Correspondence between Modified Gravity and Generalized Uncertainty Principle

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Motivation and plan of the talk

Motivation:

- To understand effects of gravity on thermodynamic systems
- To constrain theories of modified and quantum gravity

Plan of the talk:

- Description of thermodynamic systems in the presence of gravity
- Modified Gravity in the lab?
- Seismology as a tool to test fundamental interactions: new COST Action FuSe CA24101

Gravity vs matter: motivation based on a number of indications

- Effective quantities: opacity[1], ...
- Modifications introduced by modified gravity to pressure[2]
- Chemical reactions rates depend on gravity[3]
- Specific heat and crystallization depend on modified gravity[4]
- Chemical potential depends on gravity[5]
- Elementary particle interactions modified by modified gravity (dependence of the metric on the local energy-momentum distributions[6]
- EoS depends on relativistic effects introduced by GR[7]
- Thermonuclear processes...?[8]
- Fermi and Bose equations of state depend on (modified/quantum) gravity[9]

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[1] J. Sakstein, PRD 92 (2015) 124045; ...

[2] H-Ch. Kim, PRD 89 (2014) 064001

[3] P. Lecca, J. Phys.: Conf. Ser. 2090 (2021) 012034

[4] S. Kalita, L. Sarmah, AW, PRD 107 (2023) 4, 044072

[5] I.K. Kulikov, P.I. Pronin, Int. J. Theor. Phys. 34, (1995) 9

[6] A.D.I Latorre, G.J. Olmo, M. Ronco, PRB 780, 294 (2018)

[7] G.M. Hossain, S. Mandal, JCAP 02 (2021) 026; PRD 104 (2021) 123005

[8] J. Sakstein, PRD 92 (2015) 124045; AW, PRD 103 (2021) 4, 044037; M. Guerrero, AW, in preparation

[9] AW, PRD 107 (2023) 4, 044025; A. Pachol, AW, Class.Quant.Grav. 40 (2023) 19, 195021; AW, PRD 109 (2024) 2, 024011; AW PRD 109 (2024) 124031

Quantum gravity[10]

Observation 1:

Modifies Heisenberg uncertainty principle (GUP, EUP)

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\Big(1 + \text{modification}\Big)$$

or/and dispersion relation

$$E^2 + p^2\Big(1 + \text{modification}\Big) = m^2$$

[10]LQG, Doubly Special Relativity, String Theory, Noncommunative geometry,...

Quantum gravity and thermodynamics

Observation 2:

The weighted phase space volume is modified (D - dim of the phase space).

$$\frac{d^D\mathbf{x}d^D\mathbf{p}}{1+\text{modification}}$$

Consequence: modified partition function ($z = e^{\mu/k_B T}$)

$$\ln \mathcal{Z} = \frac{V}{(2\pi\hbar)^3}\frac{g}{\pm 1}\int \ln\left(1 \pm z e^{-E/k_B T}\right)\frac{d^3p}{1+\text{modification}}$$

Conclusion: Quantum Gravity modifies equations of state since

$$P = k_B T \frac{\partial}{\partial V}\ln \mathcal{Z},$$
$$n = k_B T \frac{\partial}{\partial \mu}\ln \mathcal{Z}\ |_{T,V}\,,$$
$$U = k_B T^2 \frac{\partial}{\partial T}\ln \mathcal{Z}\ |_{z,V}$$

Observation 3: MG as an effective theory derived from QG

Palatini $f(R)$ and EiBI gravity[11] - effective approach

It turns out that Palatini-like gravity in the weak limit corresponds to linear GUP

Poisson equation - the additional term can be interpreted as a modification to the matter fluid

$$\nabla^2\phi = \frac{\kappa}{2}\left(\rho + \bar{\alpha}\nabla^2\rho\right)$$

The partition function in the grand-canonical ensemble:

$$\ln Z = \frac{V}{(2\pi\hbar)^3}\frac{g}{a}\int f(E)\frac{d^3p}{(1-\sigma p)^b}$$

So the deformation of the phase space is

$$\frac{1}{(2\pi\hbar)^3}\int\frac{d^3x d^3p}{(1-\sigma p)^b},$$

$\rightarrow$ linear GUP with $b = 1$.

