Advancing superluminal neutrino constraints with UHE events

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- Energy-independent LIV (n=0) :

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$$E^2 = \vec{p}^2 \left[1 + \left(\frac{|\vec{p}|}{\Lambda} \right)^n \right] \qquad v = 1 + \frac{(n+1)}{2} \left(\frac{|\vec{p}|}{\Lambda} \right)^n$$

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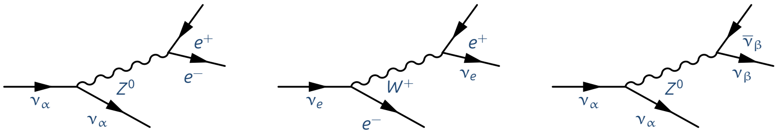

Vacuum pair emission (VPE)

Neutrino splitting (n>0 only)

Cohen & Glashow (CG) Result and Later Approaches

- Vacuum pair emission $\nu_\alpha \to \nu_\alpha e^- e^+$ occurs above a threshold

$$E > E_{\rm th}^{(n)} \doteq \left(4m_e^2\,\Delta_{(n)}^{-1}\right)^{1/(n+2)}\,, \quad \text{where}\ \ \Delta_{(0)} \doteq \delta,\ \ \Delta_{(n\neq 0)} \doteq \Lambda^{-n}$$

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- [Cohen:2011hx]: For the $n = 0$ case

$$\Gamma_{\rm CG}(E > E_{\rm th}^{(0)}) = \frac{1}{14}\frac{G_F^2}{192\pi^3}E^5\delta^3 \quad \textit{(no explicit calculation is given)}$$

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- [Bezrukov:2011qn], [Carmona:2012tp]: $\Gamma_{\rm CG}$ results from a prescription that is not derived from a Lagrangian. A correct calculation gives:

$$\Gamma_{(n=0)}(E \gg E_{\rm th}^{(0)}) = \frac{17}{420}\frac{G_F^2}{192\pi^3}E^5\delta^3$$

The computation assumes $m_e = 0$: it is an asymptotic result only valid sufficiently above the threshold

Generalization to high-energy LIV

- [Carmona:2012tp] generalized the VPE decay width of SuL neutrinos to the $n \neq 0$ case, through the neutral current only
  - *This was used by* [Stecker:2014oxa] *to compare the IceCube spectrum with LIV predictions*

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- [Carmona:2022dtp] used the $m_e = 0$ approximation to include the charge current contribution and the neutrino splitting decay process:

$$\Gamma^{(i)}_{\alpha,n}(E) \approx K^{(i)}_{\alpha,n} \frac{G_F^2}{192\pi^3} E^{5+3n} \Delta^3_{(n)} \qquad (i) = \{\text{VPE, NSpl}\}$$

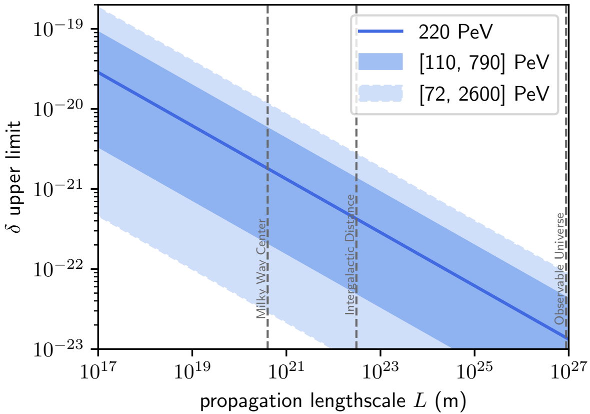
$K^{(i)}_{\alpha,n}$	VPE, $\alpha = (\mu, \tau)$	VPE, $\alpha = e$	NSpl, α
$n = 0$	17/420	221/420	0
$n = 1$	121/1680	1573/1680	22/75
$n = 2$	81/910	81/70	1422/5005

