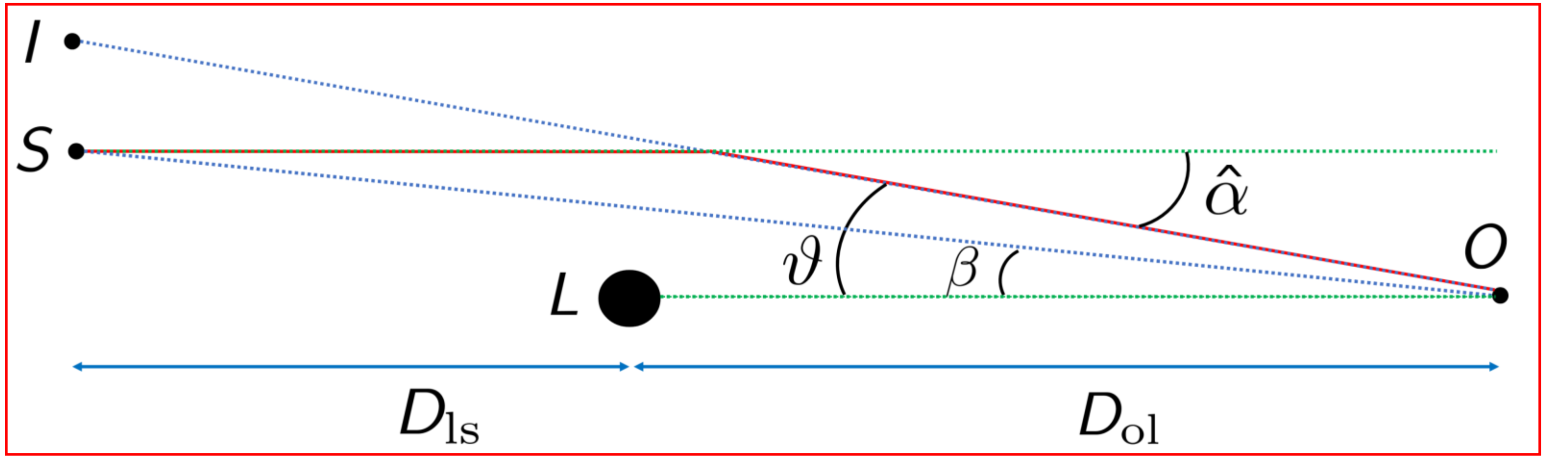

$$\sin(\vartheta) = \frac{1}{D_{\mathrm{ol}}}\sqrt{\frac{h(r_0)}{f(r_0)}}$$

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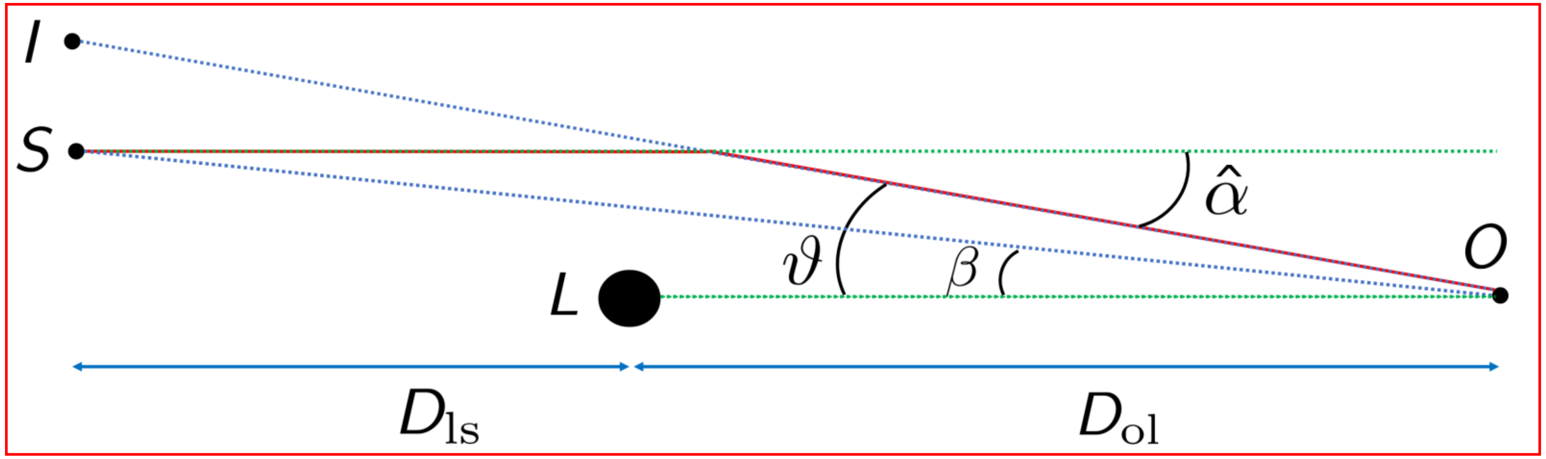

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Radius of closest distance of approach

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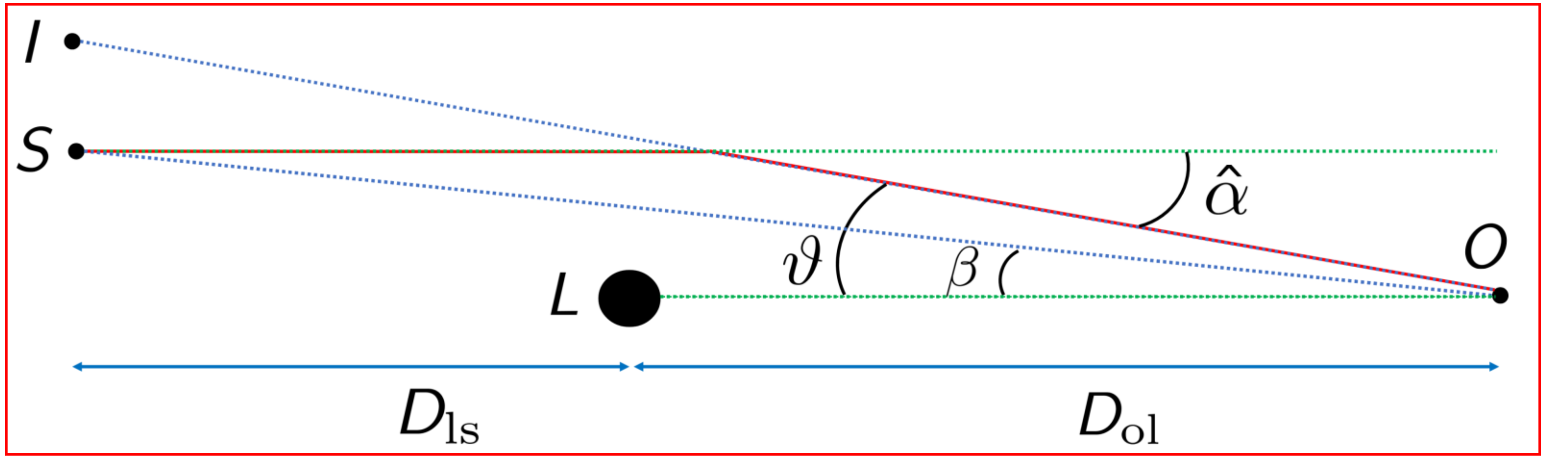

$$\sin(\vartheta) = \frac{1}{D_{\rm ol}}\sqrt{\frac{h(\boxed{r_0})}{f(r_0)}}$$

Radius of closest distance of approach

$$\hat{\alpha}(r_0) = 2\int_{r_0}^{\infty}\left[\frac{h(r)}{g(r)}\left(\frac{h(r)}{h(r_0)}\frac{f(r_0)}{f(r)} - 1\right)\right]^{-\frac{1}{2}} dr - n\pi$$

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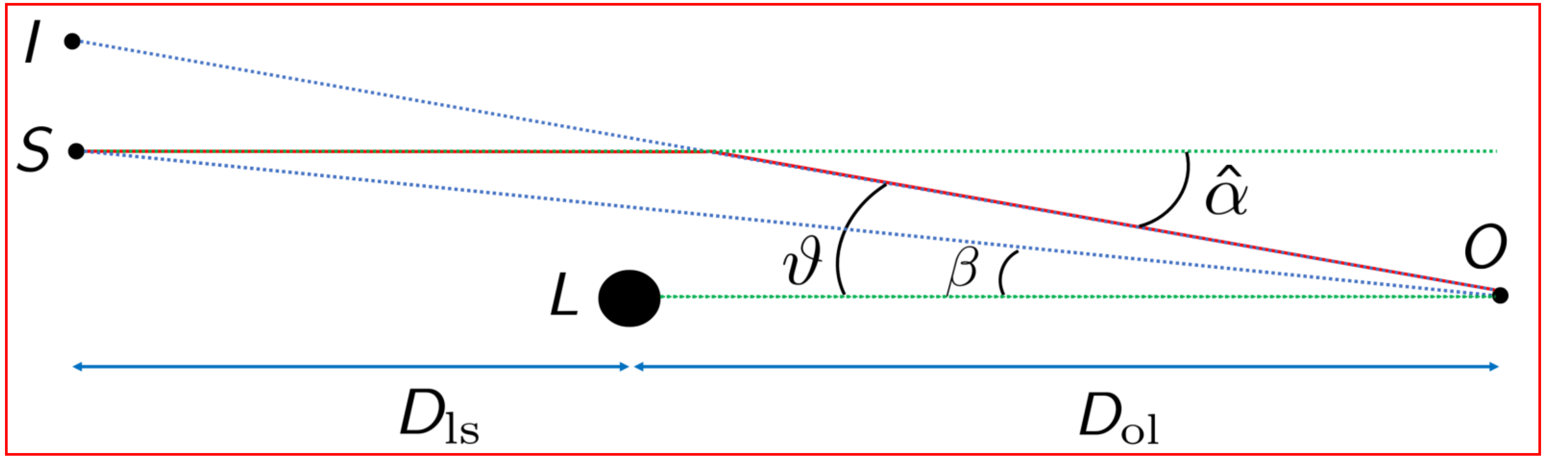

$$\sin(\vartheta) = \frac{1}{D_{\rm ol}}\sqrt{\frac{h(r_0)}{f(r_0)}}$$

controls the number of rotations of rays around the lens

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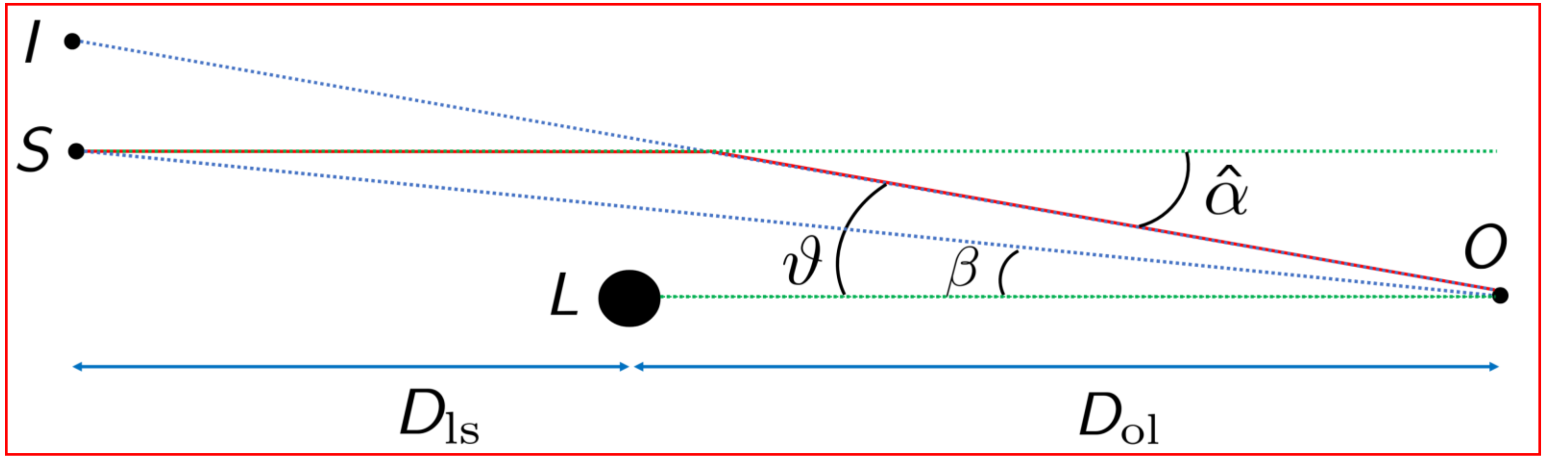

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$$\tan(\beta) = \tan(\vartheta) - \frac{D_{\text{ls}}}{D_{\text{os}}}\left[\tan(\vartheta) + \tan\left(\hat{\alpha} - \vartheta\right)\right]$$

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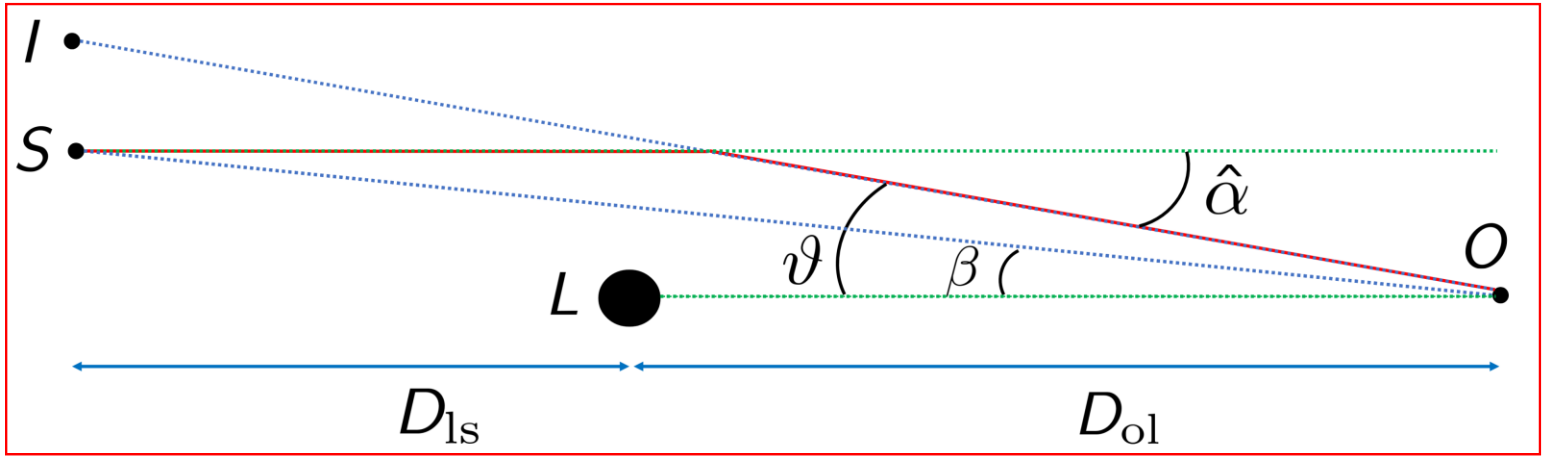
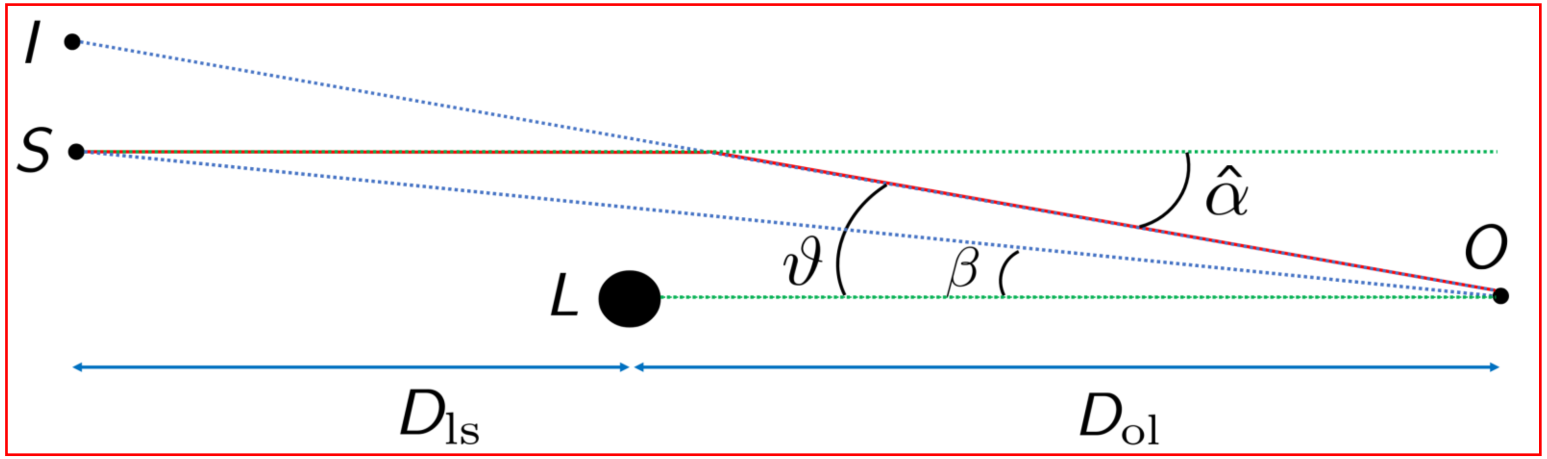


