MAGNETIC COMPACTIFICATIONS

Based on collaborations with :

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arXiv:1611.03798 [hep-th], [arXiv:1804.07497 [hep-th]] and arXiv:1909.03007 [hep-th]

LIA-Naturalness
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few papers since mostly in Japan

Outline

1) Magnetic compactifications

2) Effective field theory

- Invisible symmetry and goldstone bosons

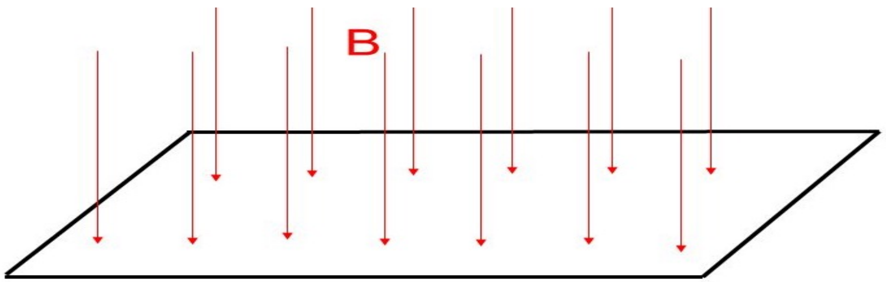
3) Light scalars with no 4d symmetry ?

4) Perspectives

1) Magnetic compactifications

Consider a 6-dim. theory : $x_0 x_1 x_2 x_3 x_4 x_5$

An internal magnetic field $\langle F_{45} \rangle = B H_I = f H_I$



Cartan gauge group generator

- Charged states: turns KK states k_1, k_2 into Landau levels n , mass (Bachas)

$$\delta M^2 = (2n+1)|qf| + 2qf\Sigma_{45}$$

where Σ_{45} is the internal helicity of particles.

- Uncharged states : standard KK masses

An internal magnetic field is quantized

$\int_{T^2} F = 2\pi N \longrightarrow f = \frac{N}{2\pi R_1 R_2} \sim M_{\mathrm{GUT}}^2$; N = int.

- Each Landau level is N times degenerate.

- chiral fermion zero modes (index theorem) :

$$n_L - n_R = \frac{1}{2\pi} \int_{T^2} F = N$$

- It breaks SUSY due to the spin-magnetic field coupling

$$H = -\mu \mathbf{B} = -\frac{q}{m} \mathbf{SB}$$

- It adds a vacuum energy (Fayet-Iliopoulos term in SUSY)

breaks SUSY, due to the magnetic moment coupling

$$D = f \quad \rightarrow \quad V = \frac{1}{2} D^2 = \frac{1}{2} f^2 \sim M_{\mathrm{GUT}}^4$$

More general case: compactification (S)YM theory from 10d to 4d, fluxes in each tori $i = 1, 2, 3$

torus

$$F^i = f_I^i H_I$$

Cartan generator

flux integers

$$\int_{T_i^2} F = 2\pi N_i \quad , \quad f_I^i = \frac{N_i}{2\pi R_1 R_2}$$

Number of 4d chiral fermions is given by an index theorem, determined by the magnetic fluxes

$$n_L - n_R = \left(\frac{1}{2\pi}\right)^3 \int_{T^6} F \wedge F \wedge F = N_1 N_2 N_3$$

The mass of a charged state given in general by

$$\delta M_{\mathbf{q}}^2 = \sum_{i=1}^3 \left[(2n_i+1) | \sum_I q_I f_I^i | + 2 \sum_I q_I f_I^i \Sigma_i \right]$$

charge

f_q^i

internal helicity torus i

Whenever a charged scalar becomes of zero mass, there is some SUSY in the spectrum (Berkooz-Douglas-Leigh, T-dual language).

$$f_q^1 \pm f_q^2 \pm f_q^3 \neq 0 \ \rightarrow \ \mathcal{N}=0 \ \ \mathrm{SUSY}$$