KM3-230213A constraints

KM3NeT analysis

- KM3-230213A: $E_{\text{UHE}} = 220^{+570}_{-110}$ PeV
- n=0 case, upper limit on δ: *[KM3NeT:2025mfl]*

$$\boxed{L\Gamma_{\text{CG}} < 10} \Rightarrow \delta < \delta_{\text{u.l.}} = \left(\frac{10 \cdot 192\pi^3}{K_{\text{CG}} \cdot G_F^2 \cdot E^5 \cdot L}\right)^{1/3}, \quad K_{\text{CG}} = \frac{1}{14}$$



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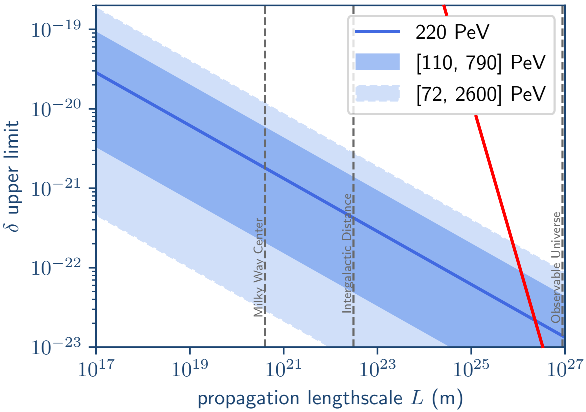

Alternative analyses and open issues

[KM3NeT:2025mfl]

- *uses an incorrect expression for the decay with* Γ
- *does not take into account the cosmic expansion*
- *does not incorporate the VPE threshold in the plot*
- *only considers the* $n = 0$ *scenario*

[Satunin:2025uui]

- *does not take into account the cosmic expansion*
- *derives incorrect limits for the* $n = 1$ *case*

[Yang:2025kfr]

- *only studies the* $n = 2$ *case*
- *uses an incorrect expression for the decay with* Γ

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Further limitations of all previous studies:

- *limits focused on the KM3-230213A event*
- *they ignore* m_e *effects, relevant close to the threshold*
- *they ignore neutrino flavor effects*

Addressing limitations in previous approaches

Including cosmic expansion effects

- The survival condition $L\Gamma < 10$ can be written as

$$\mathcal{P}_n^{\mathrm{SV}}(E, L) = \exp\Big(-L \cdot \Gamma_n(E)\Big) > \exp(-10)$$

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- With cosmic expansion, this needs to be replaced by

$$\mathcal{P}_n^{\mathrm{SV}}(E_d, z_s) = \exp\left(-\int_0^{z_s} dz\, \frac{\Gamma_n\big((1+z)E_d\big)}{(1+z)H_0\, h(z)}\right) > \exp(-10),$$

with $h(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$

In the local universe, $L = z_s/H_0$. If $L \gg 10^{17}$ m, neutrinos oscillate several times during propagation, and we can define a *mean* VPE decay width:

$$\Gamma_n^{(\mathrm{VPE})}(E) \approx K_n^{(\mathrm{VPE})} \frac{G_F^2}{192\pi^3} E^{5+3n} \Delta_{(n)}^3 \qquad \boxed{K_n^{(\mathrm{VPE})} = \frac{1}{3}\left(K_{e,n}^{(\mathrm{VPE})} + K_{\mu,n}^{(\mathrm{VPE})} + K_{\tau,n}^{(\mathrm{VPE})}\right)}$$

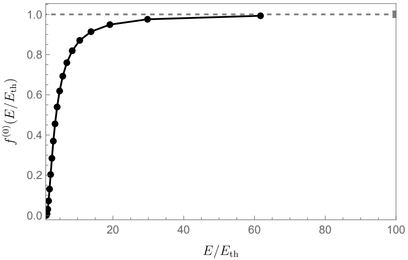
Including threshold effects

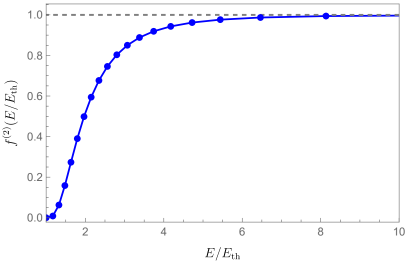
- Threshold effects in the VPE decay width can be encoded in the function $f^{(n)}(x)$, $0 < x \leqslant 1$:

$$\Gamma_n^{\mathrm{VPE}}(E) = K_n^{\mathrm{VPE}} \frac{G_F^2}{192\pi^3} \cdot E^{5+3n} \cdot \Delta_{(n)}^3 \cdot f^{(n)}\left(\frac{E}{E_{\mathrm{th}}}\right)$$

where $f^{(n)}(1) = 0$ and $\lim_{x\to\infty} f^{(n)}(x) = 1$.

- The function $f^{(n)}$ takes into account a change in the partial decay widths and in the integration limits. The result is *[in preparation]*:



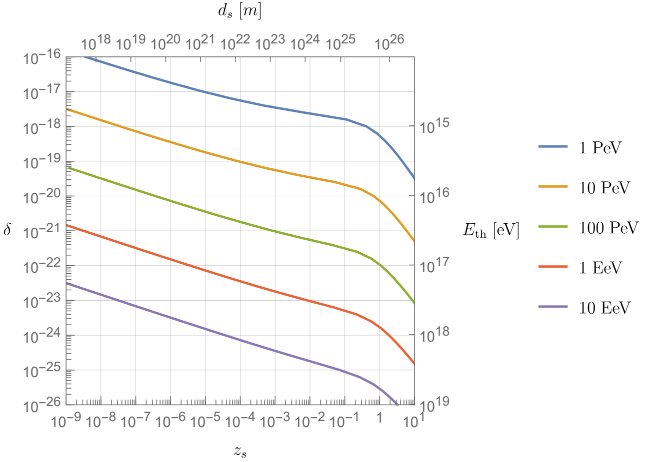


Improved constraints with future UHE neutrino events

New analysis in the constant-velocity scenario

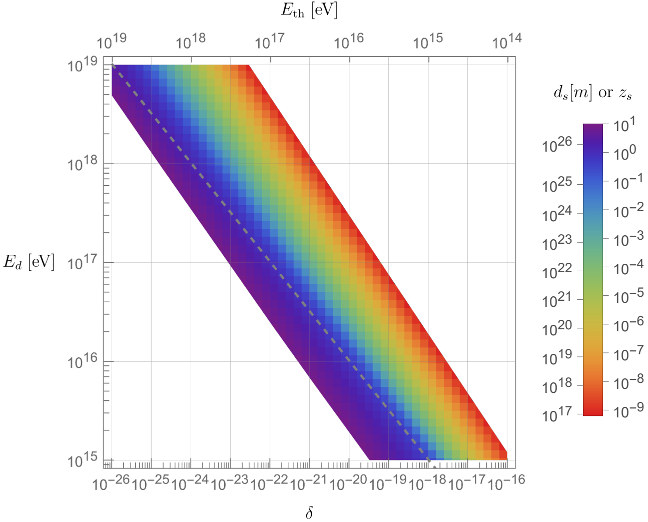
Curves $\delta(z_s)$ for fixed detected energy $E_d = 1\,\text{PeV} - 10\,\text{EeV}$

- The curves are no longer straight lines
- Threshold effects appear when $E_d \sim E_{\text{th}}(\delta)$
- Cosmic expansion changes slope at high redshift



