Holographic analysis of near-conformal dynamics and light dilaton

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1. Introduction and motivations

- Despite of tremendous efforts at LHC, probing $\sim 10\ \mathrm{TeV}$, we have not seen yet any hint of new particles.
- Is this because the scale of BSM is much higher than the EW scale, $\Lambda_{\rm ew} = (\sqrt{2}G_F)^{-1/2} \sim 246\ \mathrm{GeV}$?

$$\frac{\Lambda_{\rm ew}}{\Lambda_{\rm bsm}} \ll 1\,.$$

- If so then, why Higgs, uv sensitive, is unnaturally so light?

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- Composite Higgs model explains light Higgs by embedding it to be a part of pseudo NG bosons of UV theory, assuming the symmetry breaking scale is much higher than EW scale, $\Lambda_{\rm SB} \gg \Lambda_{\rm ew}$ to be consistent with current experimental constraints.
- Higgs mass is generated by explicit symmetry breaking interactions like pions in QCD.

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- If the new physics of CHM is near conformal, one may have a large scale separation and naturally light Higgs as well.

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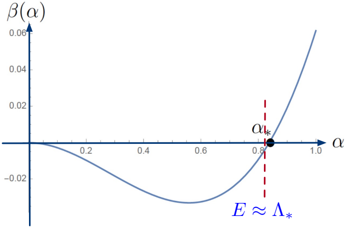
- For certain sets of N_c and N_f, some gauge theories might flow into an IR-fixed point (Banks-Zaks theory, 1982):



$$\alpha_* = \alpha(\Lambda_*)$$

- Near the IR fixed point ($E \approx \Lambda_*$), the theory is approximately conformal. If the scale symmetry is broken spontaneously at $\Lambda_{\rm SB}$ near IR fixed point, the theory will rest very close to the conformal edge, since $\beta(\alpha) \approx 0$, until fermions are decoupled.

Near Conformal Window

- Conformal windows from the beta-functions:

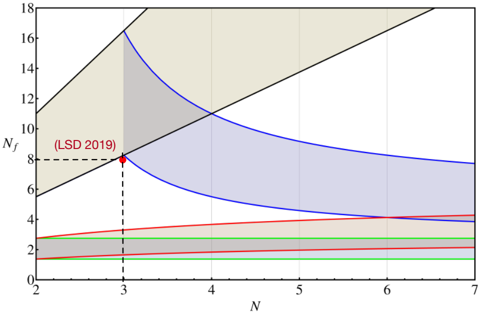
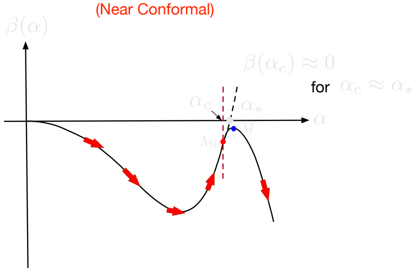


Figure: $SU(N)$ gauge theories with various rep's. (Ryttov+Sannino 2007)

Near conformal dynamics

- The theory can be slightly deformed to have $\alpha_c \approx \alpha_*$ in the large n_f and N_c limits or by introducing additional gauge interactions to any BZ theories (DKH 2018).
- The near conformal dynamics may be realized in a deformed BZ theory, having the dynamical generation of fermion mass.



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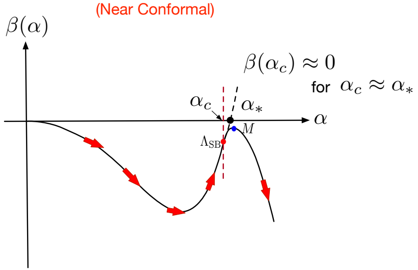

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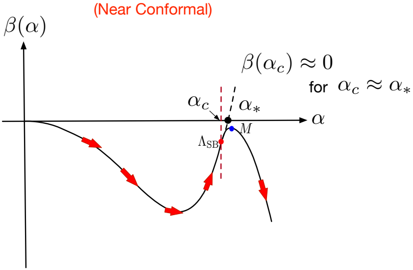
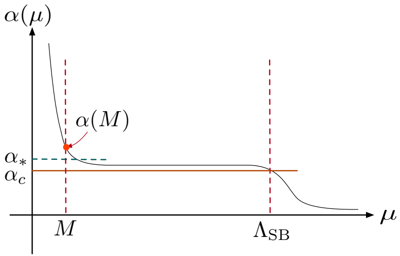


- The beta function near the IR fixed point?