Magnification:

$$\mu = \left(\frac{\sin(\beta)}{\sin(\vartheta)} \frac{d\beta}{d\vartheta} \right)^{-1}$$

Phenomenology – Time Delay

Difference between time-of-travel from the source to the observer for the light rays in a curved spacetime compared to flat spacetime



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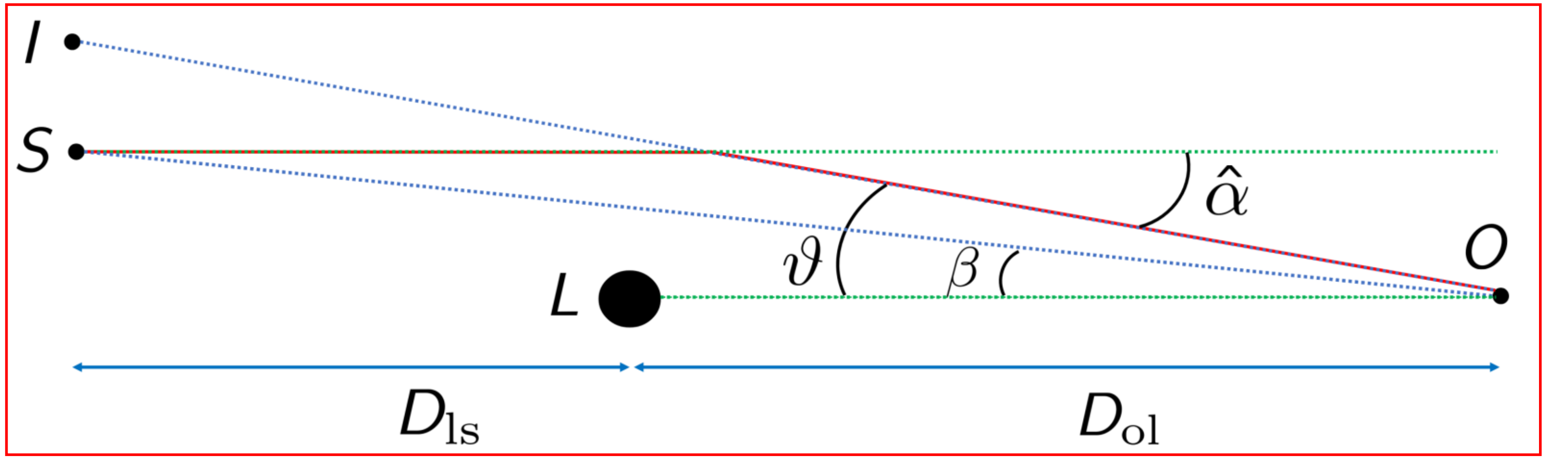

$$\tau = \left[\int_{r_0}^{r_{\rm ls}} dr + \int_{r_0}^{D_{\rm ol}} dr\right]\left[\frac{f(r)}{g(r)}\left(1 - \frac{f(r)}{f(r_0)}\frac{h(r_0)}{h(r)}\right)\right]^{-\frac{1}{2}} - D_{\rm os}\sec(\beta)$$

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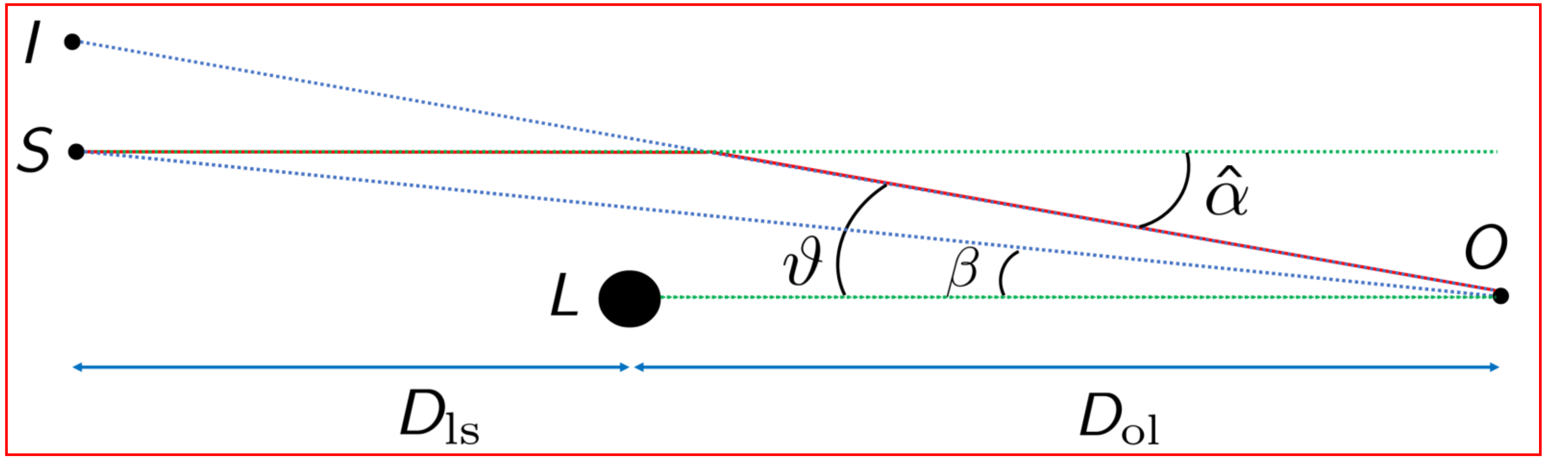

$$r_{\mathrm{ls}} = \sqrt{D_{\mathrm{ls}}^2 + D_{\mathrm{os}}^2 \tan^2(\beta)}$$

$$\tau = \left[\int_{r_0}^{r_{\mathrm{ls}}} dr + \int_{r_0}^{D_{\mathrm{ol}}} dr\right]\left[\frac{f(r)}{g(r)}\left(1 - \frac{f(r)}{f(r_0)}\frac{h(r_0)}{h(r)}\right)\right]^{-\frac{1}{2}} - D_{\mathrm{os}} \sec(\beta)$$

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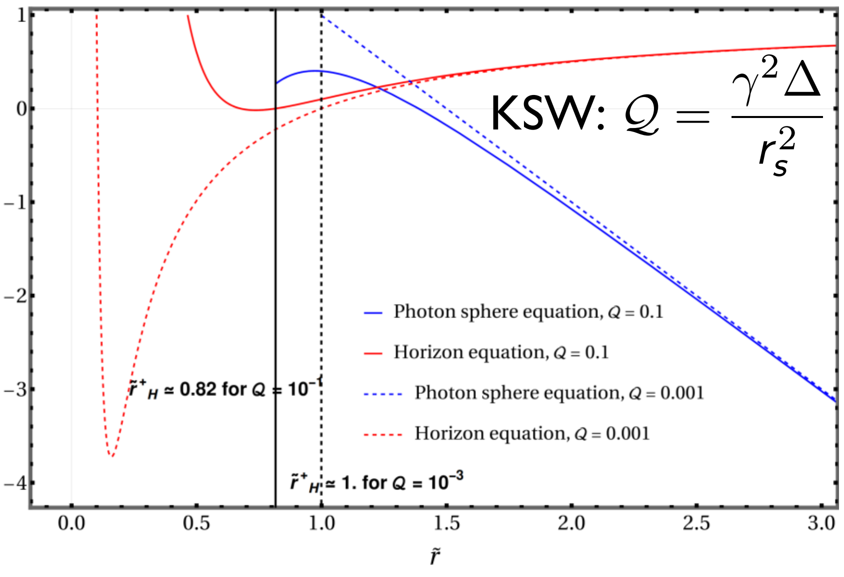
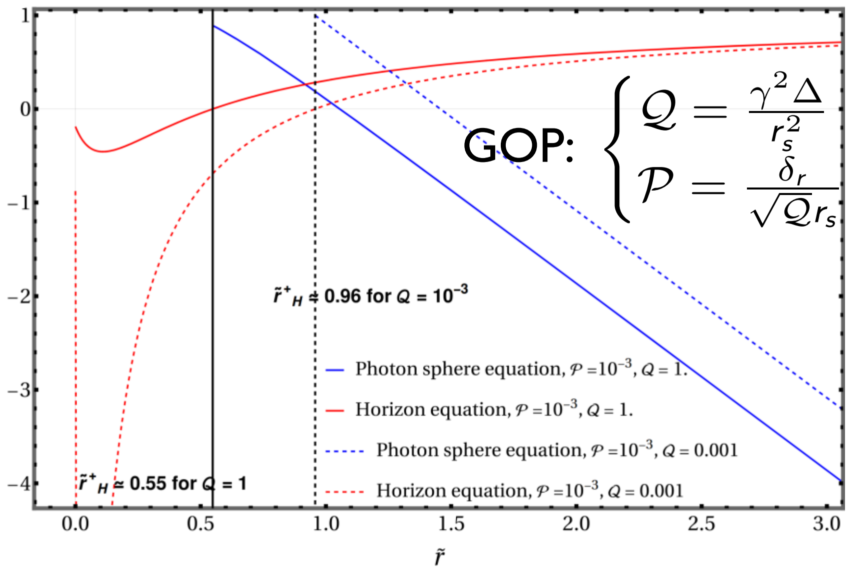
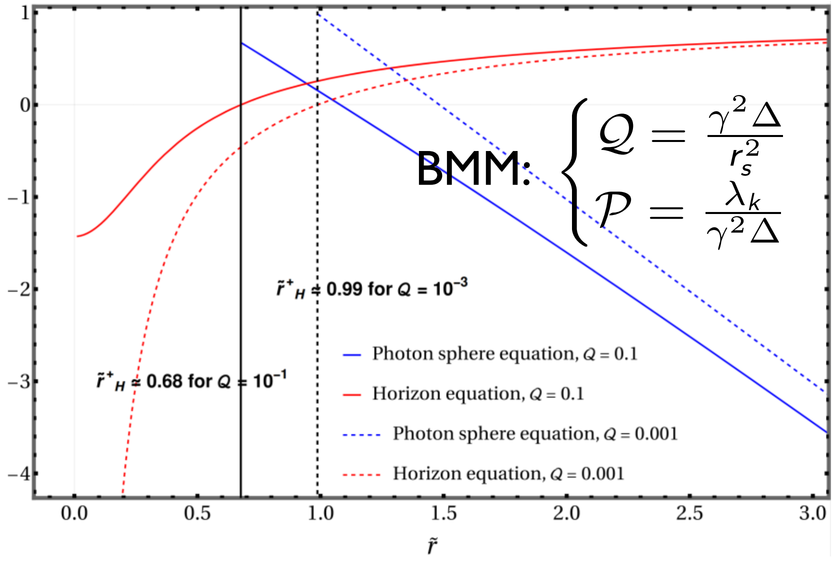
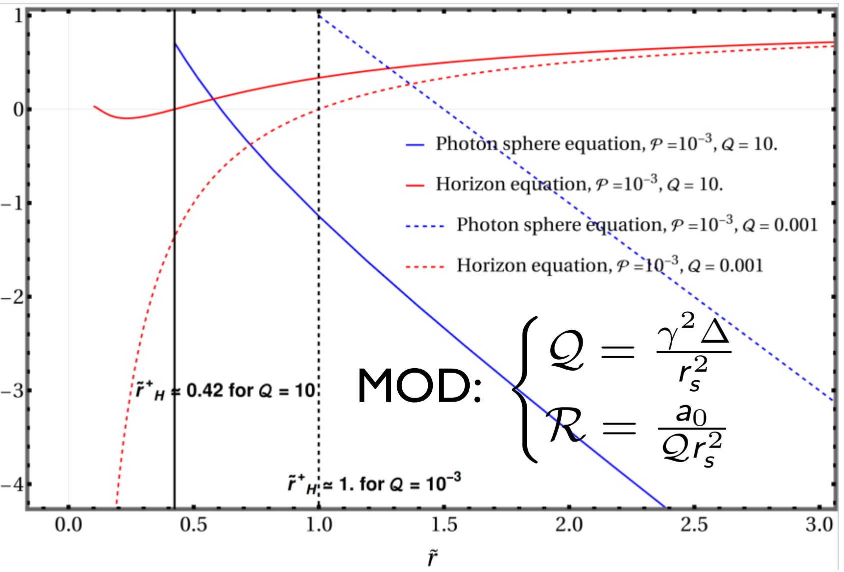
  - BH Shadow
  - Effects of LQG BH on the shadow

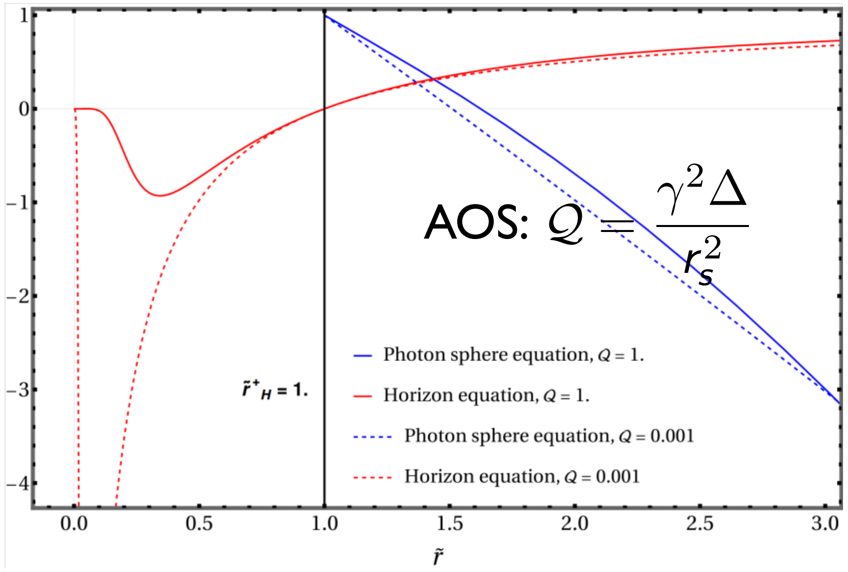
LQG Phenomenology - Photon Sphere











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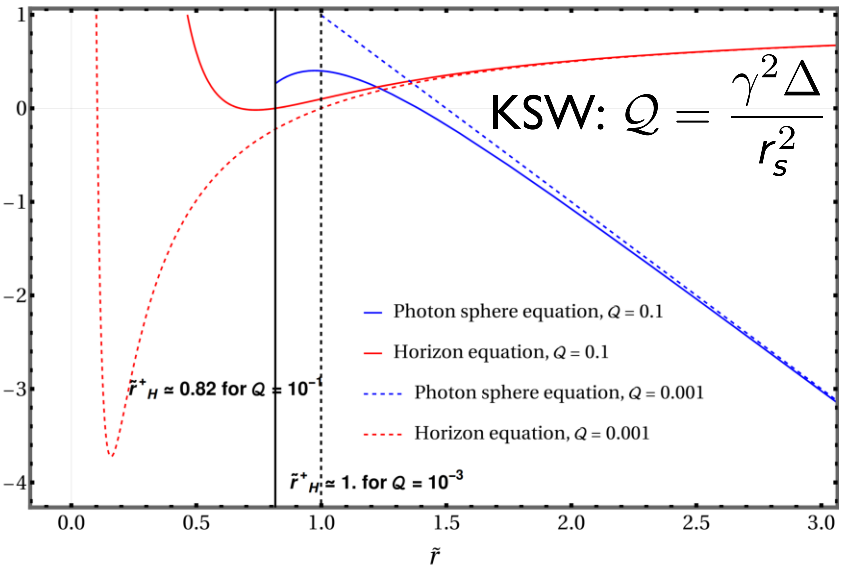

