# Detectability of post-Newtonian classical and quantum gravity via quantum clock interferometry

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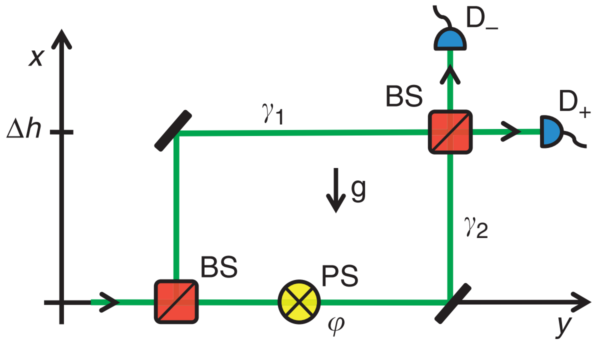
# Background

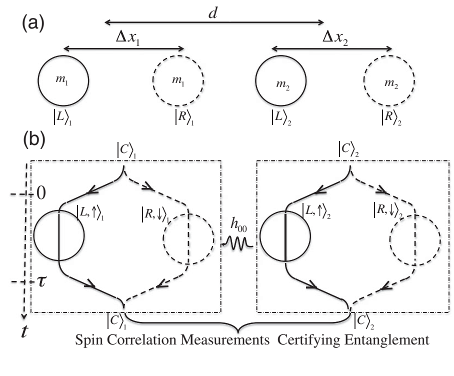
- Intersection b/w GR+QM is still unknown
- Low-energy experiments have been proposed (“table-top”)

Quantum probe of
general relativistic proper time




(Zych et al. Nat. Comm. 2, 505 (2011))

Witnessing
quantumness of gravity




(Bose et al. PRL 119, 240401 (2017))
(Marletto&Vedral PRL 119, 240402 (2017))

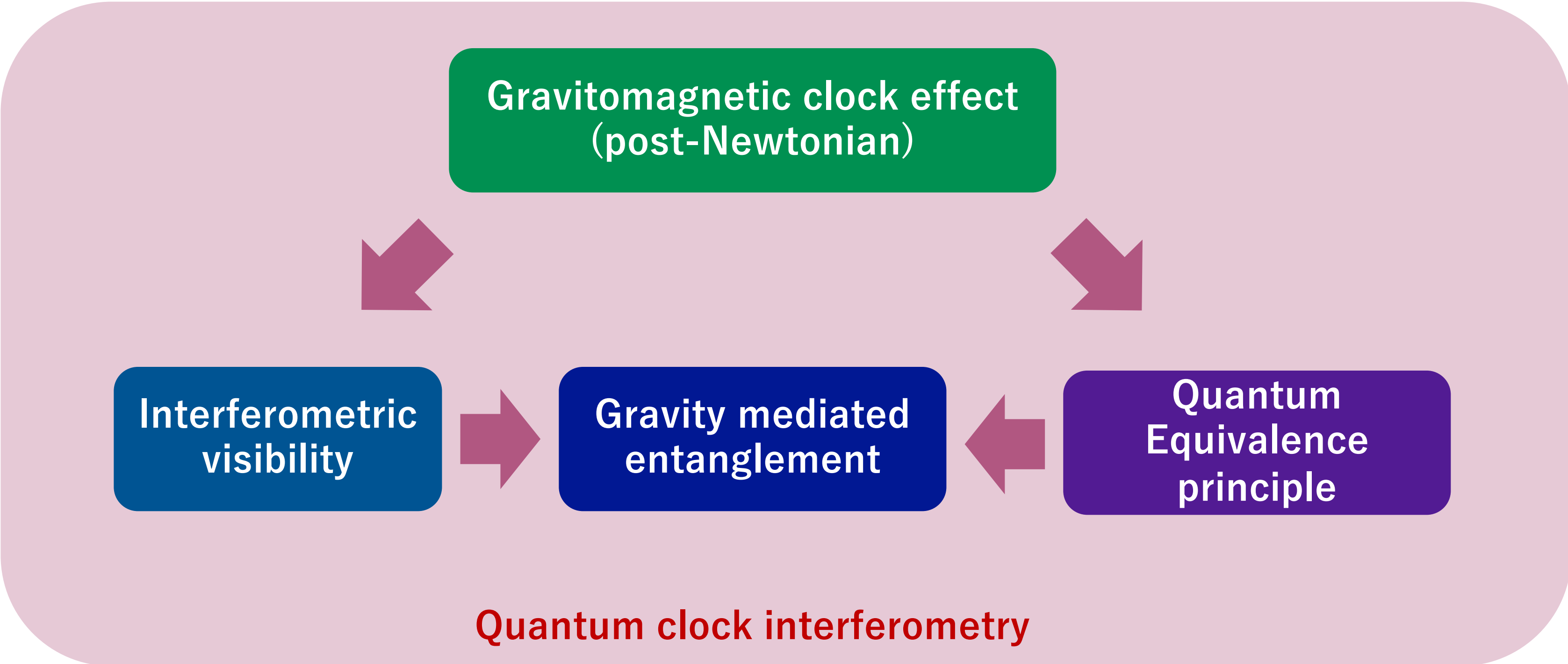
# Post-Newtonian gravity + QM?