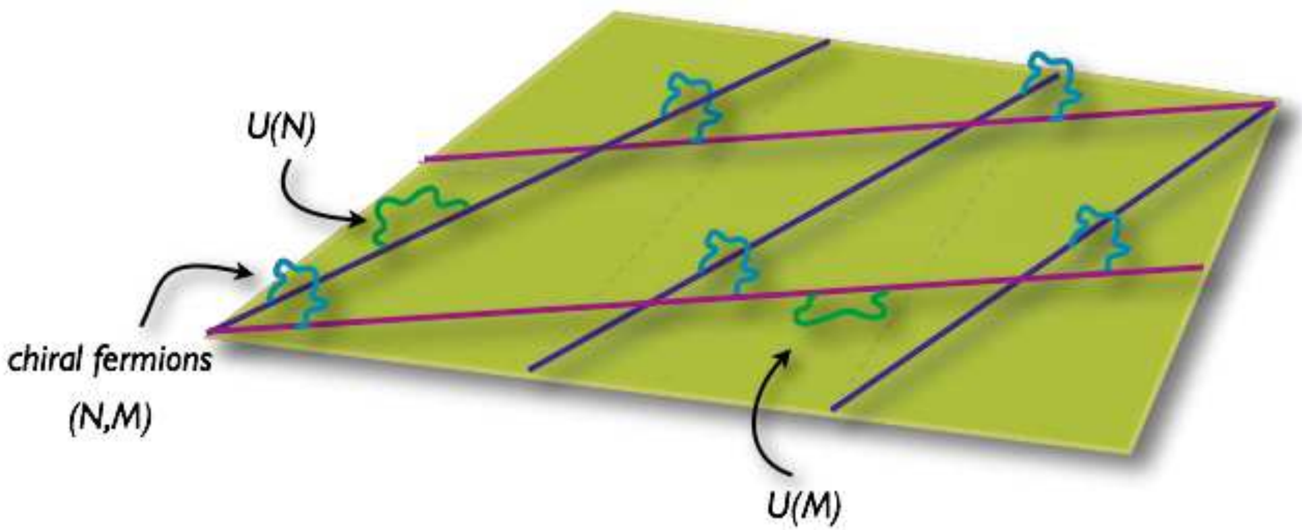
$$f_q^1 \pm f_q^2 \pm f_q^3 = 0 \ \rightarrow \ \mathcal{N}=1 \ \ \mathrm{SUSY}$$

$$f_q^1 \pm f_q^2 = 0 \ , \ \ f_q^3 = 0 \ \rightarrow \ \mathcal{N}=2 \ \ \mathrm{SUSY}$$

Widely studied in type I/II string theory :

T-dual

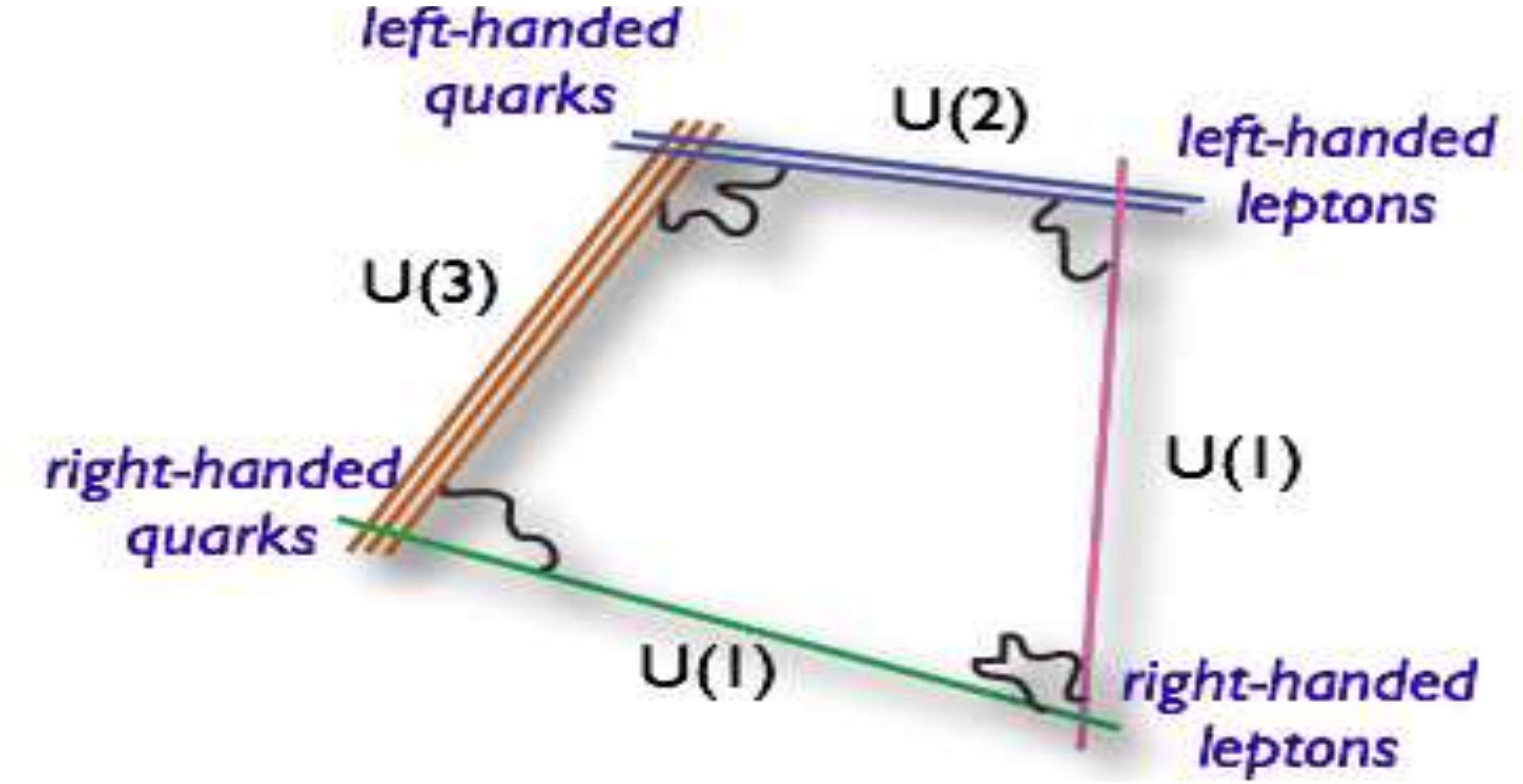
Internal magnetic fields ⟷ intersecting branes



