The regularization of spacetime singularities

Vania Vellucci

BridgeQG 2025, Paris

SDU ħQTC

General Relativity

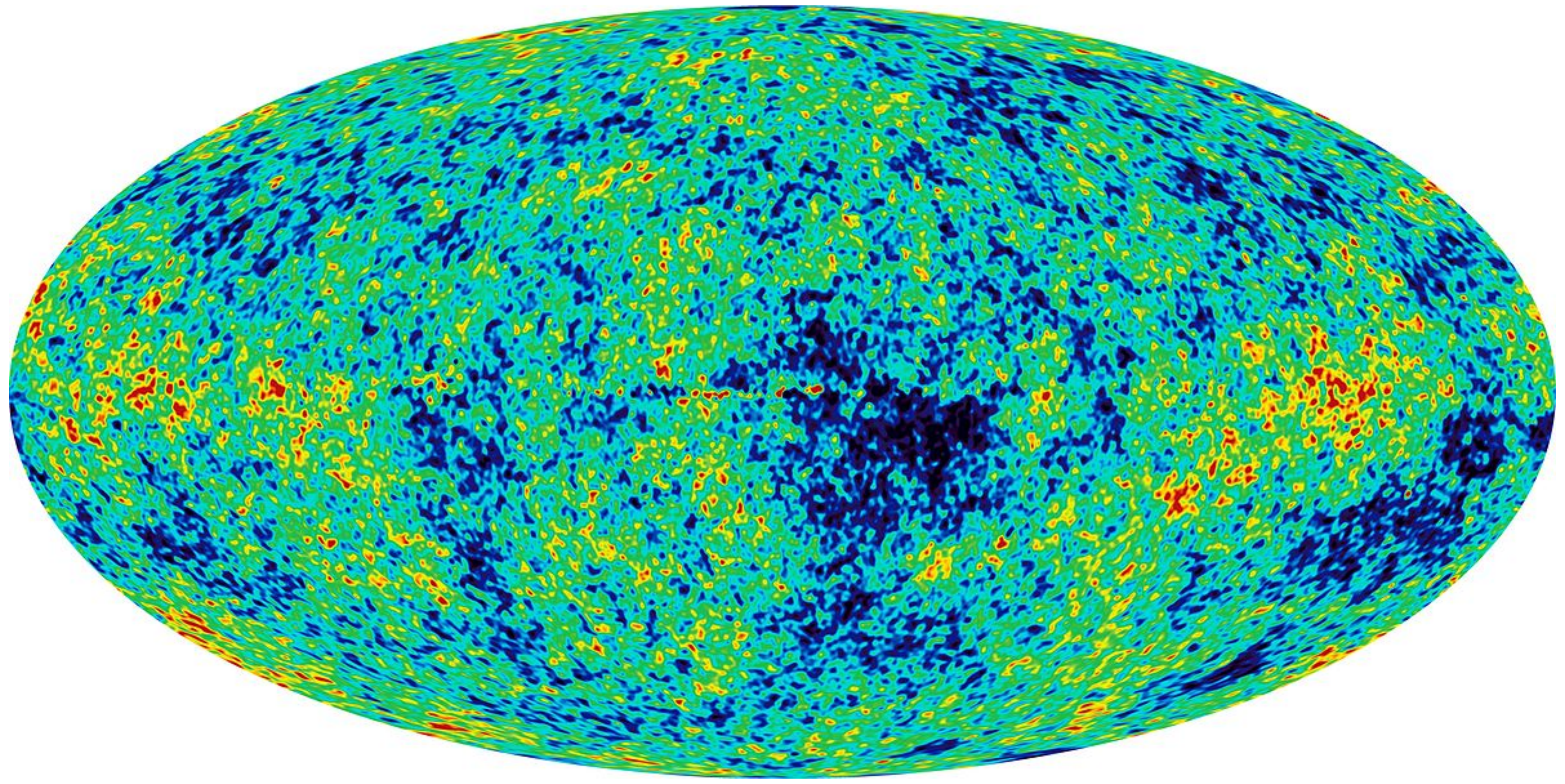
Gravitational collapse

Cosmological evolution

Singularity theorems

Black Hole singularities

Big Bang singularity



A singular space-time is **geodesic incomplete**:
there exist at least one geodesic that cannot be extended beyond a finite proper time or affine parameter (particles seem to disappear from existence!)

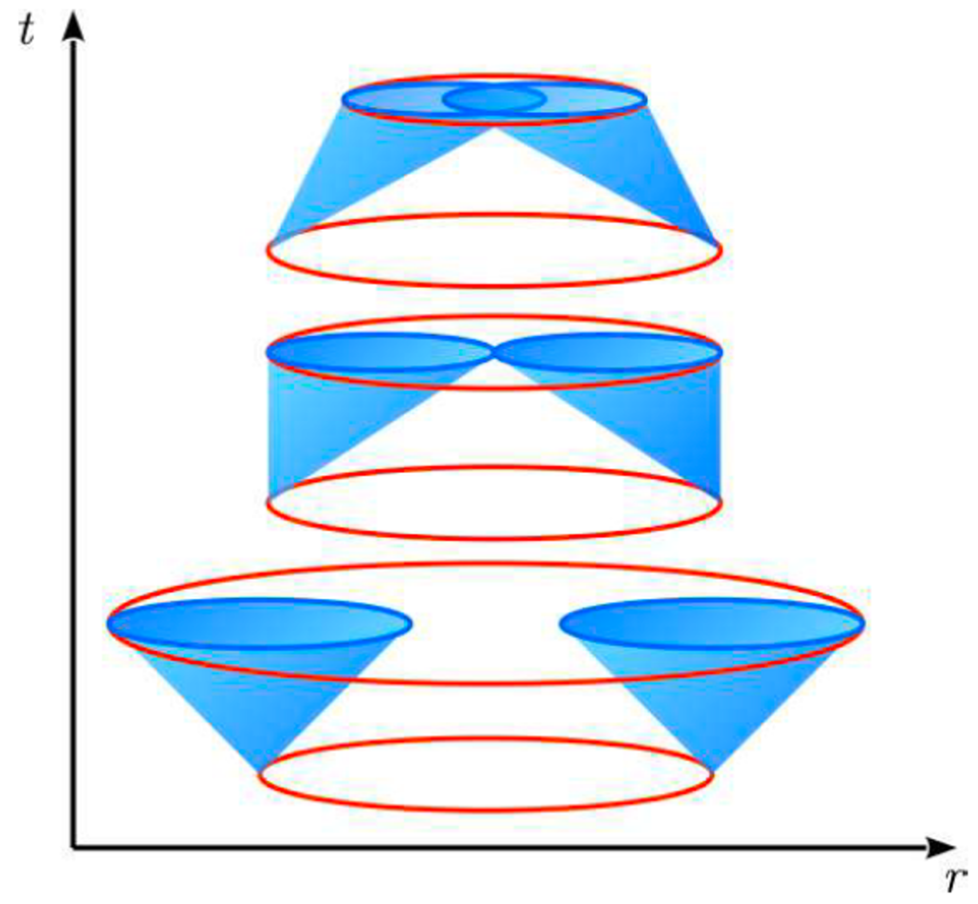
Black holes Mimickers

In a complete theory of **quantum gravity** we expect the formation of **spacetime singularities to be prevented**

