Emergent Scalar Field Dynamics on Curved Spacetime in Group Field Theory

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In collaboration with Federico Greco, Stefano Liberati and Daniele Oriti

Bridging high and low energies in search of quantum gravity - 2025
Cost Action CA23130 First Annual Conference

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Outline

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1. The program of Quantum Gravity

2. Group field theory cosmology

3. Emergent scalar field dynamics on curved spacetime from GFT: background contribution

4. Emergent field theory on curved spacetime from GFT: scalar perturbations

1. The program of (background independent) Quantum Gravity

Causality
Symmetries
Relationalism
Observables

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Symmetries
Relationalism
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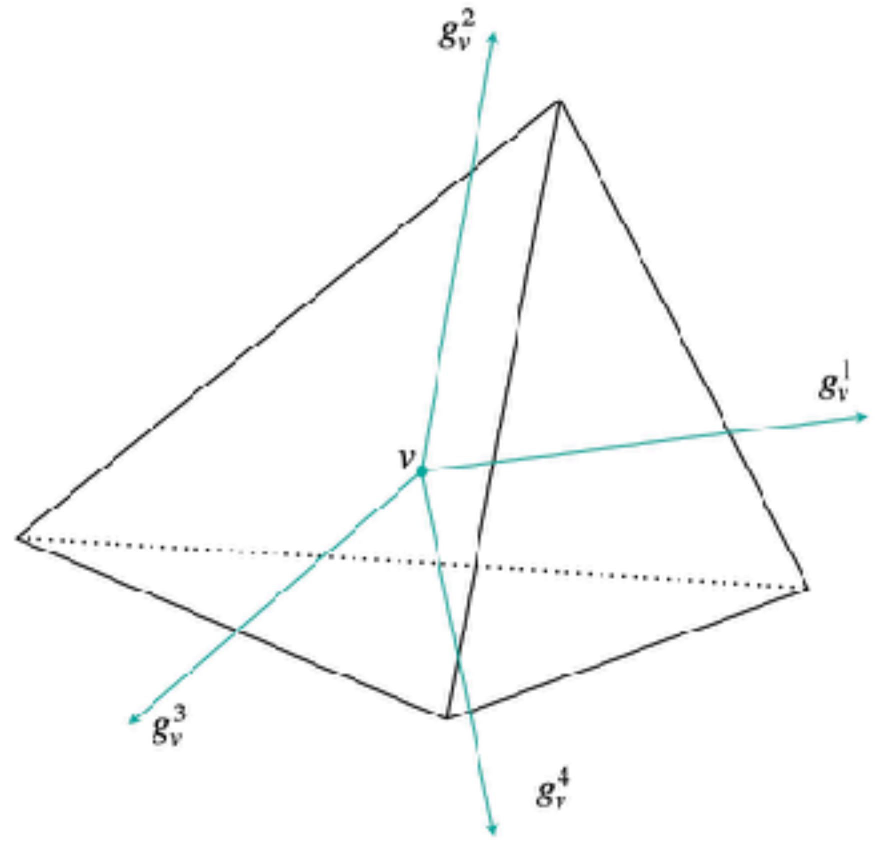
Pre-geometric entities

Atoms of spacetime



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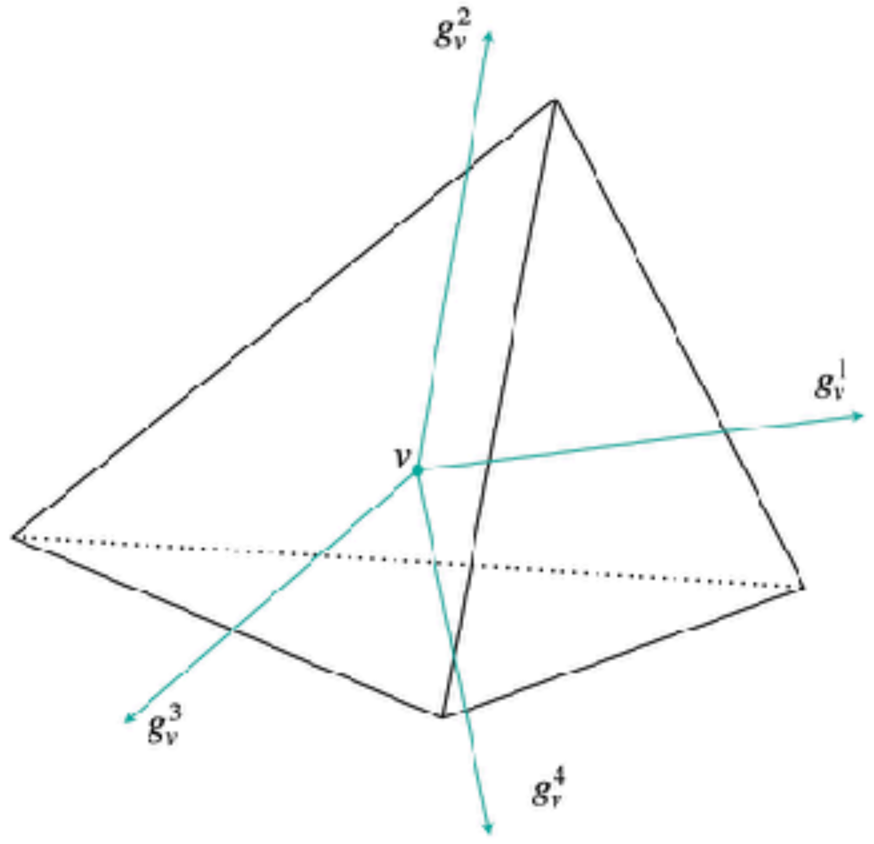

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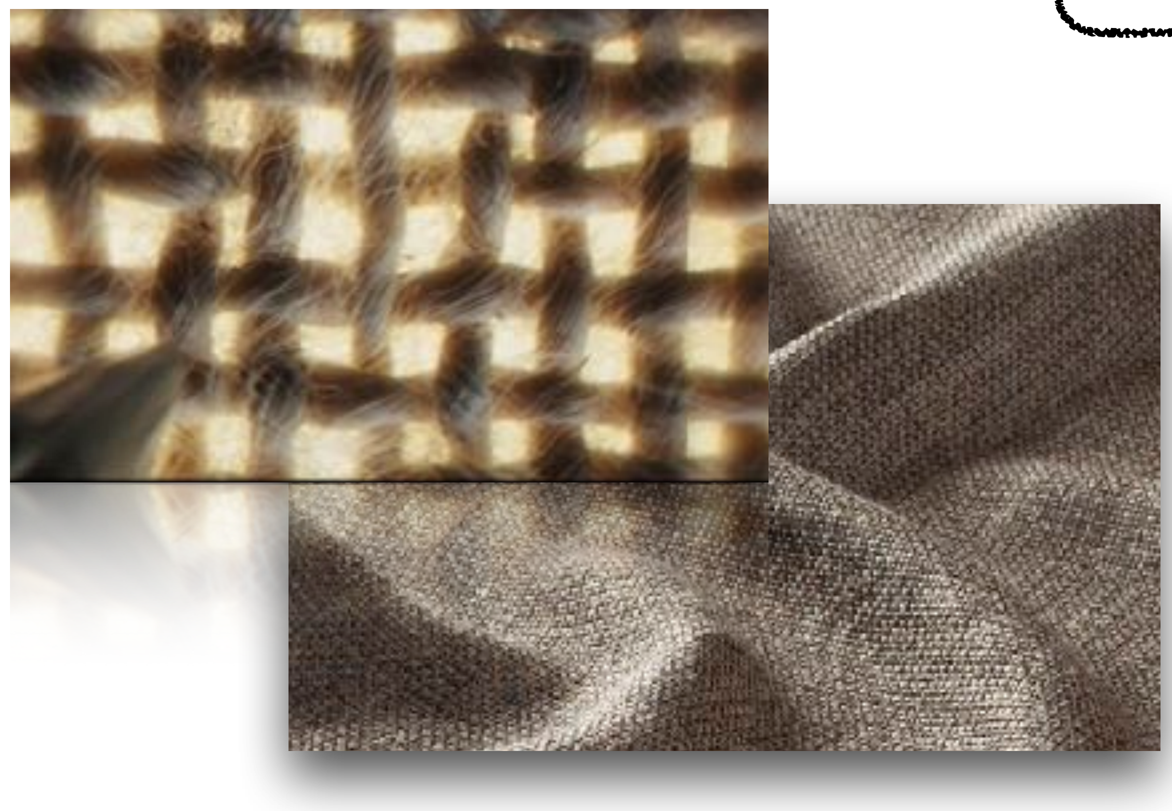