- Miransky argued that, if chiral symmetry is broken near IR fixed point or $\alpha_c \approx \alpha_*$, one has a wide separation of scales, because $\beta(\alpha) \approx 0$: Miransky-KT scaling,

$$M = \Lambda_{\rm SB} \exp\left(-\frac{\rm const.}{\sqrt{\alpha_*/\alpha_c - 1}}\right) \ll \Lambda_{\rm SB} \quad ({\rm Miransky\ '85})$$

- The theory is almost scale-invariant for $M < \mu < \Lambda_{\rm SB}$, exhibiting walking dynamics, since $\beta(\alpha) \approx 0$.



Miransky-BKT scaling

- When $\alpha_c \approx \alpha_*$, we have $\beta(\alpha) \approx \beta^{\rm np}(\alpha)$. (Miransky '85)
- In the walking region we have approximate scale invariance and ladder approximation is good. The BS equation for the scalar bound-state then becomes

$$\left[P^2 + \partial^2 + \frac{\alpha/\alpha_c}{r^2}\right]\chi_P(x) = 0.$$

- Since the potential is singular, we need to regularize it:

$$V(r) = \begin{cases} -\frac{\alpha/\alpha_c}{r^2} & \text{if } r \geq a, \\ -\frac{\alpha/\alpha_c}{a^2} & \text{if } r \leq a. \end{cases}$$

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Miransky-BKT scaling

- For bound states to be the cutoff-independent, we require the coupling to depend on the cutoff. (DKH+Rajeev '90)

$$\alpha(a) = \alpha_c + \alpha_c \frac{(\pi - \bar{\theta})^2}{[\ln(a\mu)]^2} .$$

- The non-perturbative beta function is then

$$\beta^{\rm np}(\alpha) = a\frac{\partial}{\partial a}\alpha(a) = -\frac{1}{(\pi - \bar{\theta})}(\alpha - \alpha_c)^{3/2}$$

- The gap equation has a nontrivial solution with this beta function for $\alpha \geq \alpha_c$. (Bardeen et al '86):

$$M \simeq \Lambda(\alpha)\exp\left[\int_{\alpha_0}^{\alpha} \frac{d\alpha}{\beta^{\rm np}(\alpha)}\right] = \Lambda_{\rm SB}\, e^{-\frac{\pi - \bar{\theta}}{\sqrt{\alpha_* - \alpha_c}}} .$$

Miransky-KT scaling

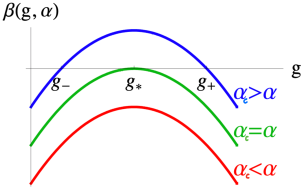
- How to understand the non-pertubative β-function?
- Non-perturbative renormalization requires a new scale.
- In the walking region $\gamma_{\bar{\psi}\psi} \simeq 1$ new marginal operator emerges and therefore generates the new scale, $M \ll \Lambda_{UV}$ (DKH+Rajeev '90):

$$\frac{\lambda}{\Lambda_{UV}^2} \left(\bar{\psi}\psi\right)^2 .$$

- We need to solve the coupled RGE equations for the gauge coupling and for the four-Fermi coupling near the IR fixed point. (Work needed to be done!)