There are few possible **alternatives to singular black holes** to describe the ultra-compact objects that we see in the sky

Horizonless Compact Objects	Regular Black holes	BHs with integrable singularites
Horizon ❌	Horizon ✓	Horizon ✓
Singularity ❌	Singularity ❌	Singularity ~

Regular Black holes



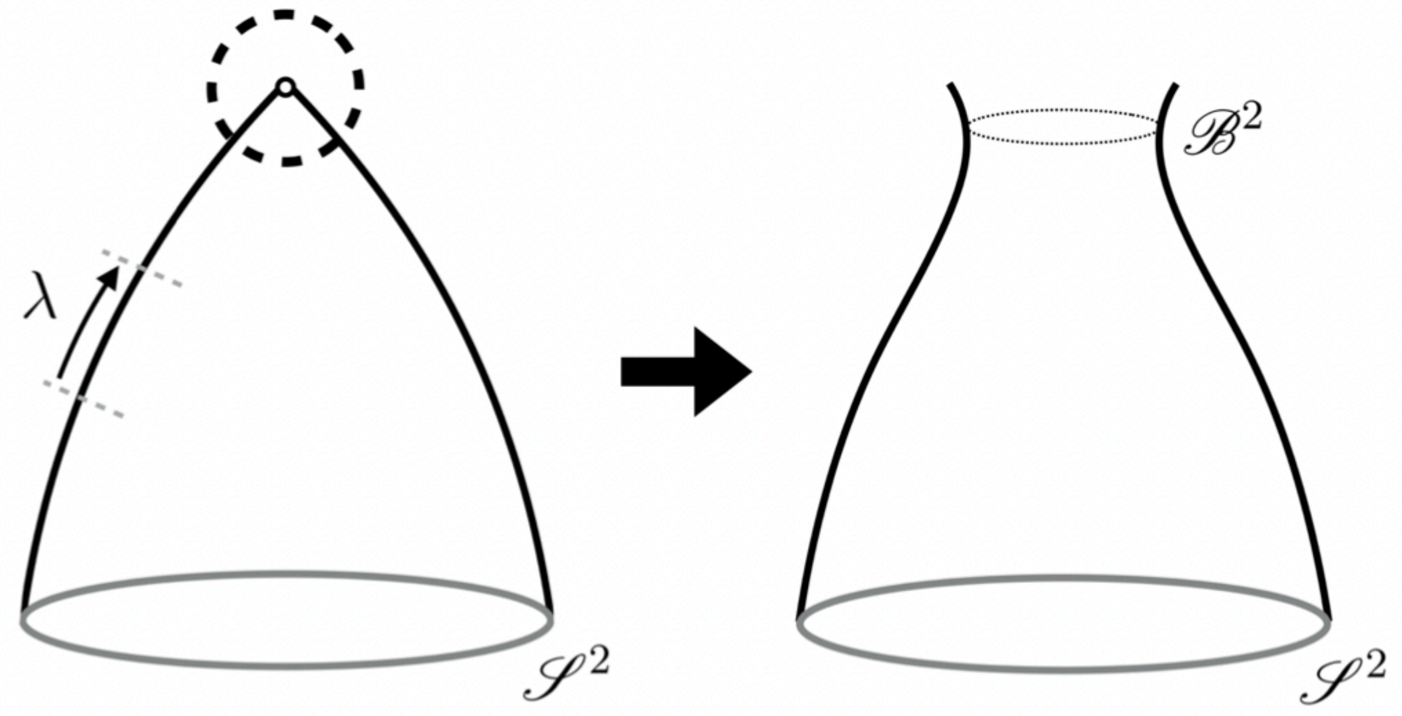
When both the expansion of null
ingoing θ_- and outgoing θ_+
geodesics become negative,
a trapped surface (horizon) is formed

$\theta_+ < 0$ in some points

GR + NEC +
non-compact
Cauchy
surface

Singular focusing point!

