Accidentally light scalars

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Motivation: Light scalar fields

Scalars are heavy.*

* Conditions apply.



The natural mass scale of an elementary scalar field is the mass of the heaviest states it couples to (directly or indirectly).

- SM Higgs mass: electroweak hierarchy problem
- Flatness of inflaton potential in small-field inflation

Zoology of light scalar fields

Nambu-Goldstone bosons (NGB) (exactly massless)

SUSY _{moduli} (exactly massless)

pseudo-NGBs

pseudo-moduli

accidentally massless scalars

Pseudo-Nambu-Goldstone bosons

= Goldstone bosons, were it not for interactions with higher-spin fields

Consider a model of elementary scalar fields and (symmetry of the renormalizable scalar potential) > (symmetry of the model)

Example: scalar field $\phi = \mathbf{3}_1$ of gauged SU(2) × U(1)

$$V = \frac{\lambda}{4} \left(|\phi|^2 - v^2 \right)^2$$

= the most general renormalizable V; has a "custodial" SO(6) symmetry

2 tree-level massless scalars (3 Goldstones eaten by gauge bosons): pseudo-NGBs of spontaneous $SO(6) \rightarrow SO(5)$ breaking \rightarrow Weinberg '73

Loop corrections will give them masses $\sim \frac{g}{4\pi}v$.



Pseudo-moduli

Appear in models of spontaneous supersymmetry breaking

A model with 3 complex scalars φ , Φ_1 , Φ_2 :

$$V = \left|h \, arphi \Phi_1 + m \Phi_2
ight|^2 + m^2 \, \left|\Phi_1
ight|^2 + rac{h^2}{4} \left|\Phi_1^2 - f^2
ight|^2 \,, \quad m^2 > h^2 f^2/2$$

Minimized at $\langle \Phi_i \rangle = 0$, $\langle \varphi \rangle =$ undetermined (complex flat direction).

This potential looks non-generic at first sight.

In fact it's the most general V allowed by SUSY and $\mathrm{U}(1)_R\times\mathbb{Z}_2$ (O'Raifeartaigh model)

2 tree-level massless modes in φ :

- one NGB of spontaneous U(1) breaking = R-axion
- one pseudomodulus which obtains a one-loop mass $\sim \frac{h^2}{4\pi} \frac{f^2}{m}$.

Accidentally light scalars

= none of the above

Accidentally light scalars are neither pseudo-NGBs (no enhanced symmetry of V) nor pseudomoduli (no SUSY). Still they share some of their features:

- The most general renormalizable *V*, after spontaneous symmetry breaking, has more tree-level flat direction than predicted by Goldstone's theorem
- The extra flat directions are lifted by radiative corrections.
- There is a little hierarchy between tree-level massive and tree-level massless states.

Prehistoric examples exist \rightarrow Bars&Lane '73, Georgi&Pais '75, ... but are complicated.

We have constructed simple examples and used them for a model of natural hybrid inflation.

(Little) Hierarchy problem

A model with a light scalar weakly coupled to new physics:



$$\delta m_{\phi}^2 \sim rac{g^2}{16\pi^2} M_{
m NP}^2$$

Hierarchy unexplained. But 1-loop little hierarchy radiatively stable.

$\phi = \mathsf{NGB}$

A model with a Nambu-Goldstone boson (only derivative couplings):

 $M_{\rm NP}$

$$m_{\phi} = 0$$

 $\delta m_{\phi}^2 = 0$ at all orders.

"Infinite" mass hierarchy natural, but useless for model-building.

$\phi = \mathsf{pNGB}$

A model with a pseudo-Nambu-Goldstone boson:

 $M_{\rm NP}$

 m_{ϕ}

 $\delta m_{\phi}^2 \sim rac{g^2}{16\pi^2} M_{
m NP}^2$

g weakly breaks the symmetry. 1-loop little hierarchy predicted.

