UV/IR Mixing and Naturalness

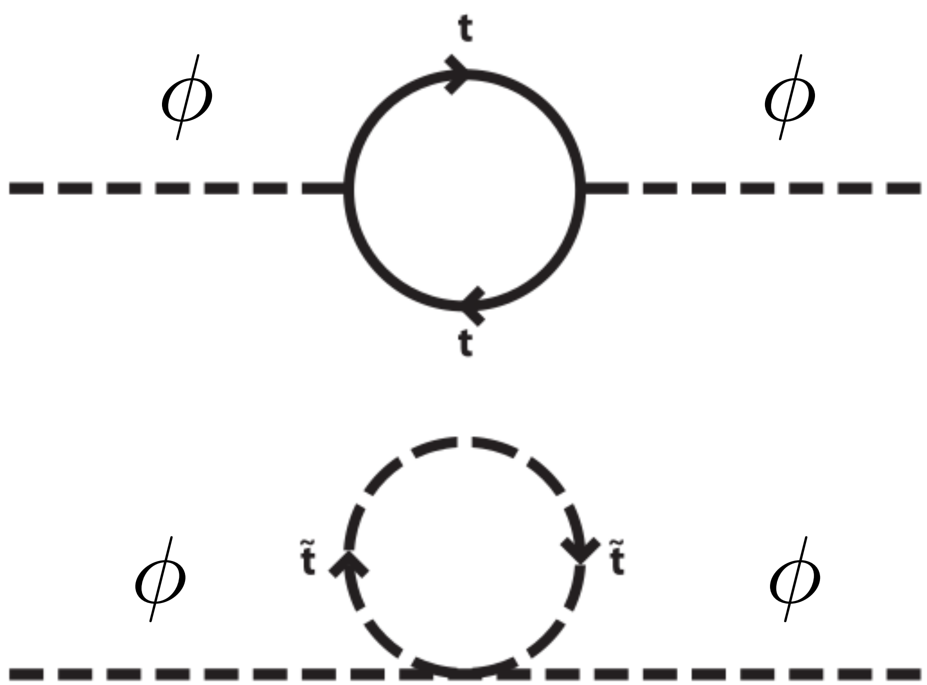
Steve Abel *(IPPP, Durham Uni.)*

Based on the following set of papers …

- w/ Dienes and Nutricati — Phys.Rev.D 110 (2024) 12, 126021; arXiv:2407.11160
- w/ Dienes and Nutricati — *Phys.Rev.D* 107 (2023) 12, 126019 ; arXiv:2303.08534
- w/ Dienes — *Phys.Rev.D* 104 (2021) 12, 126032; arXiv:2106.04622

LIO New approaches to Naturalness,
2025, Lyon

Hierarchy problem



$$V(\phi) = \frac{M_{UV}^2}{32\pi^2}(\lambda_b^2 - \lambda_f^2)\phi^2$$
$$= \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}_{EFT}M^2$$

Quadratic UV divergence — cancels automatically in (softly broken) supersymmetry (SUSY). Does theory with a scalar even make sense without SUSY??

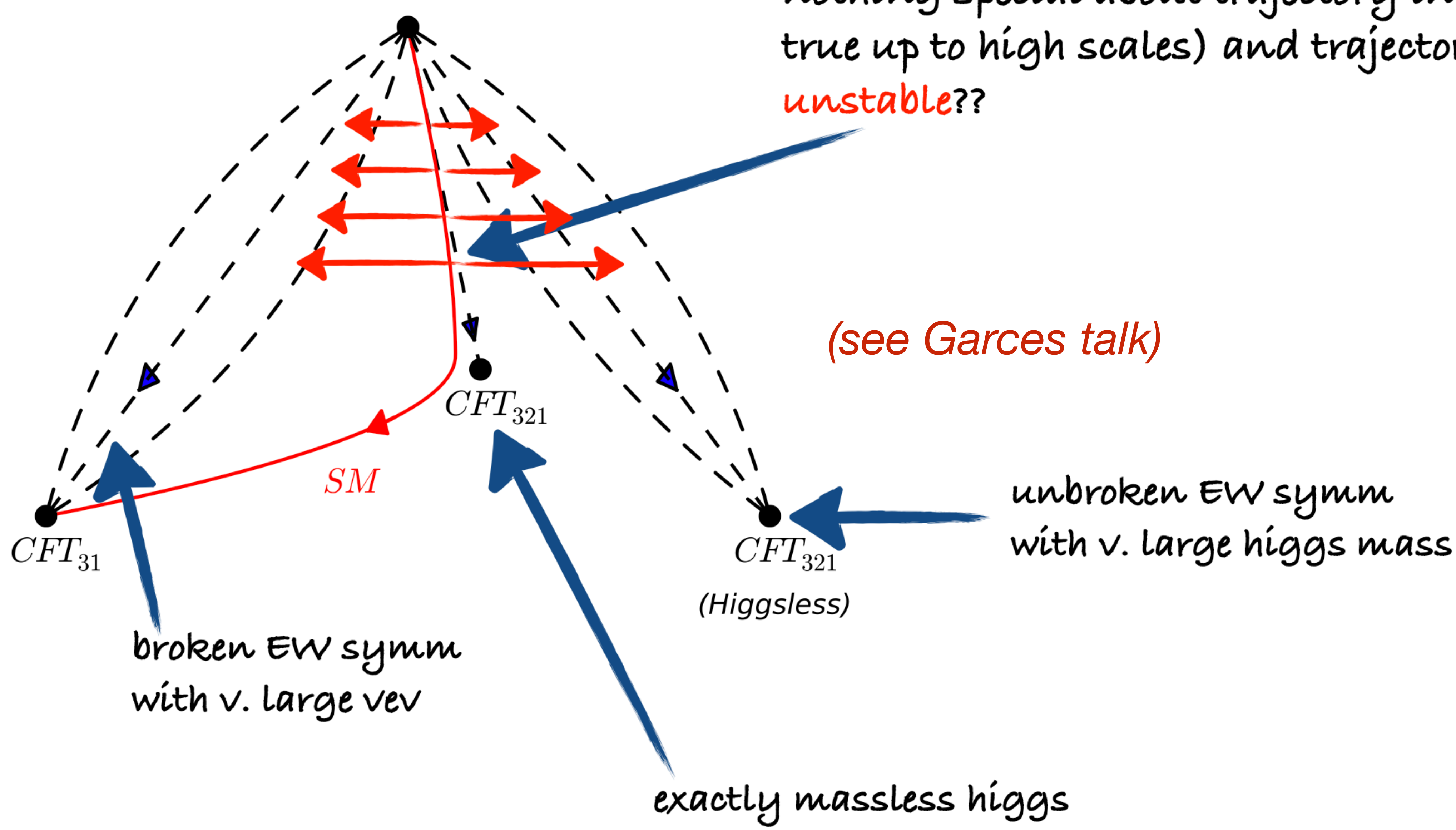
Venerable old idea that didn't work: Veltman condition: A consistent theory with a light scalar should have

$$\mathrm{Str}_{EFT}\partial_\phi^2 M^2 \stackrel{?}{=} 0$$

Hierarchy problems redux *(see James Wells)*

Technical hierarchy problem (intrinsic hierarchy) not the same as naturalness problem (extrinsic hierarchy) — Cartoon stolen from John March-Russell (who stole it from Dubovsky)

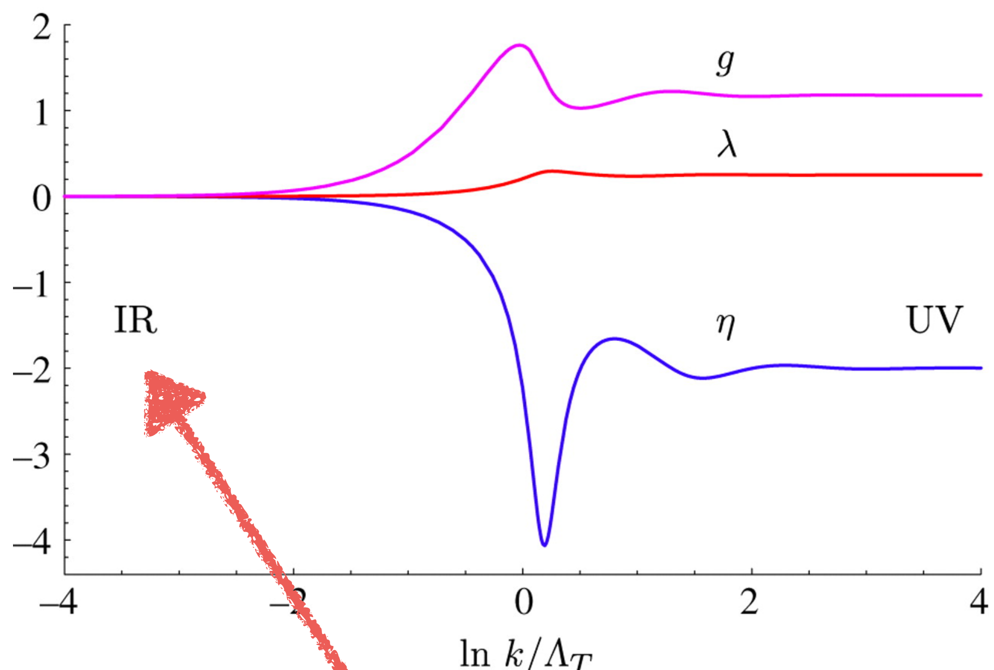
Why does trajectory of SM so closely approach zero, -0.0000000000000000000000000001, Higgs mass-squared in IR when there is nothing special about trajectory in UV (if SM true up to high scales) and trajectory is unstable??

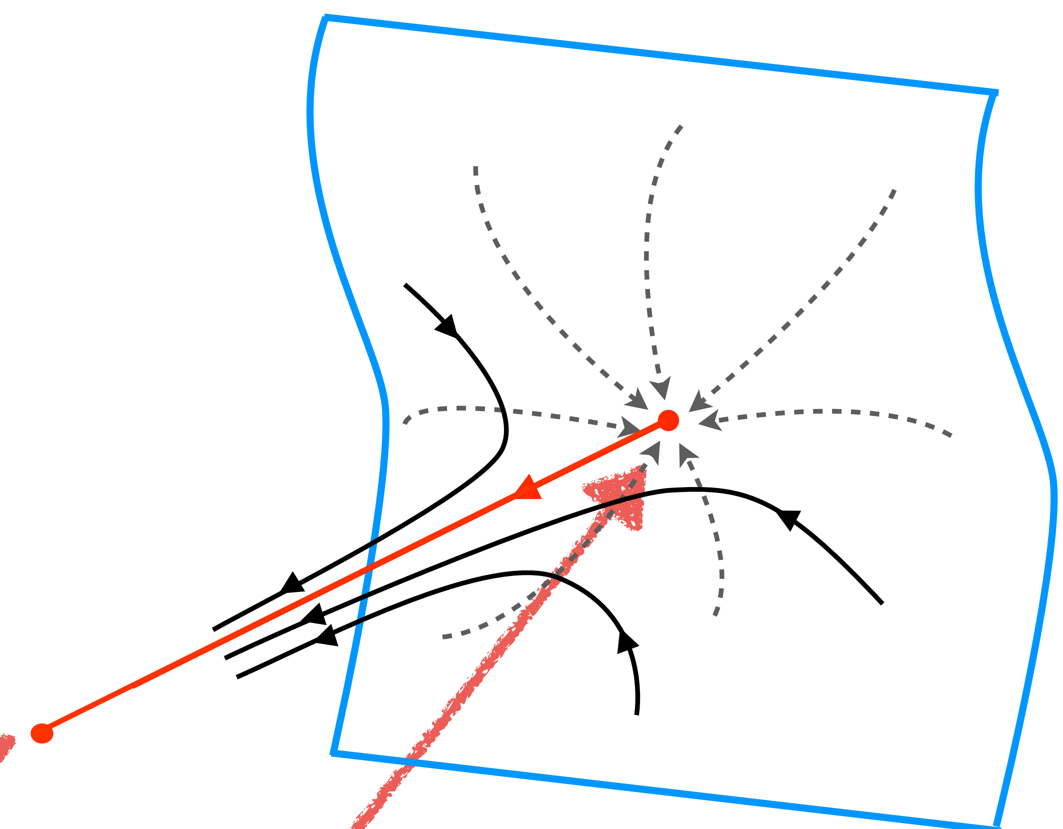


An old debate ...

Gastmans et al ’78
Weinberg ’79
Peskin
Reuter, Wetterich
Gawedski, Kupiainen
Kawai et al,
de Calan et al ’,
Litim
Morris

Weinberg et al’s proposal of UV completion by Asymptotic Safety:





Gaussian IR fixed point => perturbative

Interacting UV fixed point

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***B)* Yes! (Wetterich)** Heuristic argument — string theory has only one dimensionful parameter (which goes into defining the units by which we measure energy). A *second* energy scale is needed to observe scale violation. This could be the Planck scale, or the dynamical scale of some field theory. But well above the physics at which this second scale is generated, the theory should return to scale invariance if it is really finite

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So probably B). How does string theory do this?

