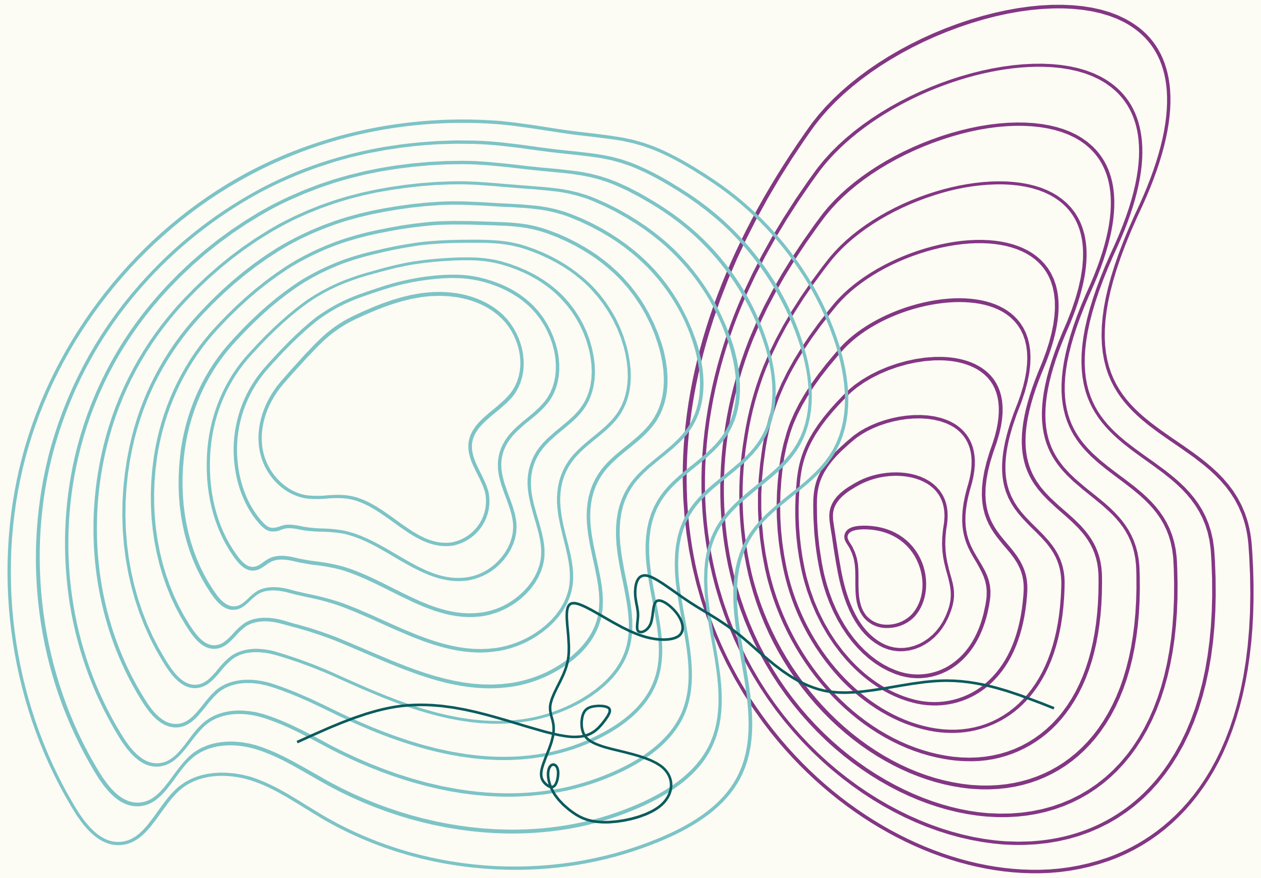
PERTURBATIVE SIGNATURES OF A SUPERIMPOSED QUANTUM UNIVERSE

... an alternative description for quantum cosmology

Bridge QG, 10 July 2025



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INTRO: QUANTUM COSMOLOGY

- **Minisuperspace models**: Wheeler-de-Witt quantisation on the reduced phase space of general relativity to obtain a quantum description of the universe

approximation to a full theory of quantum gravity

- **Problem of time** [Isham, ('93)]
    - GR is a fully constrained system → no external time parameter
    - Evolution happens w.r.t. an internal degree of freedom serving as a clock (here: perfect fluid)

[Małkiewicz, Peter, ('19)]
[Gielen, Menéndez-Pidal, ('20, `21)]
[de Cabo Martin, Małkiewicz, Peter, ('22)]
[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]
....

- **Ambiguities**
    - Quantisation: choice of clock degree of freedom, canonical variables, quantisation scheme
    - Extraction of an effective evolution of the scale factor
    - (Semiclassical) state of the universe

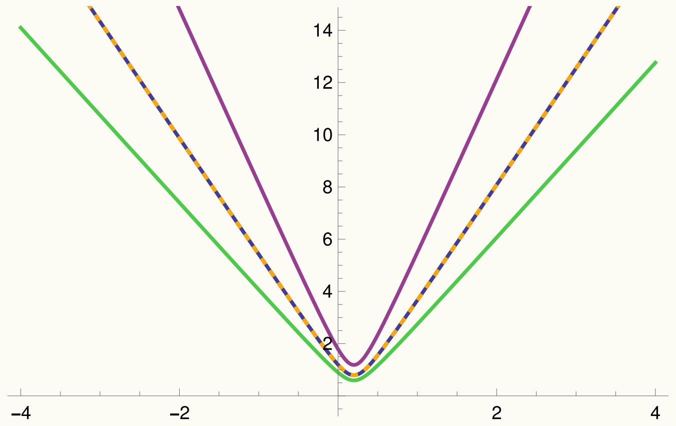
different phenomenology (e.g. singularity resolution)

OVERVIEW

Minisuperspace quantisation of FLRW spacetime

bounce

Quantum trajectories to obtain quantum corrected evolution of scale factor



Universe in a superposition $\Psi = \mathcal{N}(\psi_0 + \rho e^{\mathrm{i}\delta}\psi_1)$

Influence on perturbations?