- Previously
  - Only Newtonian (with few exceptions)
  - Low-energy regime accessible in laboratories

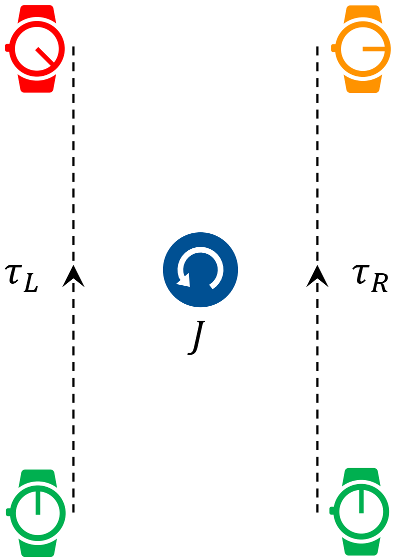
- But if restricted in the Newtonian regime
  - ✗ "Dynamical" aspects of gravity
  - ✗ Quantumness of gravity ⇒ quantumness of spacetime

- Detectability of QM+gravity ? in a regime where
  - Lower-order post-Newtonian effect
  - Relatively low-energy scale

# Outline of this research



# Gravitomagnetic clock effect




- Frame dragging
  - Spacetime around a rotating mass is “dragged” (Lense-Thirring 1918)
  - Genuinely post-Newtonian effect
  - Caused by $g_{0i} \neq 0$

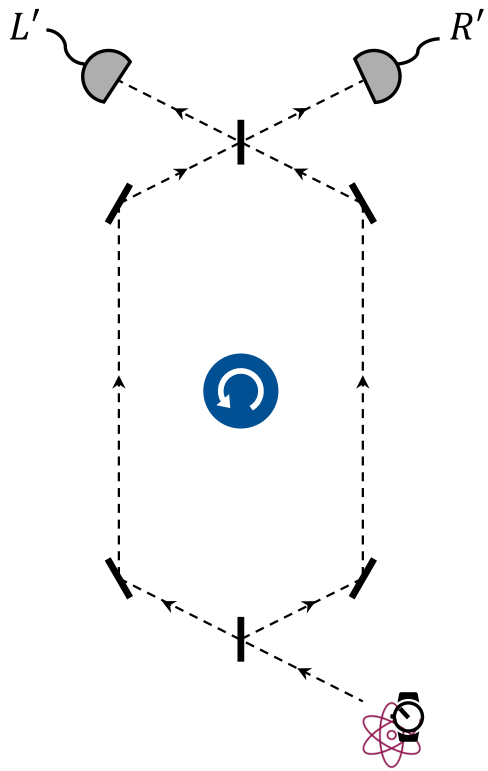
- Gravitomagnetic clock effect
  - Proper time: corotating<counterrotating

Spacetime metric at a distance from the source:

$$-(cd\tau)^2 \approx \underbrace{-\left(1-\frac{2GM}{c^2r}\right)(cdt)^2+\left(1+\frac{2GM}{c^2r}\right)dr^2+r^2\left(d\theta^2+\sin\theta^2\,d\phi^2\right)}_{\text{Schwarzschild components }(\supset\text{Newtonian})}\underbrace{-\frac{4GJ}{c^3r}\sin\theta^2\,(cdt)d\phi}_{\text{Frame dragging}}$$

# Interferometric visibility experiment

(An extension of IV experiment in homogeneous gravity by Zych et al. Nat. Comm. 2, 505 (2011) )




