# HIERARCHY PROBLEM REDUX

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May 2025

*"Why is the Higgs Boson/weak scale so much smaller than the Planck scale?"*

"It just is. Now off you go. Chop, chop!"

Lenny Susskind gave us something more specific to worry about in 1979:

*"The need for fundamental scalar fields in the theory of the weak and electromagnetic forces is a serious flaw. Aside from the subjective esthetic argument, there exists a real difficulty connected with the quadratic mass divergences which always accompany scalar fields. These divergences violate a concept of naturalness which requires the observable properties of a theory to be stable against minute variations of the fundamental parameters."*

L. Susskind. "Dynamics of SSB in the Weinberg-Salam Model"
Physical Review D20, 20 (1979).
Received July 1978, published Nov 1979 [credited Wilson for explaining it to him]

Susskind, 1979:

To illustrate a case of an unnatural adjustment, consider a particle which receives a self-energy which is quadratic in $\kappa$. To make the discussion simple, suppose the form of the mass correction is

$$m^2 = m_0^{\ 2} + \Delta m^2$$
$$= m_0^{\ 2} + \kappa^2 g_0^{\ 2} . \tag{2}$$

Solving for $\mu_0^{\ 2}$ gives

$$\mu_0^{\ 2} = \frac{m_0^{\ 2}}{\kappa^2} = \frac{m^2}{\kappa^2} - g_0^{\ 2} . \tag{3}$$

Now if $m$ is a physical mass of order 1 GeV and $\kappa \sim 10^{19}$ GeV, then

$$\mu_0^{\ 2} = -g_0^{\ 2}(1 - 10^{-38}) . \tag{4}$$

Equation (4) means that $\mu_0^{\ 2}$ must be adjusted to the 38th decimal place. What happens if it is not? Then the mass will come out to be of order $10^{19}$ GeV.

Such adjustments are unnatural and will be assumed absent in the correct theory. Unfortunately, Dimopoulos, Susskind, 1979:

*"Indeed, it seems that to establish a hierarchy of mass scales, beginning at the Planck mass ($10^{18}$ GeV) and ending at ordinary particle masses requires fundamental unrenormalized masses to be adjusted to 30 decimal places! Perhaps in some future theory such adjustments will appear natural, but at present divine intervention is the only available explanation."*

S. Dimopoulos, L. Susskind. "Mass without Scalars."
Nucl. Phys. B155, 237 (1979).
Received February 1979

This strong sentiment against fundamental scalars entered key textbooks:

> One more aspect of $\phi^4$ theory deserves comment. Since the mass term, $m^2\phi^2$, is a relevant operator, its coefficient diverges rapidly under the renormalization group flow. We have seen above that, in order to end up at the desired value of $m^2$ at low momentum, we must imagine that the value of $m^2$ in the original Lagrangian has been adjusted very delicately. This adjustment has a natural interpretation in a magnetic system as the need to sensitively adjust the temperature to be very close to the critical point. However, it seems quite artificial when applied to the quantum field theory of elementary particles, which purports to be a fundamental theory of Nature. This problem appears only for scalar fields, since for fermions the renormalization of the mass is proportional to the bare mass rather than being an arbitrary additive constant. Perhaps this is the reason why there seem to be no elementary scalar fields in Nature. We will return to this question in the Epilogue.

Peskin, Schroeder, 1996

These works are emphasizing something **intrinsic** to scalar theory in the Wilsonian renormalization group evolution picture.

As one integrates out higher momentum shells the scalar receives quadratic divergence contributions due to its self-interactions and its interactions **within the theory**.

$$\sim \Lambda^2$$

Therefore, the mere existence of an interacting fundamental scalar looks lethal to any such theory that has a large hierarchy between the scalar’s mass and the edge of the cutoff of the effective theory.


This, we will call the
*Intrinsic Hierarchy Problem*

The *Intrinsic Hierarchy Problem* takes very seriously Wilsonian picture.

Even taking literally the physical implication of tuning $\Lambda$ against a bare mass within a theory that has scalar masses.

Standard Model with fundamental scalar Higgs boson looks impossible!

Not coincidental that the *apotheosis of Wilsonian EFT thinking led to widespread derision of a naked Higgs boson*.