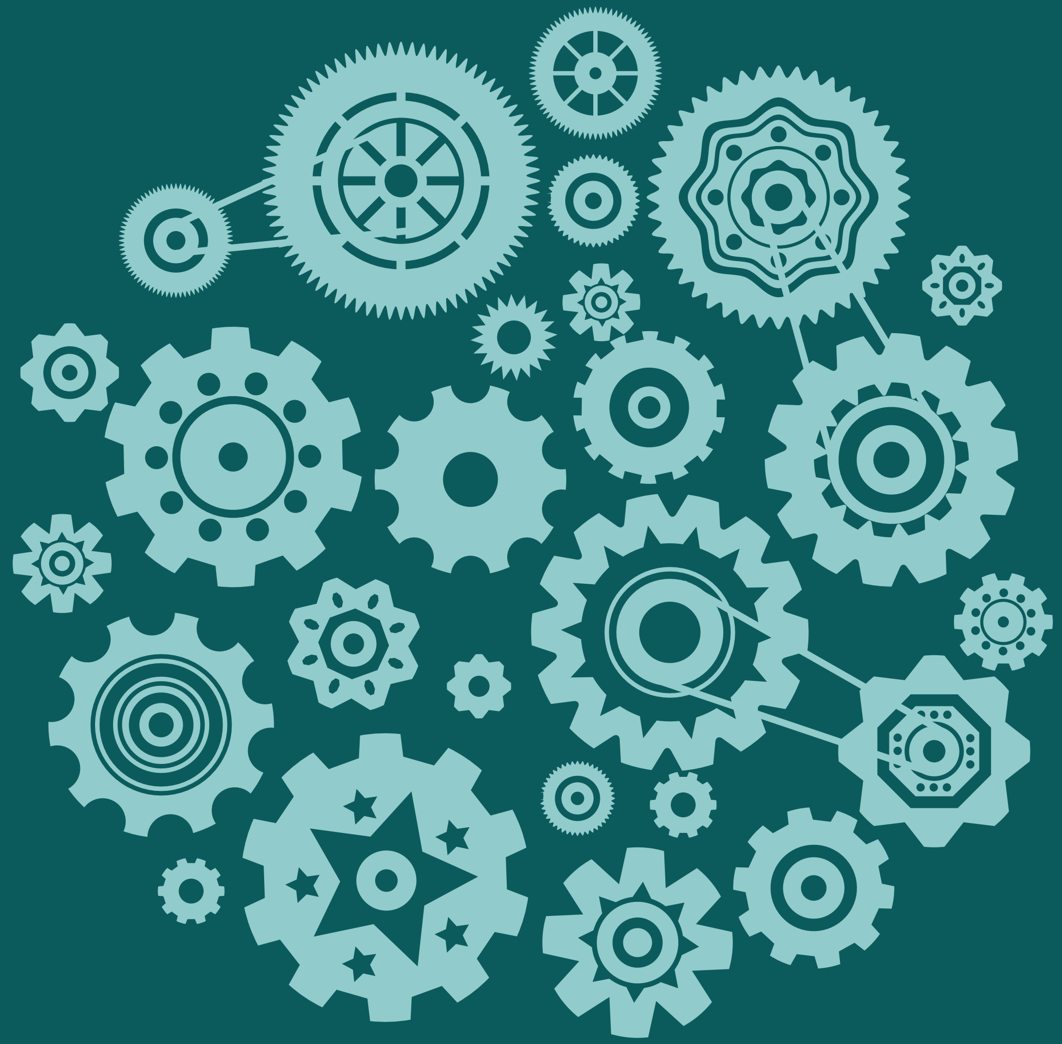
Interaction between multiple background states and their perturbations leads to non Gaussianities in perturbations

[Bergeron, Małkiewicz, Peter, ('24)]
[Bergeron, Małkiewicz, Peter, ('25)]

Trajectories give drastically different background evolution that alters the dynamics of perturbations

SETUP

Quantising the universe



THE SYSTEM

- Quantise phase space of FLRW spacetime $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

lapse

$\kappa = 8\pi G$

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$$

momentum conjugate to scale factor

scale factor

spatial section volume

- Perfect fluid as matter clock fixes the lapse $N = -a^{3w}$

equation of state parameter

- Canonical transformation to convenient variables
  - $(a, p_a) \to (q, p)$ with $p \propto a^{\frac{3}{2}(1-w)}H$, $q \propto a^{\frac{3}{2}(1-w)}$

Hubble rate

- Total Hamiltonian after deparametrisation: $\mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_T$

matter clock

QUANTISATION $\mathcal{H} \to \hat{\mathcal{H}}$

- Quantisation based on the Weyl-Heisenberg group $x \in \mathbb{R}$, $p \in \mathbb{R}$
  - $U(q,p)\psi(x) = e^{\mathrm{i}p(x-q/2)}\psi(x-q)$

- Here: $x \geq 0$, $p \in \mathbb{R}$ $\longrightarrow$ use affine group $U(q,p)\psi(x) = \frac{e^{ipx}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$

- Quantisation map $\hat{A}_f = \mathcal{N}\int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\, |q,p\rangle\, f(p,q)\, \langle q,p|$ where $|q,p\rangle = U(q,p)|\psi_0\rangle$

- Quantisation of FLRW Hamiltonian leads to a repulsive potential

$$\boxed{\hat{\mathcal{H}} \propto \hat{p}^2 + \frac{K}{\hat{q}^2} + \hat{p}_T}$$

time evolution

introduces a bounce

[Klauder, ('99)]