- Setup:
  - Mach-Zehnder type interferometry
  - Quantum clock particle
  - Source mass is rotating in a fixed direction

# Interferometric visibility experiment

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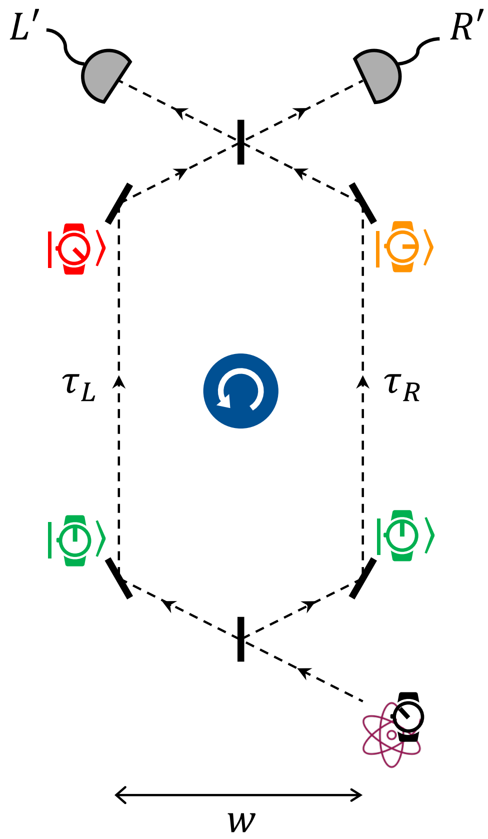
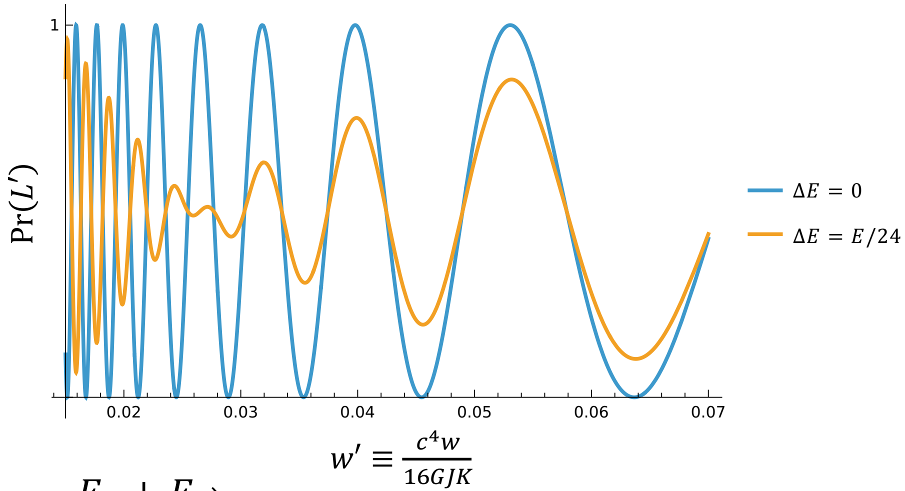



- Setup:
  - Mach-Zehnder type interferometry
  - Quantum clock particle
  - Source mass is rotating in a fixed direction
- Idea:
  - Interference pattern as a function of width $w$
  - Gravitomagnetic clock effect $\tau_L > \tau_R$, $|\ \rangle \neq |\ \rangle$
  - Which-path information → visibility modulation
- Point:
  - Newtonian contributions cancel.
    → Any observed effect is post-Newtonian.

