

Quantum Bound on Gravitational Wave Luminosity

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Simple observation: in $D = 4$ spacetime dimensions, the Planck power (luminosity) is independent of \hbar

$$\mathcal{L}_P = \frac{m_P c^2}{t_P} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Physical significance: Only in $D = 4$, can we have a formula

$$\mathcal{L}_{peak} = \mathcal{L}_P \times f(\text{scale-independent observables}).$$

We came close to observing such power

$$\mathcal{L}_P = \frac{c^5}{G} \approx 3,63 \times 10^{52} \text{ W},$$
$$\mathcal{L}_{peak} \Big|_{\text{GW150914}} \approx 3,6 \times 10^{49} \text{ W}.$$

Comments:

- In $D = 4$, gravitational wave luminosity non-extensive.
- **This talk:** Evidence for luminosity bound in quantum gravitational processes.
- Invisible in S -matrix approach, where we have a $\sqrt{\hbar G}$ -expansion.

Light Cone Gauge of Local Gravitational Subsystems

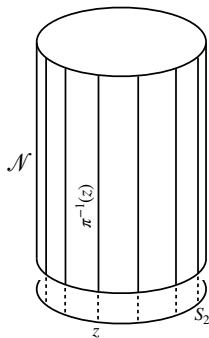
Signature (0++) metric.

$$q_{ab} = \delta_{ij} e^i_a e^j_b, \quad i, j = 1, 2.$$

Parametrisation of the dyad

$$e^i = \Omega S^i_m e^m_{(o)}.$$

- **Conformal factor** Ω parametrizes the overall scale.
- **$SL(2, \mathbb{R})$ -Holonomy** S^i_m determines the shape degrees of freedom.
- Fiducial background dyad $e^j_{(o)}$.



Choice of clock

We consider a null strip \mathcal{N} with two corners as our subsystem. No unique clock along \mathcal{N} . Convenient choice

Boundary condition at $\partial\mathcal{N} = \mathcal{C}_+ \cup \mathcal{C}_-$,

$$\mathcal{U}(\partial\mathcal{N}, z, \bar{z}) = \pm 1,$$

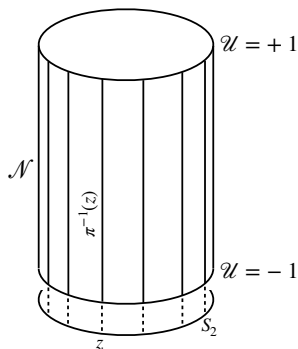
Affinity proportional to expansion

$$\partial_{\mathcal{U}}^b \nabla_b \partial_{\mathcal{U}}^a = -\frac{1}{2} \left(\Omega^{-2} \frac{d}{d\mathcal{U}} \Omega^2 \right) \partial_{\mathcal{U}}^a$$

Parametrize physical clock \mathcal{U} relative to unphysical coordinate u .

$$\partial_u \mathcal{U} \equiv \dot{\mathcal{U}} = e^{\chi} > 0.$$

The *chronoton* χ becomes a quantum reference frame (part of phase space).



Gauge group and gauge fixing:

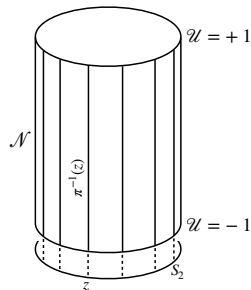
- Keep light rays fixed (a form of relationalism).
- Residual diffeomorphisms: angle dependent reparametrizations of \mathcal{U} .
- Canonical generator on phase space: Raychaudhuri equation.

Raychaudhuri equation

$$\frac{d^2}{d\mathcal{U}^2} \Omega^2 = -2\sigma\bar{\sigma} \Omega^2 e^{-2\chi}.$$

$SL(2, \mathbb{R})$ holonomy

$$\frac{d}{du} S = \left(\varphi J + (\sigma \bar{X} + \text{cc.}) \right) S.$$



$SL(2, \mathbb{R})$ generators split into $U(1)$ complex structure J and shear generators:

$$[J, X] = -2i X, \quad [J, \bar{X}] = +2i \bar{X}, \quad [X, \bar{X}] = i J.$$

Starting from the γ -action,* we obtain the symplectic structure on \mathcal{N} .

$$\Theta_{\mathcal{N}}(\delta) = \frac{1}{8\pi G} \int_{\mathcal{N}} d^3 v_o \frac{d}{du} [\Omega^2] \delta\chi + \\ + \frac{1}{8\pi\gamma G} \int_{\mathcal{N}} d^3 v_o \Omega^2 \delta\varphi + \int_{\mathcal{N}} d^3 v_o \text{Tr} (\Pi(\delta S) S^{-1}).$$

Key Observations:

- Radiative modes encoded into $T^*SL(2, \mathbb{R})$ symplectic structure.
- Second-class constraints for $\mathfrak{sl}(2, \mathbb{R})$ momentum $\Pi = LJ + c\bar{X} + \bar{c}X$

$$L = -\frac{1}{16\pi\gamma G} \frac{d}{du} \Omega^2, \quad c = -\frac{1}{8\pi\gamma G} (\gamma + i)\Omega^2 \sigma.$$

- Scalar constraint $H[N] = \Theta_{\mathcal{N}}(L_N) = 0$ generates a Virasoro algebra. See also recent results by Freidel and Ciambelli.
- Barbero-Immirzi parameter γ alters symplectic structure.

