Covariance in Spherically Symmetric Effective Models of Quantun Gravity and Black Holes

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In collaboration with Jerzy Lewandowski, Jinsong Yang and Cong Zhang [PRD 111, L081504 (2025) and arXiv:2412.02487]

The Issue of Covariance

- Different ideas of QG are being tested in spherically symmetric models, as they contain local degrees of freedom rather than the Mini-superspace models.
- In some spherically symmetric models of LQG, the effetive metrics give the spacetime structures dramatically differennt from those of classical metrics [Haggard and Rovelli 2015, Ashetekar el 2018, Lewandowski el 2023].

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- Could the BH to WH transition happens in some effective dynamics of spherically symmetric QG model?
- If so, whether is the spherically symmetric effective theory covariant?
- How to formulate the conditions of covariance into tractable equations in the models?

Kinematical Setting of Spherically Symmetric Models

- The kinematics of the spherically symmetric GR is defined on a 4-dimensional manifold $\mathcal{M}_2 \times \mathbb{S}^2$, where $\mathbb{S}^2$ denots the 2-sphere and $\mathcal{M}_2 \cong \mathbb{R} \times \Sigma$ with Σ being an 1-dimensional manifold.

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- The phase space of the model contains the canonical pairs (K_2, E^2) for the 2-dimensional gravity and (K_1, E^1) for the dilaton, where E^2 and K_1 are scalar densities of weight 1, while K_2 and E^1 are scalars on Σ [Bojowald, Swiderski, 2005].
- The nontrivial Poisson brackets read: $\{K_1(x), E^1(y)\} = 2\delta(x, y)$ and $\{K_2(x), E^2(y)\} = \delta(x, y)$, where the geometric units with $G = 1 = c$ are applied.

Ansatz of the Effective Models

- For an arbitrary effective model of some canonical QG theory, we assume that the diffeomorphism constraint keeps the same as the classical expression, but the effective Hamiltonian constraint H_{eff} deviates from the classical one.

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- The constraint algebra is expected to mirror the classical one but with a correction factor μ to account for QG effects:

$$\begin{aligned}
\{H_x[N_1^x], H_x[N_2^x]\} &= H_x[N_1^x \partial_x N_2^x - N_2^x \partial_x N_1^x], \\
\{H_x[N_1^x], H_{eff}[N_1]\} &= H_{eff}[N_1^x \partial_x N_1], \\
\{H_{eff}[N_1], H_{eff}[N_2]\} &= H_x[S(N_1 \partial_x N_2 - N_2 \partial_x N_1)],
\end{aligned}$$

with $S \equiv \mu E^1 (E^2)^{-2}$ being the structure function.

Covariance Issue in Effective Models

- Assuming that the function S in the constraint algebra represents the (x,x)-component of the inverse spatial metric of $g^{(\mu)}_{\rho\sigma}$, the algebra can retain the same geometric interpretation as in the classical case. Then

$$ds^2 = -N^2 dt^2 + \frac{(E^2)^2}{\mu E^1}(dx + N^x dt)^2 + E^1 d\Omega^2.$$

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- To ensure the covariance with respect to $g^{(\mu)}_{\rho\sigma}$, we seek the effective Hamiltonian constraint H_{eff} such that

$$\delta g^{(\mu)}_{\rho\sigma} = \mathcal{L}_{\alpha\mathfrak{N}} g^{(\mu)}_{\rho\sigma},$$

up to terms proportional to constraints. Here, $\mathfrak{N} = \partial_t - N^x \partial_x$, and $\delta g^{(\mu)}_{\rho\sigma}$ represents the infinitesimal gauge transformation generated by $H_{eff}[\alpha N]$.

Conditions for the Covariance

- By the covariance condition, one can employ E^2 and the following basic scalars to construct H_{eff}

$$s_1 = E^1,\ s_2 = K_2,\ s_3 = \frac{K_1}{E^2},\ s_4 = \frac{\partial_x s_1}{E^2},\ s_5 = \frac{\partial_x s_4}{E^2}.$$

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- The covariance condition and the form of the constraint algebra require that H_{eff} takes the form

$$H_{eff} = -2E^2\left[\partial_{s_1} M_{\rm eff} + \frac{\partial_{s_2} M_{\rm eff}}{2} s_3 + \frac{\partial_{s_4} M_{\rm eff}}{s_4} s_5 + \mathcal{R}\right],$$

where $\mathcal{R}$ is an arbitrary function of s_1 and $M_{\rm eff}$, and $M_{\rm eff}$ depending on s_1, s_2, s_4 is a solution to:

$$\frac{\mu s_1 s_4}{4} = (\partial_{s_2} M_{\rm eff})\partial_{s_2}\partial_{s_4} M_{\rm eff} - (\partial_{s_4} M_{\rm eff})\partial_{s_2}^2 M_{\rm eff},$$
$$(\partial_{s_2}\mu)\partial_{s_4} M_{\rm eff} - (\partial_{s_2} M_{\rm eff})\partial_{s_4}\mu = 0.$$

The Effective Model I

- The first effective mass, satisfying the above equations with $\mu \equiv \mu_1 = 1$ and a quantum parameter $\zeta \propto \sqrt{\hbar}$, reads

$$M_{\rm eff}^{(1)} = \frac{\sqrt{s_1}}{2} + \frac{\sqrt{s_1}^3 \sin^2\left(\frac{\zeta s_2}{\sqrt{s_1}}\right)}{2\zeta^2} - \frac{\sqrt{s_1}(s_4)^2}{8} e^{\frac{2i\zeta s_2}{\sqrt{s_1}}} \, .$$