Little Higgs



 $\delta m_\phi^2 \sim rac{g^2}{16\pi^2} rac{h^2}{16\pi^2} M_{
m NP}^2$

g and h collectively break the symmetry. 2-loop little hierarchy predicted.

Accidents

A model with an accidentally light scalar:

 $M_{\rm NP}$



μ

$$\delta m_{\phi}^2 \sim \frac{g^2}{16\pi^2} \, \delta \mu^2 \sim \frac{g^2}{16\pi^2} \frac{h^2}{16\pi^2} M_{\rm NP}^2$$

Doesn't explain $\mu \ll M_{\rm NP}$. But 2-loop hierarchy radiatively stable.

This talk

- Accidentally light scalars can be found in simple models
 e.g. G = SU(N) × U(1) (1 multiplet) or G = SO(3) × Z₂ (2 multiplets)
- Key ingredient: (moderately) large representations, e.g. 5 of SO(3)
- Vacuum manifolds have an interesting structure
- An Accident field can be used as the inflaton to marry natural inflation with hybrid inflation
 - natural inflation = inflaton potential flat at LO
 - hybrid inflation = inflation ends with the fast-roll of a second scalar field
- A nice-to-have example would of course be electroweak symmetry breaking with an accidentally light Higgs. But so far we only have an abelian Higgs ©
- Other applications ?

Accidentally light scalar fields

A simple Accident® model

 $G = SU(2) \times U(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

Write V in terms of bilinears:

$$V = -\mu^2 S + \frac{1}{2} \left(\lambda S^2 + \kappa (S^2 - |S'|^2) + \delta A^a A^a \right)$$

where

•
$$S = \phi^{\dagger} \phi$$

•
$$S' = \phi^{\mathrm{T}} \phi$$

• $S' = \phi^1 \phi$ • $A^a = \phi^{\dagger} T^a \phi$ $(a = 1, 2, 3), T^a = SU(2)$ 5-plet generators

- This is the most general renormalizable G-invariant potential
- For positive quartics it is easily minimized:

$$\langle \phi \rangle = c \, \hat{\varphi}$$

with $\hat{\varphi}$ = any real 5-component unit vector, $c \in \mathbb{C}$, $|c|^2 = \mu^2 / \lambda$.

Counting flat directions

 $G = \mathrm{SU}(2) imes \mathrm{U}(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

$$\langle \phi_j \rangle = v_j e^{i\theta}$$
, $v_j \in \mathbb{R}$ $(j = 1...5)$, $v^2 = \mu^2 / \lambda$

- 5 flat directions: rotate v, shift θ
- 4 of these are Goldstone modes (eaten if G is gauged) since G fully broken for generic v: 3 + 1 broken generators
- One flat direction goes unexplained.
 This is not a pNGB since V has no enhanced symmetry > G.
- This is what we call an Accident.



What happened?

The most general $SU(2) \times U(1)$ -invariant quartic potential does not give a mass to all states that should get one.

Violation of naive expectation from symmetry selection rules.

Intuitively: many d.o.f. (10 real scalars) but "not enough invariants for all of them"



Tree-level vacuum manifold

Assume G is gauged.

Then there is a tree-level flat direction: a one-parameter family of physically inequivalent vacua parameterized by $\alpha \in [0, \pi/6]$.



At $\alpha = 0$, ν is a null eigenvector of $T^3 \Rightarrow U(1)' \subset SU(2)$ unbroken

Tree-level mass spectrum



Sum rules: $\sum m_{\text{scalar}}^2$ = constant,

$$\sum m_{\rm vector}^2$$
 = constant

One-loop lifting of the flat direction

The Coleman-Weinberg effective potential can be used to compute one-loop corrections to the Accident mass.

$$\Delta V_{\rm CW} = rac{1}{64\pi^2} \, {
m Str} \left({\cal M}^4 \log rac{{\cal M}^2}{\Lambda^2}
ight)$$