The covariant form of linear GUP which may correspond to the Palatini-like gravity could take the following form:

$$[x_\mu, p_\nu] = i\hbar\left[g_{\mu\nu} - \alpha\left(p g_{\mu\nu} + \frac{p_\mu p_\nu}{p}\right)\right].$$

[11] AW, PRD 109 (2024) 2, 024011; A Farag Ali, AW, CGQ 41 (2024) 10, 105001

Modified Gravity and tabletop experiments[12] - liquid helium

The non-interacting Bose-Einstein condensate imposes $-10^{12} \lesssim \sigma \lesssim 3\times 10^{24}$ s/kg m for the linear GUP and $-10^{-1} \lesssim \bar{\beta} \lesssim 10^{11}$ m^2 for Palatini gravity.

Landau model (in An Introduction to the Theory of Superfluidity (CRC Press, 2018) pp. 185-204.)

$$\hbar\omega = \begin{cases} \hbar c k & \text{if } k << k_0, \\ \Delta + \frac{\hbar^2 (k-k_0)^2}{2\gamma} & \text{if } k \approx k_0, \end{cases}$$

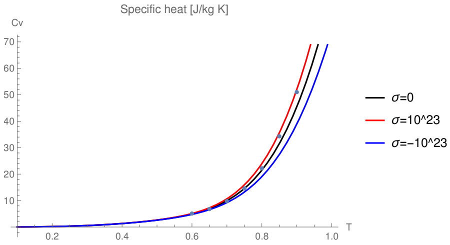
The quantum states of He^4 close to the ground state → the states of a non-interacting gas with energy levels

$$U = E_0 + \frac{V}{2\pi^2}\int_0^\infty \frac{k^2\hbar\omega_k}{e^{\beta\hbar\omega_k}-1}\frac{dk}{(1-\sigma\hbar k)}.$$

Total specific heat $C_V = \frac{\partial U}{\partial T}|_V$ (in $\text{Jkg}^{-1}\text{K}^{-1}$)

$$C_{He^4} = 20.7T^3 + \frac{387\times 10^3}{T^{3/2}}e^{-8.85/T} + \sigma\left(5.73\times 10^{-24}T^4 + \frac{7.83\times 10^{-19}}{T^{3/2}}e^{-8.85/T}\right)$$

$-10^{23} \lesssim \sigma \lesssim 10^{23}$ s/kg m and $-10^{9} \lesssim \bar{\beta} \lesssim 10^{9}$ m^2



Specific heat of liquid helium in low temperatures. The data points taken from H. Kramers, in Progress in Low Temperature Physics, Vol. 2 (Elsevier, 1957) pp. 59-82.

12 AW, PRD 109 (2024) 12, 124031

Testing Fundamental Physics with Seismology (FuSe) CA24101

Credit: Gerardo Tejada Saracho

CA24101 FuSe: join us!

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CA24101 - Testing Fundamental Physics with Seismology (FuSe)

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Description | Management Committee | Main Contacts and Leadership | Working Groups and Membership

Description

The FuSe Action tackles challenges in fundamental physics by exploring seismic phenomena and earthquake precursors, providing new opportunities for testing. It aims to bridge the gap between fundamental physicists and Earth scientists, leveraging advanced technologies such as Big Data, machine learning, and AI, and working with small technological enterprises to translate theoretical insights into practical applications.