Themes of this talk …

- **In a UV/IR mixed theory like string theory there is no technical hierarchy problem (a.k.a. no intrinsic hierarchy problem)**

- **In this talk I will demonstrate this by looking at Higgs mass corrections and its renormalisation in *any* closed string theory — equivalent of Coleman-Weinberg potential in field theory**

- **We will see how this conclusion is tied up with the way that a "Wilsonian" EFT can emerge from a very UV/IR mixed theory.**

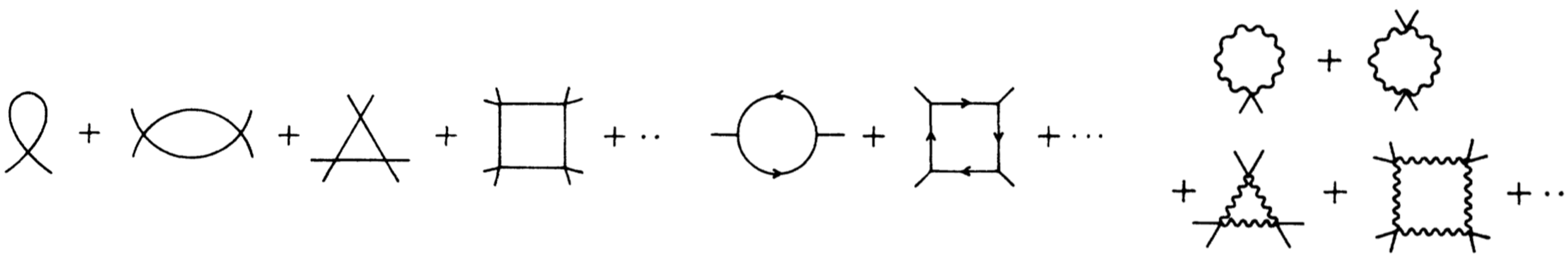
Outline

- Understanding UV/IR mixing from a string perspective
- A non-renormalisation theorem
- Higgs mass
- Renormalisation *(c.f. Garces)*
- Implications for Naturalness

Understanding UV/IR mixing via string theory

Understanding UV/IR mixing: the one-loop cosmological constant done in a stringy way

As a useful laboratory let's derive Λ the one-loop cosmological constant: we can do this as an integral over all distinct loops of massive propagators of mass M as



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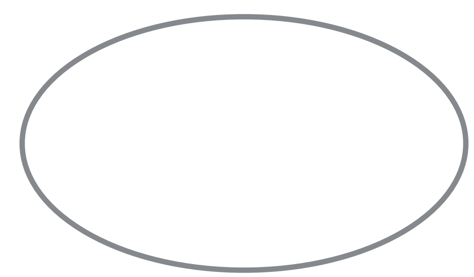

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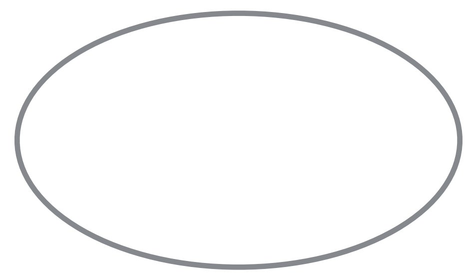

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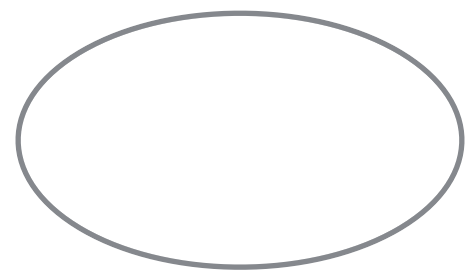

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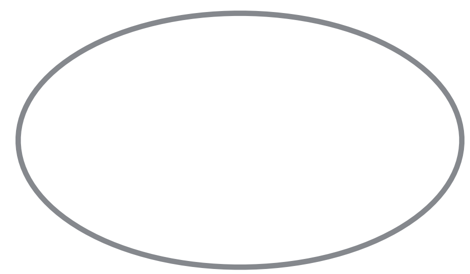

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$$= \; -\frac{1}{2}\sum_{\text{states}}\int \frac{d^4k}{(2\pi)^4}\int_0^\infty \frac{dt}{t}(-1)^F e^{-t(k^2+M^2_{\text{state}})} \qquad = \; -\frac{1}{32\pi^2}\int_{M_{UV}^{-2}}^\infty \frac{dt}{t^2} g(t)$$

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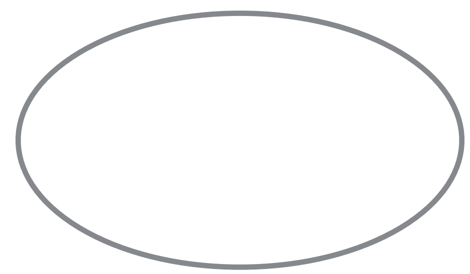

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We can identify a "particle partition function" which is a graded sum over the spectral density: THIS WILL BE THE HERO IN OUR DISCUSSION

$$g(t) = \sum_{\text{states}} \frac{1}{t}(-1)^F e^{-tM_{\text{state}}^2}$$

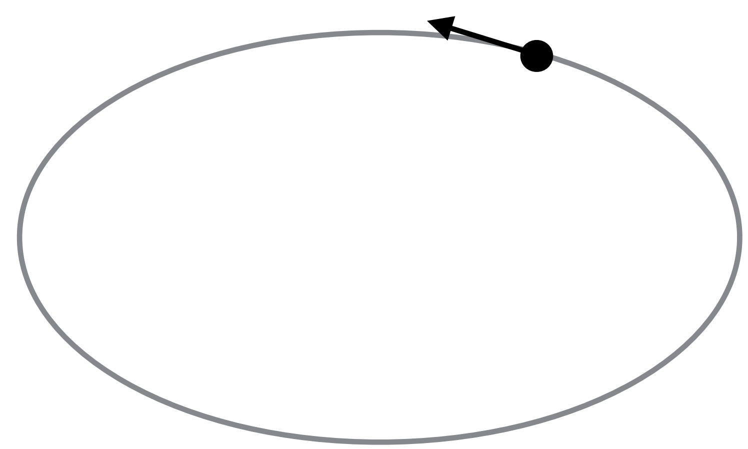
To orient you: if I perform this with cut-off it gives the precursor to the Coleman-Weinberg potential:

$$\Lambda = -\frac{M_{UV}^4}{64\pi^2}\mathrm{Str}_{EFT}\mathbf{1} + \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}_{EFT}M^2 - \mathrm{Str}_{EFT}\left[\frac{M^4}{64\pi^2}\log c\frac{M^2}{M_{UV}^2}\right]$$

where here $\mathrm{Str}_{EFT}\mathcal{X} = \sum_{\mathrm{states}}(-1)^F \mathcal{X}_{\mathrm{state}}$ is the graded sum over states in the theory.

How does string theory get to be UV-complete and so avoid the need for the cut-off M_{UV}? Importantly I want to think about the theory generically TODAY, when SUSY (if it was ever there) is absent: I am not interested in model specific things.

Instead of a circle, closed string theory instead maps out a torus (as I am sure you know):



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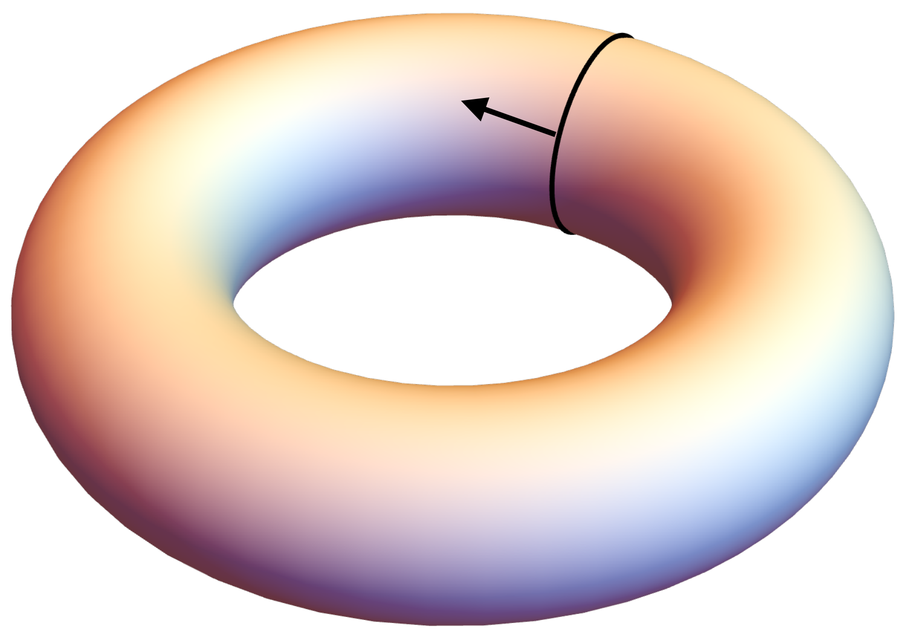
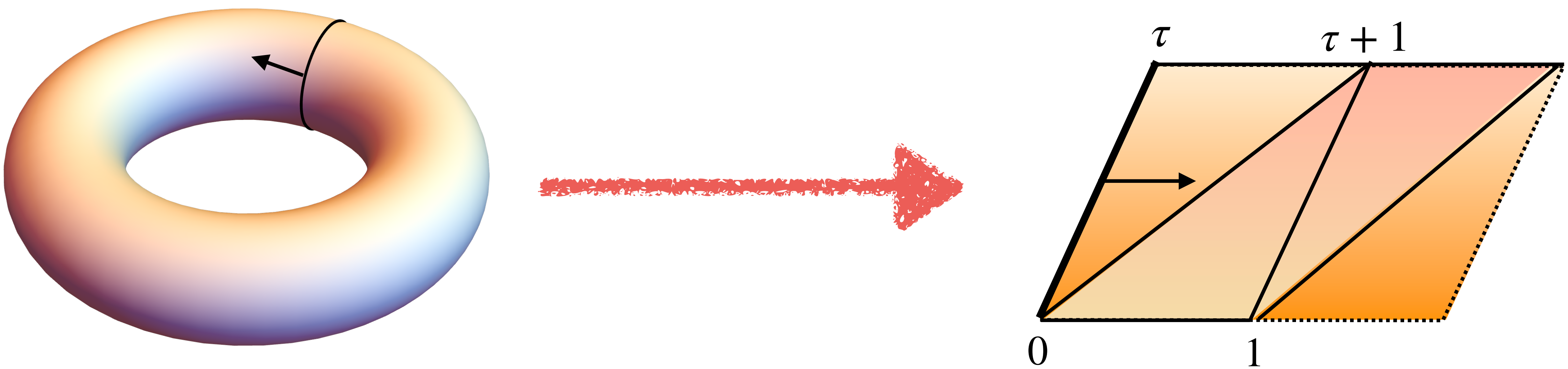


Instead of a circle, closed string theory instead maps out a torus:

But conformal invariance implies torus can be mapped to parallelogram in complex plane, defined by single parameter τ,

(see King talk —but this is world sheet modular invariance)



Modular invariance from residual transformations: $\tau \to \tau + 1$ redefines the torus and $\tau \to -1/\tau$ swaps σ_1 and σ_2 and reorients torus …

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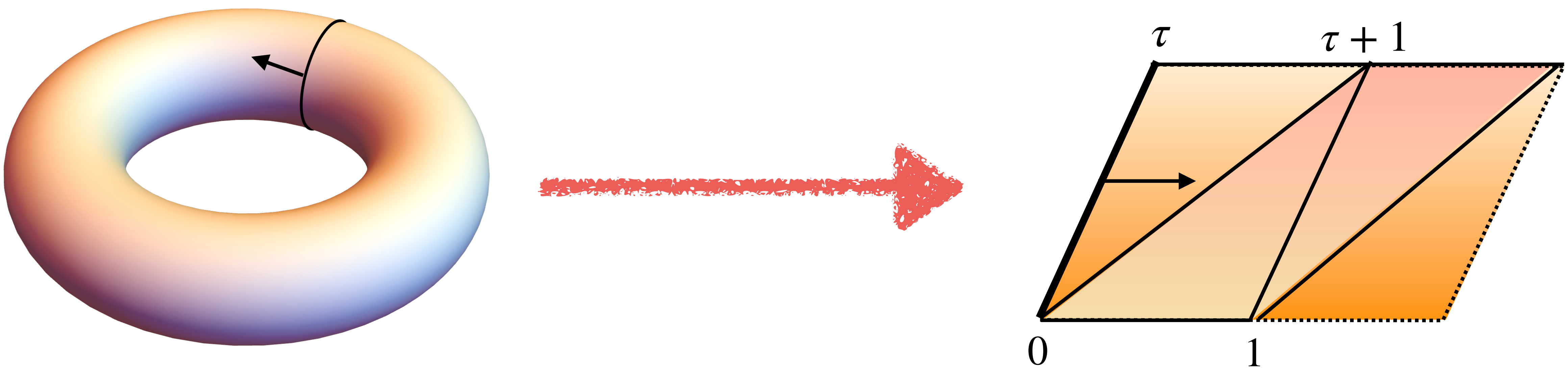

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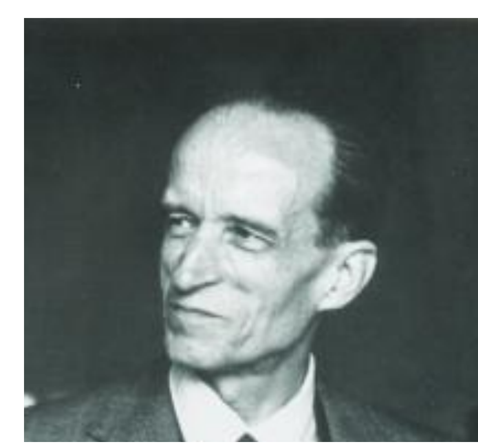
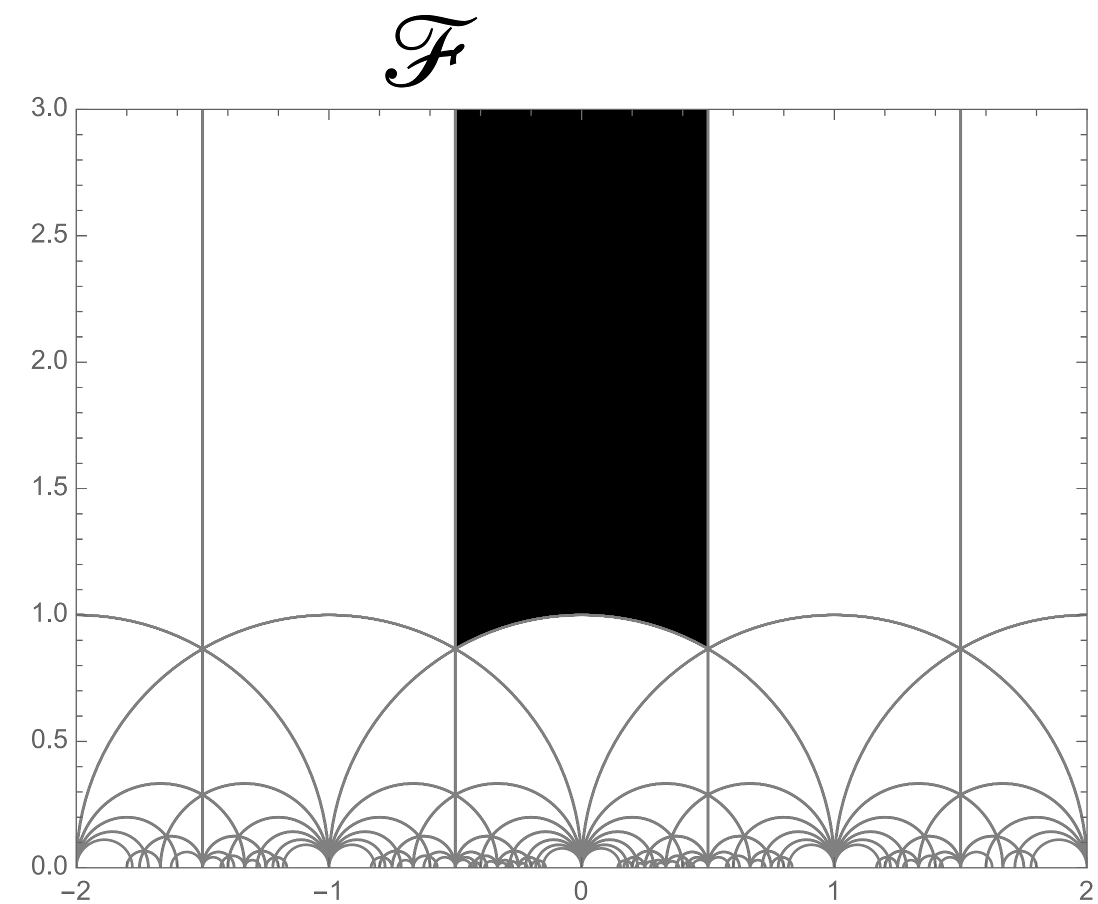
"The fifth fundamental mathematical operation after $+ \; - \; \times \; /$"



Thus the integral over all diagrams does not cover the whole τ plane but takes the form $(\mathcal{M} = M_s/2\pi)$...