$\theta_+ \to -\infty$

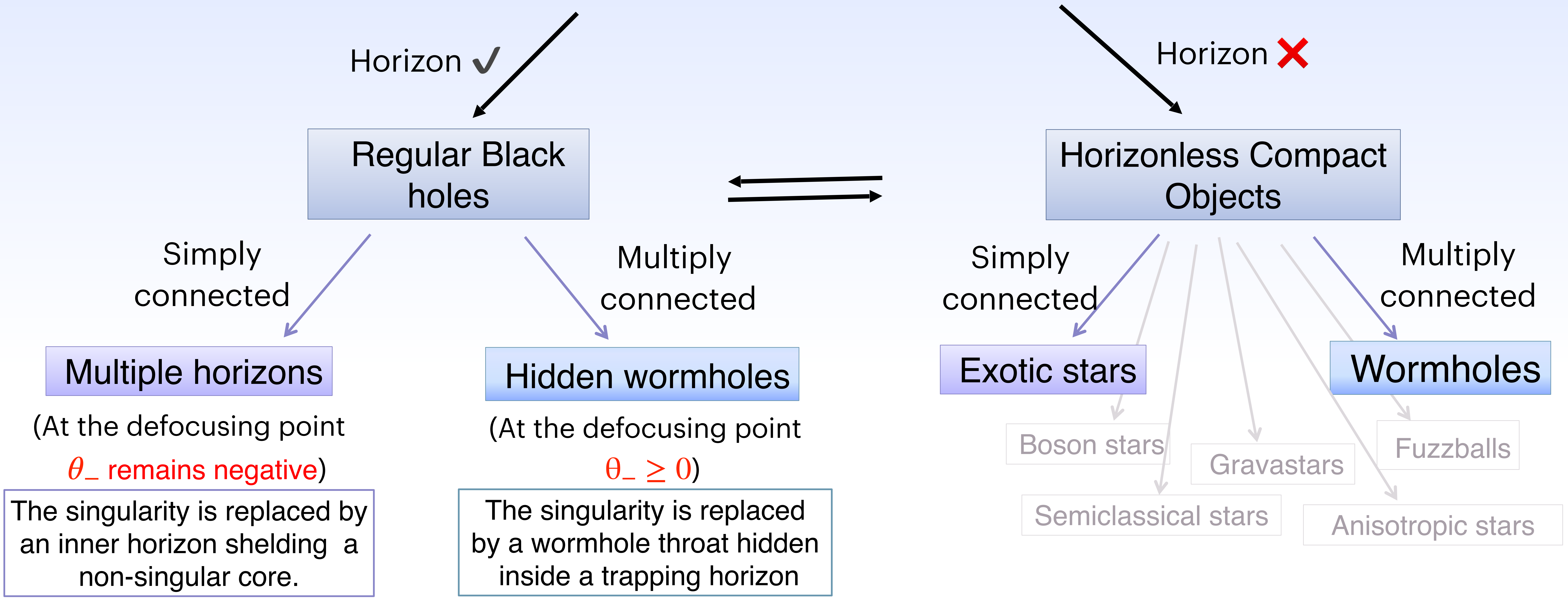


$\theta_+ < 0$ in some points

~~GR + NEC +
non-compact
Cauchy
surface~~

Defocusing point at which the
expansion
changes sign again
$\theta_+ = 0$

Fully regular Black holes Mimickers



Carballo Rubio et al.,2019

*we are considering spherical symmetric objects

The metric of a Black hole Mimicker

$$ds^2 = -e^{-2\phi(r)} f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right), \quad f(r) = 1 - \frac{2m(r)}{r}.$$

From Multiple horizons to Exotic stars (varying ℓ)

$$\phi(r) = 0$$

and

$$m(r) = M\frac{r^3}{r^3 + 2M\ell^2}$$ (Hayward metric)

or

$$m(r) = M\frac{r^3}{(r^2 + \ell^2)^{3/2}}$$ (Bardeen metric)

or...

From Hidden wormholes to traversable Wormholes (varying ℓ)

$$\phi(r) = \frac{1}{2}\log\left(1 - \frac{\ell^2}{r^2}\right)$$

and

$$m(r) = M\left(1 - \frac{\ell^2}{r^2}\right) + \frac{\ell^2}{2r}$$ (Simpson-Visser)

Carballo-Rubio et al. 2023

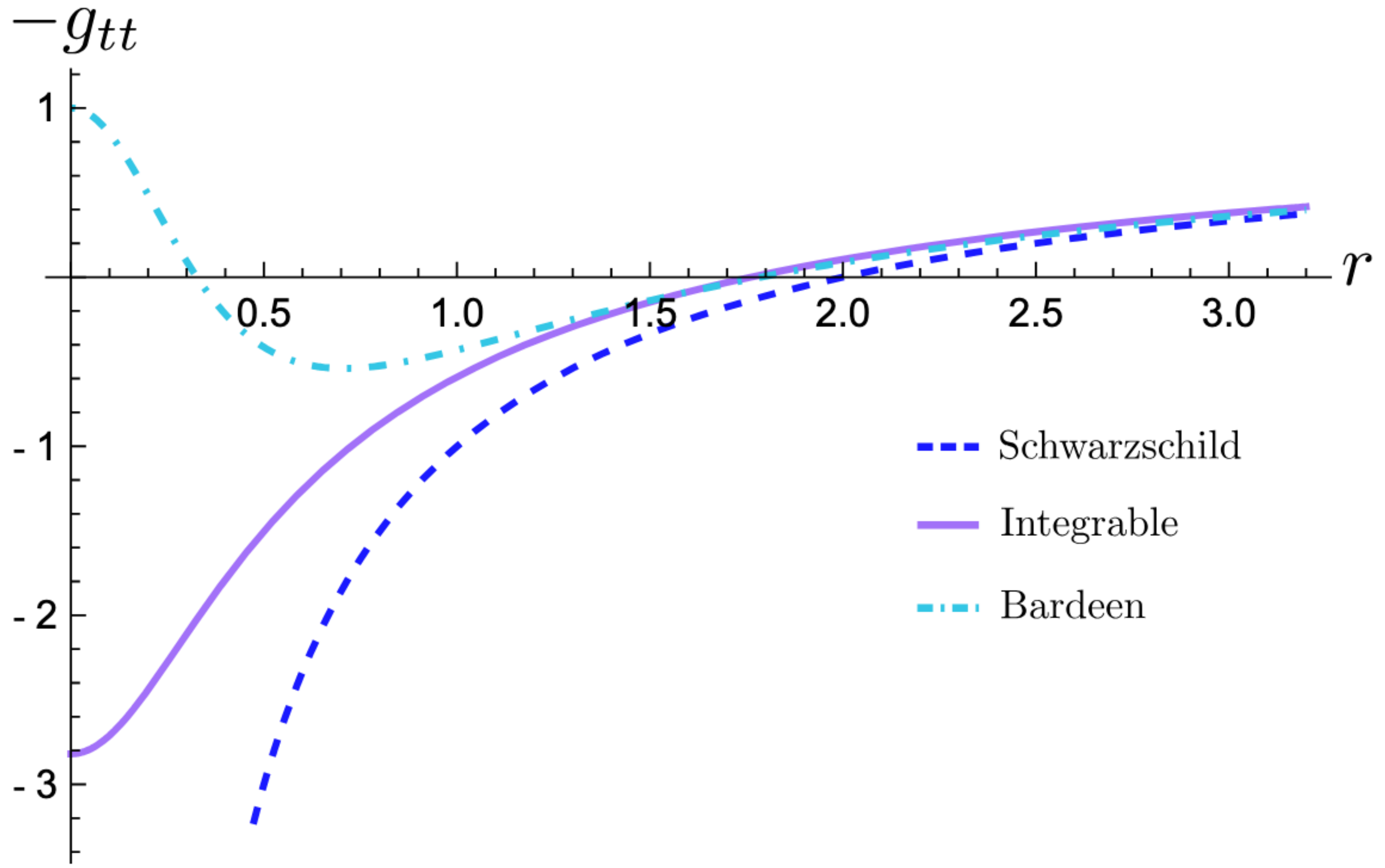
Black holes with integrable singularities