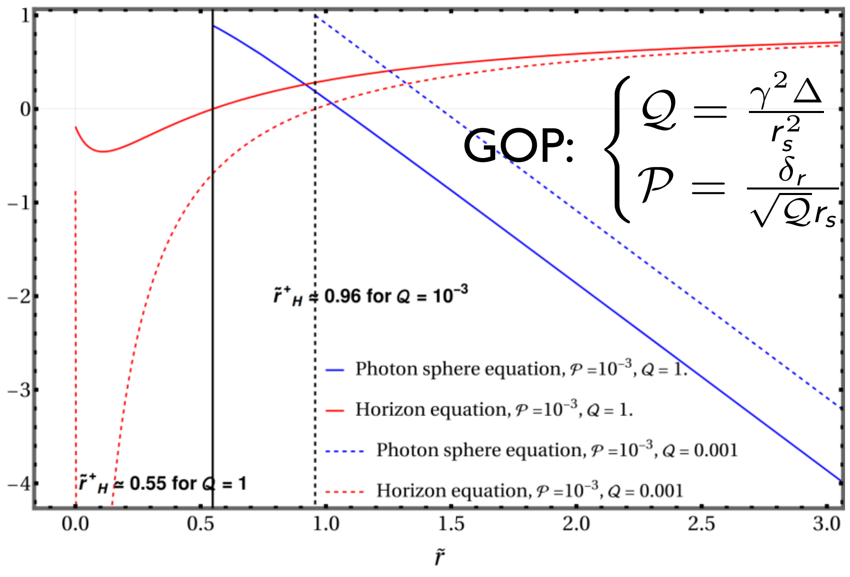

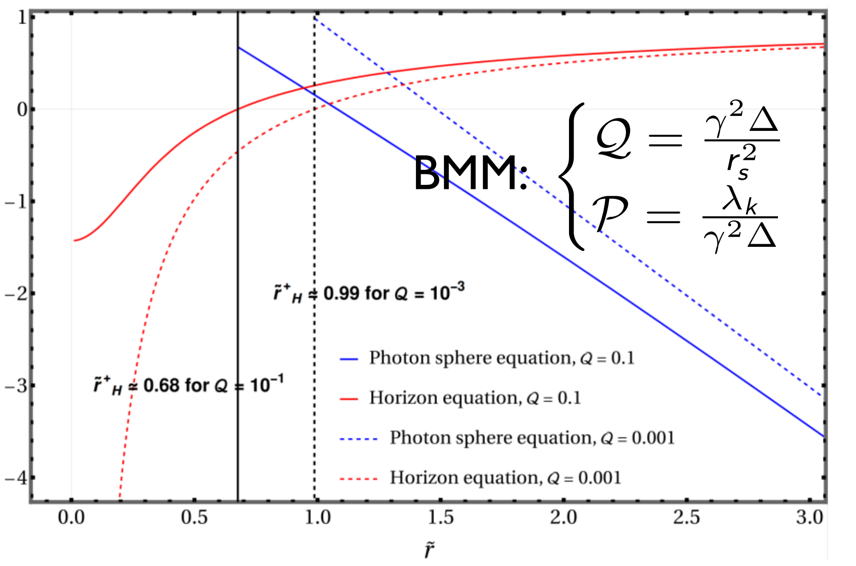

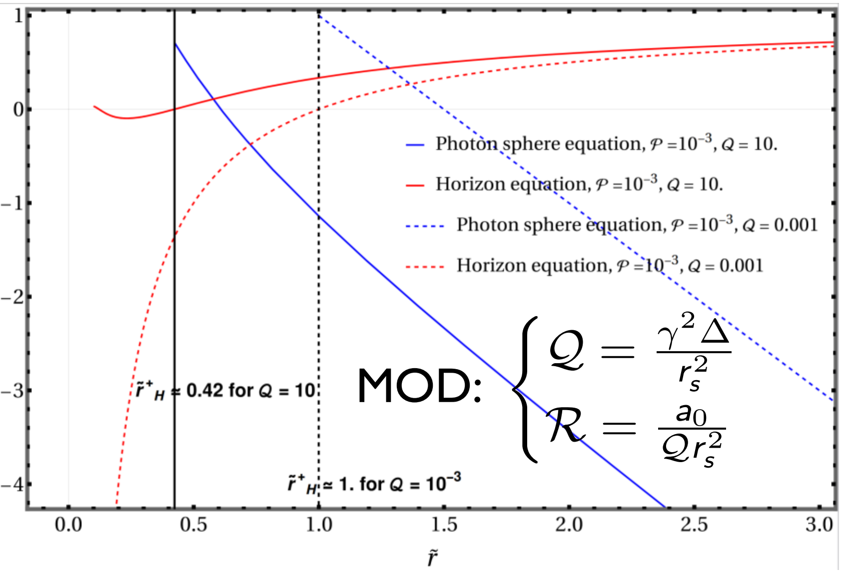

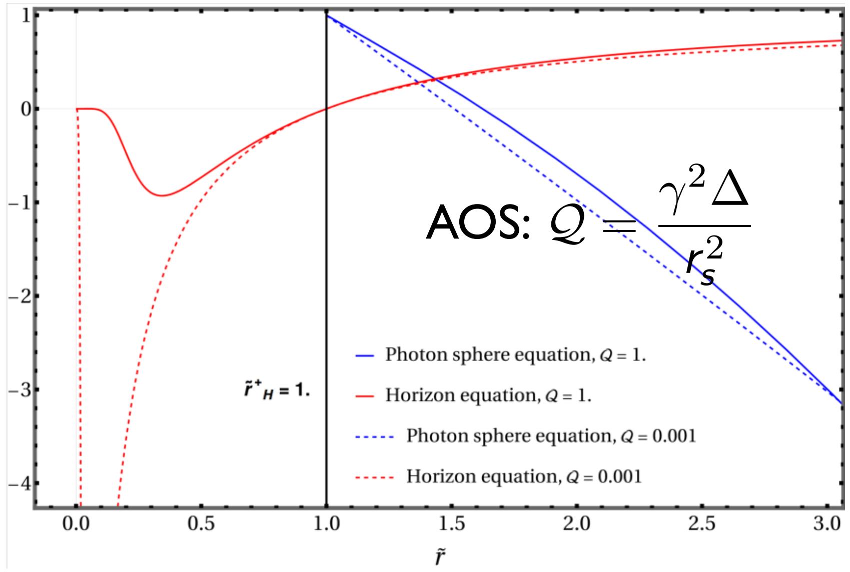

All have only one photon sphere.

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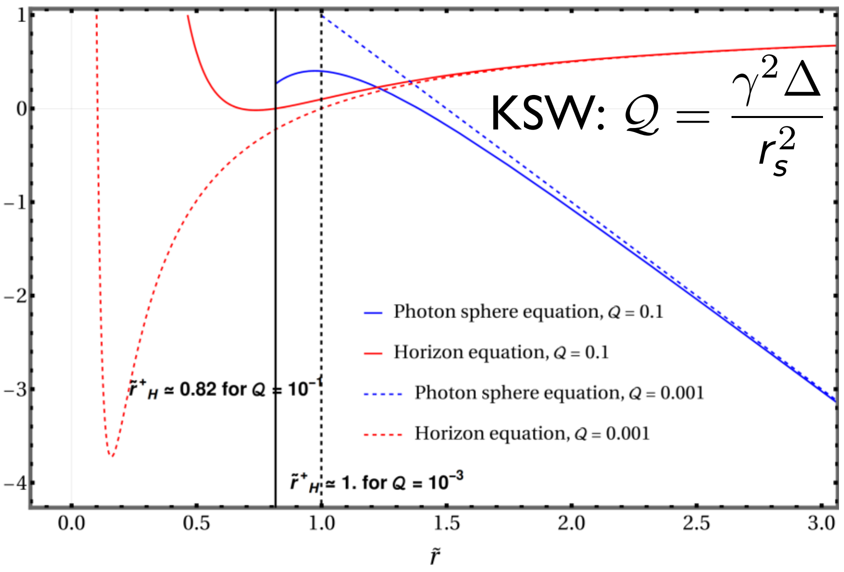
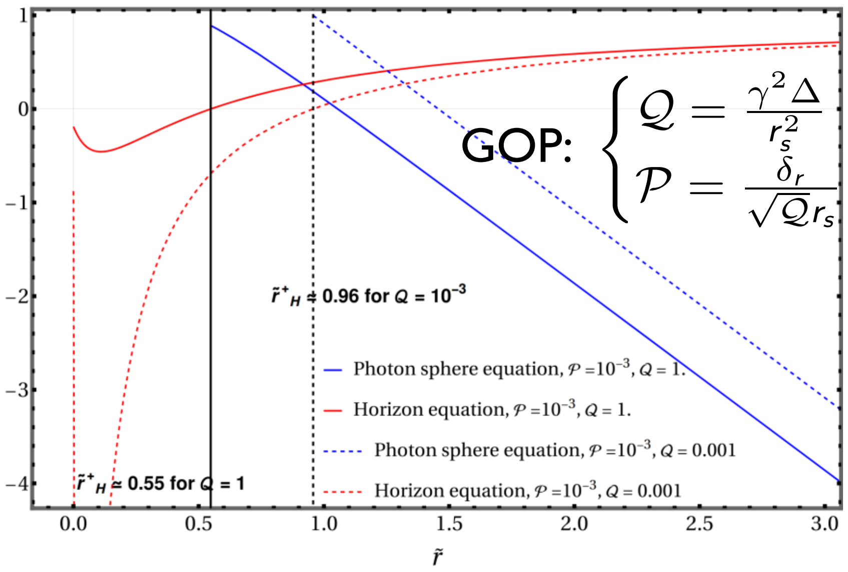
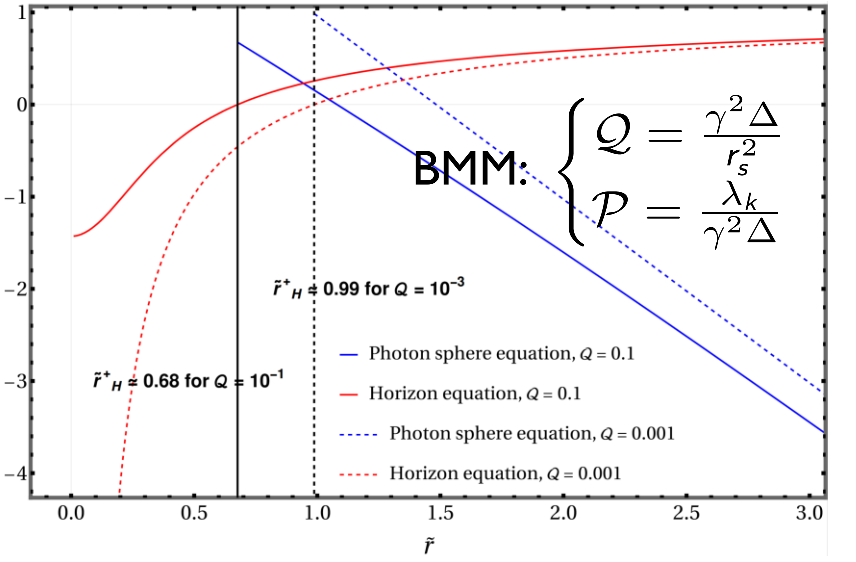
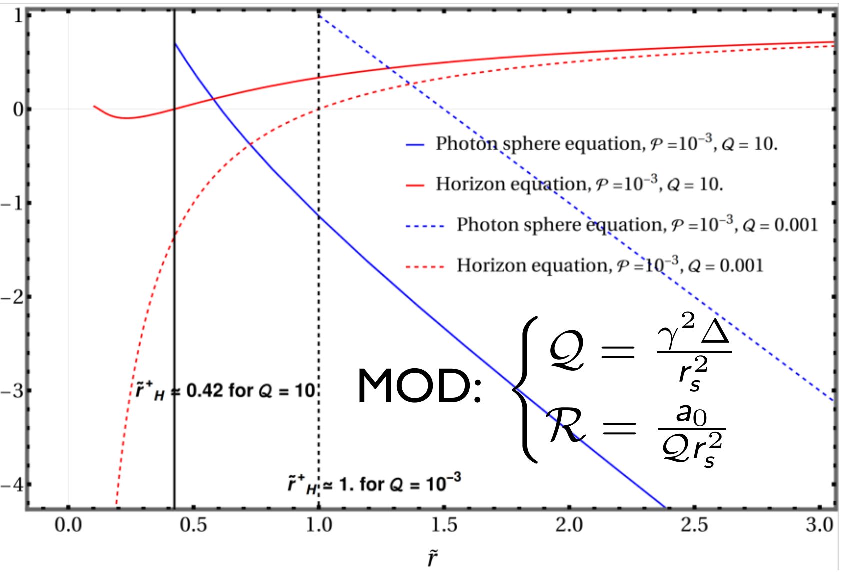
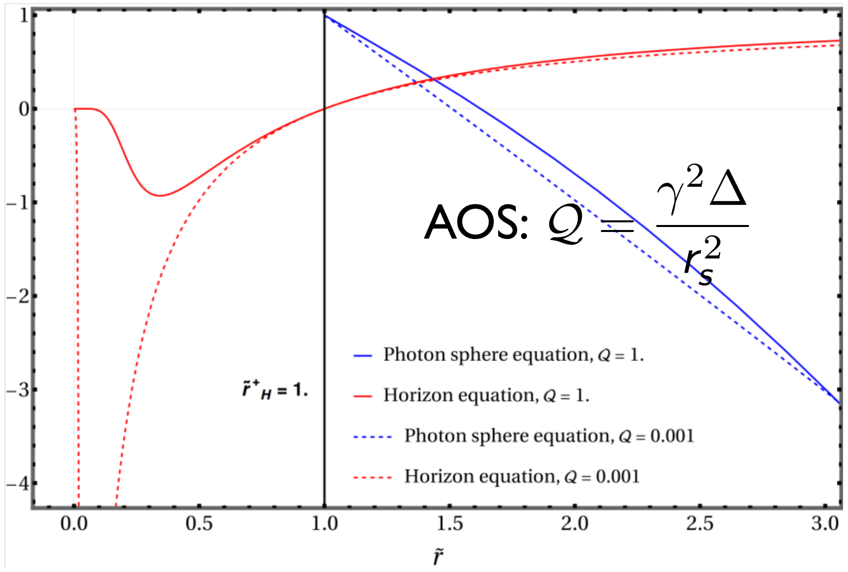










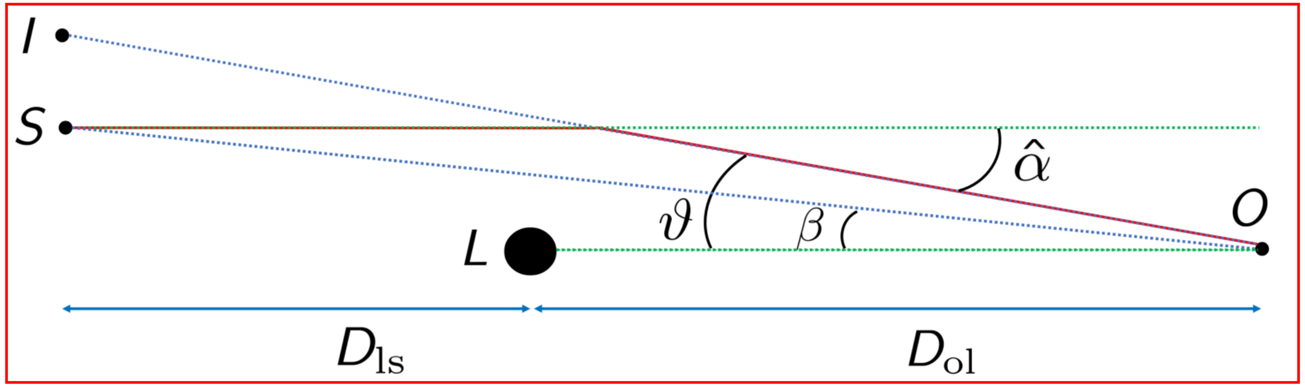
All have only one photon sphere.