$$\Lambda = -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, Z(\tau,\overline{\tau})$$

where $Z(\tau) = Z(\tau')$ when $\tau' = \dfrac{a\tau + b}{c\tau + d}$



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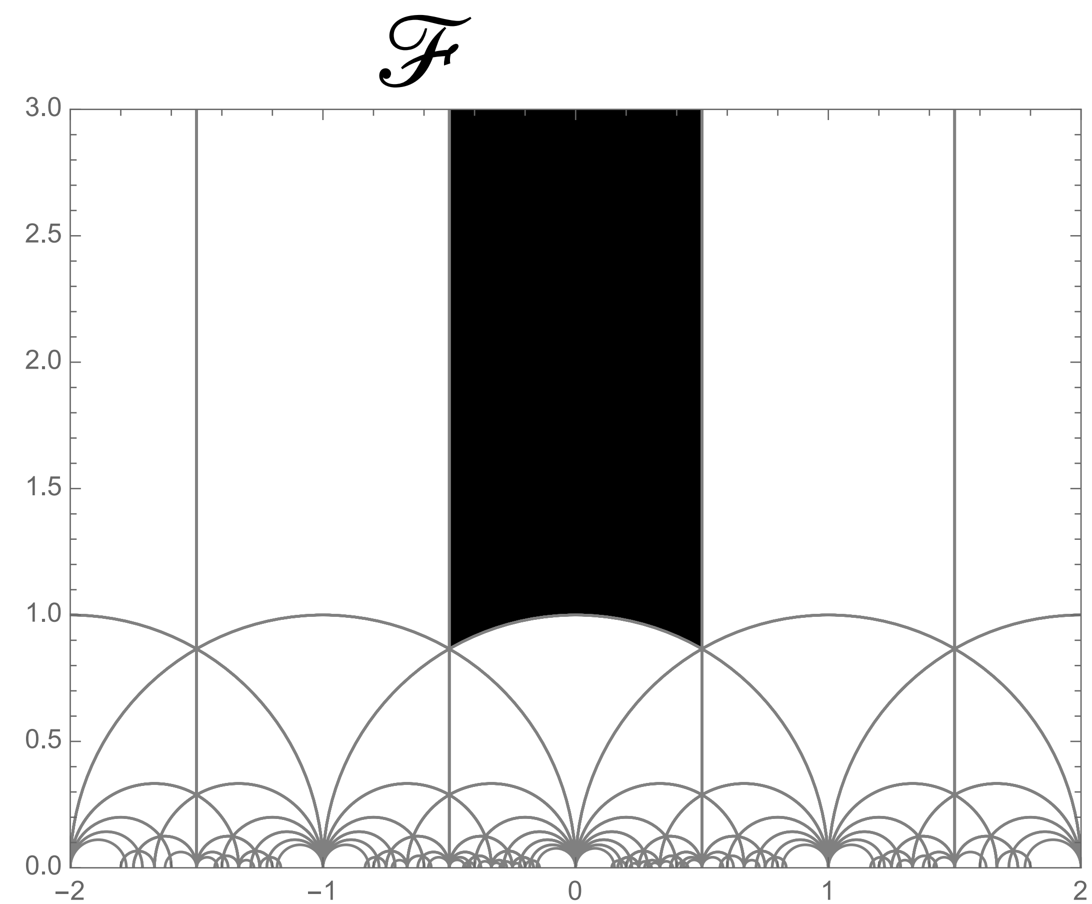
$Z(\tau)$ is the string version of the particle $g(t)$ and holds all the information about the spectrum. *All one-loop amplitudes look similar to this.*



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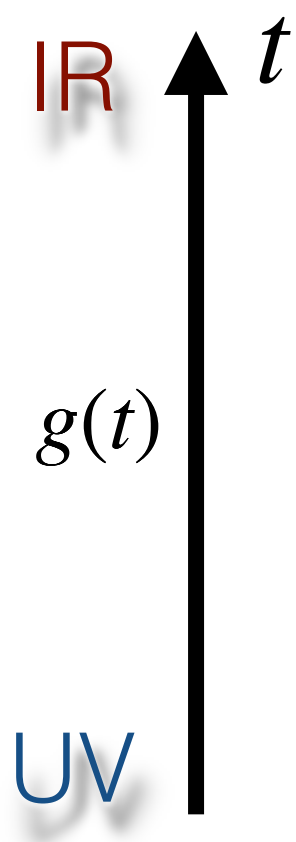

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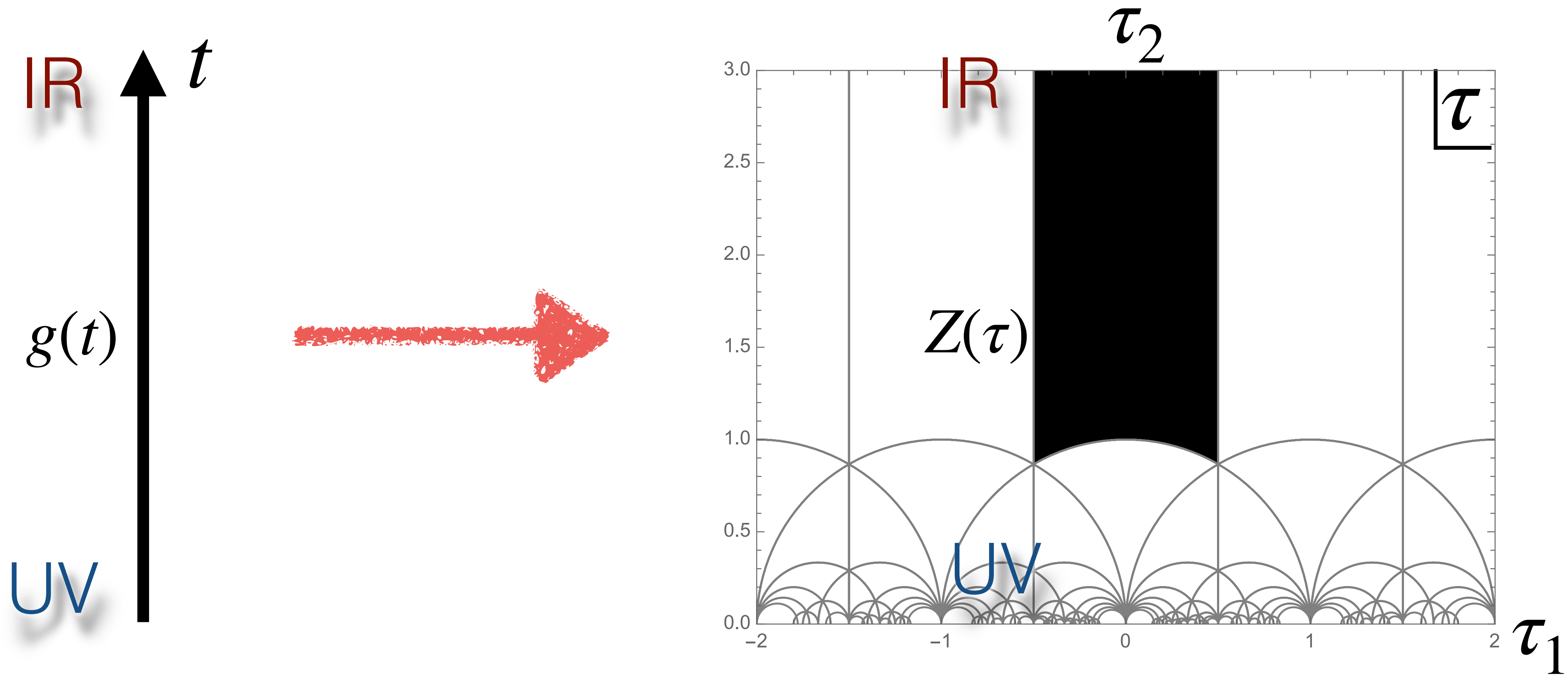

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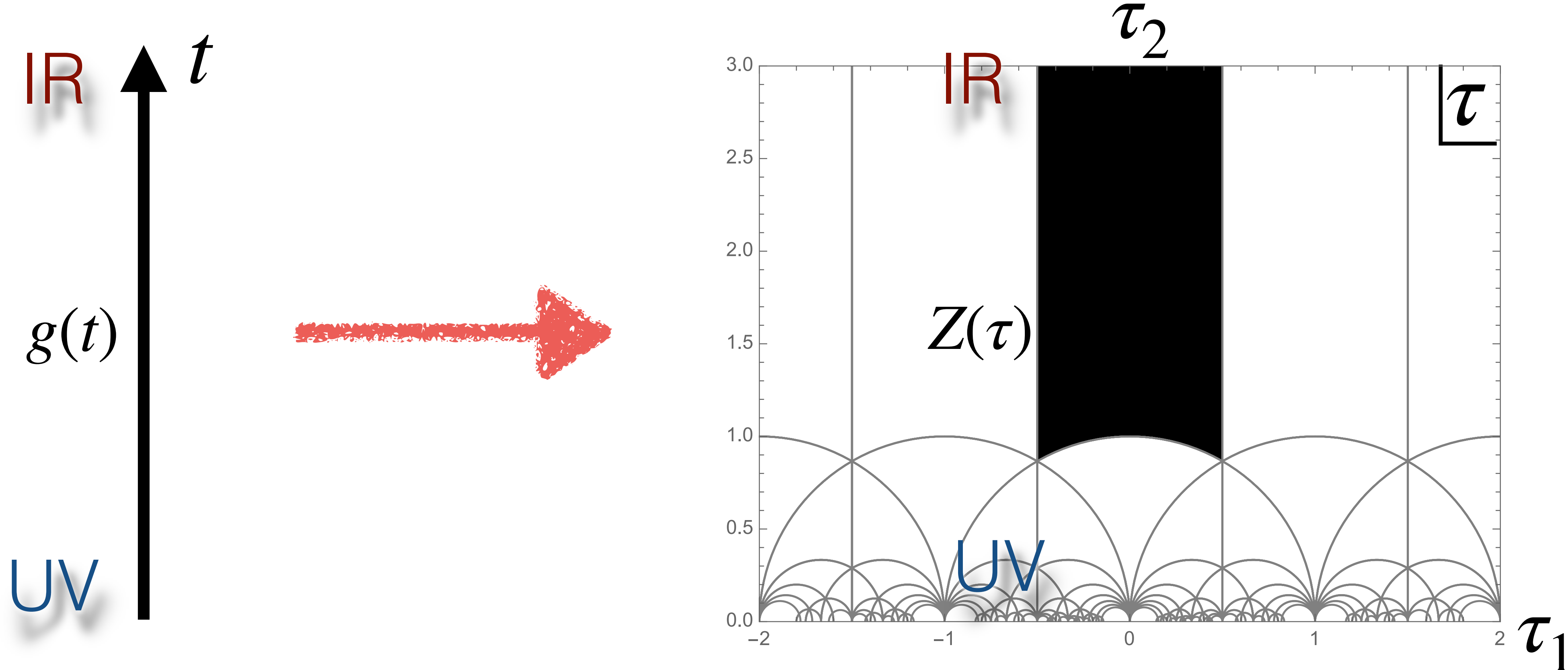

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Strings: “UV is missing”

Note there is a tendency for physicists to label $\tau_2 \to 0$ the “UV” but we are about to see that this is very misleading …

Indeed a method due to Rankin and Selberg (1939/40) expresses the integral in terms of our previous particle theory partition function $g(\tau_2)$ of the **physical (level-matched) states —**

$$g(\tau_2) = -\frac{\mathcal{M}^4}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \; Z(\tau)$$

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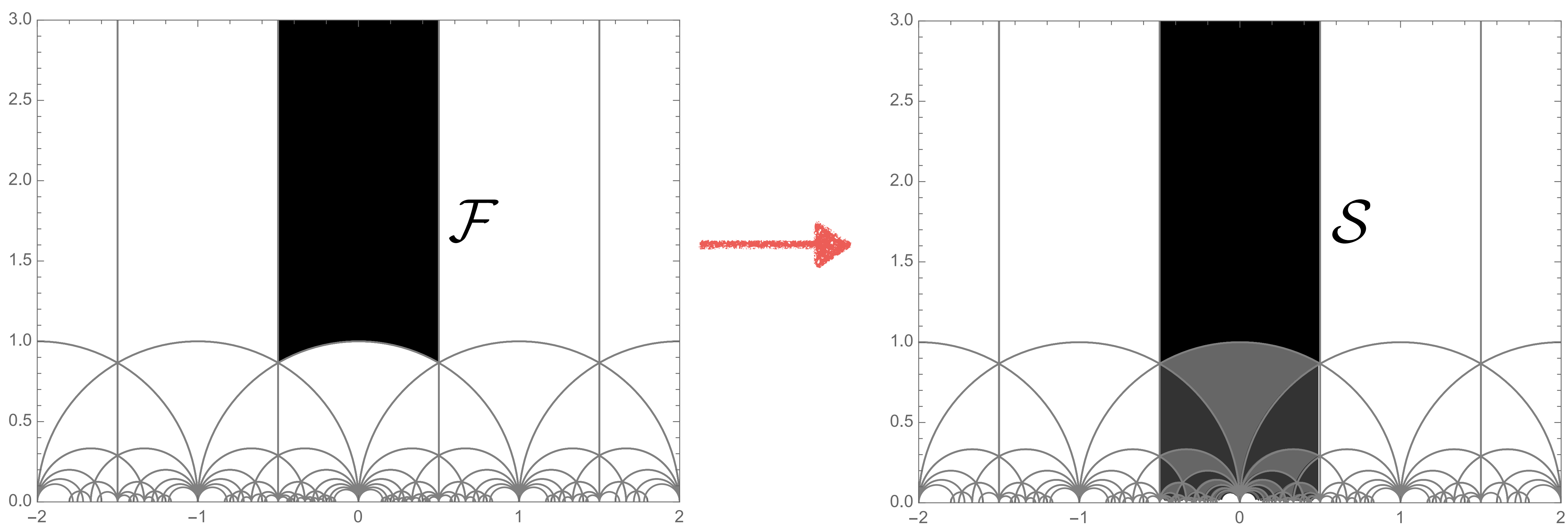
RS devise a transform in which $\mathscr{F}$ gets unfolded to the critical strip $\mathscr{S}$



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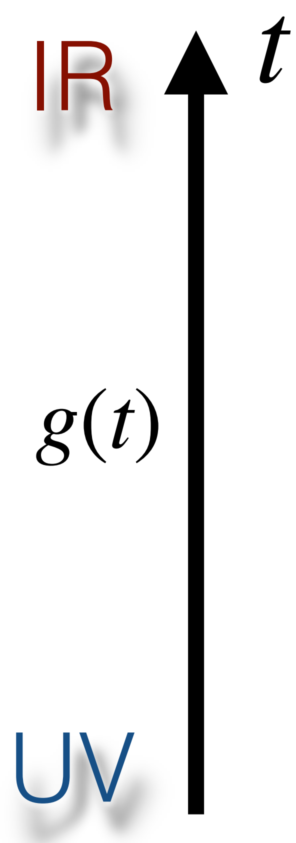
This yields the integral in this form …