Elegant geometrical intepretations :

- chiral fermions live at the intersection of branes
- Number of generations: intersection numbers
- Yukawa couplings : governed by areas

Among the most succesful quasi-realistic Standard Model realizations in String Theory



Fluxes are not arbitrary. The are constrained by:

- Field Theory: anomaly cancelation conditions

- String Theory: RR tadpole conditions. For toroidal comp.:

$$\sum_a M_a = 16 \ , \ \sum_a M_a N_2^a N_3^a = 0 \ ,$$
$$\sum_a M_a N_1^a N_3^a = 0 \ , \ \sum_a M_a N_1^a N_2^a = 0$$

Why effective field theory action ?

- If broken SUSY, quantum corrections difficult in string theory (NS-NS tadpoles)
- Chirality, (split) SUSY breaking: different sectors have different SUSY
- Electroweak symmetry breaking (Nielsen-Olesen instability) ?
- Magnetic field breaks spontaneously a global symmetry invisible from four dimensions.
- Subtlety: No mass gap : masses given by the magnetic field of the same order ($1/R$) as Landau levels → one needs an effective theory for the whole tower.

  Truncation to « zero modes » inconsistent.

2) Effective field theory (simple ex.)

Consider a 6d Weyl fermion interacting with an abelian gauge field

$$S_6 = \int d^6x \left(-\frac{1}{4} F^{MN} F_{MN} + i \overline{\Psi} \Gamma^M D_M \Psi \right)$$

4d notation, two Weyl fermions of charges q,-q ; fermionic part of the action is

$$S_{6f} = \int d^6x \Big(- i\psi\sigma^\mu \overline{D}_\mu \overline{\psi} - i\chi\sigma^\mu D_\mu \overline{\chi} \\ - \chi \left(\partial_z + \sqrt{2} q \phi \right) \psi - \overline{\chi} \left(\partial_{\bar{z}} + \sqrt{2} q \overline{\phi} \right) \overline{\psi} \Big) ,$$

where $D_\mu = \partial_\mu + iqA_\mu$, $\overline{D}_\mu = \partial_\mu - iqA_\mu$ and

$$\phi = \frac{1}{\sqrt{2}} (A_6 + iA_5) , \quad z = \frac{1}{2} (x_5 + ix_6) , \quad \partial_z = \partial_5 - i\partial_6 .$$