First reaction: NO SCALARS. (Big effort in technicolor, for example)

No-fundamental-scalar theories ran into much difficulty, theoretically and experimentally.

*Theoretically*: hard to give mass to the fermions
*Experimentally*: FCNC, strong dynamics, Precision EW deviations, etc., not there.

Doubt in the soundness of the suspect physical reasoning behind the *Intrinsic Hierarchy Problem* arose.

For example: "There is no quadratic divergence with dimensional regularization. No big cancellations are seen that can be interpreted as physical. The [Intrinsic] Hierarchy Problem is a fake problem!"

Supersymmetry to the rescue!

Instead of banishing scalars just make them subject to a higher law: supersymmetry!

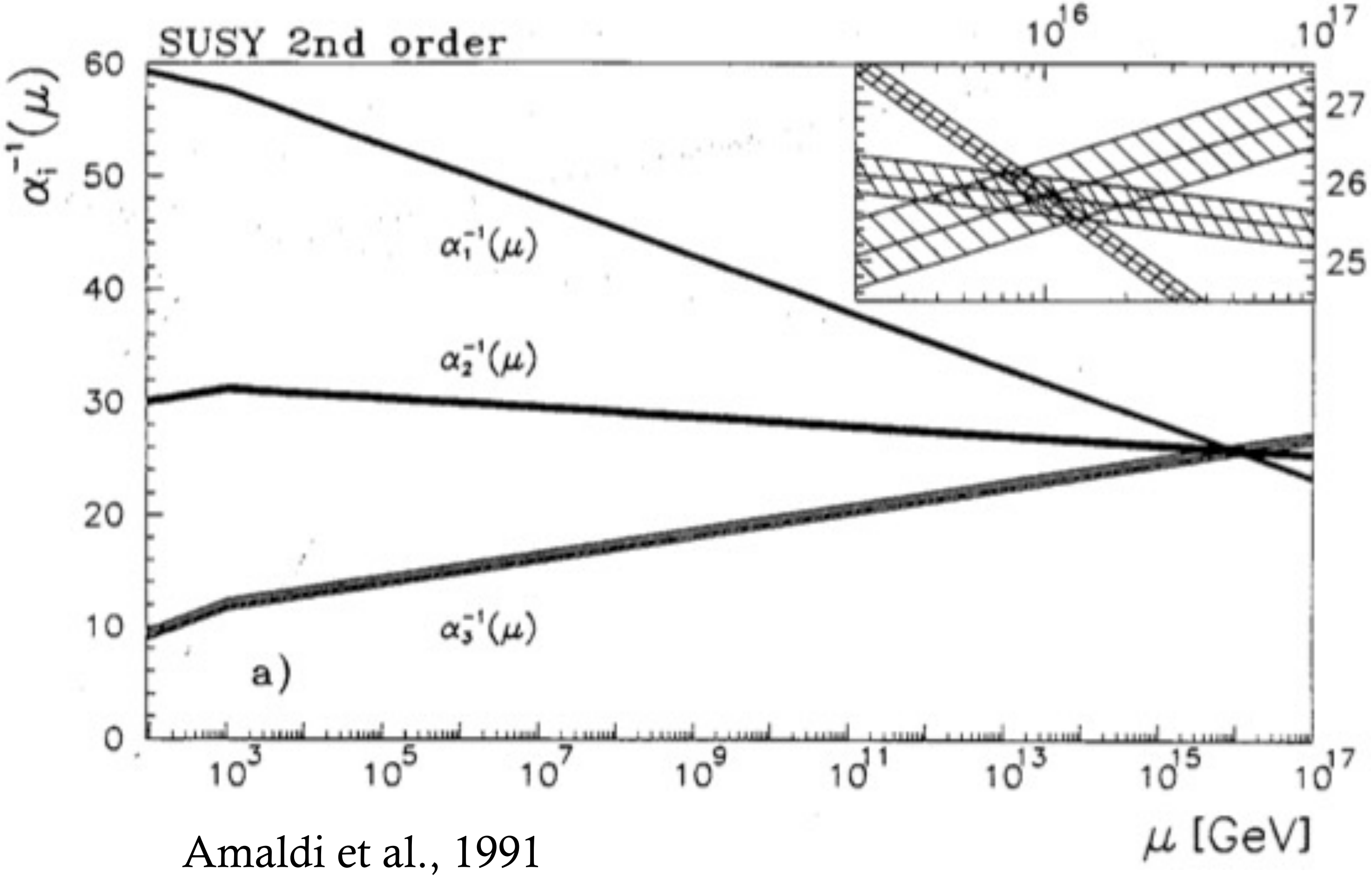
$\Delta m_H^2 \sim y_t^2\,(m_t^2 + \Lambda^2 + m_{susy}^2) - y_t^2\,(m_t^2 + \Lambda^2) \sim y_t^2\, m_{susy}^2$