Emergence of continuum and classical spacetime

Appropriate definition of continuum limit and approximations to recover GR

Quantum dynamics

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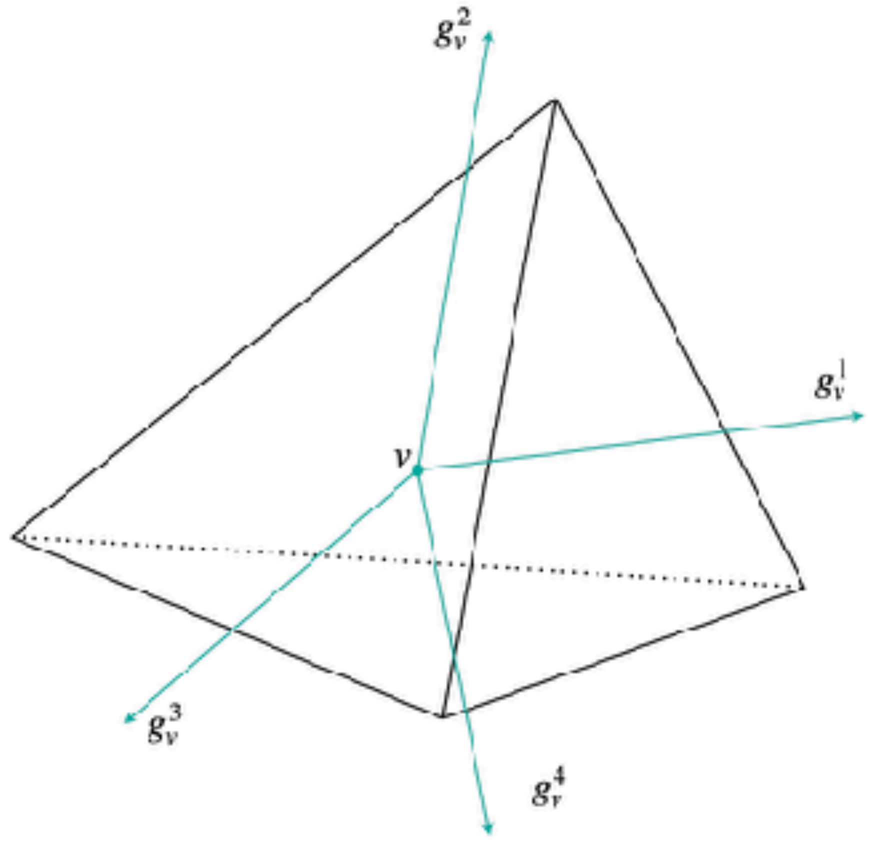
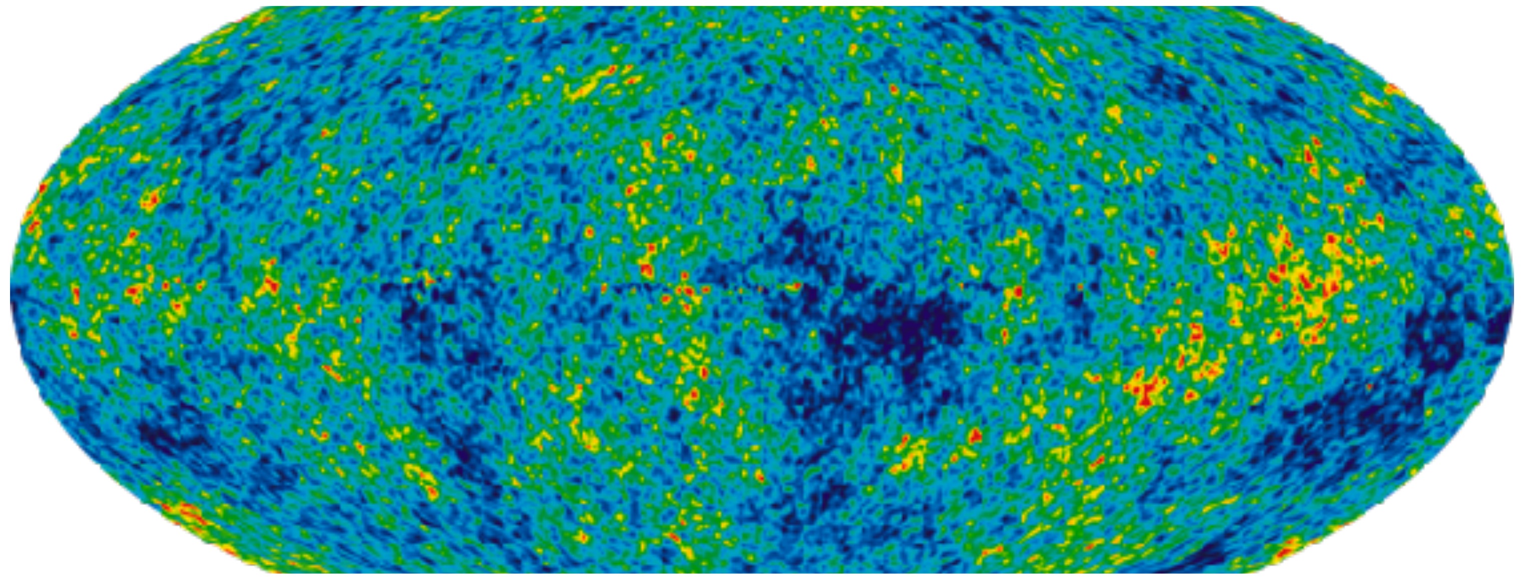


Atoms of spacetime

1. The program of (background independent) Quantum Gravity

Contact with observations and providing answer to open questions



Causality
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Generating a dictionary between the QG language

and the observational one

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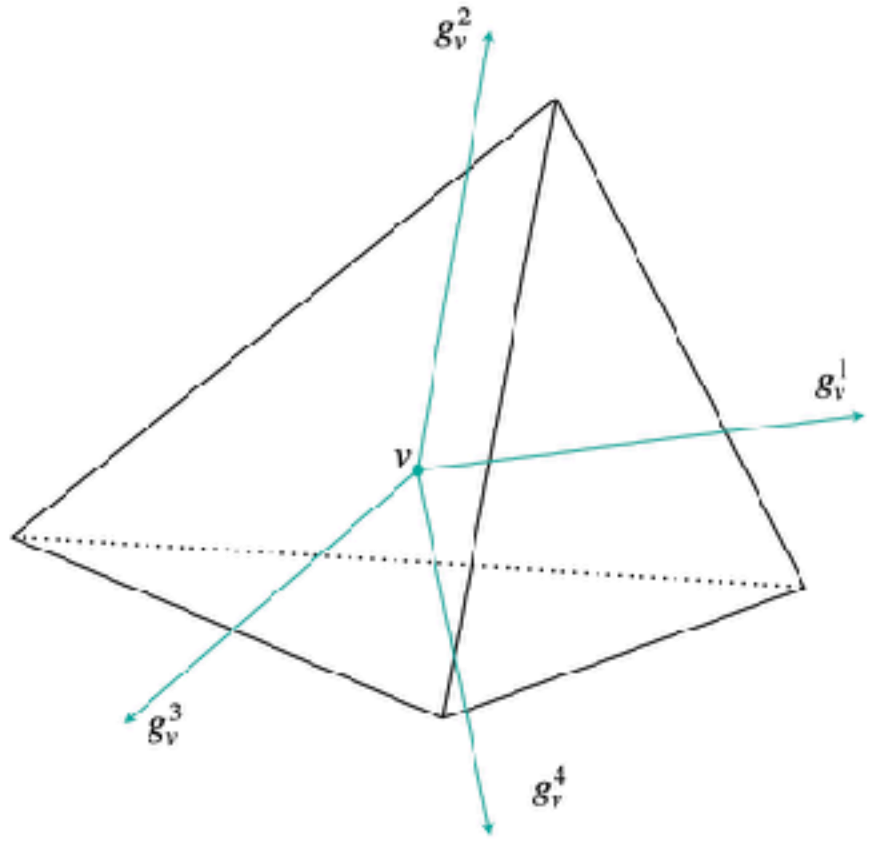

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Atoms of spacetime

2. Group field theories

Quantum field theory

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Field theories *on* spacetime