Lukash+Strokov 2011, Casadio 2024, Bonanno+Koch+Platania 2017…

Near the center

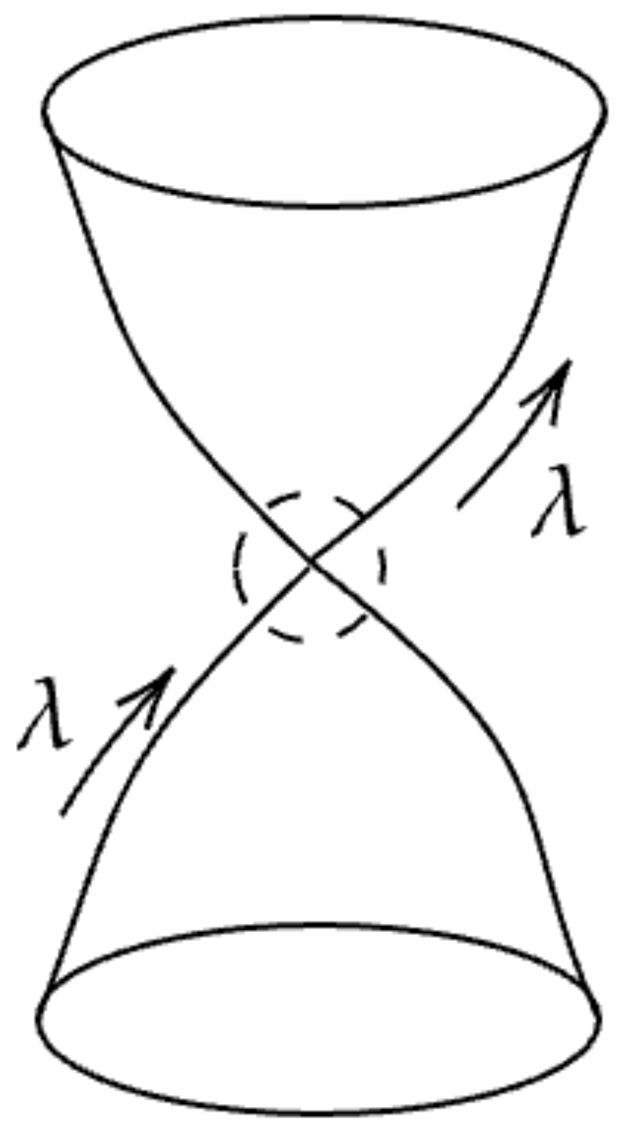
$$m(r) \sim r$$

the metric is regular but the curvature scalars diverges at $r = 0$



No need of an inner Cauchy horizon!

The **focusing point** is still there but **point** particles feel **finite tidal forces**, so you can "continue" their path after the focusing point



However when you extend the spacetime beyond the focusing point, global hyperbolicity is still lost: it is a sort of "**Cauchy point**"

Arrechea et al. 2025

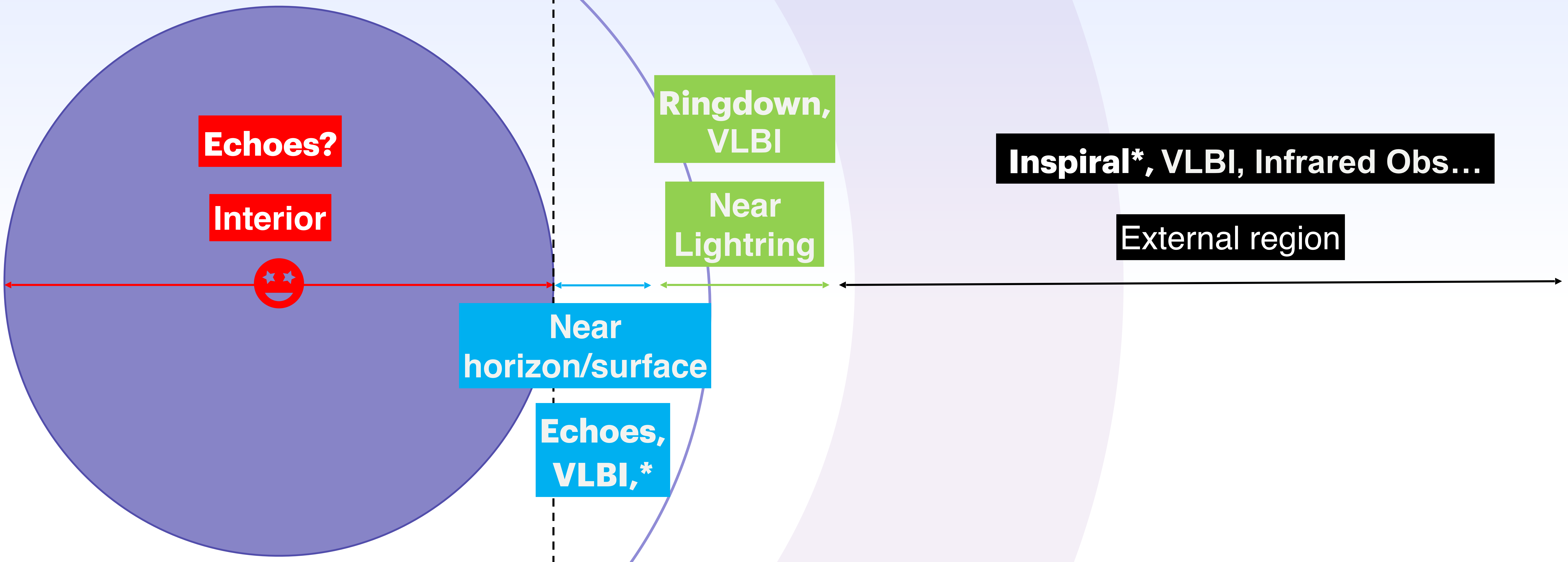
Disclaimer: instabilities

Within classical dynamics (GR) all these alternatives are unstable

BHs wiht integrable singularites	Regular Black holes	Horizonless Compact Objects
Horizon ✓	Horizon ✓	Horizon ❌
Singularity ~	Singularity ❌	Singularity ❌
↓	↓	↓
Unstable under $l \neq 0$ test field perturbations	Unstable under mass Inflation (at the Inner horizon)	Lightring instability (?)
Arrechea et al. 2025	Carballo-Rubio et al. 2018...	Cardoso et al. 2014...

(Quantum) corrected dynamics should prevent the collapse of these objects in singular BHs, taming these instabilities

Observational signatures of the regularization?