Complex CFT

- Suppose the beta-function of the coupling of the marginal four-Fermi operator is given as



$$\beta(g,\alpha) \overset{?}{=} -(\lambda-\lambda_*)^2-\alpha+\lambda_*^2$$

Marginal deformation of CFT by four-Fermi operator

$$(\lambda = g,\ \alpha_c = \lambda_*^2)$$

- Conformality is lost when the UV fixed point collides with the IR fixed point. (Kaplan-Lee-Son-Stephanov, ’09)

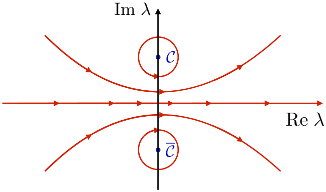
Complex CFT

- The walking dynamics is complex CFT. (V. Gorbenko, S. Rychkov, B. Zan 2018)

$$M = \Lambda \exp\left[-\oint_C \frac{d\lambda}{\beta(\lambda)}\right] = \Lambda \exp^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}$$

Near conformal window a new marginal operator rises

whose coupling λ



2. Light dilaton at conformal edge

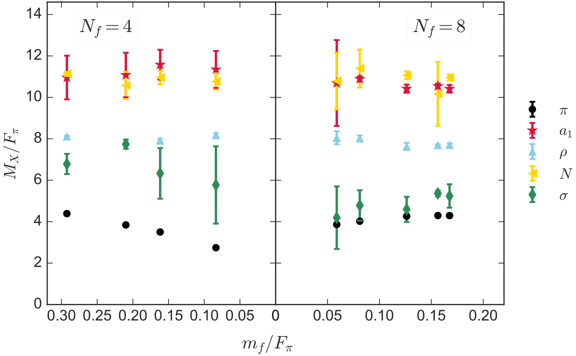
- Dilaton is a spin-0 particle that transforms nonlinearly under the scale transformations or dilatations:

$$D/f_D \mapsto D/f_D + c\,.$$

- Near the IR fixed point ($E \approx \Lambda_*$), the theory is approximately conformal. If the scale symmetry is broken spontaneously at $\Lambda_{\rm SB}$ near IR fixed point, the theory will rest very close to the conformal edge, since $\beta(\alpha) \approx 0$, until fermions are decoupled.

Spectrum Near Conformal Window

- $SU(3)$ with $N_f = 4, 8$ (LSD collaboration 2019)



Very light dilaton

- When χSB occurs at $\alpha = \alpha_c$ or at $\Lambda_{\rm SB}$, generating massless pions, the scale symmetry is also spontaneously broken.

$$0 \neq 3\left\langle \bar{\psi}\psi \right\rangle = \left\langle [D, \bar{\psi}\psi] \right\rangle$$

- In the chirally broken phase therefore we should also have light dilaton, associated the spontaneously broken scale symmetry,

$$\left\langle 0 \right| D_\mu(x) \left| \sigma(p) \right\rangle = -ifp_\mu e^{-ip\cdot x} \,,$$

where the dilatation current $D_\mu = x^\nu \theta_{\mu\nu}$, if the scale anomaly is small $|\langle \theta_\mu^\mu \rangle| \sim M^4 \ll \Lambda_{\rm SB}^4$, which is the salient feature of near conformal window, unlike QCD.

Very light dilaton

- Consider WT identity:

$$0 = \int_x \partial^\mu \langle 0| \mathrm{T} D_\mu(x)\theta^\nu_\nu(y) |0\rangle = \langle 0|[D, \theta^\nu_\nu]|0\rangle + \int_x \langle 0| \mathrm{T}\partial^\mu D_\mu(x)\theta^\nu_\nu |0\rangle$$

- We define the dilaton decay constant f as

$$i \langle 0|D^\mu(x)|\sigma(p)\rangle \equiv f p^\mu e^{-ip\cdot x} .$$

- Partially conserved dilatation current (PCDC) hypothesis:

$$f^2 m_D^2 = -4 \langle \theta^\mu_\mu \rangle \approx -16\, \mathcal{E}_{\rm vac} \sim M^4 \sim m^4_{\rm dyn} .$$

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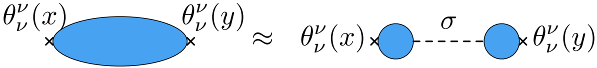