where $m_{susy}^2$ is the scale of supersymmetry breaking.

For Wilsonian literalists, it was a relief to see the cutoff dependence cancel out.

Maybe the *Intrinsic Hierarchy Problem* is real **and** fundamental scalars can still exist.

Supersymmetry’s appeal grew even more with gauge coupling unification.




Amaldi et al., 1991

## GUTs created new hierarchy challenges, like doublet triplet splitting.

In the supersymmetric case the $\Sigma$ also couples to the 5- and $\bar{5}$-dimensional Higgs representation $H_5$ and $H_{\bar{5}}$ respectively. Within the $H_{5,\bar{5}}$ are the Higgs doublets $H_{u,d}$ and the Higgs triplet $H_{3,\bar{3}}$ representations. The relevant GUT-scale superpotential for $H_5$ is

$$W_{(+)} = \mu_5 H_{\bar{5}} H_5 + \lambda H_{\bar{5}} \Sigma H_5 \tag{29}$$

After symmetry breaking the superpotential splits the $H_{5,\bar{5}}$ into $H + u, d, 3, \bar{3}$ terms:

$$W = \mu_3 H_{\bar{3}} H_3 + \mu H_u H_d \Longrightarrow W_{(-)} = \mu H_u H_d + \cdots \tag{30}$$

where

$$\mu_3 = \mu_5 + 2\lambda v_\Sigma, \quad \text{and} \tag{31}$$
$$\mu = \mu_5 - 3\lambda v_\Sigma. \tag{32}$$

We know that $v_\Sigma \simeq 10^{16}$ GeV for the unification of couples, and we also know that $\mu$ needs to be $10^{2-3}$ GeV for weak scale supersymmetry. Thus, there is an extraordinary finetuning in the cancellation that must occur in eq. 32 to realize these constraints. Upon symmetry breaking and assessing the finetuning of $\mu$ with respect to the high-scale theory parameter $\mu_5$ one finds

$$\mathrm{FT}[\mu] = \left| \frac{\mu_5}{\mu} \frac{\partial \mu}{\partial \mu_5} \right| = \left| \frac{\mu_5}{\mu} \right| \simeq \left| \frac{2\lambda v_\Sigma}{\mu} \right| \sim 10^{13} \tag{33}$$

The Hierarchy problem again is associated with the scalar bosons. Pesky!

This time the problem is not intrinsic to the Standard Model Higgs boson. Rather it was brought on by particles external to it.

Over time it was recognized that many different additions to the SM can cause problems. For example, merely adding a heavy real singlet can be disastrous.

One of the simplest ways to extend the SM is to add a real singlet scalar $\sigma$ to the spectrum. One can call this theory SM+$\sigma$ for short. The lagrangian is

$$\mathcal{L}_{SM+\sigma} = \mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\eta_\sigma}{2} H^\dagger H \sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 \tag{12}$$

Let us suppose that the mass of the $\sigma$-particle is higher than the masses of the other particles in the spectrum, and let's also call the effective theory that includes the $\sigma$ particle $\mathcal{L}_{\sigma+} = \mathcal{L}_{SM+\sigma}$.

