“Time Dilation in Deformed Special Relativity"

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based on arXiv: 2506.08111

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Contents

One Question from Quantum Gravity

The Wish-List of Quantum Gravity

- Quantize General Relativity
- Avoid BH and BB singularities
- **Explain role of Planck scale**
- Predict Hawking Radiation
- Solve Information Paradox
- Explain Dark Energy
- Solve world hunger
- ……
- ……

But we know **Lorentz Invariance**

↓

Lorentz Transformations

↓

Lengths (and energies) change depending on the inertial observer

The two seem **incompatible**: If QG-scale, for whom? If minimal length, for whom?

The Question: *What happens to Lorentz Invariance at the Planck scale??*

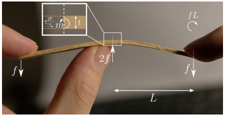
Breaking vs Deforming *Lorentz Invariance*

- **Breaking LI**: symmetry broken at the Planck scale
  - *Preferred frame*: laws of physics not invariant
  - *Modified dispersion relation*: changes in any frame

LIV - Theories

- **Deforming LI**: symmetry modified, ok at Planck scale
  - *Postulate 1*: Relativity principle holds, no preferred frame
  - *Postulate 2*: Two invariant scales, velocity c and length ℓ

DSR - Theories



Phenomenology:[1] time delays (GRB,ν), modified thresholds, life times..

[1] A. Addazi et al. ,*"Quantum gravity phenomenology at the dawn of the multi-messenger era"* (2022)

Deformed Special Relativity: *the model*

Possible realization of DSR from bicrossproduct κ-Poincaré Hopf Algebra, in $1+1D$

- *Algebra*: $[\mathscr{P}_0, \mathscr{P}_1] = 0, \quad [\mathscr{N}, \mathscr{P}_0] = \mathscr{P}_1, \quad [\mathscr{N}, \mathscr{P}_1] = \frac{1-e^{-2\ell\mathscr{P}_0}}{2\ell} - \frac{\ell}{2}\mathscr{P}_1^2$
- *Casimir*: $\mathscr{C} = \left(\frac{2}{\ell}\right)^2 \sinh^2\left(\frac{\ell}{2}\mathscr{P}_0\right) - \mathscr{P}_1^2 e^{\ell\mathscr{P}_0}$
- *Additional structures*: *co-product, antipode, ..*

Representation on commutative **phase-space coordinates**: $\{p_\nu, x^\mu\} = \delta_\nu^\mu$, $\{x_\nu, x^\mu\} = 0$

- *Algebra*: $\{p_0, p_1\} = 0, \quad \{N, p_0\} = p_1, \quad \{N, p_1\} = \frac{1}{2\ell}\left(1 - e^{-2p_0\ell}\right) - \frac{\ell}{2}p_1^2$
- *Boost*: $N = x^0 p_1 + x^1\left(\frac{1-e^{-2\ell p_0}}{2\ell} - \frac{\ell}{2}p_1^2\right)$

Relativistic Kinematics:[2] infinitesimal action of Hamiltonian/boost on generic $f(x^\alpha, p_\alpha)$

- *Dynamic evolution*: $\boxed{\partial_\tau f = \{f, C\}}$ $\quad \tau$ *evolution* $\longrightarrow$ *worldlines*
- *Boost evolution*: $\boxed{\partial_\eta f = \{f, N\}}$ $\quad \eta$ *rapidity* $\longrightarrow$ *transformations*

[2] G. Amelino-Camelia *et al.*, *"Theories with Planck-scale-deformed Lorentz symmetry"* (2012)

Deformed Special Relativity: *momentum transformations*

- To obtain the **finite action of boost** on **momentum sector** we must integrate

$$\partial_\eta p_0 = \{p_0, N\} = -p_1$$
$$\partial_\eta p_1 = \{p_1, N\} = (e^{-2\ell p_0} - 1)/2\ell + \ell p_1^2/2$$

- *Solutions $p_\mu(\eta)$: **momentum transformations*** [3]

$$p_0(\eta) = \bar{p}_0 - \frac{1}{\ell}\ln\frac{1}{1-\ell\bar{p}_1(A - A\cosh\eta - \sinh\eta)},$$
$$p_1(\eta) = \bar{p}_1\frac{(\cosh\eta + A\sinh\eta)}{1-\ell\bar{p}_1(A - A\cosh\eta - \sinh\eta)},$$

"Invariant max momentum"

$$p_1(\eta) \to \frac{1}{\ell} \quad \text{for} \quad \eta \to \infty$$

[3] G. Amelino-Camelia *et al.* *"Deformed boost transformations that saturate at the Planck scale"* (2001)

Spacetime sector: finite action of the boost on spacetime coordinates [4]

[4] *"DSR spacetime picture and phenomenology of Planck-scale-modified time dilation"*, arXiv:2506.08111