- Subtract one-loop tadpole for radial mode; fixes Λ
- Resulting correction to Accident mass is finite
- The symmetry-enhanced point $\alpha = 0$ becomes a minimum

$$\begin{split} m_{\rm acc}^2 &= \frac{1}{4\pi^2} \left[3 g_2^2 m_{W+}^2 + \delta m_+^2 f\left(\frac{m_0^2}{m_+^2}\right) \right] \bigg|_{\alpha=0},\\ f(x) &= 1 - x + x \log x \quad (\geq 0) \,. \end{split}$$

The opposite point $\alpha = \pi/6$ becomes a saddle point

One-loop lifting of the flat direction



One-loop effective potential dominated by first Fourier mode,

$$V_{\rm CW}(\alpha) \approx V_0 - V_6 \cos(6\alpha)$$

Application I: Dark Matter

If this is a dark sector:

- the Accident is a natural DM candidate: lightest charged state under U(1)'
- interactions with SM thermal bath through Higgs portal
- coupling to U(1)' dark photon \Rightarrow interesting thermal history possible (reannihilation/dark-sector freeze-out...)
 - ightarrow Chu/Hambye/Tytgat '12, Hambye et al. '19, Bharucha/FB/Desai/Mutzel '22,

Frigerio/Grinbaum-Yamamoto/Hambye '22...



Application II: The Accident as an abelian Higgs field Add to the model a Dirac fermion Ψ (= mock top quark) in the $3_{\frac{1}{2}}$:

$$\mathcal{L} \supset -M\overline{\Psi}\Psi - rac{1}{2}y\,\overline{\Psi}\phi\Psi^c + ext{ h.c.}$$

The Ψ contribution to the one-loop Accident mass is negative for $M \gtrsim y |v|$, and so tends to destabilize the U(1)'-preserving point at $\alpha = 0$:



Spontaneous U(1)' breaking by a light abelian Higgs field (= the Accident) But: small ν'/ν needs fine-tuning \rightarrow Maya Hager's, Matthew McCullough's talks It would be very interesting to find an accident model where U(1)' becomes $SU(2)_L \times U(1)_Y$ or $SO(4)_{cust} \rightarrow$ composite Higgs, little Higgs models

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Accidents and inflation

Summary of $G = SU(2) \times U(1)$ model

With ϕ = complex SU(2) 5-plet:

- 10 real scalars
- 5 are massive at the tree level
- away from α = 0:
 4 are NGBs of SU(2) × U(1) → nothing (eaten if *G* is gauged) and one is an Accident
- at $\alpha = 0$: 3 are NGBs of SU(2) × U(1) → U(1)' (eaten if *G* is gauged) and two form a light complex scalar with U(1)' charge 2
- At one loop, the enhanced-symmetry point $\alpha = 0$ becomes a minimum. EFT around this point = U(1)' gauge theory + light charged scalar
- Can add a fermion to destabilize the enhanced-symmetry point ⇒ break residual U(1)' at scale v' ≪ v (at the cost of some tuning).
 EFT = abelian Higgs model.

More Accidents

Another Accident model has a single scalar $\phi = 10_1$ of SU(3) × U(1):

• 2-dimensional tree-level vacuum manifold, 2 Accidents



- At a generic point, $SU(3) \times U(1) \rightarrow$ nothing
- At a special point,
 - $SU(3) \times U(1) \rightarrow U(1)^2$
 - this is the minimum of the one-loop effective potential
 - now 6 light scalars
- Application: e.g. dark matter (see also \rightarrow Frigerio/Grinbaum-Yamamoto/Hambye '22)
- Can generalize to $\phi = \Box \Box \Box$ of SU(N):
 - $\bullet\,$ many more Accidents, and U(1)s at the symmetry-enhanced point. . .