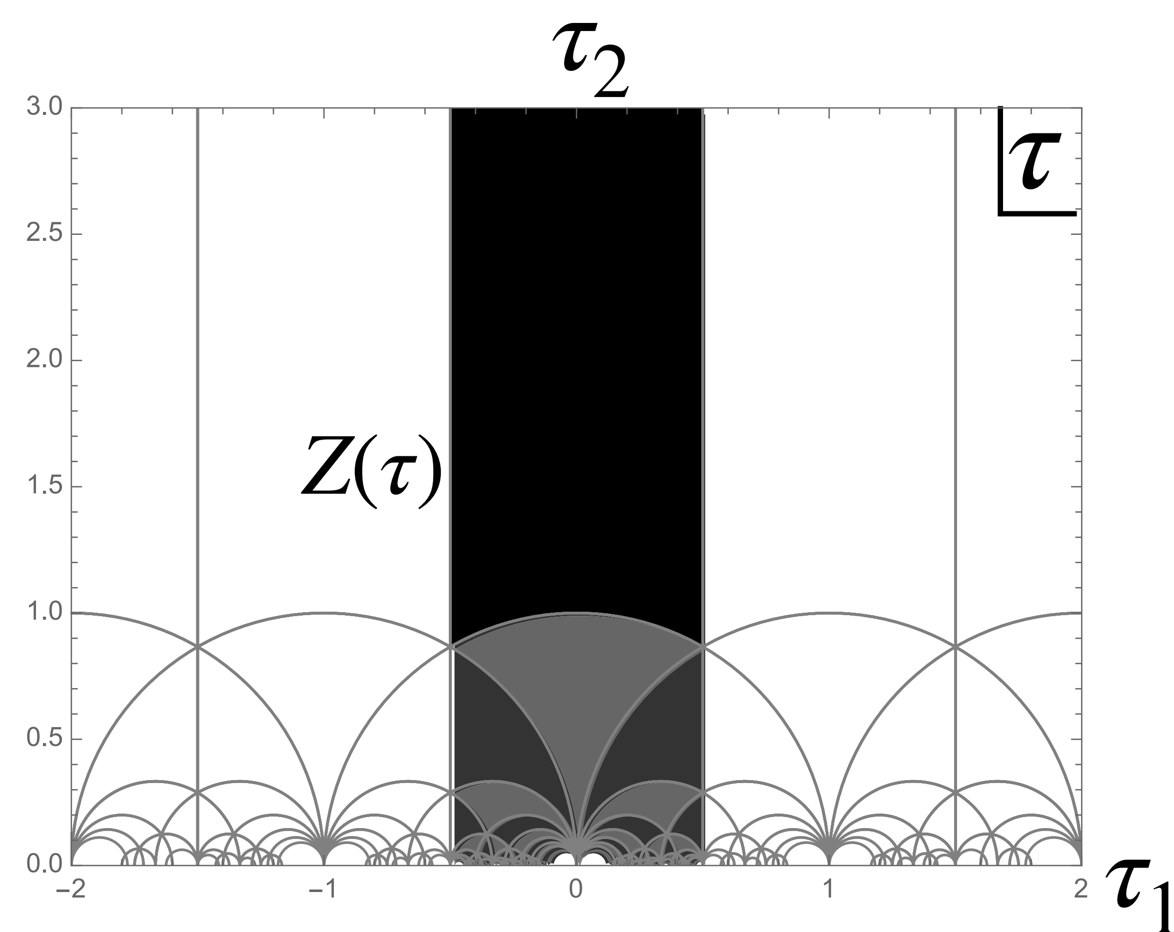
$$-\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\,Z(\tau,\overline{\tau}) = \frac{\pi}{3}\lim_{\tau_2\to 0} g(\tau_2)$$

- Rankin, Selberg (1939/40)
- Zagier (1981)

In string theory: Kutasov, Seiberg; McClain, Roth, O'Brien, Tan; Dienes; Angelantonj, Florakis, Pioline, Rabinovici

Thus we reach a less familiar cartoon …





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Strings according to RS: infinite sum over fundamental domains divided by infinite overcounting

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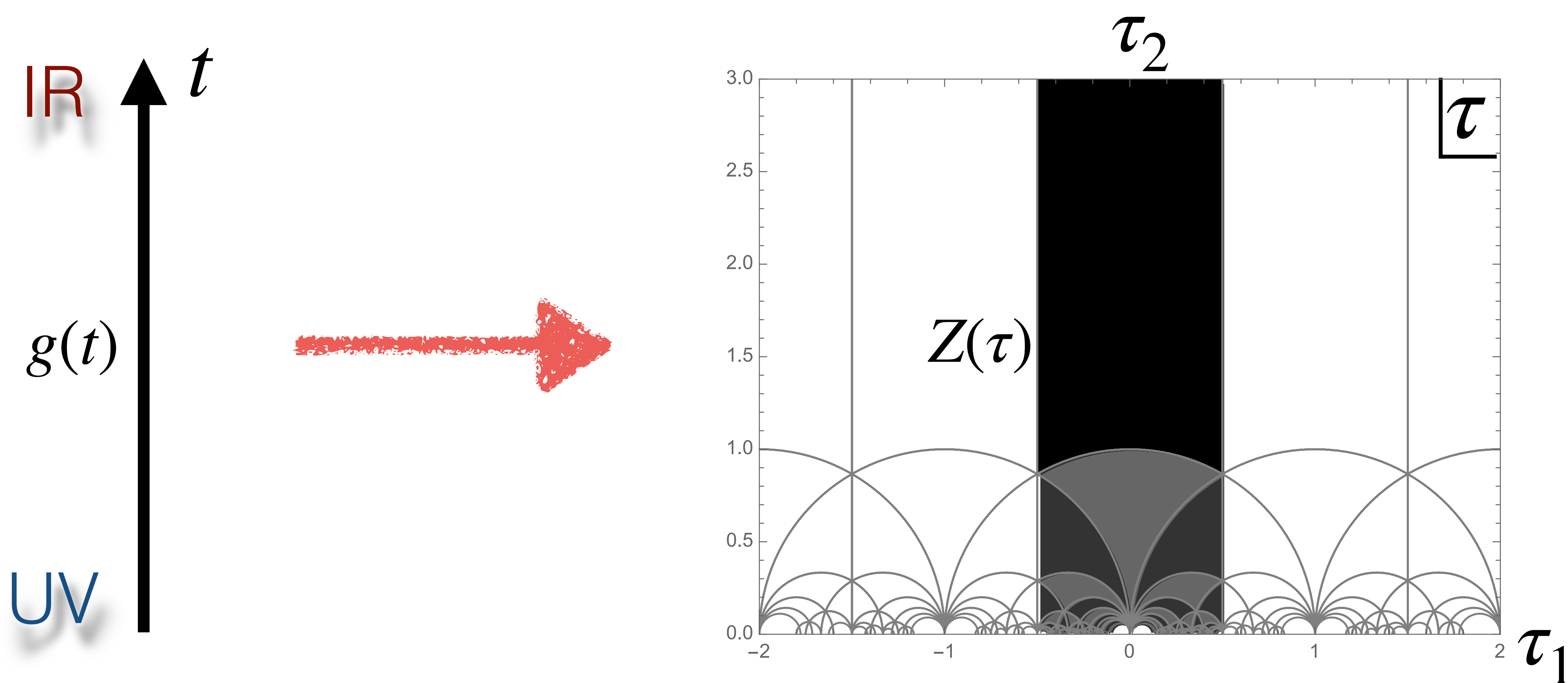

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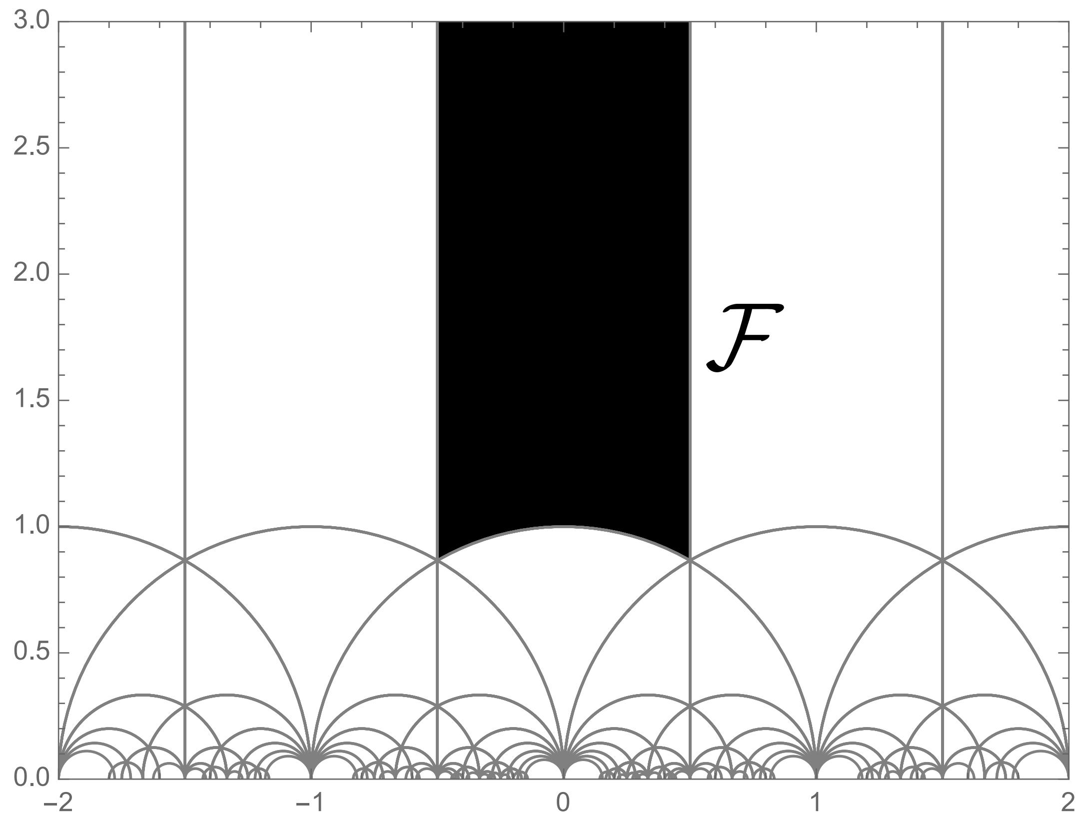
Now we see the labels “UV” and an “IR” on the string integral no longer make sense precisely because of the UV/IR mixing of the modular transformation.

Let's pause for a minute to see (as physicists) why this is remarkable:

$\pi\alpha'\tau_2$ clearly plays the role of the Schwinger parameter t when $\tau_2 \geq 1$: by naively integrating over the fundamental domain, we physicists see a result that mimics EFT …



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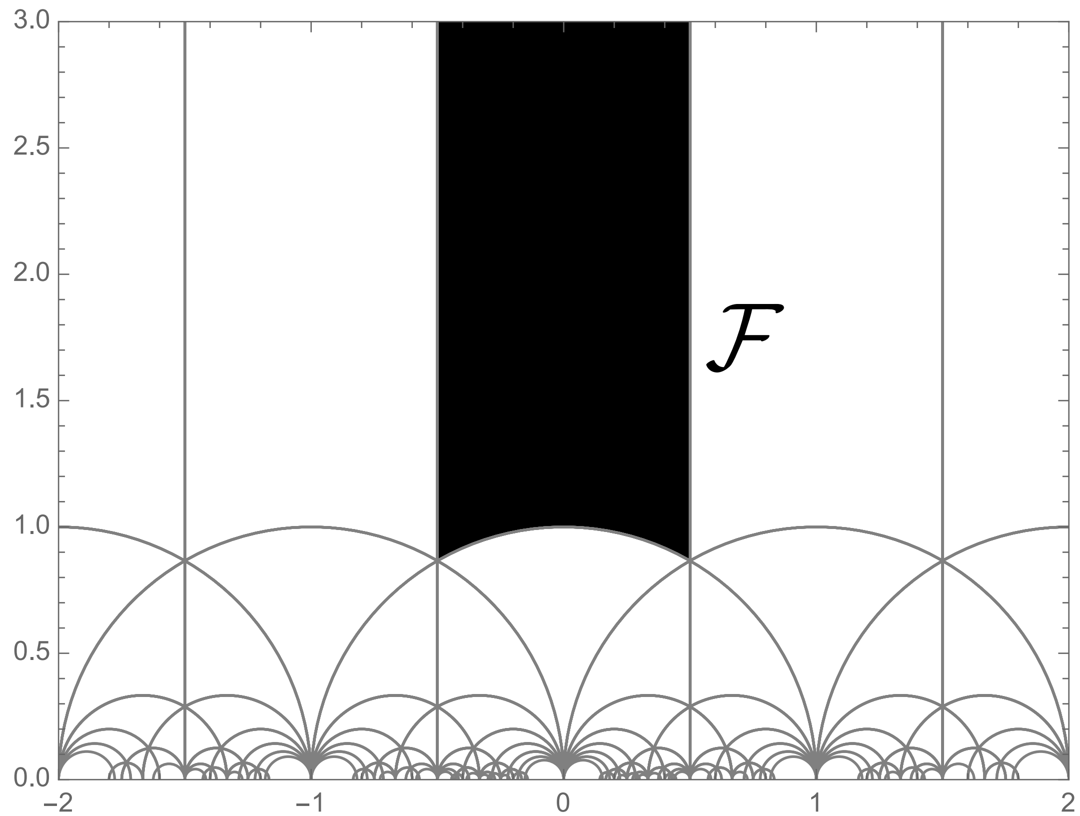
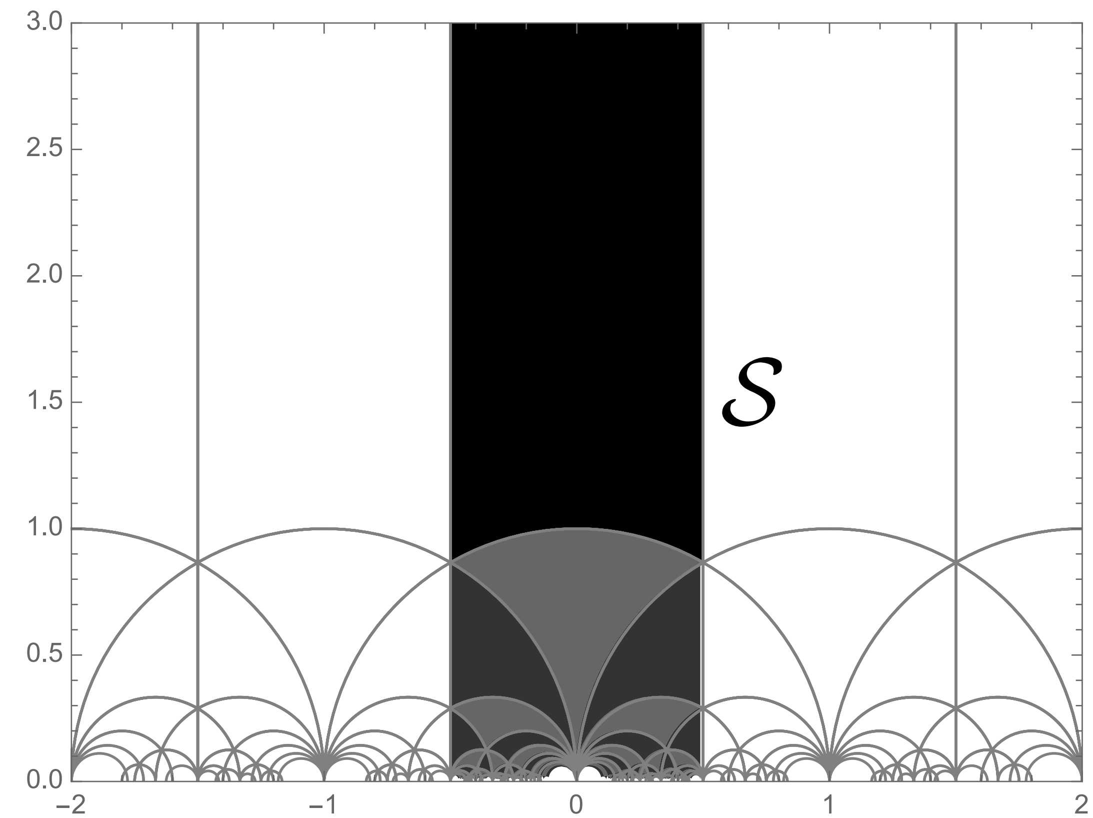
But this is equal to a ***very not EFT-like limit*** - it instead looks like a deep UV limit!!





$$\Lambda \approx \int_1^\infty \frac{d\tau_2}{\tau_2^2} g(\tau_2)$$

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A non-renormalisation theorem!