[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

QUANTUM TRAJECTORIES

- Use semiclassical states that satisfy the Schrödinger equation

$$\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$$

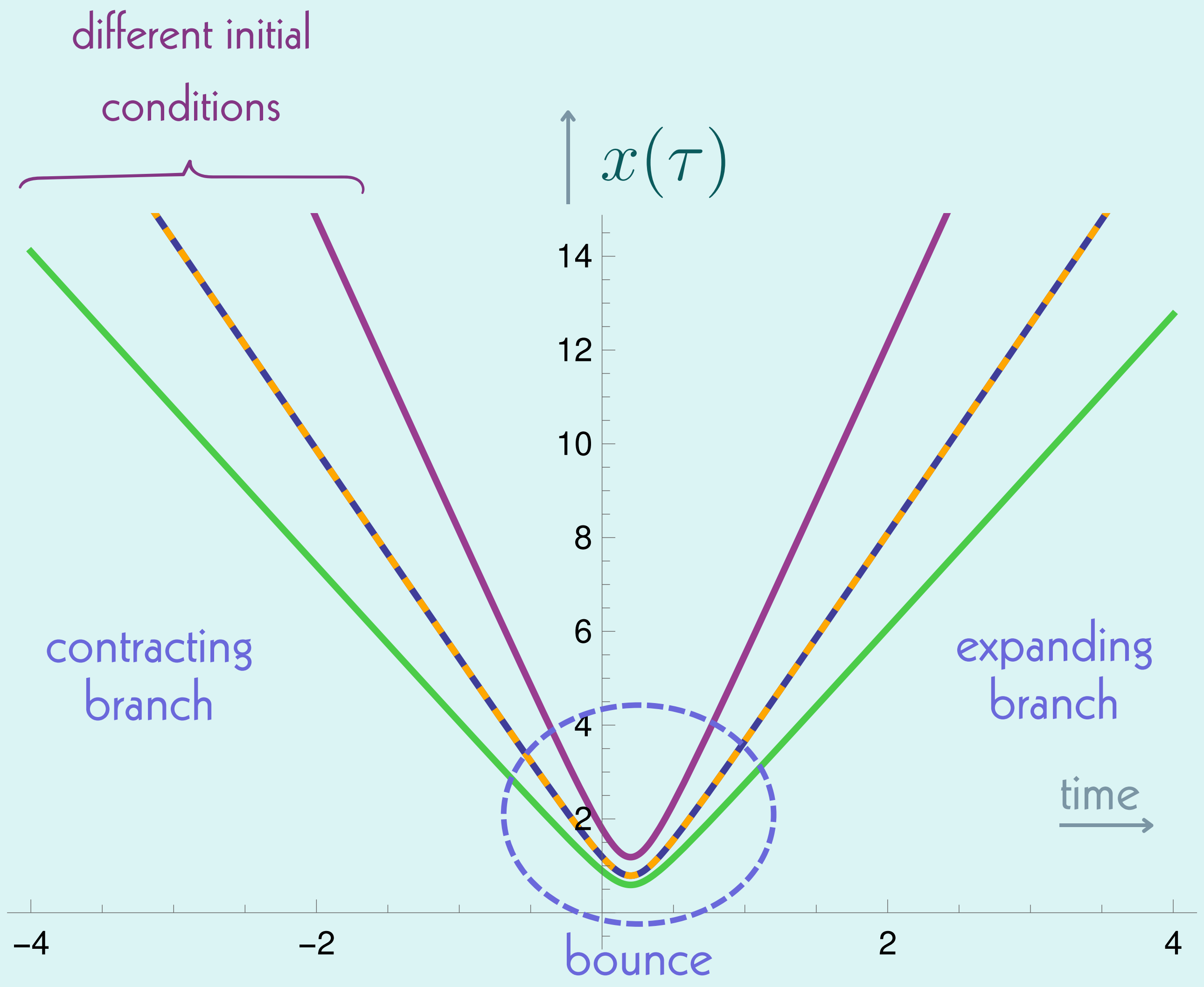
- State $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$ follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \dfrac{K}{q^2}$

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

- Continuous ensemble of trajectories can be obtained from the wave function:

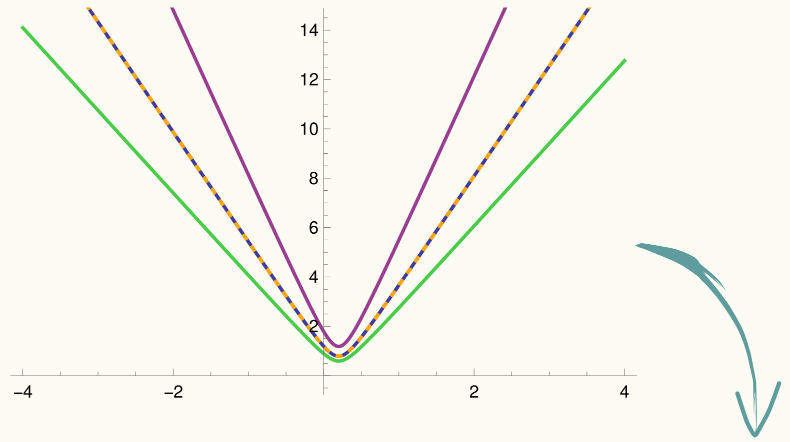
$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln\frac{\psi}{\psi^\star} \quad \text{with} \quad x \propto a^{\frac{3}{2}(1-w)}$$

  - quantum uncertainty in the initial conditions
  - assign a concrete value to the effective scale factor at all times
  - classical dynamics away from bounce