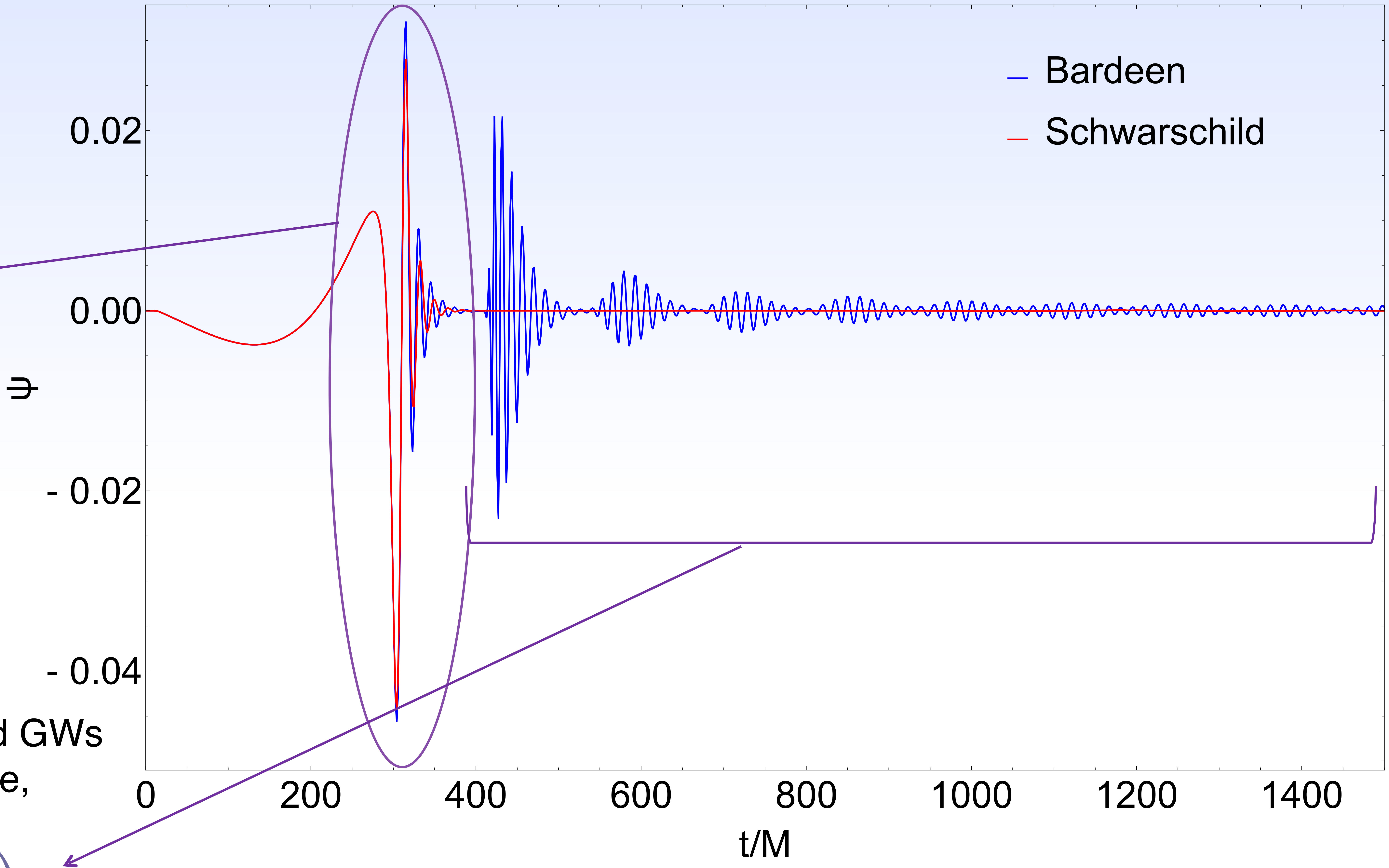


- Regularization effects!
- Observability?

- Regularization effects?
- Observability!

The post merger signal of a regular compact object

Observationally different ringdown signal for macroscopic values of ℓ / M



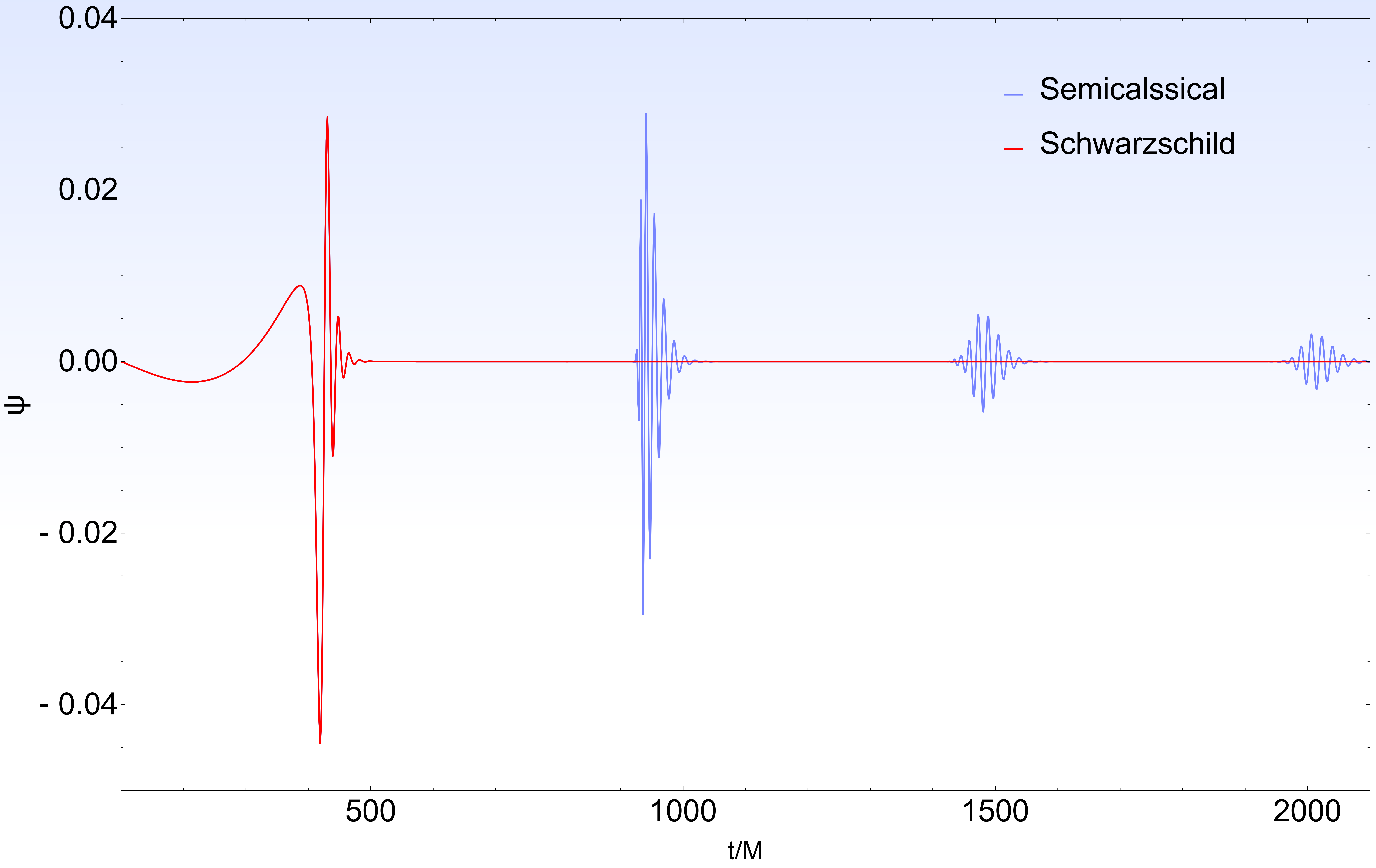
(ℓ/M here is big enough to be in the horizonless branch)

If the object is horizonless and GWs are reflected at the surface, detectable **echoes**!

$$\Delta t_{echo} \sim \log\left(\frac{r_{surf}}{2M} - 1\right)$$

But…

If $\ell \approx \ell_{planck}$ corrections to the ringdown are usually too small to be detected



If the GWs also propagate in the (high curvature) interior →

$$\Delta t_{\text{echo}} \approx 2M - 4M\sigma - 4M\ln(2\sigma) + t_{\text{int}}$$

Huge!