So this is the ultimate UV/IR mixing. And it in turn implies something spectacular about the supertrace over the physical states …

- Dienes, Misaligned SUSY, 1994
- Dienes, Moshe, Myers 1995

To see this let's try and evaluate this RS limit expression:

$$\frac{\pi}{3} \lim_{\tau_2 \to 0} g(\tau_2) \;\; = -\frac{\mathcal{M}^4}{2} \lim_{\tau_2 \to 0} \sum_{\text{states}} (-1)^F \frac{1}{\tau_2} e^{-\pi \tau_2 \alpha' M^2_{\text{state}}}$$

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It looks like it diverges because of the $1/\tau_2$ prefactor in $g(\tau_2)$!!! unless ….

$$\lim_{\tau_2 \to 0} \sum_{\text{states}} (-1)^F e^{-\pi \tau_2 \alpha' M^2_{\text{states}}} = 0$$

Thus — if we define a stringy ***regulated supertrace*** appropriate for *infinite* towers of states for any operator $\mathcal{X}$,

$$\mathrm{Str}\,\mathcal{X} = \lim_{\tau_2\to 0} \sum_{\text{states}} (-1)^F \mathcal{X}_{\text{state}}\, e^{-\pi\tau_2\alpha' M^2_{\text{state}}}$$

then here (where $\mathcal{X} = const$ for the case of Λ) we see that any modular invariant 4D theory with a finite Λ obeys

$$\boxed{\mathrm{Str}\,\mathbf{1} = 0}$$

Any tachyon-free modular invariant theory in 4D has $\mathrm{Str}(\mathbf{1}) = 0$ *even when no SUSY!* *(Note also we can see immediately that any consistent theory has to have both fermions and bosons)*

Or to put it another way … if we expand $g(\tau_2)$ around $\tau_2 = 0$ in a generic particle theory it would go like

$$g(\tau_2) = \frac{1}{\tau_2} \times \left(C_0 + C_1\tau_2 + C_2\tau_2^2 + \ldots\right)$$

but in a modular invariant theory we have $C_0 = 0$ and it must instead go like

$$g(\tau_2) = \frac{1}{\tau_2} \times \left(C_1\tau_2 + C_2\tau_2^2 + \ldots\right)$$

Note we can express the integral as $\Lambda = \pi C_1/3$, where by expanding the exponential around τ_2 and picking off the first term C_1: we have

$$\Lambda \quad = \quad \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

- Dienes, Misaligned SUSY, 1994
- Dienes, Moshe, Myers 1995

This looks exactly like the leading piece in the Coleman Weinberg potential if we were to assume that the quartic M_{UV}^4 term magically vanishes. *i.e. the condition* $\mathrm{Str} 1 = 0$ *forces the quartic divergence term to vanish in any modular invariant theory.*

Many more supertrace relations for theories with higher dimensions. e.g. sectors that feel ≥ 4 extra dimensions are scale invariant!

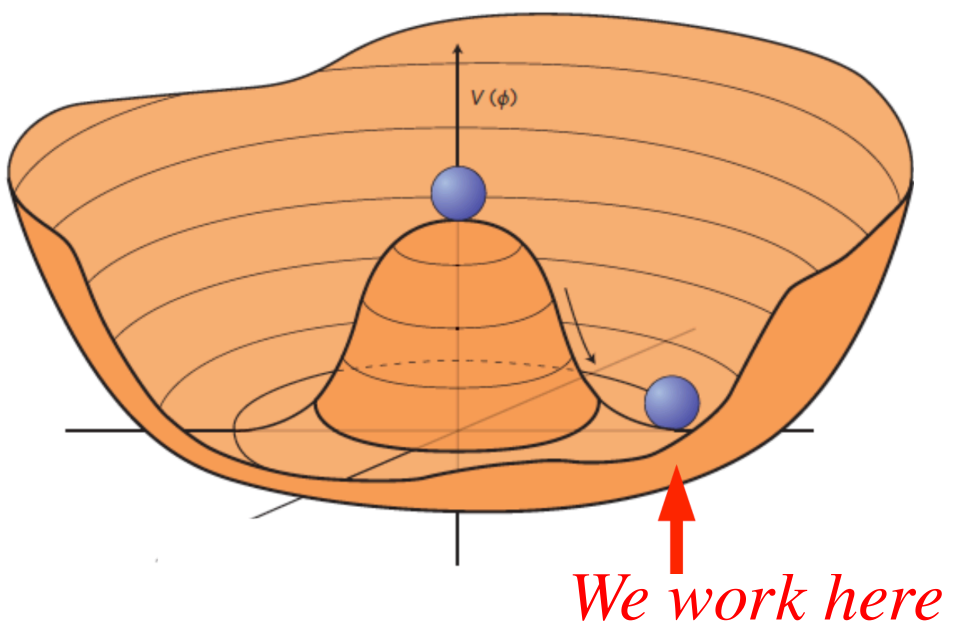
- SAA, Dienes, Nutricati 2024

Let’s talk about the Higgs mass

Let's turn to the Higgs mass. How can we use this technology to express it generally, and express the hierarchy problem?

Assume that the partition function is a function of the higgs ϕ. Then begin with the naive expression:

$$m_\phi^2 \equiv \left.\frac{d^2\Lambda(\phi)}{d\phi^2}\right|_{\phi=0}$$



Thus to get m_ϕ^2 we replace integrand with

$$\frac{\partial^2 Z(\tau)}{\partial \phi^2} = \tau_2^{-1} \sum_{\text{states}} (-1)^F \, X \, e^{-\pi\alpha'\tau_2 M^2} e^{-i\pi\alpha'\tau_1 \Delta M^2/2}$$

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Hence the relevant summand X for the Higgs mass is **almost**

$$X = -\pi\alpha'\tau_2 \, \partial_\phi^2 M^2 + (\pi\alpha'\tau_2)^2 \left(\partial_\phi M^2\right)^2$$

Need an additional piece to modular complete the ϕ-derivatives:

$$X \longrightarrow X + \frac{\xi}{4\pi^2\mathcal{M}^2}$$

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What?!

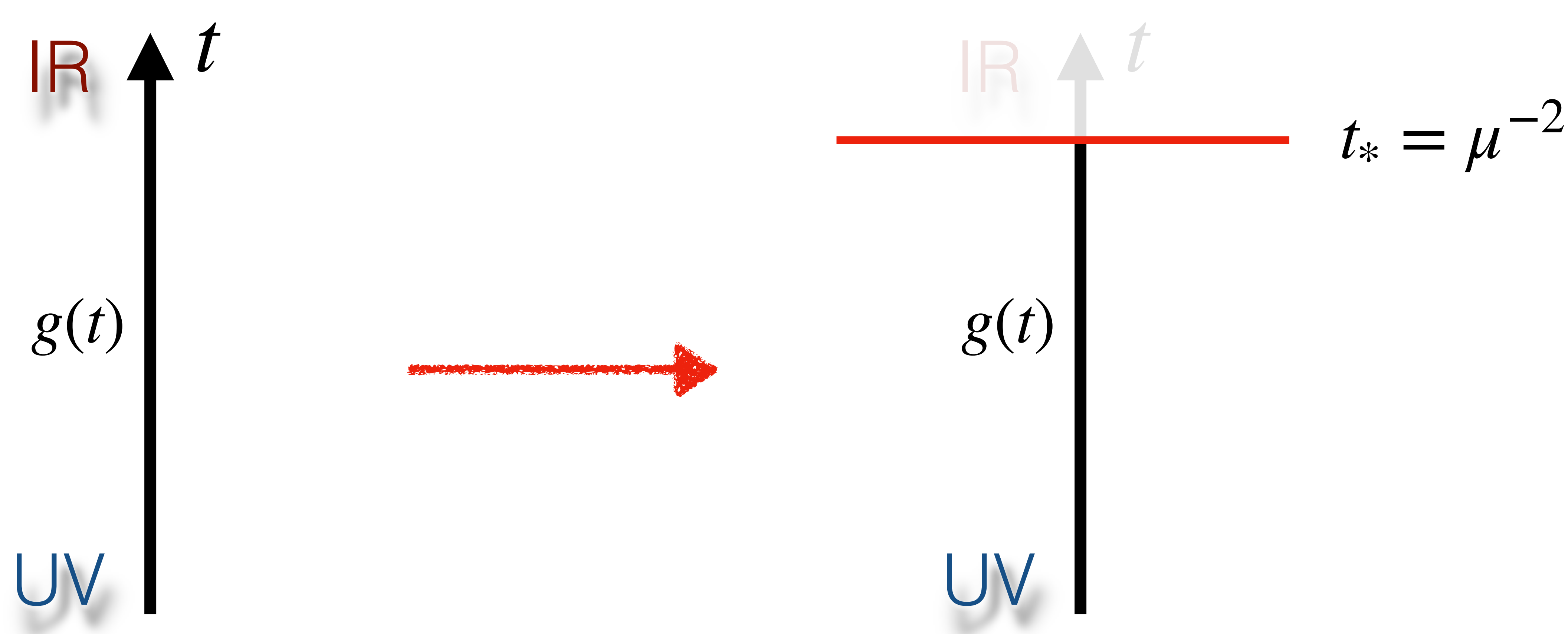
Ah yes - the amplitude doesn't yet have any energy scale in it!
These log divergences will give log running to the Higgs mass.

$\Longrightarrow$

Renormalisation

Let's talk about energy scales μ

To see energy dependence we must of course decide how to insert an energy scale μ: it is easiest to use the Wilson "lattice-cut-off" interpretation of RG and insert a cut-off function $\mathscr{G}(\mu, t)$ into integrals which crushes the IR limits for all $t \gg \mu^{-2}$:



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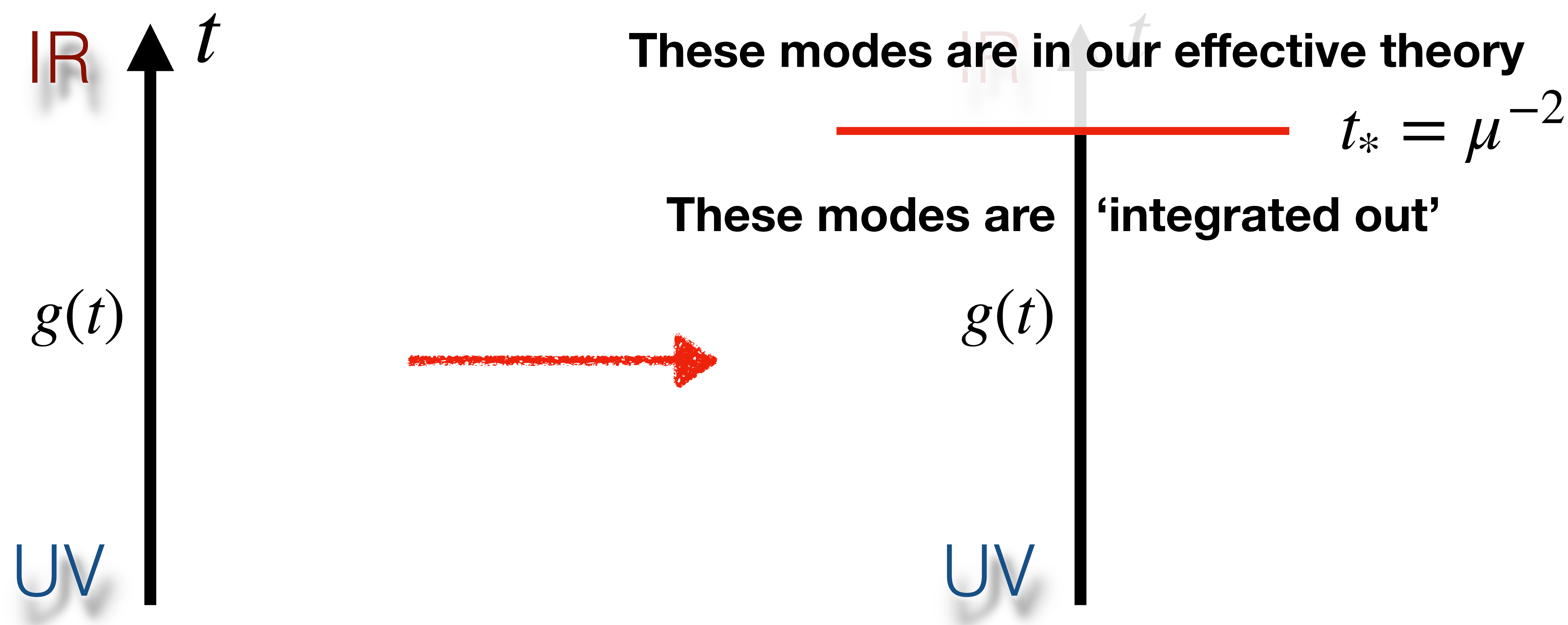