Are there models with non-abelian residual symmetries? \rightarrow EWSB

Inflation with Accidents

Inflation

Inflation = a period of accelerated expansion of the early universe Driven by the potential energy of a slowly-rolling scalar field (inflaton)



"Slow-roll":

$$\epsilon = \frac{1}{2}M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1 \qquad |\eta| = M_P^2 \left|\frac{V''}{V}\right|^2 \ll 1$$

Inflaton potential should be near-flat (or field excursions transplanckian)

Not stable under radiative corrections \Rightarrow naturalness problem for "small-field models" with $\phi \ll M_P$. Often SUSY is invoked to preserve flatness. Can we do without SUSY?

Another model: \rightarrow Wanda Isnard's talk

Swampland disclaimer

Slow-roll inflation is in tension with certain swampland conjectures.



For the purposes of this talk, I Conjecture that

"This model is not in the swampland."

(the Unrefined This-Model-Is-Not-In-The-Swampland Conjecture).

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Accidents and inflation

Inflation: CMB observables

During inflation, the universe grows by e^{N} e-folds

$$N = \int_{t_{\text{start}}}^{t_{\text{end}}} H(t) \, \mathrm{d}t \,, \qquad H^2 = \frac{V}{3M_P^2}$$

About 60 e-folds before inflation ends, CMB observables are produced, which allow to constrain the shape of the potential at this scale (*)

- Amplitude of the scalar power spectrum $A_s = H_*^2/8\pi^2 M_P^2 \epsilon_*$
- Spectral index $n_s = 1 + 2\eta_* 6\epsilon_*$
- "running" = scale derivative of n_s , also "running of running" etc.
- Tensor-to-scalar ratio $r = 16\epsilon_*$

PLANCK and other experiments have measured A_s and n_s , constrained $r \leq 4 \times 10^{-2}$, |running| \leq few per mille

An inflating Accident model

 $G = \mathrm{SO}(3)$ gauge symmetry, with real scalars $\phi = \mathbf{5}$ and $\chi = \mathbf{3}$

• Most general renormalizable $G \times \mathbb{Z}_2$ -invariant potential

$$V = -\frac{1}{2}\mu_{\phi}^{2}\phi^{2} - \frac{1}{2}\mu_{\chi}^{2}\chi^{2} + \frac{\lambda_{\phi}}{4}(\phi^{2})^{2} + \frac{\lambda_{\chi}}{4}(\chi^{2})^{2} + \frac{\epsilon}{4}\phi^{2}\chi^{2} + \frac{\zeta}{4}T_{AC}^{a}T_{CB}^{b}\phi_{A}\phi_{B}\chi^{a}\chi^{b}$$

 $A, B = 1 \dots 5; a, b = 1 \dots 3; T_{AB}^a$ = generators of 5 representation.

No continuous symmetry larger than G

• For $\mu_{\phi}^2 > 0$, $\mu_{\chi}^2 < 0$ and quartics > 0, vacuum is at $\langle \chi \rangle = 0$ and $\langle \phi \rangle \neq 0$

4 flat directions = 3 Goldstones from SO(3) breaking (eaten as G gauged)
 + 1 accidentally flat direction a

One-loop effective potential

The tree-level flat direction a is lifted by scalar and vector boson loops



Here m^4 is calculable and finite.

This looks like Natural Inflation: → Freese/Frieman/Olinto '90

- inflaton = pNGB of approximate global symmetry
- $m \ll \Lambda$ is natural (if symmetry is good)
- Inflaton potential ~ cosine
- $\gtrsim 60$ e-folds require $f \sim M_P$ 😕
- Planck (constraints on r and n_s) rules out NI completely Θ



Hybrid inflation

But our model includes a second scalar field χ : can promote it to waterfall field of hybrid inflation \rightarrow Linde '93

- Single-field inflation ends as the inflaton starts fast-rolling
- Hybrid inflation ends as a second field starts fast-rolling: waterfall transition



Hybrid inflation with Accidents



Hybrid inflation with Accidents

In terms of accident-inflaton *a* and waterfall field χ_3 , now for $\mu_{\chi}^2 > 0$:

$$V_{\text{infl}} = V_0 - m^4 \cos\frac{a}{f} + \frac{1}{2} \left(-\mu_{\chi}^2 + 36\zeta f^2 \sin^2\frac{a}{6f} \right) \chi_3^2 + \frac{\lambda_{\chi}}{4} \chi_3^4$$