At the heart of FuSe is the belief that seismic phenomena could reveal new aspects of fundamental interactions and lead to the discovery of new physics. By analysing seismic data and studying the underlying physical principles, FuSe aims to explore imprints of unknown physics that may be embedded in these natural processes. On the other hand, the study of fundamental physics can also improve our knowledge of the Earth. This effort draws on interdisciplinary expertise, with a focus on how seismic events could deepen our understanding of the fundamental forces that govern the universe.

FuSe's innovative approach combines diverse scientific fields to pursue both theoretical and practical advancements. This synergy has the potential to transform our knowledge of both fundamental physics and seismic activity, contributing to a broader understanding of Earth's interior and the cosmos.

Action keywords

Fundamental physics - Seismology - Geophysics - Material science - Big Data

Action Details

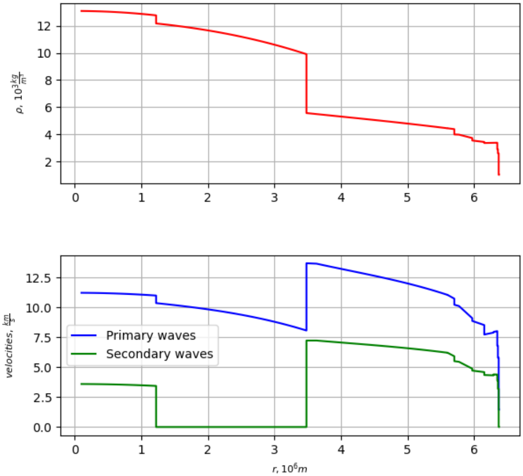
- MoU - 010/25
- CSO Approval date - 19/05/2025
- Start date - 13/10/2025
- End date - 12/10/2029

How can I participate?

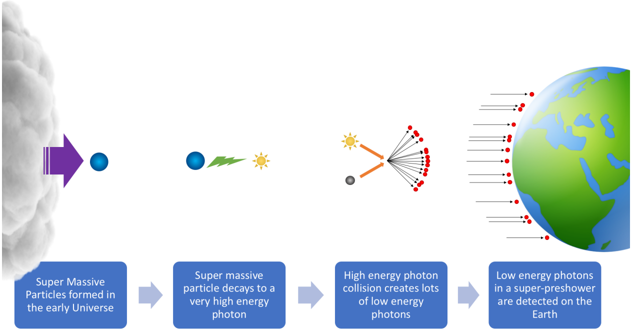
- Read the Action Description MoU
- Inform the Main Proposer/Chair of your interest (email)
- Apply to join your Working Groups of interest
- Please note, Management Committee nominations are carried out through the COST National Coordinators

Studying fundamental theories using seismic waves

- Gravity (modifications to Einstein's theory, quantum gravity)
- Fifth force? (additional field(s), extra dimensions,...)
- Dark Matter
- Particle Physics (neutrinos, ...)
- Matter properties



A statistically significant correlation exceeding 6σ between cosmic ray intensity variations and global seismic activity

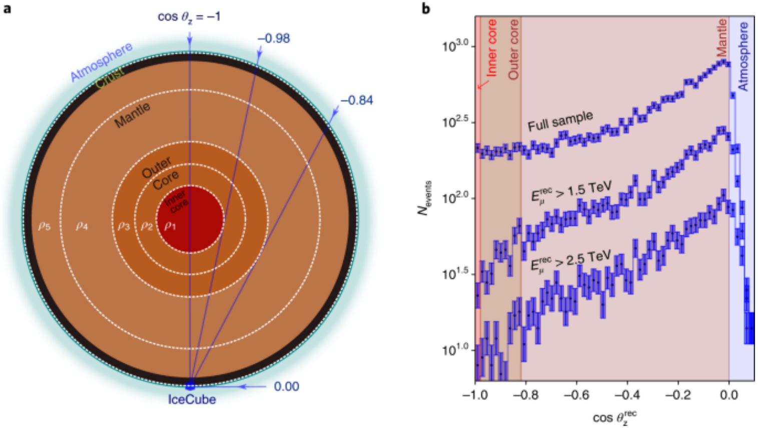


Hypothesis

- Tectonic stress $\rightarrow$ Core dynamics perturbation
- Core flow changes $\rightarrow$ Geomagnetic field variations
- Magnetic field changes $\rightarrow$ Cosmic ray trajectory alterations
- Trajectory changes $\rightarrow$ Surface detection anomalies (gravity?) $\leftarrow$ Aneta sticks her nose into these matters