Different r_{ps}

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
$$D_{\mathrm{ol}} = 7.86 \pm 0.14\,\mathrm{kpc}$$
$$D_{\mathrm{ls}}/D_{\mathrm{os}} = 0.0005$$
$$\beta = 0$$



Note: EHT resolution $\approx 20\mu$ arcsec

For KSW:

$\mathcal{Q}$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.052532147129347	26.266073564674	2258.8482
10^{-4}	0.052530623721600	26.265311860801	2258.8355
10^{-3}	0.052516895696650	26.258447848326	2258.7218
10^{-2}	0.052377857975272	26.188928987636	2257.5739

LQG Phenomenology – Light Deflection

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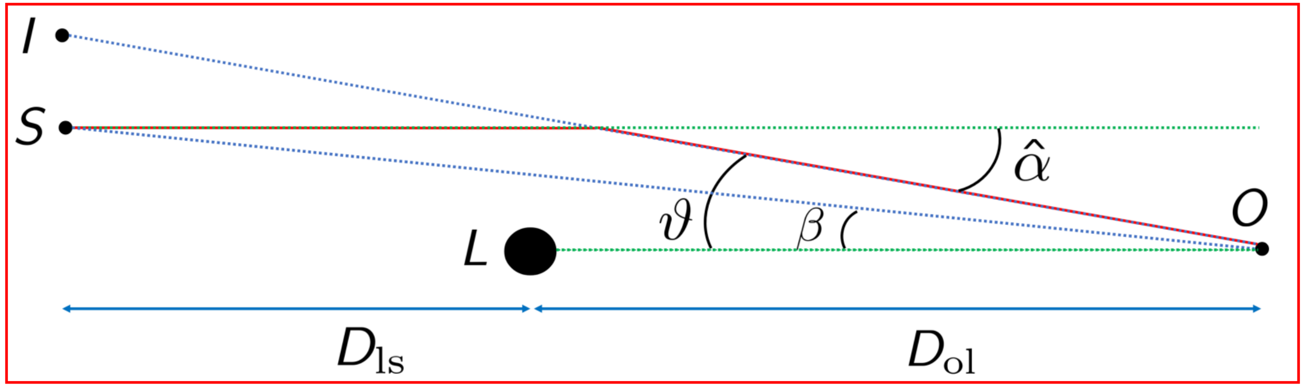

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LQG Phenomenology – Light Deflection

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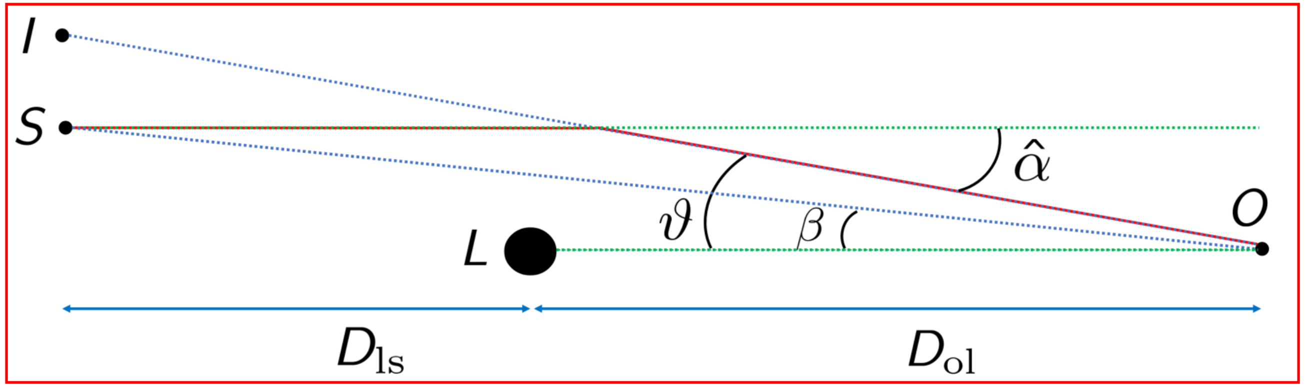
\end{aligned}
$$



Note: EHT resolution $\approx 20\mu$ arcsec

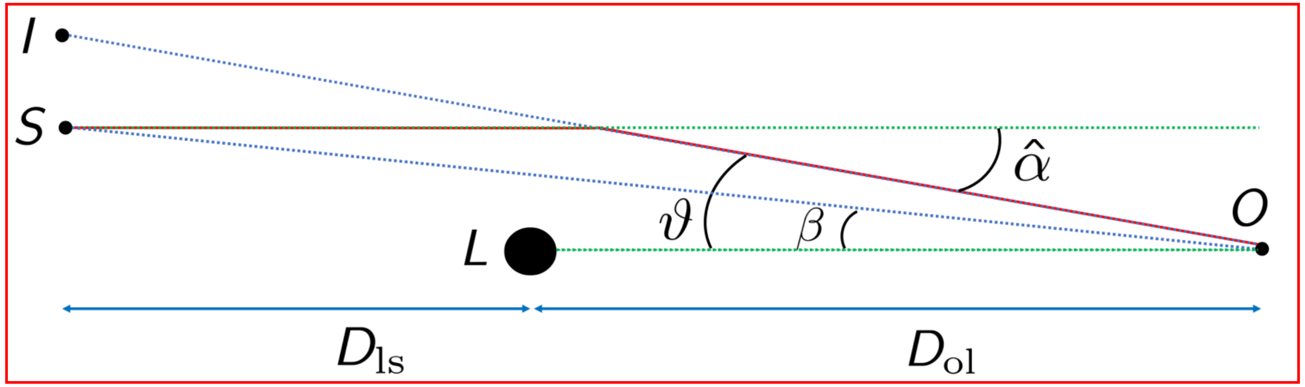
For GOP:

$\mathcal{Q}(\mathcal{P} = 10^{-3})$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.052532147129347	26.266073564674	2258.8482
10^{-4}	0.052532129474428	26.266064737214	2260.4371
10^{-3}	0.052531960195466	26.265980097733	2262.2781
10^{-2}	0.052530244543314	26.265122271658	2266.2513

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

$$
\begin{aligned}
M &= 4.02 \times 10^6 M_\odot \\
D_{\mathrm{ol}} &= 7.86 \pm 0.14\,\text{kpc} \\
D_{\mathrm{ls}}/D_{\mathrm{os}} &= 0.0005 \\
\beta &= 0
\end{aligned}
$$



Note: EHT resolution $\approx 20\mu$ arcsec

For GOP:

$\mathcal{Q}(\mathcal{P}=10^{-3})$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.052532147129347	26.266073564674	2258.8482
10^{-4}	0.052532129474428	26.266064737214	2260.4371
10^{-3}	0.052531960195466	26.265980097733	2262.2781
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LQG Phenomenology – Light Deflection

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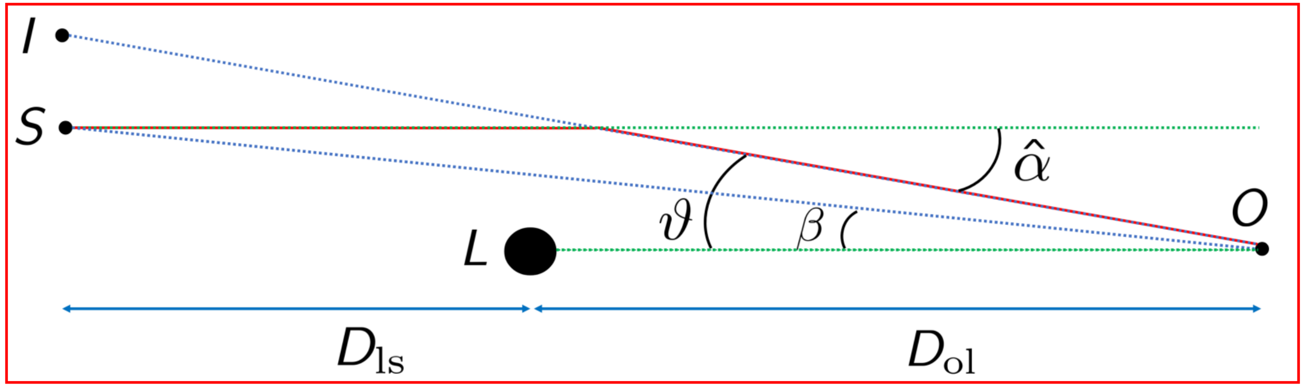

Note: EHT resolution $\approx 20\mu$ arcsec

For MOD:

$\mathcal{Q}(\mathcal{P}=1)$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.052532147129347	26.266073564674	2258.8482
10^{-4}	0.052528650489760	26.264325244881	2258.7274
10^{-3}	0.052497220593418	26.248610296710	2257.6414
10^{-2}	0.052186852210251	26.093426105126	2246.8951

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
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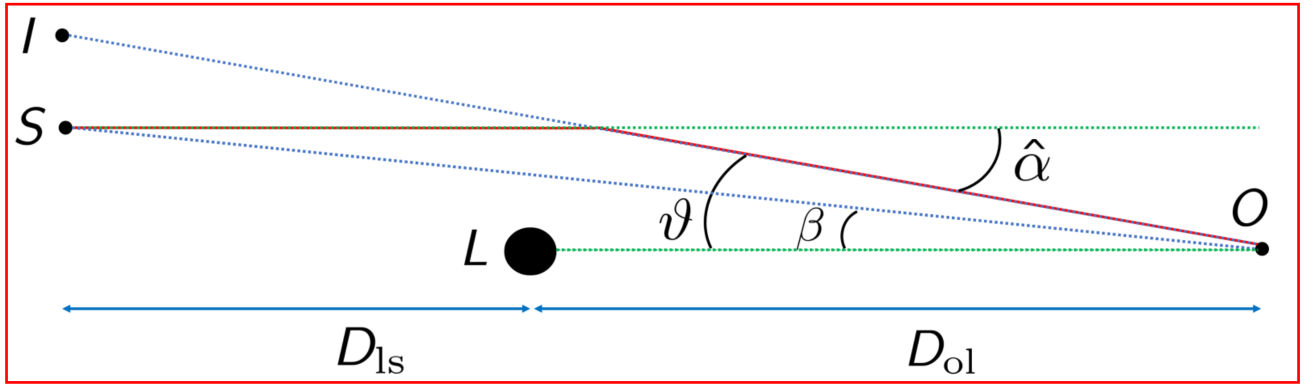

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$\mathcal{Q}(\mathcal{P}=1)$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
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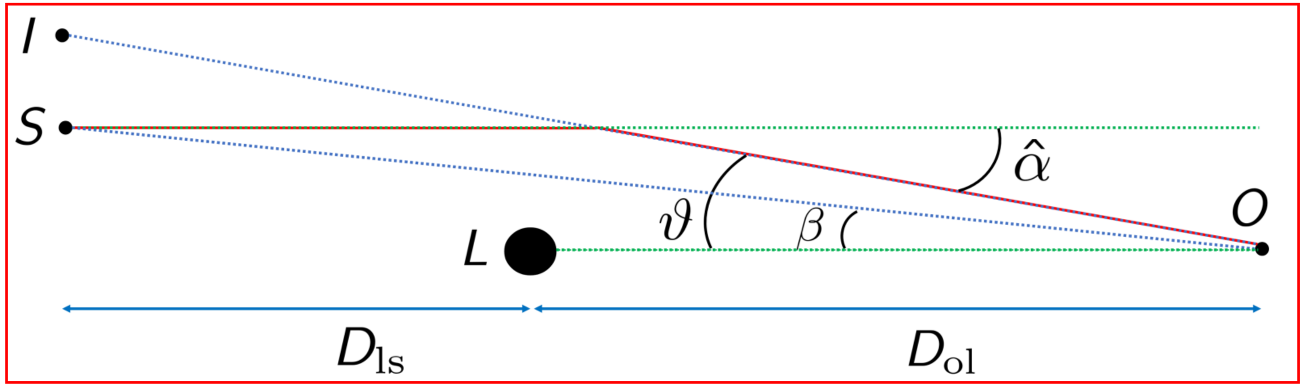


Note: EHT resolution $\approx 20\mu$ arcsec

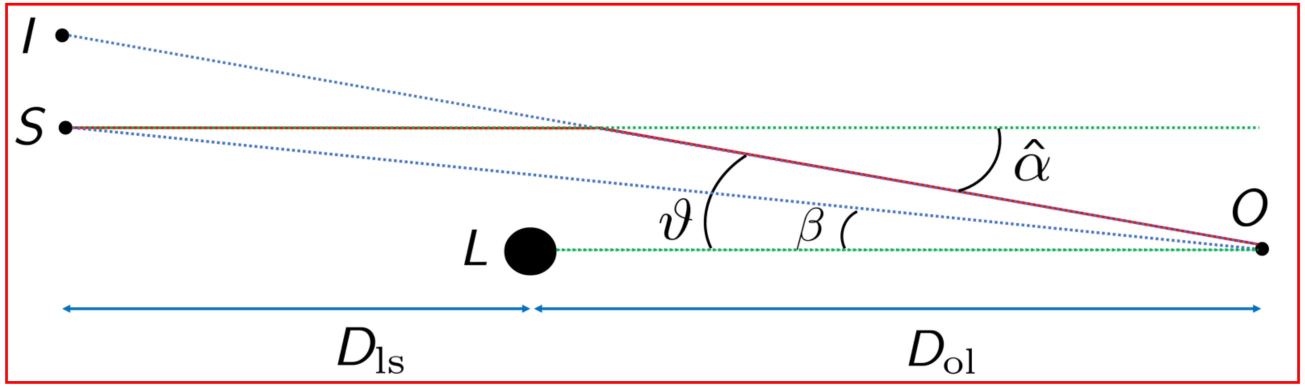
For AOS:

Q	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.05253214712934 7	26.266073564674	2258.8482
10^{-4}	0.05238479741774 0	26.192398708871	NTBC
10^{-3}	0.05221437107666 1	26.107185538331	NTBC
10^{-2}	0.05184623279377 8	25.923116396889	NTBC

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
$$D_{\rm ol} = 7.86 \pm 0.14\,\text{kpc}$$
$$D_{\rm ls}/D_{\rm os} = 0.0005$$
$$\beta = 0$$



Note: EHT resolution $\approx 20\mu$ arcsec

For AOS:

Q	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.05253214712934	26.26607356467	2258.8482
10^{-4}	0.05238479741774	26.19239870887	NTBC
10^{-3}	0.05221437107666	26.10718553833	NTBC
10^{-2}	0.05184623279377	25.92311639688	NTBC

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

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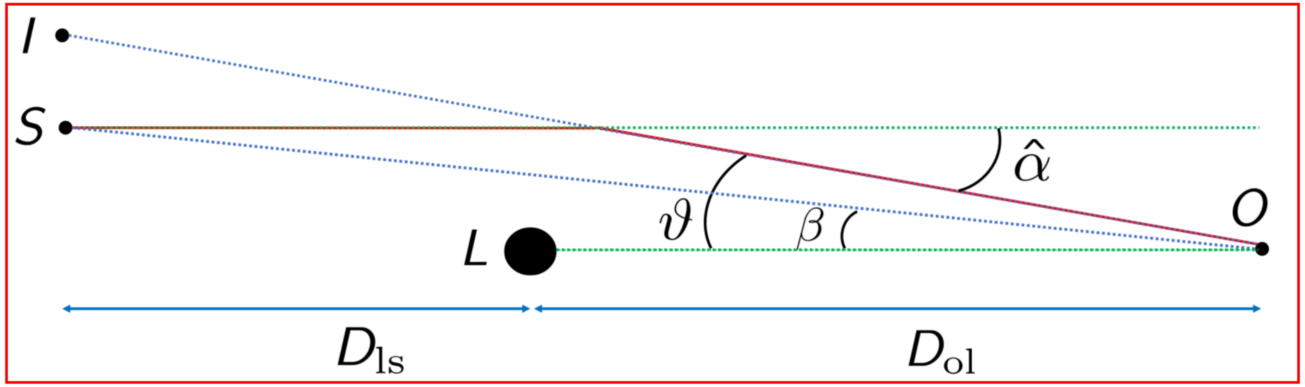

Note: EHT resolution $\approx 20\mu$ arcsec

For BMM:

$\mathcal{Q}$	$\hat{\alpha}_{1E}$ (arcsec)	ϑ_{1E} (μ arcsec)	τ_{1E} (sec)
0	0.052532147129347	26.266073564674	2258.8482
10^{-4}	0.052437291042223	26.218645521112	2257.7148
10^{-3}	0.052089916722053	26.044958361027	2253.5669
10^{-2}	0.050435523476276	25.217761738136	2233.8687

LQG Phenomenology – Light Deflection

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
$$D_{\text{ol}} = 7.86 \pm 0.14\,\text{kpc}$$
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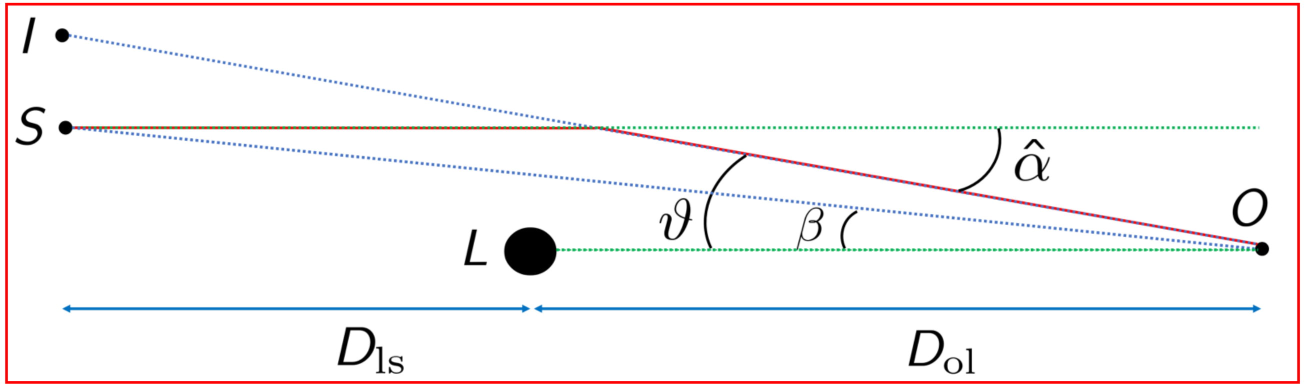
$$\beta = 0$$



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LQG Phenomenology – Time Delay

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
$$D_{\rm ol} = 7.86 \pm 0.14\,\text{kpc}$$
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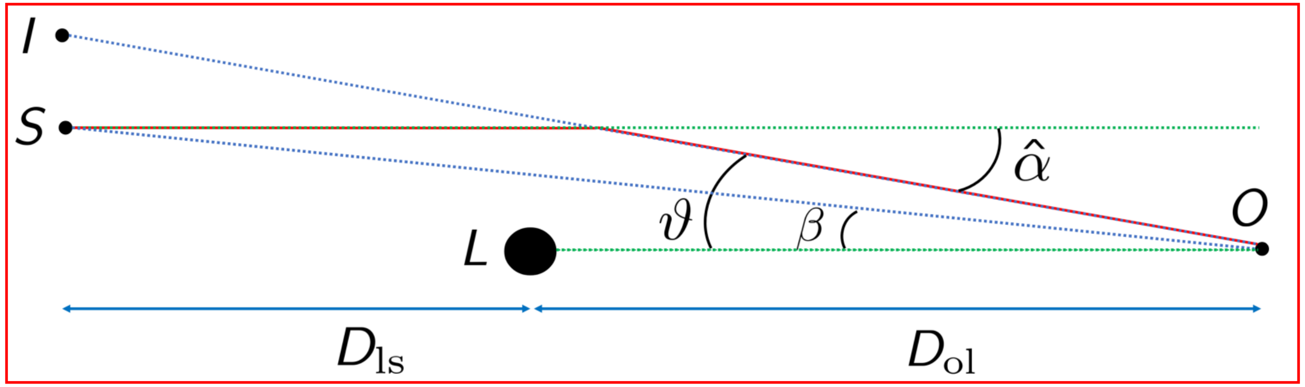


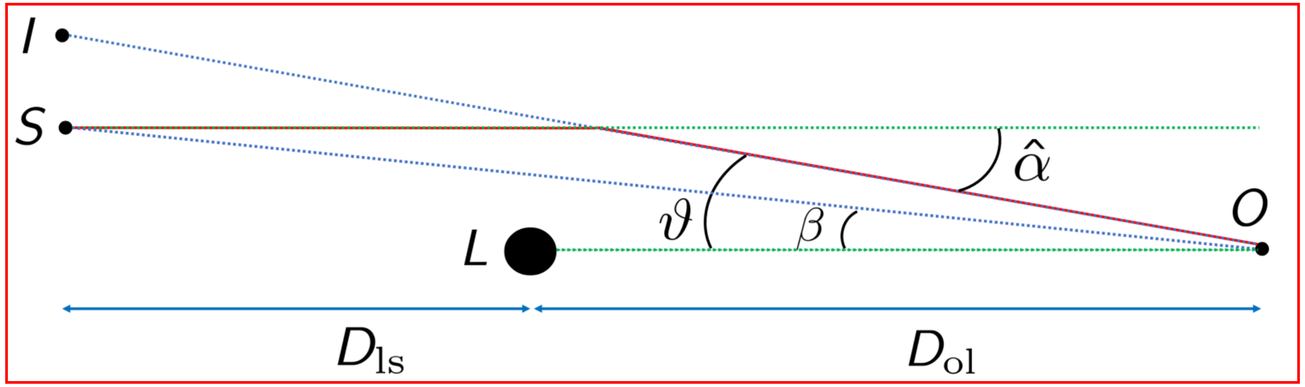
For KSW:

$\mathcal{Q}$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
0	8.4229548234213	-8.4229546587852	2258.8481752017	1.9978 $\times 10^{-6}$
10^{-4}	8.4259813166652	-8.4259811519841	2258.8355522852	1.9978 $\times 10^{-6}$
10^{-3}	8.4533306193543	-8.4533304542658	2258.7218385704	1.9973 $\times 10^{-6}$
10^{-2}	8.7382737779299	-8.7382736086276	2257.5738659314	1.9919 $\times 10^{-6}$

LQG Phenomenology – Time Delay

For Galactic black hole Sgr A* with

$$
\begin{aligned}
M &= 4.02 \times 10^6 M_\odot \\
D_{\rm ol} &= 7.86 \pm 0.14 \,\text{kpc} \\
D_{\rm ls}/D_{\rm os} &= 0.0005 \\
\beta &= 10^{-6}
\end{aligned}
$$



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$\mathcal{Q}$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
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10^{-2}	8.7382737779299	-8.7382736086276	2257.5738659314	1.9919 $\times 10^{-6}$

LQG Phenomenology – Time Delay

For Galactic black hole Sgr A* with

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$$D_{\mathrm{ol}} = 7.86 \pm 0.14\,\mathrm{kpc}$$
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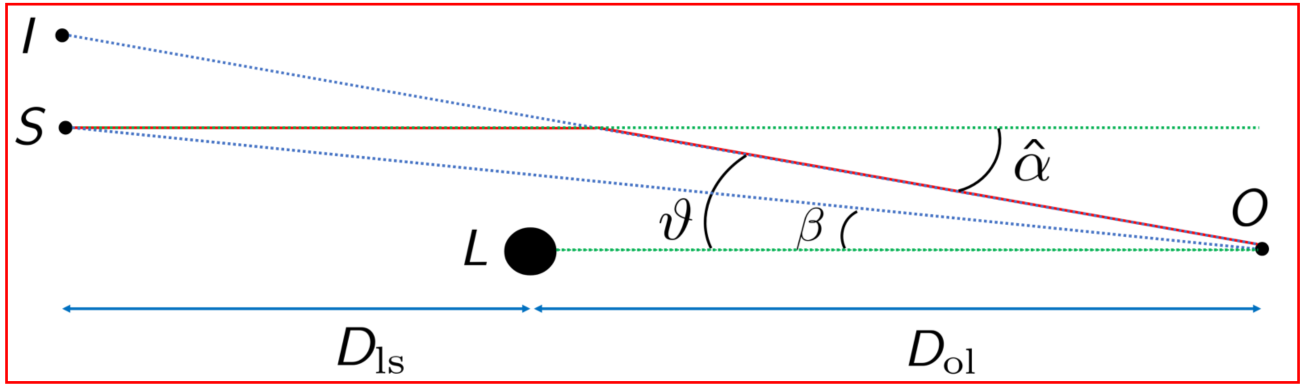

For GOP:

$\mathcal{Q}$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
0	8.4229548234213	-8.4229546587852	2258.8481752017	1.9978×10^{-6}
10^{-4}	8.4231822397741	-8.4231820751344	2260.4370973397	1.9978×10^{-6}
10^{-3}	8.4240892262330	-8.4240890615794	2262.2781038585	1.9978×10^{-6}
10^{-2}	8.4306992344965	-8.4306990697438	2266.2513367918	1.9978×10^{-6}

LQG Phenomenology – Time Delay

For Galactic black hole Sgr A* with

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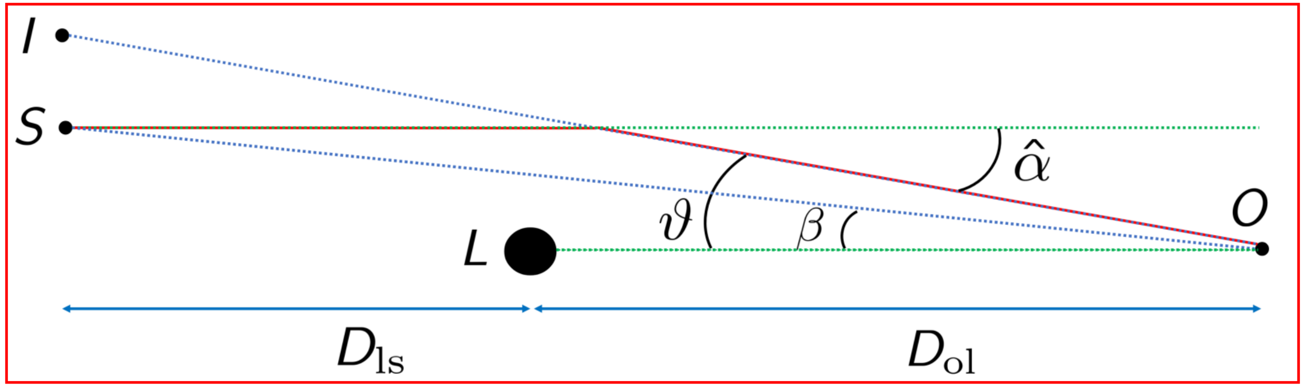

For MOD:

$\mathcal{Q}(\mathcal{P}=1)$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
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10^{-4}	8.4223874235739	-8.4223872589507	2258.7273833433	1.9977×10^{-6}
10^{-3}	8.4172813913273	-8.4172812268196	2257.6414113330	1.9965×10^{-6}
10^{-2}	8.3662633514398	-8.4229489847231	2246.8951020461	1.9847×10^{-6}

LQG Phenomenology – Time Delay

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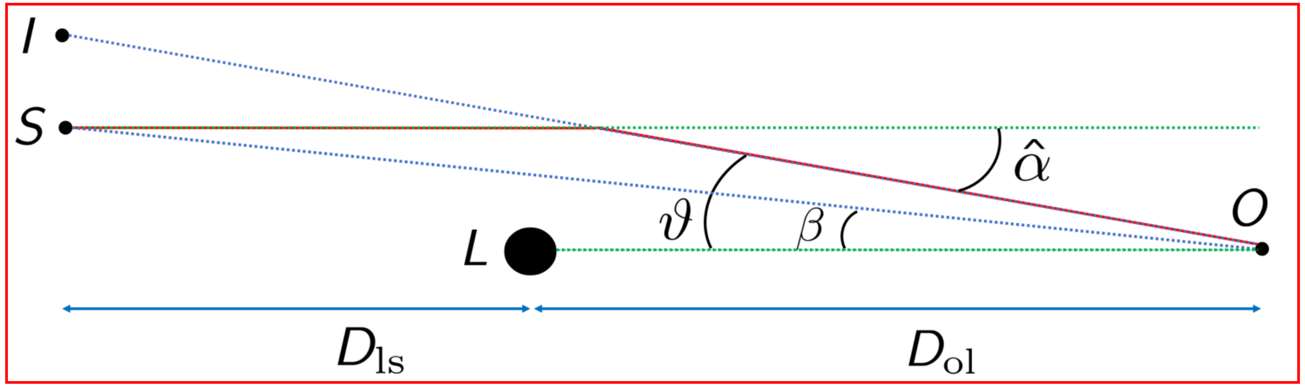

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$\mathcal{Q}$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
0	8.4229548234213	-8.4229546587852	2258.8481752017	1.9978×10^{-6}
10^{-4}	8.4285496713328	-8.4285495066820	2257.7148178629	1.9942×10^{-6}
10^{-3}	8.4491882273035	-8.4491880625983	2253.5669027987	1.9810×10^{-6}
10^{-2}	8.5507897131404	-8.5507895481661	2233.8686874139	1.9181×10^{-6}