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PCDC and Very light dilaton

- Very light dilaton from quasi-conformal UV sector ($f \sim \Lambda_{SB}$):

$$m_D^2 = -\frac{4\langle\theta_\nu^\nu\rangle}{f^2} \sim \frac{M^4}{f^2} \ll M^2 \, .$$

with the dynamical mass or the IR scale

$$M = \Lambda_{\rm SB}(\alpha_c)\,\exp\left(-\frac{\pi - \bar{\theta}}{\sqrt{\alpha_* - \alpha_c}}\right) \, .$$

- For $M \sim 10\ \rm TeV$ and $f \sim 10^{10}\ \rm TeV$, dilaton can be a warm dark mater (DKH '18)

$$m_D \sim \rm keV \, .$$

dilaton DM

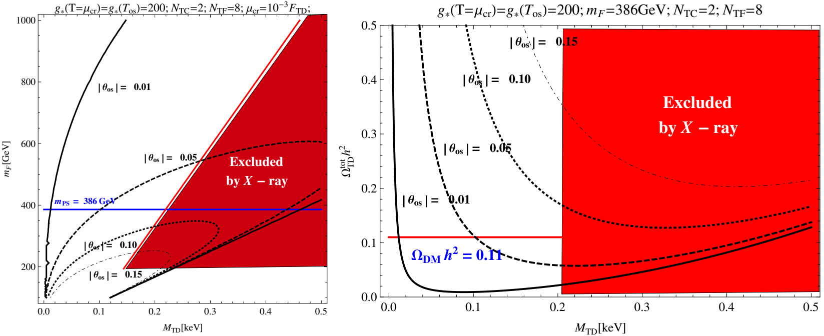

Figure: Choi+DKH+Matsuzaki 2013

3. Holographic light dilaton

J. Rojas, DKH, S.H. Im, M.Järvinen (2302.08112, 2504.18623)

Holographic dilaton

- By the AdS/CFT conjecture (Maldacena 1997), the strongly coupled gauge theories in 4D are dual to weakly interacting 5D gravity.
- It provides a tool to study strongly coupled gauge theories, which are in general hard to analyze.
- We analyze the particle spectrum at low energy, especially dilaton, of near-conformal gauge theory, using this gauge/gravity duality.

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Holographic dilaton

- Consider a gravity dual of near conformal gauge theory:

$$S = S_{\rm g}[g_{\mu\nu},\phi] + S_{\rm m}[g_{\mu\nu},\phi,X]\ ,$$

where ϕ and X are dual to $\mathrm{Tr}\left(G_{\mu\nu}^2\right)$ and $q\bar{q}$, respectively.

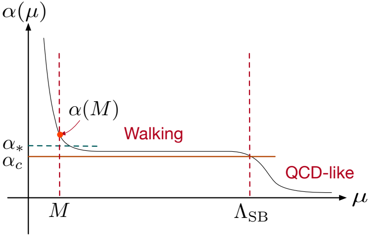
$$S_{\rm g} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R - g^{MN}\partial_M\phi\partial_N\phi + V(\phi)\right] + S_{\rm GH}$$

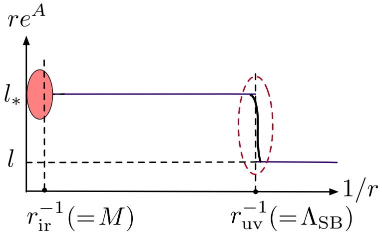
$$ds^2 = e^{2A(r)}\left(dr^2 - dt^2 + d\mathbf{x}^2\right)\,,\quad \left(r_{\rm UV} < r < r_{\rm IR}\right),$$

$$S_{\rm m} = -\frac{1}{2\kappa^2 N_c}\int d^5x\sqrt{-g}\ \mathrm{Tr}\left[g^{MN}\partial_M X^\dagger\partial_N X - m_X(r)^2 X^\dagger X\right]$$

Holographic dilaton

- Geometry of near-conformal gauge theory:





Parametrically light dilaton near the conformal edge

- As the IR scale M or $m_{\rm dyn}$ goes to zero, the scale anomaly $\langle \theta^\mu_\mu \rangle = -cM^4$ vanishes. The dilaton should be massless in that limit, $\nu = \sqrt{\alpha_*/\alpha_c - 1} \to 0$.:

$$m_D \sim \nu^n \to 0, \quad \text{as} \;\; \nu \to 0\,.$$

- The expectation is that dilaton should be parametrically light as we approach the conformal edge.