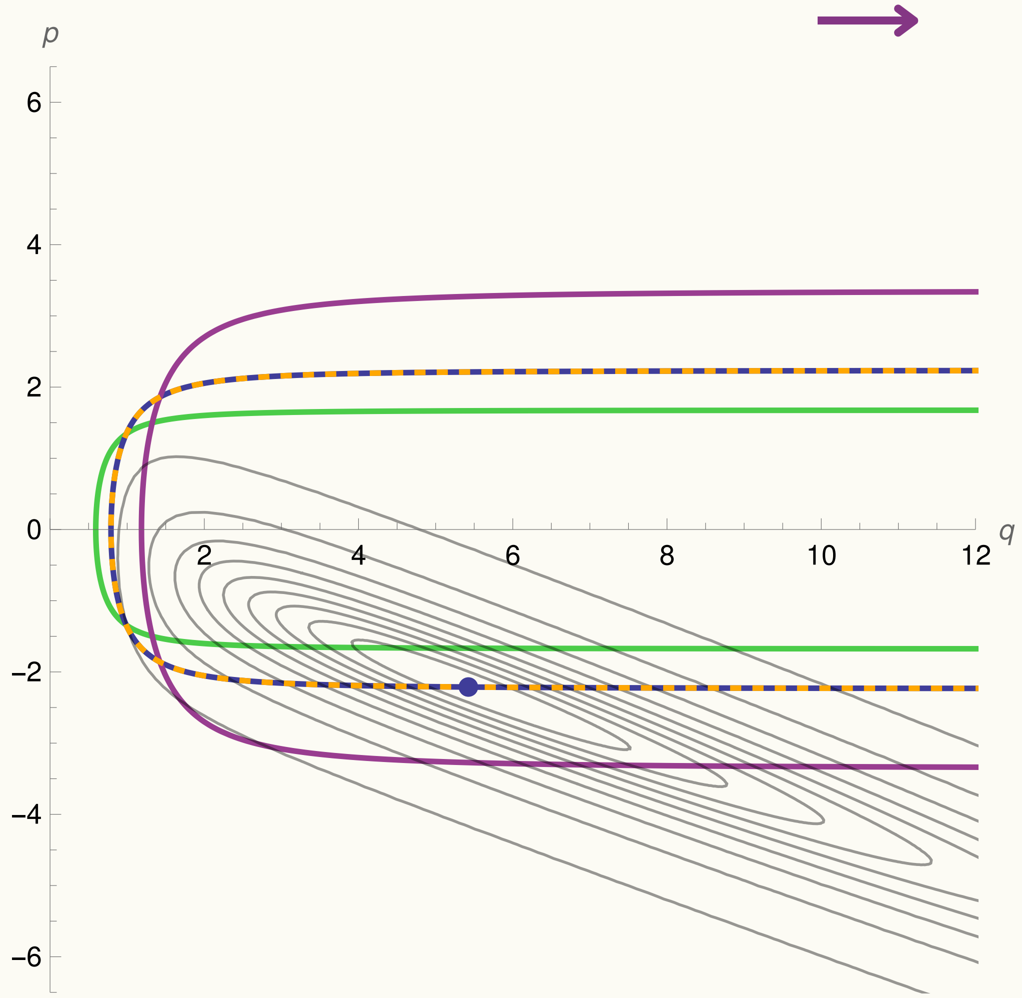


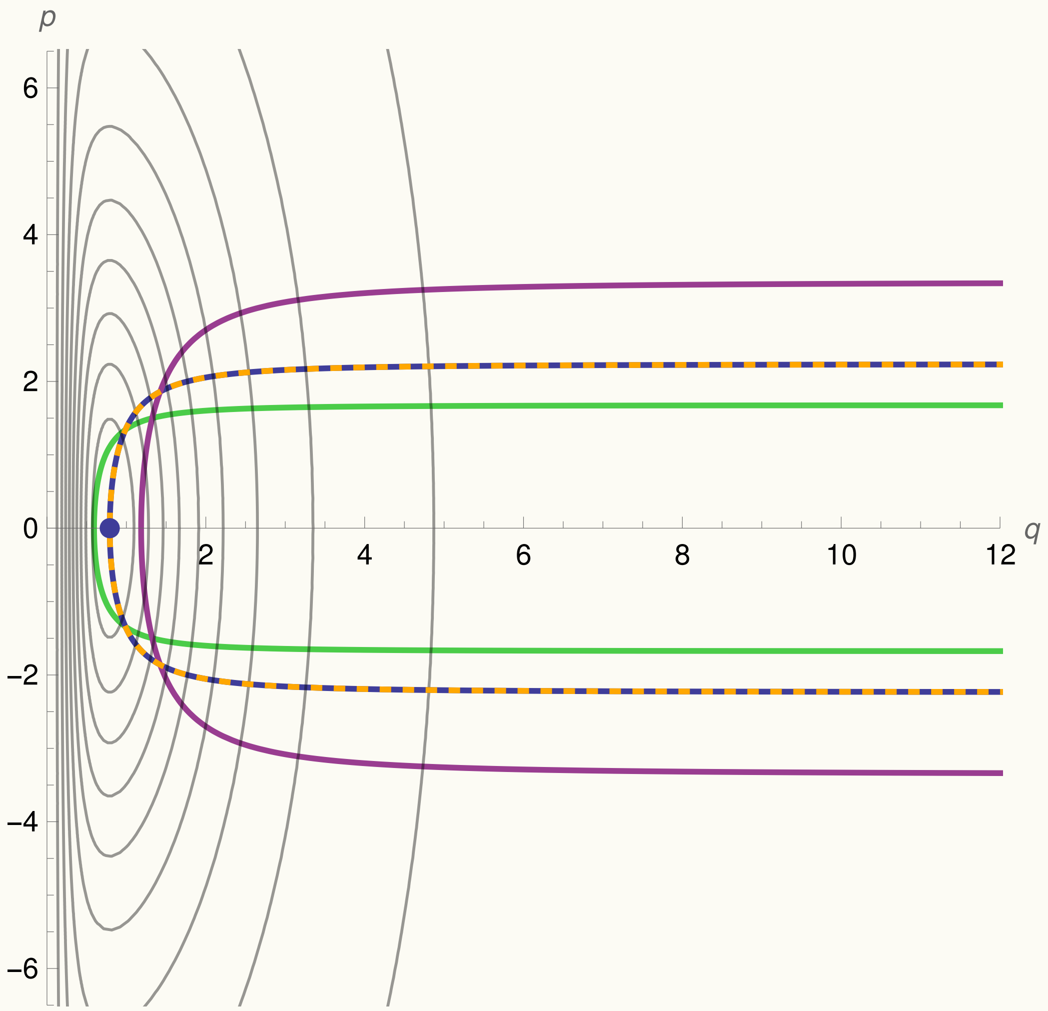
BOUNCING SINGLE STATE TRAJECTORIES

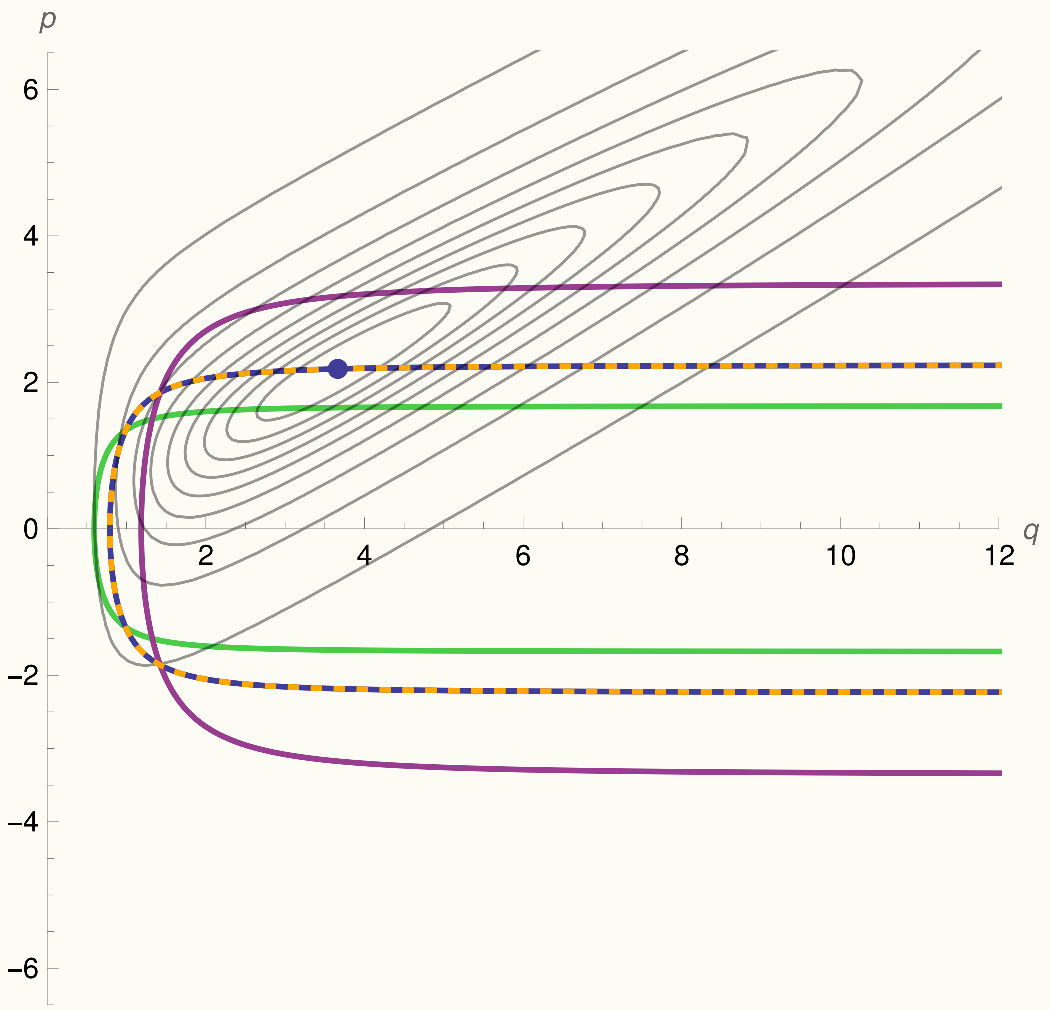
- State and trajectories follow dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\text{sem}} = p^2 + \dfrac{K}{q^2}$