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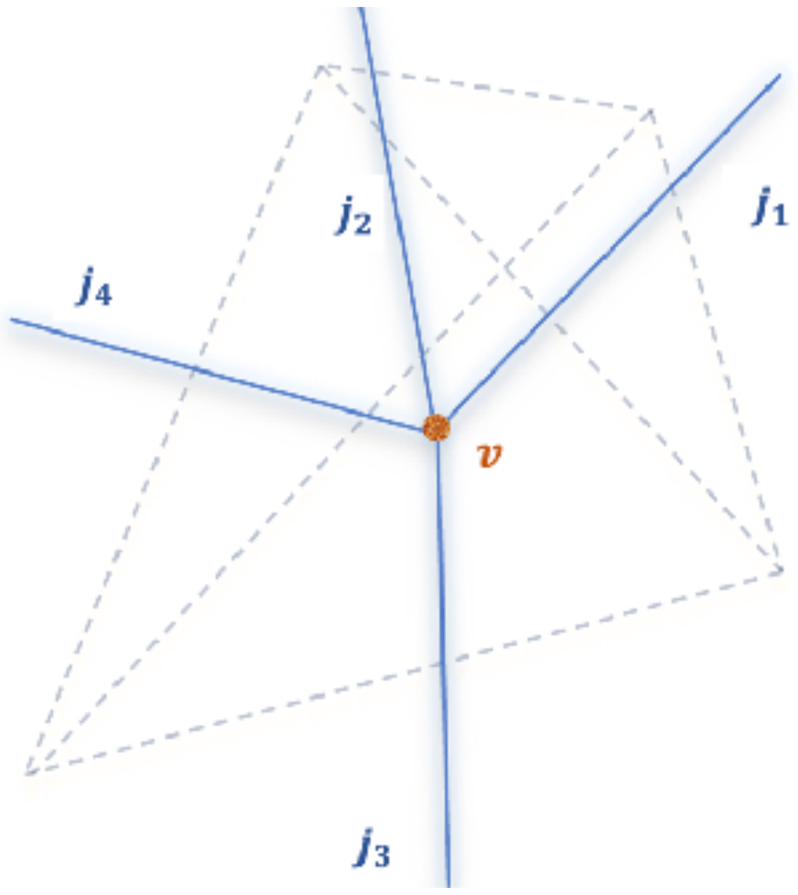
Functions of group elements that give rise to spacetime

$$\varphi : G^4 \to \mathbb{C}$$

$$(g_1, \dots, g_4) \mapsto \varphi(g_1, \dots, g_4) \equiv \varphi(g_I) \, .$$



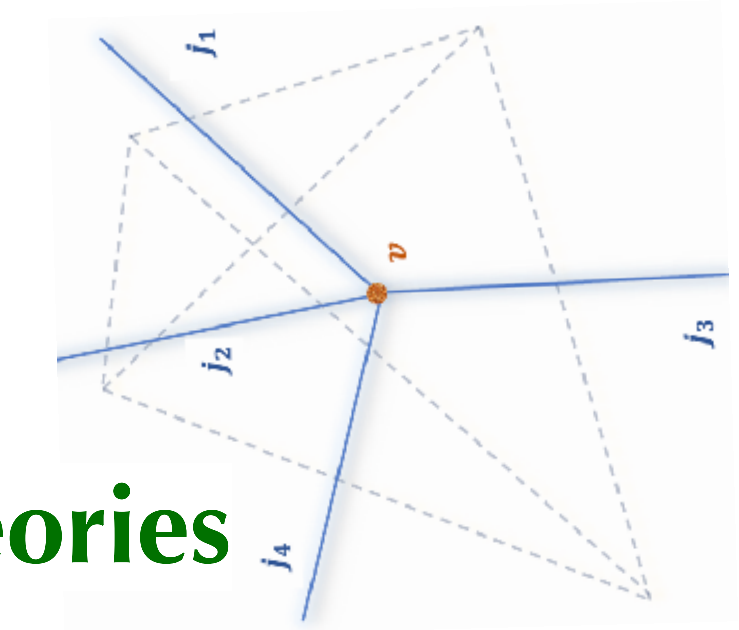
2. Group field theories

Quantum field theory

Field theories *on* spacetime

Functions of the spacetime coordinates

Action: interactions between particles on spacetime



Group field theories

Field theories *of* spacetime atoms

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Action (non-local interactions): generates quantum geometries

$$S_{\mathrm{GFT}}\left[\varphi^*, \varphi\right] = \int \left[\mathrm{d}g_I\right]^2 \varphi^*\left(g_I\right) \mathscr{K}\left(g_I, g_I'\right) \varphi\left(g_I'\right)$$

$$+ \sum_i \frac{\lambda_i}{D_i} \int \left[\mathrm{d}g_I\right]^{D_i} \varphi^*\left(g_{I_1}\right) \ldots \mathscr{U}_i\left(g_{I_1}, \ldots, g_{I_{D_i}}\right) \ldots \varphi\left(g_{I_{D_i}}\right)$$

+ c.c. ,

2. Group field theory cosmology: *do they have all the ingredients?*

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Causality: Group

$SL(2,\mathbb{C}), SU(2), ..$

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Relational evolution *with respect to scalar matter fields (clock and rods)*

Quantum dynamics

Dynamical evolution and action principle

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Emergence of continuum and classical spacetime

Effective: mean-field approach

GFT condensate

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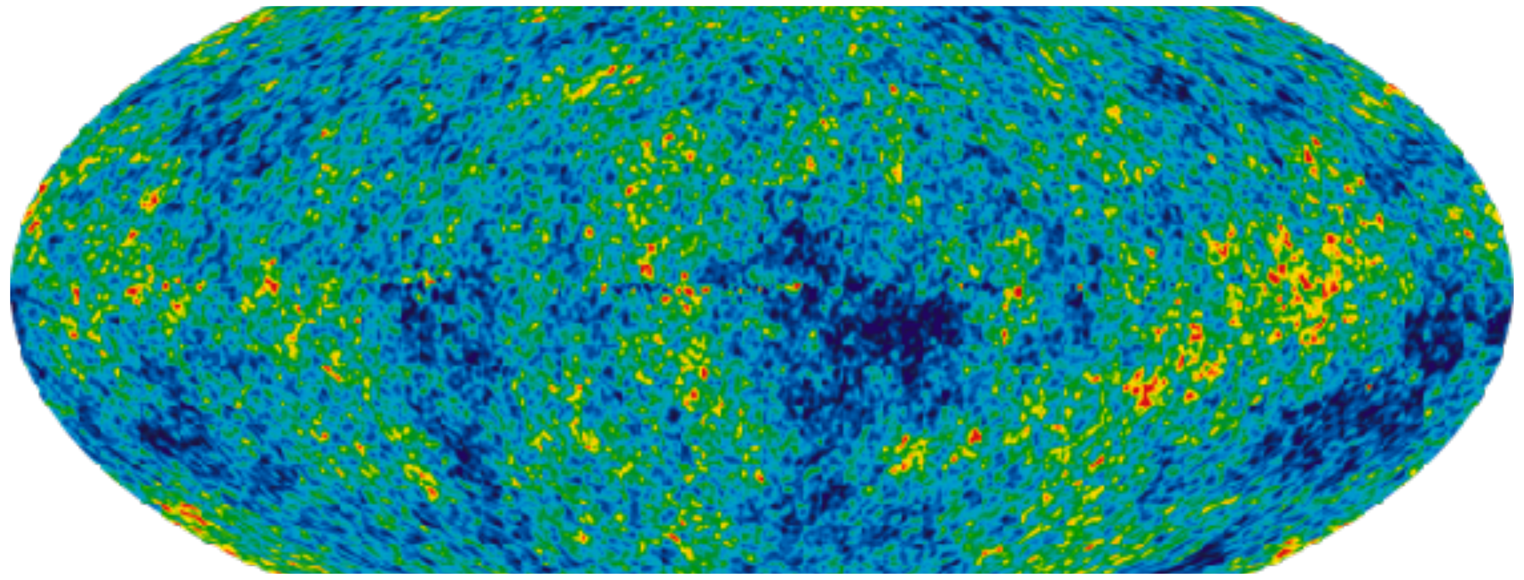

- **Coupling to scalar field: matter content of the universe**
- **Scalar perturbations**