A constant magnetic flux

$$\langle A_5 \rangle = -\tfrac{1}{2} f x_6 , \; \langle A_6 \rangle = \tfrac{1}{2} f x_5 \quad \Longleftrightarrow \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} f \bar{z}$$

is a solution of field eqs.
The effective 4d action can be easily found by using an oscillator algebra

$$a_+ = \frac{i}{\sqrt{2qf}} \left(\partial_z + q f \bar{z} \right) , \quad a_+^\dagger = \frac{i}{\sqrt{2qf}} \left(\partial_{\bar{z}} - q f z \right)$$

$$a_- = \frac{i}{\sqrt{2qf}} \left(\partial_{\bar{z}} + q f z \right) , \quad a_-^\dagger = \frac{i}{\sqrt{2qf}} \left(\partial_z - q f \bar{z} \right)$$

Ground state wavefunctions determined by

$$a_+ \xi_{0,j} = 0 \, , \quad a_- \overline{\xi}_{0,j} = 0$$

$j = 0 \cdots, |N| - 1$ is the degeneracy of the ground state.

An orthonormal set of higher mode functions is

$$\xi_{n,j} = \frac{i^n}{\sqrt{n!}} \left(a_+^\dagger\right)^n \xi_{0,j} \,, \qquad \bar{\xi}_{n,j} = \frac{i^n}{\sqrt{n!}} \left(a_-^\dagger\right)^n \bar{\xi}_{0,j}$$

The mode expansion of the fermionic fields of charge +q,-q is

$$\psi = \sum_{n,j} \psi_{n,j} \xi_{n,j} \,, \qquad \chi = \sum_{n,j} \chi_{n,j} \bar{\xi}_{n,j}$$

Gauge fields A_μ and φ have no charge, standard Kaluza-Klein modes

One can prove the invariance of the 4d action under
a symmetry mixing the whole tower

$$\delta\varphi_0 = \sqrt{2}\bar{\epsilon} f \ ,$$

$$\delta\psi_{n,j} = \sqrt{2qf}\left(\epsilon\sqrt{n+1}\,\psi_{n+1,j} - \bar{\epsilon}\sqrt{n}\,\psi_{n-1,j}\right) ,$$

$$\delta\chi_{n,j} = \sqrt{2qf}\left(-\epsilon\sqrt{n}\,\chi_{n-1,j} + \bar{\epsilon}\sqrt{n+1}\,\chi_{n+1,j}\right) ,$$

$$\delta\varphi_{l,m} = (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m})\varphi_{l,m} \ ,$$

$$\delta A_{\mu,l,m} = (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m}) A_{\mu,l,m} \ .$$

SUSY 6d example

- Abelian 6d SUSY theory compactified on a torus.

N=2 SUSY in 4d before adding the magnetic flux;

4d multiplets: vector (V, ϕ)

charged hyper $(Q, \tilde{Q})$

- 6d effective action in superfields: (Marcus,Sagnotti,Siegel ; Arkani-Hamed,Gregoire,Wacker)

$$S_6 = \int d^6x \Big\{ \frac{1}{4} \int d^2\theta \, W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left(\partial V \bar{\partial} V + \phi \bar{\phi} + \sqrt{2} V \left(\bar{\partial}\phi + \partial \bar{\phi} \right) \right)$$

$$+ \int d^2\theta \, \tilde{Q}(\partial + \sqrt{2} g q \phi) Q + \text{h.c.} + \int d^4\theta \left(\overline{Q} e^{2gqV} Q + \overline{\tilde{Q}} e^{-2gqV} \tilde{Q} \right) \Big\}$$

$$\partial = \partial_5 - i\partial_6 \, , \quad \phi|_{\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}} (A_6 + i A_5)$$

ϕ are internal components of gauge fields = Wilson lines

Mode expansions with flux :

$$\phi_0|_{\theta=\bar{\theta}=0} = \frac{f}{2\sqrt{2}}(x_5 - ix_6) + \varphi\,, \qquad \varphi = \frac{1}{\sqrt{2}}(a_6 + ia_5)$$

$$Q(x_M, \theta, \bar{\theta}) = \sum_{n,j} Q_{n,j}(x_\mu, \theta, \bar{\theta})\, \psi_{n,j}(x_m)\,,$$

$$\tilde{Q}(x_M, \theta, \bar{\theta}) = \sum_{n,j} \tilde{Q}_{n,j}(x_\mu, \theta, \bar{\theta})\, \overline{\psi}_{n,j}(x_m)\,.$$