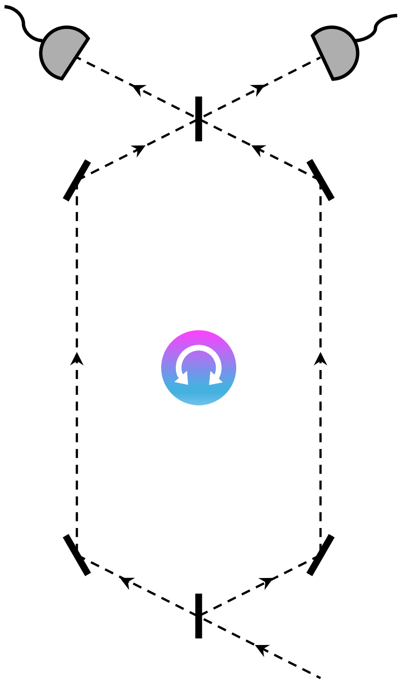
# Interference pattern

- Description of the clock:
  - Rest Hamiltonian $\widehat{H}_{rest} = E_g|g\rangle\langle g| + E_e|e\rangle\langle e|$
  - Initial state $\frac{|g\rangle+|e\rangle}{\sqrt{2}}$
- Evolution of Internal State:
  - $U_L = \exp\left(\frac{1}{i\hbar}\widehat{H}_{rest}\tau_L\right),\ U_R = \exp\left(\frac{1}{i\hbar}\widehat{H}_{rest}\tau_R\right)$
  - Relative evolution $U_L^{\dagger}U_R = \exp\left(\frac{1}{i\hbar}\widehat{H}_{rest}\Delta\tau\right)$
  - $\Delta\tau \equiv \tau_L - \tau_R = \frac{16GJK}{c^4 w}$ (post-Newtonian)
- Detection Probabilities:
  - $\Pr(L') = \frac{1}{2}\left(1 + \cos\frac{\Delta E\Delta\tau}{\hbar}\cos\frac{\bar{E}\Delta\tau}{\hbar}\right)$
  - $\Pr(R') = \frac{1}{2}\left(1 - \cos\frac{\Delta E\Delta\tau}{\hbar}\cos\frac{\bar{E}\Delta\tau}{\hbar}\right)$

$$\left(\Delta E \equiv E_e - E_g, \bar{E} \equiv \frac{E_g + E_e}{2}\right)$$



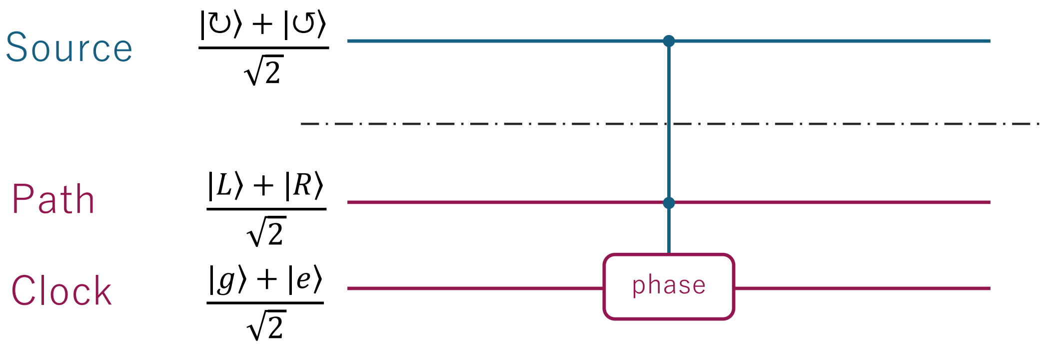
# GME experiment



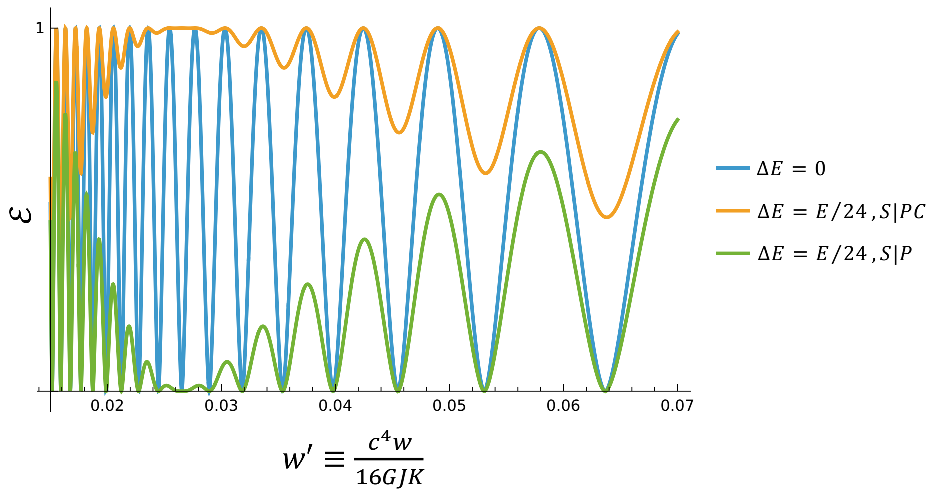
- Setup:
  - Source mass is in a superposition of opposite rotational directions $\frac{|\circlearrowright\rangle+|\circlearrowleft\rangle}{\sqrt{2}}$
  - Proper times $\tau_L, \tau_R$ depend on the rotational directions
    $\rightarrow$ entanglement b/w source & clock particle
  - Field-disentangling scheme
    (Higgins et al. PRD 110, L101901 (2024))