Arrechea et al. 2024

Conclusions

- In a complete theory of quantum gravity we expect the formation of **spacetime singularities to be prevented**
- There are **few regular alternatives** to describe ultra-compact objects
- They all presents **issues/instabilities** when analysed in GR

  ↓

  QG dynamics needed to really asses viability
- They all present **phenomenological signatures** with respect to singular BHs even if their **observability is not guaranteed**

Thank You

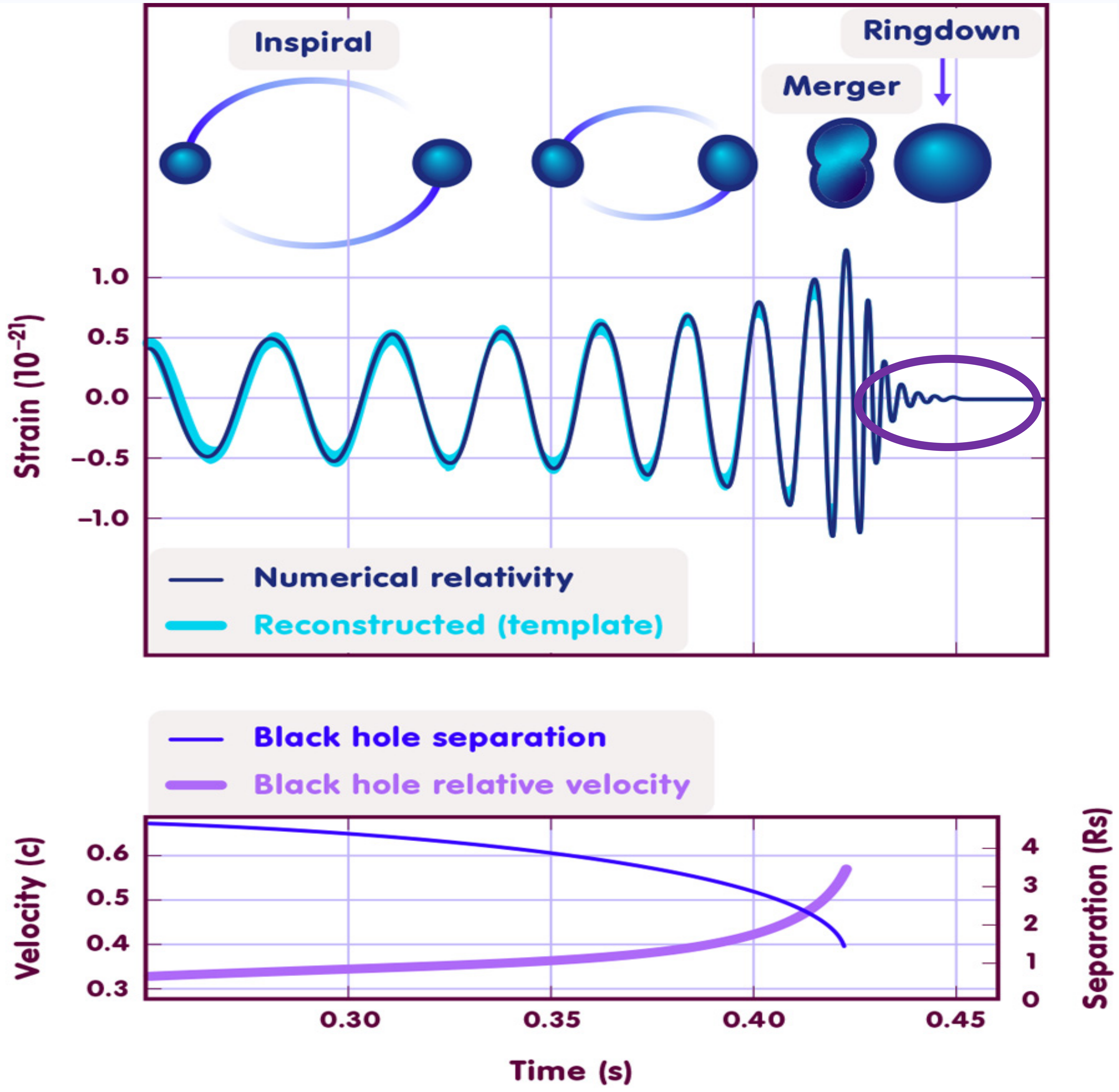
An example: the post-merger GWs signal

Ringdown signal:

- caused by the relaxation of the remnant object to its final, stationary state

- can be studied in **perturbation theory:**

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$$



LIGO / Redesign: Daniela Leitner

Echoes

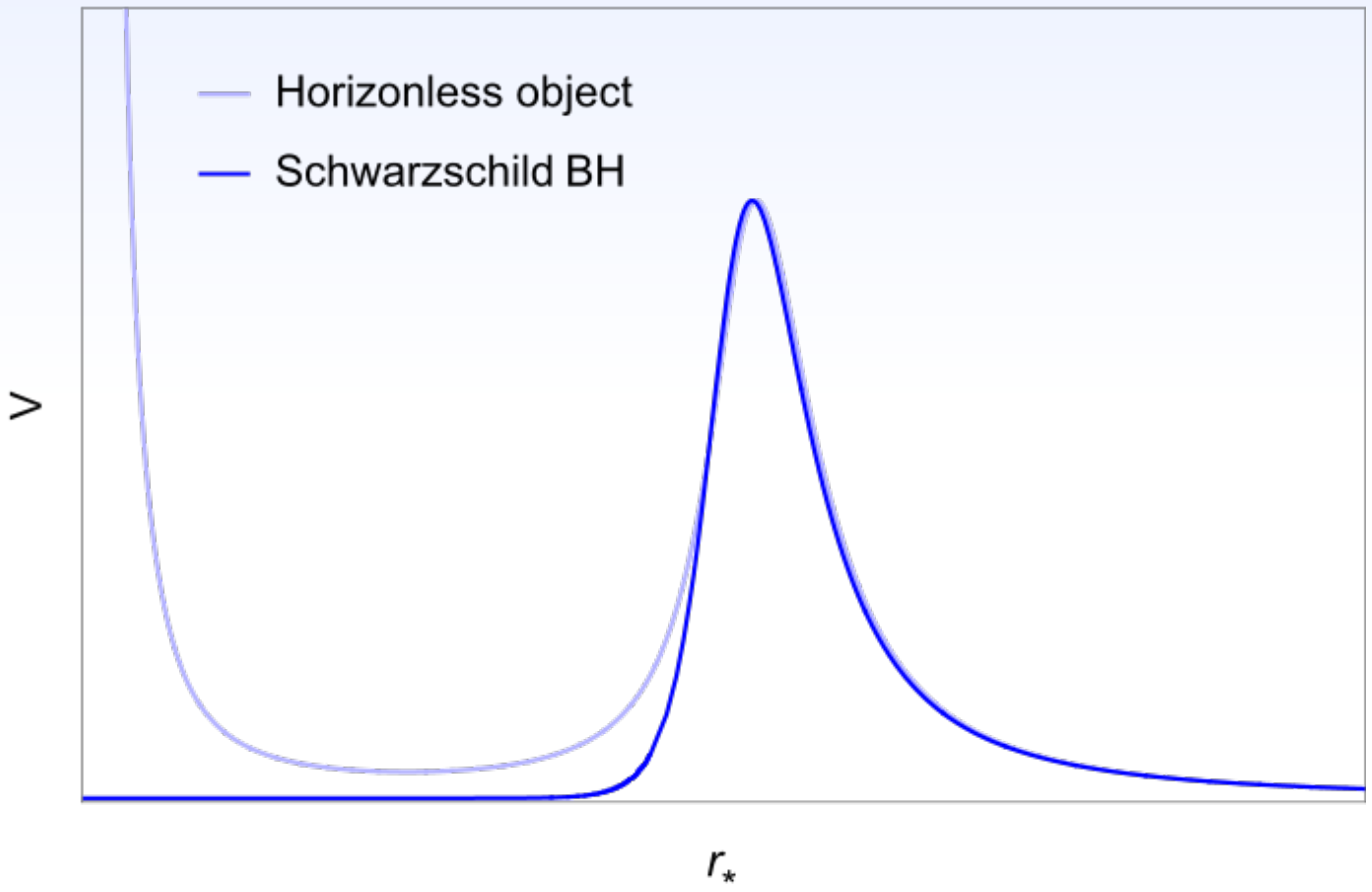
Perturbations around a spherically symmetric BH or an Horizonless Compact Object

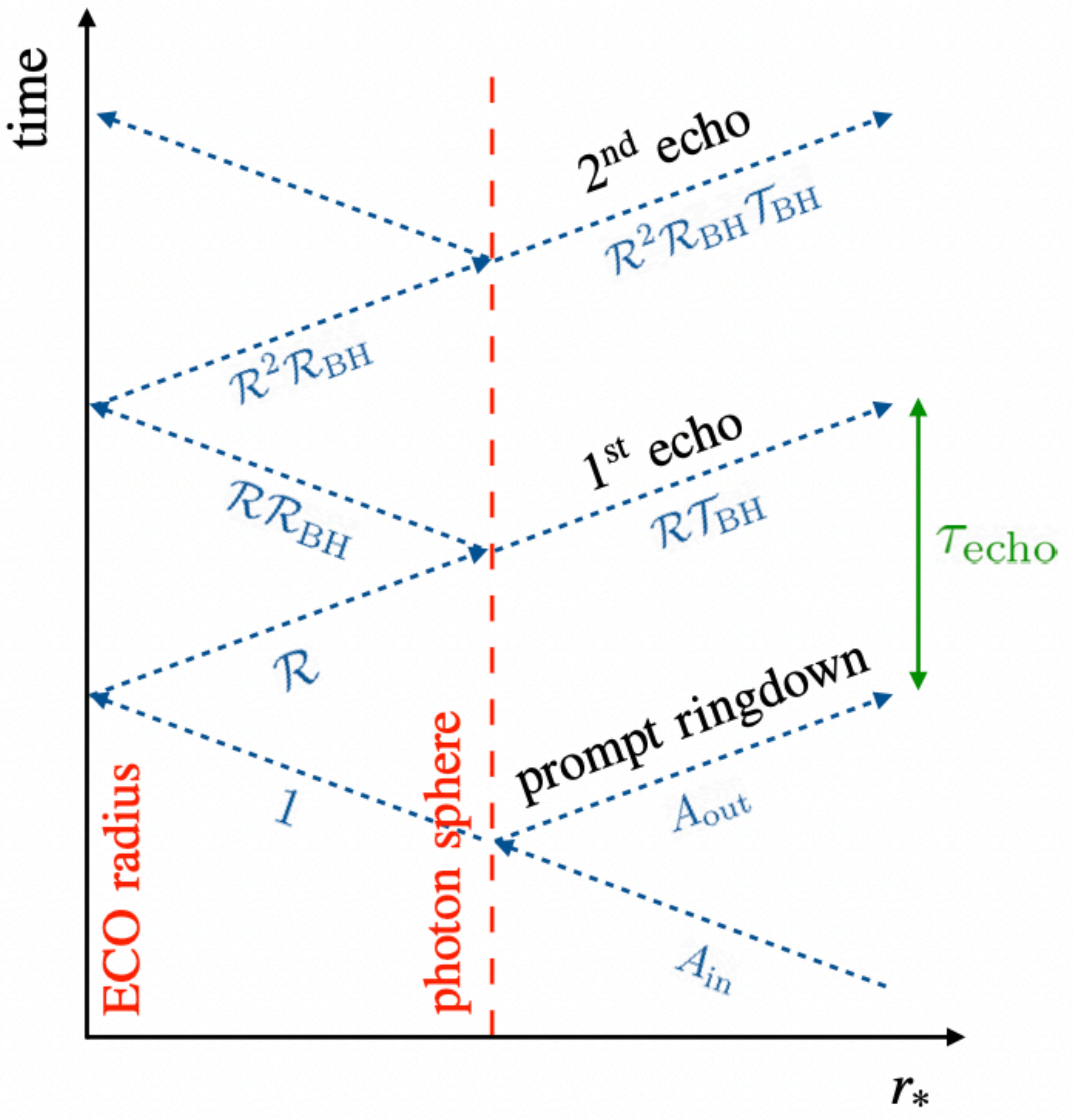
$$\left(\frac{d^2}{dr_*^2} - \frac{d^2}{dt^2} - V_l(r)\right)h_{lm}(r) = 0$$

Feel very different potentials (different boundary conditions)