Several nice properties:

- Hybrid inflation \Rightarrow can take $f < M_P$. No transplanckian field excursions.
- No continuous global symmetries whose fate at the Planck scale is unclear
- Previous models of Natural Hybrid Inflation (→ Kaplan/Weiner '03, German/Ross '00s...) need tuning parameters or engineering non-abelian discrete symmetries to couple waterfall field to inflaton without spoiling flatness. Hybrid inflation with accidents is much more elegant.
- Or both the inflationary and the waterfall direction are pNGB directions (→ Randall/Sojacic/Guth 95,..., Cacciapaglia/Cheong/Deandrea/Isnard/Park '23)
- V is sensitive to Planck-scale quantum corrections only at two loops

Testing the model: CMB



A prediction on CMB parameters: $n_r > 0$, $n_{rr} > 0$.

Present constraints: $n_r = 10^{-3} \pm 10^{-2}$, $n_{rr} = 10^{-2} \pm 10^{-2}$. May be testable soon.

Testing the model: Gravitational waves What happens after inflation?

"Tachyonic preheating" \rightarrow Felder et al. '00s, waterfall instability excites inhomogeneities, these scatter off each other until the universe thermalizes and the inflationary energy is transferred to radiation

This is a violent process producing a stochastic gravitational wave background which could be observable today \rightarrow Garcia-Bellido et al, Dufaux et al '07

Frequencies too large to be measured unless inflationary energy scale $V_0^{1/4}$ is $\ll M_P$ (unnatural). Projected sensitivity of Einstein Telescope:



Testing variations of the model: Topological defects

Simple variations of the model can give rise to cosmic strings or domain walls. Their collisions will again source gravitational waves which may be detectable with pulsar timing arrays. A stochastic GW background has recently been discovered \rightarrow NanoGrav collaboration '23 (but it's unclear if this is primordial or astrophysics)

The model: an SO(3) five-plet and two SO(3) triplets, constrained by a suitable $\mathbb{Z}_2 \times \mathbb{Z}_4$, with \mathbb{Z}_4 softly broken.

- The five-plet contains an accidentally near-flat inflationary direction
- Two distinct vacua, degenerate in the limit of exact $\mathbb{Z}_4 \Rightarrow$ domain walls
- Degeneracy is lifted by soft Z₄ breaking ⇒ domain walls unstable. (Necessary, since otherwise they will overclose the universe.)
- Before they decay, domain walls can wiggle and collide, producing a stochastic GW background.

Testing variations of the model: GWs from DWs



Benchmark points show that present and future experiments are sensitive to different parts of parameter space.

•
$$\sigma^{1/3} = 3.2 \times 10^5 \text{ GeV}$$

• $\sigma^{1/3} = 3.2 \times 10^8 \text{ GeV}$

• $\sigma^{1/3} = 1.2 \times 10^{11} \text{ GeV}$

where $\sigma^{1/3} \propto V_0^{1/4}$ (domain wall tension). Gray lines = observed stochastic GW background. $f_{\rm peak} \propto \sqrt{\Delta V/\sigma}$ while $h^2 \Omega_{\rm GW} \propto \sigma^4/\Delta V^2$

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Conclusions

Conclusions

- Accidents
 - = accidentally light scalars
 - = light scalar fields whose mass suppression does not follow (obviously?) from selection rules/NDA
- They appear in models with larg(ish) representations because of the restrictive structure of the scalar potential
- We are still missing pieces of the puzzle:
 - What are the conditions to find Accidents in some given model?
 - Is there a structural reason why the residual continuous symmetry always seems to be abelian?
- Related: application to EWSB?
- The inflaton can be an Accident: elegant realization of small-field hybrid natural inflation