Given the high mass of the $\sigma$ particle we can integrate it out and are left with a low energy lagrangian $\mathcal{L}_{\sigma-}$ below the $\sigma$-mass threshold which is the SM lagrangian plus many higher dimensional operators, such as $\mathcal{O}_6 = |H|^6$. After some analysis we can see that no operator in $\mathcal{L}_{\sigma-}$ suffers from a finetuning of matching across the $m_\sigma$ threshold except possibly the coefficient $m^2$ of the operator $|H|^2$. In that case the matching is

$$m^2_{(-)} = m^2_{(+)} - \frac{\eta_\sigma m_\sigma^2}{16\pi^2}\left[1 - \ln\left(\frac{m_\sigma^2}{\mu^2}\right)\right] \tag{13}$$

Extreme finetuning if $m_\sigma >> m_H$

where for clarity we have defined

$$m^2_{(\pm)} = m^2 \text{ evaluated at } q^2 = m_\sigma^2(1 \pm \epsilon), \text{ where } \epsilon \ll 1. \tag{14}$$

In other words $m^2_{(-)}$ is the coefficient of $|H|^2$ in the low-energy effective theory just below the $m_\sigma$ threshold after the $\sigma$-particle has been integrated out, and $m^2_{(+)}$ is the coefficient of $|H|^2$ in the high-energy theory above the $m_\sigma$ threshold that includes the $\sigma$ particle.

One can make a general statement, which we call the

*Extrinsic Hierarchy Problem*:

The Standard Model EFT, because of its fundamental scalar boson, is unstable to generically expected states that should exist between the weak scale and the Planck scale and that should interact with the Higgs boson.

But the *Extrinsic Hierarchy Problem* is also controversial.

*Who says there cannot be small numbers made from big cancellations?* (Even Wilson said that in 2004)

*What is the rigorous basis to say* that a technically correct theory I form to solve a problem (baryogenesis, dark matter, etc.) or take hints seriously (grand unified theory) is **improbable** or **impossible** and should not be pursued any longer**?**

*What exactly are the assumptions and reasonings* that tell us that the Standard Model has a **problem** (the Extrinsic Hierarchy Problem) that requires deliberate consideration to solve?

The *Extrinsic Hierarchy Problem* can be stated in the form of a paradox.

{Premises}+{Reasoning} → {Absurd Conclusion}

where the absurd conclusion is one that violates observation.

Articulating premises and reasoning allows attacking them.

For example, *Zeno's paradoxes*, *Olbers paradox, etc.*

*Premises*: infinite space, uniform population of stars, etc.
*Reasoning*: Inverse square law, etc.
*Conclusion*: The night sky should not be dark!

## Premise 1

P1 **Conventional Ur-Theory.** Nature is well described by an Ur-Theory just below the Planck scale with its Ur-Action and Ur-Langrangian density that is comprised of non-zero coefficients for all symmetry-allowed operators. The Ur-Theory is $3+1$ dimensional with standard spacetime symmetries already recognized (diffeomorphism invariance, Poincaré symmetry,) endowed with internal gauge symmetries characterized by $SU(3) \times SU(2) \times U(1)$ of the SM as well as other possible gauge symmetries and global symmetries applicable to the Ur-Theory's collection of conventional quantum fields and their interactions.

P2 **Aleatory Parameters.** Coefficients of the operators of the Ur-Theory are aleatorily assigned to each of the symmetry-allowed operators. The assigned parameters are given to the theory in the ultraviolet and have no teleological designs on the properties or implications of the theory in the deep infrared.

## Premise 3

P3 **Multitude of States.** Nature has many more scalars, fermions (both chiral and vectorlike), and vector bosons in its spectrum than just those of the Standard Model. In addition, many of these states have masses significantly higher than the weak scale, including masses near the highest mass scale of the Ur-Theory, just below the Planck mass.

R1 **Logical Reasoning.** Correct standard reasoning applied to mathematics, language, logic rules, quantum field theory, etc.

R2 **Premises Acceptance.** Acceptance and implementation of the implications of each premise.

R3 **Naturalness.** No extreme unprincipled coincidence in couplings or finetuning across energy thresholds when passing from the UV to IR scales.

By unprincipled coincidence or finetuning we mean one where there is no argument revealing its necessity. For example, the equivalent electromagnetic coupling of the electron and the muon is not an extreme coincidence since electromagnetic gauge invariance necessitates it (a principled coincidence). To be concrete, we consider an extreme finetuning to be one part in $10^6$ [13].

