Simulations and the Resulting Spectra for Reactions in Astrophysical Electromagnetic Cascades with Lorentz Invariance Violation

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SME and Modified Dispersion Relations

A possible consequence of LIV Standard Model Extensions are modified dispersion relations of the form

$$E_i = m_i^2 + p_i^2 + \chi_n^{(i)} \frac{p_i^{n+2}}{M_{\mathrm{Pl}}^n} \, , \; n \geq 0$$

This modifies existing reactions and allows new reactions which are forbidden when Lorentz invariance is conserved, such as

- Photon Decay: $\gamma \rightarrow e^+ + e^-$
- the Vacuum Cherenkov effect (for nuclei and charged fermions): $X \rightarrow X + \gamma$
- spontaneous decay (for nuclei): ${}_Z^AX \rightarrow {}_{Z'}^{A'} X' + {}_{Z''}^{A''} X''$

Reaction Thresholds (and Their Modifications)

The (modified) thresholds may be calculated from energy and momentum conservation, which (in the simplest case) may be reformulated as

$$s_{\mathrm{in}}^{\mathrm{head-on}} = s_{\mathrm{out}}^{\mathrm{parallel}} .$$

and results (for electromagnetic cascades) in

$$s_{\mathrm{thr}} = \begin{cases} k_\gamma^2 \left[\chi_n^\gamma \left(\frac{k_\gamma}{M_{\mathrm{Pl}}}\right)^n\right] & \text{for photon decay} \\ k_\gamma^2 \left[4\left(\frac{m_e}{k_\gamma}\right)^2 + \frac{\chi_n^e}{2^n}\left(\frac{k_\gamma}{M_{\mathrm{Pl}}}\right)^n\right] & \text{for pair production,} \\ p_e^2 \left[\left(\frac{m_e}{p_e}\right)^2 + \chi_n^e \left(\frac{p_e}{M_{\mathrm{Pl}}}\right)^n\right] & \text{for the Vacuum Cherenkov effect,} \\ p_e^2 \left[\left(\frac{m_e}{p_e}\right)^2 + \chi_n^e \left(\frac{p_e}{M_{\mathrm{Pl}}}\right)^n\right] & \text{for inverse Compton scattering.} \end{cases}$$

Modified Mean Free Paths

The (inverse) mean free path is given by

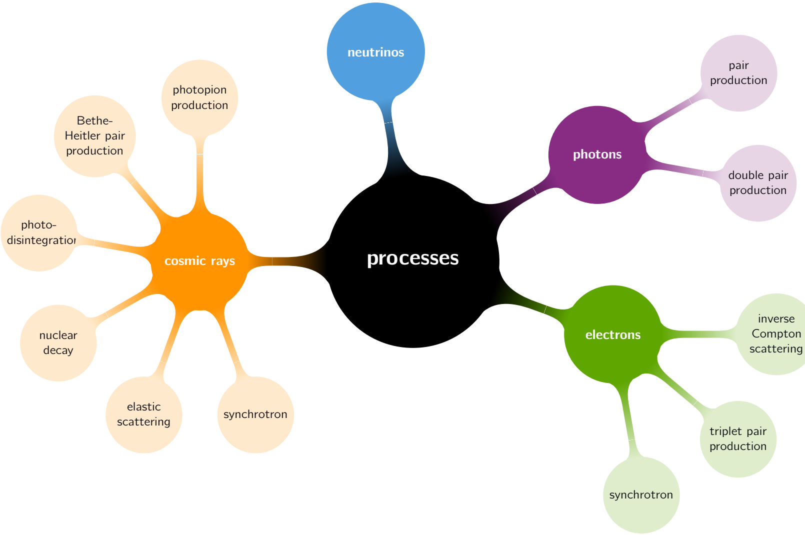
$$\lambda^{-1} = \frac{1}{2p_{\rm in}} \int_{s_{\rm thr}}^{\infty} \frac{n_{\rm bg}\left(\frac{s^* - \mathfrak{S}(p_{\rm in})}{4p_{\rm in}}\right)}{\left[s^* - \mathfrak{S}(p_{\rm in})\right]^2} \int_{\mathfrak{S}(p_{\rm in})}^{s^*} \sigma(s)\left[s - \mathfrak{S}(p_{\rm in})\right] \, {\rm d}s \, {\rm d}s^* \,,$$

with

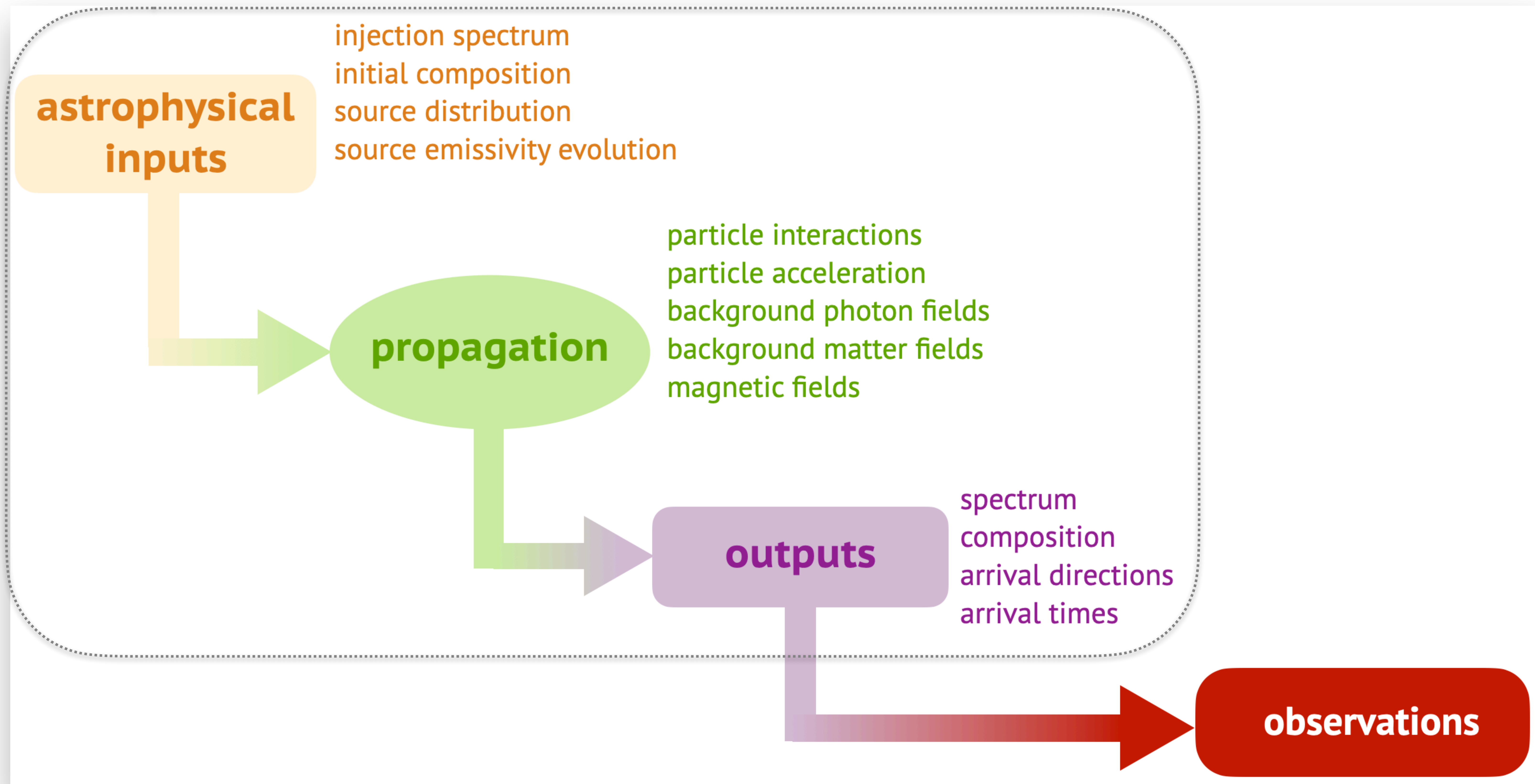
$$\mathfrak{S}(p_{\rm in}) \equiv p_{\rm in}^2 \left[\left(\frac{m_{\rm in}}{p_{\rm in}}\right)^2 + \chi_n^{\rm in}\left(\frac{p_{\rm in}}{M_{\rm Pl}}\right)^n\right] .$$

