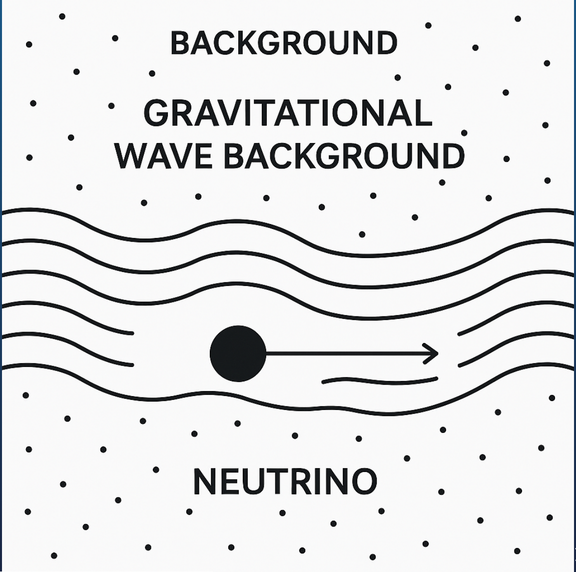
Understanding gravitationally induced decoherence parameters in neutrino oscillations using a microscopic quantum mechanical model

Bridging high and low energies in search of quantum gravity
2025 Cost Action CA23130 First Annual Conference

Roman Kemper
Institute for Quantum Gravity, Theoretical Physics III, FAU

joint work with Alba Domi, Thomas Eberl, Max Joseph Fahn, Kristina Giesel, Lukas Hennig, Ulrich Katz, Michael Kobler, JCAP 11 (2024) 006, arXiv:2403.03106

Set up: study neutrinos with a gravitational wave background



[1]

[1] Renata Ferrero

Motivation

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- Interaction with environment may alter the oscillation behaviour
- Neutrinos have energy $\rightarrow$ interact with gravity $\rightarrow$ investigate gravity as environment
- Understanding these effect may help to:
  - Enhance understanding of neutrino oscillations
  - Gain insight into gravitational wave environments

Setting the Stage

- Previous studies primarily use phenomenological models [2]
  $\rightarrow$ less trackable connection to underlying microscopic model

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  - Interpretation of the system within an open quantum system framework
  - Construct microscopic model
  - Derive master equation within this framework

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  - Derive master equation within this framework
  - Identify decoherence parameters
  - Analyse the neutrino oscillation probabilities

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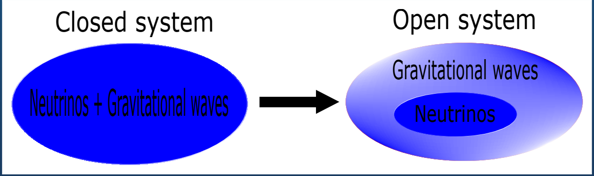
Open quantum systems approach

- We are mainly interested in the neutrino oscillation system $\hat{\rho}_{\mathrm{S}}$ $\rightarrow$ use open quantum system approach



- Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{int}}, \qquad \hat{H}_{\mathrm{int}} = \eta \hat{A}_{\mathrm{S}} \otimes \hat{B}_{\mathrm{E}} \tag{1}$$

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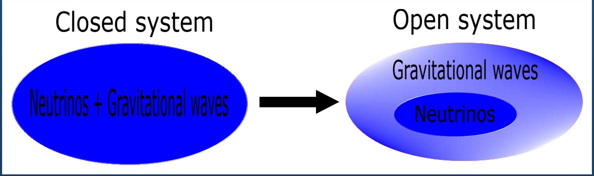

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- Tracing out environmental degrees of freedom $\rightarrow$ master equation

Open quantum systems approach

- Master equation: evolution of the neutrino oscillation system under the effective influence of the environment

$$\frac{\partial}{\partial t}\hat{\rho}_{\mathrm{S}}(t) = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{add}}, \hat{\rho}_{\mathrm{S}}(t)\right] + \mathcal{D}[\hat{\rho}_{\mathrm{S}}(t)] \tag{2}$$

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- Dissipator $\mathcal{D}[\hat{\rho}_{\mathrm{S}}]$ encodes decoherence and dissipation effects

Microscopic model for gravitationally induced decoherence

Field theory perspective:

- Assume system forces to be way stronger than gravity $\rightarrow$ weak coupling $\rightarrow$ linearised gravity
- Use inspiration from field-theoretic models (scalar[3]- or vectorfield[4])

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  - mimic perturbed metric by position operator $\hat{q}$
  - $\hat{H}_{\mathrm{S}}$ as substitute for $\hat{T}^{\mu\nu}$

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$$\begin{aligned} \hat{H}_{\mathrm{tot}} &= \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{int}} \\ &= \underbrace{\hat{H}_{\mathrm{S}}}_{\text{neutrinos}} + \underbrace{\frac{1}{2}\sum_{i=1}^{N}\left[\hat{p}_i^2 + \omega_i^2 \hat{q}_i^2\right]}_{\text{gravitational waves}} \underbrace{- \eta \hat{H}_S \otimes \sum_{i=1}^{N} \hat{q}_i}_{\text{interaction}} \end{aligned} \tag{3}$$