Generating a dictionary between the QG language

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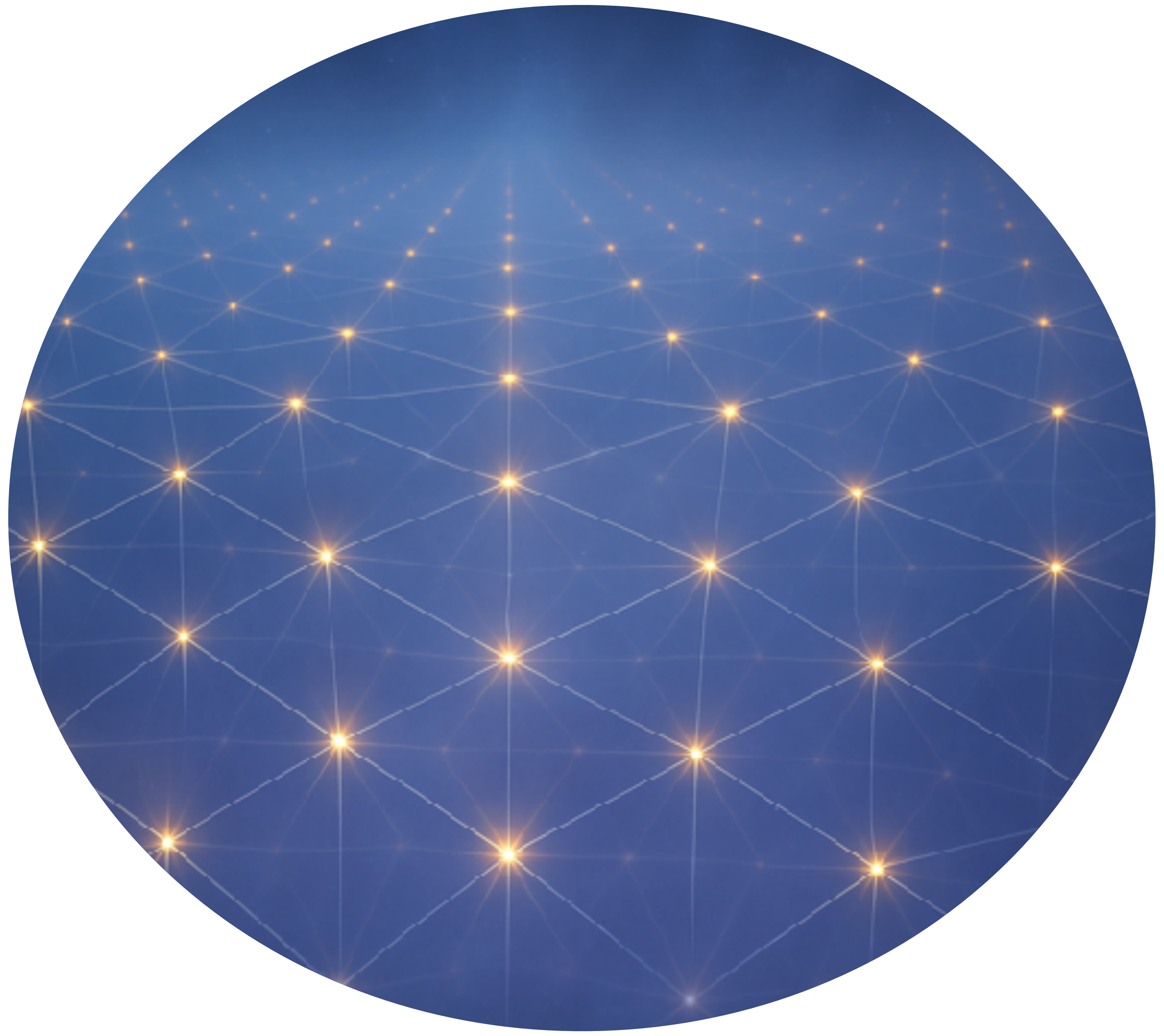
Ingredients of the GFT condensate (kinematics)

Classical set up

GFT condensate setting



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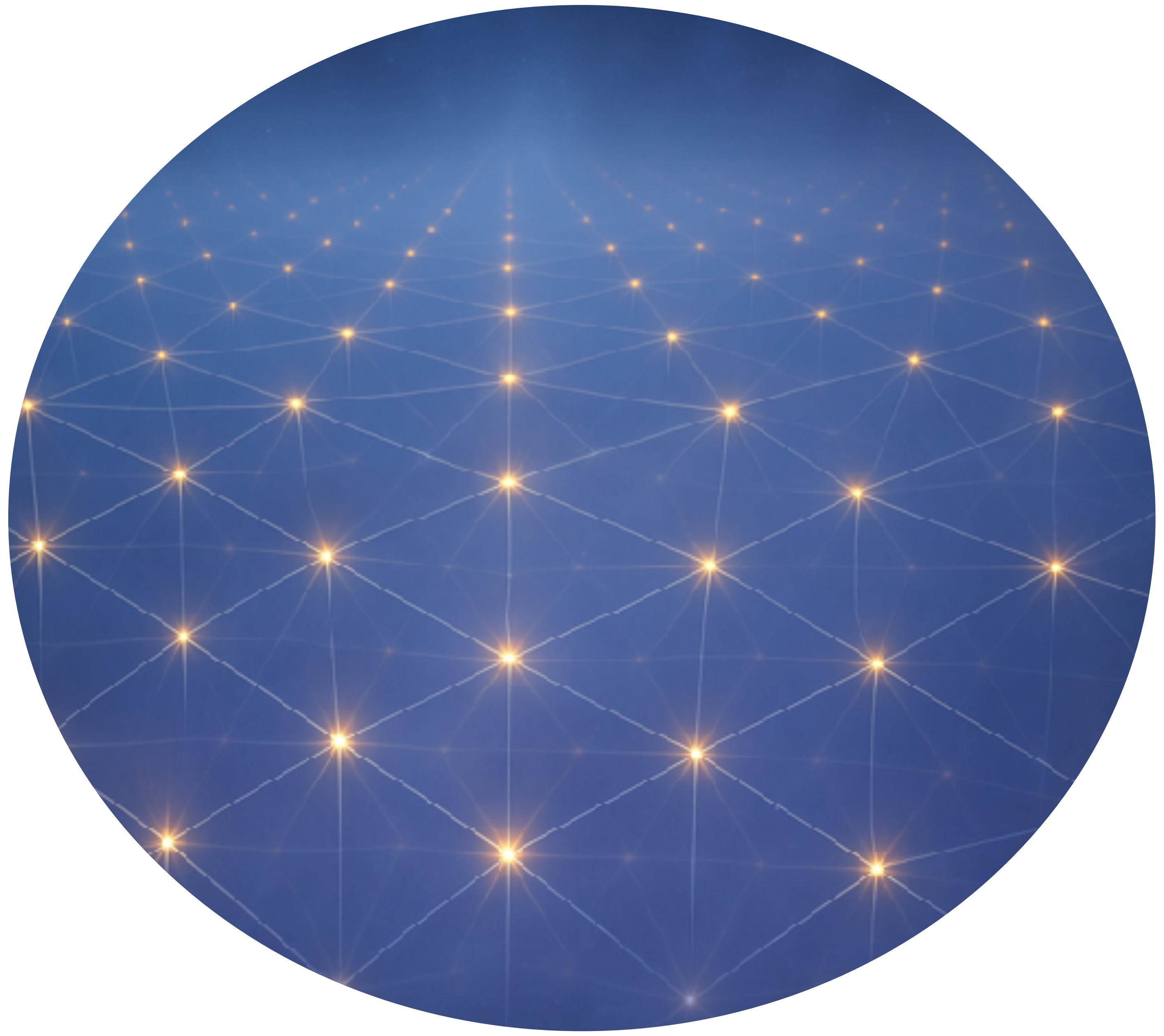