Publicité

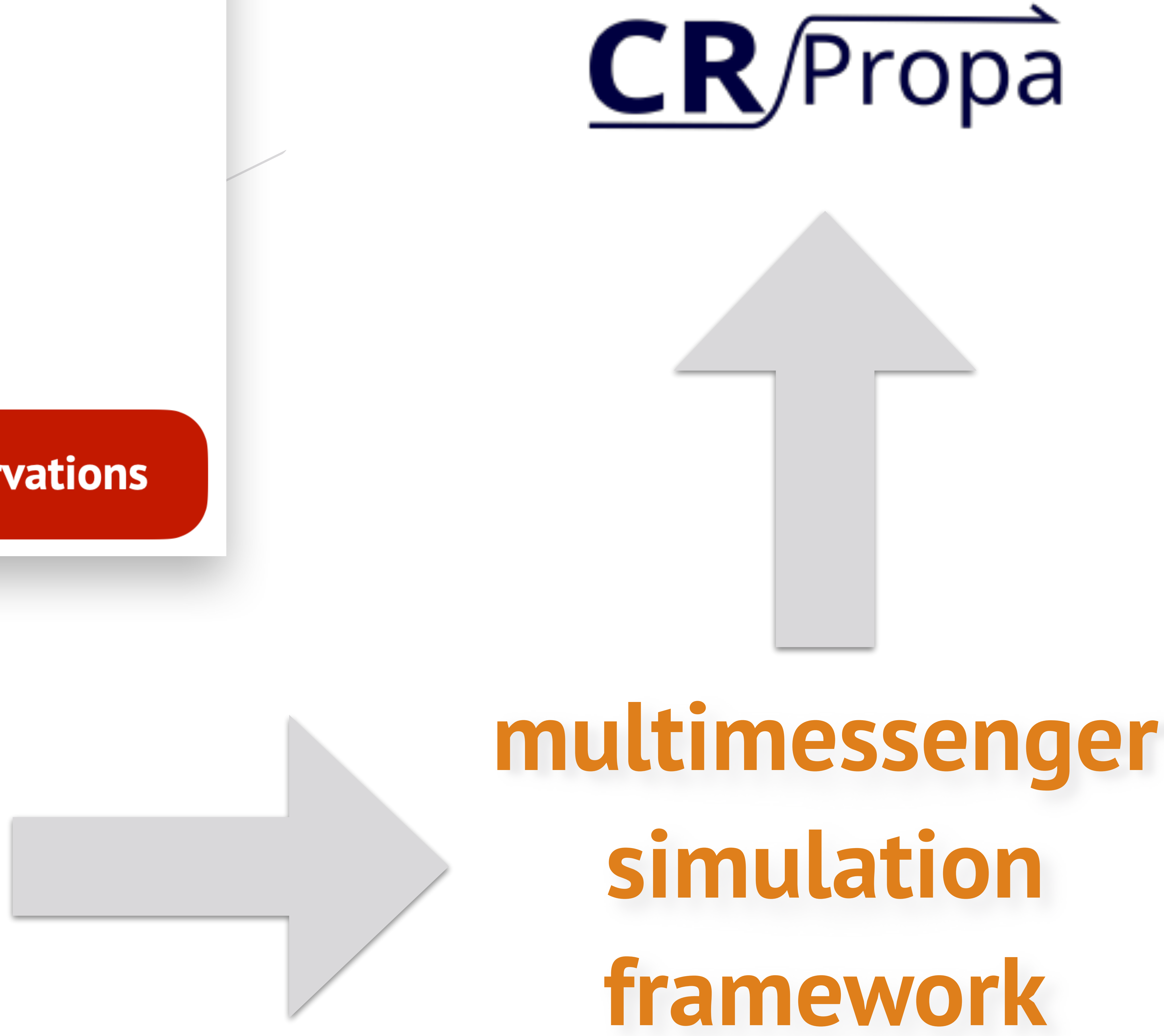
CRPropa – a Universal Tool for galactic and extragalactic Propagation [Alves Batista et al., 2022]



modelling the propagation of cosmic messengers



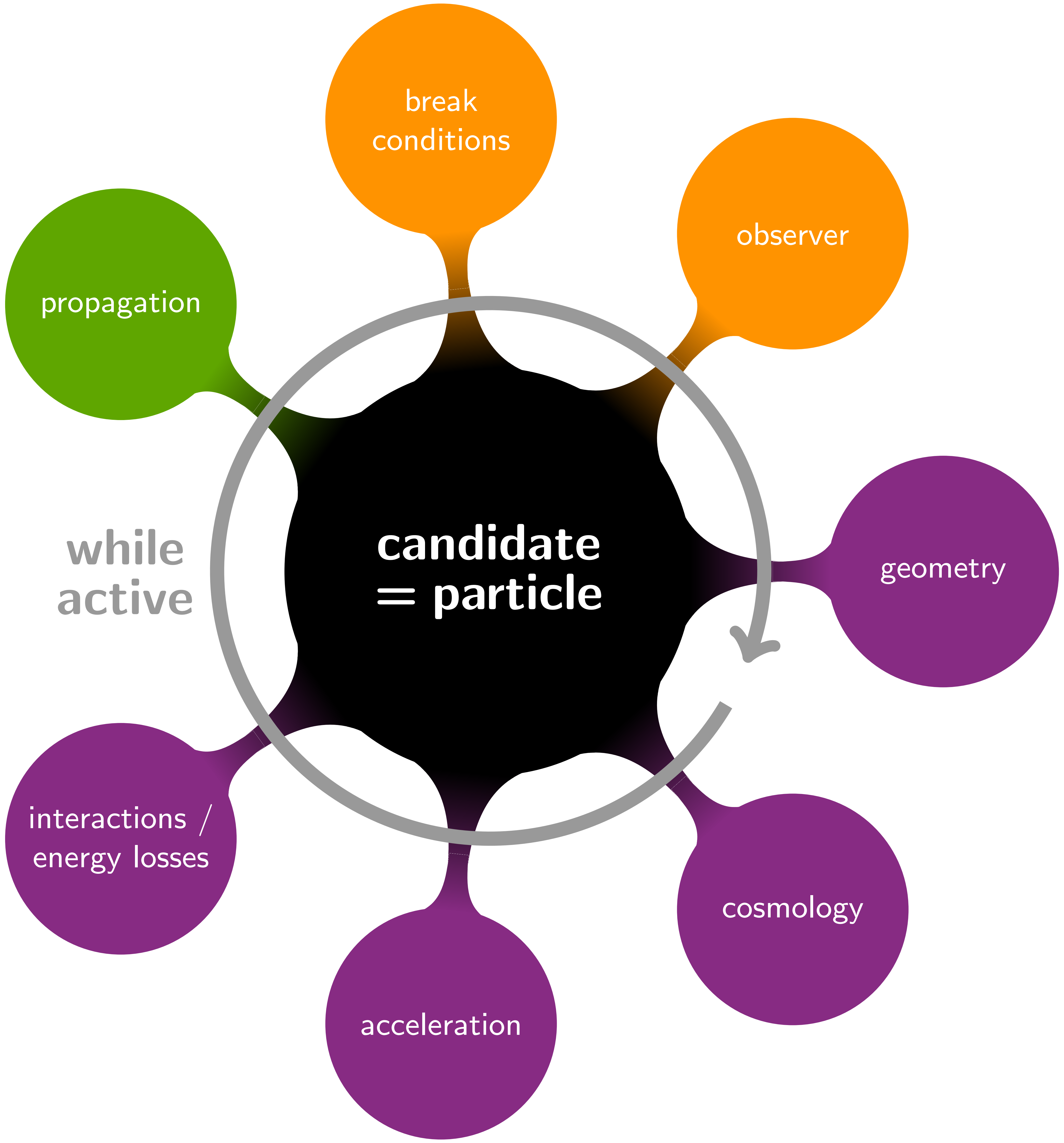
- mixing all ingredients → interpret (fit) observations based on models
- this should be done ***self-consistently for all messengers***
- need to ***scan full parameter space*** of uncertainties