Boost action on coordinates

- To obtain the **finite action of boost** on **spacetime sector** we must integrate

$$\partial_\eta x^0 = \{x^0, N\} = -e^{-2\ell p_0} x^1$$
$$\partial_\eta x^1 = \{x^1, N\} = -x^0 + \ell p_1 x^1$$

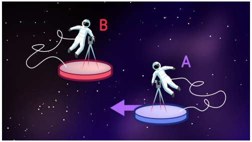
- Solutions $x^\mu(\eta)$: **coordinate transformations**

$$x^0(\eta) = \frac{\overline{x}^0}{4} e^{-\eta} \frac{\left(1-\overline{p}_1\ell + e^\eta \overline{p}_1 \ell + e^\eta\right)^2 + e^{-2\ell\bar{p}_0}\left(e^\eta - 1\right)^2}{1-\overline{p}_1\ell(A - A\cosh\eta) - \sinh\eta)} - \frac{\overline{x}^1}{2} \frac{\left(e^\eta - 1\right)\left(1-\overline{p}_1\ell + e^\eta\overline{p}_1\ell + e^\eta\right) e^{-\eta-2\ell\bar{p}_0}}{1-\overline{p}_1\ell(A - A\cosh\eta - \sinh\eta)},$$
$$x^1(\eta) = \frac{\overline{x}^1}{4} e^{-\eta}\left[\left(1-\overline{p}_1\ell + e^\eta\overline{p}_1\ell + e^\eta\right)^2 + \left(e^\eta - 1\right)^2 e^{-2\ell\bar{p}_0}\right] - \overline{x}^0(\sinh\eta + \overline{p}_1\ell\cosh\eta - \overline{p}_1\ell),$$

Boost action on coordinates

Take **free particle** P, observer A boosted wrt B with rapidity η

- For B it has (ε, p) and moves on coordinates (t, x)
- For A it has (ε', p') and moves on coordinates (t', x')



"Transformations"

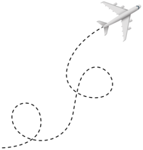
$$
\boxed{\begin{aligned} t' &= g_t(\varepsilon, p)\ t + g_x(\varepsilon, p)\ x \\ x' &= f_t(\varepsilon, p)\ t + f_x(\varepsilon, p)\ x \end{aligned}}
$$

"Properties"

- They mix coordinates in the whole phase-space !
- They reduce to ordinary Lorentz in the $\ell \to 0$ limit !
- They correctly preserve the Poisson brackets !

Worldline covariance and Spacetime metric

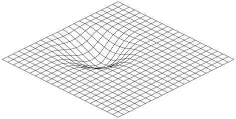
- The Hamilton's eqs $\frac{dx^\mu}{d\tau} = \{x^\mu, C\}$ give the **worldline** of P



$$x_{\varepsilon,p}(t) = \frac{2\ell p e^{2\ell\varepsilon}}{e^{2\ell\varepsilon}\left(\ell^2 p^2 - 1\right) + 1} t$$

we confirm its **covariance** under finite boost transformation

- We can now determine the **spacetime metric** for P



$$ds^2 = dt^2 - e^{-2\ell\varepsilon} dx^2$$

validating the guess from **relative locality**[5]

[5] G. Amelino-Camelia, L. Friedel, J. Kowalski-Glikman, L. Smolin, *"The principles of Relative Locality"* (2011)

- Using our *transformations* or the *metric* we find the **time dilation**
  - *"Special Relativity"* $\Delta t = \frac{p_0}{M}\ \Delta\tau$
  - *"Deformed SR"* $$\boxed{\Delta t = \frac{\cosh(\ell M) - e^{-\ell p_0}}{\sinh(\ell M)} \Delta\tau}$$

- This **DSR-dilation factor** has nice properties
  - *Lorentz limit*: for $\ell \to 0$ it reduces to standard Lorentz factor p_0/M
  - *No pathologies*: it is positive and well-defined for all boosts
  - *Saturation*: for infinite boost, it saturates to finite value $\coth M\ell$

- **Phenomenology**: new DSR-observable $\longrightarrow$ **life times**[6]
  - *LHC muons*: $\frac{\Delta t}{\Delta\tau} \simeq \frac{p_0}{M}\left(1 + \frac{\ell p_0}{2}\right)$, current accuracy requires $\sim 10^{10}\,TeV$ muons

[6] I.P. Lobo, C. Pfeifer, *"Reaching the Planck scale with muon lifetime measurements"* (2021)

The Question: *What happens to Lorentz Invariance at the Planck scale??*

A possible answer is given by **Deformed Special Relativity**:

- *Accommodates Planck scale*: in full invariant and consistent way
- *New channels QG-phenomenology*: time delays (GRB,ν), life times (μ)

Thanks for your attention