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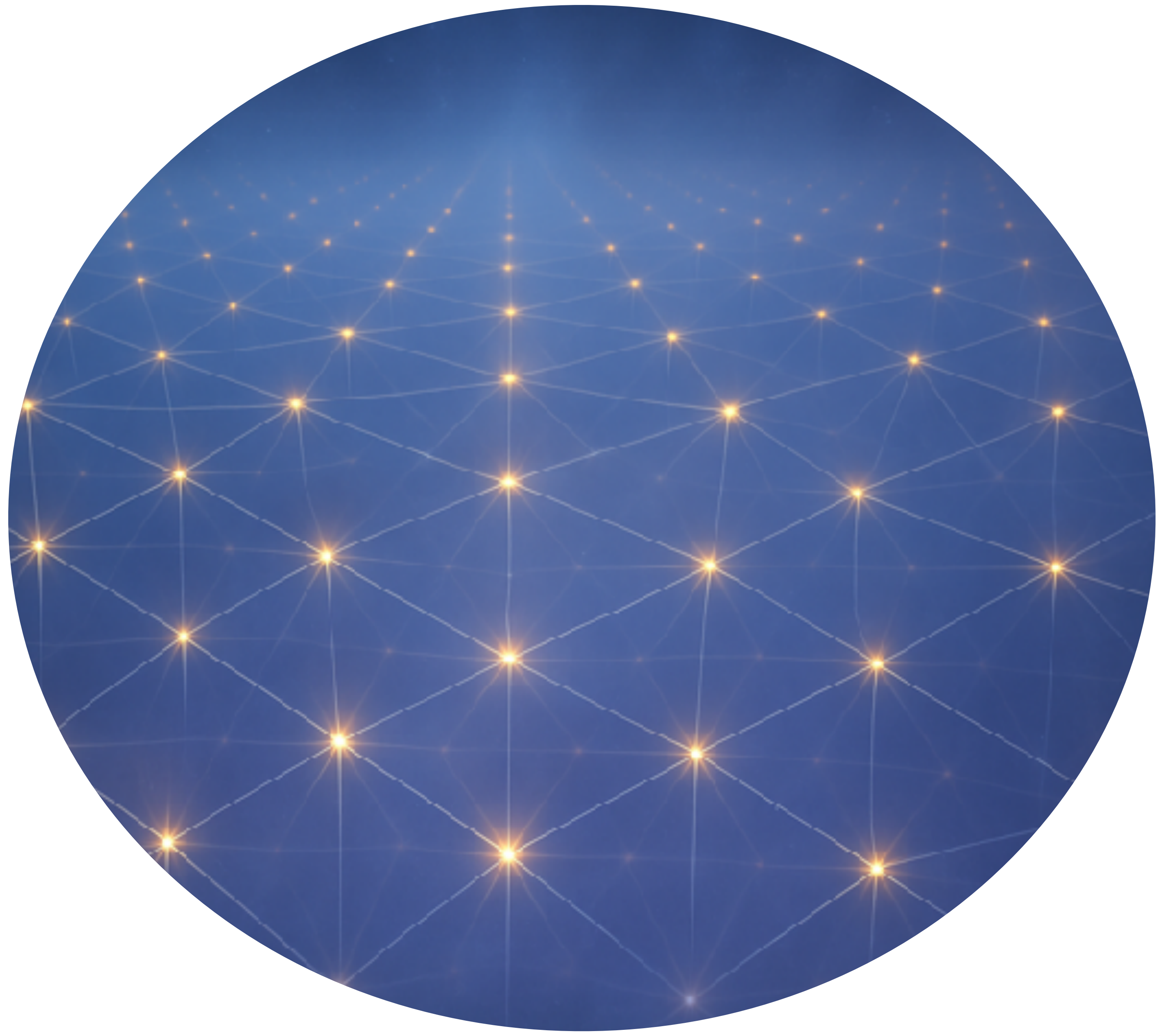

Five **massless scalar fields** **minimally** coupled to GR

Classical action for five scalar fields

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}M^{(\lambda)}_{\mu\nu}g^{ab}\partial_a\chi^\mu\partial_b\chi^\nu$$

$$-\frac{\alpha_\phi}{2}\int \mathrm{d}^4x\sqrt{-g}g^{ab}\partial_a\phi\partial_b\phi$$

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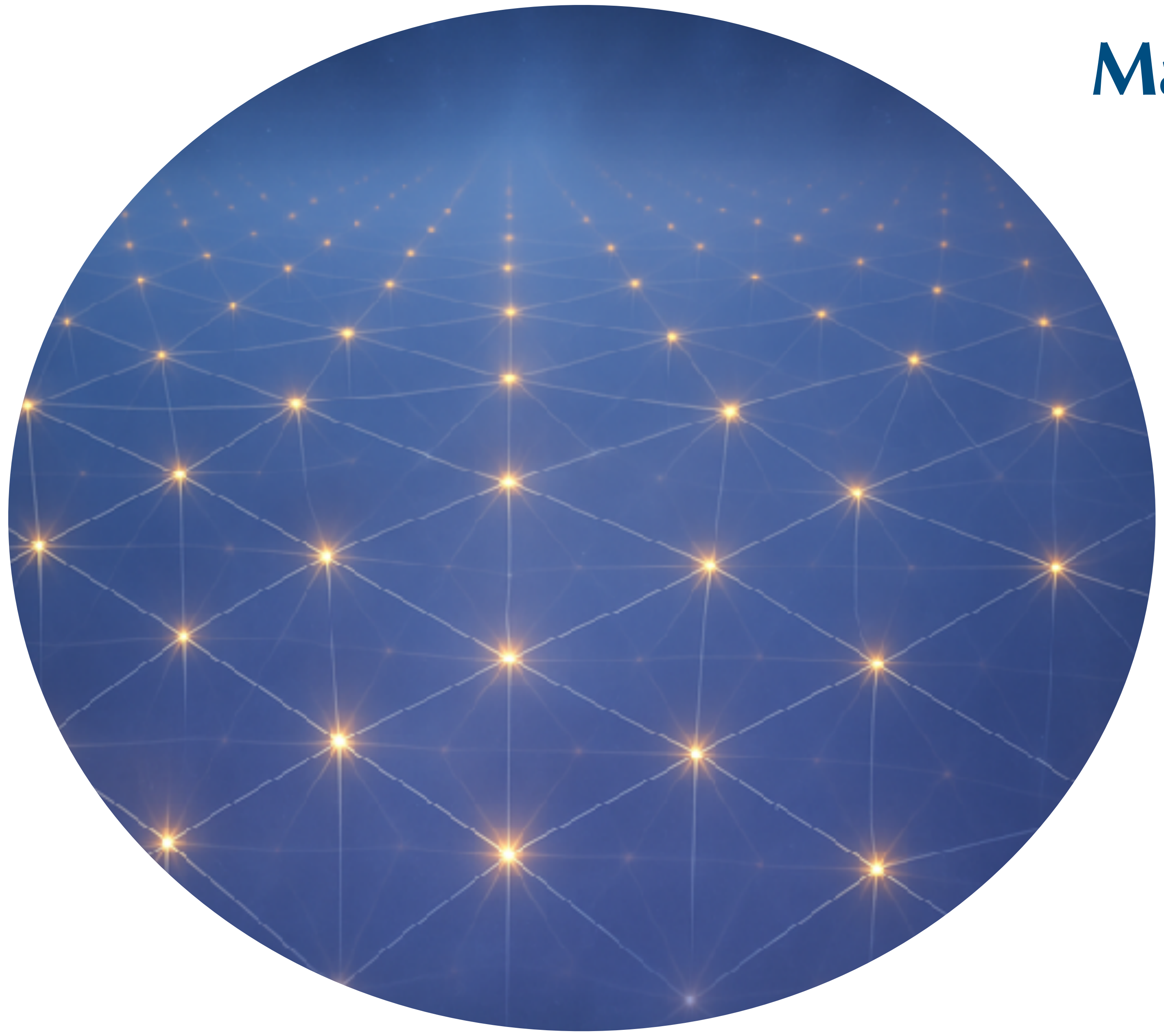

GFT condensate setting

Matter fields

- 4 scalar fields: clock and rods χ^μ
- 1 scalar field: matter content of the universe ϕ

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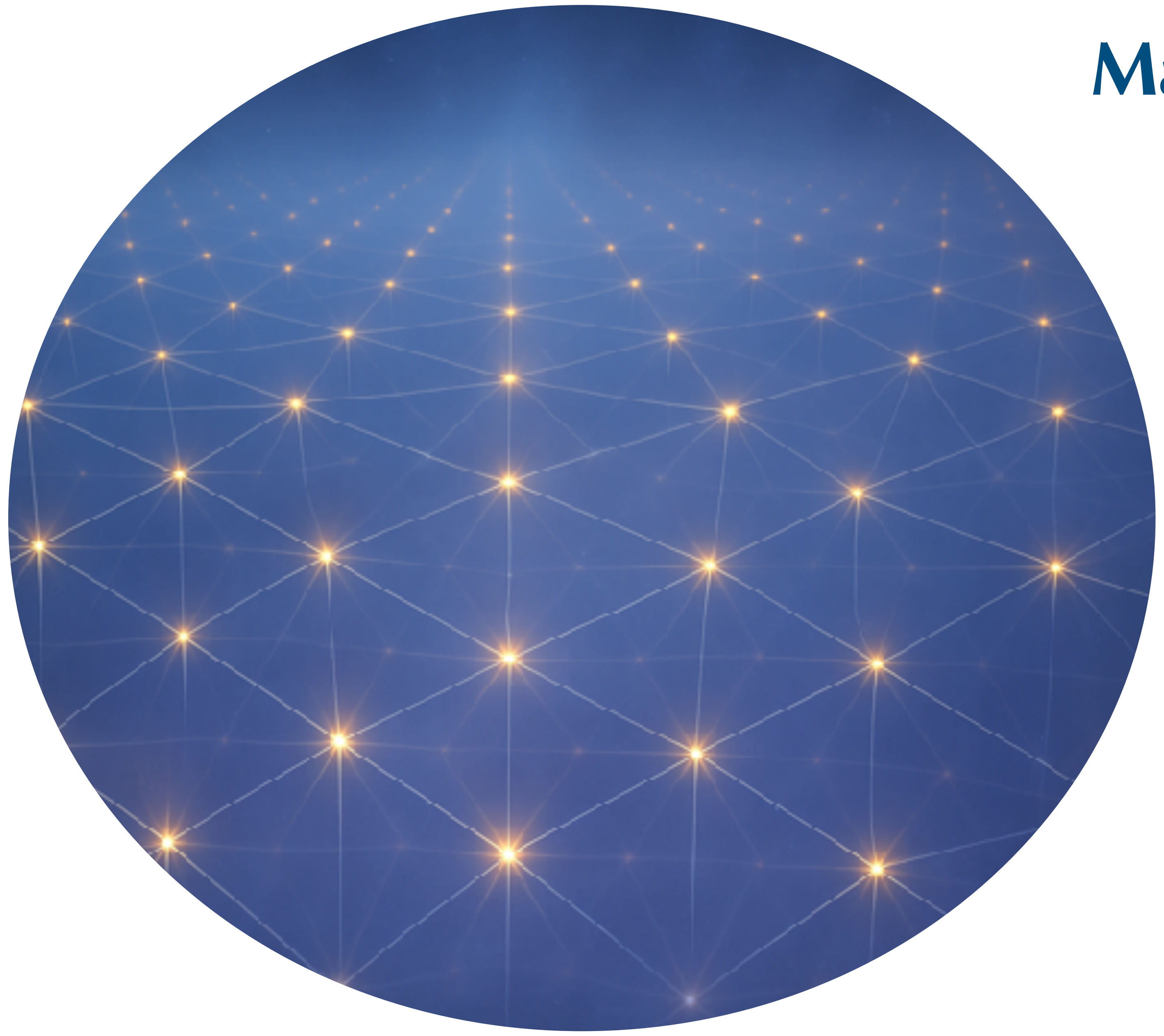