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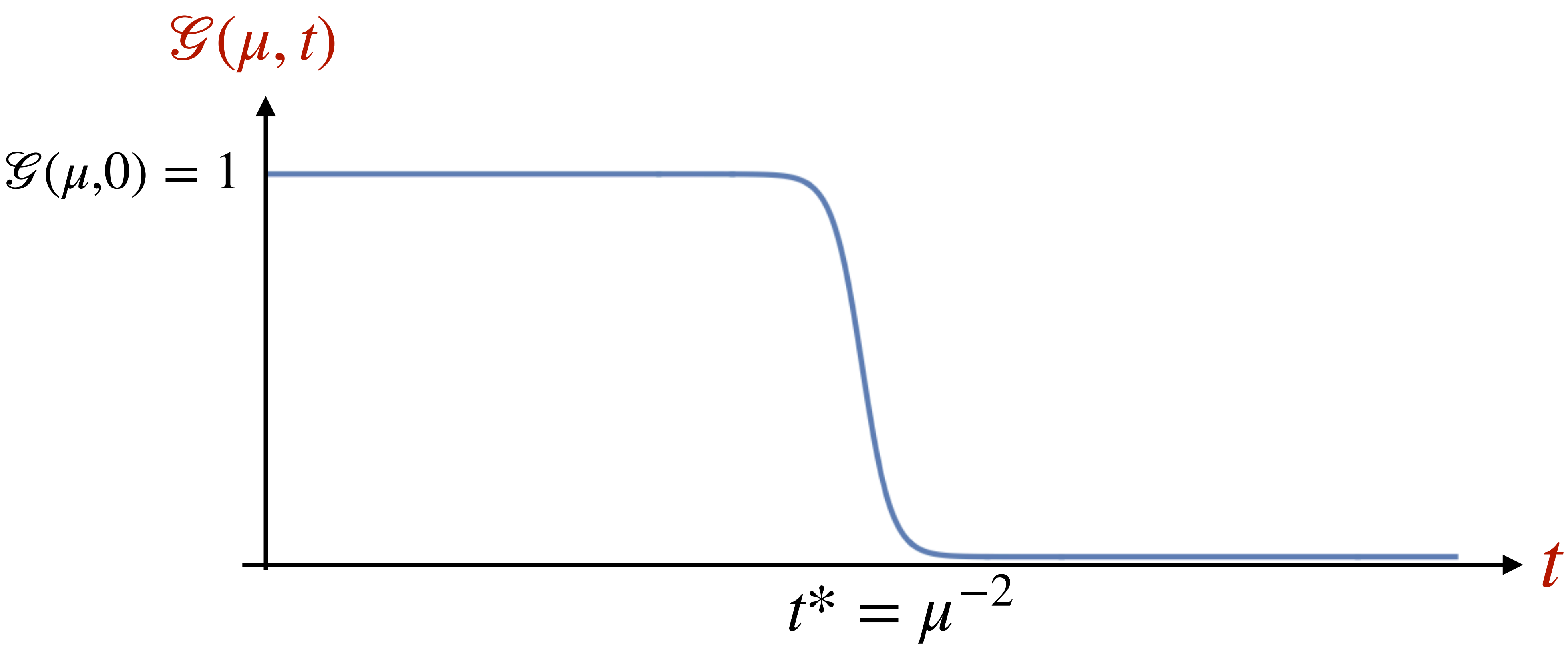
So need to insert something that looks like this into the integral:



As a simple example we can take a Heaviside function: $\mathscr{G}(\mu, t) = \theta(1 - \mu^2 t)$

so that the one-loop corrections then take the form

$$\Delta m_\phi^2(\mu) \; = \; \int_{M_{UV}^{-2}}^{\infty} \frac{dt}{t^2} \mathcal{G}(\mu, t) g(t) = \int_{M_{UV}^{-2}}^{\mu^{-2}} \frac{dt}{t^2} g(t)$$

Since in field theory we have $g(t) = \frac{1}{t}(C_0 + C_1 t + C_2 t^2 + \ldots)$

we see that logarithmic divergences at low energies come from the C_2' term, and our log divergence turn into

$$\Delta m_\phi^2 = 2C_2' \log \mu / M_{UV}$$

Let's see this in string theory:

First we need a modular invariant renormalisation:

- SAA, Dienes, 2021

To insert an energy scale μ we insert a cut-off function $\mathscr{G}(\mu, \tau)$ which removes log divergences from any massless states and **which must itself be *modular invariant***

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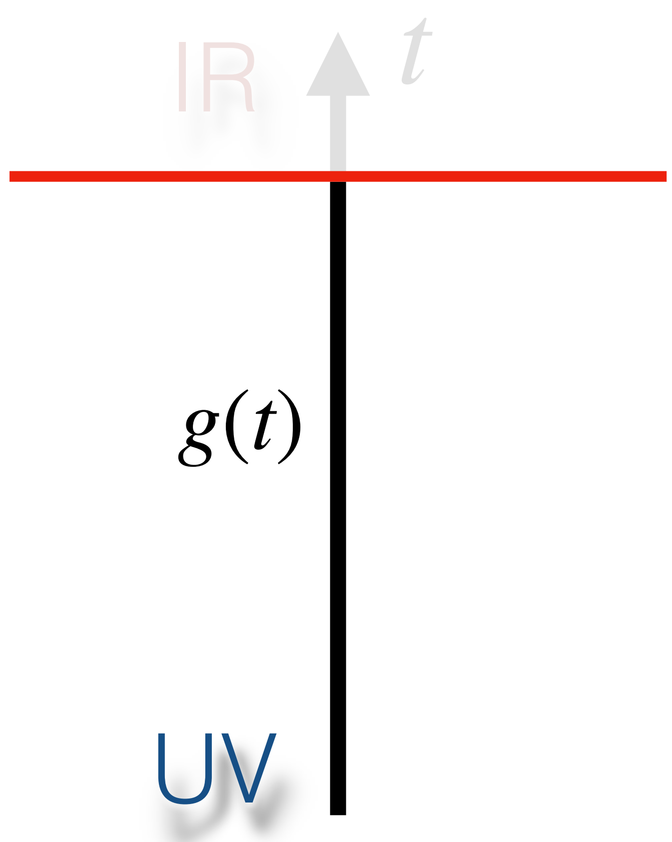

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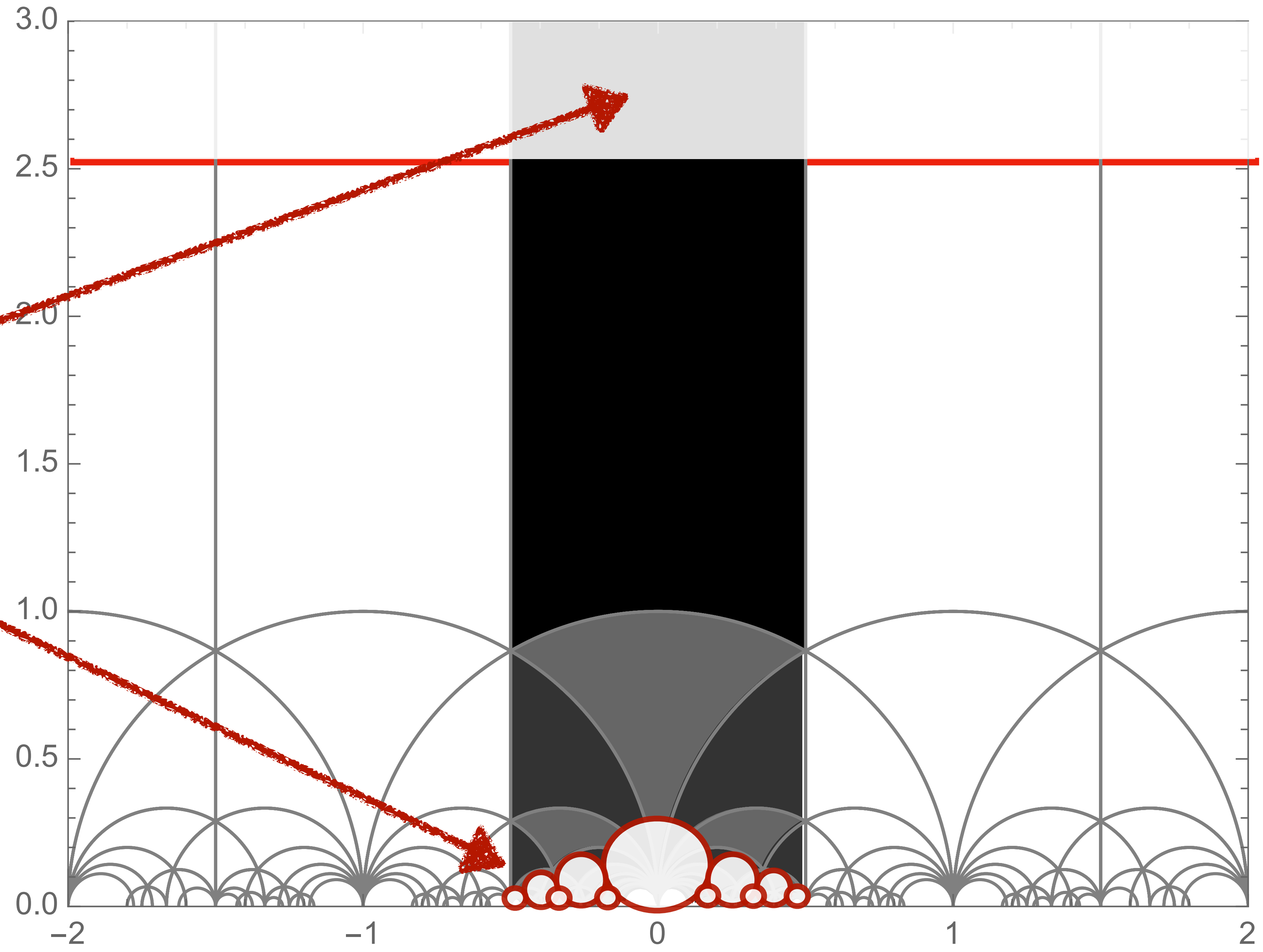
$$\tau_* \equiv M_s^2/\mu^2$$

$\mathscr{G}$ **suppresses integral in a modular invariant way around all the cusps**



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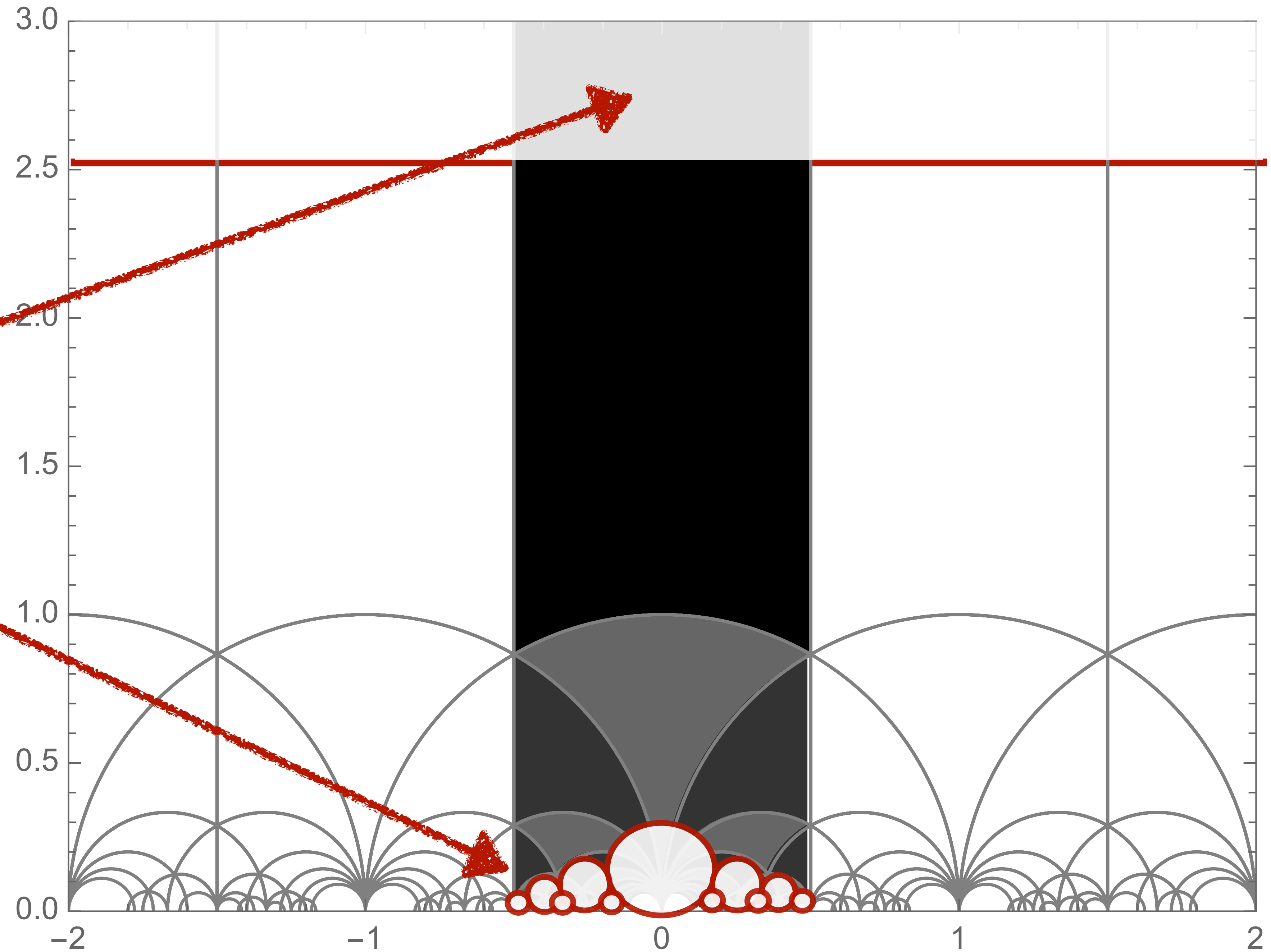

NOTE: Consistency requires $\mathscr{G}(\mu,\tau)$ invariant under

$$\mu \rightarrow M_s^2/\mu$$

The result is a smooth modular invariant stringy Coleman-Weinberg potential

Complicated infinite sum of Bessel functions, but it has the following magical behaviour …

$$\widehat{m}_\phi^2 = \frac{\xi}{4\pi^2}\frac{\widehat{\Lambda}(\mu)}{\mathcal{M}^2} + \partial_\phi^2 \widehat{\Lambda}(\mu)$$

$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2 - c'\,\underset{M\gtrsim\mu}{\mathrm{Str}}\,M^2\mu^2 \quad - \underset{0\leq M\lesssim\mu}{\mathrm{Str}}\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right) + c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$.