LQG Phenomenology – Time Delay

For Galactic black hole Sgr A* with

$$M = 4.02 \times 10^6 M_\odot$$
$$D_{\rm ol} = 7.86 \pm 0.14\,\text{kpc}$$
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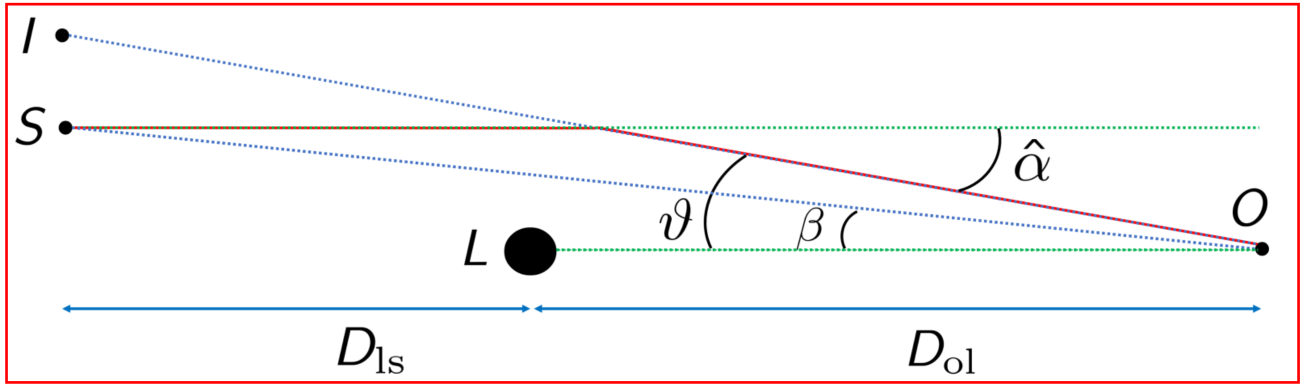

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$\mathcal{Q}$	$\tilde{\mu}_{1p}(\times 10^{-9})$	$\tilde{\mu}_{1s}(\times 10^{-9})$	$\tilde{\tau}_{1s}$(sec)	$d\tilde{\tau} = \tilde{\tau}_{1s} - \tilde{\tau}_{1p}$(sec)
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10^{-4}	8.4285496713328	-8.4285495066820	2257.7148178629	1.9942 $\times 10^{-6}$
10^{-3}	8.4491882273035	-8.4491880625983	2253.5669027987	1.9810 $\times 10^{-6}$
10^{-2}	8.5507897131404	-8.5507895481661	2233.8686874139	1.9181 $\times 10^{-6}$

Outline

- Spherically symmetric BHs in LQG
  - Interesting phenomena in the above BH
  - Effects of LQG on the above phenomena in BHs
- Rotating BH in LQG
  - BH Shadow and LQG constraints

Rotating LQG BH

One can construct a rotating metric from a static one (no guarantee that it is the solution of the modified theory!) using Janis-Newman formalism:

$$ds^2 = -F(r)dt^2 - 2a\sin^2(\theta)\left(\sqrt{\frac{F(r)}{G(r)}} - F(r)\right)dtd\phi + \frac{H(r)}{\Delta(r)}dr^2$$

$$+ H(r)d\theta^2 + \sin^2(\theta)\left[H(r) + a^2\sin^2(\theta)\left(2\sqrt{\frac{F(r)}{G(r)}} - F(r)\right)\right]d\phi^2$$

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Angular momentum of rotating BH

Rotating LQG BH

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$$\boxed{F(r) = \frac{\frac{h(r)}{g(r)} + a^2\cos^2(\theta)}{\left(\frac{h(r)}{\sqrt{g(r)f(r)}} + a^2\cos^2(\theta)\right)^2}H(r)}$$

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$$+ H(r)d\theta^2 + \sin^2(\theta)\left[H(r) + a^2\sin^2(\theta)\left(2\sqrt{\frac{F(r)}{G(r)}} - F(r)\right)\right]d\phi^2$$

$$F(r) = \frac{\frac{h(r)}{g(r)} + a^2\cos^2(\theta)}{\left(\frac{h(r)}{\sqrt{g(r)f(r)}} + a^2\cos^2(\theta)\right)^2}\boxed{H(r)}$$

Free; not needed for shadows

Rotating LQG BH

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$$ds^2 = -F(r)dt^2 - 2a\sin^2(\theta)\left(\sqrt{\frac{F(r)}{\boxed{G(r)}}} - F(r)\right)dtd\phi + \frac{H(r)}{\Delta(r)}dr^2$$

$$+H(r)d\theta^2 + \sin^2(\theta)\left[H(r) + a^2\sin^2(\theta)\left(2\sqrt{\frac{F(r)}{G(r)}} - F(r)\right)\right]d\phi^2$$

$$\boxed{G(r) = \frac{\frac{h(r)}{g(r)} + a^2\cos^2(\theta)}{H(r)}}$$

Outline

- Spherically symmetric BHs in LQG
  - Interesting phenomena in the above BH
  - Effects of LQG on the above phenomena in BHs
- Rotating BH in LQG
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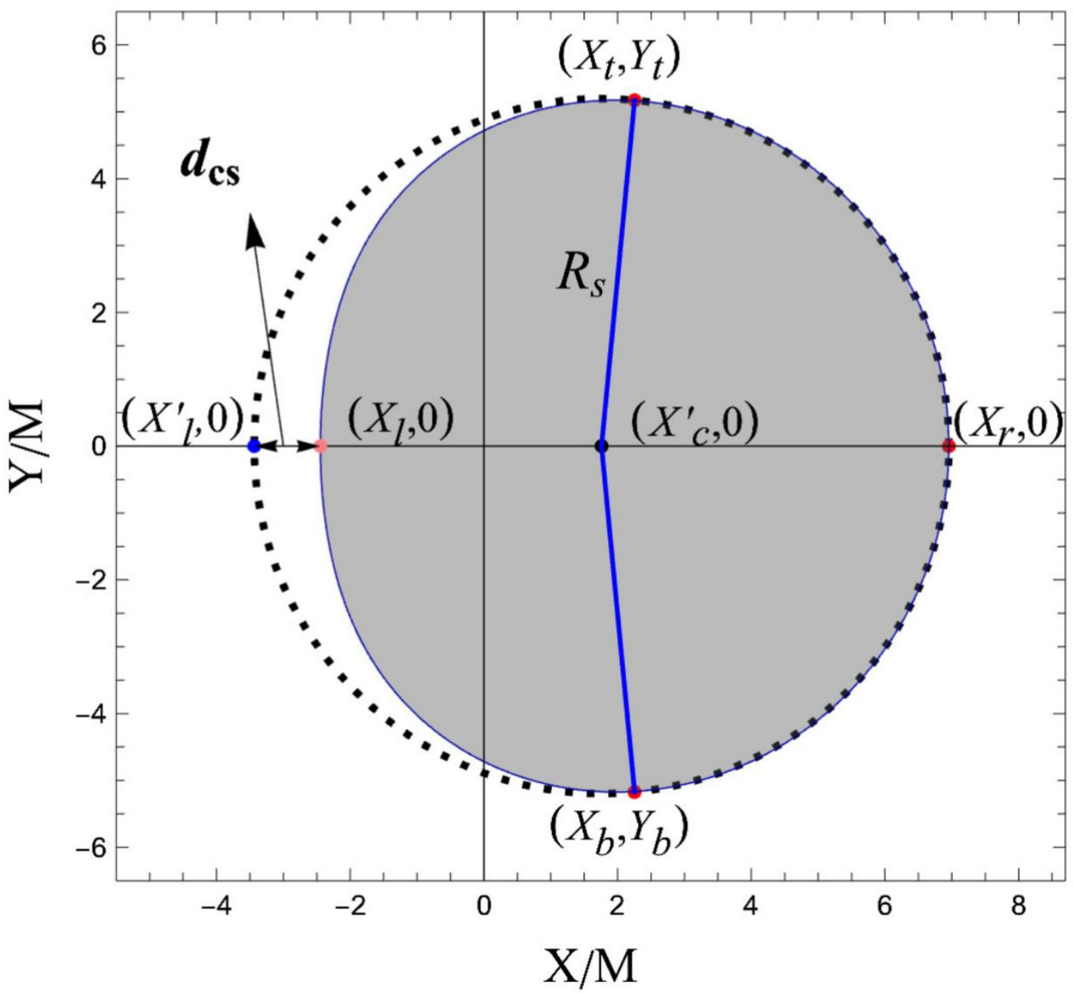
Constraint From Shadow

Approximate reference circle with a radius denoted as R_{sh} and a distortion parameter represented by D_{sh}

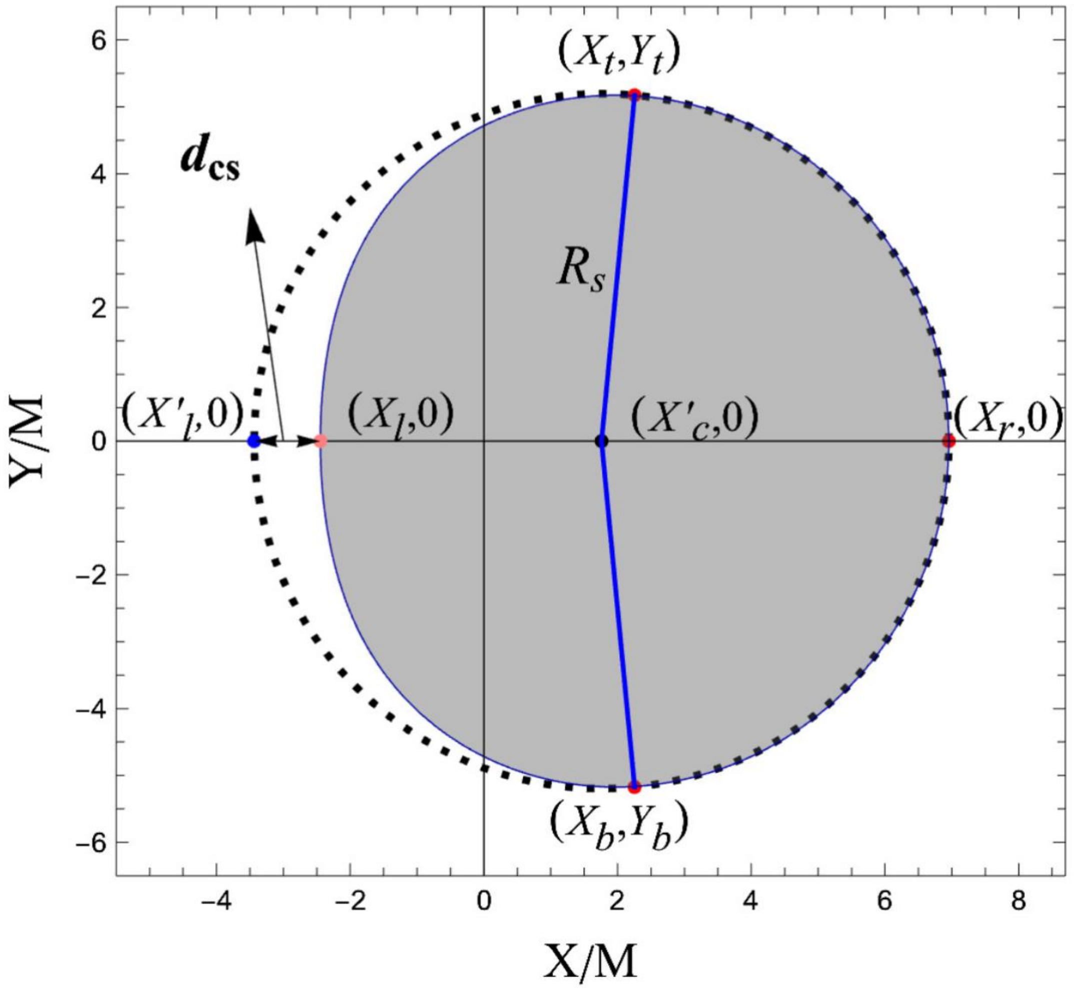


Constraint From Shadow

Approximate reference circle with a radius denoted as R_{sh} and a distortion parameter represented by D_{sh}

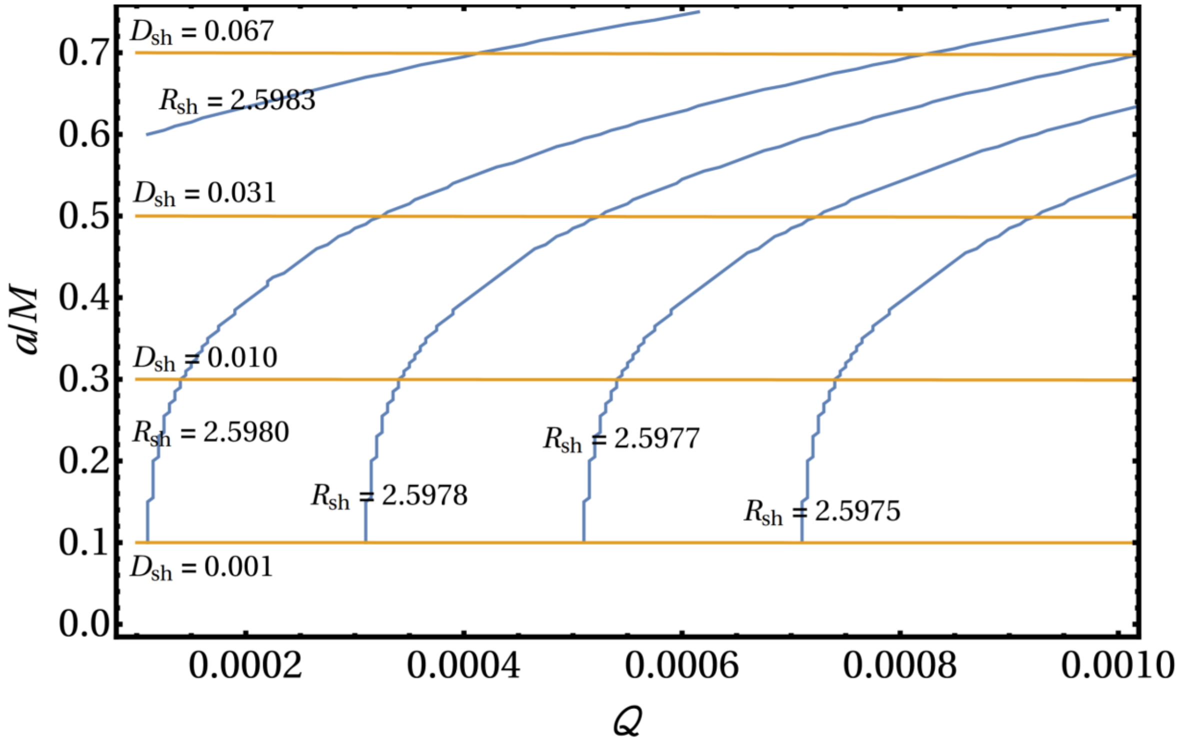
$$R_{\text{sh}} = \frac{(X_t - X_r)^2 + Y_t^2}{2|X_r - X_t|}$$