- Assumptions:
  - “superposition of classical gravitational fields”
    (Giacomini et al. arXiv:2012.13754 (2020))
  - Negligible backaction of the clock

# Entanglement generation




- $\mathcal{E}^{S|PC} = H\big(\Pr(L'), \Pr(R')\big)$
- $\mathcal{E}^{S|P} = H\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \left(\cos\frac{\Delta E \Delta\tau}{\hbar}\sin\frac{\bar{E}\Delta\tau}{\hbar}\right)^2}\right)$
- If $\Delta E = 0$ : $\mathcal{E}^{S|PC} = \mathcal{E}^{S|P} = H\left(\frac{1}{2} \pm \frac{1}{2}\cos\left(\frac{\bar{E}\Delta\tau}{\hbar}\right)\right)$
- ($H$: Shannon entropy)



# QEP in nonstatic spacetime

(An extension of QEP in static spacetime by Zych et al. Nat. Phys. 14, 1027 (2018))

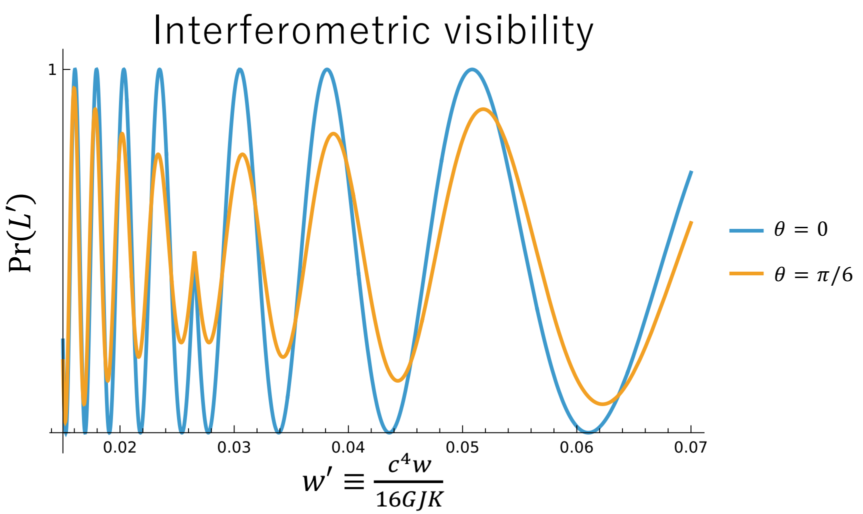
- Time evolution of the clock particle: "Routhian"
  - Internal DoF → Hamiltonian, External DoF → Lagrangian
  - $\hat{R} = \hat{H}_{rest} \frac{d\tau}{dt} = \hat{H}_{rest} \left(1 \underbrace{- \frac{v^2}{c^2}}_{\text{Time dilation}} \underbrace{+ \frac{\Phi}{c^2} - \frac{\Phi^2}{c^4}}_{\text{Gravitational redshift}} \underbrace{- \sum_i \frac{g_{0i}}{c g_{00}} \frac{dx^i}{dt}}_{\text{Frame dragging}}\right)$
  - Test theory: $\hat{R} = \underbrace{\hat{H}_N \left(1 - \frac{v^2}{c^2} + \frac{\Phi}{c^2} - \frac{\Phi^2}{c^4}\right)}_{\text{QEP holds in Newtonian limit}} - \hat{H}_f \sum_i \frac{g_{0i}}{c g_{00}} \frac{dx^i}{dt}$
- QEP in nonstatic spacetime
  - $\hat{H}_N = \hat{H}_f$, in particular $[\hat{H}_N, \hat{H}_f] = 0$