[6] [Blencowe, Xu '22]

Derivation of the master equation

- Assumptions
  - Weak coupling
  - Gravitational waves follow a Bose-Einstein distribution with temperature parameter T
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$$\begin{aligned}\frac{d}{dt}\hat{\rho}_S(t) = &-\frac{i}{\hbar}\left[\hat{H}_S, \hat{\rho}_S(t)\right] \\ &+\frac{8\eta^2}{\hbar^2}\frac{k_B T}{\hbar}\left(\hat{H}_S\hat{\rho}_S(t)\hat{H}_S - \frac{1}{2}\left\{\hat{H}_S^2, \hat{\rho}_S(t)\right\}\right)\end{aligned} \tag{4}$$

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- Free parameters: T, $\eta \hat{=}$coupling parameter

Neutrino oscillations

- Neutrinos are created and interact as distinct flavour states $|\nu_\alpha\rangle$
- Propagate in the mass basis $|\nu_i\rangle$, connected to the flavour basis by a unitary transformation $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$ (PMNS-matrix)

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- Propagate in the mass basis $|\nu_i\rangle$, connected to the flavour basis by a unitary transformation $|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle$ (PMNS-matrix)
- Oscillation formula (relativistic neutrinos)

$$P(\nu_\alpha \to \nu_\beta) = \sum_{ij} U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j} e^{-i \frac{\Delta m^2_{ij} L}{2E} - \Gamma_{ij} L} \qquad (5)$$

  - Observed oscillations depend on mass differences $\Delta m^2_{ij} = m^2_i - m^2_j$
  - Environmental coupling can lead to a damping

Neutrino setup

- Relativistic neutrinos that travel through earth with three flavours
- Hamiltonian

$$\hat{H}_S = \hat{H}_{vac} + \hat{U}^\dagger \hat{H}_{mat} \hat{U} \tag{6}$$

$$\hat{H}_{vac} = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} = E\mathbb{1}_3 + \frac{c^4}{6E} \begin{pmatrix} -\Delta m_{21}^2 - \Delta m_{31}^2 & 0 & 0 \\ 0 & \Delta m_{21}^2 - \Delta m_{32}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 + \Delta m_{32}^2 \end{pmatrix}$$

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- Matter part: PREM Model[7] takes varying electron density N_e of the different layers of the Earth into account

$$\hat{H}_{mat} = \pm\sqrt{2} G_f N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

[7] [Dziewonski, Anderson 1981]

Results

- Solution of the master equation

$$\rho_{ij}(t) = \rho_{ij}(0) \cdot e^{-\frac{i}{\hbar}(H_i - H_j)t - \frac{4\eta^2 k_B T}{\hbar^3}(H_i - H_j)^2 t}$$ **microscopic model**

$$H_i \approx E_{\mathrm{mean}} + \frac{m_i^2 c^4}{2E_{\mathrm{mean}}}$$ **ultra relativistic neutrinos**

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- Damping dependents on energy squared differences $(H_i - H_j)^2$ scaled by $\frac{4\eta^2 k_B T}{\hbar^3}$
- Free parameters
  - T: "temperature" parameter characterising the gravitational waves environment
  - η: coupling strength

Phenomenological models

- Often: starting point Lindblad equation[8]

$$\frac{d}{dt}\hat{\rho}_S(t) = -\frac{i}{\hbar}\left[\hat{H}_S, \hat{\rho}_S(t)\right] + \sum_{i,j} a_{ij}\left(\hat{L}_S^i \hat{\rho}_S(t) \hat{L}_S^j - \frac{1}{2}\left\{\hat{L}_S^i \hat{L}_S^j, \hat{\rho}_S(t)\right\}\right)$$

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- Equation is solved by

$$\rho_{ij}(t) = \rho_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(H^i_S - H^j_S\right)t - \gamma_{ij}E^n_{\mathrm{vac}}t} \tag{9}$$

- Energy dependence: vacuum energy $E_{\mathrm{vac}}, \quad n \in \{-2, 1, 0, 1, 2, 3\}$

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- Energy dependence: vacuum energy $E_{\mathrm{vac}}, \; n \in \{-2, 1, 0, 1, 2, 3\}$
- decoherence parameters γ_{ij}: 3 independent parameters if energy conservation holds ($[\hat{H}_S, \hat{L}_S^i] = 0$)
- n, γ_{ij} are postulated

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Results: Comparison to phenomenological models

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$$\rho_{ij}(t) = \rho_{ij}(0) \cdot e^{-\frac{i}{\hbar}\left(H_i - H_j\right)t - \frac{4\eta^2 k_B T}{\hbar^3}\left(H_i - H_j\right)^2 t}$$ **microscopic model**