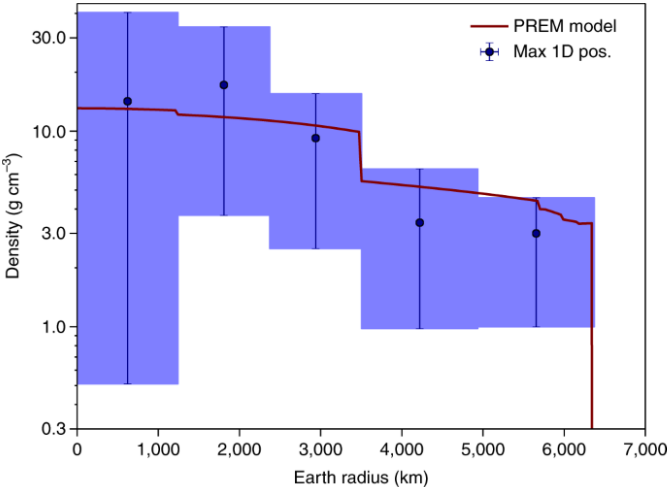
Credit: Cosmic Ray Extremely Distributed Observatory (CREDO);
Homola, P., et al. (2023). Observation of large-scale precursor correlations between cosmic rays and earthquakes with a periodicity similar to the solar cycle

How to use seismic waves to understand gravity?



Neutrino tomography of Earth: Zenith angular distribution of the atmospheric muon neutrino events in the IC86 sample.
(Donini et al., Nature Physics vol. 15, p. 37-40, 2019)

How to use seismic waves to understand gravity?



Neutrino tomography of Earth: Fit of the density profile of the Earth with IC86 data. (Donini et al., Nature Physics volume 15, pages 37-40, 2019)

How to use seismic waves to understand gravity?

geometry (based on a model of gravity)

$$+$$

mass & moment of inertia (observational constraints):

$$=$$

gravity $\sim$ terms including *seismic data*

Result:
Constraints on (quantum) gravitational theories **40 order of magnitude better** than from cosmological data and about **50 order of magnitude better** than data from black holes (shadows, quasi normal modes).

Kozak et al (2021, 2023, 2024).

Non-relativistic equations of modified and quantum gravity

Modified Poisson equation

$$\nabla^2\Phi \approx \frac{1}{2}(\rho + \text{modification})$$

For spherical-symmetric spacetime the gravitational potential the hydrostatic equilibrium equation

$$\frac{d\Phi}{dr} = -\rho^{-1}\frac{dP}{dr},$$
$$M = \int 4\pi' \tilde{r}^2 \rho(\tilde{r}) d\tilde{r},$$

+ matter description (EoS or **seismic data**, temperature dependence,...)
+ eventual equations for additional fields

A new method of testing theories of gravity proposed[13]

[13] A. Kozak, AW, Phys.Rev.D 104 (2021) 8, 084097

Terrestrial planets - seismology vs gravity II [14]