the CRPropa framework

Alves Batista et al. JCAP 05 (2016) 038. arXiv:1603.07142
Alves Batista et al. JCAP 09 (2022) 035. arXiv:2208.00107

- publicly available Monte Carlo code
- propagation of high-energy cosmic rays, gamma rays, neutrinos, and electrons
- propagation in *any* environment (Galactic, extragalactic, around sources)
- modular structure
- parallelisation with OpenMP
- development on Github: https://github.com/CRPropa/CRPropa3

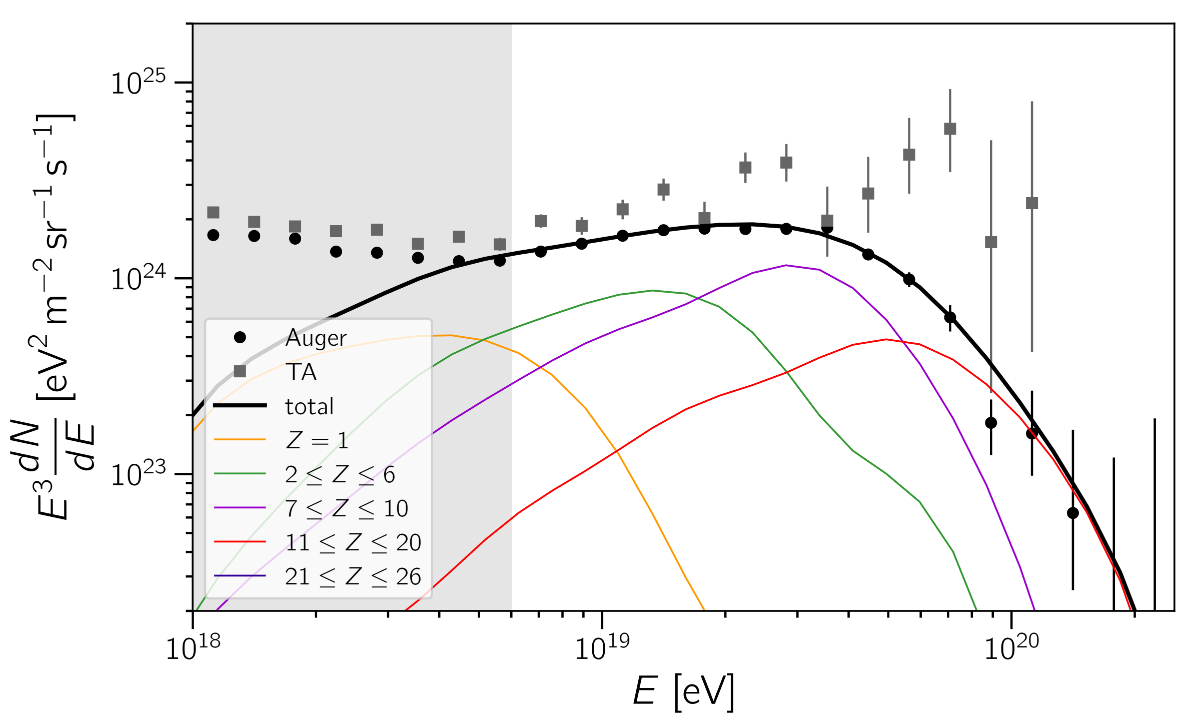


crpropa.desy.de

CRPropa

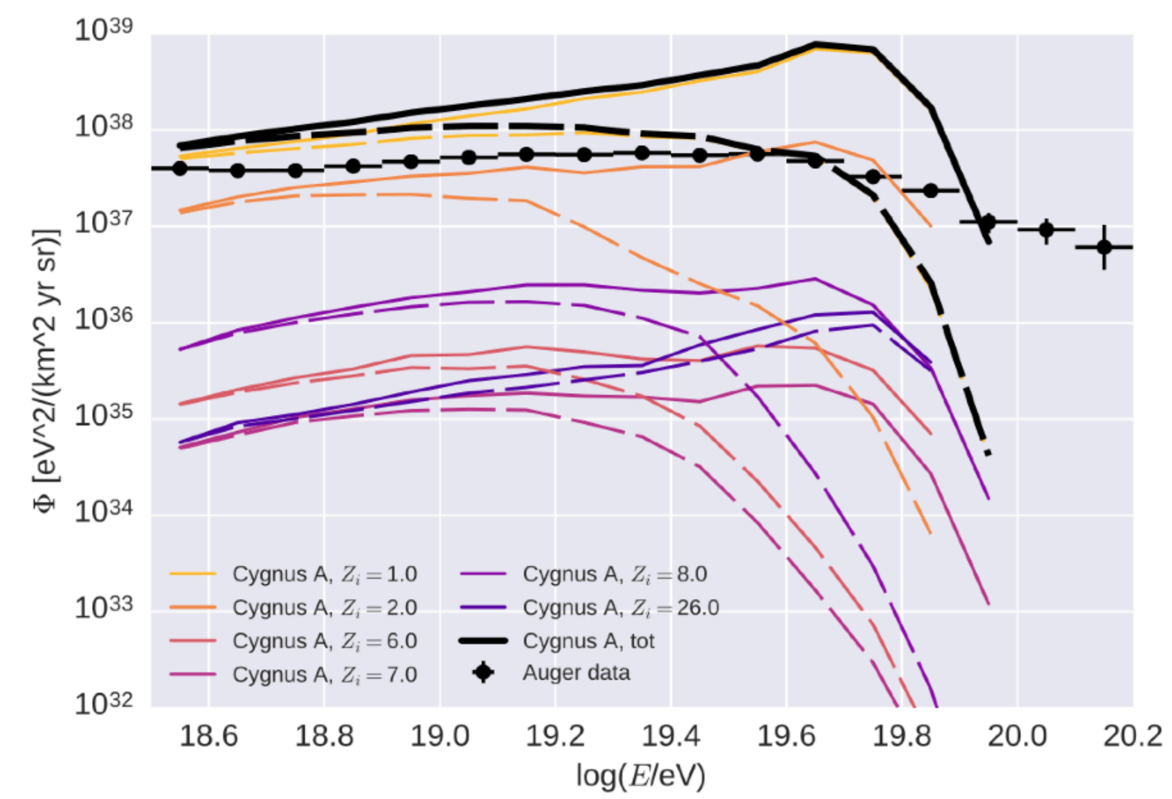
the CRPropa framework: applications

fit UHECR measurements



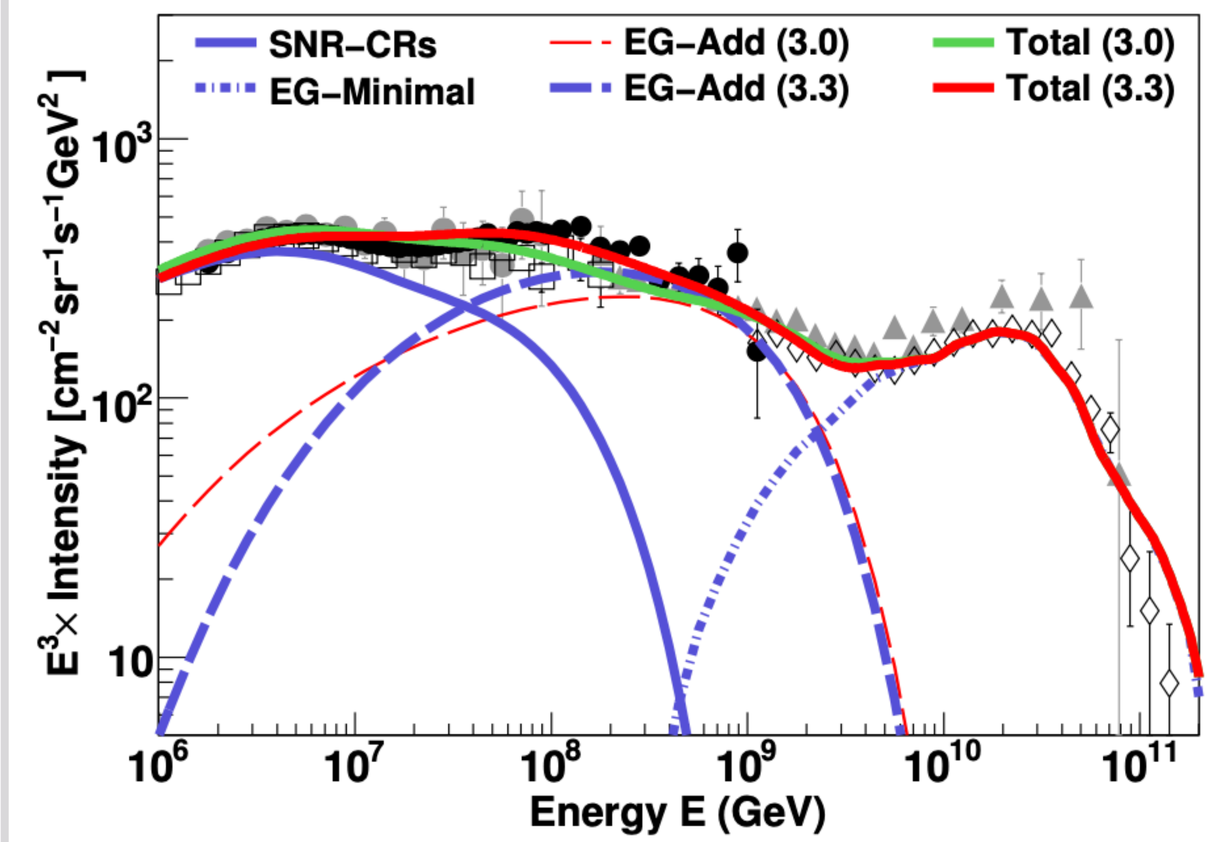
Alves Batista, de Almeida, Lago, Kotera. JCAP 01 (2019) 002. arXiv:1806.10879

test UHECR source models



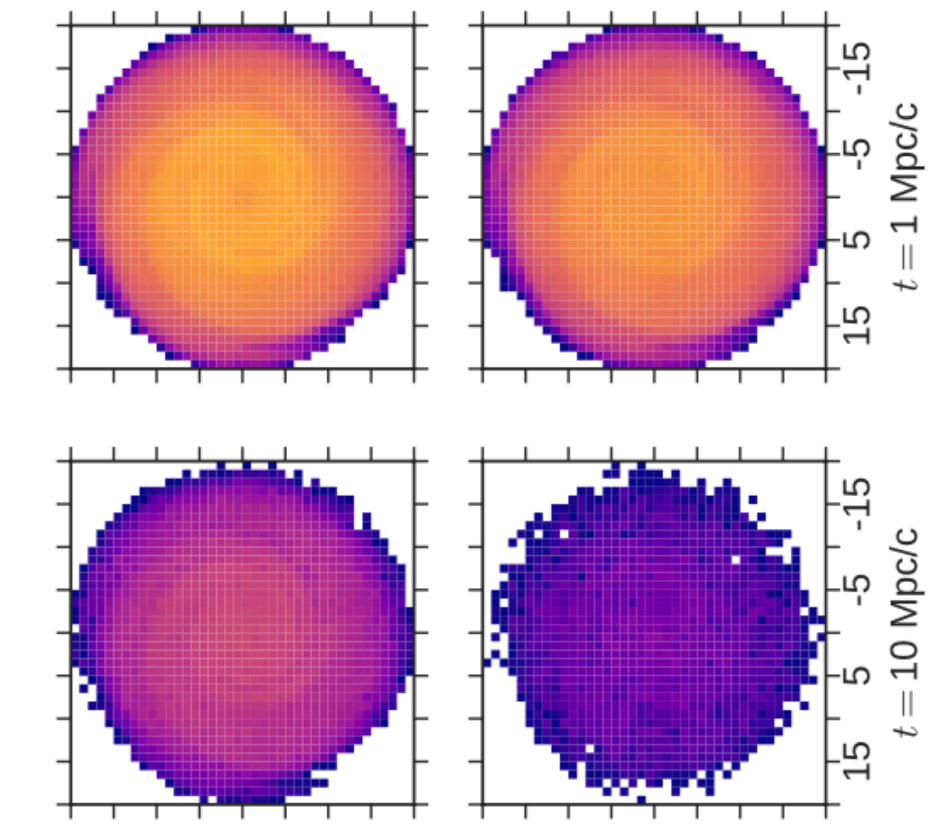
Eichmann et al. JCAP 02 (2018) 036. arXiv:1701.06792