*The Extrinsic Hierarchy Problem is the unresolved paradox with credible premises* $\{P_1, P_2, P_3\}$ *and sound reasoning* $\{R_1, R_2, R_3\}$ *that leads to the absurd conclusion that the Higgs boson mass should be many orders of magnitude above its measured value.*

The Hierarchy Paradox says that if each of these premises is true then the Higgs boson mass should experience a condition where its mass $m_H^2$ is related to the difference between two (or many more!) very heavy mass thresholds $M_X^2$ and $M_Y^2$ that must cancel:

$$m_H^2 = M_X^2 - M_Y^2. \tag{3}$$

If $M_X^2$ and $M_Y^2$ are aleatory parameters that are very large $M_X^2, M_Y^2 \sim M_{\rm Pl}^2$ it is an extreme unexpected finetuned coincidence ("divine intervention!" as Dimopoulos and Susskind might say) that they should cancel and given $M_X^2 - M_Y^2 \ll M_{\rm Pl}^2$. Thus, the paradox is born that the premises and the reasoning have lead to a conclusion ($m_H \sim M_{\rm Pl}$) that is absurd because it is in conflict with observation ($m_H \simeq 10^2\,{\rm GeV} \ll M_{\rm Pl}$).

## Violating premise 1: Conventional Ur-Theory

Many of our solutions violate this premise.

**Supersymmetry**: Fields live in superspace! Superpoincaré invariance. All quadratic sensitivities are cutoff by susy breaking mass $m_{susy} \sim m_{weak}$.

**Xdim**: Fields/Gravity lives in extra spatial dimensions! The Higgs boson cannot experience any high modes due to large or warped extra dimension:

$$M_{\rm Pl}^2 \sim R^n M_D^{2+n} \qquad\qquad ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

Lack of discovery at the LHC has put these ideas under pressure.

Violating premise 2: aleatory parameters (contingency)

Premise is necessary. Without it there can be no sense of probability at all.

Contingency in our universe is a necessary assumption for us to judge anything as **unlikely**, which is what the Hierarchy Problem is all about. Otherwise, we commit a fallacy of *illicit probabilistic inference*.

But how did the universe become teeming with contingency?

- Wotan throwing dice
- Baby universes born in eternal inflation (multiverse on the landscape)
- ?

Merely assuming non-contingent universe would mean we are done.

## Violating premise 3: multitude of states

There are several ways to violate this premise and solve the *Extrinsic Hierarchy Problem.* Violating a "Copernican principle" of nature though – we are not special.

- There are no new states beyond the Standard Model, period.

- There are no states derivable from the Ur-Theory that couple to the Standard Model Higgs boson.

- The only states that couple to the Standard Model Higgs are those that participate in a conspiracy of symmetries and/or interactions to keep the Higgs boson parametrically light.

## Violating the reasoning: Naturalness

It is fashionable to say "Naturalness is dead" and "Woe is Naturalness!" and "It's not scientific!" etc., but perhaps too flippant and imprecise.

My view: *Naturalness should not be controversial.*

What is controversial is the aleatory premise. However, if the aleatory premise is correct then Naturalness is merely an attempt to turn uneasy feelings of finetuning into statistical probability propositions.

Attacking Naturalness reasoning is not a fruitful direction in my view.

## Non-solutions

By articulating the *Extrinsic Hierarchy Problem* with some care we can see that some solutions proposed for the "Hierarchy Problem" are not solutions to EHP.

One example is Little Higgs Theories.

Little Higgs enthusiasts take seriously Wilsonian EFT, aleatory premise, and Ur-Theory premise even. But they have a restricted ad hoc approach to the multitude of states premise.

They believe that "business as usual" type of states and symmetries will do. There is nothing particularly special, except for being restricted by a single goal, which only solves partially the *Intrinsic Hierarchy Problem.*

*"To understand the requirements on this new physics better we must look at the source of the Higgs mass instability. The three most dangerous radiative corrections to the Higgs mass in the Standard Model come from one-loop diagrams with top quarks, SU(2) gauge bosons, and the Higgs itself running in the loop."* (Schmaltz, Tucker-Smith, '05)

In other words, this is their key worry (*Intrinsic Hierarchy Problem*):

$$m_H^2 = m_{\mathrm{bare}}^2 + \frac{y_t^2}{16\pi^2}\Lambda^2 + \delta\mathcal{O}(m_{\mathrm{weak}}^2)$$

New states and new symmetries have seemingly only one raison d'être: Make sure humans only see a Higgs boson and nothing else that conspires to keep it light.