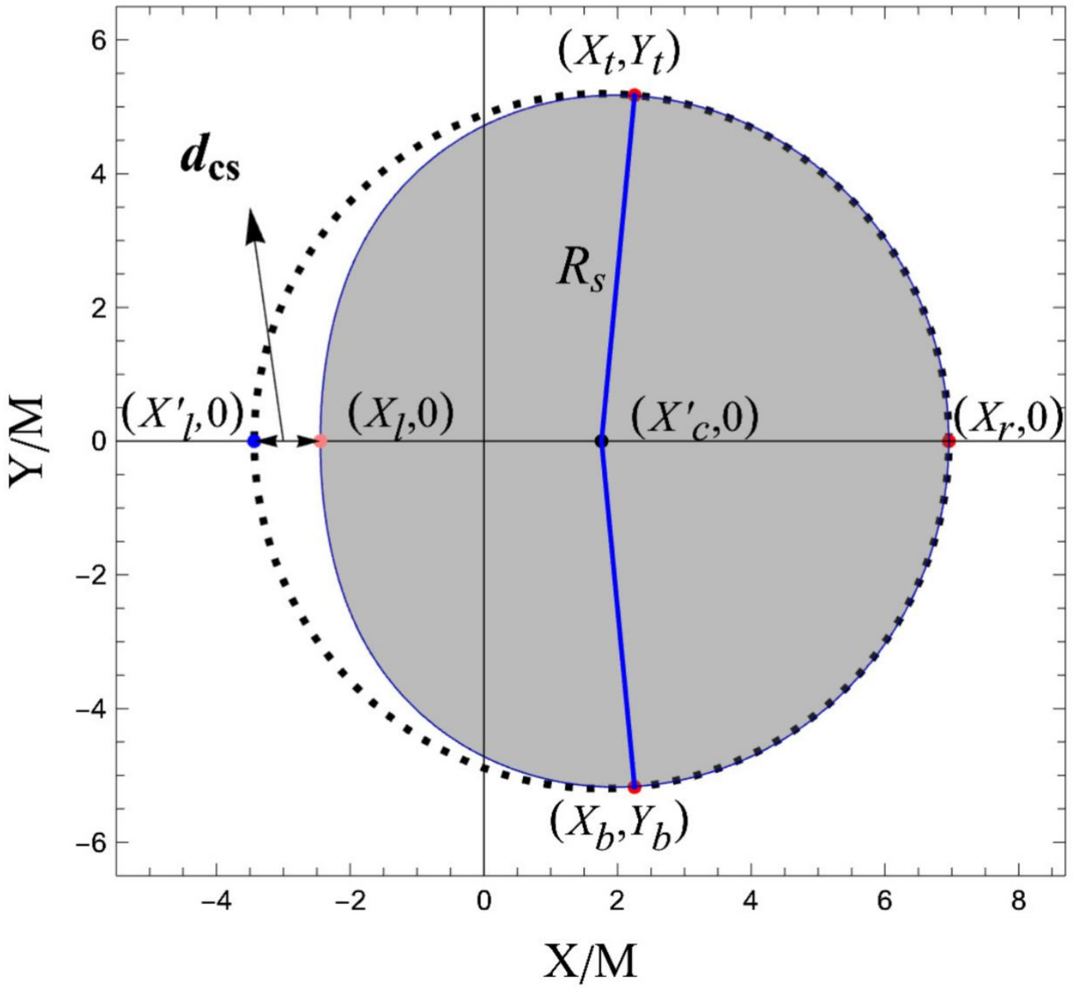
$$D_{\text{sh}} = \frac{|X_l' - X_l|}{R_{sh}}$$



Constraint From Shadow

Can constructs plots such as below to set bounds on quantum parameters





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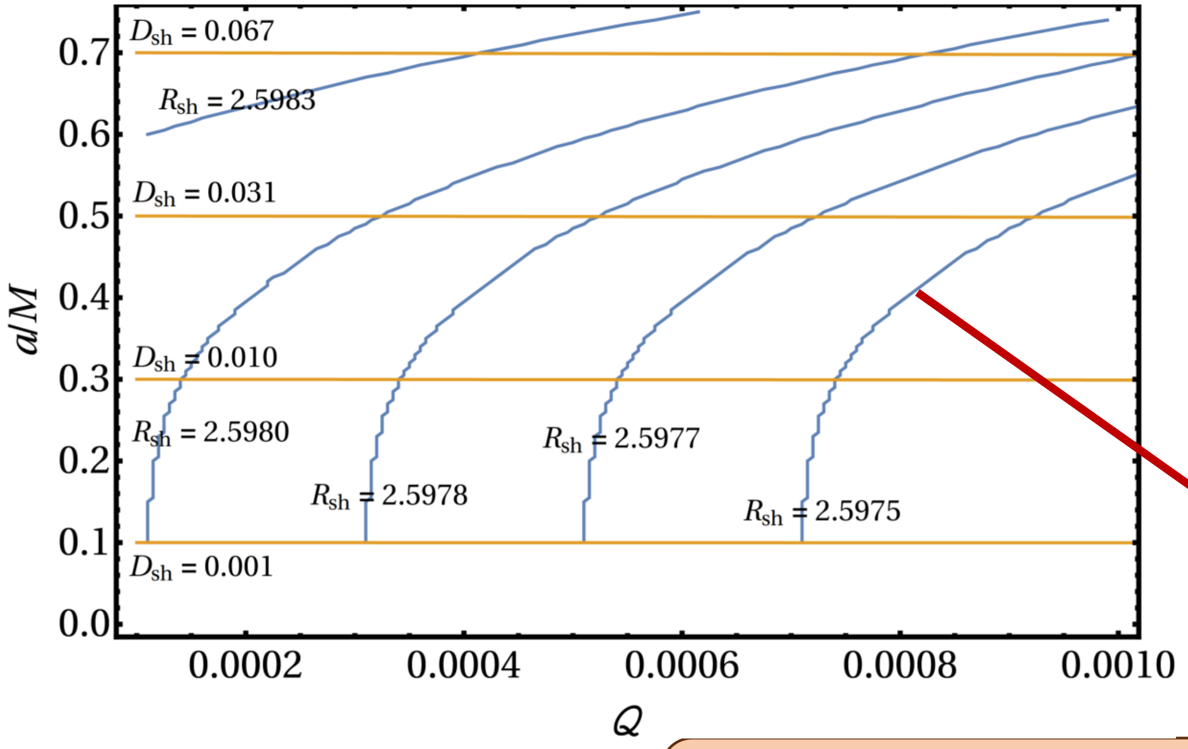
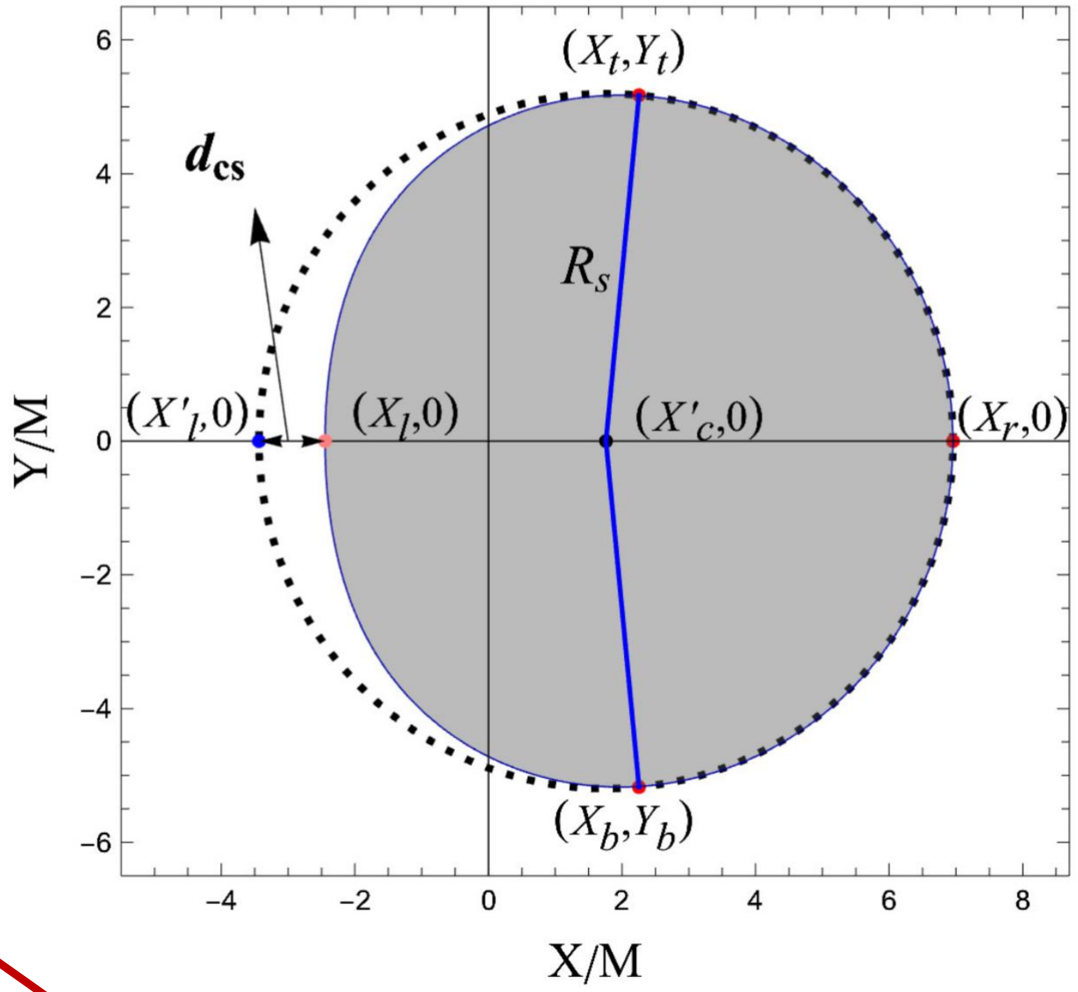
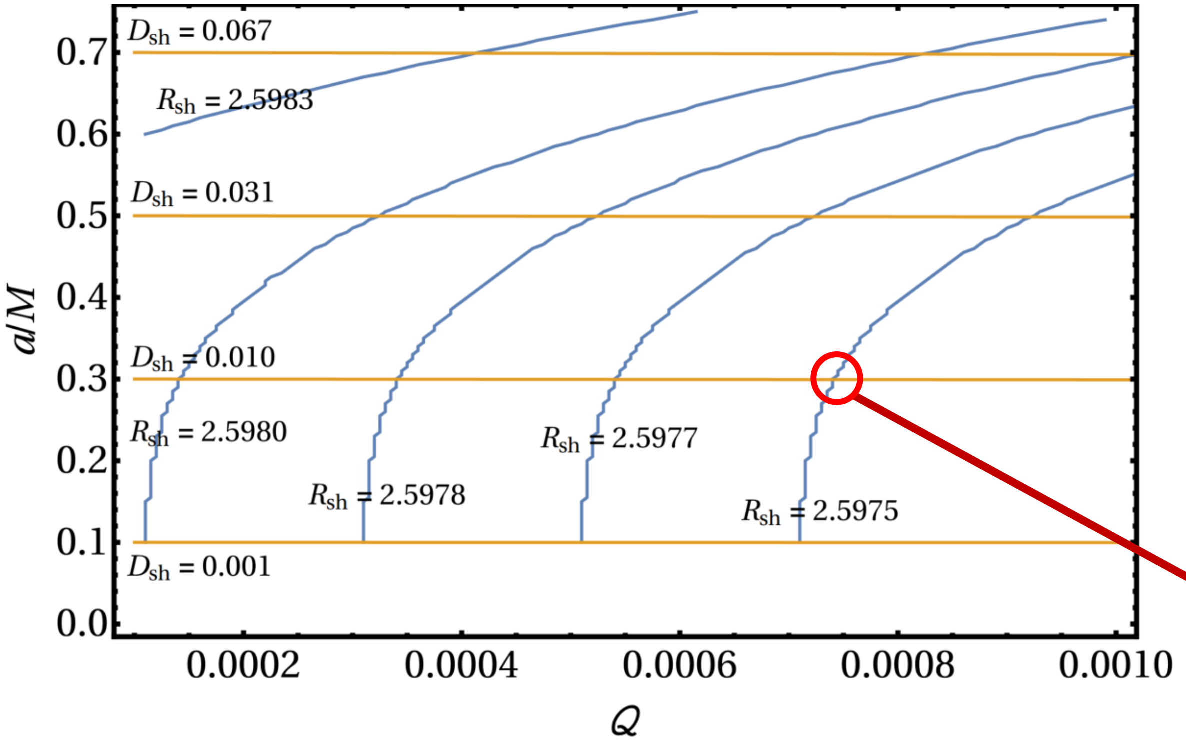


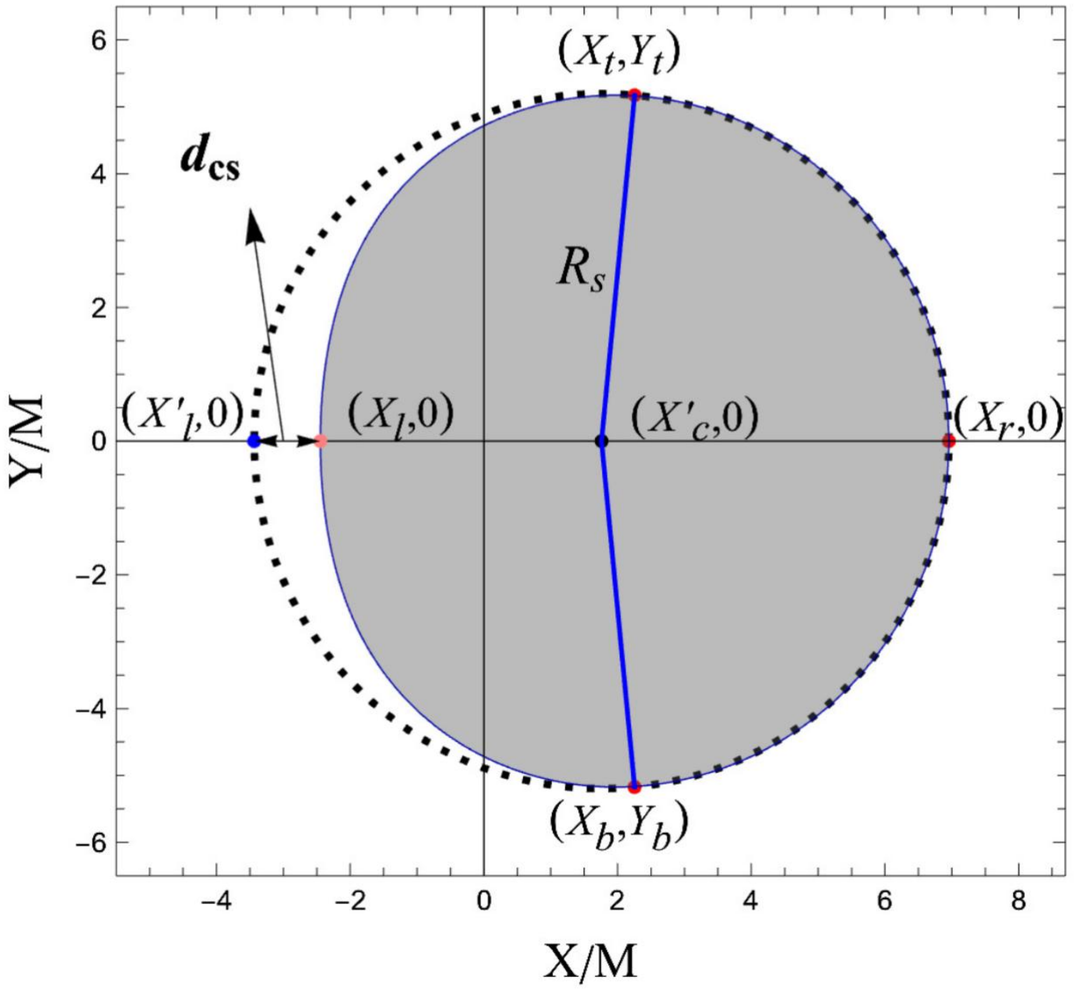


Represents a unique value for R_{sh} corresponds to a wide range of values for the spin and the quantum parameter

Constraint From Shadow

Can constructs plots such as below to set bounds on quantum parameters





Degeneracy is broken intersections of the contours of R_{sh} and D_{sh}

Constraint From Shadow

BH Model	Quantum parameters	Unified Quantum Parameter	Bounds from Sgr A*	Bounds from M87
KSW	δ	$= R_s^4 \mathcal{Q}$	No bound	No bound
GOP	δr R_0	$= \mathcal{P}\sqrt{\mathcal{Q}} R_s$ $= \left(\frac{\mathcal{Q}}{4\pi}\right)^{1/3} R_s$	No bound	No bound
AOS	$\gamma L_o \delta_c$ $\gamma \delta_b$	$= \frac{\mathcal{Q}^{2/3}}{2^{4/3}\pi^{2/3}} R_s$ $= \left(\frac{2\mathcal{Q}}{\pi}\right)^{1/6}$	No bound*	No bound*
MOD	ϵ a_0	$= \sqrt{Q}$ $= \mathcal{R}\mathcal{Q}R_s$	$\mathcal{P} \lesssim 6.5$ and $\mathcal{Q} = 0.1$	$\mathcal{Q} \lesssim 0.05$ and $\gamma = 10^{-3}$
BMM	λ_M	$(4\mathcal{Q})^{2/3}/2$	$\mathcal{Q} \lesssim 0.056$	$\mathcal{Q} \lesssim 0.008$
*Upper bound $\mathcal{Q} \lesssim 1$ can be extracted for $a/M = 0.8$				