The final 4d effective action for Landau levels is

FI term

$$S_4^* = \int d^4x \left[\int d^4\theta \left(\overline{\varphi}\varphi + \sum_{n,j} (\overline{Q}_{n,j} e^{2gqV_0} Q_{n,j} + \overline{\tilde{Q}}_{n,j} e^{-2qgV_0} \tilde{Q}_{n,j}) + 2fV_0 \right) \right.$$
$$+ \int d^2\theta \left(\frac{1}{4} \mathcal{W}_0^\alpha \mathcal{W}_{\alpha,0} \right.$$
$$\left. \left. + \sum_{n,j} \left(-i\sqrt{-2qgf(n+1)} \tilde{Q}_{n+1,j} Q_{n,j} + \sqrt{2} qg \tilde{Q}_{n,j}\, \varphi\, Q_{n,j} \right) \right) + \text{h.c.} \right]$$

Coupled mass terms

- SUSY broken like in the FI model, with an infinite number of fields. Truncation to a finite number inconsistent.

The mass formula for charged states is nontrivial in the effective theory (ex. below 10d comp. of SYM theory):

- Physical eigenstates are linear combination of Landau levels. Ex (fluxes in two tori T_2^2 , T_3^2)

$$\phi^-_{n,n'} = \frac{1}{\mu_{n,n'}} \left(\sqrt{2gf_2 n}\ \phi^{3-}_{n-1,n'} - \sqrt{2gf_3 n'}\ \phi^{2-}_{n,n'-1} \right)$$

- Part of the charged scalar tower is unphysical: Goldstone bosons absorbed by the massive charged gauge bosons

$$\Pi^-_{n,n'} = \frac{1}{\sqrt{2}M_{n,n'}} \left(\mu_{n,n'} \bar{\chi}^+_{n,n'} + \mu_{n+1,n'+1} \chi^-_{n,n'} \right) \quad , \quad \text{where}$$

$$\chi^-_{n,n'} = \frac{1}{\mu_{n+1,n'+1}} \left(\sqrt{2gf_3(n'+1)}\ \phi^{3-}_{n,n'+1} + \sqrt{2gf_2(n+1)}\ \phi^{2-}_{n+1,n'} \right) \quad , \chi^+_{n,n'} = \frac{1}{\mu_{n,n'}} \left(\sqrt{2gf_2 n}\ \phi^{2+}_{n-1,n'} + \sqrt{2gf_3 n'}\ \phi^{3+}_{n,n'-1} \right)$$

3) Light scalars with no 4d symmetry ?

Masses of elementary particles in the Standard Model have various mysteries:

The smallness of fermion masses is technically natural due to chiral symmetries $\Psi \to e^{i\gamma_5 \alpha}\Psi$, which protect quantum corrections

$$\delta m \sim \frac{g^2}{16\pi^2} m \ln \frac{\Lambda}{m}$$

Masses of elementary scalars are a bigger puzzle,
UV sensitivity → the hierarchy problem
(widely studied sols. SUSY, compositeness, symmetries, anthropics/landscape, one-loop accidents. Many talks workshop)

Can Higher-dim. symmetries protect quantum corrections in a way invisible from 4d ? YES

Ex: Internal comp. of a gauge field protected by gauge symmetry (gauge-Higgs unification)

$$\delta m^2 \sim (\mathrm{loop}) \times \frac{1}{R^2}$$ (Antoniadis,Benakli,Quiros,2001…)

In magnetic compactifications with SUSY broken by magnetic flux, dimensional arguments fix quantum corrections

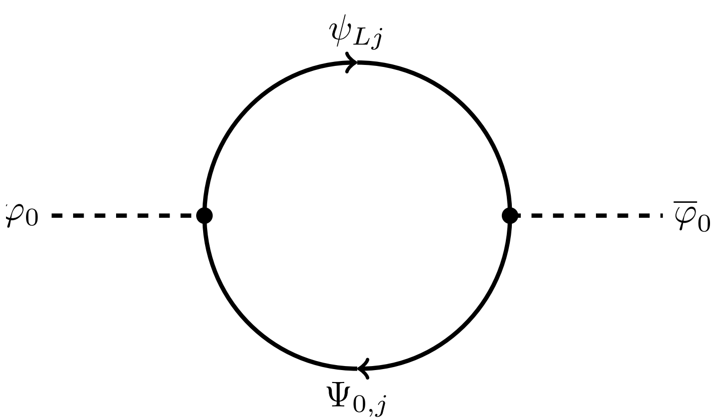
$$\delta m^2 \sim \frac{g^2}{16\pi^2} B \sim \frac{g^2}{16\pi^2} \frac{1}{R^2}$$