In the "Simplest Little Higgs" model there is little contemplation of a multitude of other states $\Phi$ that can destabilize the theory by, e.g., $|\phi_i|^2|\Phi|^2$.

The theory is good, however, as a partial solution to the *Intrinsic Hierarchy Problem*, which takes Wilsonian cutoff issues very seriously.

Twin Higgs theories are similarly non-solutions to the *Extrinsic Hierarchy Problem.*

The model has an $SU(3)_{\rm color} \times SU(3)_{\rm weak} \times U(1)_X$ gauge group with three generations transforming as

$$\begin{aligned} \Psi_Q = (3,3)_{\frac{1}{3}} \qquad & \Psi_L = (1,3)_{-\frac{1}{3}} \\ d^c = (\bar{3},1)_{\frac{1}{3}} \qquad & e^c = (1,1)_1 \\ 2\times u^c = (\bar{3},1)_{-\frac{2}{3}} \qquad & n^c = (1,1)_0 \end{aligned} \tag{55}$$

The triplets $\Psi_Q$ and $\Psi_L$ contain the Standard Model quark and lepton doublets, the singlets are $u^c, d^c, e^c, n^c$.[3] The $SU(3)_{weak} \times U(1)_X$ symmetry is broken by expectation values for scalar fields $\phi_1 = \phi_2 = (1,3)_{-1/3}$.

The Lagrangian of the model contains the usual kinetic terms, Yukawa couplings and a tree level Higgs potential

$$\mathcal{L}_{kin} \sim \Psi_Q^\dagger \not{D} \Psi_Q + \cdots + |D_\mu \phi_1|^2 + \cdots \tag{56}$$

$$\begin{aligned} \mathcal{L}_{yuk} \sim\ & \lambda_1^u \phi_1^\dagger \Psi_Q u_1^c + \lambda_2^u \phi_2^\dagger \Psi_Q u_2^c + \frac{\lambda^d}{f}\phi_1\phi_2\Psi_Q\, d^c \\ & + \lambda^n \phi_1^\dagger \Psi_L n^c + \frac{\lambda^e}{f}\phi_1\phi_2\Psi_L\, e^c \end{aligned} \tag{57}$$

$$\mathcal{L}_{pot} \sim \mu^2 \phi_1^\dagger \phi_2\ . \tag{58}$$

Schmaltz, Tucker-Smith, '05

## Conclusions 1/2

The Hierarchy Problem of the SM may best be thought of as two Hierarchy Problems.

*Intrinsic Hierarchy Problem*: Wilsonian quadratic divergence taken seriously. If applicable, it is an intrinsic problem to the SM that it cannot avoid.

*Extrinsic Hierarchy Problem*: Reasonably expected new states and interactions in nature disallow the Higgs boson being so light.

The first might be a fake problem (dim reg, experimental results, etc.). If real, then the Wilsonian paradigm must be attacked (UV/IR connection important).

The second persists! It is a conceptual paradox, because we do not know which premises are wrong. And whatever resolves the paradox will change/augment radically our currently conceived view of nature.

## Conclusions 2/2 (only if "in principle" solutions – may have empirical/finetuning pressures)

| Theory | Intrinsic Hierarchy Problem Solution? | Extrinsic Hierarchy Problem Solution? |
|---|---|---|
| SM is everything | No | Yes |
| Wilsonian cutoff problems are fake (e.g. dim reg) | Yes | No |
| Supersymmetry | Yes | Yes |
| Extra Dimensions | Yes | Yes |
| No fundamental scalars | Yes | Yes |
| Little Higgs/Twin Higgs | Yes (partial) | No |
| Relaxion | Yes (partial) | No (exotic $\phi$ issue) |
| Non-Wilsonian UV/IR | Yes (by assumption) | ? |
| Anthropics | ? | ? |
| Non-contingent universe (Wotan's design) | Yes (by N/A) | Yes (by N/A) |