Holographic dilaton

- Near the conformal edge, the scaling dimension of $q\bar{q}$ is $\Delta_{\mathrm{IR}} = 2 \pm i\nu$ $(\nu \ll 1)$, slightly violating the BF bound,

$$m_X^2 = -4 - \nu^2 < -4 \,.$$

- The background solution is then

$$X(r) = m_q r + \sigma r^3 \,, \qquad (r < r_{\mathrm{uv}})$$

$$X(r) = X_0 \left(\frac{r}{r_{\mathrm{uv}}}\right)^2 \sin\left(\nu \log \frac{r}{r_{\mathrm{uv}}} + \alpha\right), \quad (r_{\mathrm{uv}} < r < r_{\mathrm{ir}})$$

- For the stability, the background solution should not have a node for $r_{\mathrm{ir}} < r < r_{\mathrm{uv}}$:

$$\frac{r_{\mathrm{ir}}}{r_{\mathrm{uv}}} = e^{(\pi-\alpha)/\nu} \quad (\text{Miransky scaling})\,.$$

Holographic dilaton

- For the UV boundary conditions at $r = r_{\rm uv}$, we match two solutions smoothly to get

$$\begin{aligned} \tan\alpha &= \nu \frac{\sigma r_{\rm uv}^3 + m_q r_{\rm uv}}{\sigma r_{\rm uv}^3 - m_q r_{\rm uv}}, \\ X_0 &= \frac{\sigma r_{\rm uv}^3 + m_q r_{\rm uv}}{\sin\alpha}, \end{aligned}$$

where we see that the phase $\alpha \sim \mathcal{O}(\nu)$ for $m_q \approx 0$..

- As an approximation, we cut-off the IR geometry and impose boundary conditions

$$\mathcal{A}X(r_{\rm ir}) + \mathcal{B}r_{\rm ir}X'(r_{\rm ir}) = 0 \ ,$$

Holographic dilaton

- We now consider small fluctuations to analyze the spectrum of physical states. Among them relevant ones for O^+ states are

$$\psi = \frac{1}{6}\left(h^\mu_\mu - \frac{\partial^\mu\partial^\nu}{\partial^2}h_{\mu\nu}\right)$$

and from the bulk scalars

$$\phi(z,x) = \bar{\phi}(z) + \varphi(z,x)\ , \qquad X_{ij}(z,x) = \delta_{ij}\bar{X}(z) + \delta_{ij}\chi(z,x)\ .$$

Holographic dilaton

- The relevant gauge invariant combination is (Kiritsis+Nitti '07)

$$\xi = \psi - \frac{A'}{\bar{X}'}\chi$$

- Solving the equations of motion, we get in the probe approximation

$$\xi(r) = \frac{C_1 \Re[J_{i\nu}(\omega r)] + C_2 \Re[Y_{i\nu}(\omega r)]}{2\sin(\nu \ln \frac{r}{r_{\rm uv}} + \alpha) + \nu \cos(\nu \ln \frac{r}{r_{\rm uv}} + \alpha)} \ .$$

Holographic dilaton

- From the boundary conditions that parametrize the theory near the conformal edge one finds for $\mathcal{A}/\mathcal{B} \ll 1$ or $\mathcal{A} = 0$

$$\omega^2 r_{\mathrm{ir}}^2 = \frac{2\nu \sin\alpha}{\sin(\beta-\alpha)\sin\beta} + \frac{\nu^2}{\sin^2\beta} + \mathcal{O}(\nu^3)$$

where β is defined as $\frac{r_{\mathrm{ir}}}{r_{\mathrm{uv}}} \equiv e^{(\pi-\beta)/\nu}$.