transition G-EG CRs



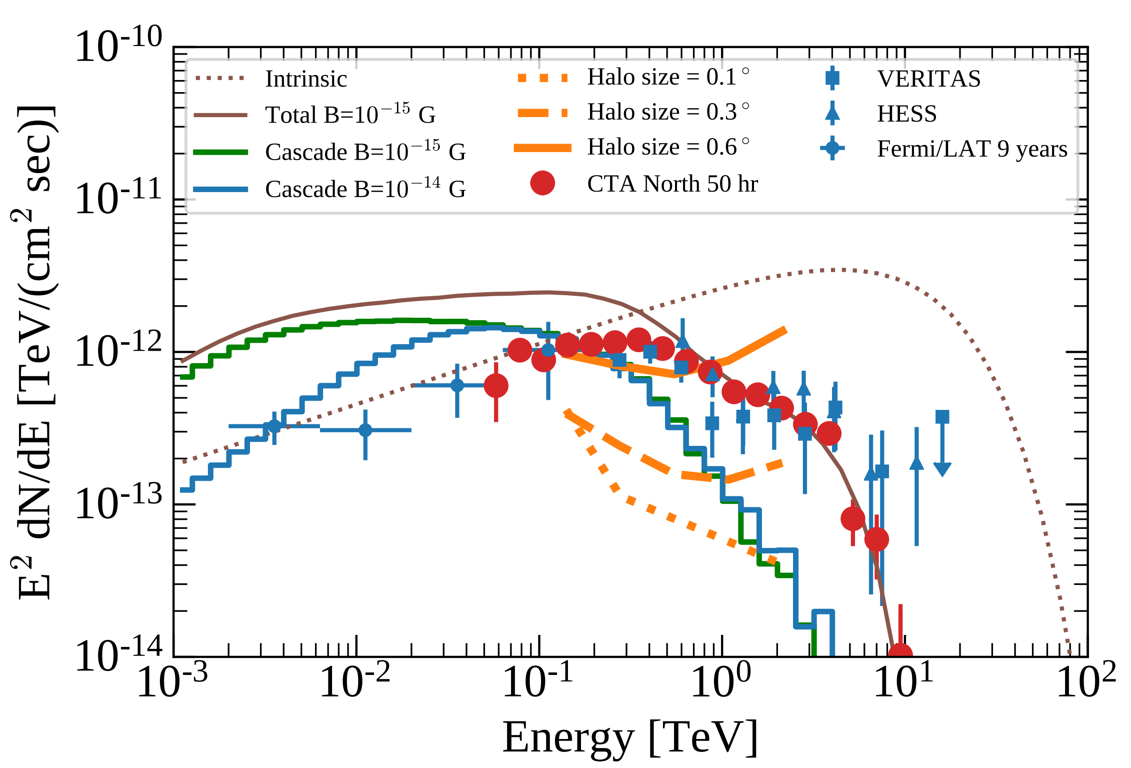
Thoudam et al. Astron. Astrophys. 595 (2016) A33. arXiv:1605.03111

diffusion of Galactic CRs



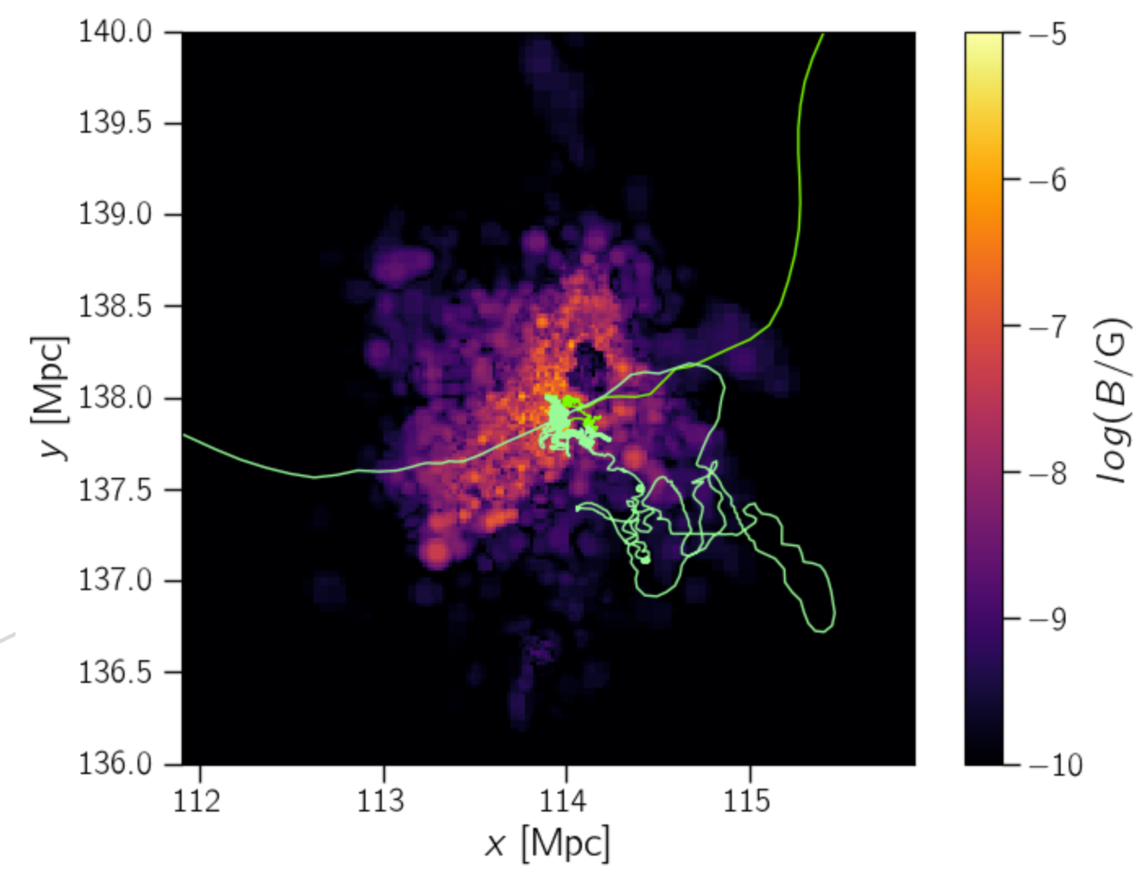
Merten et al. JCAP 06 (2016) 046. arXiv:1704.07484