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Fully UV complete one-loop effective potential for any modular invariant theory

Below the mass of all states (that couple to the Higgs) there is no further running

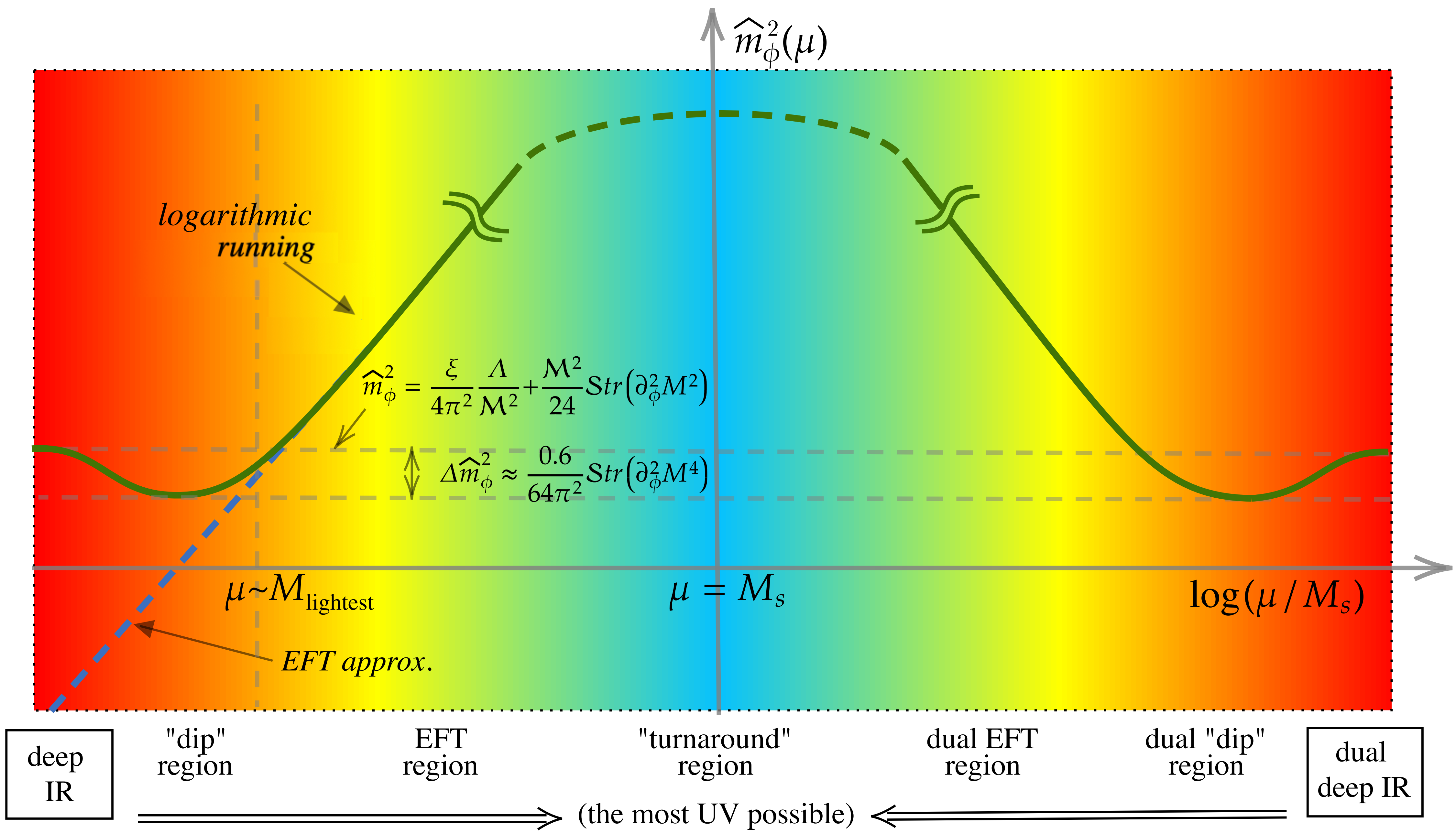
Parameter c, c', c'' depends on the choice of $\mathscr{G}(\mu,\tau)$ $\equiv$ different RG schemes.

At some intermediate energy scale the result is a sum over all states *as **if they had all logarithmically run up from their mass**.*

It is by construction symmetric around the string scale.

Implications for Naturalness?

The Higgs mass begins at a UV value we can calculate, has RG running, maybe GUT breaking, EW and QCD phase transition, yada yada yada. But then it must eventually wind up at the exact same value in the IR. And everything is finite. Like this…



$$\lim_{\mu\to 0} \hat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\partial_\phi^2 M^2$$

Similar behaviour for all couplings in fact regardless of extra dimensions (no power law running!)

Using such a regulator cut-off function with a 2-torus volume factor we can compare $\Delta_G(\mu)$ with the famous result of Dixon, Kaplunovsky and Louis, but recovering energy dependence and the EFT …

SAA, Dienes, Nutricati

$$\Delta_G = \frac{-1}{1+a^2\rho}\Bigg\{\log(cT_2U_2|\eta(T)\eta(U)|^4) + 2\log\sqrt{\rho}a + \frac{8}{\rho-1}\sum_{\gamma,\gamma'\in\Gamma_\infty\backslash\Gamma}\Bigg[\tilde{\mathcal{K}}_0^{(0,1)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right) - \frac{1}{\rho}\tilde{\mathcal{K}}_1^{(1,2)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right)\Bigg]\Bigg\},$$

where

$$\tilde{\mathcal{K}}_\nu^{(n,p)}(z,\rho) = \sum_{k,r=1}^{\infty}(krz)^n\Big(K_\nu(krz/\rho) - \rho^p K_\nu(krz)\Big)$$

$$c(\rho) \equiv 16\pi^2\rho^{-\frac{\rho+1}{\rho-1}}e^{-2(\gamma+1)}$$

$$\mu = \sqrt{\rho}aM_s$$

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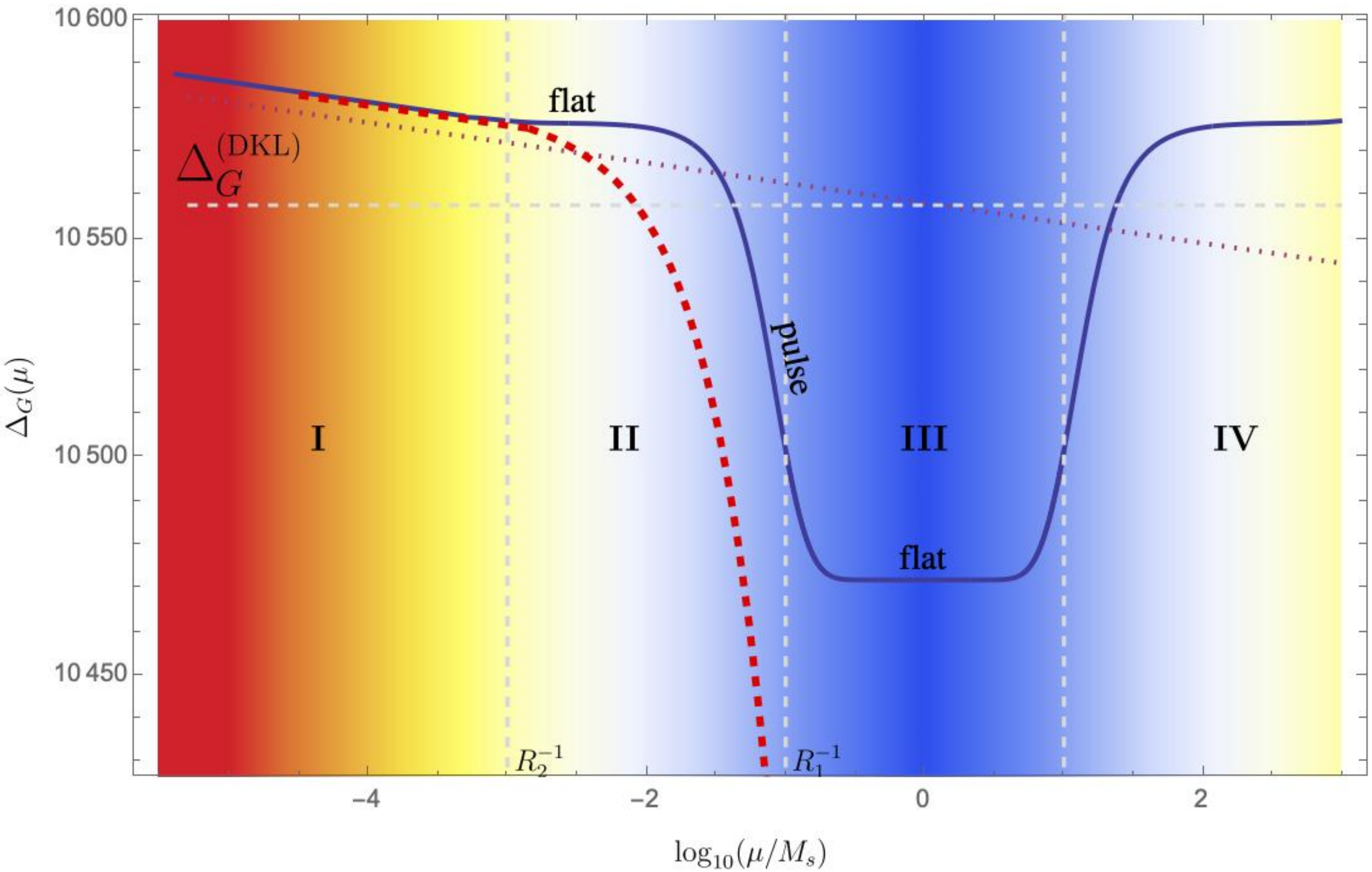
Rectangular torus:

$T_2 = R_2 R_1 = 10000$

$U_2 = R_2/R_1 = 100$



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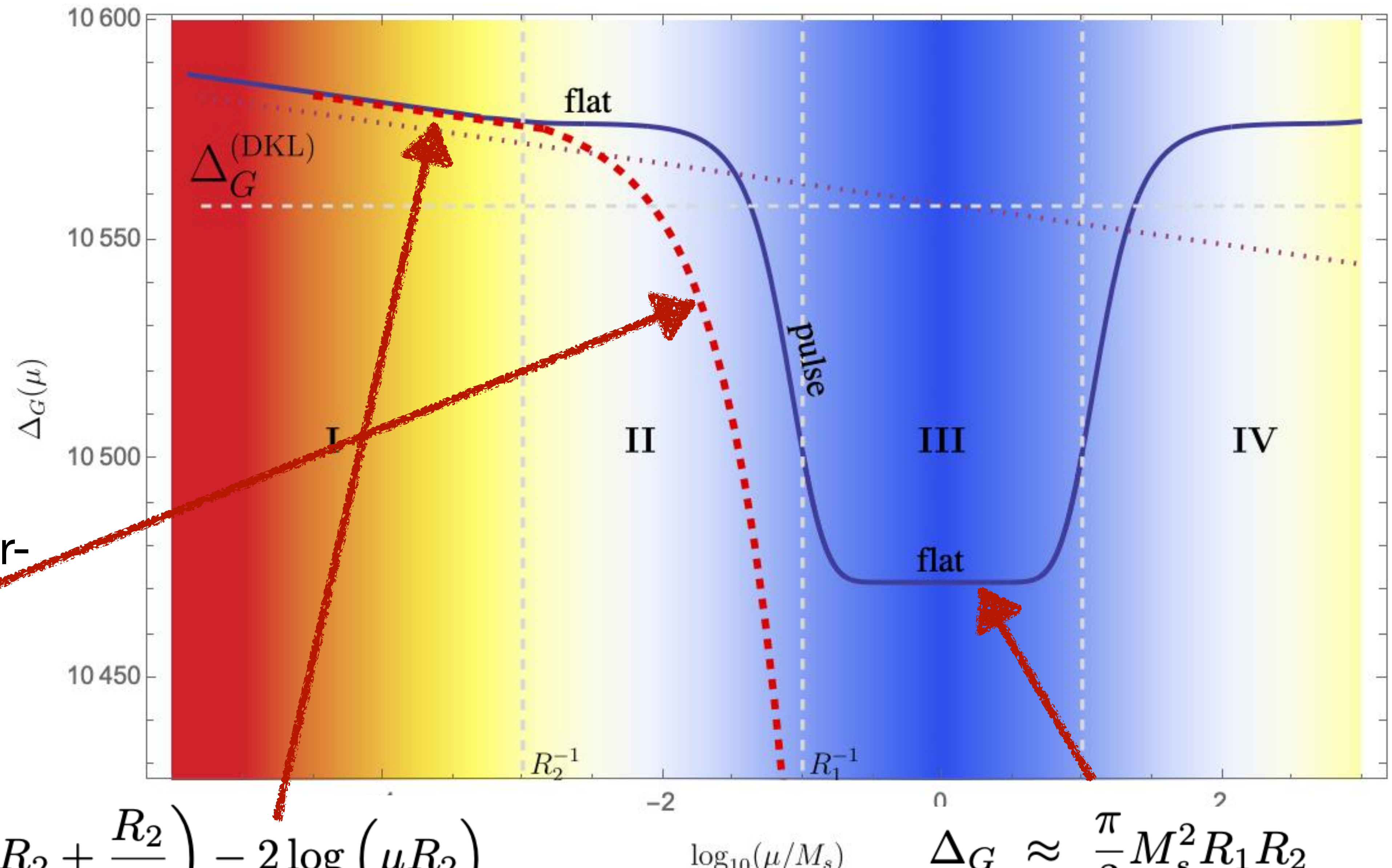


Traditional power-law running

$$\Delta_G \approx \frac{\pi}{3}\left(M_s^2 R_1 R_2 + \frac{R_2}{R_1}\right) - 2\log\left(\mu R_2\right)$$

$$\Delta_G \approx \frac{\pi}{3} M_s^2 R_1 R_2$$

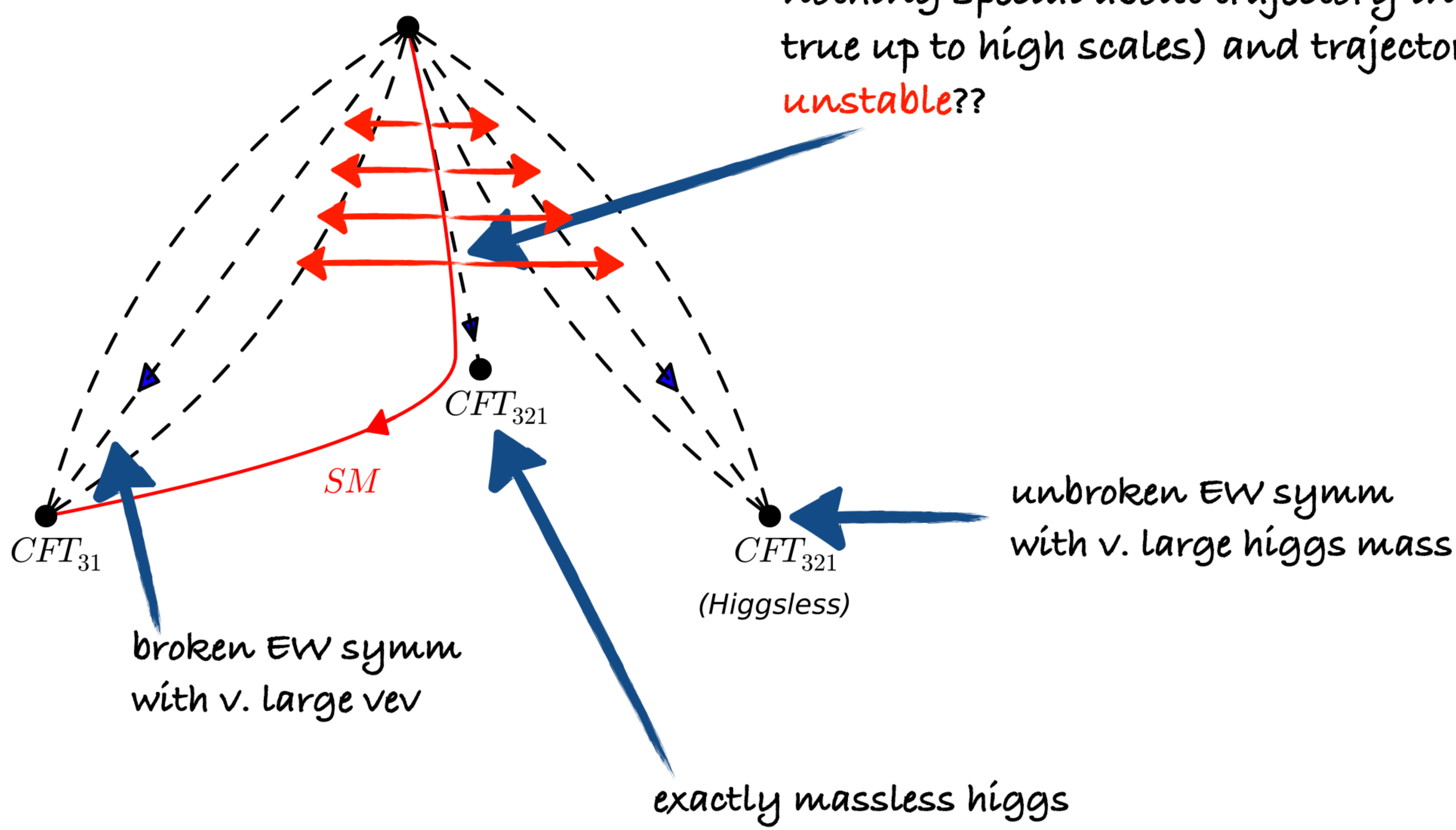
Comments

- The maximum energy scale we can describe in a fully modular invariant way is $\mu \sim M_s$
- Below this scale different RG-definitions agree - while above M_s (or even $1/R$) it is unclear if a universal concept of an "RG scale" can make sense
- A crucial aspect of all this is that the heavily UV/IR mixed theory admits a consistent insertion of an energy scale (one which is invariant under the UV/IR mixing)
- Thus the definition of energy-scale inherits the same symmetries as the UV/IR mixing
- These statements are *probably* independent of it being string theoretic and *probably* apply to any UV/IR mixed theory (e.g. scale invariance at the fundamental scale is probably unavoidable, e.g. in non-commutative theories)

Comments

Our picture solves the intrinsic hierarchy problem — although not the extrinsic one

Why does trajectory of SM so closely approach zero, -0.0000000000000000000000000001, Higgs mass-squared in IR when there is nothing special about trajectory in UV (if SM true up to high scales) and trajectory is unstable??