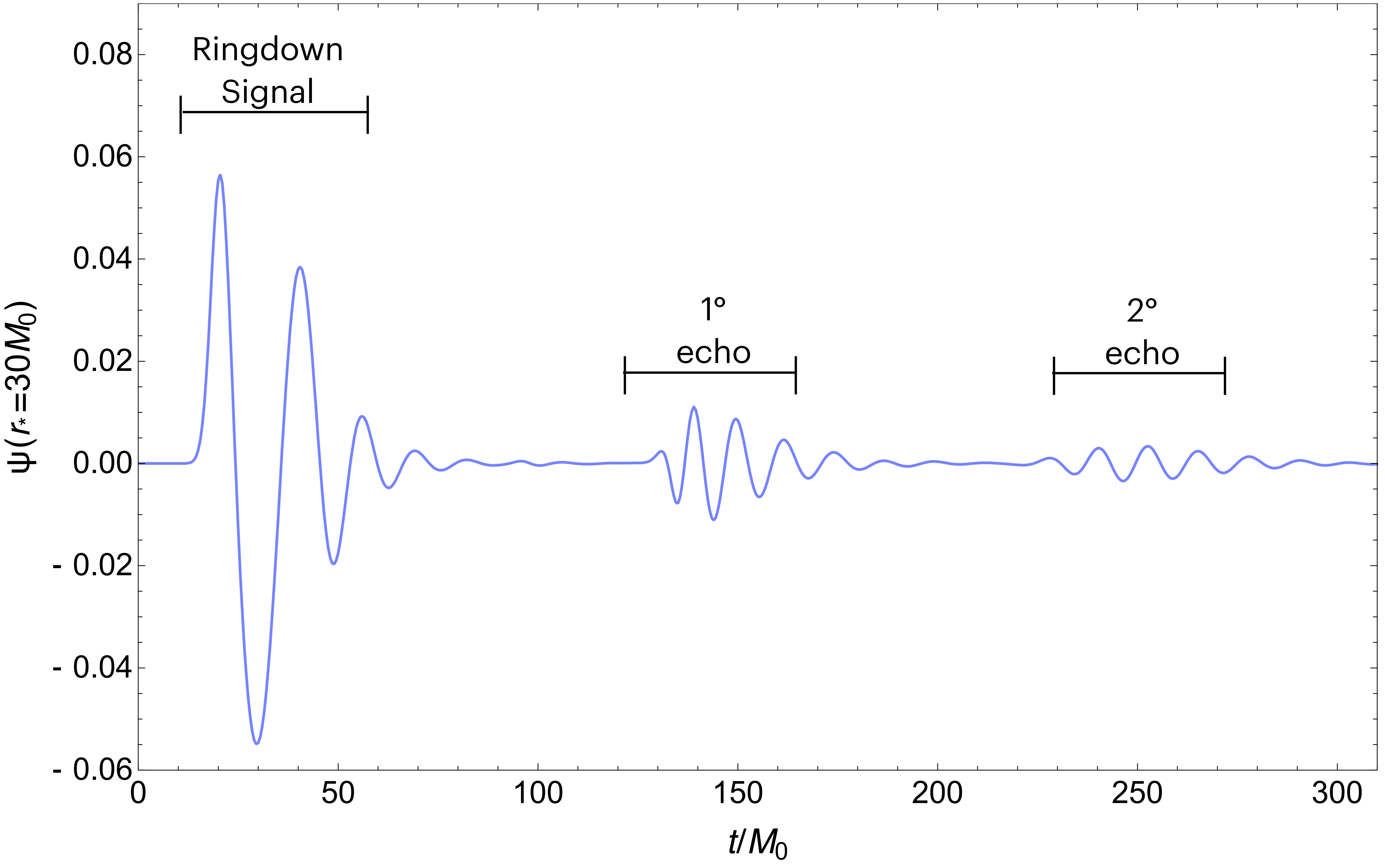
- We see **echoes** of the original ringdown signal
- We have a complete different spectrum of (**trapped**) QNMs

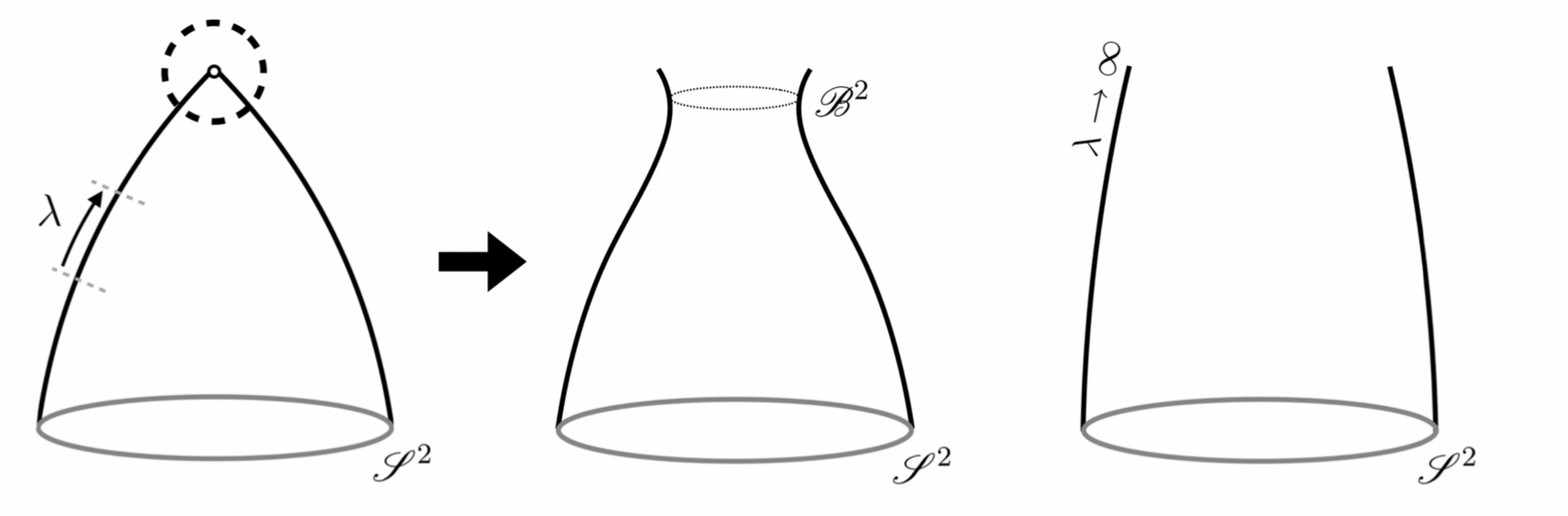
Kokkotas (1996); Ferrari & Kokkotas (2000); Cardoso, Franzin, Pani (2016); Cardoso+ (2016)





Maggio et al., 2021




λ
$\mathscr{S}^2$
$\mathscr{B}^2$
$\mathscr{S}^2$
λ → ∞
$\mathscr{S}^2$