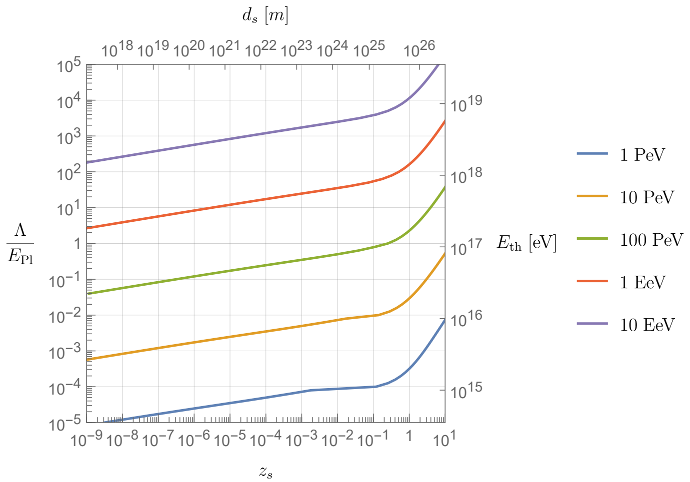
New analysis in the constant-velocity scenario

A heat-map plot (E_d, δ, z_s) allows to get a limit on δ for a specific E_d event, depending on its source redshift z_s



Extending the analysis to the energy-dependent case

$n = 2$ case: curves $\Lambda(z_s)$ for fixed detected energy E_d



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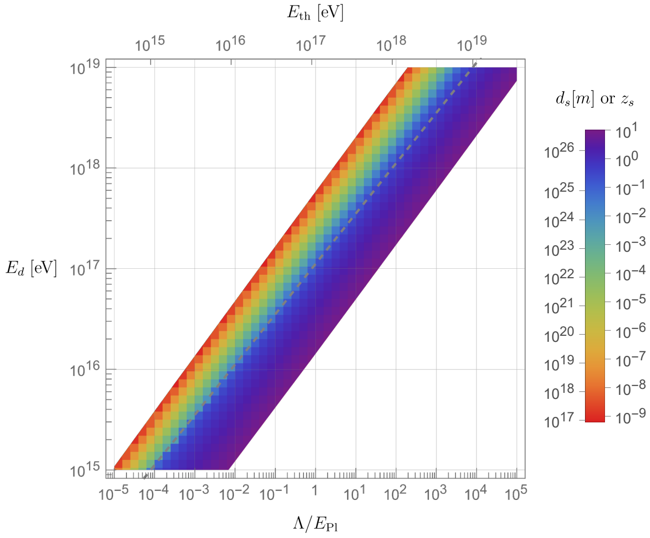
$n = 2$ case: heat-map plot (E_d, Λ, z_s)



Conclusions and future work

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Many works in the literature still rely on the Cohen & Glashow (C&G) expression to study LIV-induced superluminal neutrino decay, overlooking the fact that:

- It assumes a decay amplitude not derived from a Lagrangian
- It neglects the charged current contribution to the decay — important for flavor-sensitive simulations
- It is limited to the $n = 0$ case, whereas more general, quantum-gravity-motivated scenarios with $n > 0$ allow for additional processes such as neutrino splitting

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- Moreover, the C&G expression — like other formulas for the decay width — is an asymptotic result, valid only well above threshold. Near threshold, corrections can be significant, especially in the $n = 0$ case.

- Our reanalysis may become relevant with the detection of future UHE neutrino events — a population we now know exists.

Outlook: superluminal cascades

- The previous analysis is based on the survival probability of a neutrino detected with energy E_d. However, the probability of detection after propagation from a source at a (local) distance L is

$$\mathcal{P}^{(0)}(E_d, L) = \mathcal{P}_e(E_d)\, e^{-L\,\Gamma(E_d)}$$

where $\mathcal{P}_e(E_d)$ is the emission probability and one is assuming that the neutrino has propagated from the source without any decay

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$$\mathcal{P}^{(1)}(E_d, L) \;=\; \int_0^1 dy\, f(y)\, \mathcal{P}_e(E_d/y) \int_0^1 dx\, e^{-(1-x)\,L\,\Gamma(E_d/y)} L\,\Gamma(E_d/y)\, e^{-xL\,\Gamma(E_d)}$$

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- The sum of the probabilities $\sum_n \mathcal{P}^{(n)}(E_d, L)$ will give the spectral flux of detected neutrinos from a source at a distance L

Thank you for your attention