- No exchange of heat between different layers (adiabatic compression)
- The planet is a spherical-symmetric ball in hydrostatic equilibrium
- The planet consists of radially symmetric shells with the given density jump between the inner and outer core $\Delta\rho = 600$, central density $\rho_c = 13050$ and density at the mantle's base $\rho_m = 5563$ (in $\mathrm{kg/m^3}$) - PREM
- Mass $M = 4\pi \int_0^R r^2 \rho(r) dr$ and moment of inertia $I = \frac{8}{3}\pi \int_0^R r^4 \rho(r) dr$ where R is Earth's radius, play a role of the constraints (given by observations with a high accuracy)
- The outer layers' density profile described by Birch law $\rho = a + bv_p$

v_p is the longitudinal elastic wave. It contributes, together with the transverse elastic wave v_s, to the seismic parameter Φ_s and the elastic properties of an isotropic material

$$\Phi_s = v_p^2 - \frac{4}{3}v_s^2 = \frac{K}{\rho}, \quad K = \frac{dP}{d\ln\rho}$$

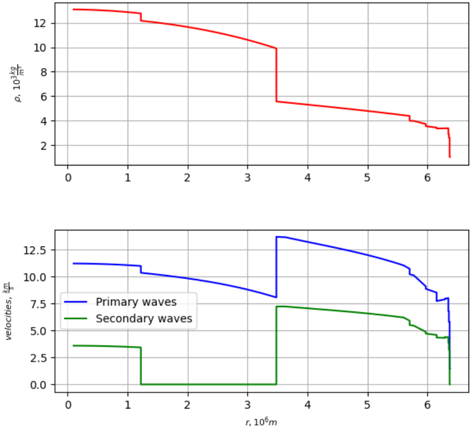
The hydrostatic equilibrium equation in MG:

$$\frac{d\rho}{dr} = -\rho \left(\frac{GM(r)}{r^2} + \text{modification} \right) \Phi_s^{-1},$$

[14] A. Kozak, AW, Phys.Rev.D 108 (2023) 4, 044055

The density profile given by the PREM (Newtonian)

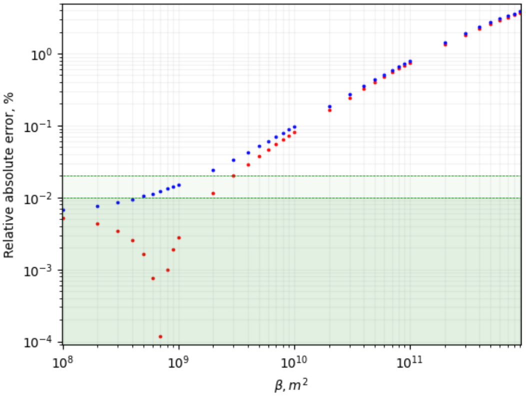
- The density profile given by the preliminary reference Earth model in which Newtonian gravity is assumed.
- The velocities' plots are obtained from data without using any theory of gravity.
- The primary waves are the same as the longitudinal waves, while the secondary waves are transverse in nature.
- The units are in km/s for velocities, while the densities are in kg/m^3.



Terrestrial planets - seismology vs gravity III [15]

Constraining theory (moment of inertia $I = 8.01736 \pm 0.00097 \times 10^{37}$ kg m^2 and mass $M = 5.9722 \pm 0.0006 \times 10^{24}$ kg)

- Relative absolute error for the mass and the moment of inertia of Earth. Red dots represent errors for the moment of inertia, while blue ones correspond to the mass.
- The dark green stripe represents a 1σ region for both quantities, while the light green denotes a 2σ region.
- The green region denotes the uncertainties for both mass and moment of inertia because, for either of them, the ratio of σ to the mean value is similar ($\approx 0.01\%$).
- The values of $(\rho_m, \rho_c, \Delta\rho)$ chosen for numerical calculations are $(5563, 13050, 600)$ kg/m^3, respectively.