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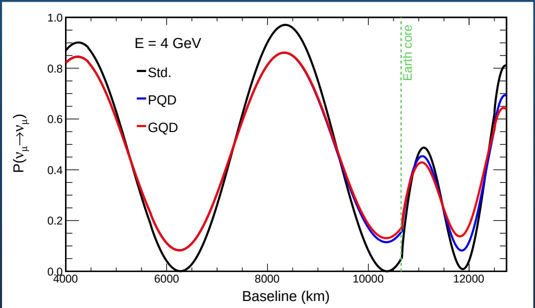
- Oscillation probabilities ($L = ct$)

$$P(\nu_\alpha \to \nu_\beta) = \sum_{ij} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} e^{-i\frac{\Delta m_{ij}^2 L}{2E} - \frac{\eta^2 c^8 k_B T}{\hbar^3 E^2}(\Delta m_{ij}^2)^2 L}$$

Results: Oscillation probabilities

- Oscillation probabilities (for $T = 0.9K$, $\eta = 10^{-8}s$, $n = 2$ and fitting values for γ_{ij} using PREM[9] and OscProb[10])



- In vacuum: match with certain phenomenological models
- In matter: no match possible with phenomenological models with constant decoherence parameters

[9] [Dziewonski, Anderson 1981]

[10] [Coelho et al. 24]

Conclusion

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  - Microscopic model inspired by linearised gravity and QFT
  - Derived a master equation of Lindblad form
  - Only two free parameters
- Comparison with phenomenological models
  - In vacuum: decoherence parameters match phenomenological models, with certain energy dependence
  - Non-vacuum: decoherence parameters depend on matter effects in contrast to many phenomenological models

Outlook

More experimental perspective:

- Establish experimental bounds on the decoherence parameters using experimental data
- First bounds set using the KamLAND experiment and the corresponding phenomenological models[11]

[11] [Romeri, Giunti, Stuttard, Ternes 2023]

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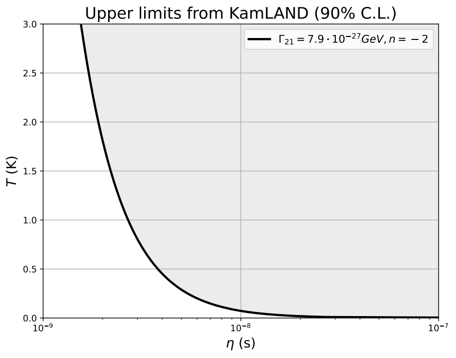
More theoretical perspective:

- Full field theoretic model
- Study different quantisation techniques, like polymere QM (LQG inspired quantisation)

[11] [Romeri, Giunti, Stuttard, Ternes 2023]

Results: upper bounds for free parameter?

- Similar set up in the KamLAND experiment
  $\rightarrow$ bounds reported in [Romeri, Giunti, Stuttard, Ternes 2023] can be translated into constraints onto T, η



- grey area excluded

Lamb shift renormalisation

- Master equation includes Lamb shift contribution

$$\frac{d}{dt}\hat{\rho}_S(t) = -\frac{i}{\hbar}\left[\hat{H}_S, \hat{\rho}_S(t)\right] + \frac{i\Omega\eta^2}{\hbar^2}\left[\hat{H}_\mathrm{S}, \hat{\rho}_\mathrm{S}\right] + D[\hat{\rho}_\mathrm{S}] \quad (10)$$

- Depends on the cut off frequency Ω, where $\Omega \to \infty$
- Field theorie[12]: Correspond to a vacuum effect which needs to be renormalised
- Introduce counter term in $\hat{H}_\mathrm{tot}$ to deal with this divergence

$$\hat{H}_\mathrm{C} = \frac{1}{\hbar}\int_0^\infty d\omega \frac{J(\omega)}{\omega} H_\mathrm{S}^2 = \frac{\Omega\eta^2}{\hbar}\hat{H}_\mathrm{S} \quad (11)$$

with the spectral density $J(\omega)$

[12] [Fahn, Giesel, 24]

First approximation and interpretation of the free parmeters

- η (comparison with field theoretical models[13])
  - Around $10^{-42}s$, which is near Planck scale.
- T (cosmological motivated [14])
  - Radiation-dominated era before inflation $\rightarrow$ thermal gravitational wave background
  - Thermal gravitational wave background from graviton decoupling at $T \sim T_{\text{Planck}}$
  - Black-body spectrum preserved; temperature redshifts with expansion
  - Present-day estimate: $T_{\text{gw}} \simeq 0.9\,\text{K} < T_\gamma \simeq 2.72\,\text{K}$

13 [Oniga, Wang 16], [Anastopoulos, Hu 13], [Fahn, Giesel, Kobler 22], [Fahn, Giesel, 24], [Lagouvardos, Anastopoulos 21],[Fahn, Giesel, Kemper 25]

14 [Kolb, Turner 90], [Gasperini, Giovannini, Veneziano 93], [Giovannini 19]