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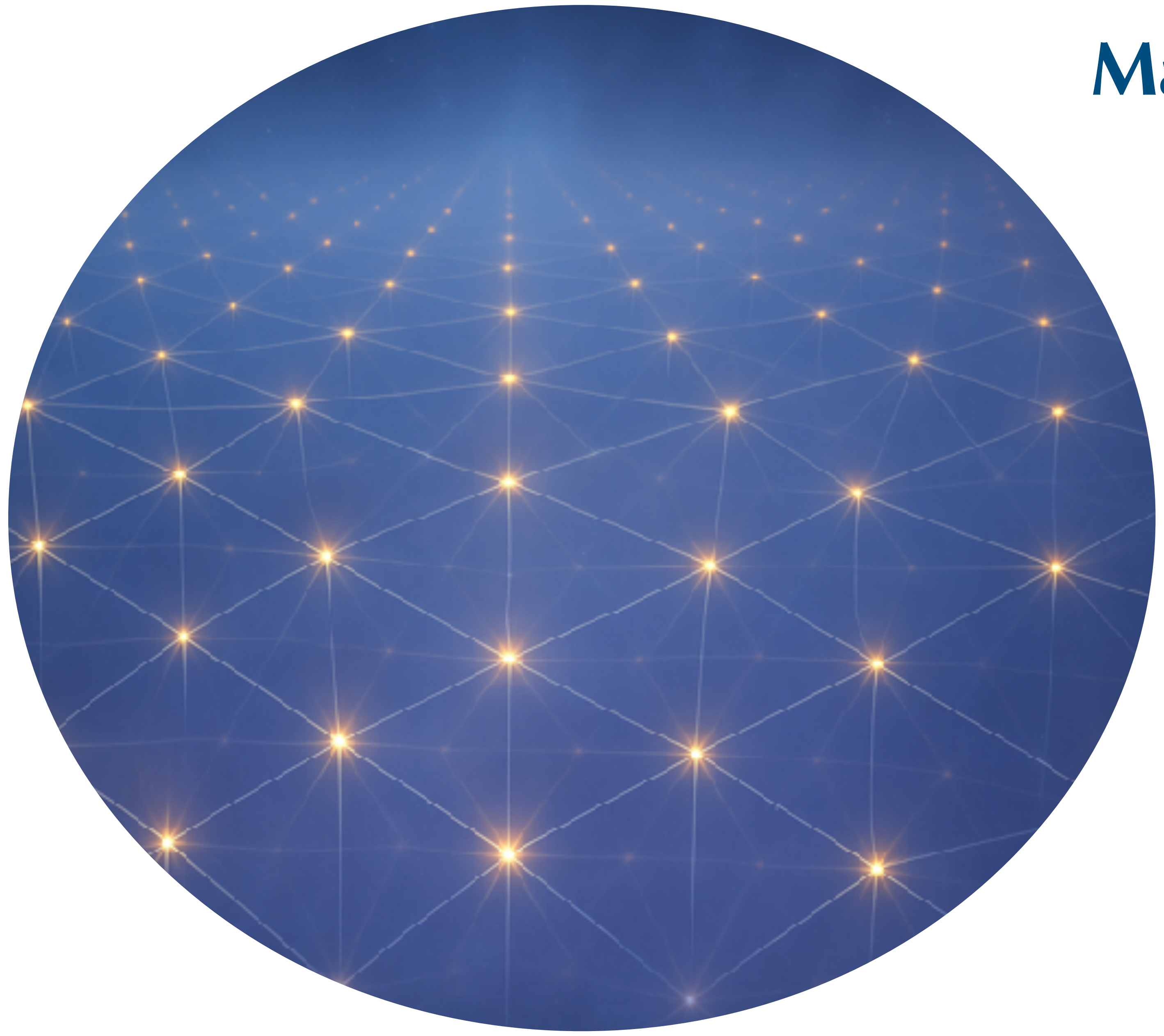

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$\sigma\left(g_I, \chi^\mu, \phi\right)$: Wavefunction on minisuperspace

The domain of the condensate wavefunction $\simeq$ minisuperspace of homogeneous geometries

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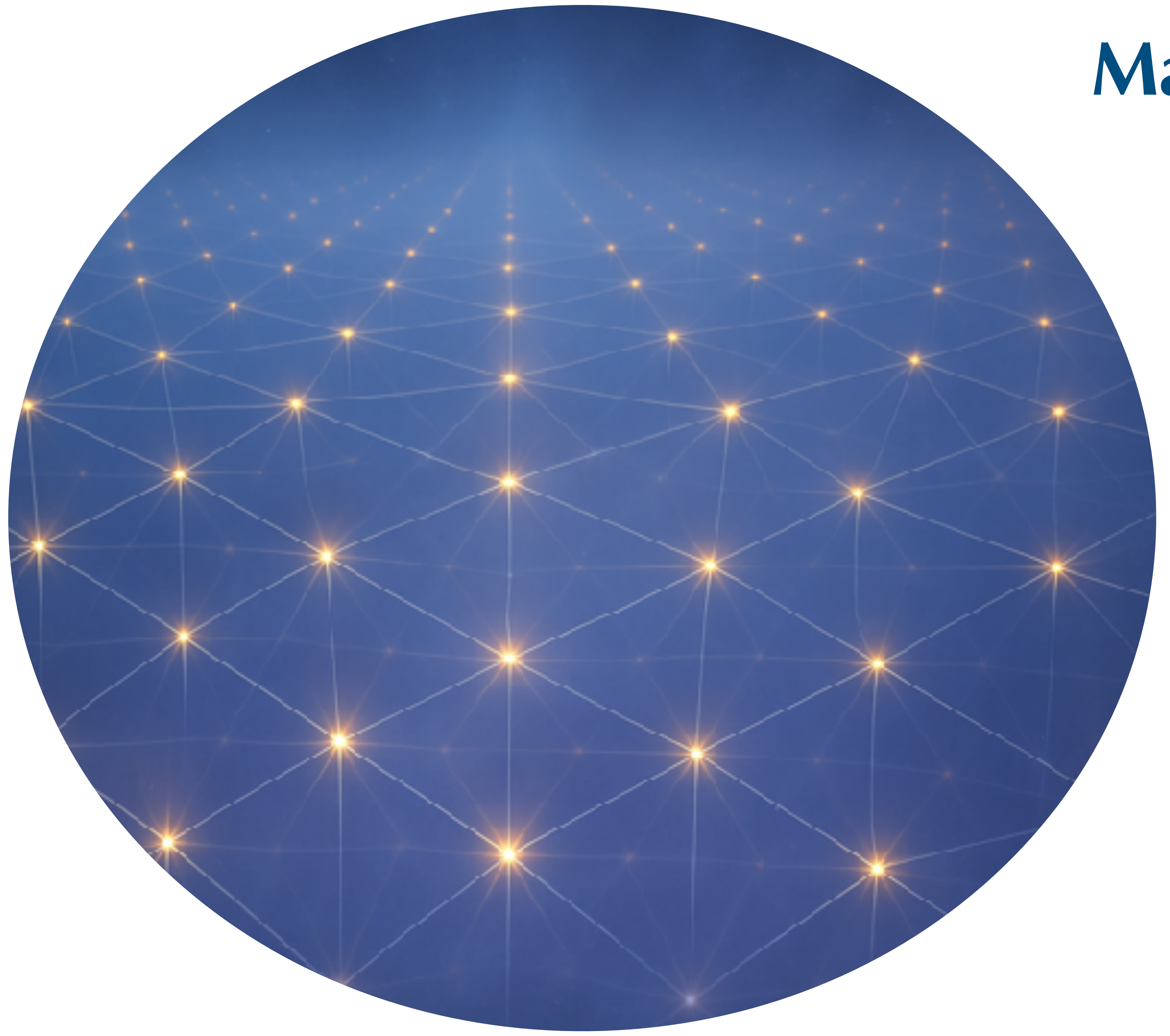