gamma rays + IGMFs



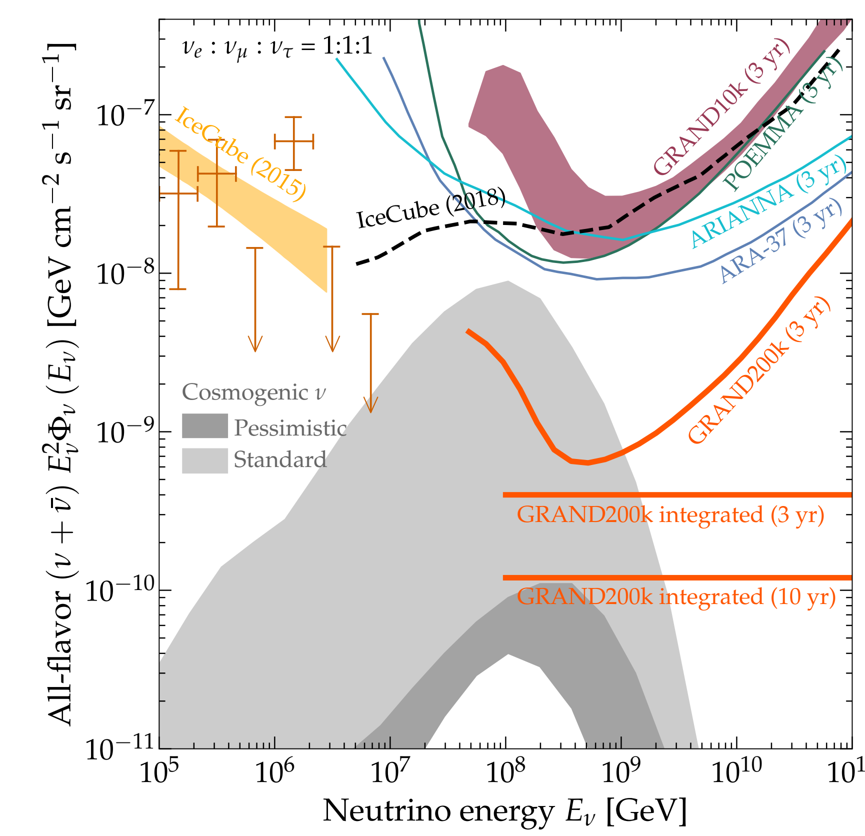
CTA Consortium. JCAP 02 (2021) 048. arXiv:2010.01349

neutrinos from galaxy clusters



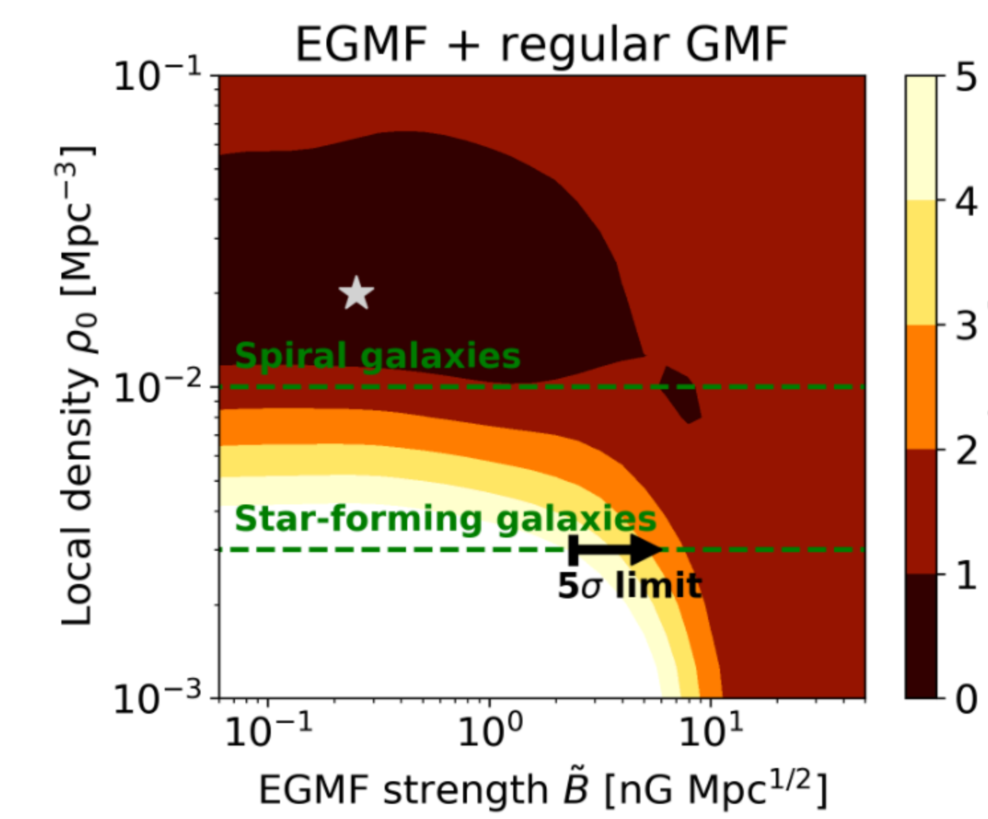
Hussain, Alves Batista, de Gouveia Dal Pino. Nature Comms 14 (2023) 2486. arXiv:2101.07702

cosmogenic neutrinos



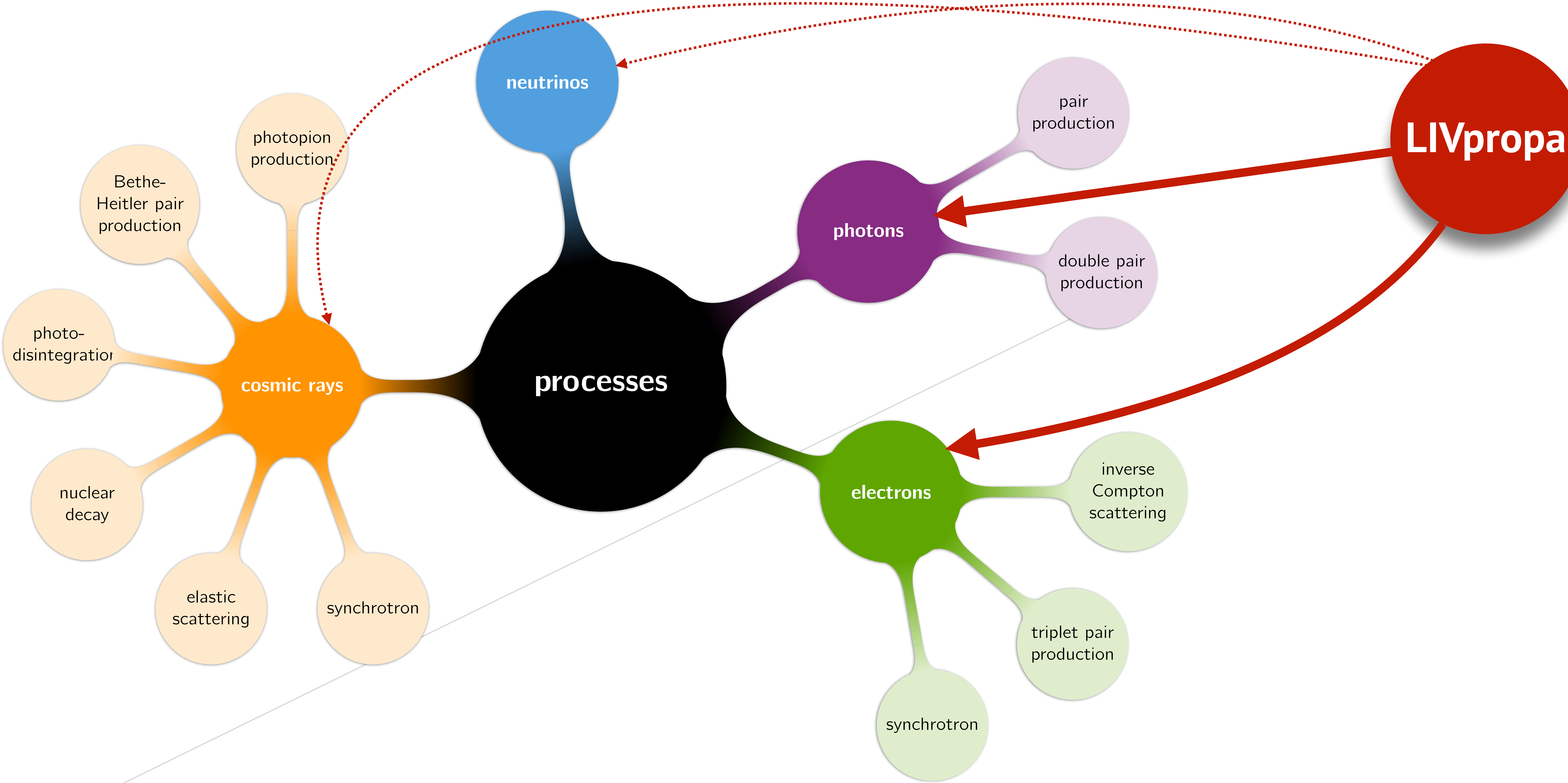
GRAND Collaboration. Science China Phys 63 (2020) 219501. arXiv:1810.09994