$p \propto a^{\frac{3}{2}(1-w)} H$ $\quad$ $q \propto a^{\frac{3}{2}(1-w)}$







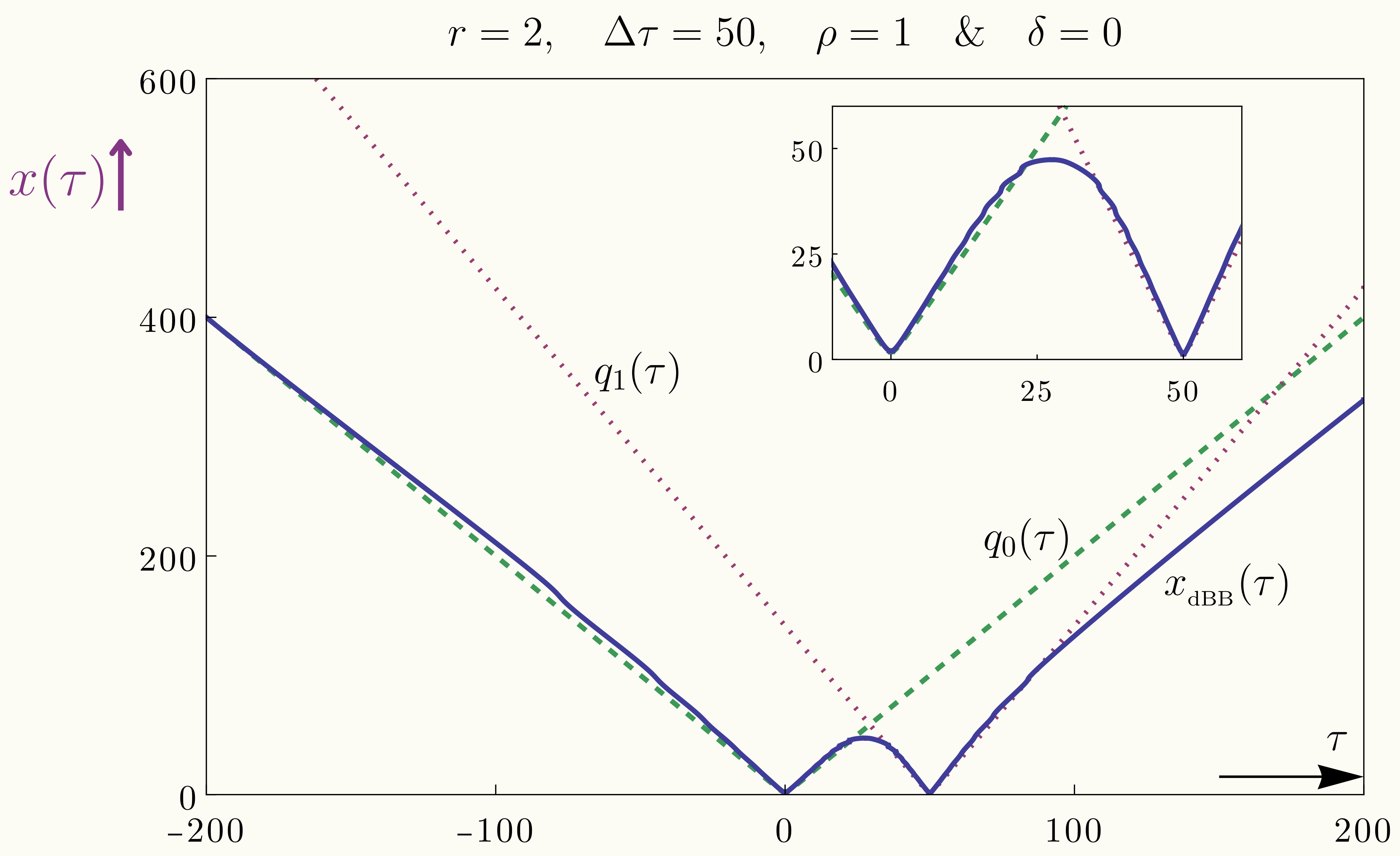
SUPERPOSITION UNIVERSE

Bouncing biverse

UNIVERSE IN A SUPERPOSITION

$$\Psi = \mathcal{N} \sum_n \alpha_n \psi_n$$

- Biverse: $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

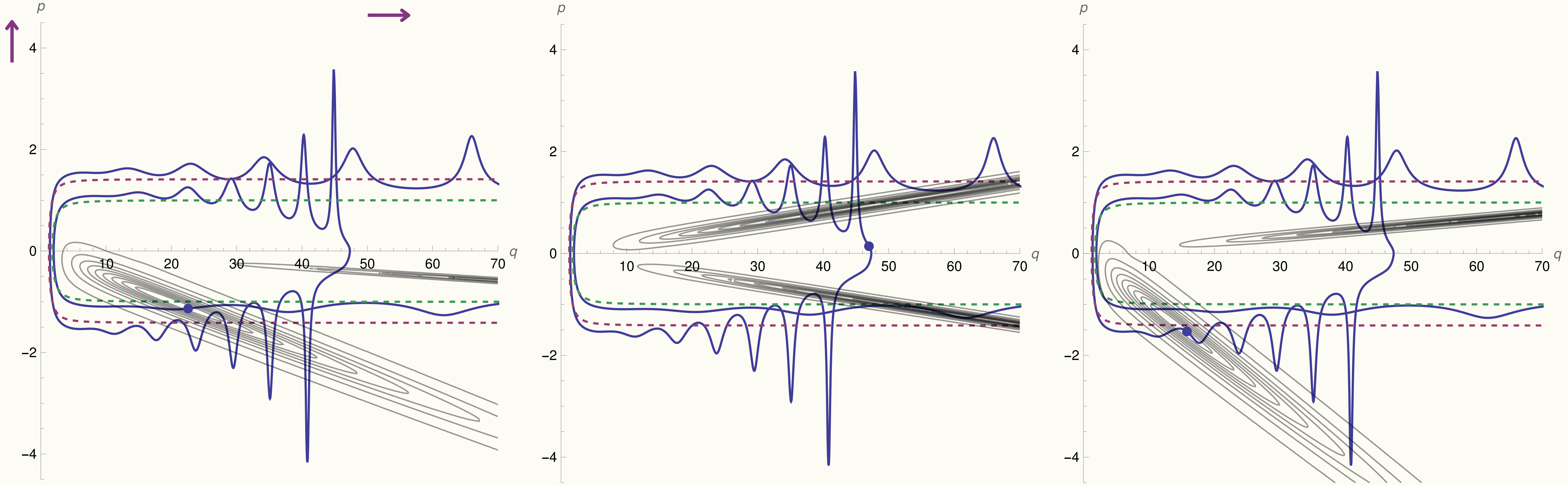


- Start on semiclassical trajectory
- Parameters:
  - r ◦ ratio of late time momenta of semiclassical solutions
  - $\Delta\tau$ ◦ difference in bounce times
  - $\rho,\ \delta$ ◦ contribution of second wave function