Summary

- Quantum BH: important bridges between theoretical quantum gravity and experimental searches
- We have provided an extensive database of expected values, ready for comparison with near-future quantum gravity experimental searches via BH observations
- Static spherically symmetric BH
  - Orbital phenomena: Photon sphere, perihelion shift, lensing, time delay
- Rotating BH (via Newman-Janis algorithm)
  - Shadow

Backup Slides

LQG BH Metrics: KSW

The 5 mainstream models can be written in the form

$$ds^2 = -fdt^2 + gdr^2 + h\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

KSW model: [Kelly, Santacruz, Wilson-Ewing, PRD 102, 106024 (2020)]

$$f(r) = 1 - \frac{R_s}{r} + \frac{\gamma^2\Delta}{r^2}\left(\frac{R_s}{r}\right)^2$$

$$g(r) = \frac{1}{f(r)}$$

$$h(r) = r^2$$

LQG BH Metrics: KSW

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$$h(r) = r^2$$

Schwarzschild radius

$$R_s = 2M$$

LQG BH Metrics: KSW

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$$h(r) = r^2$$

Minimum area in LQG

LQG BH Metrics: KSW

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Berbero-Immirzi parameter

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$$g(r) = \frac{1}{f(r)}$$

$$h(r) = r^2$$

Quantum parameter for phenomenology

$$Q = \frac{\gamma^2\Delta}{R_s^2}$$

LQG BH Metrics: GOP

The 5 mainstream models can be written in the form

$$ds^2 = -fdt^2 + gdr^2 + h\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

GOP model: [Gambini, Olmedo, Pullin, CQG 37 (2020), 205012]

$$f(r) = 1 - \frac{R_s}{r + r_0} + \frac{R_s^3 r_0^3}{(r + r_0)^6 \left(1 + \frac{R_s}{r+r_0}\right)^2}$$

$$g(r) = \frac{1}{f(r)}\left(1 + \frac{\delta r}{2(r + r_0)}\right)^2$$

$$h(r) = (r + r_0)^2$$

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$$g(r) = \frac{1}{f(r)} \left(1 + \frac{\delta r}{2\left(r + r_0\right)}\right)^2$$

$$h(r) = (r + r_0)^2$$

$$\boxed{r_0 = \left(\frac{R_s \gamma^2 \Delta}{4\pi}\right)^{\frac{1}{3}}}$$

LQG BH Metrics: GOP

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$$g(r) = \frac{1}{f(r)}\left(1 + \frac{\boxed{\delta r}}{2\,(r + r_0)}\right)^2$$

$$h(r) = (r + r_0)^2$$

spin networks lattice spacing

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$$g(r) = \frac{1}{f(r)}\left(1 + \frac{\delta r}{2(r + r_0)}\right)^2$$

$$h(r) = (r + r_0)^2$$

Quantum parameter for phenomenology

$$\mathcal{Q} = \frac{\gamma^2 \Delta}{R_s^2}$$

$$\mathcal{P} = \frac{\delta_r}{\sqrt{\mathcal{Q}} R_s}$$

LQG BH Metrics: MOD

The 5 mainstream models can be written in the form

$$ds^2 = -fdt^2 + gdr^2 + h\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

MOD model: [Modesto, CQG 23 (2006) 5587]

$$f(r) = \frac{(r - r_+)(r - r_-)(r + \sqrt{r_+ r_-})^2}{r^4 + a_0^2}$$

$$g(r) = \frac{1}{f(r)} \frac{(r + r_*)^4}{r^4}$$

$$h(r) = r^2 + \frac{a_0^2}{r^2}$$

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Outer horizon

$$g(r) = \frac{1}{f(r)} \frac{(r + r_*)^4}{r^4}$$

$$h(r) = r^2 + \frac{a_0^2}{r^2}$$

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$$f(r) = \frac{(r-r_+)(r-\boxed{r_-})(r+\sqrt{r_+r_-})^2}{r^4+a_0^2}$$

Inner horizon

$$g(r) = \frac{1}{f(r)}\frac{(r+r_*)^4}{r^4}$$

$$h(r) = r^2 + \frac{a_0^2}{r^2}$$

LQG BH Metrics: MOD

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$$h(r) = r^2 + \frac{a_0^2}{r^2}$$

$$\boxed{a_0 = \frac{\Delta}{8\pi}}$$

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$$g(r) = \frac{1}{f(r)}\frac{(r+r_*)^4}{r^4}$$

$$h(r) = r^2 + \frac{a_0^2}{r^2}$$

Quantum parameter for phenomenology

$$\mathcal{Q} = \frac{\gamma^2\Delta}{R_s^2}$$

$$\mathcal{R} = \frac{a_0}{\mathcal{Q}r_s^2}$$

LQG BH Metrics: AOS

The 5 mainstream models can be written in the form

$$ds^2 = -fdt^2 + gdr^2 + h\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

AOS model: [Ashtekar, Olmedo, Singh, PRD 98, 126003 (2018)]

$$f(r) = \left(\frac{r}{R_s}\right)^{2\epsilon} \frac{\left(1 - \left(\frac{R_s}{r}\right)^{1+\epsilon}\right)\left(2 + \epsilon + \epsilon\left(\frac{R_s}{r}\right)^{1+\epsilon}\right)^2\left((2+\epsilon)^2 - \epsilon^2\left(\frac{R_s}{r}\right)^{1+\epsilon}\right)}{16\left(1 + \frac{\delta_c^2 L_0^2 \gamma^2 R_s^2}{16 r^4}\right)(1+\boxed{\epsilon})^4}$$

$$g(r) = \frac{1}{f(r)}\left(\frac{r}{R_s}\right)^{-2(2+\epsilon)} \frac{\left(\epsilon + \left(\frac{r}{R_s}\right)^{1+\epsilon}(2+\epsilon)\right)^4}{16(1+\epsilon)^4}$$

$$h(r) = r^2\left(1 + \frac{\gamma^2 L_0^2 \delta_c^2 R_s^2}{16 r^4}\right)$$

$$\epsilon = \sqrt{1 + \gamma^2\left(\frac{\sqrt{2\Delta}}{\sqrt{\pi}R_s\gamma^2}\right)^{\frac{2}{3}}} - 1$$

LQG BH Metrics: AOS

The 5 mainstream models can be written in the form

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$$g(r) = \frac{1}{f(r)}\left(\frac{r}{R_s}\right)^{-2(2+\epsilon)} \frac{\left(\epsilon+\left(\frac{r}{R_s}\right)^{1+\epsilon}(2+\epsilon)\right)^4}{16(1+\epsilon)^4}$$

$$h(r) = r^2\left(1+\frac{\gamma^2 \boxed{L_0^2\delta_c^2} R_s^2}{16r^4}\right)$$

$$\boxed{L_0\delta_c = \frac{1}{2L_0}\left(\frac{\gamma\Delta^2}{2\pi^2 R_s}\right)^{\frac{1}{3}}}$$

LQG BH Metrics: AOS

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$$g(r) = \frac{1}{f(r)}\left(\frac{r}{R_s}\right)^{-2(2+\epsilon)} \frac{\left(\epsilon+\left(\frac{r}{R_s}\right)^{1+\epsilon}(2+\epsilon)\right)^4}{16(1+\epsilon)^4}$$

$$h(r) = r^2\left(1+\frac{\gamma^2 L_0^2 \delta_c^2 R_s^2}{16r^4}\right)$$

Quantum parameter for phenomenology

$$\mathcal{Q} = \frac{\gamma^2 \Delta}{R_s^2}$$

LQG BH Metrics: BMM

The 5 mainstream models can be written in the form

$$ds^2 = -f dt^2 + g dr^2 + h\left(d\theta^2 + \sin^2(\theta) d\phi^2\right)$$

BMM model: [Bodendorfer, Mele, Munch, Phys. Lett. B, 819:136390, 2021]

$$f(r) = \left(1 - \sqrt{\frac{1}{2\lambda_M}} \frac{1}{\sqrt{1+r^2}}\right) \frac{1+r^2}{h(r)}$$

$$g(r) = \frac{1}{f(r)}$$

$$h(r) = \frac{\lambda_M}{\sqrt{1+r^2}} \frac{M_{\mathrm{BH}}^2 \left(r + \sqrt{1+r^2}\right)^6 + M_{\mathrm{WH}}^2}{\left(r + \sqrt{1+r^2}\right)^3}$$

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Black hole mass

LQG BH Metrics: BMM

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$$g(r) = \frac{1}{f(r)}$$

$$h(r) = \frac{\lambda_M}{\sqrt{1+r^2}}\frac{M_{\mathrm{BH}}^2\left(r+\sqrt{1+r^2}\right)^6 + \boxed{M_{\mathrm{WH}}^2}}{\left(r+\sqrt{1+r^2}\right)^3}$$

White hole mass

LQG BH Metrics: BMM

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$$ds^2 = -fdt^2 + gdr^2 + h\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)$$

BMM model: [Bodendorfer, Mele, Munch, Phys. Lett. B, 819:136390, 2021]

$$f(r) = \left(1 - \sqrt{\frac{1}{2\lambda_M}}\frac{1}{\sqrt{1+r^2}}\right)\frac{1+r^2}{h(r)}$$

$$g(r) = \frac{1}{f(r)}$$

$$h(r) = \frac{\boxed{\lambda_M}}{\sqrt{1+r^2}}\frac{M_{\mathrm{BH}}^2\left(r+\sqrt{1+r^2}\right)^6 + M_{\mathrm{WH}}^2}{\left(r+\sqrt{1+r^2}\right)^3}$$

$$\boxed{\lambda_M = \frac{1}{2}\left(\frac{\gamma^2\Delta}{M_{BH}^2}\right)^{\frac{2}{3}}}$$

LQG BH Metrics: BMM

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Quantum parameter for phenomenology

$$\mathcal{Q} = \frac{\gamma^2\Delta}{R_s^2}$$

$$\mathcal{P} = \frac{M_{\mathrm{WH}}}{M_{\mathrm{BH}}}$$

LQG Phenomenology - Photon Sphere

Model	$\frac{r_H}{R_s}$	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Numeric)
KSW	$1-\mathcal{Q}$	$\frac{3}{2}-\frac{8}{9}\mathcal{Q}$	1.49911	1.49910
GOP	$1-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	$\frac{3}{2}-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	1.4077	0.9587
AOS	$\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{4}{3}}$	$\frac{3}{2}+\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{2}{3}}\left[\frac{5}{3}-\ln\left(\frac{27}{8}\right)\right]$	1.5008	1.5099
MOD	$1-\frac{1}{2}\sqrt{\mathcal{Q}}$	$\frac{3}{2}-\frac{5}{6}\sqrt{\mathcal{Q}}$	1.4736	1.4992
BMM	$1-\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	$\frac{3}{2}-\frac{2}{3}\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	1.4916	1.4831

LQG Phenomenology - Photon Sphere

Model	$\frac{r_H}{R_s}$	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Numeric)
KSW	$1-\mathcal{Q}$	$\frac{3}{2}-\frac{8}{9}\mathcal{Q}$	1.49911	1.49910
GOP	$1-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	$\frac{3}{2}-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	1.4077	0.9587
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LQG Phenomenology - Photon Sphere

Model	$\frac{r_H}{R_s}$	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Pert., 1st order)	$\frac{r_{ps}}{R_s}$ (Numeric)
KSW	$1-\mathcal{Q}$	$\frac{3}{2}-\frac{8}{9}\mathcal{Q}$	1.49911	1.49910
GOP	$1-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	$\frac{3}{2}-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	1.4077	0.9587
AOS	$\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{4}{3}}$	$\frac{3}{2}+\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{2}{3}}\left[\frac{5}{3}-\ln\left(\frac{27}{8}\right)\right]$	1.5008	1.5099
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BMM	$1-\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	$\frac{3}{2}-\frac{2}{3}\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	1.4916	1.4831

LQG Phenomenology - Photon Sphere

Model	$\frac{r_H}{R_s}$	$\frac{r_{\text{ps}}}{R_s}$ (Pert., 1st order)	$\frac{r_{\text{ps}}}{R_s}$ (Pert., 1st order)	$\frac{r_{\text{ps}}}{R_s}$ (Numeric)
KSW	$1-\mathcal{Q}$	$\frac{3}{2}-\frac{8}{9}\mathcal{Q}$	1.49911	1.49910
GOP	$1-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	$\frac{3}{2}-\left(\frac{\pi\mathcal{Q}}{4}\right)^{\frac{1}{3}}$	1.4077	0.9587
AOS	$\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{4}{3}}$	$\frac{3}{2}+\left(\frac{\mathcal{Q}}{4\pi}\right)^{\frac{2}{3}}\left[\frac{5}{3}-\ln\left(\frac{27}{8}\right)\right]$	1.5008	1.5099
MOD	$1-\frac{1}{2}\sqrt{\mathcal{Q}}$	$\frac{3}{2}-\frac{5}{6}\sqrt{\mathcal{Q}}$	1.4736	1.4992
BMM	$1-\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	$\frac{3}{2}-\frac{2}{3}\left(2\mathcal{Q}^2\right)^{\frac{1}{3}}$	1.4916	1.4831

In strong gravitational field near photon sphere, perturbative analysis of GOP breaks down.

Not quite in other models.

LQG Phenomenology – Perihelion Shift

KSW:

Up to first order in Δ, R_s

$$\delta\phi = \frac{3\pi}{a\left(e^2 - 1\right)} R_s$$

LQG Phenomenology – Perihelion Shift

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Semi-major axis

LQG Phenomenology – Perihelion Shift

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Up to first order in Δ, R_s

$$\delta\phi = \frac{3\pi}{\boxed{a}(\boxed{e}^2 - 1)} R_s$$

Eccentricity

Semi-major axis

LQG Phenomenology – Perihelion Shift

KSW:

Up to first order in Δ, R_s

$$\delta\phi = \frac{3\pi}{\boxed{a}(\boxed{e}^2 - 1)} R_s$$

Eccentricity

Semi-major axis

$$\boxed{R_\pm = (1 \pm e)a}$$

LQG Phenomenology – Perihelion Shift

GOP:

Up to first order in R_s, $R_0 = \left(\frac{\mathcal{Q}}{4\pi}\right)^{1/3} R_s$

$$\delta\phi = \frac{3\pi R_s}{a\left(e^2-1\right)} + \frac{3\pi\left(e^2+1\right)R_0 R_s}{a^2(e^2-1)^2} + \frac{\pi\delta r}{a\left(e^2-1\right)}\left(1+\frac{\left(e^2+1\right)R_0}{a(e^2-1)} - \frac{\left(e^2+6\right)R_s}{4a(e^2-1)}\right)$$

LQG Phenomenology – Perihelion Shift

MOD:

Up to first order in $P = \dfrac{\sqrt{1+\mathcal{Q}}-1}{\sqrt{1+\mathcal{Q}}+1}$, $a_0 = \mathcal{R}\mathcal{Q}R_s$, R_s

$$\Delta\phi = \frac{3\pi R_s}{a\left(e^2-1\right)} - \frac{4\pi P R_s}{a\left(e^2-1\right)} + \frac{\pi a_0^2}{4a^4\left(e^2-1\right)^4}\left(\frac{\left(9e^4+844e^2+696\right)R_s}{4a\left(e^2-1\right)} - \left(\frac{9e^4}{2}+66e^2+52\right) + \frac{\left(9e^4-332e^2-296\right)PR_s}{a\left(e^2-1\right)}\right)$$