EGMF constraints



van Vliet, Palladino, Taylor, Winter. MNRAS 510 (2022) 1289. arXiv:2104.05732

... and much more!


neutrinos
photopion production
Bethe-Heitler pair production
photo-disintegration
cosmic rays
nuclear decay
elastic scattering
synchrotron
processes
photons
pair production
double pair production
LIVpropa
electrons
inverse Compton scattering
triplet pair production
synchrotron

Modified Mean Free Paths

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$$\lambda^{-1} = \frac{1}{2p_{\rm in}} \int_{s_{\rm thr}}^{\infty} \frac{n_{\rm bg}\left(\frac{s^* - \mathfrak{S}(p_{\rm in})}{4p_{\rm in}}\right)}{[s^* - \mathfrak{S}(p_{\rm in})]^2} \int_{\mathfrak{S}(p_{\rm in})}^{s^*} \sigma(s)\,[s - \mathfrak{S}(p_{\rm in})]\;\mathrm{d}s\,\mathrm{d}s^*\,, \tag{1}$$

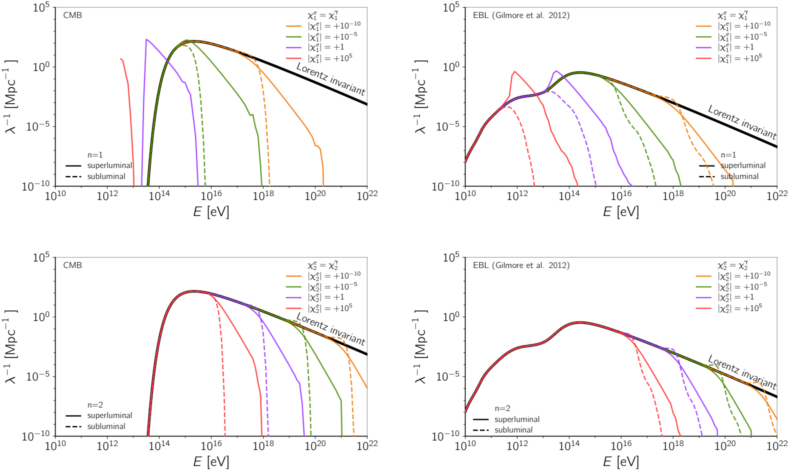
with

$$\mathfrak{S}(p_{\rm in}) \equiv p_{\rm in}^2 \left[\left(\frac{m_{\rm in}}{p_{\rm in}}\right)^2 + \chi_n^{\rm in}\left(\frac{p_{\rm in}}{M_{\rm Pl}}\right)^n\right], \tag{2}$$

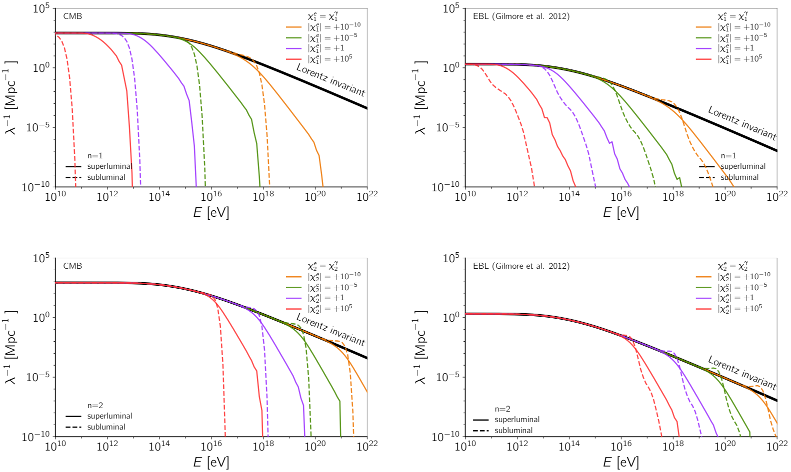
such that we can calculate the LIV-modified propagation lengths and then simulate the propagation using the LIVpropa[2] plugin and the "binary" approach [Saveliev et al., 2024].

[2] https://github.com/rafaelab/LIVpropa

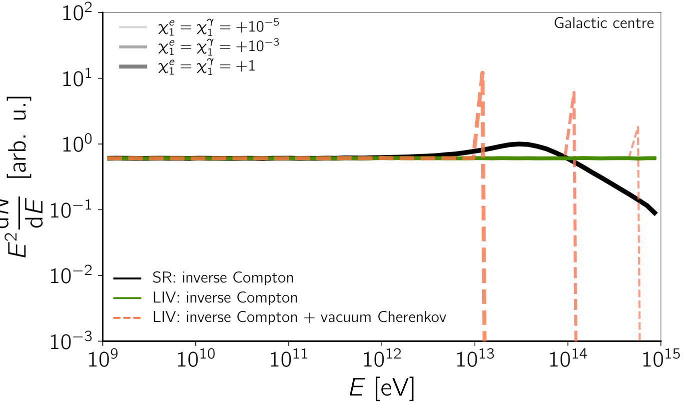
Modified MFP for Pair Production [Saveliev et al., 2024]



Modified MFP for Inverse Compton [Saveliev et al., 2024]

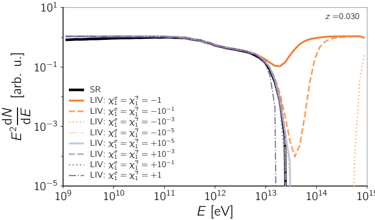


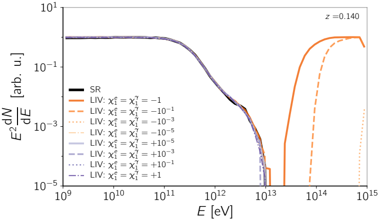
Modified Spectra [Saveliev et al., 2024]



- Simulated gamma-ray flux for a source at the Galactic center, considering LIV with $n = 1$ for an intrinsic spectrum with a spectral index of -2 and cut-off at $10^{15}\,\mathrm{eV}$ considerng IC and VC

Modified Spectra [Saveliev et al., 2024]





- Simulated gamma-ray flux for a source at $z = 0.03$ (left panel) and another at $z = 0.14$ (right panel), considering LIV with $n = 1$ for an intrinsic spectrum with a spectral index of -2 and cut-off at $10^{15}\,\mathrm{eV}$ considerng PP and IC