[15] A. Kozak, AW, Phys.Rev.D 108 (2023) 4, 044055

Theories of gravity constrained so far

Modified Poisson equation

$$\nabla^2\phi(\mathbf{x}) = 4\pi G\Big(\rho(\mathbf{x}) + \nabla^2\alpha\big(\mathbf{x},\rho(\mathbf{x})\big)\Big),$$

- Palatini $f(R)$ and Eddington-inspired Born-Infeld gravity (Ricci-based)[16]: $\alpha(r,\rho) = \epsilon/2\rho(r)$, and $\epsilon = 4\beta$

$$-2\times10^9 \lesssim \beta \lesssim 10^9\mathrm{m}^2 \text{ for Palatini }, -8\times10^9 \lesssim \epsilon \lesssim 4\times10^9\mathrm{m}^2 \text{ for EiBI}$$

- DHOST theories $\alpha(r,\rho) = \frac{\Upsilon}{4}r^2\rho(r)$

$$-10^{-3} \lesssim \Upsilon \lesssim 10^{-3}$$

- Quantum gravity: Snyder and qGUP ($\beta_0 := \beta M_P^2 c^2$): $\beta_0 < 4.67\times10^{44}$
- Quantum gravity: linear GUP: $-6\times10^{22} \lesssim \sigma \lesssim 3\times10^{22}$ s/kg m

[16] New cosmological data provides bounds $|\beta| < 10^{49}$ m^2, Aguiar Gomes+, JCAP 01 (2024) 011

The uncertainties for the models' parameters I

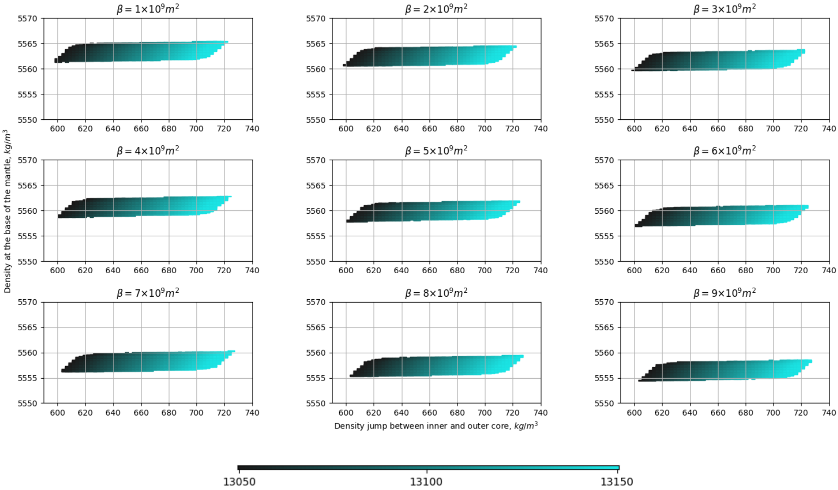


Figure: 1σ confidence regions of the theory parameters $(\rho_c, \rho_m, \Delta\rho)$ for different values of the β parameter, being of order 10^9 m^2. The darker color corresponds to lower values of the central density, while the brighter one - to higher. The range of the central density is shown in the color bar below the figures. The units are kg/m^3.

The uncertainties for the models' parameters II

- There always exists a region for a given value of the theory parameter for which all three density parameters result in a good agreement with experimental measurements
- $\Delta\rho$ and ρ_c admit much wider ranges of their values, not taking out of the 1σ region.
- ρ_m can differ by no more than $2-3$ kg m^{-3} from the value assumed in our calculations in order to remain within the 1σ region
- To incorporate bigger uncertainty of ρ_m, increase in the range of ρ_m and $\Delta\rho$, and/or the range of β would be necessary
- Large uncertainty in the determination of ρ_m is related to a bigger range of β parameter's allowed values
- Example: for $\beta = 10^9$m^2, deviations from the PREM ρ_m ($\beta = 0$) leading to the same values of M and I, is 0.02% while, in the worst case, for the uncertainty of the PREM model 50 kg m^{-3}, is 0.9% ($\Delta\rho$ and ρ_c unchanged). It increases the bound to 10^{11}m^2.