UNIVERSE IN A SUPERPOSITION

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{\mathrm{i}\delta}\psi_1)$

$p \propto a^{\frac{3}{2}(1-w)} H$ $\quad$ $q \propto a^{\frac{3}{2}(1-w)}$



- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

PERTURBATIONS FROM BIVERSE TRAJECTORIES

- Consider tensor perturbations $$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \left[\delta_{ij} + h_{ij}(\eta,\vec{x})\right]\mathrm{d}x^i\mathrm{d}x^j\right)$$

[Peter, Pinho, Nelson Pinto-Neto, ('05)]

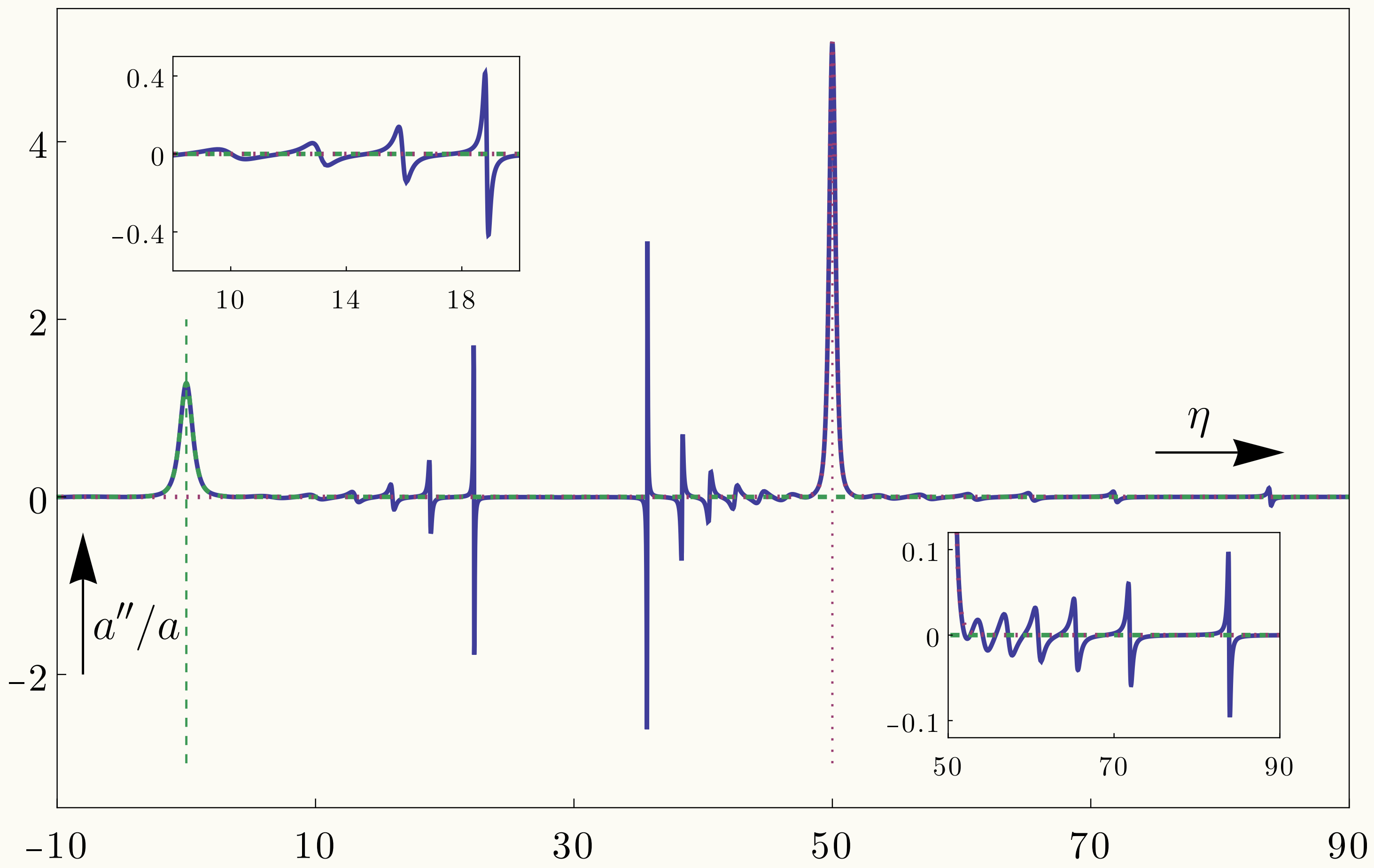
[Peter, Pinho, Nelson Pinto-Neto, ('06)]