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Factorisation

$$\sigma_{\epsilon,\delta,\pi_0,\pi_x;x^\mu}\left(g_I,\chi^\mu,\phi\right)=\eta_\epsilon\left(\chi^0-x^0;\pi_0\right)\eta_\delta\left(|\boldsymbol{\chi}-\mathbf{x}|;\pi_x\right)\tilde{\sigma}\left(g_I,\chi^\mu,\phi\right)$$

Effective dynamics of GFT condensates (dynamics)

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- Extract effective hydrodynamic equations: **averaged** form of the equations of motion

$$\left\langle \sigma_{\epsilon^\mu}; x^\mu, \pi_\mu \left| \frac{\delta S_{\mathrm{GFT}}\left[\hat{\varphi}, \hat{\varphi}^\dagger\right]}{\delta \hat{\varphi}^\dagger\left(g_I, x^\mu\right)} \right| \sigma_{\epsilon^\mu}; x^\mu, \pi_\mu \right\rangle = 0\,.$$

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$$\partial_0^2 \tilde{\sigma}_j\left(x, \pi_\phi\right) - i\gamma\partial_0\tilde{\sigma}_j\left(x, \pi_\phi\right) - E_j\left(\pi_\phi\right)\tilde{\sigma}_j\left(x, \pi_\phi\right) + \alpha^2\nabla^2\tilde{\sigma}_j\left(x, \pi_\phi\right) = 0$$

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- Physical entities of interest correspond to **hydrodynamic averages**: expectation values of basic operators

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Perturbative framework with respect to the spatial gradient (slightly inhomogeneous relational quantities)

$$\rho_j = \rho_0 + \delta\rho\,, \quad \theta_j = \theta_0 + \delta\theta\,.$$

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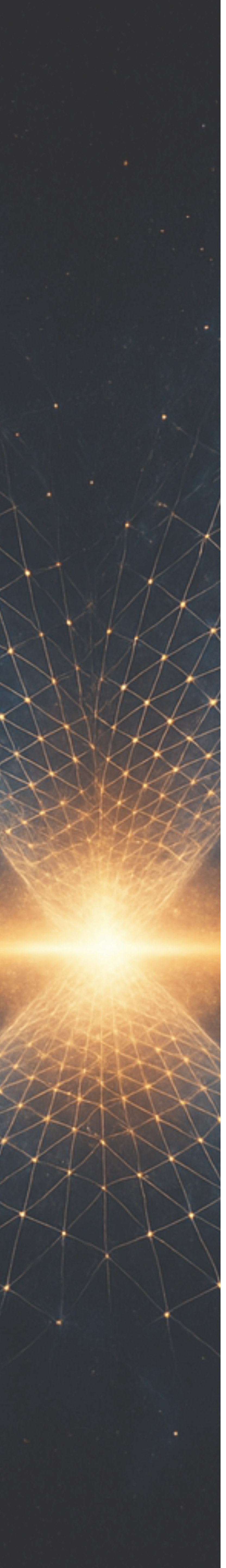
Dynamics of the background quantities

Dynamics of the perturbed quantities



3. Emergent scalar field theory on curved spacetime from GFT

$a^3 = \langle \hat{V} \rangle = V_j \rho_0^2$

Background level

Evolution equation for the background GFT phase

$$\ddot{\theta}_0 + \frac{3\dot{a}}{a}\left(\dot{\theta}_0 - \frac{\gamma}{2}\right) = 0$$

- Function of the clock and the scalar field momentum

- Minisuperspace

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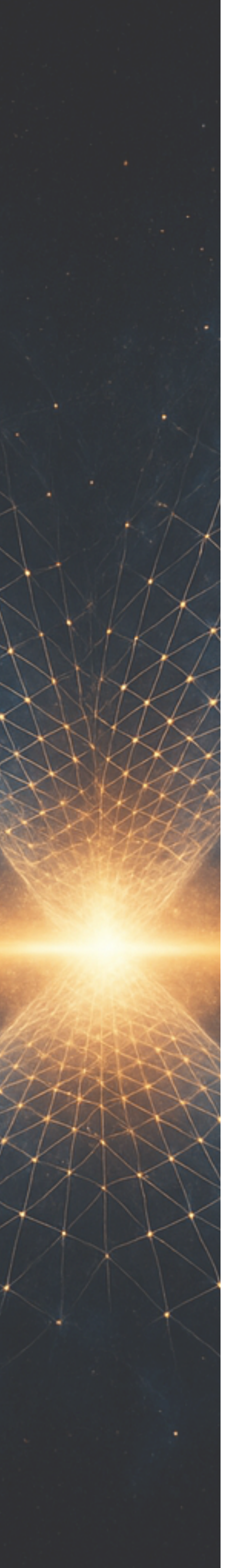
- Study the dynamics of the background homogeneous scalar field (necessary for the scalar perturbation treatment)



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- **Large volume regime (large GFT densities): this can be matched to GR results**

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For the small volume regime: we have a quantum bounce replacing the classical singularity

3. Emergent scalar field theory on curved spacetime from GFT

Near the bounce

Background level

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- Function of the clock and the scalar field momentum

Not observable: adequate mathematical formulation!

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Effective metric?

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Effective metric?

Closed equation of motion of the scalar field with no condensate parametrisation

Matter field dynamics near the bounce

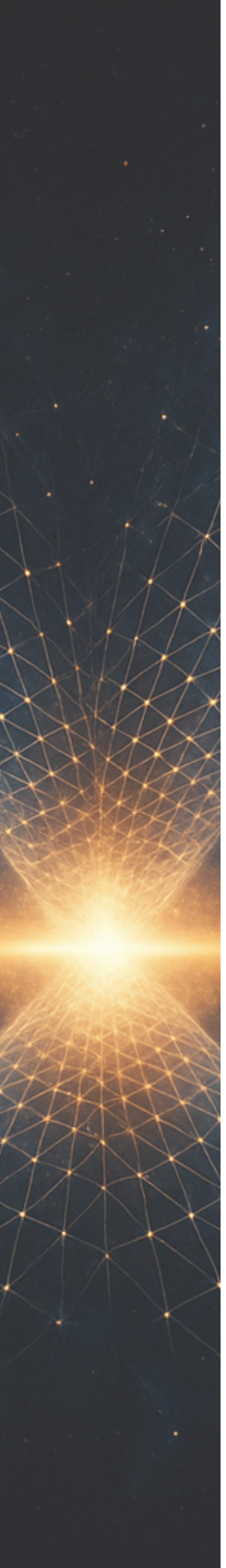
Background level

Matter field dynamics near the bounce **Background level**

1 Inversion relation: $\theta_0 = \left(a^3 f(\pi_\phi, x^0)\right)^{-1} \phi_0 + \frac{\gamma x^0}{2}$

2 Fundamental dynamics of the GFT phase:

$$\ddot{\theta}_0 + \frac{3\dot{a}}{a}\left(\dot{\theta}_0 - \frac{\gamma}{2}\right) = 0$$