*underlying action: $S[A, e] = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4 v \left[F^{\alpha\beta}_{\alpha\beta} - \frac{1}{2\gamma} F_{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \right].$

$SU(1,1)$ Casimir and central charge

Canonical momentum dual to the $SL(2, \mathbb{R}) \simeq SU(1, 1)$ (shape) modes:

$$\Pi = LJ + c\bar{X} + \bar{c}X \in \mathfrak{su}(1, 1)$$

$SU(1, 1)$ Casimir in terms of the geometric data:

$$L^2 - c\bar{c} = \frac{1}{(16\pi\gamma G)^2} \Omega^4 (\vartheta^2 - 4(1 + \gamma^2)\sigma\bar{\sigma}).$$

What we find is:

- Bose statistics for CFT modes along light rays:
 - CFT has negative central charge.
 - Both $L^2 \leq c\bar{c}$ and $L^2 \geq c\bar{c}$ possible.
 - But resulting CFT is non-unitary.
- Fermi statistics for CFT modes along light rays:
 - CFT has positive central charge.
 - Only $L^2 \geq c\bar{c}$ *infra-Planckian* modes occur.
 - violation of unitarity can be avoided.

On physical grounds, we are led to choose Fermi statistics.

This implies

$$\vartheta^2 - 4(1 + \gamma^2)\sigma\bar{\sigma} \geq 0.$$

For semi-classical states, we expect (as expectation values)

$$\frac{\sigma\bar{\sigma}}{\vartheta^2} \leq \frac{1}{4} \frac{1}{1 + \gamma^2}.$$

This must hold for all null hypersurfaces.

We obtain luminosity bound (power per solid angle)

$$\mathcal{L}_B(u, \zeta, \bar{\zeta}) = \frac{4c^5}{G} \lim_{r \rightarrow \infty} \frac{\bar{\sigma}_{(\ell)}(u, r, \zeta, \bar{\zeta})\sigma_{(\ell)}(u, r, \zeta, \bar{\zeta})}{(\vartheta_{(\ell)}(u, r, \zeta, \bar{\zeta}))^2} \leq \frac{c^5}{G} \frac{1}{1 + \gamma^2}.$$

Utilize Bondi expansion to characterize gravitational radiation.

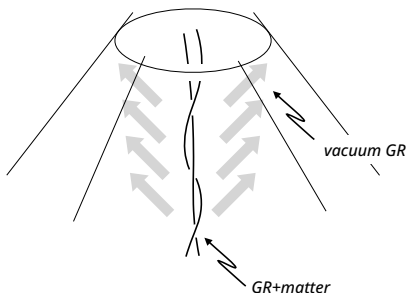
■ Bondi mass loss formula

$$\dot{M}_B(u) = -\frac{1}{4\pi G} \oint_{S_u^2 \subset \mathcal{I}_+} d^2v_o \dot{\sigma}^{(0)} \dot{\bar{\sigma}}^{(0)}.$$

■ Falloff conditions

$$\sigma_{(\ell)}(u, r, \zeta, \bar{\zeta}) = -\frac{\dot{\sigma}^{(0)}(u, \zeta, \bar{\zeta})}{r} + \mathcal{O}(r^{-2}),$$

$$\vartheta_{(\ell)}(u, r, \zeta, \bar{\zeta}) = -\frac{2}{r} + \mathcal{O}(r^{-1}).$$



Asymptotic expansion

$$\mathcal{L}_B(u, \zeta, \bar{\zeta}) = \frac{4c^5}{G} \lim_{r \rightarrow \infty} \frac{\bar{\sigma}_{(\ell)}(u, r, \zeta, \bar{\zeta}) \sigma_{(\ell)}(u, r, \zeta, \bar{\zeta})}{(\vartheta_{(\ell)}(u, r, \zeta, \bar{\zeta}))^2}.$$

In the *S-matrix* approach, the $\mathcal{O}(r^{-1})$ term in $\vartheta_{(\ell)}$ is a commuting *c-number*. In the quasi-local quantisation of gravity, it becomes a *q-number* akin to LQG area operator.

Conclusion

Take home messages

- Non-perturbative quantisation of null initial data at finite distance.
 - Spectra for geometric observables reproduce LQG discreteness of area.
 - Difference of the area at initial and final cut: number operator.
 - Barbero–Immirzi parameter activates otherwise irrelevant $SU(1, 1)$ representations.
- We strengthened earlier conjecture on Planck luminosity bound.
- Results implicitly proof that $r \rightarrow \infty$ and $\hbar \rightarrow 0$ may not commute.