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The value of the Higgs mass here is a choice of charges in the unregulated fundamental theory

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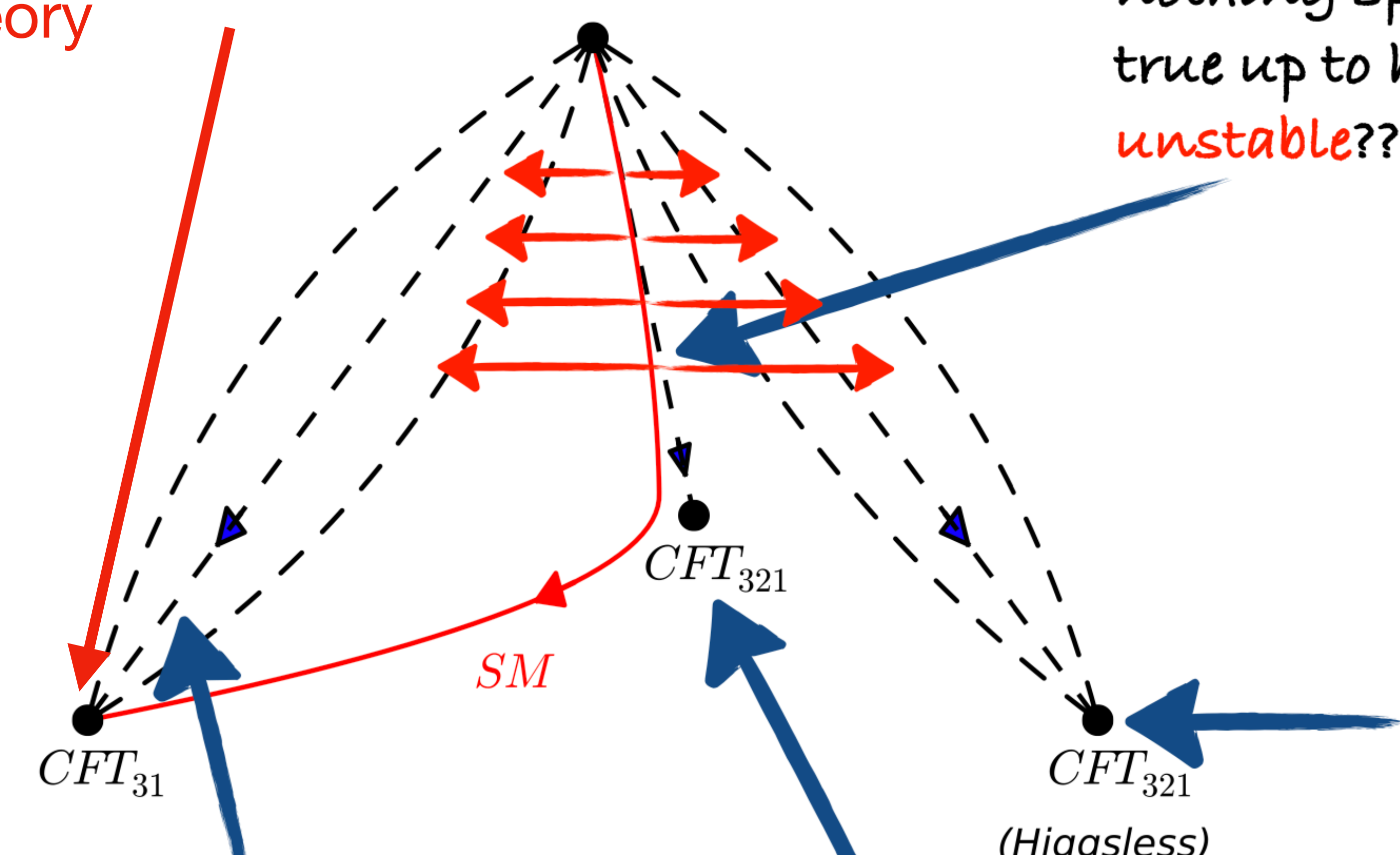

unbroken EW symm with v. large higgs mass

broken EW symm with v. large vev

exactly massless higgs

Comments

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In other words the deep “UV” fixed point is identified with the IR one

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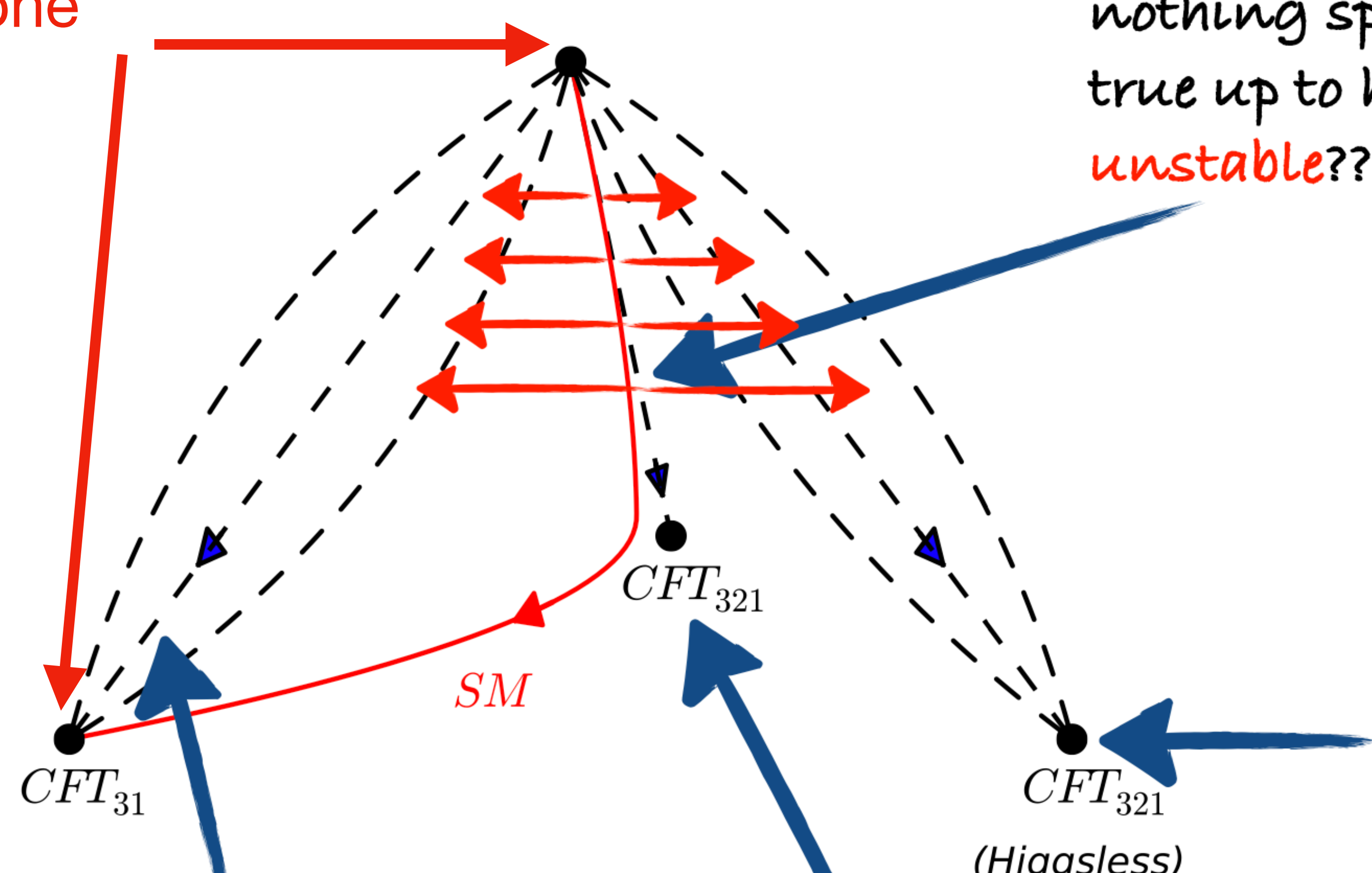

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Summary

- UV/IR mixing (via modular invariance) is the divine intervention required for "Stability against minute variations of the fundamental parameters"
- "Small numbers" in the IR are stabilised because they are *input parameters*.
- There is no "bare UV theory" that can be reached at higher and higher energy. We simply return to the IR theory.
- Stringy "Veltman condition" involving just fundamental charges:

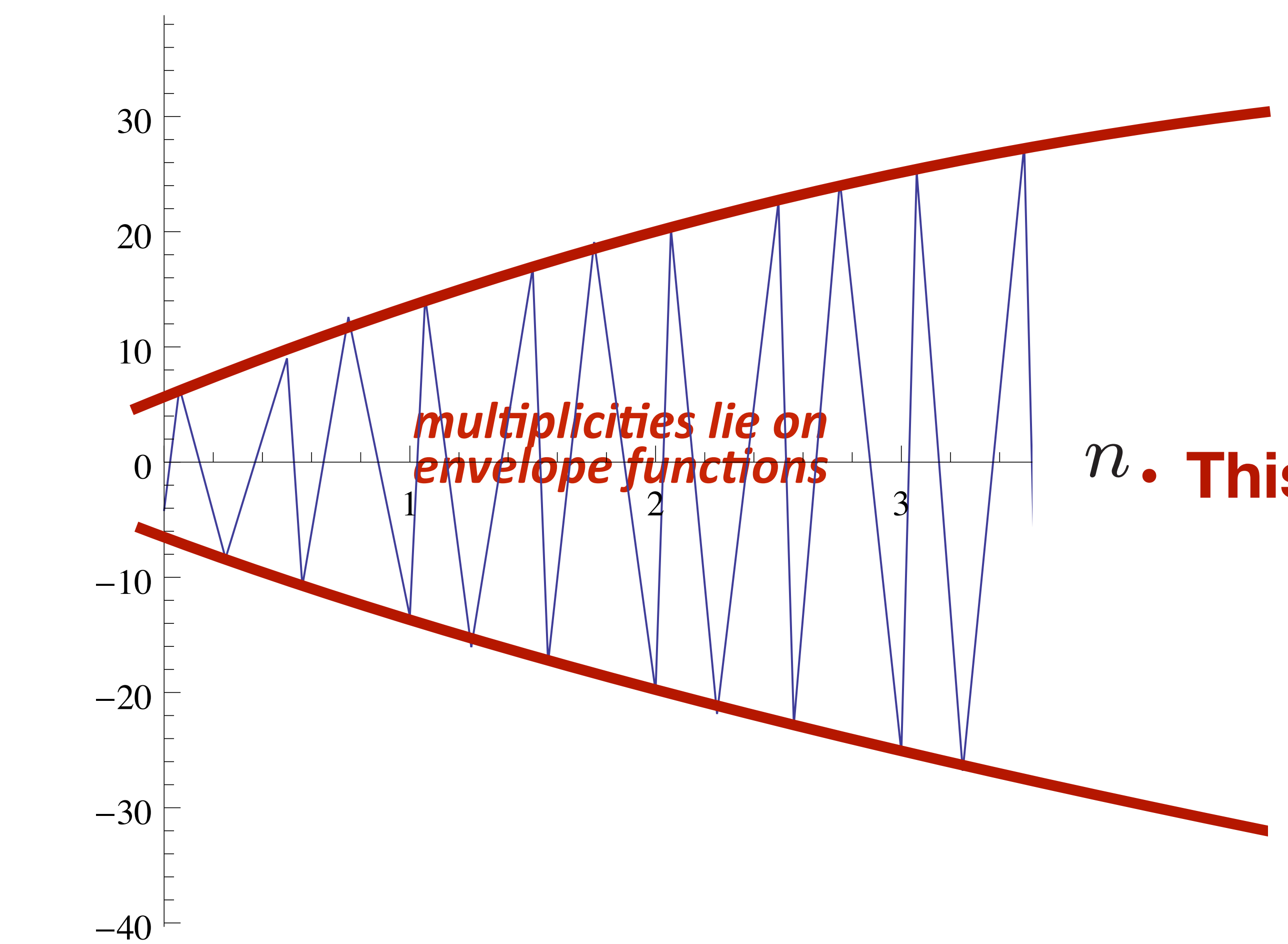
$$\mathrm{Str}\,\partial_\phi^2 M^2 \lesssim \frac{24}{\mathcal{M}^2} M_W^2$$

- "Extrinsic hierarchy" problem solved in the sense that if we find a theory with $\partial_\phi^2 \mathrm{Str} M^2 \approx 0$ (and higher loop equivalents) then the Higgs mass correction is small.
- Can any UV based solution to anything, such as a UV neutrino model, make sense? (Probably not - no extensive hierarchy problem if no UV solution to anything)

Back-ups

Note …

$$\Lambda \quad = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$



- **This crazy spectrum has finite Λ !!!**

NB: this spectrum is SO(16)xSO(16) theory: much more that just “asymptotic SUSY” which is more like

$$\Lambda \sim \frac{Tr(n_B - n_F)}{Tr(n_B + n_F)}$$

Comments

- for example in non-commutative field theory the symmetric limiting energy behaviour is also seen ...

SAA + Jaeckel, Khoze, Ringwald (2006)