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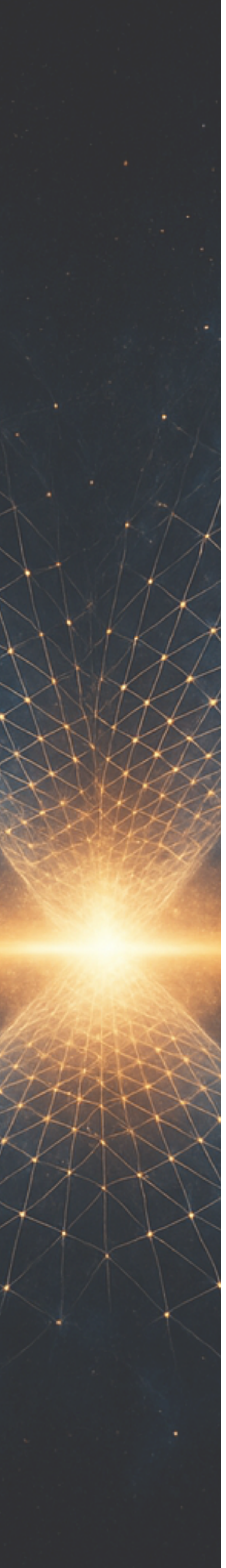
3 Clock is perfect

4 Assume an effective d′Alembert operator:

$$\Box = \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g}\, g^{\mu\nu} \partial_\nu \phi_0\right)$$



Matter field dynamics near the bounce

Background level

1 Inversion relation: $\theta_0 = \left(a^3 f(\pi_\phi, x^0)\right)^{-1} \phi_0 + \dfrac{\gamma x^0}{2}$

2 Fundamental dynamics of the GFT phase:

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$\Rightarrow$ **Closed evolution equation for the homogeneous scalar field**

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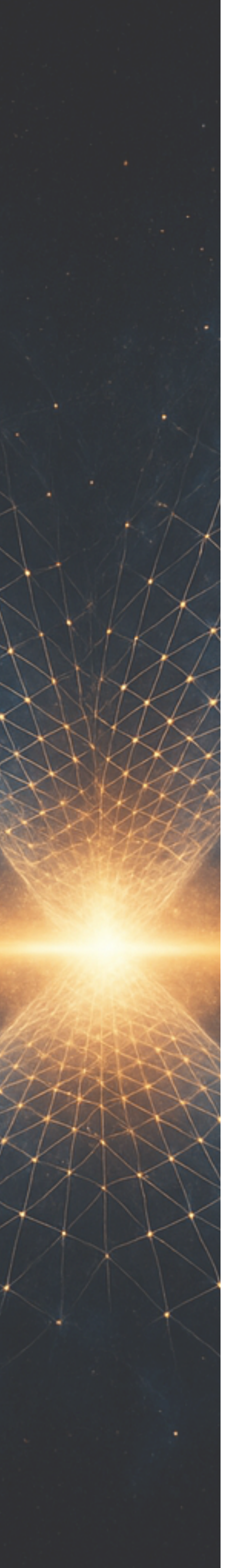

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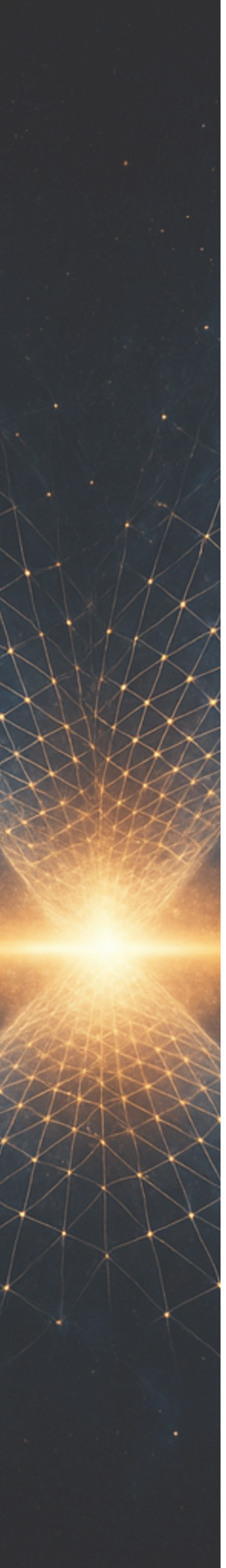

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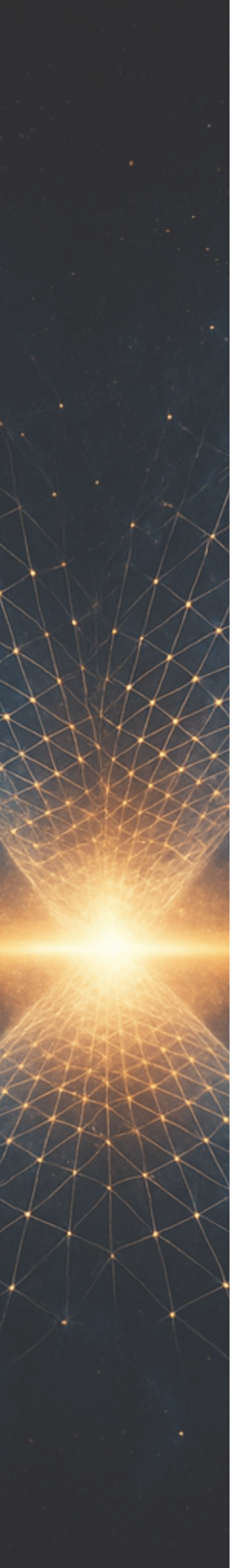
And additional terms …

Modification with respect to the initial assumptions/classical system



What do we make of this?

Near the bounce

$$\ddot{\phi}_0 - \left(3H + 2\frac{\dot{f}}{f}\right)\dot{\phi}_0 + g_{00}\left(R + \xi m^2\right)\phi_0 = 0\,.$$

- **Non-minimal coupling:** standard field theory on curved spacetime tells us that such coupling to gravity occurs due to the natural presence of strong gravitational interaction between the geometrical degrees of freedom and those of matter.

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Modification with respect to classical system (**massless minimally coupled scalar field**)

4. Scalar perturbations: Prescription to extract a field theory on curved spacetime

Near the bounce

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Background level

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- ☑ Dynamics of the homogenous scalar field
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Near the bounce

Background level

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<u>Goal:</u> Express the effective evolution equation of the perturbed scalar field as a field theory on curved spacetime

We need to get rid of the condensate parametrisation !

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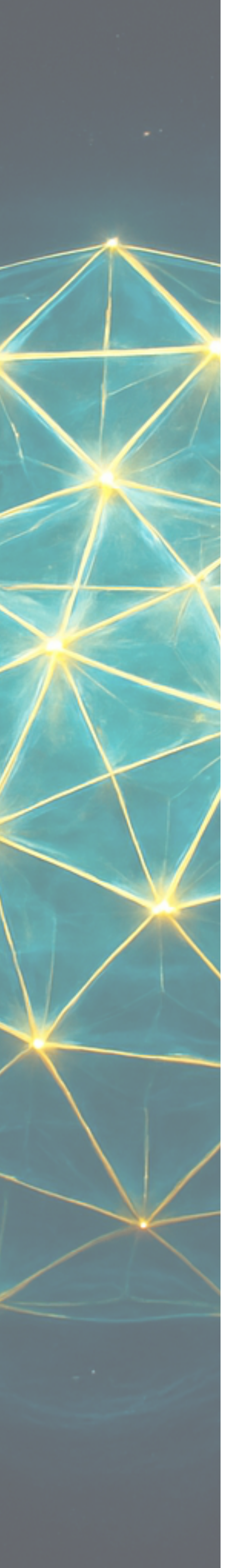

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$$\delta\theta \equiv \left(\Psi(x^0, \pi_\phi) + \Phi(x, \pi_\phi)\right)^{-1} \delta\phi \qquad \delta\phi = \delta\langle\hat{\Phi}\rangle_\sigma = \left[\frac{\delta N}{N_0}\phi_0 + N_0 \partial_{\pi_\phi}\delta\theta\right]_{\pi_\phi = \tilde{\pi}_\phi}$$

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- Modified dispersion relation
- Extract the Bardeen Potentials from the effective dynamics

Indications of possible phenomenology

What did we achieve?

- ☑ Starting from a full quantum gravity setting: we extracted an effective scalar field theory on a curved spacetime.

- ☑ At early times, this resulted in the emergence of possibly:
  - Non-minimal coupling to gravity at early times —> Modified theory of gravity
  - Mass/matter or potential term that can be studied in the inflationary scenario or Chameleon mechanism

- ☑ The ground now is set for phenomenology and contact with observations!

Open questions and outlook

- Identify modified gravity theories that are produced from the effective dynamics outlined above .

- Classify such theories according to the QG parameters.

- **<u>Phenomenological investigations:</u>** Study the Inflationary potential, Mass term (chameleon mech.), the modified dispersion relation (dissipation effects) ..

- **<u>Canonical quantization of the scalar field</u> :** study all phenomena that a standard QFT theory produces: Correlation functions, CMB spectrum —> observational cosmology.

Thank you for your attention!