- Dynamics of perturbation modes $$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$$

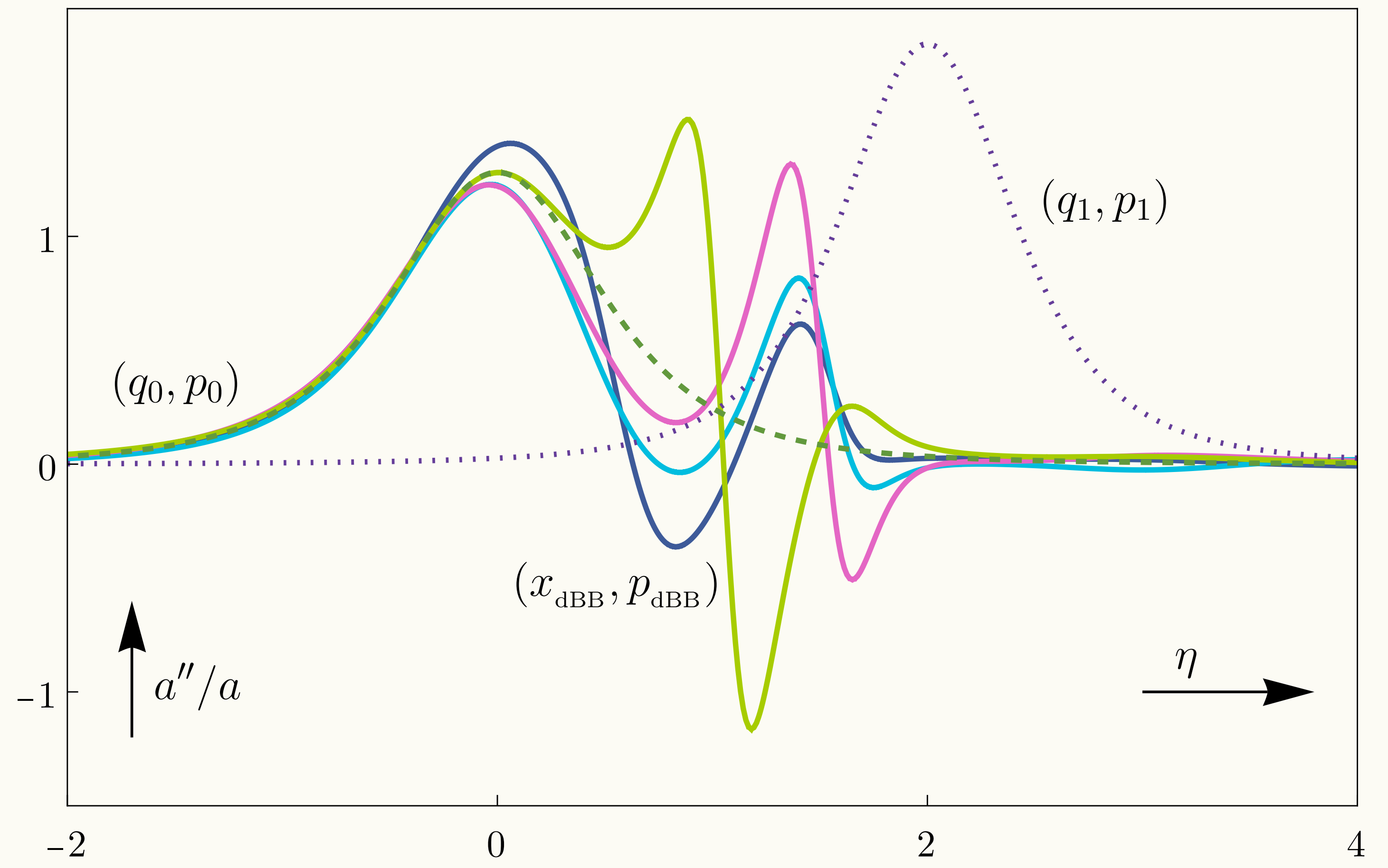
$h_{ij} \propto \mu_{ij}/a$

modified by trajectories

$r = 2, \quad \Delta\tau = 50, \quad \rho = 1 \quad \& \quad \delta = 0$



$r = 1.2, \quad \Delta\tau = 2, \quad \rho = 0.2 \quad \& \quad \delta \in \{0, \pi/2, \pi, 3\pi/2\}$