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Holographic spectrum

- We find that a light dilaton exists if $\mathcal{A} = 0$, namely for Neunmann IR boundary condition:

$$m_D \sim r_{\rm ir}^{-1}\sqrt{\nu}\,.$$

- The dilaton can be parametrically lighter than all other hadrons which are $\mathcal{O}\left(r_{\rm ir}^{-1}\right)$. (2302.08112 and 2504.18623)
- For massive gravitons, $h_{\mu\nu}^{TT}$, for example, we have

$$m_G^{(n)} \approx \frac{\pi}{r_{\rm ir}}\left(n+\frac{1}{4}\right) \gtrsim m_{\rm dyn}\,,$$

PCDC and dilaton decay constant

- We calculate the dilaton decay constant from PCDC:

$$\lim_{q^\mu \to 0} \int d^4x e^{iq\cdot x} \langle 0 | T\theta^\mu_\mu(x)\theta^\nu_\nu(0)|0\rangle = -if_D^2 m_D^2 \,.$$

- For the gauge-invariant fluctuation ξ we have

$$\mathcal{S} = -\frac{\mathcal{N}}{2\kappa^2}\int d^4x dr \sqrt{-\det \bar{g}} \left(\frac{\bar{X}'}{A'}\right)^2 \bar{g}^{MN}\partial_M\xi\partial_N\xi,$$

- The generating functional is then by AdS/CFT

$$W[\xi] = \frac{\mathcal{N}}{2\kappa^2}\int d^4x\, e^{3A(r)}\left(\frac{\bar{X}'}{A'}\right)^2 \xi\xi'\Big|_{r=r_{\rm UV}} \,.$$

PCDC and dilaton decay constant

- The two-point function at $q \to 0$ then becomes

$$\lim_{q^\mu \to 0} \int d^4x e^{iq\cdot x} \langle 0| T\theta^\mu_\mu(x)\theta^\nu_\nu(0)|0\rangle \simeq -i\frac{\mathcal{N}\ell^3}{2\kappa^2}\frac{X_0^2}{r_{\rm uv}^2}\frac{2\nu}{\sin 2\beta}\,.$$

- The PCDC relation therefore tells us that

$$f_D^2 m_D^2 \sim r_{\rm ir}^{-4}\left(\frac{\nu}{\sin\beta\cos\beta\sin^2(\beta-\alpha)}\right).$$

- We find $\beta - \alpha \sim \sqrt{\nu}$ while $\beta \sim \mathcal{O}(1)$ or $\beta \sim \nu^{1/3} > \alpha$ to have the scale anomaly, $\langle\theta^\mu_\mu\rangle = -c_1 M^4$.

Conclusion

- Near conformal CHM might explain the Higgs mass hierarchy:

$$m_H \sim v_{\rm ew} \ll M = \Lambda_{\rm SB}(\alpha_c) \exp\left(-\frac{\pi - \bar{\theta}}{\sqrt{\alpha_* - \alpha_c}}\right).$$

- The holographic analysis shows (2302.08112 and 2504.18623), since $\langle \theta^\mu_\mu \rangle \sim M^4$ (Gusynin+Miransky '89)

$$m_D = c_1 \frac{M^2}{f} \sim M \cdot \sqrt{\nu} \quad \text{or} \quad f \sim M \cdot \nu^{-1/2}.$$

- Light dilaton can be a warm DM, $m_D \sim \text{keV}$ for $M = 10$ TeV and $\Lambda_{\rm UV} \simeq \Lambda_{\rm SB} \sim 10^{10}$ TeV.
- The Miransky scaling is mildly violated:

$$\frac{r_{\rm ir}}{r_{\rm uv}} \simeq \exp\left(\frac{\pi - \beta}{\nu}\right), \quad \beta \sim \nu^{1/3}.$$

Merci !