Astrophysical bounds on Generalized Uncertainty Principle[17]

Our bound when more realistic physics taken into account

$\beta_0 \leq 1.36 \times 10^{48}$ from low-mass stars (A. Pachol, AW, Eur.Phys.J.C 83 (2023) 12, 1097)

$\beta_0 < 4.67 \times 10^{44}$ from Earthquakes (A. Kozak, A. Pachol, AW, Annals of Physics, 2025)

experiment	ref.	upper bound on β
equivalence principle (pendula)	[240]	~~10^{20}~~ 10^{73}
gravitational bar detectors	[387, 388]	~~10^{33}~~ 10^{93}
equivalence principle (atoms)	[389]	10^{45}
perihelion precession (solar system)	[123, 155]	10^{69}
perihelion precession (pulsars)	[123]	10^{71}
gravitational redshift	[155]	10^{76}
black hole quasi normal modes	[251]	10^{77}
light deflection	[123, 155]	10^{78}
time delay of light	[155]	10^{81}
black hole shadow	[247]	10^{90}
black hole shadow	[251, 259]	10^{90}

17 See review by Bosso+, CQG 40.19 (2023): 195014

Tabletop experiment bounds on Generalized Uncertainty Principle[18]

Our bound when more realistic physics taken into account

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Experiment	Reference	Upper bound on β
Phonon cavity	[399]	10^{46}
Harmonic oscillators	[400, 401]	10^{60}
LIV in hydrogen atom	[304]	10^{30}
Scanning tunneling microscope	[65, 373]	10^{33}
μ anomalous magnetic moment	[65, 402]	10^{33}
Hydrogen atom	[54, 57, 95, 395, 396]	10^{34}
Lamb shift	[65, 403]	10^{36}
^{87}Rb interferometry	[404, 405]	10^{39}
Kratzer potential	[90]	10^{46}
Stimulated emission	[110]	10^{46}
Landau levels	[65, 69, 403]	10^{50}
Quantum noise	[112]	10^{57}

[18] See review by Bosso+ 2023 Class. Quantum Grav. 40 195014

Improving the method and future constraints

- Spherical-symmetric 1-dim Earth with adiabatic compression:
  - to introduce the complexities of Earth's true geometry (it rotates)
  - to estimate the equatorial moment of inertia relative to the polar moment by applying travel time ellipticity corrections to PREM[19]
  - to recognize the imperfections of layers and accounting for variable density jumps
  - to take into account a temperature variation with depth.
- Core description:
  - PREM does not describe well the boundaries of the outer and inner core
  - to use a more precise model like AK135-F[20] - it incorporates the complexities of core waves
  - to use equations of state for modeling core density and bulk moduli[21] (improving the uncertainties in density jumps at the inner and outer core boundaries).
- Birch law - a probable reevaluation when dealing with seismic data from Mars (the coefficients obtained experimentally).

[19] B. L. N. Kennett, O. Gudmundsson, Geophysical Journal International 127.1 (1996): 40-48.

[20] B. L. N. Kennett, E. R. Engdahl, R. Buland, Geophysical Journal International 122.1 (1995): 108-124.

[21] J. C. E. Irving, S. Cottaar, V Lekic, Science advances 4.6 (2018): eaar2538.

Improving the thermodynamic description and future plans

- To consider gravity effects in the elasti moduli and lattice description of the Earth's materials - corrections to the thermal energy (in progress)
- To take into account gravity effects in equations of state, melting and transport properties (in progress)
- To consider modified dispersion relation in the above calculations

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Summary and conclusions

- Tests of gravity with the use of stars and substellar objects (BD, (exo)-planets, seismology)
- We must be consistent in describing physical systems in different scales
- We should consider more realistic models on both sides: gravity and matter - rotating bodies, magnetic fields, ..., opacities (atmosphere), microphysics description - to obtained better bounds and understand the gravity effects
- More research on matter properties in the MG and QG frameworks is necessary

Fantastic Advantures of Maika and Laika:
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Thanks!

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