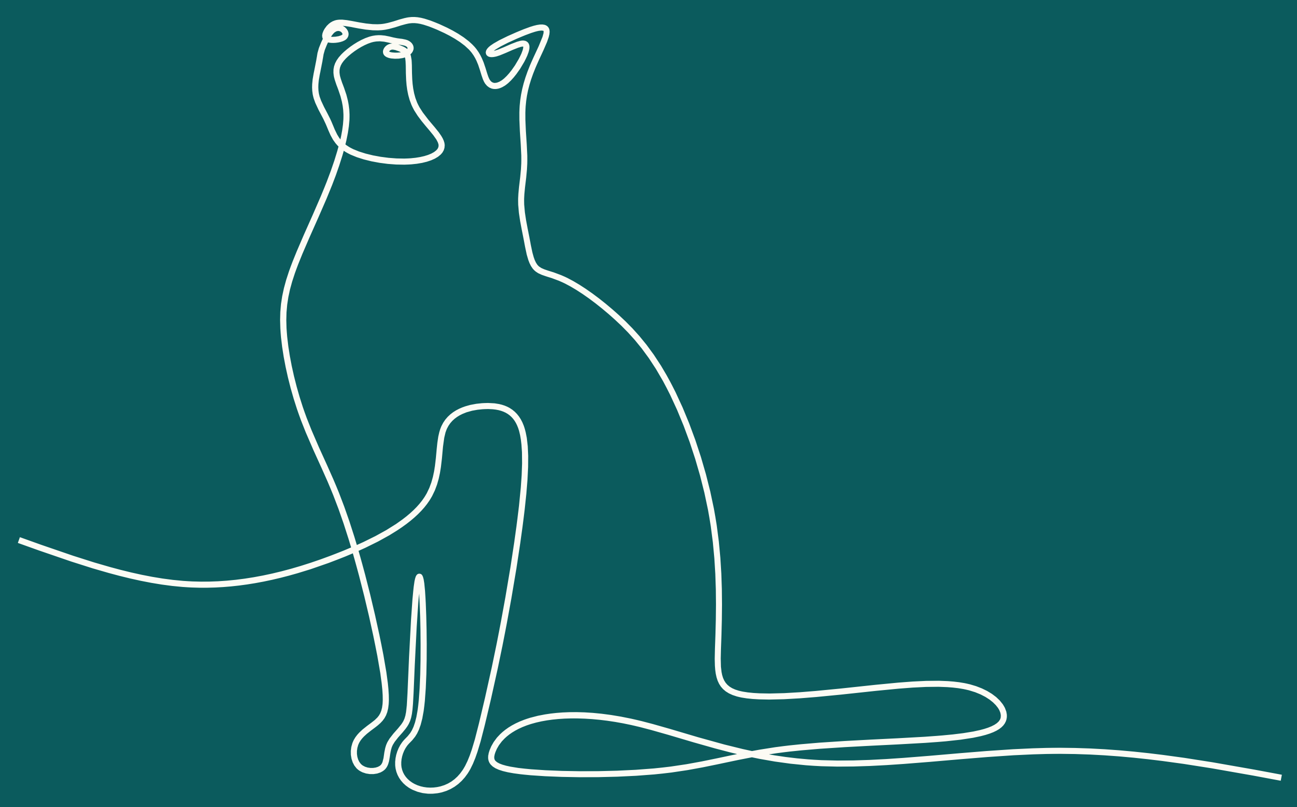


- Potential resulting from the biverse trajectories differs from single state case which would dictate perturbative dynamics in each universe separately if effective scale factor was obtained from projection onto a state

[Bergeron, Małkiewicz, Peter, ('24)]

CONCLUSION

...and next steps

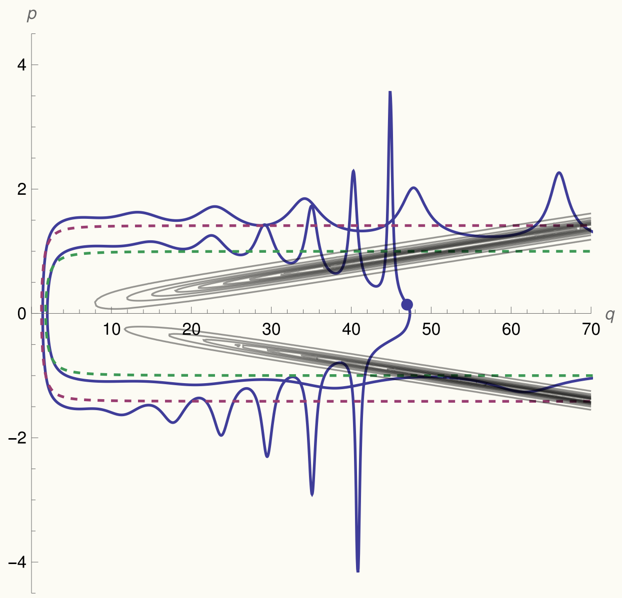


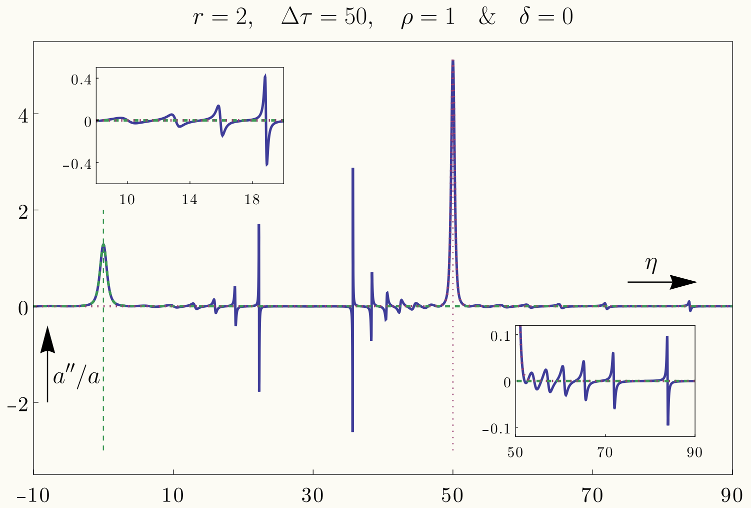
CONCLUSION

- Affinely quantise an FLRW spacetime to obtain bouncing trajectories
  - trajectories assign unambiguous value to the scale factor at all times

- Trajectories for a universe in a superposition introduce distinct features in the scale factor evolution

- These features affect the evolution of perturbations and may therefore have phenomenological consequences





→ *Next:* detailed study of perturbations and the resulting power spectra

Quantum cosmology offers a unique scenario where quantum trajectories could lead to different phenomenological implications

THANK YOU!

Questions...?

BACKUP