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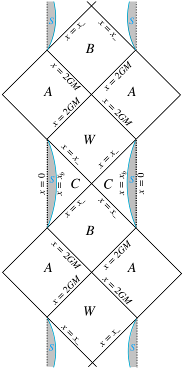
- A stationary BH solution to this model reads

$$\begin{aligned} ds^2_{(1)} =& - f_1 dt^2 + f_1^{-1} dx^2 + x^2 d\Omega^2, \\ f_1 =& 1 - \frac{2M}{x} + \frac{\zeta^2}{x^2}\left(1 - \frac{2M}{x}\right)^2. \end{aligned}$$

- The function f_1 has two positive roots for all $M > 0$: $x_+ = 2M$ and $x_- = \zeta^2/\beta - \beta/3$, with $\beta^3 = 3\zeta^2\left(\sqrt{81M^2 + 3\zeta^2} - 9M\right)$.

Spacetime Stucture of the BH in Model I



- While the classical Schwarzschild singularity is resolved in this spacetime by a transition region connecting a BH to a WH, the timelike singularities persist at $x = 0$.

The Effective Model II

- The second effective mass, satisfying the above equations with $\mu \equiv \mu_2 = 1 + \frac{\zeta^2}{\sqrt{s_1}^3}\left(\sqrt{s_1} - 2M_{\rm eff}^{(2)}\right)$, reads

$$M_{\rm eff}^{(2)} = \frac{\sqrt{s_1}}{2} + \frac{\sqrt{s_1}^3 \sin^2\left(\frac{\zeta s_2}{\sqrt{s_1}}\right)}{2\zeta^2} - \frac{\sqrt{s_1}(s_4)^2 \cos^2\left(\frac{\zeta s_2}{\sqrt{s_1}}\right)}{8}.$$

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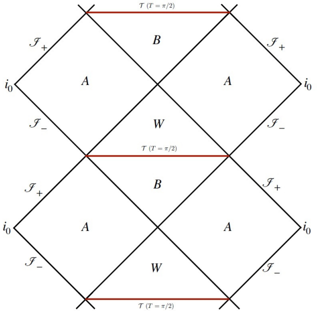
- A stationary BH solution to this model reads

$$\begin{aligned} ds_{(2)}^2 =& - f_2 dt^2 + \mu_2^{-1} f_2^{-1} dx^2 + x^2 d\Omega^2, \\ f_2 =& 1 - \frac{2M}{x}, \;\; \mu_2 = 1 + \frac{\zeta^2}{x^2}\left(1 - \frac{2M}{x}\right). \end{aligned}$$

Spacetime Structure of the BH in Model II



- The classical singularity is replaced by a transition surface $\mathcal{T}$ connecting the regions B and W, where the Kretschmann scalar is bounded by $\mathcal{K}\big|_{\mathcal{T}} = \frac{81}{4\zeta^4} + O((M\zeta^5)^{-2/3})$.

The Effective Model III

- The third effective mass, satisfying the above equations with $\mu \equiv \mu_3 = 1 - \frac{4\zeta^4 (M_{\rm eff}^{(3)})^2}{s_1^3}$, reads

$$M_{\rm eff}^{(3)} = \frac{{s_1}^{3/2}}{2\zeta^2} \sin\left(\frac{\zeta^2}{s_1}(1 + (s_2)^2 - \frac{(s_4)^2}{4})\right)$$

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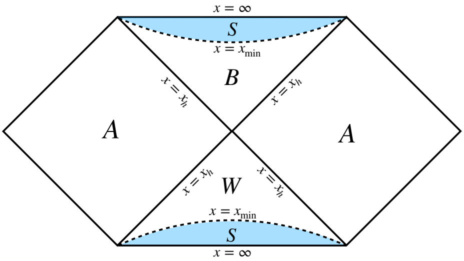
- A stationary BH solution to this model reads

$$\begin{aligned} ds_{(3)}^2 =& - f_3^{(n)} dt^2 + \mu_3^{-1} (f_3^{(n)})^{-1} dx^2 + x^2 d\Omega^2, \\ f_3^{(n)} =& 1 - (-1)^n \frac{x^2}{\zeta^2} \arcsin(\frac{2M\zeta^2}{x^3}) - \frac{n\pi x^2}{\zeta^2}, \\ \mu_3 =& 1 - \frac{4\zeta^4 M^2}{x^6}. \end{aligned}$$

Spacetime Structure of the BH in Model III



- The case of $M > \frac{\zeta}{2}\left(\frac{2}{\pi}\right)^{3/2}$: The spacetime is extended beyond the singularity into an asympototic Schwarzschild-de Sitter one with negative mass and does not contain any Cauchy horizons.

Summary

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Summary

- General covariance is precisely formulated into a set of equations in spherically symmetric models, leading to the tractable conditions for ensuring the covariance.
- Alternative candidates of effective Hamiltonian constraints satisfying the covaraince conditions are proposed.
- The quantum modified BH solutions of the effective models are obtained. Some of them capture qualitative characters of the previous effective models of LQG, including the singularity resolution and BH to WH transitions, while preserving the covariance.
- In a particular covariant model, the BH spacetime is extended beyond the singularity into an asympototic Schwarzschild-de Sitter one with negative mass and does not contain any Cauchy horizons.

!Thanks!