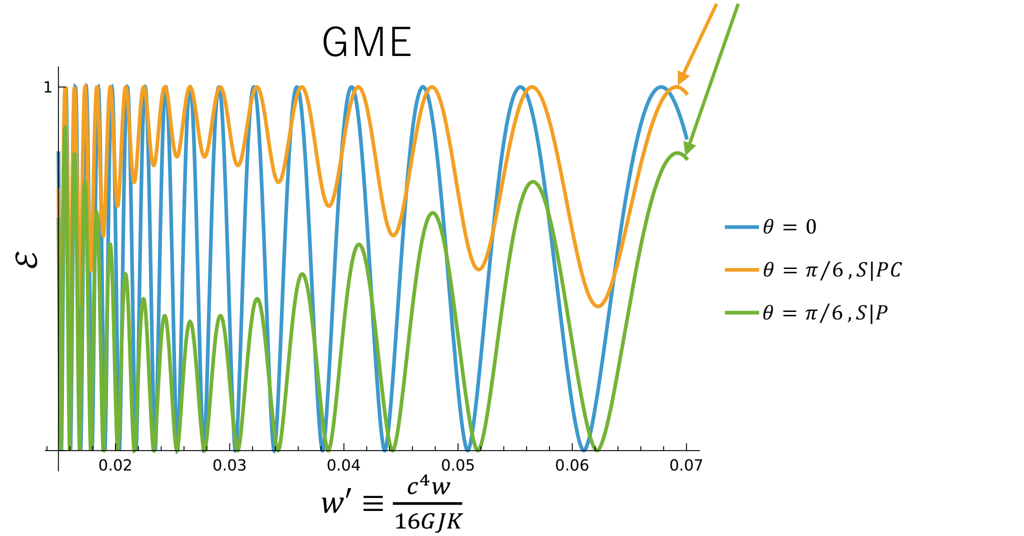
**QEP ensures geometric picture of gravity**

# QEP violations in clock interferometry

- Evolution of Internal State
  - Hamiltonians $\hat{H}_N = E_g|g\rangle\langle g| + E_e|e\rangle\langle e|,\ \hat{H}_f = E_g|g'\rangle\langle g'| + E_e|e'\rangle\langle e'|$
  - Initial state $|g\rangle = \cos\theta|g'\rangle + e^{i\varphi}\sin\theta\,|e'\rangle$
  - Relative evolution $U_L^\dagger U_R \approx \exp\left(\frac{1}{i\hbar}\hat{H}_f\Delta\tau\right)$
  - $[\hat{H}_N, \hat{H}_f] \neq 0 \iff |\langle g|U_L^\dagger U_R|g\rangle| < 1$

Gravity is
✓ quantum
✗ spacetime geometry



# Feasibility?

- Back-of-the-envelope calculation
  - Typical transition frequency of quantum clock: $\frac{\Delta E}{\hbar} \sim 10^{15}$
  - Typical rest-mass energy of quantum clock: $\frac{E}{\hbar} \sim 10^{26}$
  - The width of the interferometer $w \sim 1mm$
  - Angular momentum of the source $J$
  - Amplitude modulation $\frac{\Delta E \Delta \tau}{\hbar} \sim J \cdot 10^{-26}$, phase shift $\frac{\bar{E} \Delta \tau}{\hbar} \sim J \cdot 10^{-15}$
  - Cf. Earth's angular momentum $J \sim 10^{33}$

The effect is too small in any realistic parameter regions.
No more than *gedankenexperiment*!

# What does the unfeasibility possibly mean?

1. Our scheme is not good.
   → Alternative experimental scheme would be necessary.

2. Quantum nature of post-Newtonian gravity cannot be detected at low energy (even if Newtonian gravity can be).
   → High-energy experiment is necessary.

3. Post-Newtonian gravity is classical (even if Newtonian gravity is quantum).
   → Quantumness of gravity is regime-dependent.

# Summary

arXiv:2506.15014

## What we have done:

- Proposed an experimental schemes to detect post-Newtonian gravitational effect in quantum clock interferometry
- Interferometric visibility / GME / quantum equivalence principle
- The effects are too small to be detected in realistic scenario
- Discussed possible interpretations of the unfeasibility

## Related researches:

- Interferometric detection of QCE by spin ½ particle
  (Basso&Maziero GRG 53, 70 (2021))
- Gravitational interaction between angular momenta to mediate entanglement
  (Lantaño et al. arXiv:2409.01364)
- Post-Newtonian gravitational effects from delocalized quantum source
  (Chen&Giacomini arXiv:2402.10288)

# GME Experiment: Assumptions

(Adaptation from Giacomini et al. arXiv:2012.13754 (2020))

- Distinguishable states of the gravitational field are assigned different quantum state vectors.
- Each well-defined gravitational field is described by general relativity.
- The superposition principle holds for such gravitational fields.