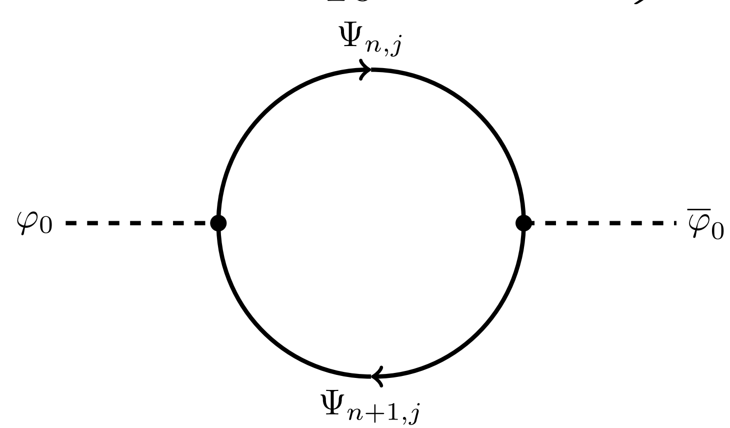
- Compactification scale $M_c = R^{-1}$ could be the GUT/unification scale, but also much lower.

Can one get $\delta m^2 << \frac{1}{R^2}$?

Usually, Yukawa and gauge interactions generate scalar masses after quantum corrections. Fermionic loop would generate

$$\delta m^2_{\varphi_0} = -2q^2|N| \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{k^2\,(k^2+2qf)}$$
$$= -\frac{q^2|N|}{4\pi^2}\left(\Lambda^2 - 2qf\ln\left(\frac{\Lambda^2}{2qf}\right) + \dots\right)$$





However, summing over the whole tower, one gets

$$\delta m^2_{\varphi_0} = -2q^2|N| \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{(k^2 + 2qfn)\,(k^2 + 2qf(n+1))}$$

$$= 4q^2|N| \sum_n \int \frac{d^4k}{(2\pi)^4} \left(\frac{n}{k^2 + 2qfn} - \frac{n+1}{k^2 + 2qf(n+1)} \right) = 0$$

(regularization issues discussed in the literature)

We believe the higher-dimensional, spontaneously broken symmetry, mixing the whole tower, protects the scalar, Goldstone boson. The symmetry is invisible from 4d.

- The symmetry ensures that the cancelation is valid to all orders in perturbation theory.

See also Ghilencea+H.M.Lee, Hirose+Maru, Honda+Shibasaki, etc

Another way to get a light scalar: tuning of charged scalar masses (10d comp.)

10d gauge field 4d fields

$$A_N \quad \rightarrow \quad A_\mu, \phi_1, \phi_2, \phi_3$$

Mass of lightest field in the ϕ_1 tower is

$$M_{\phi_1}^2 = -|f_q^1| + |f_q^2| + |f_q^3|$$

ϕ_1 can be light for specific values of moduli fields: moduli stabilization details or landscape.

$M_{\phi_1}^2 = 0$ $\Longrightarrow$ SUSY

Magnetic compactifications are generically unstable:
Internal components of charged gauge fields are often tachyonic:
Nielsen-Olesen instabilities. Stable field-theory models possible.

- Tachyons imply condensation of charged scalars. Simplest cases : restoration of full $\mathcal{N}=4$ SUSY after condensation. Not clear that always true.

If no tachyon condensation $\Rightarrow$ R-symmetry, no gaugino masses.
Tachyon condensation therefore needed.

- Flux leading to $\mathcal{N}=1$ or $\mathcal{N}=2$ SUSY assumed

to be stable. However, the SUSY flux conditions depend on tori areas (moduli fields). If no additional potential for moduli, dynamics drives the system towards decompactification limit.

Perspectives

- Magnetized compactifications generate chirality and can break SUSY (in some sectors) such that

$$M_{SUSY} \sim M_{GUT} \sim R^{-1}$$

- Magnetic fields breaks spontaneously symmetries invisible from 4d → (pseudo) Goldstones from higher-dim. symmetries. Tuning/moduli stabilization close to SUSY point another way to get light scalars.

- Various open questions: tachyon condensation, stability of SUSY vacua. Quantum corrections ?

- Applications: SM/hierarchy problem, moduli stabilization, inflation, orbifold GUT's, flavor symmetries (talk S.King)

Another Xtra dims. scenario for LHC for $R^{-1} \sim 10 - 50$ TeV