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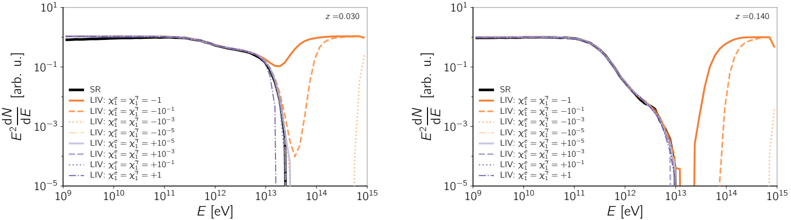

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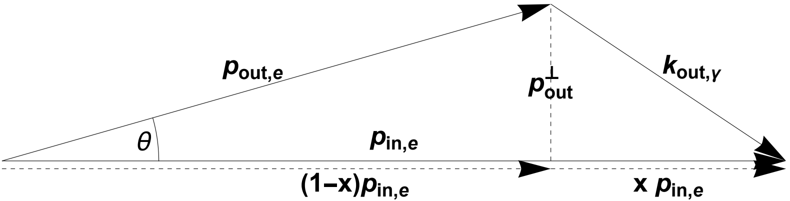
- Can we do better?

$$\lambda^{-1} = \frac{1}{2p_{\rm in}} \int_{s_{\rm thr}}^{\infty} \frac{n_{\rm bg}\left(\frac{s^* - \mathfrak{S}(p_{\rm in})}{4p_{\rm in}}\right)}{\left[s^* - \mathfrak{S}(p_{\rm in})\right]^2} \int_{\mathfrak{S}(p_{\rm in})}^{s^*} \sigma(s)\left[s - \mathfrak{S}(p_{\rm in})\right] \mathrm{d}s\, \mathrm{d}s^* \,,$$

Modified Differential Reaction Rates [Rubtsov et al., 2012]

For $n = 2$:



$$\Gamma_{\mathrm{VC}} \equiv \int \mathrm{d}x \frac{\mathrm{d}\Gamma_{\mathrm{VC}}}{\mathrm{d}x}$$

$$= \frac{\alpha}{2p} \int \frac{\mathrm{d}x\mathrm{d}p^2_{\mathrm{out},\perp}}{px(1-x)}\, \delta\left[\omega^{\mathrm{VC}}_{\mathrm{LIV}}(x) - \frac{p^2_{\mathrm{out},\perp}}{2px(1-x)}\right] \overline{|\mathcal{M}^{\mathrm{VC}}_{\mathrm{LIV}}|^2}$$

for

$$\omega^{\mathrm{VC}}_{\mathrm{LV}}(x) = -\frac{\chi^{\gamma}_2}{2} \frac{p^3_{\mathrm{in},e} x^3}{M^2_{\mathrm{Pl}}} + \frac{\chi^{e}_2}{2} \frac{p^3_{\mathrm{in},e}\left(x^3 - 3x^2 + 3x\right)}{M^2_{\mathrm{Pl}}}$$

Modified Differential Reaction Rates [Rubtsov et al., 2012]

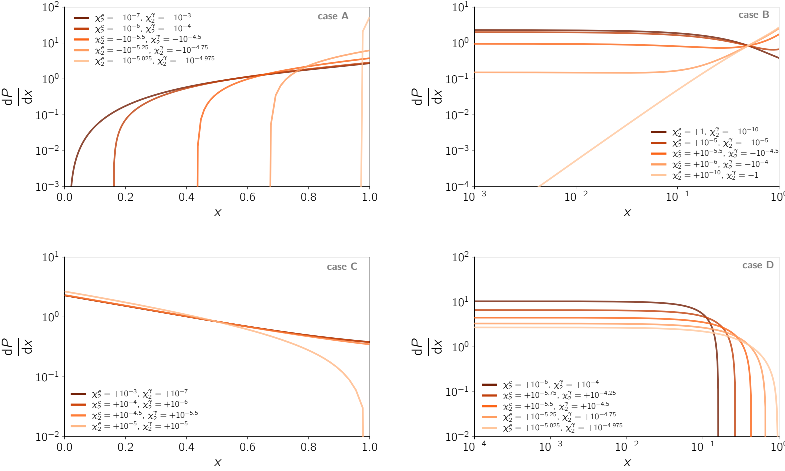
$$\omega_{\mathrm{LV}}^{\mathrm{VC}}(x) = -\frac{\chi_2^{\gamma}}{2}\frac{p_{\mathrm{in},e}^3 x^3}{M_{\mathrm{Pl}}^2} + \frac{\chi_2^{e}}{2}\frac{p_{\mathrm{in},e}^3\left(x^3 - 3x^2 + 3x\right)}{M_{\mathrm{Pl}}^2}$$

which results in the differential probability

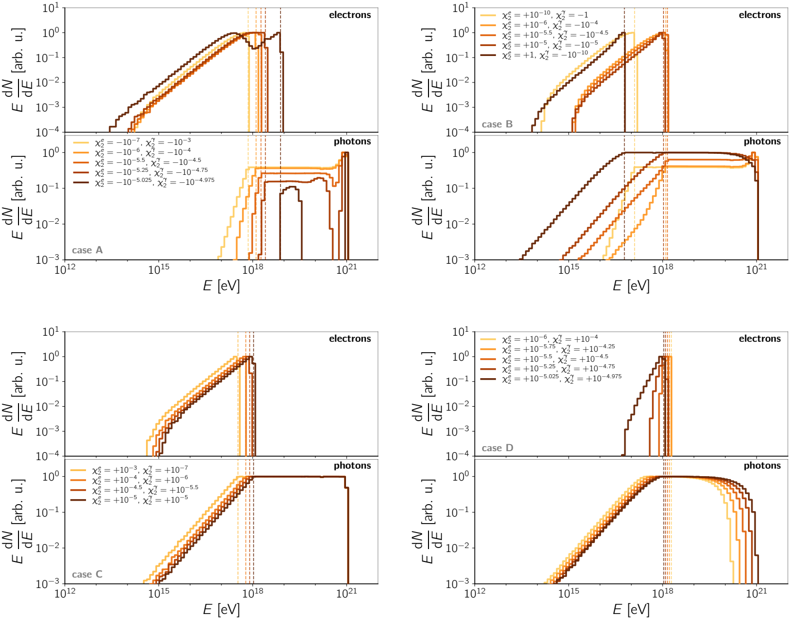
$$\frac{\mathrm{d}P_{\mathrm{VC}}}{\mathrm{d}x} = \frac{\alpha}{\Gamma_{\mathrm{VC}}}\left(\frac{2}{x} - 2 + x\right)\omega_{\mathrm{LV}}^{\mathrm{VC}}(x)\,.$$

With that we can then carry out simulations, in the following with incoming electrons with energies $E_{\mathrm{in},e} = 10^{21}\,\mathrm{eV}$ [Saveliev et al., 2025].

Modified Differential Reaction Rates [Rubtsov et al., 2012]



Full VC spectra [Saveliev et al., 2025]



Conclusions and Outlook

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- CRPropa (together with its LIV extension LIVpropa) provides all the right tools to carry out detailed propagation of any multimessenger
- Using it, one can calculate very precise spectra to compare with observational data
- We are happy to test any proposed model!

Alves Batista, R. et al. (2022).
CRPropa 3.2 — an Advanced Framework for High-Energy Particle Propagation in Extragalactic and Galactic Spaces.
J. Cosmol. Astropart. Phys., 09:035.

Rubtsov, G., Satunin, P., and Sibiryakov, S. (2012).
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