Quantum Coherence from Quantum Spacetime

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$$(p \oplus q)_i = p_i + q_i + \mathcal{O}(\kappa^{-1})$$
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We will focus here on deformations of the energy composition law





Entanglement Induced by Deformed Energy Composition

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Analogy with Gravitationally-Induced Entanglement





When particles interact gravitationally, the uncertainty in the position reflects in entanglement generation due to gravity.



Does uncertainty in the direction of momenta reflect in entanglement due to a deformed composition law?

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Two-Particle Toy Model (1 Dimension)

 H_0 is the Hamiltonian of a single particle and P_i is the momentum components operator that we should define

$$H = H_0 \otimes \mathbb{1} + \mathbb{1} \otimes H_0 + \ell \sum_i P_i \otimes P_i$$



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Suppose that each particle is a two-level system, for example defined by a non-minimal coupling with an electromagnetic field

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$$H_{\text{und}} = \epsilon \mathbb{1} \otimes \mathbb{1} + \frac{\omega}{2} (\sigma_z \otimes \mathbb{1} + \mathbb{1} \otimes \sigma_z)$$







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For the **deformed** case we have also **two** directions in momenta. Our system needs **four** indices for the energies and directions $|i, j, \alpha, \beta\rangle$, where $\{\alpha, \beta\} = \{+-\}$

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The operator that has $|\pm\rangle$ as eigenstates is the x-Pauli matrix σ_x

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$$\mathcal{H} = \epsilon(\mathbb{1})^4 + \frac{\omega}{2}$$
 (

$(\sigma_z \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} + \mathbb{1} \otimes \sigma_z \otimes \mathbb{1} \otimes \mathbb{1}) + \ell |P| \otimes |P| \otimes \sigma_x \otimes \sigma_x,$

Reference energy

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Moduli of momenta

$$\mathcal{H} = \begin{pmatrix} H_{11} & 0 & 0 & 0 \\ 0 & H_{-11} & 0 & 0 \\ 0 & 0 & H_{1-1} & 0 \\ 0 & 0 & 0 & H_{-1-1} \end{pmatrix}$$





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Each block H_{ab} is a 4 × 4 matrix of the kind

$$H_{ab} = \begin{pmatrix} \epsilon + \frac{a+b}{2}\omega & 0 & 0 \\ 0 & \frac{\epsilon + \frac{a+b}{2}\omega}{2}\omega & \ell m\sqrt{(\epsilon+a\omega)(\epsilon+b\omega)} & 0 \\ 0 & \ell m\sqrt{(\epsilon+a\omega)(\epsilon+b\omega)} & \epsilon + \frac{a+b}{2}\omega & 0 \\ \ell m\sqrt{(\epsilon+a\omega)(\epsilon+b\omega)} & 0 & 0 & \epsilon + \frac{a+b}{2}\omega \end{pmatrix}$$

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Due to the quantum algebra, this system behaves as an open quantum system

[Arzano, PDR (2014)] [Arzano, D'Esposito, Gubitosi, Comm. Phys. (2023)]

$$\partial_t \rho = -i[P_0, \rho] - \frac{\ell}{2} \left(\mathbf{P}^2 \rho + \rho \mathbf{P}^2 - 2 \sum_i P_i \rho \right)$$



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The Lindblad equation can be written as 4 blocks of X-shaped matrices

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Lindblad equation for each of the 4 subsystems

There is 25% chance for each value of $\{a, b\}$ with elements

 $\partial_t \rho_{ab} + i \left[H_{ab}, \rho_{ab} \right] + \frac{\ell}{2} \left(\mathcal{P}_{ab}^2 \rho_{ab} + \rho_{ab} \mathcal{P}_{ab}^2 - 2 \mathcal{P}_{ab} \rho_{ab} \mathcal{P}_{ab} \right) = 0.$

 $\bar{\rho}_{ij} \doteq [\bar{\rho}]_{ij} = 0.25 \sum_{a,b} [\rho_{ab}]_{ij}$



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We considered the initial state

$$|\Psi\rangle = \sin\theta|01\rangle +$$

which when $\theta = 0$ is unentangled, and when $\theta = \frac{\pi}{4}$ is maximally entangled (Bell state).

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 $\cos \theta |10\rangle$,



Entanglement generation

$\mathcal{C} = 2 \max \left\{ |\bar{\rho}_{23}| - \sqrt{\bar{\rho}_{11}\bar{\rho}_{44}}, |\bar{\rho}_{14}| - \sqrt{\bar{\rho}_{22}\bar{\rho}_{33}}|, 0 \right\},\$

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The modified composition law creates entanglement while the Lindblad evolution destroys it.



Figure 1: Concurrence for different initial conditions determined by θ . The parameters used are $\ell = 1, m = 1, \epsilon = 1$, and $\omega = 0.5$.



Other quantum correlations (coherence and quantum discord)

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Figure 4: Comparison of the different quantifiers for coherence, discord and entanglement studied in the paper for $\theta = 0$. The red/dotted curve represents the l_1 -norm of coherence. The blue/dash-dotted curve represents quantum discord. The black/solid curve represents concurrence.

We have a **competition** between the action of the deformed Hamiltonian, and due to the Lindblad-like contact of the quantum system with the "quantum" spacetime environment"



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The entanglement time scale depends on the uncertainty in the potential (momentum in our case) $T_{ent} = (\ell \Delta P^2)^{-1}$ [Yang, PRD (2018)]. In our case, $T_{ent} = \hbar E_{QG}/2mc^2 E$ (similar to decoherence time scale)

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A pair of ions with mass of the order $m \sim 10^{10} \text{ eV}$ [Krutyanskiy et al., PRL (2022)], and energy scale of the order $E \sim 1$ eV, $E_{OG} = E_P \sim 10^{28}$ eV, the entanglement time scale is $T_{ent} \approx 3$ min (grows to a few hours if $E_{OG} \sim NE_P$)



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Thank you!













